Hindawi
Journal of Function Spaces
Volume 2020, Article ID 5042724, 8 pages
https://doi.org/10.1155/2020/5042724

*Research Article*

# Analytical Solutions to the Caudrey-Dodd-Gibbon-Sawada-Kotera Equation via Symbol Calculation Approach

**Yongyi Gu** [1,2]

[1] *Big Data and Educational Statistics Application Laboratory, Guangdong University of Finance and Economics, Guangzhou 510320, China*
[2] *School of Statistics and Mathematics, Guangdong University of Finance and Economics, Guangzhou 510320, China*

Correspondence should be addressed to Yongyi Gu; gdguyongyi@163.com

Received 24 March 2020; Accepted 22 April 2020; Published 5 May 2020

Guest Editor: Chuanjun Chen



In this paper, we derive analytical solutions of the Caudrey-Dodd-Gibbon-Sawada-Kotera (CDGSK) equation via symbol calculation approach. Applying the $\exp(-\varphi(z))$-expansion method, we achieve the trigonometric, exponential, hyperbolic, and rational function solutions for the CDGSK equation. By choosing the appropriate parameters, we give some computer simulation to the analytical solutions of the CDGSK equation.

## 1. Introduction

Nonlinear fractional and integer order differential equations are widely utilized in fluid dynamics, solid state physics, plasma physics, biology, nonlinear optics, chemistry, etc. It has aroused the strong interest of researchers to these differential equations [1–18]. The study to exact solutions of various NLDEs is extremely important in modern mathematics with ramifications to some areas of physics, mathematics, and other sciences. There are many systematic methods to seek exact solutions of nonlinear differential equations, for example, the Hirota bilinear method [19], Tanh method [20], Lie symmetry [21], modified Kudryashov method [22], Exp-function method [23], sine-Gordon expansion method [24], complex method [25–29], and $\exp(-\varphi(z))$-expansion method [30–32].

Sawada and Kotera [33] proposed one of basic models in soliton theory as follows:

$$u_{xxxxx} + 15uu_{xxx} + 15u_x u_{xx} + 45u^2 u_x + u_t = 0, \tag{1}$$

which is also introduced by Caudrey et al. independently [34]; so in literature, Equation (1) is called the Caudrey-Dodd-Gibbon-Sawada-Kotera equation. Many years have passed by, lots of research results for the CDGSK equation have been developed. As to this equation, the finite dimensional reduction was investigated by Enolski et al. [35], and N-soliton solutions were discovered by Parker [36] via the dressing method. Darboux transformation [37] and Bäcklund transformation in bilinear forms [38] were applied to study the CDGSK equation. Riemann theta function solutions of the CDGSK equation were also established [39]. In this paper, we employ the $\exp(-\varphi(z))$-expansion method [30–32] to obtain the exact solutions of the CDGSK equation.

## 2. The $\exp(-\varphi(z))$-Expansion Method

In this section, we give the main steps of the mentioned method.

*Step 1.* Inserting traveling wave transform

$$u(x,t) = u(z), \quad z = \lambda x + \omega t \tag{2}$$

into a nonlinear PDE

$$P\left(u, u_x, u_y, u_t, u_{xx}, u_{yy}, u_{tt}, \cdots\right) = 0 \tag{3}$$

yields

$$F\left(u, u', u'', \cdots\right) = 0, \quad (4)$$

in which $F$ is the polynomial of $u$ along with its derivatives.

*Step 2.* Assume that Equation (4) has analytical solutions as follows:

$$u(z) = \sum_{\tau=0}^{m} B_\tau (\exp(-\varphi(z)))^\tau, \quad (5)$$

where $B_\tau (0 \le \tau \le m)$ are constants which will be ascertained subsequently, such that $B_\tau \ne 0$ and $\varphi = \varphi(z)$ satisfy

$$\varphi'(z) = \gamma + \exp(-\varphi(z)) + \vartheta \exp(\varphi(z)). \quad (6)$$

The solutions of Equation (6) are given in the following: When $\gamma^2 - 4\vartheta > 0, \vartheta \ne 0$,

$$\varphi(z) = \ln\left(\frac{-\sqrt{(\gamma^2 - 4\vartheta)}\tanh\left(\left(\sqrt{\gamma^2 - 4\vartheta}/2\right)(z+c)\right) - \gamma}{2\vartheta}\right), \quad (7)$$

$$\varphi(z) = \ln\left(\frac{-\sqrt{(\gamma^2 - 4\vartheta)}\coth\left(\left(\sqrt{\gamma^2 - 4\vartheta}/2\right)(z+c)\right) - \gamma}{2\vartheta}\right). \quad (8)$$

When $\gamma^2 - 4\vartheta < 0, \vartheta \ne 0$,

$$\varphi(z) = \ln\left(\frac{\sqrt{(4\vartheta - \gamma^2)}\tan\left(\left(\sqrt{4\vartheta - \gamma^2}/2\right)(z+c)\right) - \gamma}{2\vartheta}\right),$$

$$\varphi(z) = \ln\left(\frac{\sqrt{(4\vartheta - \gamma^2)}\cot\left(\left(\sqrt{4\vartheta - \gamma^2}/2\right)(z+c)\right) - \gamma}{2\vartheta}\right). \quad (9)$$

When $\gamma^2 - 4\vartheta > 0, \gamma \ne 0, \vartheta = 0$,

$$\varphi(z) = -\ln\left(\frac{\gamma}{\exp(\gamma(z+c)) - 1}\right). \quad (10)$$

When $\gamma^2 - 4\vartheta = 0, \gamma \ne 0, \vartheta \ne 0$,

$$\varphi(z) = \ln\left(-\frac{2(\gamma(z+c)+2)}{\gamma^2(z+c)}\right). \quad (11)$$

When $\gamma^2 - 4\vartheta = 0, \gamma = 0, \vartheta = 0$,

$$\varphi(z) = \ln(z+c). \quad (12)$$

In Equations (7)–(12), $B_\tau \ne 0, \gamma, \vartheta, c$ are constants. Taking the homogeneous balance between the nonlinear terms and highest order derivatives of Equation (4) yields the positive integer $m$.

*Step 3.* Insert Equation (5) into Equation (4), and collect the function $\exp(-\varphi(z))$ to yield the polynomial to $\exp(-\varphi(z))$. Let all coefficients with the same power of $\exp(-\varphi(z))$ be zero to obtain a system of algebraic equations. In solving these equations, we achieve the values of $B_\tau \ne 0, \gamma, \vartheta$ and substitute them into Equation (5) as well as Equations (7)–(12) to accomplish the determination for analytical solutions of the original PDE.

## 3. Exact Solutions of the CDGSK Equation

Inserting

$$u(x,t) = u(z), \quad z = \lambda x + \omega t \quad (13)$$

into Equation (1) and then integrating it, we obtain

$$\lambda^5 u'''' + 15\lambda^3 u u'' + 15\lambda u^3 + \omega u + \delta = 0, \quad (14)$$

where $\delta$ is the integration constant.

Taking the homogeneous balance between $u^3$ and $u''''$ of Equation (14) yields

$$u(z) = B_0 + B_1 \exp(-\varphi(z)) + B_2(\exp(-\varphi(z)))^2, \quad (15)$$

where $B_2 \ne 0$ and $B_1$ and $B_0$ are constants.

Substituting $u'''', uu'', u^3, u$ into Equation (14) and letting the coefficients about $\exp(-\varphi(z))$ be zero, we obtain

$$\begin{aligned} e^{0(-\varphi(z))} &: \lambda^5 B_1 \gamma^3 \vartheta + 14\lambda^5 B_2 \gamma^2 \vartheta^2 + 8\lambda^5 B_1 \gamma \vartheta^2 \\ &+ 16\lambda^5 B_2 \vartheta^3 + 15\lambda^3 B_0 B_1 \vartheta \gamma + 30\lambda^3 B_0 B_2 \vartheta^2 \\ &+ 15\lambda B_0^3 + \omega B_0 + \delta = 0, \end{aligned}$$

$$\begin{aligned} e^{1(-\varphi(z))} &: B_1 \omega + B_1 \gamma^4 \lambda^5 + 30 B_2 \gamma^3 \lambda^5 \vartheta + 22 B_1 \gamma^2 \lambda^5 \vartheta \\ &+ 120 B_2 \gamma \lambda^5 \vartheta^2 + 30\lambda^3 B_0 B_1 \vartheta + 15 B_0 B_1 \gamma^2 \lambda^3 \\ &+ 90 B_0 B_2 \gamma \lambda^3 \vartheta + 15 B_1{}^2 \gamma \lambda^3 \vartheta + 30 B_1 B_2 \lambda^3 \vartheta^2 \\ &+ 16 B_1 \lambda^5 \vartheta^2 + 45 B_0{}^2 B_1 \lambda = 0, \end{aligned}$$

$$\begin{aligned} e^{2(-\varphi(z))} &: 16 B_2 \gamma^4 \lambda^5 + 15\lambda^5 B_1 \gamma^3 + 232 B_2 \gamma^2 \lambda^5 \vartheta \\ &+ 60 B_1 \gamma \lambda^5 \vartheta + 136 B_2 \lambda^5 \vartheta^2 + 15 B_1{}^2 \gamma^2 \lambda^3 \\ &+ 60 B_0 B_2 \gamma^2 \lambda^3 + 105 B_1 B_2 \gamma \lambda^3 \vartheta + 30 B_2{}^2 \lambda^3 \vartheta^2 \\ &+ 45 B_0 B_1 \gamma \lambda^3 + 120 B_0 B_2 \lambda^3 \vartheta + 30 B_1{}^2 \lambda^3 \vartheta \\ &+ 45 B_0{}^2 B_2 \lambda + 45 B_0 B_1{}^2 \lambda + B_2 \omega = 0, \end{aligned}$$

$$\begin{aligned} e^{3(-\varphi(z))} &: 130 B_2 \gamma^3 \lambda^5 + 50 B_1 \gamma^2 \lambda^5 + 440 B_2 \gamma \lambda^5 \vartheta \\ &+ 75 B_1 B_2 \gamma^2 \lambda^3 + 40 B_1 \lambda^5 \vartheta + 15 B_1^3 \lambda \\ &+ 150 B_0 B_2 \gamma \lambda^3 + 45 B_1^2 \gamma \lambda^3 + 150 B_1 B_2 \lambda^3 \vartheta \\ &+ 30 B_0 B_1 \lambda^3 + 90 B_0 B_1 B_2 \lambda + 90 B_2^2 \gamma \lambda^3 \vartheta = 0, \end{aligned}$$

$$\begin{aligned}
e^{4(-\varphi(z))}&: 330\,B_2\,\gamma^2\lambda^5 + 60\,B_1\,\gamma\,\lambda^5 + 60\,B_2^2\gamma^2\lambda^3\\
&+ 240\,B_2\,\lambda^5\vartheta + 195\,B_1\,B_2\,\gamma\,\lambda^3 + 30\,B_1^2\lambda^3\\
&+ 120\,B_2^2\lambda^3\vartheta + 90\,B_0\,B_2\,\lambda^3 + 45\,B_0\,B_2^2\lambda\\
&+ 45\,B_1^2B_2\,\lambda = 0,\\
e^{5(-\varphi(z))}&: 336\,B_2\,\gamma\,\lambda^5 + 24\,B_1\,\lambda^5 + 150\,B_2^2\gamma\,\lambda^3\\
&+ 120\,B_1\,B_2\,\lambda^3 + 45\,B_1\,B_2^2\lambda = 0,\\
e^{6(-\varphi(z))}&: 120\,B_2\,\lambda^5 + 90\,B_2^2\lambda^3 + 15\,B_2^3\lambda = 0.
\end{aligned} \tag{16}$$

We solve the above algebraic equations and then obtain two different cases:

*Case 1.*

$$\begin{aligned}
B_2 &= -2\lambda^2,\\
B_1 &= -2\gamma\lambda^2,\\
B_0 &= \frac{-5\,\lambda^3\left(\gamma^2+8\,\vartheta\right)+\sqrt{5\,\lambda^6(\gamma^2-4\,\vartheta)^2-20\,\lambda\,\omega}}{30\lambda},\\
\delta &= \frac{\lambda^7\left(\gamma^2-4\,\vartheta\right)^3}{9}\\
&\quad - \frac{\sqrt{5\lambda\left(\lambda^5(\gamma^2-4\,\vartheta)^2-4\,\omega\right)}\left(2\,\lambda^5\left(\gamma^2-4\,\vartheta\right)^2+\omega\right)}{45\lambda},
\end{aligned} \tag{17}$$

where $\vartheta$ and $\gamma$ are arbitrary constants.

Substituting Equation (17) into Equation (15) yields

$$\begin{aligned}
u(z) &= \frac{-5\,\lambda^3\left(\gamma^2+8\,\vartheta\right)+\sqrt{5\,\lambda^6(\gamma^2-4\,\vartheta)^2-20\,\lambda\,\omega}}{30\lambda}\\
&\quad - 2\gamma\lambda^2\exp\left(-\varphi(z)\right) - 2\lambda^2(\exp\left(-\varphi(z)\right))^2.
\end{aligned} \tag{18}$$

Applying Equations (7)–(12) into Equation (18), respectively, yields analytical solutions of the CDGSK equation as follows:

*Case 1.1.* When $\gamma^2-4\vartheta>0$, $\vartheta\neq 0$,

$$\begin{aligned}
u_{11}(z) &= \frac{-5\,\lambda^3\left(\gamma^2+8\,\vartheta\right)+\sqrt{5\,\lambda^6(\gamma^2-4\,\vartheta)^2-20\,\lambda\,\omega}}{30\lambda}\\
&\quad + \frac{4\lambda^2\gamma\vartheta}{\sqrt{(\gamma^2-4\vartheta)}\tanh\left(\left(\sqrt{\gamma^2-4\vartheta}/2\right)(z+c)\right)+\gamma}\\
&\quad - \frac{8\lambda^2\vartheta^2}{\left(\sqrt{(\gamma^2-4\vartheta)}\tanh\left(\left(\sqrt{\gamma^2-4\vartheta}/2\right)(z+c)\right)+\gamma\right)^2},\\
u_{12}(z) &= \frac{-5\,\lambda^3\left(\gamma^2+8\,\vartheta\right)+\sqrt{5\,\lambda^6(\gamma^2-4\,\vartheta)^2-20\,\lambda\,\omega}}{30\lambda}\\
&\quad + \frac{4\lambda^2\gamma\vartheta}{\sqrt{(\gamma^2-4\vartheta)}\coth\left(\left(\sqrt{\gamma^2-4\vartheta}/2\right)(z+c)\right)+\gamma}\\
&\quad - \frac{8\lambda^2\vartheta^2}{\left(\sqrt{(\gamma^2-4\vartheta)}\coth\left(\left(\sqrt{\gamma^2-4\vartheta}/2\right)(z+c)\right)+\gamma\right)^2}.
\end{aligned} \tag{19}$$

*Case 1.2.* When $\gamma^2-4\vartheta<0$, $\vartheta\neq 0$,

$$\begin{aligned}
u_{13}(z) &= \frac{-5\,\lambda^3\left(\gamma^2+8\,\vartheta\right)+\sqrt{5\,\lambda^6(\gamma^2-4\,\vartheta)^2-20\,\lambda\,\omega}}{30\lambda}\\
&\quad - \frac{4\lambda^2\gamma\vartheta}{\sqrt{(4\vartheta-\gamma^2)}\tan\left(\left(\sqrt{4\vartheta-\gamma^2}/2\right)(z+c)\right)-\gamma}\\
&\quad - \frac{8\lambda^2\vartheta^2}{\left(\sqrt{(4\vartheta-\gamma^2)}\tan\left(\left(\sqrt{4\vartheta-\gamma^2}/2\right)(z+c)\right)-\gamma\right)^2},\\
u_{14}(z) &= \frac{-5\,\lambda^3\left(\gamma^2+8\,\vartheta\right)+\sqrt{5\,\lambda^6(\gamma^2-4\,\vartheta)^2-20\,\lambda\,\omega}}{30\lambda}\\
&\quad - \frac{4\lambda^2\gamma\vartheta}{\sqrt{(4\vartheta-\gamma^2)}\cot\left(\left(\sqrt{4\vartheta-\gamma^2}/2\right)(z+c)\right)-\gamma}\\
&\quad - \frac{8\lambda^2\vartheta^2}{\left(\sqrt{(4\vartheta-\gamma^2)}\cot\left(\left(\sqrt{4\vartheta-\gamma^2}/2\right)(z+c)\right)-\gamma\right)^2}.
\end{aligned} \tag{20}$$

*Case 1.3.* When $\gamma^2-4\vartheta>0$, $\gamma\neq 0$, $\vartheta=0$,

$$\begin{aligned}
u_{15}(z) &= \frac{-5\lambda^3\gamma^2+\sqrt{5\lambda\left(\lambda^5\gamma^4-4\omega\right)}}{30\lambda} - \frac{2\lambda^2\gamma^2}{\exp\left(\gamma(z+c)\right)-1}\\
&\quad - \frac{2\lambda^2\gamma^2}{\left(\exp\left(\gamma(z+c)\right)-1\right)^2}.
\end{aligned} \tag{21}$$

*Case 1.4.* When $\gamma^2-4\vartheta=0$, $\gamma\neq 0$, $\vartheta\neq 0$,

$$\begin{aligned}
u_{16}(z) &= \frac{-30\lambda^3\vartheta+\sqrt{-5\lambda\omega}}{15\lambda} + \frac{\lambda^2\gamma^3(z+c)}{\gamma(z+c)+2}\\
&\quad - \frac{\lambda^2\gamma^4(z+c)^2}{2(\gamma(z+c)+2)^2}.
\end{aligned} \tag{22}$$

*Case 1.5.* When $\gamma^2-4\vartheta=0$, $\gamma=0$, $\vartheta=0$,

$$u_{17}(z) = \frac{\sqrt{-5\lambda\omega}}{15\lambda} - \frac{2\lambda^2}{(z+c)^2}. \tag{23}$$

*Case 2.*

$$B_2 = -2\lambda^2,$$
$$B_1 = -2\gamma\lambda^2,$$
$$B_0 = -\frac{5\lambda^3(\gamma^2+8\vartheta)+\sqrt{5\lambda^6(\gamma^2-4\vartheta)^2-20\lambda\omega}}{30\lambda},$$
$$\delta = \frac{\lambda^7(\gamma^2-4\vartheta)^3}{9} + \frac{\sqrt{5\lambda\left(\lambda^5(\gamma^2-4\vartheta)^2-4\omega\right)}\left(2\lambda^5(\gamma^2-4\vartheta)^2+\omega\right)}{45\lambda}, \tag{24}$$

where $\vartheta$ and $\gamma$ are arbitrary constants.

Substituting Equation (24) into (15) yields

$$u(z) = -\frac{5\lambda^3(\gamma^2+8\vartheta)+\sqrt{5\lambda^6(\gamma^2-4\vartheta)^2-20\lambda\omega}}{30\lambda} - 2\lambda^2\gamma\exp(-\varphi(z)) - 2\lambda^2(\exp(-\varphi(z)))^2. \tag{25}$$

Applying Equations (7)–(12) into Equation (25), respectively, yields analytical solutions of the CDGSK equation as follows:

*Case 2.1.* When $\gamma^2-4\vartheta>0$, $\vartheta\neq 0$,

$$\begin{aligned} u_{21}(z) &= -\frac{5\lambda^3(\gamma^2+8\vartheta)+\sqrt{5\lambda^6(\gamma^2-4\vartheta)^2-20\lambda\omega}}{30\lambda}, \\ &+ \frac{4\lambda^2\gamma\vartheta}{\sqrt{(\gamma^2-4\vartheta)}\tanh\left(\left(\sqrt{\gamma^2-4\vartheta}/2\right)(z+c)\right)+\gamma} \\ &- \frac{8\lambda^2\vartheta^2}{\left(\sqrt{(\gamma^2-4\vartheta)}\tanh\left(\left(\sqrt{\gamma^2-4\vartheta}/2\right)(z+c)\right)+\gamma\right)^2}, \\ u_{22}(z) &= -\frac{5\lambda^3(\gamma^2+8\vartheta)+\sqrt{5\lambda^6(\gamma^2-4\vartheta)^2-20\lambda\omega}}{30\lambda}, \\ &+ \frac{4\lambda^2\gamma\vartheta}{\sqrt{(\gamma^2-4\vartheta)}\coth\left(\left(\sqrt{\gamma^2-4\vartheta}/2\right)(z+c)\right)+\gamma} \\ &- \frac{8\lambda^2\vartheta^2}{\left(\sqrt{(\gamma^2-4\vartheta)}\coth\left(\left(\sqrt{\gamma^2-4\vartheta}/2\right)(z+c)\right)+\gamma\right)^2}. \end{aligned} \tag{26}$$

*Case 2.2.* When $\gamma^2-4\vartheta<0$, $\vartheta\neq 0$,

$$\begin{aligned} u_{23}(z) &- \frac{5\lambda^3(\gamma^2+8\vartheta)+\sqrt{5\lambda^6(\gamma^2-4\vartheta)^2-20\lambda\omega}}{30\lambda}, \\ &- \frac{4\lambda^2\gamma\vartheta}{\sqrt{(4\vartheta-\gamma^2)}\tan\left(\left(\sqrt{4\vartheta-\gamma^2}/2\right)(z+c)\right)-\gamma} \\ &- \frac{8\lambda^2\vartheta^2}{\left(\sqrt{(4\vartheta-\gamma^2)}\tan\left(\left(\sqrt{4\vartheta-\gamma^2}/2\right)(z+c)\right)-\gamma\right)^2}, \\ u_{24}(z) &= -\frac{5\lambda^3(\gamma^2+8\vartheta)+\sqrt{5\lambda^6(\gamma^2-4\vartheta)^2-20\lambda\omega}}{30\lambda}, \\ &- \frac{4\lambda^2\gamma\vartheta}{\sqrt{(4\vartheta-\gamma^2)}\cot\left(\left(\sqrt{4\vartheta-\gamma^2}/2\right)(z+c)\right)-\gamma} \\ &- \frac{8\lambda^2\vartheta^2}{\left(\sqrt{(4\vartheta-\gamma^2)}\cot\left(\left(\sqrt{4\vartheta-\gamma^2}/2\right)(z+c)\right)-\gamma\right)^2}. \end{aligned} \tag{27}$$

*Case 2.3.* When $\gamma^2-4\vartheta>0$, $\gamma\neq 0$, $\vartheta=0$,

$$u_{25}(z) = -\frac{5\lambda^3\gamma^2+\sqrt{5\lambda\left(\lambda^5\gamma^4-4\omega\right)}}{30\lambda} - \frac{2\lambda^2\gamma^2}{\exp(\gamma(z+c))-1} - \frac{2\lambda^2\gamma^2}{(\exp(\gamma(z+c))-1)^2}. \tag{28}$$

*Case 2.4.* When $\gamma^2-4\vartheta=0$, $\gamma\neq 0$, $\vartheta\neq 0$,

$$u_{26}(z) = -\frac{30\lambda^3\vartheta+\sqrt{-5\lambda\omega}}{15\lambda} + \frac{\lambda^2\gamma^3(z+c)}{\gamma(z+c)+2} - \frac{\lambda^2\gamma^4(z+c)^2}{2(\gamma(z+c)+2)^2}. \tag{29}$$

*Case 2.5.* When $\gamma^2-4\vartheta=0$, $\gamma=0$, $\vartheta=0$,

$$u_{27}(z) = -\frac{\sqrt{-5\lambda\omega}}{15\lambda} - \frac{2\lambda^2}{(z+c)^2}. \tag{30}$$

Figures 1–5 show the properties of the solutions.

## 4. Conclusions

In this paper, abundant analytical solutions of the CDGSK equation are obtained via symbol calculation approach. The properties of the solutions are shown by some graphs. It shows that the $\exp(-\varphi(z))$-expansion method is an effective method to seek analytical solutions for nonlinear differential equations.

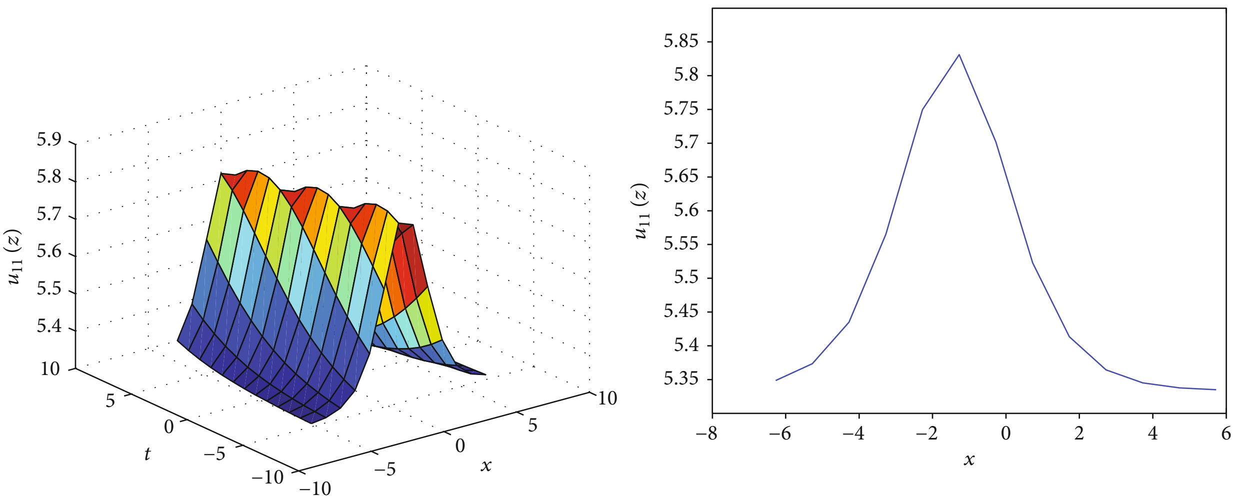


Figure 1: The 3D and 2D graphics of $u_{11}(z)$ by choosing the values $\gamma = 5$, $\vartheta = 6$, $\lambda = 1$, $\omega = 1/4$, $c = 1$, and $t = 0$ for the 2D graphic.

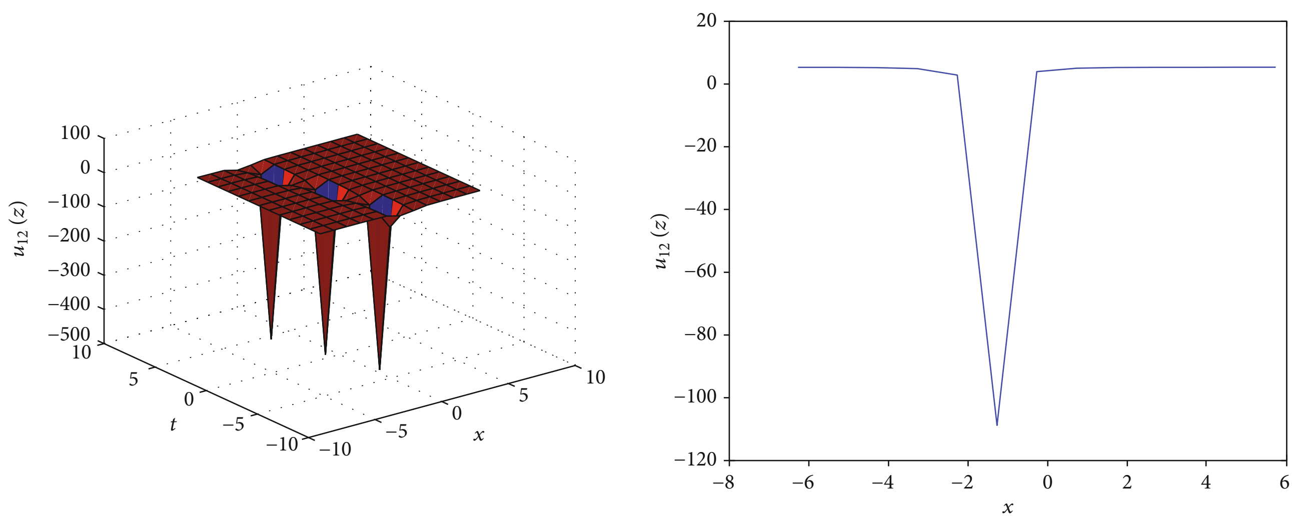


Figure 2: The 3D and 2D graphics of $u_{12}(z)$ by choosing the values $\gamma = 5$, $\vartheta = 6$, $\lambda = 1$, $\omega = 1/4$, $c = 1$, and $t = 0$ for the 2D graphic.

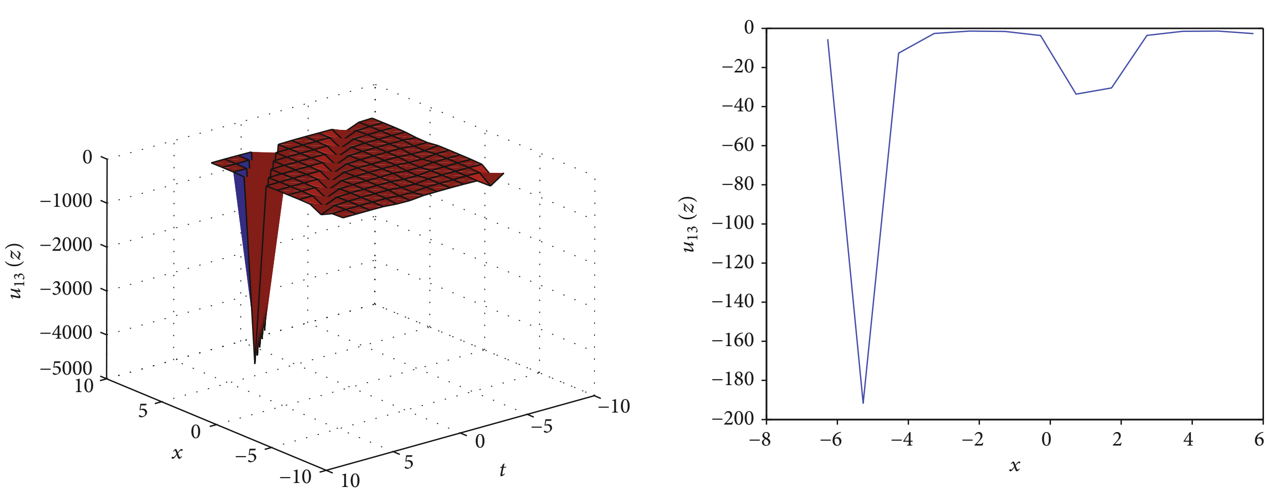


Figure 3: The 3D and 2D graphics of $u_{13}(z)$ by choosing the values $\gamma = 4$, $\vartheta = 5$, $\lambda = 1$, $\omega = 1$, $c = 1$, and $t = 0$ for the 2D graphic.

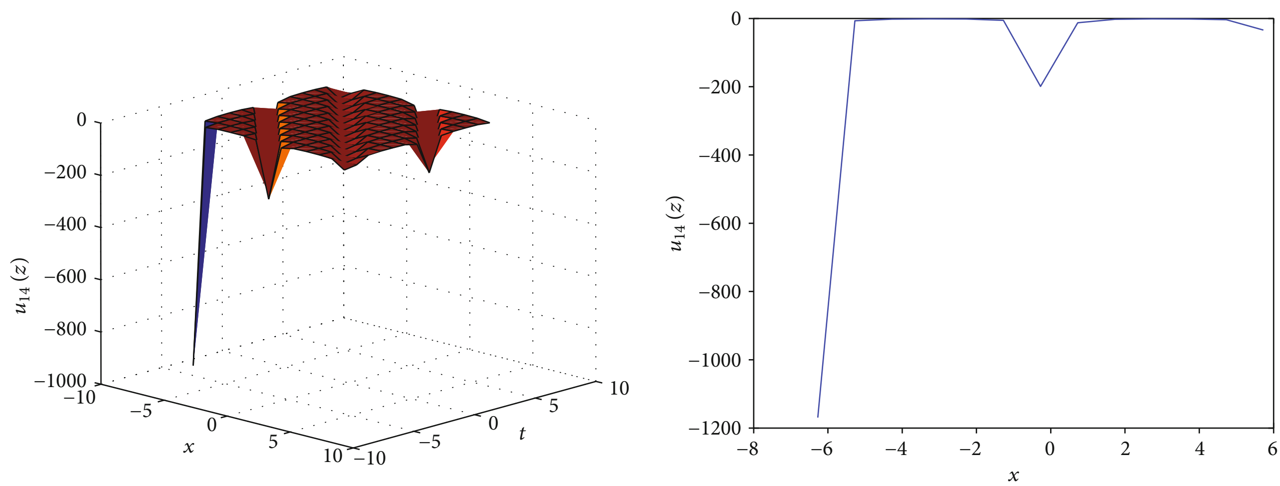


Figure 4: The 3D and 2D graphics of $u_{14}(z)$ by choosing the values $\gamma = 4$, $\vartheta = 5$, $\lambda = 1$, $\omega = 1$, $c = 1$, and $t = 0$ for the 2D graphic.

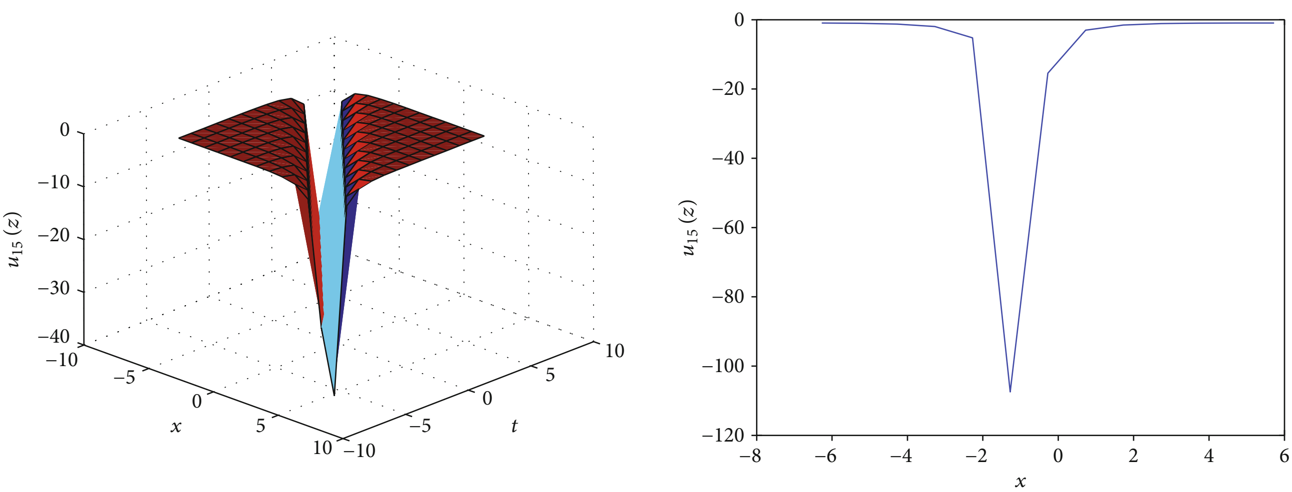


Figure 5: The 3D and 2D graphics of $u_{15}(z)$ by choosing the values $\gamma = 2$, $\vartheta = 0$, $\lambda = 1$, $\omega = 1$, $c = 1$, and $t = 0$ for the 2D graphic.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

## Acknowledgments

This work is supported by the NSF of China (11901111) and the Visiting Scholar Program of Chern Institute of Mathematics.

## References


[1] Y. Gu and J. Qi, "Symmetry reduction and exact solutions of two higher-dimensional nonlinear evolution equations," *Journal of Inequalities and Applications*, vol. 2017, no. 1, 2017.

[2] Y. Gu, X. Zheng, and F. Meng, "Painlevé analysis and abundant meromorphic solutions of a class of nonlinear algebraic differential equations," *Mathematical Problems in Engineering*, vol. 2019, Article ID 9210725, 11 pages, 2019.

[3] C. Chen, W. Liu, and C. Bi, "A two-grid characteristic finite volume element method for semilinear advection-dominated diffusion equations," *Numerical Methods for Partial Differential Equations*, vol. 29, no. 5, pp. 1543–1562, 2013.

[4] C. Chen, X. Zhang, G. Zhang, and Y. Zhang, "A two-grid finite element method for nonlinear parabolic integro-differential equations," *International Journal of Computer Mathematics*, vol. 96, no. 10, pp. 2010–2023, 2018.

[5] C. Chen and X. Zhao, "A posteriori error estimate for finite volume element method of the parabolic equations," *Numerical Methods for Partial Differential Equations*, vol. 33, no. 1, pp. 259–275, 2017.

[6] C. Chen, K. Li, Y. Chen, and Y. Huang, "Two-grid finite element methods combined with Crank-Nicolson scheme for nonlinear Sobolev equations," *Advances in Computational Mathematics*, vol. 45, no. 2, pp. 611–630, 2019.

[7] C. Chen, H. Liu, X. Zheng, and H. Wang, "A two-grid MMOC finite element method for nonlinear variable-order time-

fractional mobile/immobile advection–diffusion equations," *Computers and Mathematics with Applications*, vol. 79, no. 9, pp. 2771–2783, 2020.

[8] X. Zhang, L. Liu, and Y. Wu, "Multiple positive solutions of a singular fractional differential equation with negatively perturbed term," *Mathematical and Computer Modelling*, vol. 55, no. 3-4, pp. 1263–1274, 2012.

[9] X. Zhang, Y. Wu, and L. Caccetta, "Nonlocal fractional order differential equations with changing-sign singular perturbation," *Applied Mathematical Modelling*, vol. 39, no. 21, pp. 6543–16552, 2015.

[10] X. Zhang, L. Liu, Y. Wu, and Y. Cui, "The existence and nonexistence of entire large solutions for a quasilinear Schrödinger elliptic system by dual approach," *Journal of Mathematical Analysis and Applications*, vol. 464, no. 2, pp. 1089–1106, 2018.

[11] X. Zhang, L. Liu, Y. Wu, and Y. Cui, "Existence of infinitely solutions for a modified nonlinear Schrodinger equation via dual approach," *Electronic Journal of Differential Equations*, vol. 2147, pp. 1–15, 2018.

[12] X. Zhang, J. Jiang, Y. Wu, and Y. Cui, "Existence and asymptotic properties of solutions for a nonlinear Schrödinger elliptic equation from geophysical fluid flows," *Applied Mathematics Letters*, vol. 90, pp. 229–237, 2019.

[13] J. He, X. Zhang, L. Liu, and Y. Wu, "Existence and nonexistence of radial solutions of the Dirichlet problem for a class of general k-Hessian equations," *Nonlinear Analysis: Modelling and Control*, vol. 23, no. 4, pp. 475–492, 2018.

[14] J. He, X. Zhang, L. Liu, Y. Wu, and Y. Cui, "A singular fractional Kelvin-Voigt model involving a nonlinear operator and their convergence properties," *Boundary Value Problems*, vol. 2019, no. 1, 2019.

[15] X. Zhang, Y. Wu, and Y. Cui, "Existence and nonexistence of blow-up solutions for a Schrödinger equation involving a nonlinear operator," *Applied Mathematics Letters*, vol. 82, pp. 85–91, 2018.

[16] X. Zhang, L. Liu, Y. Wu, and Y. Cui, "A sufficient and necessary condition of existence of blow-up radial solutions for a k-Hessian equation with a nonlinear operator," *Nonlinear Analysis: Modelling and Control*, vol. 25, no. 1, pp. 126–143, 2020.

[17] X. Zhang, J. Xu, J. Jiang, Y. Wu, and Y. Cui, "The convergence analysis and uniqueness of blow-up solutions for a Dirichlet problem of the general k-Hessian equations," *Applied Mathematics Letters*, vol. 102, p. 106124, 2020.

[18] X. Zhang, J. Jiang, Y. Wu, and Y. Cui, "The existence and nonexistence of entire large solutions for a quasilinear Schrödinger elliptic system by dual approach," *Applied Mathematics Letters*, vol. 100, p. 106018, 2020.

[19] R. Hirota, *The Direct Method in Soliton Theory*, Cambridge University Press, London, 2004.

[20] H. Tariq and G. Akram, "New approach for exact solutions of time fractional Cahn–Allen equation and time fractional Phi-4 equation," *Physica A: Statistical Mechanics and its Applications*, vol. 473, pp. 352–362, 2017.

[21] H. Jafari, N. Kadkhoda, and D. Baleanu, "Fractional lie group method of the time-fractional Boussinesq equation," *Nonlinear Dynamics*, vol. 81, no. 3, pp. 1569–1574, 2015.

[22] K. Hosseini, F. Samadani, D. Kumar, and M. Faridi, "New optical solitons of cubic-quartic nonlinear Schrödinger equation," *Optik*, vol. 157, pp. 1101–1105, 2018.

[23] K. Hosseini, A. Zabihi, F. Samadani, and R. Ansari, "New explicit exact solutions of the unstable nonlinear Schrödinger's equation using the $\exp_a$ and hyperbolic function methods," *Optical and Quantum Electronics*, vol. 50, no. 2, 2018.

[24] H. M. Baskonus, T. A. Sulaiman, and H. Bulut, "New solitary wave solutions to the (2+1)-dimensional Calogero–Bogoyavlenskii–Schiff and the Kadomtsev–Petviashvili hierarchy equations," *Indian Journal of Physics*, vol. 91, no. 10, pp. 1237–1243, 2017.

[25] W. Yuan, Y. Li, and J. Lin, "Meromorphic solutions of an auxiliary ordinary differential equation using complex method," *Mathematical Methods in the Applied*, vol. 36, no. 13, pp. 1776–1782, 2013.

[26] Y. Gu, W. Yuan, N. Aminakbari, and Q. Jiang, "Exact solutions of the Vakhnenko-Parkes equation with complex method," *Journal of Function Spaces*, vol. 2017, Article ID 6521357, 6 pages, 2017.

[27] Y. Gu, N. Aminakbari, W. Yuan, and Y. Wu, "Meromorphic solutions of a class of algebraic differential equations related to Painlevé equation III," *Houston Journal of Mathematics*, vol. 43, no. 4, pp. 1045–1055, 2017.

[28] Y. Gu, W. Yuan, N. Aminakbari, and J. Lin, "Meromorphic solutions of some algebraic differential equations related Painlevé equation IV and its applications," *Mathematical Methods in the Applied Sciences*, vol. 41, no. 10, pp. 3832–3840, 2018.

[29] Y. Gu and F. Meng, "Searching for analytical solutions of the (2 +1)-dimensional KP equation by two different systematic methods," *Complexity*, vol. 2019, Article ID 9314693, 11 pages, 2019.

[30] K. Khan and M. A. Akbar, "The exp(−$\Phi(\xi)$)-expansion method for finding travelling wave solutions of Vakhnenko-Parkes equation," *International Journal of Dynamical Systems and Differential Equations*, vol. 5, no. 1, pp. 72–83, 2014.

[31] N. Kadkhoda and H. Jafari, "Analytical solutions of the Gerdjikov–Ivanov equation by using $\exp(-\varphi(\xi))$-expansion method," *Optik*, vol. 139, pp. 72–76, 2017.

[32] S. M. Mabrouk and A. S. Rashed, "N-Solitons, kink and periodic wave solutions for (3 + 1)-dimensional Hirota bilinear equation using three distinct techniques," *Chinese Journal of Physics*, vol. 60, pp. 48–60, 2019.

[33] K. Sawada and T. Kotera, "A method for finding N-soliton solutions of the K.d. V. equation and K.d.V.-like equation," *Progress of Theoretical Physics*, vol. 51, no. 5, pp. 1355–1367, 1974.

[34] P. J. Caudrey, R. K. Dodd, and J. D. Gibbon, "A new hierarchy of Korteweg–de Vries equations," *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, vol. 351, no. 1666, pp. 407–422, 1997.

[35] V. Z. Enolski, Y. N. Fedorov, and A. N. W. Hone, "Generic hyperelliptic prym varieties in a generalized Henon-Heiles system," *Journal of Geometry and Physics*, vol. 87, pp. 106–114, 2014.

[36] A. Parker, "A reformulation of the dressing method for the Sawada-Kotera equation," *Inverse Problems*, vol. 17, no. 4, pp. 885–895, 2001.

[37] R. N. Aiyer, B. Fuchssteiner, and W. Oevel, "Solitons and discrete eigenfunctions of the recursion operator of non-linear evolution equations. I. The Caudrey-Dodd-Gibbon-Sawada-Kotera equation," *Journal of Physics A*, vol. 19, no. 18, pp. 3755–3770, 1986.

[38] J. Satsuma and D. J. Kaup, “A Bäcklund transformation for a higher order Korteweg-de Vries equation,” *Journal of the Physical Society of Japan*, vol. 43, no. 2, pp. 692–697, 1977.

[39] X. Geng, G. He, and L. Wu, “Riemann theta function solutions of the Caudrey–Dodd–Gibbon–Sawada–Kotera hierarchy,” *Journal of Geometry and Physics*, vol. 140, pp. 85–103, 2019.