Research Article

Numerical Analysis of the Fractional-Order Nonlinear System of Volterra Integro-Differential Equations

Pongsakorn Sunthrayuth, 1 Roman Ullah, 2 Adnan Khan, 3 Rasool Shah, 3 Jeevan Kafle, 4 Ibrahim Mahariq, 5 and Fahd Jarad 6, 7

1Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Thanyaburi, Pathum Thani, Thailand
2Department of Computing, Muscat College, Muscat, Oman
3Department of Mathematics, Abdul Wali Khan University Mardan, Mardan 23200, Pakistan
4Central Department of Mathematics, Tribhuvan University, Kirtipur, Kathmandu, Nepal
5College of Engineering and Technology, American University of the Middle East, Kuwait
6Department of Mathematics, Cankaya University, Etimesgut, Ankara, Turkey
7Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan

Correspondence should be addressed to Jeevan Kafle; jeevan.kafle@cdmath.tu.edu.np and Fahd Jarad; fahd@cankaya.edu.tr

Received 15 July 2021; Revised 30 August 2021; Accepted 8 October 2021; Published 21 October 2021

Academic Editor: Emanuel Guariglia

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This paper presents the nonlinear systems of Volterra-type fractional integro-differential equation solutions through a Chebyshev pseudospectral method. The proposed method is based on the Caputo fractional derivative. The results that we get show the accuracy and reliability of the present method. Different nonlinear systems have been solved; the solutions that we get are compared with other methods and the exact solution. Also, from the presented figures, it is easy to conclude that the CPM error converges quickly as compared to other methods. Comparing the exact solution and other techniques reveals that the Chebyshev pseudospectral method has a higher degree of accuracy and converges quickly towards the exact solution. Moreover, it is easy to implement the suggested method for solving fractional-order linear and nonlinear physical problems related to science and engineering.

1. Introduction

Fractional calculus has a long history as classical calculus. The concept of fractional calculus arouse when Leibnitz used a proper representation $d^nf/\,dx^n$ for the $n$th derivative in his publications. L'Hopital raises a question on the particular notation on what happens if “$n$” is a noninteger. It was the beginning of fractional calculus [1]. Recently, mathematicians focused on fractional calculus due to its numerous applications in every field of science; viscoelastic materials [2], economics [3], continuum and statistical mechanics [4], dynamics of interfaces between soft nanoparticles and rough substrates [5], solid mechanics [6], and much more [7–14].

Mathematical formulations solve many problems of nature with the help of converting the physical phenomena to the equation form. Differential equations (DEs) are among those that play the main role in modeling various phenomena. However, some problems are complex and cannot be handled with the help of a differential equation. In this regard, the researchers utilized fractional differential equations (FDEs) that model the phenomenon more accurately than differential equations having order integers. Nowadays, FDEs got the importance of real-world modeling problems: such as electrode-electrolyte polarization [15], electrochemistry of corrosion [16], circuit systems [17], optics and signal processing [18], heat conduction [19], diffusion wave [20], control theory of dynamical systems [21],
2. Definitions and Preliminary Concept

This unit shows the preliminary concept and some essential definitions taken from fractional calculus and used in our present research work.

2.1. Definition. The definition for fractional derivative by Caputo of order \( \alpha \) is showed by the following mathematical expression [48]:

\[
D^\alpha j(s) = \frac{1}{\Gamma(n-\alpha)} \int_0^s (s-t)^{n-\alpha-1} j^{(n)}(t) dt,
\]

for \( n-1<\alpha\leq n, n \in \mathbb{N}, s>0, j \in \mathcal{C}^m_{-1} \).

2.2. Definition. The fractional derivatives by Jin-Hunan He are described as [48]

\[
\frac{D^\alpha j(s)}{D s^\alpha} = \Gamma(1+\alpha) \lim_{\Delta s \to 0} \frac{f(s_1)-f(s_2)}{(s_1-s_2)^\alpha},
\]

where \( \Delta s \) does not approach zero.

2.3. Definition. Xiao-Jun explains derivatives having fractional order as [48]

\[
D^\alpha j(s_0) = j^{(n)}(s_0) = \frac{d^n j(s)}{ds^n} \bigg|_{s=s_0} = \lim_{s \to s_0} \frac{\Delta^n (j(s) - j(s_0))}{(s-s_0)^n},
\]

where

\[
\Delta^n (j(s) - j(s_0)) \equiv I^n (1+\alpha) \Delta (j(s) - j(s_0)).
\]

2.4. Definition. The integral operator by Riemann-Liouville for order \( \alpha \) is [48]

\[
I^\alpha j(s) = \frac{1}{\Gamma(\alpha)} \int_0^s (s-t)^{\alpha-1} j(t) dt.
\]

The Caputo derivative operator and Riemann-Liouville integral operator have the following properties

\[
D^\alpha I^n j(s) = j(s),
\]

\[
I^n D^\alpha j(s) = j(s) - \sum_{k=0}^{n-1} \frac{j^{(k)}(0^+)}{k!} s^k, s \geq 0, n-1 < \alpha < n.
\]

3. Chebyshev Pseudospectral Method (CPM)

The Chebyshev polynomials are defined in the \([-1,1]\) interval and can be described by the following recurrence formula:

\[
R_{n+1}(t) = 2aR_n(s) - R_{n-1}(s), \quad n = 1, 2, \cdots,
\]

where

\[
R_0(s) = 1,
\]

\[
R_1(s) = s.
\]

To apply the Chebyshev polynomials in the \([0,1]\) interval, we define the Chebyshev shifted polynomials \( \tilde{R}_n(s) \) which are defined in the manner of Chebyshev polynomials \( R_n(s) \) by relation

\[
\tilde{R}_n(s) = R_n(2s-1).
\]

And the recurrence formula is as follows:

\[
\tilde{R}_{n+1}(s) = 2(2s-1)\tilde{R}_n(s) - \tilde{R}_{n-1}(s), \quad n = 1, 2, \cdots,
\]

where

\[
\tilde{R}_0(s) = 1,
\]

\[
\tilde{R}_1(s) = 2s-1.
\]
Table 1: Exact versus CPM solution of problem 1 at \( m = 10 \).

<table>
<thead>
<tr>
<th>( s )</th>
<th>Exact ( j(s) )</th>
<th>Exact ( k(s) )</th>
<th>CPM solution ( j(s) )</th>
<th>CPM solution ( k(s) )</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>0.0000000000</td>
<td>1.0000000000</td>
<td>0.0000000000</td>
<td>1.0000000000</td>
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<tr>
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<tr>
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<td>1.0810723718</td>
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</tr>
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<td>1.127629652</td>
</tr>
<tr>
<td>0.6</td>
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<td>1.1854652182</td>
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<td>1.251690056</td>
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<td>1.251690056</td>
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<tr>
<td>0.8</td>
<td>0.8881059821</td>
<td>1.337439463</td>
<td>0.8881059821</td>
<td>1.337439463</td>
</tr>
<tr>
<td>0.9</td>
<td>1.0265167257</td>
<td>1.4330863854</td>
<td>1.0265167257</td>
<td>1.4330863854</td>
</tr>
<tr>
<td>1.0</td>
<td>1.1752011936</td>
<td>1.5430806348</td>
<td>1.1752011936</td>
<td>1.5430806348</td>
</tr>
</tbody>
</table>

Table 2: Error comparison of CPM versus other methods of Section 4.1 at \( m = 10 \).

<table>
<thead>
<tr>
<th>( s )</th>
<th>Error ( j_{\text{CPM}} )</th>
<th>Error ( k_{\text{CPM}} )</th>
<th>Error ( j_{\text{OTM}} )</th>
<th>Error ( k_{\text{OTM}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000000000E + 00</td>
<td>0.0000000000E + 00</td>
<td>0.00E + 00</td>
<td>0.00E + 00</td>
</tr>
<tr>
<td>0.1</td>
<td>8.2692537238E - 17</td>
<td>1.4541999963E - 17</td>
<td>1.39E - 17</td>
<td>0.00E + 00</td>
</tr>
<tr>
<td>0.2</td>
<td>9.1822482293E - 16</td>
<td>1.6662883612E - 16</td>
<td>5.27E - 16</td>
<td>0.00E + 00</td>
</tr>
<tr>
<td>0.3</td>
<td>1.2443994903E - 15</td>
<td>2.2699378388E - 16</td>
<td>4.45E - 14</td>
<td>1.11E - 15</td>
</tr>
<tr>
<td>0.4</td>
<td>1.2607906760E - 15</td>
<td>2.311311594E - 16</td>
<td>1.05E - 12</td>
<td>3.51E - 14</td>
</tr>
<tr>
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<td>7.2897283903E - 16</td>
<td>1.2454060032E - 16</td>
<td>1.23E - 11</td>
<td>5.10E - 13</td>
</tr>
<tr>
<td>0.6</td>
<td>2.5175064758E - 13</td>
<td>5.5241090380E - 14</td>
<td>9.11E - 11</td>
<td>4.55E - 12</td>
</tr>
<tr>
<td>0.7</td>
<td>5.8189005235E - 12</td>
<td>1.3209327312E - 12</td>
<td>4.97E - 10</td>
<td>2.90E - 11</td>
</tr>
<tr>
<td>0.8</td>
<td>5.8725732247E - 11</td>
<td>1.3786485036E - 11</td>
<td>2.16E - 9</td>
<td>1.44E - 10</td>
</tr>
<tr>
<td>0.9</td>
<td>3.7796170567E - 10</td>
<td>9.1669608328E - 11</td>
<td>7.90E - 9</td>
<td>5.92E - 10</td>
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<td>1.0</td>
<td>1.8157658168E - 09</td>
<td>4.5450903393E - 10</td>
<td>2.52E - 8</td>
<td>2.10E - 9</td>
</tr>
</tbody>
</table>

Figure 1: The solution graph of example 1. (a) Exact solution and (b) CPM solution.

Figure 2: The solution graph of example 1. (a) Exact solution and (b) CPM solution.
A function \( j(s) \in L_2[0, 1] \), in terms of Chebyshev shifted polynomials described as

\[
j(s) = \sum_{n=1}^{\infty} c_n \tilde{R}_n(s).
\]  

(12)
Table 3: Exact and CPM solutions of Section 4.2 at $m = 3$.  

<table>
<thead>
<tr>
<th>$s$</th>
<th>Exact $j(s)$</th>
<th>Exact $k(s)$</th>
<th>CPM solution $j(s)$</th>
<th>CPM solution $k(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.047500000000000</td>
<td>-0.002375000000000</td>
<td>0.047500000050000</td>
<td>-0.002375000000000</td>
</tr>
<tr>
<td>0.15</td>
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<td>-0.019125000000000</td>
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<td>-0.01912499997000</td>
</tr>
<tr>
<td>0.25</td>
<td>0.187500000000000</td>
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<td>0.187500000100000</td>
<td>-0.04687499994000</td>
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<tr>
<td>0.35</td>
<td>0.227500000000000</td>
<td>-0.079625000000000</td>
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</tr>
<tr>
<td>0.45</td>
<td>0.247500000000000</td>
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<td>-0.11374999990000</td>
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<tr>
<td>0.55</td>
<td>0.247500000000000</td>
<td>-0.136125000000000</td>
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<td>-0.13612500000000</td>
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<tr>
<td>0.65</td>
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<td>-0.14787500000000</td>
</tr>
<tr>
<td>0.75</td>
<td>0.187500000000000</td>
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<td>-0.14062500000000</td>
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<tr>
<td>0.85</td>
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<tr>
<td>0.95</td>
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<td>-0.045125000010000</td>
</tr>
</tbody>
</table>

Table 4: Error comparison of CPM versus other methods of Section 4.2.  

<table>
<thead>
<tr>
<th>$s$</th>
<th>Error($j_{CPM}$)</th>
<th>Error($k_{CPM}$)</th>
<th>Error($j_{OM}$)</th>
<th>Error($k_{OM}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>5.00000000000E-11</td>
<td>0.00000000000E+00</td>
<td>8.17886E-8</td>
<td>1.71222E-7</td>
</tr>
<tr>
<td>0.15</td>
<td>1.00000000000E-10</td>
<td>3.00000000000E-11</td>
<td>4.14502E-8</td>
<td>5.20012E-8</td>
</tr>
<tr>
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<td>1.00000000000E-10</td>
<td>6.00000000000E-11</td>
<td>3.00945E-9</td>
<td>1.53079E-7</td>
</tr>
<tr>
<td>0.35</td>
<td>1.00000000000E-10</td>
<td>5.00000000000E-11</td>
<td>5.66834E-8</td>
<td>1.82626E-7</td>
</tr>
<tr>
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<td>0.00000000000E+00</td>
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<td>3.81977E-8</td>
<td>6.42170E-7</td>
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<tr>
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<td>0.00000000000E+00</td>
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<td>0.00000000000E+00</td>
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</tr>
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<td>0.00000000000E+00</td>
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<td>1.53242E-7</td>
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<tr>
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<td>0.00000000000E+00</td>
<td>4.55344E-8</td>
<td>1.18939E-8</td>
</tr>
<tr>
<td>0.95</td>
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<td>1.00000000000E-10</td>
<td>8.32363E-8</td>
<td>9.10621E-8</td>
</tr>
</tbody>
</table>

The Chebyshev shifted polynomials first $(m + 1)$ terms are considered as

$$j_m(s) = \sum_{n=0}^{m} c_n \hat{R}_n(s),$$

$$\frac{d^a}{ds^a} \left( \sum_{n=0}^{m} c_n \hat{R}_n(s) \right) + \sum_{n=0}^{m} c_n \hat{R}_n(s) + \int_{t_0}^{t} \left( \sum_{n=0}^{m} c_n \hat{R}_n(s) \right) ds = g(s, j).$$

(13)

For finding the system of equations, we have

$$\frac{d^a}{ds^a} \left( \sum_{n=0}^{m} c_n \hat{R}_n(s) \right) + \sum_{n=0}^{m} c_n \hat{R}_n(s) + \int_{t_0}^{t} \left( \sum_{n=0}^{m} c_n \hat{R}_n(s) \right) du = g(s, j).$$

(14)

whereas

$$s_{i} = \frac{i - 0.5}{2^{k-1}M}.$$  (15)

I solved the resultant system using maple software, which provide CPM solution for the given problem.

4. Numerical Representation

4.1 Problem. Consider the nonlinear FIDE system having B.Cs $j(0) = 0$, $k(0) = 1$

$$D^a j(s) + \frac{1}{2} \left( \frac{dk}{ds} \right)^2 - \int_{0}^{a} \left[ (s-t)k(t) + k(t)j(t) \right] dt = 1,$$

$$D^a k(s) + sj(s) - \int_{0}^{a} \left[ (s-t)j(t) + k^2(t) \right] dt = 2s,$$

having $j(s) = \sinh(s)$, $k(s) = \cosh(s)$ as the exact solution at $a = 1$.

The exact solution and numerical results obtained by means of the proposed method are shown in Table 1. The absolute error comparison of our method and those obtained from OTM is given in Table 2. The behavior of the exact solution and approximate solution (our method) of this example when $a = 1$ is presented in Figures 1 and 2 whereas the error comparison of CPM and OTM can be observed in Figures 3 and 4. The graphical representation for different fractional order of $a$ is seen in Figures 5 and 6 which confirm that the solution converge to the exact solution as the value of $a$ converges from the fractional order to the integer order.
4.2. Problem. Consider the FIDE system with B.Cs $j(0) = j(1) = 0$, $k(0) = k(0) = 0$

$$D^\alpha j(s) + k^2(s) + \frac{s}{2} \frac{dk}{ds} - \int_0^s ((s-t)k(t) + j(t)k(t))dt = g_2(s),$$

$$D^\alpha k(s) + \frac{j^2}{2}(s) - \int_0^s ((s-t)j(t) - k^2(t) + j^2(t))dt = g_1(s),$$

(17)
Table 5: Exact versus CPM solution of Section 4.3 at \( m = 10 \).

<table>
<thead>
<tr>
<th>( s )</th>
<th>Exact ( j(s) )</th>
<th>Exact ( k(s) )</th>
<th>CPM solution ( j(s) )</th>
<th>CPM solution ( k(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0000000000</td>
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<td>-1.0000000000</td>
</tr>
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</tr>
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</table>

Table 6: Error comparison of CPM versus other methods of Section 4.3 at \( m = 10 \).

<table>
<thead>
<tr>
<th>( s )</th>
<th>Error(( j_{CPM} ))</th>
<th>Error(( k_{CPM} ))</th>
<th>Error(( j_{OTM} ))</th>
<th>Error(( k_{OTM} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000000000E + 00</td>
<td>0.0000000000E + 00</td>
<td>0.00E + 00</td>
<td>0.00E + 00</td>
</tr>
<tr>
<td>0.1</td>
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<td>2.22E - 16</td>
<td>0.06E + 00</td>
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<td>0.2</td>
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<td>4.44E - 16</td>
<td>6.66E - 16</td>
</tr>
<tr>
<td>0.3</td>
<td>1.9076326419E - 14</td>
<td>1.947068817E - 14</td>
<td>4.55E - 14</td>
<td>4.55E - 14</td>
</tr>
<tr>
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<td>1.28E - 11</td>
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<tr>
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<td>9.57E - 11</td>
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<td>2.30E - 9</td>
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<td>4.8069973992E - 10</td>
<td>8.49E - 9</td>
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<td>2.3502562547E - 09</td>
<td>2.73E - 8</td>
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</tr>
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</table>

Figure 11: The solution graph of example 3. (a) Exact solution and (b) CPM solution.

Figure 12: The solution graph of example 3. (a) Exact solution and (b) CPM solution.
with the exact solution \( j(s) = s - s^2, k(s) = s^3 - s^2 \) at \( \alpha = 2 \), where

\[
    g_1(s) = \frac{7}{6} s^6 - \frac{49}{20} s^5 + \frac{4}{3} s^4 + \frac{3}{2} s^3 - s^2 - 2,
\]
\[
    g_2(s) = \frac{s^7}{7} - \frac{s^6}{3} + \frac{19}{12} s^5 - \frac{5}{2} s^3 + s^2 + 6s - 2. \tag{18}
\]

In Table 3, we give the numerical values of the exact solution and CPM solution for \( m = 3 \). The absolute errors obtained by the present method are compared with SCM in Table 4. We compare the actual and estimated solution in Figures 7 and 8 which tells us that both the solutions are quite close to each other. Also, Figures 9 and 10 display the error comparison of CPM and SCM which verify that our method is in good agreement with the exact solution.
4.3. Problem. Consider the nonlinear FIDE system having
B.Cs \( j(0) = 1, \ j'(0) = 2, \ k(0) = -1, \) and \( k'(0) = 0 \)

\[
D^\alpha j(s) + \frac{1}{2} \left( \frac{dk}{ds} \right)^2 - \frac{1}{2} \int_0^s \left( j^2(t) + k^2(t) \right) dt = 1 - \frac{1}{3} s^3,
\]

\[
D^\alpha k(s) + sj(s) - \frac{1}{4} \int_0^s \left( j^2(t) - k^2(t) \right) dt = s^2 - 1,
\]

having \( j(s) = s + e^s, \ k(s) = s - e^s \) as the exact solution.

To solve this example, we implemented the method suggested in Section 4 for \( \alpha = 2 \) with \( m = 10 \). The exact solution and estimated solution by CPM are presented in Table 5. The absolute error of our method and those obtained from OTM are given in Table 6. In Figures 11 and 12, it is clear that the numerical solution of the proposed method is in good contact with the exact solution. In order to illustrate the effectiveness of CPM, the error comparison with OTM is shown in Figures 13 and 14. Also, in Figures 15 and 16, we can obtain that as \( \alpha \rightarrow 2 \) the estimated solution approach to the exact solutions.

5. Conclusion

In this work, we implemented the Chebyshev pseudospectral method for solving nonlinear fractional integral and integro-differential equation systems. The proposed technique reduces this type of systems to the solution of the system of linear and nonlinear algebraic equations. Special attention is given to study the convergence of the proposed method. The results that we get by implementing the suggested technique are in excellent agreement with the exact solution and show more accuracy than the solution obtained using other methods. Also, from the presented figures, it is easy to conclude that the CPM error converges quickly as compared to other methods. The computation work in this article is done using Maple.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors’ Contributions

All authors jointly worked on the results, and they read and approved the final manuscript.

Acknowledgments

This study is supported by the Central Department of Mathematics, Tribhuvan University, Kirtipur, Kathmandu, Nepal.

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