

Research Article

A Complete Model of Crimean-Congo Hemorrhagic Fever (CCHF) Transmission Cycle with Nonlocal Fractional Derivative

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Crimean-Congo hemorrhagic fever is a common disease between humans and animals that is transmitted to humans through infected ticks, contact with infected animals, and infected humans. In this paper, we present a boxed model for the transmission of Crimean-Congo fever virus. With the help of the fixed-point theory, our proposed system model is investigated in detail to prove its unique solution. Given that the Caputo fractional-order derivative preserves the system's historical memory, we use this fractional derivative in our modeling. The equilibrium points of the proposed system and their stability conditions are determined. Using the Euler method for the Caputo fractional-order derivative, we calculate the approximate solutions of the fractional system, and then, we present a numerical simulation for the transmission of Crimean-Congo hemorrhagic fever.

1. Introduction

Crimean-Congo hemorrhagic fever is a common disease between human and livestock. The virus that causes this disease is one of the most important Arthropod-Borne viruses of the Bunyaviridae family, and it is a genus of Nairovirus that can cause severe and deadly disease in humans, but it is not associated with any specific clinical sign in livestock. The most common vector is a tick called Hyalomma, but it is also transmitted by other ticks [1]. The average mortality rate among infected people is 30 percent [2].

The first known case of the disease was recorded in 1942 in the Crimean region of the former Soviet Union. The virus that caused the disease was also isolated from the blood of a feverish patient in 1956 in the Democratic Republic of the

Congo. The relationship between these two reported places of disease and the attention to the main symptoms of the disease (fever and bleeding) has led to the choice of the current name of the disease (Crimean-Congo hemorrhagic fever) (see [3, 4]). The disease has been reported in more than 31 countries in Africa, Asia, and Eastern Europe [5].

Numerous serological studies have confirmed infections in animals, especially domestic animals such as cattle, sheep, and goats that may occur as feverish reactions. Infection in animals occurs through the bite of ticks infected with the Crimean-Congo hemorrhagic fever virus [6]. Crimean-Congo hemorrhagic fever virus can also infect a wide range of wild animals. Among wild mammals, rabbits have been an important reservoir of the virus in the European part of the former Soviet Union and Bulgaria. In Asia, hedgehogs,

rats, and particular species of rabbits are the reservoir of this virus [7].

The most important ways of getting infected with the Crimean-Congo fever virus are as follows: the person is getting bitten by infected ticks, the contact of scratched or injured skin of a person's body with the contents of infected crushed ticks, the contact of damaged skin or human mucosa with infected animal blood or secretions, and the contact with blood and other secretions of the infected person, as well as the contact with infected surgical instruments [8–10]. Because Crimean-Congo hemorrhagic fever is more likely to come from the contact with an infected animal or human or bite by infected ticks are transmitted to humans so hunters, farmers, ranchers, health personnel, and those contact with infected animals and humans due to occupations more likely to be infected.

Clinical signs and the course of this disease include four stages:

- (i) Incubation Period: After a tick bite, the incubation period usually lasts 1-3 days and reaches a maximum of 1 day. The incubation period following the contact with infected tissues or blood is usually 5-6 days, and the maximum time is 13 days [11]
- (ii) Prehaemorrhagic: In 80 percent of cases, Crimean-Congo hemorrhagic fever infections are asymptomatic. People in whom the disease has clinical manifestations, the onset of symptoms is sudden, and it lasts about 1 up to 7 days (average 3 days). The initial symptoms are severe headache, fever, chills, joint pain, muscle cramps, dizziness, pain and stiffness of the neck, eye pain, and fear of light. Nausea, vomiting, diarrhea, abdominal pain, loss of appetite, swelling and redness of the face, decreased heart rate, and low blood pressure have also been reported [12]
- (iii) Haemorrhagic: The bleeding phase is short and usually starts on days 3 to 5 and lasts 1 to 11 days (average 4 days). Bleeding in the mucosa, hematoma, bleeding gums and nose, bleeding from the uterus, bloody sputum, and bleeding from the conjunctiva and ears are the symptoms of the disease at this stage. Bleeding from various organs worsens the patient's condition so that the patient may die in the second week of severe bleeding, intravascular coagulation, liver failure, and dehydration [13]
- (iv) Convalescence Period: Between days 7 and 20, the fever stops, and then the bleeding stops. From the tenth day, when the skin lesions fade, patients gradually recover. Most patients are discharged from the hospital in the third to sixth week after the onset of illness when blood and urine tests return to be normal [11]

Biological and mathematical researchers have conducted research studies to model the transmission of Crimean-Congo fever. Kashkynbayev et al. have used an SI Model to study tick-borne diseases, including Crimean-Congo fever

[14]. Ergena et al. have used an SIR Model to study the dynamic of tuberculosis and Crimean-Congo fever as epidemic diseases [15]. Switkes et al. have used the deterministic system of nonlinear differential equations to model the transmission of Crimean-Congo haemorrhagic fever with host immunity [16].

In recent years, extensive studies [17–19] have been conducted on the mathematical analysis of fractional derivatives and integrals. The fractional-order derivative is nonlocal and includes the historical and long-term memory effect of the system, and this is one of its most important advantages over the integer-order derivative, which helps to model natural phenomena better [20–23].

By the expansion of fractional differential calculus, researchers in many branches of science have turned to use the fractional differential equation system in their research. Mathematical modeling of the spread of viruses and the transmission of infectious diseases using systems of fractional differential equations are considered as one of the topics that has attracted the attention of researchers in recent decades [24]. Almeida et al. [25] proposed an epidemiological MSEIR model formulated in the sense of Caputo fractional derivative. Baleanu et al. [26, 27] formulated new models of the HIV-1 infection of CD4+ T-cell and human liver via Caputo-Fabrizio fractional derivative. In addition, Rezapour et al. [28, 29] introduced new models for the spread of AH1N1 influenza and the transmission of Zika virus between humans and mosquitoes via Caputo-Fabrizio and Caputo fractional derivatives, respectively. Singh analyzed the fractional blood alcohol model with a composite fractional derivative [30], and Singh et al. investigated the fractional fish farm model and fractional model of guava for biological pest control, [31, 32]. Also, Ghanbari et al. presented an efficient numerical method for the fractional model of allelopathic stimulatory phytoplankton species [33].

In this article, we model the complete Crimean-Congo fever transmission cycle between humans, animals, and ticks, which in previous articles, researchers have only modeled a part of the cycle. Due to the effect of fractional derivative memory and good results obtained in recent years from fractional mathematical modeling, in this study, we use the fractional-order differential equation system to model the Crimean-Congo fever transmission.

The structure of this paper is organized as follows: In Section 2, some basic definitions and concepts of fractional calculus are recalled. A fractional-order mathematical model for the Crimean-Congo fever transmission cycle is formulated in Section 3. In Section 4, with the help of the fixed-point theory, our proposed system (10) is proven to have a unique solution. The approximate solution of the fractional differential equation system (10) is obtained numerically, and a numerical simulation for the transmission of the Crimean-Congo fever virus is also provided in Section 5. In Section 6, we conclude our research work.

2. Preliminary Results and Definitions

In the current section, we recall the two definitions of the fractional-order derivative and corresponding integral of

each one. A concept of the Laplace transform of fractional derivative is also discussed.

Definition 1 [34]. For an integrable function w , the Caputo derivative of fractional order $\vartheta \in (0, 1)$ is given by

$${}^c D^\vartheta w(t) = \frac{1}{\Gamma(m-\vartheta)} \int_0^t \frac{w^{(m)}(v)}{(t-v)^{\vartheta-m+1}} dv, \quad m[\vartheta] + 1. \quad (1)$$

The Gamma function, denoted by $\Gamma(\cdot)$, is defined as:

$$\Gamma(\vartheta) = \lim_{m \rightarrow \infty} \frac{m! m^\vartheta}{\vartheta(\vartheta+1)(\vartheta+2) \dots (\vartheta+m)}. \quad (2)$$

Also, the corresponding fractional integral of order ϑ with $\text{Re}(\vartheta) > 0$ is given by

$${}^c I^\vartheta w(t) = \frac{1}{\Gamma(\vartheta)} \int_0^t (t-v)^{\vartheta-1} w(v) dv. \quad (3)$$

Definition 2 ([35, 36]). For $w \in H^1(c, d)$ and $d > c$, the Caputo-Fabrizio derivative of fractional order $\vartheta \in (0, 1)$ for w is given by

$${}^{CF} D^\vartheta w(t) = \frac{M(\vartheta)}{(1-\vartheta)} \int_c^t \exp\left(\frac{-\vartheta}{1-\vartheta}(t-v)\right) w'(v) dv, \quad (4)$$

where $t \geq 0$, $M(\vartheta)$ is a normalization function that depends on ϑ and $M(0) = M(1) = 1$. If $w \notin H^1(c, d)$ and $0 < \vartheta < 1$, this derivative for $w \in L^1(-\infty, d)$ as given by

$${}^{CF} D^\vartheta w(t) = \frac{\vartheta M(\vartheta)}{(1-\vartheta)} \int_{-\infty}^d (w(t) - w(v)) \exp\left(\frac{-\vartheta}{1-\vartheta}(t-v)\right) dv. \quad (5)$$

Also, the corresponding CF fractional integral is presented by

$${}^{CF} I^\vartheta w(t) = \frac{2(1-\vartheta)}{(2-\vartheta)M(\vartheta)} w(t) + \frac{2\vartheta}{(2-\vartheta)M(\vartheta)} \int_0^t w(v) dv. \quad (6)$$

The Laplace transform is one of the most important tools in solving differential equations, which has different definitions in fractional calculus. The next definition presents the Laplace transform of the Caputo fractional-order derivative.

Definition 3 [34]. The Laplace transform of Caputo Fractional differential operator of order ϑ is given by

$$L\left[{}^c D^\vartheta w(t)\right](s) = s^\vartheta Lw(t) - \sum_{i=0}^{m-1} s^{\vartheta-i-1} w^{(i)}(0), \quad m-1 < \vartheta \leq m \in N. \quad (7)$$

This can also be obtained in the following form:

$$L\left[{}^c D^\vartheta w(t)\right] = \frac{s^m L[w(t)] - s^{m-1} w(0) - s^{m-1} w'(0) - \dots - w^{(m-1)}}{s^{m-\vartheta}}. \quad (8)$$

3. Model Formulation

Mathematical models are considered as one of the most important tools in the study of disease transmission. In this section, we present a fractional-order mathematical model for the Crimean-Congo fever transmission cycle.

Crimean-Congo haemorrhagic fever (CCHF) is a feverish hemorrhagic disease that is mostly transmitted by ticks. Although the virus is specific to animals, single infection, and epidemic cases of CCHF also occurred in humans. To model the transmission of this viral disease, we consider the population of transmitting ticks N_k , the population of livestock and wild animals N_j , and the human population N_h . We divide the tick population into two groups and denote susceptible ticks with S_k and infected ticks with I_k . In the previous section, we have mentioned that livestock and some wild animals can also be infected with this disease and be a virus reservoir, which we divide into two groups, susceptible group S_l and infected group I_l . Like the previous two populations, we divide the human population into two susceptible S_h and infected I_h groups. Susceptible ticks are infected through infected ticks at the effective contact rate β_1 and through infected animals at the effective contact rate β_2 . Infected ticks transmit the virus to susceptible animals at the effective contact rate β_3 when they feed on animal body. Crimean-Congo fever virus is transmitted to humans in three ways. The virus is transmitted to humans through infected ticks at the effective contact rate β_4 , through the blood and blood products of an infected animal at the effective contact rate β_5 , and through the blood and bloody mucosa of infected human at the effective contact rate β_6 . We also consider the recruitment rate of ticks, animals, and humans as Λ_k , Λ_j , and Λ_h , respectively. The natural mortality rates of ticks, animals, and humans are d_k , d_j , and d_h , respectively.

Based on the provided explanations, we present the Crimean-Congo fever transfer model with the system of differential equations as follows:

$$\begin{cases} \frac{dS_k}{dt} = \Lambda_k - \beta_1 S_k(t) I_k(t) - \beta_2 S_k(t) I_l(t) - d_k S_k(t), \\ \frac{dI_k}{dt} = \beta_1 S_k(t) I_k(t) + \beta_2 S_k(t) I_l(t) - d_k I_k(t), \\ \frac{dS_l}{dt} = \Lambda_l - \beta_3 S_l(t) I_l(t) - d_l S_l(t), \\ \frac{dI_l}{dt} = \beta_3 S_l(t) I_l(t) - d_l I_l(t), \\ \frac{dS_h}{dt} = \Lambda_h - \beta_4 S_h(t) I_k(t) - \beta_5 S_h(t) I_l(t) - \beta_6 S_h(t) I_h(t) - d_h S_h(t), \\ \frac{dI_h}{dt} = \beta_4 S_h(t) I_k(t) + \beta_5 S_h(t) I_l(t) + \beta_6 S_h(t) I_h(t) - d_h I_h(t), \end{cases} \quad (9)$$

where all of the initial conditions $S_k(0) = S_{0k}, I_k(0) = I_{0k}, S_l(0) = S_{0l}, I_l(0) = I_{0l}, S_h(0) = S_{0h}$, and $I_h(0) = I_{0h}$ are positive.

The fractional-order system (FDEs) is related to systems with memory, history, or nonlocal effects which exist in the many biological systems that show the realistic biphasic decline behavior of infection or diseases but at a slower rate. In the above integer-order system, since the internal memory effects of the biological system of CCHF are not included, it is better that we extend the proposed ordinary model to a fractional model. In this alternative, the equality of the dimensions of both sides of the equation is disturbed, and we use an auxiliary parameter σ , with the dimension of sec., to solve this problem ([37]). Thus, the fractional-order model for the Crimean-Congo haemorrhagic fever (CCHF) is given as follows:

$$\begin{cases} \sigma^{\vartheta-1} {}^C D_t^\vartheta S_k(t) = \Lambda_k - \beta_1 S_k(t) I_k(t) - \beta_2 S_k(t) I_l(t) - d_k S_k(t), \\ \sigma^{\vartheta-1} {}^C D_t^\vartheta I_k(t) = \beta_1 S_k(t) I_k(t) + \beta_2 S_k(t) I_l(t) - d_k I_k(t), \\ \sigma^{\vartheta-1} {}^C D_t^\vartheta S_l(t) = \Lambda_l - \beta_3 S_l(t) I_k(t) - d_l S_l(t), \\ \sigma^{\vartheta-1} {}^C D_t^\vartheta I_l(t) = \beta_3 S_l(t) I_k(t) - d_l I_l(t), \\ \sigma^{\vartheta-1} {}^C D_t^\vartheta S_h(t) = \Lambda_h - \beta_4 S_h(t) I_k(t) - \beta_5 S_h(t) I_l(t) - \beta_6 S_h(t) I_h(t) - d_h S_h(t), \\ \sigma^{\vartheta-1} {}^C D_t^\vartheta I_h(t) = \beta_4 S_h(t) I_k(t) + \beta_5 S_h(t) I_l(t) + \beta_6 S_h(t) I_h(t) - d_h I_h(t), \end{cases} \quad (10)$$

where $t \geq 0$ and $0 < \vartheta < 1$.

3.1. Nonnegative Solution. To show the nonnegativity of solutions, we claim that $M = \{(S_k, I_k, S_l, I_l, S_h, I_h) \in R_6^+ : N_k(t) \leq (\Lambda_k/d_k), N_l(t) \leq (\Lambda_l/d_l), N_h(t) \leq (\Lambda_h/d_h)\}$ is the feasibility region of system (10). To prove this claim, we consider the following Lemma.

Lemma 4. *The closed set M with respect to the fractional system (10) is positively invariant.*

Proof. We first add two relations in the system (10) to obtain the fractional derivative of the total population of ticks. So,

$$\sigma^{\vartheta-1} {}^C D_t^\vartheta N_k(t) = \Lambda_k - d_k N_k(t), \quad (11)$$

where $N_k(t) = S_k(t) + I_k(t)$. We apply the Laplace transform to the parties of the above relation, then

$$N_k(t) = N_k(0) E_\vartheta(-d_k \sigma^{1-\vartheta} t^\vartheta) + \int_0^t \Lambda_k \sigma^{1-\vartheta} \eta^{\vartheta-1} E_{\vartheta,\vartheta}(-d_k \sigma^{1-\vartheta} \eta^\vartheta) d\eta. \quad (12)$$

In the above equation, $N_k(0)$ is the initial size of ticks population, and the terms $E_\vartheta, E_{\vartheta,\vartheta}$ are the Mittag-Leffler functions which are defined by

$$E_\vartheta(w) = \sum_{n=0}^{\infty} \frac{w^n}{\Gamma(1+n\vartheta)}, E_{\vartheta,\vartheta}(w) = \sum_{n=0}^{\infty} \frac{w^n}{\Gamma(\vartheta+n\vartheta)}, \quad \vartheta > 0. \quad (13)$$

By simplifying the relations, we conclude that

$$\begin{aligned} N_k(t) &= N_k(0) E_\vartheta(-d_k \sigma^{1-\vartheta} t^\vartheta) + \int_0^t \Lambda_k \sigma^{1-\vartheta} \eta^{\vartheta-1} \\ &\quad \cdot \sum_{n=0}^{\infty} \frac{(-1)^n d_k^n \sigma^{n(1-\vartheta)} \eta^{n\vartheta}}{\Gamma(n\vartheta + \vartheta)} d\eta = \frac{\Lambda_k \sigma^{1-\vartheta}}{d_k \sigma^{1-\vartheta}} \\ &\quad + E_\vartheta(-d_k \sigma^{1-\vartheta} t^\vartheta) \left(N_k(0) - \frac{\Lambda_k \sigma^{1-\vartheta}}{d_k \sigma^{1-\vartheta}} \right), \\ &= \frac{\Lambda_k}{d_k} + E_\vartheta(-d_k \sigma^{1-\vartheta} t^\vartheta) \left(N_k(0) - \frac{\Lambda_k}{d_k} \right). \end{aligned} \quad (14)$$

Now, if $N_k(0) \leq (\Lambda_k/d_k)$, then for $t > 0, N_k(t) \leq (\Lambda_k/d_k)$. At the same way for N_l and N_h , we can prove that if $N_l(0) \leq (\Lambda_l/d_l)$ and $N_h(0) \leq (\Lambda_h/d_h)$, then $N_l(t) \leq (\Lambda_l/d_l)$ and $N_h(t) \leq (\Lambda_h/d_h)$. Thus, the closed set M with respect to fractional model (2) is positively invariant. \square

3.2. Equilibrium Points. In the current section, we determine the equilibrium points of the system (10) and the basic reproduction number. We present the necessary conditions for the stability of the system at the equilibrium point. To determine the equilibrium points, we set the equations to zero in system (10),

$$\begin{aligned} {}^C D^\vartheta S_k(t) &= {}^C D^\vartheta I_k(t) = {}^C D^\vartheta S_l(t) = {}^C D^\vartheta I_l(t) = {}^C D^\vartheta S_h(t) \\ &= {}^C D^\vartheta I_h(t) = 0, \end{aligned} \quad (15)$$

We solve the resulting algebraic equations and determine the equilibrium point of the system. The disease-free equilibrium point, denoted by E_0 , is obtained as: $E_0 = ((\Lambda_k/d_k), 0, (\Lambda_l/d_l), 0, (\Lambda_h/d_h), 0)$. The second equilibrium point, called the endemic equilibrium point, is obtained as $E^* = (S_k^*, I_k^*, S_l^*, I_l^*, S_h^*, I_h^*)$,

$$\begin{aligned} S_k^* &= \frac{\Lambda_k}{\beta_1 I_k^* + \beta_2 I_l^* + d_k}, S_l^* = \frac{\Lambda_l}{\beta_3 I_k^* + d_l}, S_h^* \\ &= \frac{\Lambda_h}{\beta_4 I_k^* + \beta_5 I_l^* + \beta_6 I_h^* + d_h}. \end{aligned} \quad (16)$$

When the basic reproduction number is greater than one, and the spread of the disease continues, the endemic equilibrium point is defined. To obtain the basic reproduction number, we use the next generation method [38]. We consider the matrix form of the system (10) as follows:

$${}^C D^\vartheta v(t) = F(v(t)) - V(v(t)), \quad (17)$$

where

$$F(v(t)) = \sigma^{1-\vartheta} \begin{bmatrix} \beta_1 S_k(t) I_k(t) + \beta_2 S_k(t) I_l(t) \\ \beta_3 S_l(t) I_k(t) \\ \beta_4 S_h(t) I_k(t) + \beta_5 S_h(t) I_l(t) + \beta_6 S_h(t) I_h(t) \end{bmatrix},$$

$$V(v(t)) = \sigma^{1-\vartheta} \begin{bmatrix} d_k I_k(t) \\ d_l I_l(t) \\ d_h I_h(t) \end{bmatrix}. \tag{18}$$

By calculating the Jacobian matrix for F and V at the disease-free equilibrium point, we obtain:

$$J_F(E_0) = \sigma^{1-\vartheta} \begin{bmatrix} \frac{\beta_1 \Lambda_k}{d_k} & \frac{\beta_2 \Lambda_k}{d_k} & 0 \\ \frac{\beta_3 \Lambda_l}{d_l} & 0 & 0 \\ \frac{\beta_4 \Lambda_h}{d_h} & \frac{\beta_5 \Lambda_h}{d_h} & \frac{\beta_6 \Lambda_h}{d_h} \end{bmatrix}, \tag{19}$$

$$J_V(E_0) = \sigma^{1-\vartheta} \begin{bmatrix} d_k & 0 & 0 \\ 0 & d_l & 0 \\ 0 & 0 & d_h \end{bmatrix}.$$

The basic reproduction number R_0 is defined as the eigenvalue of next generation matrix of system (10), $R_0 = \rho(FV^{-1})$. We obtain: $R_0 = \max(R_h, R_{kl})$,

$$R_h = \frac{\beta_6 \Lambda_h}{d_h^2}, R_{kl} = \frac{\beta_1 \Lambda_k d_l + \sqrt{\beta_1^2 \Lambda_k^2 d_l^2 + 4\beta_2 \beta_3 \Lambda_k \Lambda_l d_k^2}}{2d_l d_k^2}. \tag{20}$$

In commonly used infection models, when $R_0 > 1$, the infection will be able to start spreading in a population, but not if $R_0 < 1$.

3.3. Stability of Equilibrium Points. To determine the necessary conditions for the stability of the disease-free equilibrium point, we investigate the roots of the characteristic equation of system (10). The Jacobian matrix of the system (10) is

$$J = \sigma^{1-\vartheta} \times \begin{bmatrix} -\beta_1 I_k - \beta_2 I_l - d_k & -\beta_1 S_k & 0 & -\beta_2 S_k & 0 & 0 \\ \beta_1 I_k + \beta_2 I_l & \beta_1 S_k - d_k & 0 & \beta_2 S_k & 0 & 0 \\ 0 & -\beta_3 S_l & -\beta_3 I_k - d_l & 0 & 0 & 0 \\ 0 & \beta_3 S_l & \beta_3 I_k & -d_l & 0 & 0 \\ 0 & -\beta_4 S_h & 0 & -\beta_5 S_h & -\beta_4 I_k - \beta_5 I_l - \beta_6 I_h - d_h & -\beta_6 S_h \\ 0 & \beta_4 S_h & 0 & \beta_5 S_h & \beta_4 I_k + \beta_5 I_l + \beta_6 I_h & \beta_6 S_h - d_h \end{bmatrix}. \tag{21}$$

Then, the Jacobian matrix at E_0 is obtained as:

$$J(E_0) = \sigma^{1-\vartheta} \begin{bmatrix} -d_k & -\beta_1 \frac{\Lambda_k}{d_k} & 0 & -\beta_2 \frac{\Lambda_k}{d_k} & 0 & 0 \\ 0 & \beta_1 \frac{\Lambda_k}{d_k} - d_k & 0 & \beta_2 \frac{\Lambda_k}{d_k} & 0 & 0 \\ 0 & -\beta_3 \frac{\Lambda_l}{d_l} & -d_l & 0 & 0 & 0 \\ 0 & \beta_3 \frac{\Lambda_l}{d_l} & 0 & -d_l & 0 & 0 \\ 0 & -\beta_4 \frac{\Lambda_h}{d_h} & 0 & -\beta_5 \frac{\Lambda_h}{d_h} & -d_h & -\beta_6 \frac{\Lambda_h}{d_h} \\ 0 & \beta_4 \frac{\Lambda_h}{d_h} & 0 & \beta_5 \frac{\Lambda_h}{d_h} & 0 & \beta_6 \frac{\Lambda_h}{d_h} - d_h \end{bmatrix}. \tag{22}$$

In the following theorem, we determine the necessary conditions for the stability of the disease-free equilibrium point.

Theorem 5. *The disease-free equilibrium point E^0 is locally asymptotically stable if $R_0 < 1$.*

Proof. The characteristic equation of matrix $J(E_0)$ is obtained as follows:

$$(d_l + \lambda)(d_k + \lambda)(d_h + \lambda) \left(\beta_6 \frac{\Lambda_h}{d_h} - d_h - \lambda \right) \cdot \left[(d_l + \lambda) \left(\beta_1 \frac{\Lambda_k}{d_k} - d_k - \lambda \right) - \beta_2 \beta_3 \frac{\Lambda_k \Lambda_l}{d_k d_l} \right] = 0. \tag{23}$$

Therefore, the eigenvalues of the Jacobin matrix are $\lambda_1 = -d_l$, $\lambda_2 = -d_k$, $\lambda_3 = -d_h$, $\lambda_4 = (\beta_6 \Lambda_h / d_h) - d_h$, and the roots of the following equation are:

$$\lambda^2 - \lambda \left(\frac{\beta_1 \Lambda_k}{d_k} - d_k - d_l \right) - \frac{\beta_1 \Lambda_k d_l}{d_k} + d_l d_k + \beta_2 \beta_3 \frac{\Lambda_k \Lambda_l}{d_k d_l} = 0. \tag{24}$$

The three roots λ_1, λ_2 , and λ_3 are negative. If $R_0 < 1$, then $R_h = (\beta_6 \Lambda_h / d_h^2) < 1$, we obtain $\lambda_4 < 0$. It also follows from $R_0 < 1$ that $R_{kl} < 1$, then we conclude by simplifying $\beta_1 \Lambda_k < d_k^2$. In Equation (24), which is a quadratic equation, we have:

$$P = \frac{-\beta_1 \Lambda_k d_l}{d_k} + d_l d_k + \beta_2 \beta_3 \frac{\Lambda_k \Lambda_l}{d_k d_l}, S = \frac{\beta_1 \Lambda_k}{d_k} - d_k - d_l, \tag{25}$$

since $\beta_1 \Lambda_k < d_k^2$ then $P > 0, S < 0$ so Equation (24) has 2 negative roots. Therefore, all of the eigenvalues are negative, and the disease-free equilibrium point is locally asymptotically stable. \square

4. Existence of Unique Solution

In the current section, using the fixed-point theory, we prove that system (10) has a unique solution. Fixed-point theory is essential in proving the existence of a solution to the proposed system where adequate conditions are provided by fixed-point theorems such that a unique fixed point exists for a given function. To achieve this goal, we prove that kernels are satisfied under the Lipschitz condition, and they are contraction. Then, the existence of solution to the proposed system is constructed via fixed-point theorem. From the Lipschitz condition, the uniqueness of our obtained solution is proven when the obtained condition is satisfied.

First, we consider system (10) in the following compact form:

$$\begin{cases} \sigma^{\vartheta-1} D_t^\vartheta S_k(t) = R_1(t, S_k(t)), \\ \sigma^{\vartheta-1} D_t^\vartheta I_k(t) = R_2(t, I_k(t)), \\ \sigma^{\vartheta-1} D_t^\vartheta S_l(t) = R_3(t, S_l(t)), \\ \sigma^{\vartheta-1} D_t^\vartheta I_l(t) = R_4(t, I_l(t)), \\ \sigma^{\vartheta-1} D_t^\vartheta S_h(t) = R_5(t, S_h(t)), \\ \sigma^{\vartheta-1} D_t^\vartheta I_h(t) = R_6(t, I_h(t)). \end{cases} \quad (26)$$

We apply the fractional-order integral to the parties of the above equations, so

$$\begin{cases} S_k(t) - S_k(0) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t R_1(\mu, S_k)(t-\mu)^{\vartheta-1} d\mu, \\ I_k(t) - I_k(0) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t R_2(\mu, I_k)(t-\mu)^{\vartheta-1} d\mu, \\ S_l(t) - S_l(0) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t R_3(\mu, S_l)(t-\mu)^{\vartheta-1} d\mu, \\ I_l(t) - I_l(0) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t R_4(\mu, I_l)(t-\mu)^{\vartheta-1} d\mu, \\ S_h(t) - S_h(0) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t R_5(\mu, S_h)(t-\mu)^{\vartheta-1} d\mu, \\ I_h(t) - I_h(0) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t R_6(\mu, I_h)(t-\mu)^{\vartheta-1} d\mu. \end{cases} \quad (27)$$

In the following, we prove that the kernels R_j , $j = 1, 2, 3, 4, 5, 6$ are satisfied in the Lipschitz condition, and they are contraction.

Theorem 6. *Kernel R_1 is satisfied in Lipschitz condition and contraction if we have:*

$$0 \leq \beta_1 z_1 + \beta_2 z_2 + d_k < 1. \quad (28)$$

Proof. We can write for S_k and S_{1k} ,

$$\begin{aligned} & \|R_1(t, S_k) - R_1(t, S_{1k})\| \\ &= \|-\beta_1 I_k(S_k - S_{1k}) - \beta_2 I_l(S_k - S_{1k}) - d_k(S_k - S_{1k})\|, \\ &\leq \beta_1 \|I_k\| \|S_k - S_{1k}\| + \beta_2 \|I_l\| \|S_k - S_{1k}\| + d_k \|S_k - S_{1k}\|, \\ &\leq (\beta_1 \|I_k\| + \beta_2 \|I_l\| + d_k) \|S_k - S_{1k}\|, \\ &\leq (\beta_1 z_1 + \beta_2 z_2 + d_k) \|S_k - S_{1k}\|. \end{aligned} \quad (29)$$

Consider $e_1 = \beta_1 z_1 + \beta_2 z_2 + d_k$, where $\|I_k(t)\| \leq z_1$ and $\|I_l\| \leq z_2$, are bounded functions. We get:

$$\|R_1(t, S_k) - R_1(t, S_{1k})\| \leq e_1 \|S_k(t) - S_{1k}(t)\|, \quad (30)$$

if $0 \leq \beta_1 z_1 + \beta_2 z_2 + d_k < 1$, then the kernel R_1 is satisfied in Lipschitz condition, and it is contraction. \square

In a similar way, we can show that the kernels R_j , $j = 2, 3, 4, 5, 6$ are satisfied in the Lipschitz condition as follows:

$$\begin{cases} \|R_2(t, I_k) - R_2(t, I_{1k})\| \leq e_2 \|I_k(t) - I_{1k}(t)\|, \\ \|R_3(t, S_l) - R_3(t, S_{1l})\| \leq e_3 \|S_l(t) - S_{1l}(t)\|, \\ \|R_4(t, I_l) - R_4(t, I_{1l})\| \leq e_4 \|I_l(t) - I_{1l}(t)\|, \\ \|R_5(t, S_h) - R_5(t, S_{1h})\| \leq e_5 \|S_h(t) - S_{1h}(t)\|, \\ \|R_6(t, I_h) - R_6(t, I_{1h})\| \leq e_6 \|I_h(t) - I_{1h}(t)\|, \end{cases} \quad (31)$$

so that $e_2 = \beta_1 z_4 + d_k$, $e_3 = \beta_3 z_1 + d_l$, $e_4 = d_l$, $e_5 = \beta_4 z_1 + \beta_5 z_2 + \beta_6 z_3 + d_h$, $e_6 = \beta_6 z_6 + d_h$ are bounded functions where $\|I_h(t)\| \leq z_3$, $\|S_k(t)\| \leq z_4$, $\|S_l(t)\| \leq z_5$, and $\|S_h(t)\| \leq z_6$. Also, if $0 \leq e_j < 1$, $j = 2, 3, 4, 5, 6$, then R_j are contraction for $j = 2, 3, 4, 5, 6$.

Based on system (27), we define:

$$A_{1n}(t) = S_{nk}(t) - S_{(n-1)k}(t) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \left(R_1(\mu, S_{(n-1)k}) - R_1(\mu, S_{(n-2)k}) \right) (t-\mu)^{\vartheta-1} d\mu,$$

$$A_{2n}(t) = I_{nk}(t) - I_{(n-1)k}(t) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \left(R_2(\mu, I_{(n-1)k}) - R_2(\mu, I_{(n-2)k}) \right) (t-\mu)^{\vartheta-1} d\mu,$$

$$A_{3n}(t) = S_{nl}(t) - S_{(n-1)l}(t) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \left(R_3(\mu, S_{(n-1)l}) - R_3(\mu, S_{(n-2)l}) \right) (t-\mu)^{\vartheta-1} d\mu,$$

$$A_{4n}(t) = I_{nl}(t) - I_{(n-1)l}(t) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \left(R_4(\mu, I_{(n-1)l}) - R_4(\mu, I_{(n-2)l}) \right) (t-\mu)^{\vartheta-1} d\mu,$$

$$A_{5n}(t) = S_{nh}(t) - S_{(n-1)h}(t) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \left(R_5(\mu, S_{(n-1)h}) - R_5(\mu, S_{(n-2)h}) \right) (t - \mu)^{\vartheta-1} d\mu, \quad \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} t_\varepsilon e_i < 1. \quad (37)$$

$$A_{6n}(t) = I_{nh}(t) - I_{(n-1)h}(t) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \left(R_6(\mu, I_{(n-1)h}) - R_6(\mu, I_{(n-2)h}) \right) (t - \mu)^{\vartheta-1} d\mu, \quad (32)$$

where $S_k(0) = S_{0k}, I_k(0) = I_{0k}, S_l(0) = S_{0l}, I_l(0) = I_{0l}, S_h(0) = S_{0h}$, and $I_h(0) = I_{0h}$ are initial conditions. The norm of A_{1n} in the above system is expressed as follows:

$$\|A_{1n}(t)\| = \|S_{nk}(t) - S_{(n-1)k}(t)\| \leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \|R_1(\mu, S_{(n-1)k}) - R_1(\mu, S_{(n-2)k})\| (t - \mu)^{\vartheta-1} d\mu. \quad (33)$$

According to the Lipschitz condition (30), we conclude

$$\|A_{1n}(t)\| \leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_1 \int_0^t \|A_{1(n-1)}(\mu)\| d\mu. \quad (34)$$

Similarly, we can prove that

$$\begin{aligned} \|A_{2n}(t)\| &\leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_2 \int_0^t \|A_{2(n-1)}(\mu)\| d\mu, \\ \|A_{3n}(t)\| &\leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_3 \int_0^t \|A_{3(n-1)}(\mu)\| d\mu, \\ \|A_{4n}(t)\| &\leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_4 \int_0^t \|A_{4(n-1)}(\mu)\| d\mu, \\ \|A_{5n}(t)\| &\leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_5 \int_0^t \|A_{5(n-1)}(\mu)\| d\mu, \\ \|A_{6n}(t)\| &\leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_6 \int_0^t \|A_{6(n-1)}(\mu)\| d\mu. \end{aligned} \quad (35)$$

Therefore, we get

$$\begin{aligned} S_{nk}(t) &= \sum_{i=1}^n A_{1i}(t), I_{nk}(t) = \sum_{i=1}^n A_{2i}(t), S_{nl}(t) = \sum_{i=1}^n A_{3i}(t), \\ I_{nl}(t) &= \sum_{i=1}^n A_{4i}(t), S_{nh}(t) = \sum_{i=1}^n A_{5i}(t), I_{nh}(t) = \sum_{i=1}^n A_{6i}(t). \end{aligned} \quad (36)$$

In the next theorem, we prove the existence of solution by the fixed-point theorem.

Theorem 7. *The Crimean-Congo fever transmission fractional-order model (10) has a solution, if there exists t_ε such that*

Proof. By Equation (34) and Equation (46), we obtain

$$\begin{aligned} \|A_{1n}(t)\| &\leq \|S_{nk}(0)\| \left[\frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_1 t \right]^n, \|A_{2n}(t)\| \leq \|I_{nk}(0)\| \left[\frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_2 t \right]^n, \\ \|A_{3n}(t)\| &\leq \|S_{nl}(0)\| \left[\frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_3 t \right]^n, \|A_{4n}(t)\| \leq \|I_{nl}(0)\| \left[\frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_4 t \right]^n, \\ \|A_{5n}(t)\| &\leq \|S_{nh}(0)\| \left[\frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_5 t \right]^n, \|A_{6n}(t)\| \leq \|I_{nh}(0)\| \left[\frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_6 t \right]^n. \end{aligned} \quad (38)$$

The above relations show that the system has a continuous solution. Now, it is sufficient to show that the above functions construct the solution for the fractional-order model (10). We consider the following relations:

$$\begin{aligned} S_k(t) - S_k(0) &= S_{nk}(t) - U_{1n}(t), I_k(t) - I_k(0) = I_{nk}(t) - U_{2n}(t), \\ S_l(t) - S_l(0) &= S_{nl}(t) - U_{3n}(t), I_l(t) - I_l(0) = I_{nl}(t) - U_{4n}(t), \\ S_h(t) - S_h(0) &= S_{nh}(t) - U_{5n}(t), I_h(t) - I_h(0) = I_{nh}(t) - U_{6n}(t). \end{aligned} \quad (39)$$

The norm of $U_{1n}(t)$ is obtained as follows:

$$\begin{aligned} \|U_{1n}(t)\| &= \left\| \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \left(R_1(\mu, S_k) - R_1(\mu, S_{(n-1)k}) \right) d\mu \right\| \\ &\leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \|R_1(\mu, S_k) - R_1(\mu, S_{(n-1)k})\| d\mu \\ &\leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_1 \|S_k - S_{(n-1)k}\| t. \end{aligned} \quad (40)$$

By continuing this repetitive method, we conclude:

$$\|U_{1n}(t)\| \leq \left[\frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} t \right]^{n+1} e_1^{n+1} k. \quad (41)$$

At t_ε , we have

$$\|U_{1n}(t)\| \leq \left[\frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} t_\varepsilon \right]^{n+1} e_1^{n+1} k. \quad (42)$$

If we take limit on the recent relation as n approaches to ∞ , it results $\|U_{1n}(t)\| \rightarrow 0$. Similarly, we conclude that $\|B_{jn}(t)\| \rightarrow 0, j = 2, 3, 4, 5, 6$, and the proof is complete. \square

To show the uniqueness of the solution of CCHF model, we consider that the fractional-order system (10) has another solution such as $(S_{1k}(t), I_{1k}(t), S_{1l}(t), I_{1l}(t), S_{1h}(t), I_{1h}(t))$, then for S_k, S_{1k} can be written as:

$$S_k(t) - S_{1k}(t) = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t (R_1(\mu, S_k) - R_1(\mu, S_{1k})) d\mu. \quad (43)$$

We take the norm on the above equation, so

$$\|S_k(t) - S_{1k}(t)\| = \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} \int_0^t \|R_1(\mu, S_k) - R_1(\mu, S_{1k})\| d\mu. \quad (44)$$

By Lipschitz condition (30), we obtain:

$$\|S_k(t) - S_{1k}(t)\| \leq \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_1 t \|S_k(t) - S_{1k}(t)\|. \quad (45)$$

Thus,

$$\|S_k(t) - S_{1k}(t)\| \left(1 - \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_1 t\right) \leq 0. \quad (46)$$

Theorem 8. *The solution of fractional-order system (10) is unique when the following condition is met:*

$$1 - \frac{\sigma^{1-\vartheta}}{\Gamma(\vartheta)} e_1 t > 0. \quad (47)$$

Proof. Assume that the condition (46) holds, in which case we conclude from (46) and (47) that $\|S_k(t) - S_{1k}(t)\| = 0$, and this shows that $S_h(t) = S_{1h}(t)$. In the same way, similar relationships can be reached for I_k, S_l, I_l, S_h, I_h . This completes the proof. \square

5. Numerical Simulation and Discussion

In this section, we first obtain the approximate solution of the fractional differential equation system (10) by a numerical method, and then, we present a numerical simulation for the transmission of the Crimean-Congo fever virus.

5.1. Numerical Method. We use the fractional Euler method for Caputo derivative [39] to obtain the approximate solutions of the Crimean-Congo hemorrhagic fever virus transmission model. First, we consider the compact form of the system (10) as follows:

$$\sigma^{\vartheta-1} {}^C D_t^\vartheta \varphi(t) = Q(t, \varphi(t)), \varphi(0) = \varphi_0, 0 \leq t \leq T < \infty, \quad (48)$$

where $\varphi = (S_k, I_k, S_l, I_l, S_h, I_h) \in \mathbb{R}_+^6$, $\varphi_0 = (S_{0k}, I_{0k}, S_{0l}, I_{0l}, S_{0h}, I_{0h})$, and $Q(t)$ is a continuous real vector function that is satisfied in the Lipschitz condition as follows:

$$\|Q(\varphi_1(t)) - Q(\varphi_2(t))\| \leq m \|\varphi_1(t) - \varphi_2(t)\|, m > 0. \quad (49)$$

We apply the fractional-order integral operator corresponding to the Caputo fractional-order derivative on both sides of Equation (48), so

$$\varphi(t) = \sigma^{1-\vartheta} \left[\varphi_0 + I^\vartheta Q(\varphi(t)) \right], 0 \leq t \leq T < \infty. \quad (50)$$

Set $r = (T - 0)/N$ and $t_n = nr$, where $t \in [0, T]$ and N is a natural number and $n = 0, 1, 2, \dots, N$. Let φ_n be the approximation of $\varphi(t)$ at $t = t_n$. By the fractional Euler method ([39]), we get:

$$\begin{aligned} \varphi_{n+1} &= \sigma^{1-\vartheta} \left[\varphi_0 + \frac{r^\vartheta}{\Gamma(\vartheta+1)} \sum_{p=0}^n \omega_{n+1,p} Q(t_p, \varphi_p) \right], p \\ &= 0, 1, 2, \dots, N-1, \end{aligned} \quad (51)$$

where

$$\omega_{n+1,p} = (n+1-p)^\vartheta - (n-p)^\vartheta, p = 0, 1, 2, \dots, n. \quad (52)$$

The obtained scheme is stable. Details of the stability analysis are given in Theorem (3.1) of [39]. According to the explanations provided, the answer of the system is obtained as follows:

$$\begin{aligned} S_{(n+1)k} &= \sigma^{1-\vartheta} \left[S_{0k} + \frac{r^\vartheta}{\Gamma(\vartheta+1)} \sum_{p=0}^n \omega_{n+1,p} \gamma_1(t_p, \varphi_p) \right], \\ I_{(n+1)k} &= \sigma^{1-\vartheta} \left[I_{0k} + \frac{r^\vartheta}{\Gamma(\vartheta+1)} \sum_{p=0}^n \omega_{n+1,p} \gamma_2(t_p, \varphi_p) \right], \\ S_{(n+1)l} &= \sigma^{1-\vartheta} \left[S_{0l} + \frac{r^\vartheta}{\Gamma(\vartheta+1)} \sum_{p=0}^n \omega_{n+1,p} \gamma_3(t_p, \varphi_p) \right], \\ I_{(n+1)l} &= \sigma^{1-\vartheta} \left[I_{0l} + \frac{r^\vartheta}{\Gamma(\vartheta+1)} \sum_{p=0}^n \omega_{n+1,p} \gamma_4(t_p, \varphi_p) \right], \\ S_{(n+1)h} &= \sigma^{1-\vartheta} \left[S_{0h} + \frac{r^\vartheta}{\Gamma(\vartheta+1)} \sum_{p=0}^n \omega_{n+1,p} \gamma_5(t_p, \varphi_p) \right], \\ I_{(n+1)h} &= \sigma^{1-\vartheta} \left[I_{0h} + \frac{r^\vartheta}{\Gamma(\vartheta+1)} \sum_{p=0}^n \omega_{n+1,p} \gamma_6(t_p, \varphi_p) \right], \end{aligned} \quad (53)$$

so that $\omega_{n+1,p} = (n+1-p)^\vartheta - (n-p)^\vartheta$ and the functions γ_j for $j = 0, 1, \dots, 6$ are expressed as:

$$\begin{aligned} \gamma_1(t, \varphi(t)) &= \Lambda_k - \beta_1 S_k(t) I_k(t) - \beta_2 S_k(t) I_l(t) - d_k S_k(t), \\ \gamma_2(t, \varphi(t)) &= \beta_1 S_k(t) I_k(t) + \beta_2 S_k(t) I_l(t) - d_k I_k(t), \\ \gamma_3(t, \varphi(t)) &= \Lambda_l - \beta_3 S_l(t) I_k(t) - d_l S_l(t), \\ \gamma_4(t, \varphi(t)) &= \beta_3 S_l(t) I_k(t) - d_l I_l(t), \\ \gamma_5(t, \varphi(t)) &= \Lambda_h - \beta_4 S_h(t) I_k(t) - \beta_5 S_h(t) I_h(t) \\ &\quad - \beta_6 S_h(t) I_h(t) - d_h S_h(t), \\ \gamma_6(t, \varphi(t)) &= \beta_4 S_h(t) I_k(t) + \beta_5 S_h(t) I_l(t) \\ &\quad + \beta_6 S_h(t) I_h(t) - d_h I_h(t). \end{aligned} \quad (54)$$

5.2. Simulation. In the present subsection, we present a numerical simulation to investigate the transmission of

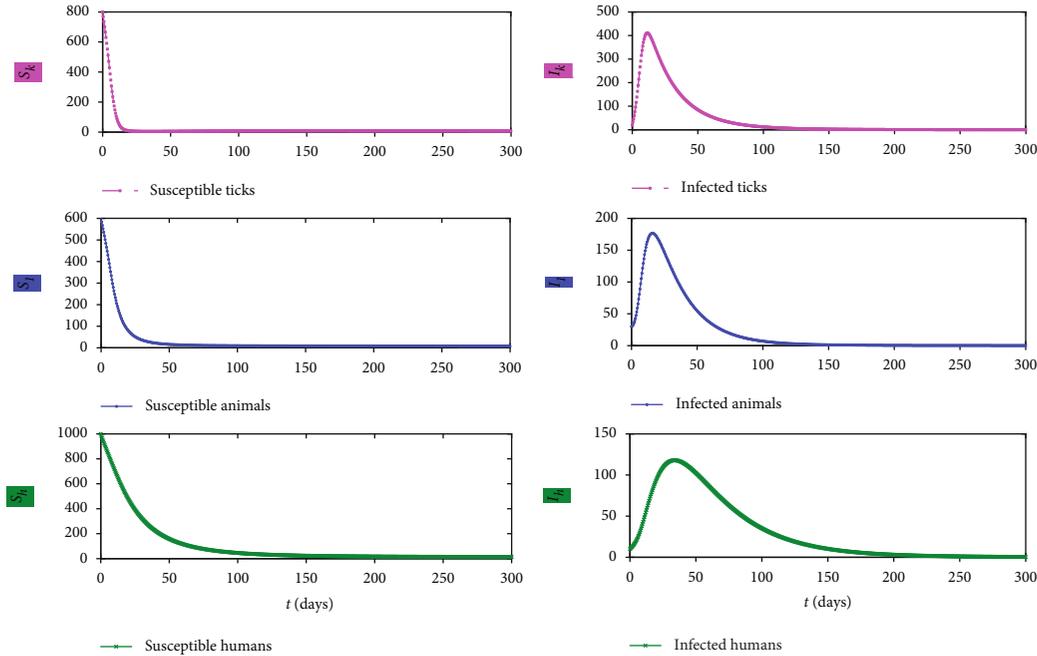


FIGURE 1: Plots of the results of model (10) with $R_0 < 1$ for $\vartheta = 0.98$.

Crimean-Congo fever virus based on the amount of reproduction number. Also, we compare the results of the integer-order and fractional-order models.

To perform the desired simulation, in two cases, we consider different values for the parameters. In the first case, we assume: $\beta_1 = 0.5 \times 10^{-4}$, $\beta_2 = 0.3 \times 10^{-4}$, $\beta_3 = 0.1 \times 10^{-3}$, $\beta_4 = 0.03 \times 10^{-4}$, $\beta_5 = 0.4 \times 10^{-4}$, $\beta_6 = 0.7 \times 10^{-4}$, $\Lambda_k = 0.6$, $\Lambda_l = 0.3$, $\Lambda_h = 0.6$, $d_k = 0.09$, $d_l = 0.07$, $d_h = 0.007$, $\sigma = 0.99$. We also consider the initial values as $S_k = 800$, $I_k = 20$, $S_l = 600$, $I_l = 30$, $S_h = 1000$, $I_h = 10$.

Using the above parameters, we obtain $R_h = 0.0233$, $R_{kl} = 0.00597$; thus, $R_0 = 0.0233 < 1$. Figure 1 shows the results of model (10) for all six groups for $\vartheta = 0.98$. In this case, $R_0 < 1$, and Figure 1 shows that over time, the number of susceptible people is decreased, and the number of infected people is increased, but in less than 20 days, the number of infected people is decreased and eventually reaches zero, and the spread of the disease stops. In this case, $S(t)$ and $I(t)$ converge to the disease-free equilibrium point E_0 .

In the second case, we assume that the disease transmission rate increases from the susceptible group to the infected group, and the transmission rates are equal to $\beta_1 = 0.5 \times 10^{-3}$, $\beta_2 = 0.3 \times 10^{-3}$, $\beta_3 = 0.1 \times 10^{-2}$, $\beta_4 = 0.7 \times 10^{-3}$, $\beta_5 = 0.4 \times 10^{-3}$, $\beta_6 = 0.7 \times 10^{-3}$. With these transfer rates, the value of the reproduction number is equal to $R_0 = 1.435 > 1$. Figure 2 shows the results of model (10) for the six groups studied in this case. Over time, the population of susceptible groups decreases and the population of infected groups increases, and finally, after 100 days, the population of infected groups decreases and converges to the endemic equilibrium point. As the rate of disease transmission increases, the value of R_0 increases, and we observe that the disease does not go away and its spread continues.

In this work, we have used the fractional-order derivative for modeling. In order to investigate the effect of derivation order, we have drawn the model results for infected groups with derivatives with integer-order $\vartheta = 1$ and fractional-order $\vartheta = 0.98$ in Figure 3. Figure 3 shows that the results of model (10) are similar for the integer-order and the Caputo fractional order, and a small change in the order of derivation has no effect on the overall trend of the results in terms of ascending and descending, but the resulting numerical values are different.

5.3. The Reproduction Number Sensitivity Analysis. We investigate the effect of parameters in Crimean-Congo hemorrhagic fever fractional model (10) on reproduction number using the method introduced by [40]. For this simulation, we use the parameters in the first case of the previous subsection. Since R_0 is defined as $R_0 = \max(R_h, R_{kl})$, therefore, we analyze the sensitivity of R_0 in two cases.

First, if $R_0 = R_h$, by the mentioned method, we have $S_{\beta_6} = (\partial R_0 / \partial \beta_6)(\beta_6 / R_0) = 1 > 0$, $S_{\Lambda_h} = (\partial R_0 / \partial \Lambda_h)(\Lambda_h / R_0) = 1 > 0$, $S_{d_h} = (\partial R_0 / \partial d_h)(d_h / R_0) = -2\beta_6 \Lambda_h / d_h^3 = -1.55 < 0$. Figure 4 shows the sensitivity of R_0 with respect to each of the parameters. As you can see, changing each of the parameters of model (10) that is involved in R_h changes the reproduction number. The reproduction number is directly related to parameters β_6 , Λ_h and inversely related to parameter d_h . From an epidemiological point of view, whenever the reproduction number decreases, then the spread of the disease is controlled. Given that β_6 has the most positive effect on the R_0 , so to control the spread of the disease, β_6 should be reduced through the reduced communication of infected and susceptible humans.

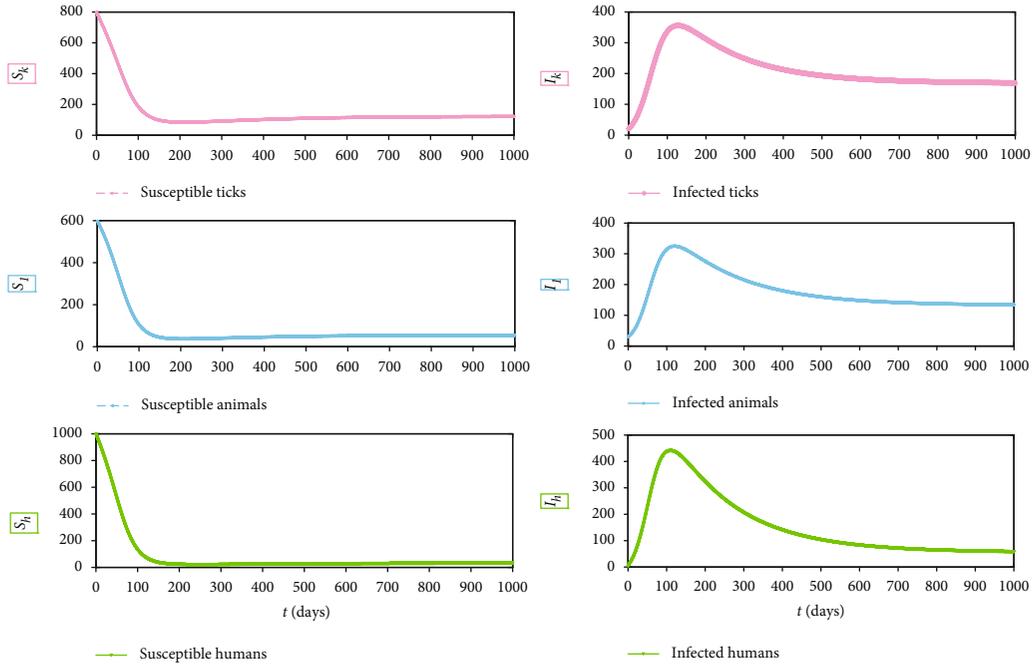


FIGURE 2: Plots of the results of model (10) with $R_0 > 1$ for $\vartheta = 0.98$.

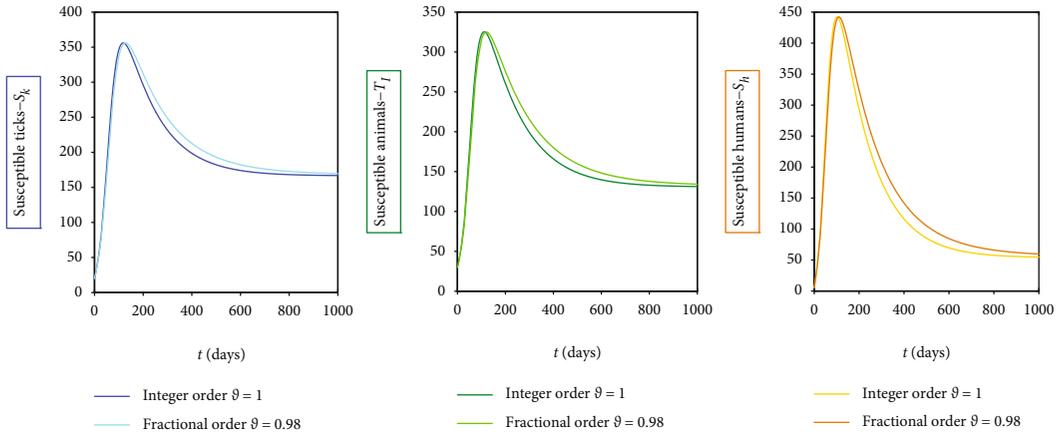


FIGURE 3: Plots of the results of model (10) for infected groups with integer-order $\vartheta = 1$ and fractional-order $\vartheta = 0.98$ in the case $R_0 > 1$.

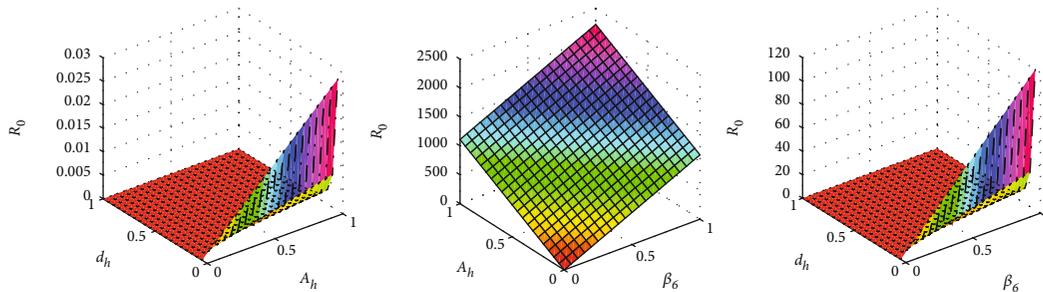


FIGURE 4: The graphs show the effect of model parameters on the reproduction number for the case $R_0 = R_h$.

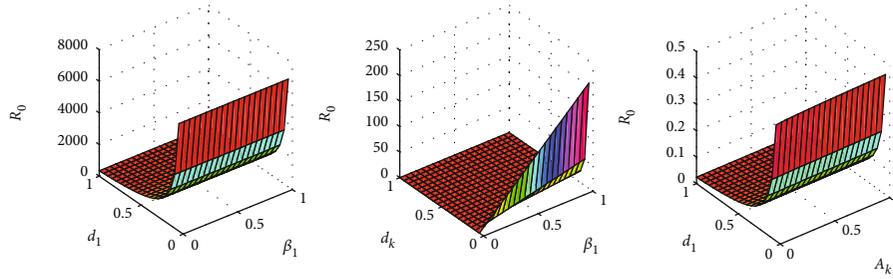


FIGURE 5: The graphs show the effect of model parameters on the reproduction number for the case $R_0 = R_{kl}$.

In the latter case if $R_0 = R_{kl}$, we obtain the same equations as above

$$\begin{aligned}
 S_{\Lambda_k} &= \frac{\partial R_0}{\partial \Lambda_k} \frac{\Lambda_k}{R_0} = 0.724 > 0, S_{\beta_1} = \frac{\partial R_0}{\partial \beta_1} \frac{\beta_1}{R_0} = 0.4486 > 0, \\
 S_{d_1} &= \frac{\partial R_0}{\partial d_1} \frac{d_1}{R_0} = -0.5513 < 0, \\
 S_{\beta_2} &= \frac{\partial R_0}{\partial \beta_2} \frac{\beta_2}{R_0} = 0.275 > 0, S_{\beta_3} = \frac{\partial R_0}{\partial \beta_3} \frac{\beta_3}{R_0} = 0.276 > 0, \\
 S_{d_k} &= \frac{\partial R_0}{\partial d_k} \frac{d_k}{R_0} = -1.448 < 0.
 \end{aligned} \tag{55}$$

Figure 5 shows the sensitivity of R_0 with respect to each of the parameters. The reproduction number is directly related to parameters $\beta_1, \beta_2, \beta_3$, and Λ_k and inversely related to parameter d_1, d_k . Among the mentioned parameters, parameters β_1, β_2 , and β_3 can be controlled, and all of which have a positive effect on causality, so to reduce the amount of reproduction number, it is enough to reduce the rate of disease transmission between ticks, animals, and humans.

6. Conclusion

In this work, we have presented a box model using the Caputo fractional-order derivative by taking into account the transmission of the Crimean-Congo hemorrhagic fever virus between ticks, animals (domestic and wild), and humans. We have calculated the feasible region and the equilibrium points of the system (10), and we have determined the necessary conditions for the stability of the equilibrium point. In the last section, using the Euler method for the Caputo fractional derivative, we have obtained the approximate solution of system (10), and then, we have provided a numerical simulation for the transmission of Crimean-Congo hemorrhagic fever virus. In two cases: $R_0 < 1$ and $R_0 > 1$, the results of the model have been plotted for the six groups in the model, which clearly show that in the case $R_0 < 1$, the transmission of the disease stops after a while, and the results of the system converge to the disease-free equilibrium point. We have increased the rate of disease transmission among the groups, and in this case, the results for $R_0 > 1$ show that the disease continues endemically, and also, the results converge to the endemic equilibrium point. The results of the model are compared with two types of deriva-

tives of integer-order and fractional-order, and the result of comparison shows that changing the type of derivative with close order has no effect on the overall trend of the results but the obtained numerical values are different.

Later, we have investigated the effect of each of the model parameters on R_0 , and the results show that the disease transmission rates among the groups have a positive effect on the value of R_0 ; therefore, to control the spread of Crimean-Congo hemorrhagic fever, the disease transmission rate should be reduced by reducing contact between different groups.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

The authors declare that the study was realized in collaboration with equal responsibility. All authors read and approved the final manuscript.

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