Research Article

Generalized Weighted Statistical Convergence for Double Sequences of Fuzzy Numbers and Associated Korovkin-Type Approximation Theorem

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We define the notions of weighted (λ, μ) -statistical convergence of order (γ_1, γ_2) and strongly weighted (λ, μ) -summability of (γ_1, γ_2) for fuzzy double sequences, where $0 < \gamma_1, \gamma_2 \leq 1$. We establish an inclusion result and a theorem presenting a connection between these concepts. Moreover, we apply our new concept of weighted (λ, μ) -statistical convergence of order (γ_1, γ_2) to prove Korovkin-type approximation theorem for functions of two variables in a fuzzy sense. Finally, an illustrative example is provided with the help of q -analogue of fuzzy Bernstein operators for bivariate functions which shows the significance of our approximation theorem.

1. Introduction and Preliminaries

The notion of weighted statistical convergence for sequences of real numbers has been studied by Karakaya and Chishti [1] as a generalization of the concept of statistical convergence which is related to the idea of asymptotic density (or natural density) of a subset of $\mathbb{N}$, the set of natural numbers, according to Fast [2]. The weighted statistical convergence was further improved by Mursaleen et al. [3] and later generalized by Belen and Mohiuddine [4] with a view of nondecreasing sequence of positive numbers. Ghosal [5] added constraints to these ideas. For some other related work, we refer the interested reader to [6–10]. Recently, Çolak [11] defined the notion of statistical convergence of order α ($0 < \alpha \leq 1$) and strong p -Cesàro summability of order α while the order of statistical was also given in [12], and it reduces to strong p -Cesàro summability for $\alpha = 1$ according to Connor [13].

The idea convergence for double sequence $y = (y_{kl})$ was presented by Pringsheim [14]; that is, (y_{kl}) is convergent to ξ in the Pringsheim sense if for every $\varepsilon > 0$, there is $N \in \mathbb{N}$ such that $|y_{kl} - \xi| < \varepsilon$ whenever $k, l > N$. The idea of statistical convergence for $y = (y_{kl})_{(k,l) \in \mathbb{N} \times \mathbb{N}}$ was introduced by

Mursaleen and Edely [15] as follows: (y_{kl}) is statistically convergent to ξ if for every $\varepsilon > 0$,

$$\lim_{k_1, l_1 \rightarrow \infty} \frac{1}{k_1 l_1} \{ (k, l), k \leq k_1, l \leq l_1 : |y_{kl} - \xi| \geq \varepsilon \} = 0, \quad (1)$$

where the limit is taken in Pringsheim's sense while the order of notion was studied in [16]. This idea was further generalized by Mursaleen et al. [17] and called it (λ, μ) -statistical convergence, and for single sequence in [18] and its weighted variant were presented by Cinar and Et [19], which were later on studied by various authors in various setups [20–22]. Throughout this paper, limit of the double sequences means limit in Pringsheim's sense.

In the very recent past, for sequence of fuzzy numbers, Mohiuddine et al. [23] weighted statistical convergence and strong weighted summability by using the idea of difference operator and established a connection between these notions while under certain conditions, these notions reduce to statistical convergence and strong p -Cesàro summability, respectively, in [24, 25]. Savas and Mursaleen [26] defined statistical convergence and statistically Cauchy for fuzzy

double sequences and obtained that these concepts are equivalent. The statistical analogue of pointwise and uniformly convergent double sequences for fuzzy-valued functions were discussed by Mohiuddine et al. [21]. These concepts were studied and extended by several authors; for instance, see [27–31].

Recall that, in [32], a fuzzy number is a fuzzy set on the real axis; that is, $u : \mathbb{R} \rightarrow [0, 1]$ which is normal, fuzzy convex, upper semicontinuous and the closure of the set $\{y \in \mathbb{R} : u(y) > 0\}$ is compact. By using the symbol $\mathcal{L}(\mathbb{R})$, we denote the set of all fuzzy numbers on $\mathbb{R}$. α -Level set $[u]_\alpha$ of u is given as follows:

$$[u]_\alpha = \begin{cases} \{y \in \mathbb{R} : u(y) \geq \alpha\} & \text{if } 0 < \alpha \leq 1, \\ \{y \in \mathbb{R} : u(y) > 0\} & \text{if } \alpha = 0. \end{cases} \quad (2)$$

For each $\alpha \in [0, 1]$, $[u]_\alpha$ is closed and bounded and non-empty interval for each $\alpha \in [0, 1]$ defined by $[u]_\alpha = [u_\alpha^-, u_\alpha^+]$, where $u_\alpha^- \leq u_\alpha^+$ and $u_\alpha^-, u_\alpha^+ \in \mathbb{R}$. For any $u_1, u_2 \in \mathcal{L}(\mathbb{R})$, the partial ordering is defined by

$$\begin{aligned} u_1 \leq u_2 &\Leftrightarrow u_{1\alpha}^- \leq u_{2\alpha}^-, \\ u_{1\alpha}^+ &\leq u_{2\alpha}^+, \end{aligned} \quad (3)$$

for all $0 \leq \alpha \leq 1$. For any $u_1, u_2 \in \mathcal{L}(\mathbb{R})$ and $\alpha \in [0, 1]$, define the following:

$$\begin{aligned} [u_1 \oplus u_2]_\alpha &= [u_1]_\alpha \oplus [u_2]_\alpha, \\ [\mu \odot u_1]_\alpha &= \mu[u_1]_\alpha \quad (\mu \in \mathbb{R}). \end{aligned} \quad (4)$$

By means of the Hausdorff metric D , the metric $d : \mathcal{L}(\mathbb{R}) \times \mathcal{L}(\mathbb{R}) \rightarrow \mathbb{R}_+$ is defined by

$$\begin{aligned} d(u_1, u_2) &= \sup_{\alpha \in [0, 1]} D([u_1]_\alpha, [u_2]_\alpha) \\ &= \sup_{\alpha \in [0, 1]} \max \{|u_{1\alpha}^- - u_{2\alpha}^-|, |u_{1\alpha}^+ - u_{2\alpha}^+|\}. \end{aligned} \quad (5)$$

It is clear from [33] that $(\mathcal{L}(\mathbb{R}), d)$ is a complete metric space.

Suppose $g_1, g_2 : [a, b] \times [a, b] \rightarrow \mathcal{L}(\mathbb{R})$ are two fuzzy number-valued functions. Then, the distance between g_1 and g_2 is defined by

$$\begin{aligned} d^*(g_1, g_2) &= \sup_{y \in [a, b] \times [a, b]} \sup_{\alpha \in [0, 1]} \max \\ &\cdot \{|g_{1\alpha}^- - g_{2\alpha}^-|, |g_{1\alpha}^+ - g_{2\alpha}^+|\}. \end{aligned} \quad (6)$$

2. Generalized Weighted Statistical Convergence of Order (γ_1, γ_2)

Suppose two nondecreasing sequence $\lambda = (\lambda_k)$ and $\mu = (\mu_l)$ of positive integers such that both λ and μ tending to infinity; that is, $\lambda_k \rightarrow \infty$ and $\mu_l \rightarrow \infty$ as $k \rightarrow \infty$ and $l \rightarrow \infty$, respectively, such that

$$\begin{aligned} \lambda_{k+1} &\leq \lambda_k + 1, \quad \lambda_k = 1, \\ \mu_{l+1} &\leq \mu_l + 1, \quad \mu_l = 1. \end{aligned} \quad (7)$$

Consider two sequences $u = (u_m)$ and $v = (v_n)$ of nonnegative numbers such that $\liminf_m u_m > 0$, $\liminf_n v_n > 0$ and

$$\begin{aligned} U_{\lambda_k} &= \sum_{m \in [k-\lambda_k+1, k]} u_m \rightarrow \infty (k \rightarrow \infty), \\ V_{\mu_l} &= \sum_{n \in [l-\mu_l+1, l]} v_n \rightarrow \infty (l \rightarrow \infty). \end{aligned} \quad (8)$$

Assume that $M \subseteq \mathbb{N} \times \mathbb{N}$ and let $0 < \gamma_1, \gamma_2 \leq 1$. We define the weighted (λ, μ) density of order (γ_1, γ_2) , denoted by $\delta_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}$, of M by

$$\delta_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}(M) = \lim_{k, l \rightarrow \infty} \frac{|A_{U_{\lambda_k}, V_{\mu_l}}|}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}}, \quad (9)$$

provided the limit in the last relation (9) exists, where

$$A_{U_{\lambda_k}, V_{\mu_l}} = \{m \leq U_{\lambda_k}, n \leq V_{\mu_l} : (m, n) \in M\}. \quad (10)$$

Definition 1. A fuzzy double sequence (s_{mn}) is said to be weighted (λ, μ) -statistically convergent of order (γ_1, γ_2) , where $0 < \gamma_1, \gamma_2 \leq 1$; in short, we shall write $S_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}$ -convergent, to a fuzzy number s_0 if for every $\varepsilon > 0$, the set

$$B = \{(m, n) \in \mathbb{N} \times \mathbb{N} : u_m v_n d(s_{mn}, s_0) \geq \varepsilon\} \quad (11)$$

has $\delta_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}$ density zero; that is,

$$\delta_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}(B) = 0. \quad (12)$$

Equivalently, we can write

$$\begin{aligned} \lim_{k, l \rightarrow \infty} \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \{(m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn}, s_0) \geq \varepsilon\} \right| &= 0. \end{aligned} \quad (13)$$

We denote this convergence by $s_{mn} \xrightarrow{S_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}} s_0$ or $S_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)} - \lim s_{mn} = s_0$. We denote the set of all $S_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}$ -convergence by $S_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}$.

The choice of $\gamma_1 = \gamma_2 = 1$ in the notion of $S_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}$ -convergence gives weighted (λ, μ) -statistically convergence for fuzzy double sequences. In addition, if we choose $\lambda_k = k$, $\mu_l = l$, $u_m = 1$, and $v_n = 1$ for all m, n , then $S_{2, \gamma_1, \gamma_2}^{\bar{N}(\lambda, \mu)}$ -convergence gives the notion of (λ, μ) -statistically convergence in a fuzzy sense [26].

For $0 < \gamma_1, \gamma_2 \leq 1$, the notion of weighted (λ, μ) -statistically convergent of order (γ_1, γ_2) is well but not well defined

for $\gamma_1, \gamma_2 > 1$. To show this assertion, we consider the following example.

Example 1. Assume that $\gamma_1, \gamma_2 > 1$, and define the fuzzy double sequence (s_{mn}) by

$$s_{mn} = \begin{cases} \bar{1} & \text{if } m+n \text{ even,} \\ \bar{0} & \text{if } m+n \text{ odd.} \end{cases} \quad (14)$$

Let $\varepsilon > 0$ be given, and let $u_m = 1$ and $v_n = 1 \forall m, n \in \mathbb{N}$. Then,

$$\begin{aligned} & \lim_{k,l \rightarrow \infty} \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn}, \bar{1}) \geq \varepsilon \right\} \right| \\ & \leq \lim_{k,l \rightarrow \infty} \frac{\lambda_k \mu_l}{2\lambda_k^{\gamma_1} \mu_l^{\gamma_2}} = 0. \end{aligned} \quad (15)$$

On the other hand,

$$\begin{aligned} & \lim_{k,l \rightarrow \infty} \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn}, \bar{0}) \geq \varepsilon \right\} \right| \\ & \leq \lim_{k,l \rightarrow \infty} \frac{\lambda_k \mu_l}{2\lambda_k^{\gamma_1} \mu_l^{\gamma_2}} = 0. \end{aligned} \quad (16)$$

We conclude from (15) and (16) that

$$\begin{aligned} S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} - \lim s_{mn} &= \bar{1}, \\ S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} - \lim s_{mn} &= \bar{0}, \end{aligned} \quad (17)$$

which is impossible.

Example 2. Define (s_{mn}) as

$$s_{mn} = \begin{cases} b & \text{if } m = k^2, n = l^2, \\ \bar{0} & \text{otherwise,} \end{cases} \quad (18)$$

for $i, j \in \mathbb{N}$, where b stands for a fixed fuzzy number. Let $\gamma_1 = \gamma_2 = 1$, and let $u_m = 1$ and $v_n = 1 \forall m, n$. Let $\lambda_k = k$ and $\mu_l = l$. Suppose b_α^+ is the upper bound of the α -cut. Then, for $\varepsilon > 0$, $0 \leq \alpha \leq 1$ and $i, j \in \mathbb{N}$, one writes

$$d(s_{mn}, \bar{0}) = \begin{cases} \sup_{0 \leq \alpha \leq 1} |b_\alpha^+| & \text{if } m = k^2, n = l^2, \\ \bar{0} & \text{otherwise.} \end{cases} \quad (19)$$

Thus,

$$\begin{aligned} & \lim_{k,l \rightarrow \infty} \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn}, \bar{0}) \geq \varepsilon \right\} \right| \\ & \leq \lim_{k,l \rightarrow \infty} \frac{\sqrt[2]{k} \sqrt[2]{l}}{kl} = 0. \end{aligned} \quad (20)$$

Hence, $s_{mn} \xrightarrow{S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)}} \bar{0}$ but (s_{mn}) is not convergent.

Theorem 2. Suppose $0 < \gamma_1, \gamma_2, \eta_1, \eta_2 \leq 1$ such that $\gamma_1 \leq \eta_1$ and $\gamma_2 \leq \eta_2$. Then, $S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} \subseteq S_{2,\eta_1,\eta_2}^{\bar{N}(\lambda,\mu)}$.

Proof. Let $\varepsilon > 0$ be given, and let $\gamma_1, \gamma_2, \eta_1, \eta_2 \in (0, 1]$. If $\gamma_1 \leq \eta_1$ and $\gamma_2 \leq \eta_2$, then, for every $\varepsilon > 0$, we can write

$$\begin{aligned} & \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn}, s_0) \geq \varepsilon \right\} \right|, \\ & \leq \frac{1}{U_{\lambda_k}^{\eta_1} V_{\mu_l}^{\eta_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn}, s_0) \geq \varepsilon \right\} \right|, \end{aligned} \quad (21)$$

which yields that $S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} \subseteq S_{2,\eta_1,\eta_2}^{\bar{N}(\lambda,\mu)}$.

Theorem 3. Suppose two fuzzy double sequence (s_{mn}) and (t_{mn}) such that $S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} - \lim s_{mn} = s_0$ and $S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} - \lim t_{mn} = t_0$. Then,

- (i) $S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} - \lim cs_{mn} = cs_0$ ($c \in \mathbb{R}$)
- (ii) $S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} - \lim (s_{mn} + t_{mn}) = s_0 + t_0$

Proof.

- (i) Let $S_{2,\gamma_1,\gamma_2}^{\bar{N}(\lambda,\mu)} - \lim s_{mn} = s_0$. It is clear for $c = 0$. Let $c \neq 0$. For $\varepsilon > 0$, we write

$$\begin{aligned} & \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(cs_{mn}, cs_0) \geq \varepsilon \right\} \right| \\ & = \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n |c| d(s_{mn}, s_0) \geq \varepsilon \right\} \right| \\ & = \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn}, s_0) \geq \frac{\varepsilon}{|c|} \right\} \right| \end{aligned} \quad (22)$$

Thus, (i) holds.

Suppose that $S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)} - \lim s_{mn} = s_0$ and $S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)} - \lim t_{mn} = t_0$. Let $\varepsilon > 0$ be given. Then,

$$\begin{aligned} & \left| \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn} + t_{mn}, s_0 + t_0) \right. \right. \\ & \quad \left. \left. \geq \varepsilon \right\} \right| \leq \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \leq V_{\mu_l} : u_m v_n d(s_{mn}, s_0) \right. \right. \\ & \quad \left. \left. \geq \frac{\varepsilon}{2} \right\} \right| + \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \right. \right. \\ & \quad \left. \left. \leq V_{\mu_l} : u_m v_n d(t_{mn}, t_0) \geq \frac{\varepsilon}{2} \right\} \right| \end{aligned} \quad (23)$$

Hence, (ii) holds.

Definition 4. Assume that $\gamma_1, \gamma_2 \in (0, 1]$. Then, a fuzzy double sequence (s_{mn}) is said to be strongly weighted (λ, μ) -summable of order (γ_1, γ_2) ; in short, we shall write $S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}$ -summable, to fuzzy number s_0 , denoted by

$$s_{mn} \xrightarrow{M_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0, \quad (24)$$

such that

$$\lim_{k,l \rightarrow \infty} \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \sum_{m \in [k-\lambda_k+1, k]} \sum_{n \in [l-\mu_l+1, l]} u_m v_n d(s_{mn}, s_0) = 0. \quad (25)$$

Theorem 5. Consider a fuzzy double sequence $s = (s_{mn})$, and let $0 < \gamma_1, \gamma_2 \leq 1$. Then,

- (i) $s_{mn} \xrightarrow{M_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0$ implies $s_{mn} \xrightarrow{S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0$
- (ii) (s_{mn}) is a bounded double sequence and $s_{mn} \xrightarrow{S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0$ imply $s_{mn} \xrightarrow{M_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0$

Proof.

- (i) Let $s_{mn} \xrightarrow{M_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0$, and let $\varepsilon > 0$ be given. Then,

$$\begin{aligned} & \sum_{m \in [k-\lambda_k+1, k]} \sum_{n \in [l-\mu_l+1, l]} u_m v_n d(s_{mn}, s_0) \\ &= \sum_{\substack{m \in [k-\lambda_k+1, k] \\ u_m v_n d(s_{mn}, s_0) \geq \varepsilon}} \sum_{\substack{n \in [l-\mu_l+1, l] \\ u_m v_n d(s_{mn}, s_0) \geq \varepsilon}} u_m v_n d(s_{mn}, s_0) \\ & \quad + \sum_{\substack{m \in [k-\lambda_k+1, k] \\ u_m v_n d(s_{mn}, s_0) < \varepsilon}} \sum_{\substack{n \in [l-\mu_l+1, l] \\ u_m v_n d(s_{mn}, s_0) < \varepsilon}} u_m v_n d(s_{mn}, s_0) \\ &\geq \sum_{\substack{m \in [k-\lambda_k+1, k] \\ u_m v_n d(s_{mn}, s_0) \geq \varepsilon}} \sum_{\substack{n \in [l-\mu_l+1, l] \\ u_m v_n d(s_{mn}, s_0) \geq \varepsilon}} u_m v_n d(s_{mn}, s_0) \end{aligned} \quad (26)$$

Therefore, we get

$$\begin{aligned} & \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \sum_{m \in [k-\lambda_k+1, k]} \sum_{n \in [l-\mu_l+1, l]} u_m v_n d(s_{mn}, s_0) \\ &\geq \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \sum_{\substack{m \in [k-\lambda_k+1, k] \\ u_m v_n d(s_{mn}, s_0) \geq \varepsilon}} \sum_{\substack{n \in [l-\mu_l+1, l] \\ u_m v_n d(s_{mn}, s_0) \geq \varepsilon}} \varepsilon \\ &\geq \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \right. \right. \\ & \quad \left. \left. \leq V_{\mu_l} : u_m v_n d(s_{mn}, s_0) \geq \varepsilon \right\} \right| \varepsilon. \end{aligned} \quad (27)$$

Letting $k, l \rightarrow \infty$ on both sides of the above relation, we get $s_{mn} \xrightarrow{S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0$.

Let $s_{mn} \xrightarrow{S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0$. Since $s = (s_{mn}) \in \mathcal{L}_{2,\infty}$, $d(s_{mn}, s_0) \leq C$ for all $m, n \in \mathbb{N}$. For a given $\varepsilon > 0$, one writes

$$\begin{aligned} & \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \sum_{m \in [k-\lambda_k+1, k]} \sum_{n \in [l-\mu_l+1, l]} u_m v_n d(s_{mn}, s_0) \\ &= \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \sum_{\substack{m \in [k-\lambda_k+1, k] \\ u_m v_n d(s_{mn}, s_0) \geq \varepsilon}} \sum_{\substack{n \in [l-\mu_l+1, l] \\ u_m v_n d(s_{mn}, s_0) \geq \varepsilon}} u_m v_n d(s_{mn}, s_0) \\ & \quad + \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \sum_{\substack{m \in [k-\lambda_k+1, k] \\ u_m v_n d(s_{mn}, s_0) < \varepsilon}} \sum_{\substack{n \in [l-\mu_l+1, l] \\ u_m v_n d(s_{mn}, s_0) < \varepsilon}} u_m v_n d(s_{mn}, s_0) \\ &\leq \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \sum_{\substack{m \in [k-\lambda_k+1, k] \\ u_m v_n d(s_{mn}, s_0) < \varepsilon}} \sum_{\substack{n \in [l-\mu_l+1, l] \\ u_m v_n d(s_{mn}, s_0) < \varepsilon}} C \\ & \quad + \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \sum_{\substack{m \in [k-\lambda_k+1, k] \\ u_m v_n d(s_{mn}, s_0) < \varepsilon}} \sum_{\substack{n \in [l-\mu_l+1, l] \\ u_m v_n d(s_{mn}, s_0) < \varepsilon}} \varepsilon \\ &\leq \frac{1}{U_{\lambda_k}^{\gamma_1} V_{\mu_l}^{\gamma_2}} \left| \left\{ (m, n), m \leq U_{\lambda_k}, n \right. \right. \\ & \quad \left. \left. \leq V_{\mu_l} : u_m v_n d(s_{mn}, s_0) \geq \varepsilon \right\} \right| C + \varepsilon. \end{aligned} \quad (28)$$

It follows that $s_{mn} \xrightarrow{M_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}} s_0$.

3. Application to Fuzzy Korovkin-Type Theorems

The fuzzy version of Korovkin theorem has been obtained by Anastassiou [34] while the classification of this result has been established by Korovkin in [35] (also see [36–42]), and then Anastassiou and Duman [43] and Karaisa and Kadak [44] studied this result in a statistical sense. In the recent past, Mohiuddine et al. [23] investigated the fuzzy Korovkin-type approximation theorem through weighted statistical convergence of fuzzy sequence based on difference operators. The fuzzy Korovkin theorem for function of two variables was discussed by Demirci and Karakus [45] with the help of A -statistical convergence for a sequence of fuzzy numbers. Here, we prove the fuzzy Korovkin-type theorem

for the function of two variables with the help of $S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)}$ -convergence.

Assume that $I = [a, b]$ ($a, b \in \mathbb{R}$) and $g : I \times I (= I^2) \rightarrow \mathcal{L}(\mathbb{R})$ is a fuzzy number-valued function. Then, g is fuzzy continuous at a point S of $I \times I$ if $D(g(s), g(S)) \rightarrow 0$ whenever $s = (s_{mn})$ is Pringsheim's convergent to S in a fuzzy sense. We denote the set of fuzzy continuous function on $I^2 = [a, b] \times [a, b]$ by $C_{\mathcal{L}}(I^2)$. Here, $C_{\mathcal{L}}(I^2)$ is only a cone but not a vector space and let $T : C_{\mathcal{L}}(I^2) \rightarrow C_{\mathcal{L}}(I^2)$ be an operator. Further, T is said to be a fuzzy linear operator if

$$T(\beta_1 \cdot g_1 \oplus \beta_2 \cdot g_2; x, y) = \beta_1 \cdot T(g_1; x, y) \oplus \beta_2 \cdot T(g_2; x, y), \quad (29)$$

for all $\beta_1, \beta_2 \in \mathbb{R}$, $g_1, g_2 \in C_{\mathcal{L}}(I^2)$, and $(x, y) \in I^2$. In addition, if $T(g_1; x, y) \circ T(g_2; x, y)$ holds for all $g_1, g_2 \in C_{\mathcal{L}}(I^2)$ and $(x, y) \in I^2$ with $g_1(x, y) \circ g_2(x, y)$, then T is called fuzzy position linear operator.

Theorem 6. Let $(A_{nr})_{n,r \in \mathbb{N}}$ be a double sequence such that $A_{nr} : C_{\mathcal{L}}(I^2) \rightarrow C_{\mathcal{L}}(I^2)$. Let $(\tilde{A}_{nr})_{n,r \in \mathbb{N}}$ be corresponding positive linear operators acting from $C(I^2)$ into itself having the relation

$$\{A_{nr}(g; x, y)\}_{\alpha}^{\pm} = \tilde{A}_{nr}(g_{\alpha}^{\pm}; x, y) \quad (\forall \alpha \in [0, 1], (x, y) \in I^2, g \in C_{\mathcal{L}}(I^2), n, r \in \mathbb{N}). \quad (30)$$

Assume further that

$$S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)} - \lim \| \tilde{A}_{nr}(g_i) - g_i \| = 0 \quad (i = 0, 1, 2, 3), \quad (31)$$

where $g_0(t_1, t_2) = 1$, $g_1(t_1, t_2) = t_1$, $g_2(t_1, t_2) = t_2$, and $g_3(t_1, t_2) = t_1 + t_2$. Then,

$$S_{2,\gamma_1,\gamma_2}^{\tilde{N}(\lambda,\mu)} - \lim d^*(A_{nr}(g), g) = 0, \quad (32)$$

for all $n, r \in \mathbb{N}$ and $g \in C_{\mathcal{L}}(I^2)$.

Proof. Suppose that (31) holds and $(n, r) \in \mathbb{N} \times \mathbb{N}$. Suppose also that $(x, y) \in I^2$, $g \in C_{\mathcal{L}}(I^2)$, and $\alpha \in [0, 1]$. Since $g_{\alpha}^{\pm} \in C(I^2)$, it follows that, for a given $\varepsilon > 0$, there exists $\delta > 0$ such that for all $(t_1, t_2) \in I^2$, we have $|g_{\alpha}^{\pm}(t_1, t_2) - g_{\alpha}^{\pm}(x, y)| < \varepsilon$ whenever $|t_1 - x| < \delta$ and $|t_2 - y| < \delta$.

By the fuzzy boundedness of g , we may write $|g_{\alpha}^{\pm}(x, y)| \leq C_{\alpha}^{\pm}$ for all $(x, y) \in I^2$, where $C_{\alpha}^{\pm} = \|g_{\alpha}^{\pm}\|$ for all $a < y < b$. We then obtain for all $(t_1, t_2), (x, y) \in I^2$ that

$$|g_{\alpha}^{\pm}(t_1, t_2) - g_{\alpha}^{\pm}(x, y)| \leq 2C_{\alpha}^{\pm}. \quad (33)$$

With the help of last relations, for all $|t_1 - x| < \delta$ and $|t_2 - y| < \delta$, we obtain

$$|g_{\alpha}^{\pm}(t_1, t_2) - g_{\alpha}^{\pm}(x, y)| < \varepsilon + \frac{2C_{\alpha}^{\pm}}{\delta^2} \{(t_1 - x)^2 + (t_2 - y)^2\}. \quad (34)$$

Since $\tilde{A}_{nr}$ is a positive linear operator, by applying this operator to (34), we get

$$\begin{aligned} & |\tilde{A}_{nr}(g_{\alpha}^{\pm}(t_1, t_2); x, y) - g_{\alpha}^{\pm}(x, y)| \\ &= \left| \tilde{A}_{nr}(g_{\alpha}^{\pm}(t_1, t_2) - g_{\alpha}^{\pm}(x, y); x, y) \right. \\ &\quad \left. + g_{\alpha}^{\pm}(x, y) [\tilde{A}_{nr}(g_0; x, y) - g_0(x, y)] \right| \\ &\leq \tilde{A}_{nr}(|g_{\alpha}^{\pm}(t_1, t_2) - g_{\alpha}^{\pm}(x, y)|; x, y) \\ &\quad + C_{\alpha}^{\pm} |\tilde{A}_{nr}(g_0; x, y) - g_0(x, y)| \\ &\leq \tilde{A}_{nr} \left(\varepsilon + \frac{2C_{\alpha}^{\pm}}{\delta^2} [(t_1 - x)^2 + (t_2 - y)^2]; x, y \right) \\ &\quad + C_{\alpha}^{\pm} |\tilde{A}_{nr}(g_0; x, y) - g_0(x, y)| \\ &\leq \varepsilon + \left(\varepsilon + C_{\alpha}^{\pm} + \frac{4C_{\alpha}^{\pm}E^2}{\delta^2} \right) |\tilde{A}_{nr}(g_0; x, y) - g_0(x, y)| \\ &\quad + \frac{4C_{\alpha}^{\pm}E}{\delta^2} |\tilde{A}_{nr}(g_1; x, y) - g_1(x, y)| \\ &\quad + \frac{4C_{\alpha}^{\pm}E}{\delta^2} |\tilde{A}_{nr}(g_2; x, y) - g_2(x, y)| \\ &\quad + \frac{2C_{\alpha}^{\pm}}{\delta^2} |\tilde{A}_{nr}(g_3; x, y) - g_3(x, y)|, \end{aligned} \quad (35)$$

where $E = \max \{|x|, |y|\}$. We thus obtain by letting

$$\Omega_{\alpha}^{\pm}(\varepsilon) = \max \left\{ \varepsilon + C_{\alpha}^{\pm} + \frac{4C_{\alpha}^{\pm}E^2}{\delta^2}, \frac{4C_{\alpha}^{\pm}E}{\delta^2}, \frac{2C_{\alpha}^{\pm}E}{\delta^2} \right\} \quad (36)$$

and taking $\sup_{(x,y) \in I^2}$ that

$$\begin{aligned} \|\tilde{A}_{nr}(g_{\alpha}^{\pm}) - g_{\alpha}^{\pm}\| &\leq \varepsilon + \Omega_{\alpha}^{\pm}(\varepsilon) \left\{ \|\tilde{A}_{nr}(g_0) - g_0\| \right. \\ &\quad \left. + \|\tilde{A}_{nr}(g_1) - g_1\| + \|\tilde{A}_{nr}(g_2) - g_2\| \right. \\ &\quad \left. + \|\tilde{A}_{nr}(g_3) - g_3\| \right\}. \end{aligned} \quad (37)$$

From (30), we can write

$$\begin{aligned} d^*(A_{mr}(g), g) &= \sup_{(x,y) \in I^2} d(A_{mr}(g; x, y), g(x, y)) \\ &= \sup_{(x,y) \in I^2} \sup_{\alpha \in [0,1]} \max \left\{ |\tilde{A}_{mr}(g_{\alpha}^{-}; x, y) - g_{\alpha}^{-}(x, y)|, \right. \\ &\quad \left. |\tilde{A}_{mr}(g_{\alpha}^{+}; x, y) - g_{\alpha}^{+}(x, y)| \right\}, \end{aligned} \quad (38)$$

and so

$$d^*(A_{nr}(g), g) = \sup_{\alpha \in [0,1]} \max \left\{ \|\tilde{A}_{nr}(g_\alpha^-) - g_\alpha^-\|, \|\tilde{A}_{nr}(g_\alpha^+) - g_\alpha^+\| \right\}. \quad (39)$$

With the help of (37) and (39), we get

$$d^*(A_{nr}(g), g) \leq \varepsilon + \Omega(\varepsilon) \left\{ \|\tilde{A}_{nr}(g_0) - g_0\| + \|\tilde{A}_{nr}(g_1) - g_1\| + \|\tilde{A}_{nr}(g_2) - g_2\| + \|\tilde{A}_{nr}(g_3) - g_3\| \right\}, \quad (40)$$

where

$$\Omega(\varepsilon) = \sup_{\alpha \in [0,1]} \max \{ \Omega_\alpha^-(\varepsilon), \Omega_\alpha^+(\varepsilon) \}. \quad (41)$$

Multiplying by the product of two nonnegative sequences (u_n) and (v_r) as details given in the previous section, one gets

$$u_n v_r d^*(A_{nr}(g), g) \leq u_n v_r \varepsilon + \Omega(\varepsilon) \left\{ u_n v_r \|\tilde{A}_{nr}(g_0) - g_0\| + u_n v_r \|\tilde{A}_{nr}(g_1) - g_1\| + u_n v_r \|\tilde{A}_{nr}(g_2) - g_2\| + u_n v_r \|\tilde{A}_{nr}(g_3) - g_3\| \right\}. \quad (42)$$

For a given $Q > 0$, choose $\varepsilon > 0$ such that $\varepsilon u_n v_r < Q$. Then, upon setting,

$$\begin{aligned} F &= \{(n, r): u_n v_r d^*(A_{nr}(g), g) \geq Q\}, \\ F_0 &= \left\{ (n, r): u_n v_r \|\tilde{A}_{nr}(g_0) - g_0\| \geq \frac{Q - \varepsilon u_n v_r}{4\Omega(\varepsilon)} \right\}, \\ F_1 &= \left\{ (n, r): u_n v_r \|\tilde{A}_{nr}(g_1) - g_1\| \geq \frac{Q - \varepsilon u_n v_r}{4\Omega(\varepsilon)} \right\}, \\ F_2 &= \left\{ (n, r): u_n v_r \|\tilde{A}_{nr}(g_2) - g_2\| \geq \frac{Q - \varepsilon u_n v_r}{4\Omega(\varepsilon)} \right\}, \\ F_3 &= \left\{ (n, r): u_n v_r \|\tilde{A}_{nr}(g_3) - g_3\| \geq \frac{Q - \varepsilon u_n v_r}{4\Omega(\varepsilon)} \right\}. \end{aligned} \quad (43)$$

It follows that

$$F \subset F_0 \cup F_1 \cup F_2 \cup F_3. \quad (44)$$

Consequently, by taking weighted (λ, μ) density of order (γ_1, γ_2) , we obtain

$$\begin{aligned} \delta_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)}(F) &\leq \delta_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)}(F_0) + \delta_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)}(F_1) \\ &\quad + \delta_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)}(F_2) + \delta_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)}(F_3). \end{aligned} \quad (45)$$

Using the hypothesis (31), we conclude that

$$S_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)} - \lim d^*(A_{nr}(g), g) = 0 \quad (46)$$

holds for all $(n, r) \in \mathbb{N} \times \mathbb{N}$ and $g \in C_{\mathcal{F}}(I^2)$.

Recall that, for any nonnegative integer j , the q -integer $[j]_q$ ($q > 0$) is given by

$$[j]_q = \begin{cases} \frac{1 - q^j}{1 - q} & \text{if } q \neq 1, \\ j & \text{if } q = 1, \end{cases} \quad (47)$$

and the q -factorial $[j]_q!$ by

$$\begin{aligned} [j]_q! &= [j]_q! [j-1]_q! \cdots [1]_q!, \\ [0]_q! &= 1. \end{aligned} \quad (48)$$

The q -binomial coefficients, for $0 \leq m \leq j$, is defined by

$$\begin{bmatrix} j \\ m \end{bmatrix}_q = \frac{[j]_q!}{[m]_q! [j-m]_q!}. \quad (49)$$

The bivariate case of classical q -Bernstein operators were introduced and studied by Barbosu [46]. We are now considering the fuzzy analogue of these operators to construct an example to illustrate our last theorem.

Example 3. Consider the fuzzy analogue of bivariate q -Bernstein operators, defined by

$$\begin{aligned} B_{nr}^{\mathcal{F}, q_1, q_2}(g; x, y) &= \bigoplus_{k=0}^n \bigoplus_{m=0}^r g \left(\frac{[k]_{q_1}}{[n]_{q_1}}, \frac{[m]_{q_2}}{[r]_{q_2}} \right) \\ &\quad \odot b_{k, m, n, r}^{q_1, q_2}(x, y) (n, r \in \mathbb{N}), \end{aligned} \quad (50)$$

for all $g \in C_{\mathcal{F}}(I^2)$ ($I^2 = [0, 1] \times [0, 1]$), $(x, y) \in [0, 1] \times [0, 1]$, and $0 < q_1, q_2 < 1$, where

$$b_{k, m, n, r}^{q_1, q_2}(x, y) = \begin{bmatrix} n \\ k \end{bmatrix}_{q_1} \begin{bmatrix} r \\ m \end{bmatrix}_{q_2} x^k (1-x)^{n-k}_{q_1} y^m (1-y)^{r-m}_{q_2}. \quad (51)$$

Note that

$$(1-x)_q^n = \prod_{k=0}^{n-1} (1 - q^k x). \quad (52)$$

The above operators $B_{nr}^{\mathcal{L}, q_1, q_2}(g; x, y)$ are positive and linear, and one writes

$$\begin{aligned} & \left\{ B_{nr}^{\mathcal{L}, q_1, q_2}(g; x, y) \right\}_{\alpha}^{\pm} \\ &= B_{nr}^{q_1, q_2}(g_{\alpha}^{\pm}; x, y) \\ &= \sum_{k=0}^n \sum_{m=0}^r b_{k, m, n, r}^{q_1, q_2}(x, y) g_{\alpha}^{\pm} \left(\frac{[k]_{q_1}}{[n]_{q_1}}, \frac{[m]_{q_2}}{[r]_{q_2}} \right), \end{aligned} \quad (53)$$

for all $0 \leq \alpha \leq 1$, $g_{\alpha}^{\pm} \in C[0, 1]$. Suppose that $q_1 = q_{1n}$ and $q_2 = q_{2r}$ such that $q_{1n} \rightarrow 1$ as $n \rightarrow \infty$ and $q_{2r} \rightarrow 1$ as $r \rightarrow \infty$. In view of this assumption, one can easily find that

$$\begin{aligned} [n]_{q_1} &\rightarrow \infty (n \rightarrow \infty), \\ [r]_{q_2} &\rightarrow \infty (r \rightarrow \infty). \end{aligned} \quad (54)$$

We define the sequence of positive linear operators by

$$\tilde{\Phi}_{nr}(g_{\alpha}^{\pm}; x, y) = (1 + s_{nr}) B_{nr}^{q_1, q_2}(g_{\alpha}^{\pm}; x, y). \quad (55)$$

In this case, (s_{nr}) is same as defined in Example 2 with $\gamma_1 = \gamma_2 = 1$, $\lambda_k = k$, $\mu_l = l$, $u_n = 1$, and $v_r = 1$ for all $n, r \in \mathbb{N}$. Therefore, we get

$$\begin{aligned} \tilde{\Phi}_{nr}(g_0; x, y) &= 1 + s_{nr}, \\ \tilde{\Phi}_{nr}(g_1; x, y) &= (1 + s_{nr})x, \\ \tilde{\Phi}_{nr}(g_2; x, y) &= (1 + s_{nr})y, \\ \tilde{\Phi}_{nr}(g_3; x, y) &= (1 + s_{nr}) \left(x^2 + y^2 + \frac{x - x^2}{[n]_{q_1}} + \frac{y - y^2}{[r]_{q_2}} \right). \end{aligned} \quad (56)$$

Since

$$S_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)} - \lim s_{nr} = 0, \quad (57)$$

we observe that

$$S_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)} - \lim \|\tilde{\Phi}_{nr}(g_i) - g_i\| = 0 \quad (i = 0, 1, 2, 3). \quad (58)$$

Consequently, by Theorem 6, we obtain

$$S_{2, \gamma_1, \gamma_2}^{\tilde{N}(\lambda, \mu)} - \lim d^*(\Phi_{nr}(g), g) = 0, \quad (59)$$

for all $g \in C_{\mathcal{L}}(I^2)$. Hence, all the conditions of Theorem 6 hold true for our operators $\tilde{\Phi}_{nr}$ but Theorem 2.2 obtained in [45] for convergence in Pringsheim's sense does not work for $\tilde{\Phi}_{nr}$ since the fuzzy double sequence (s_{nr}) is not convergent in Pringsheim's sense. Thus, we conclude that Theorem 6 is stronger than the one proved for convergence in Pringsheim's sense.

Data Availability

Not applicable.

Conflicts of Interest

The author declares there are no conflicts of interest.

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