Hindawi

Retraction

Retracted: Semigroup of Finite-State Deterministic Intuitionistic Fuzzy Automata with Application in Fault Diagnosis of an Aircraft Twin-Spool Turbofan Engine

Journal of Function Spaces

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] S. Kousar, F. Aslam, N. Kausar, and Y. U. Gaba, "Semigroup of Finite-State Deterministic Intuitionistic Fuzzy Automata with Application in Fault Diagnosis of an Aircraft Twin-Spool Turbofan Engine," *Journal of Function Spaces*, vol. 2021, Article ID 1994732, 10 pages, 2021.

Research Article

Semigroup of Finite-State Deterministic Intuitionistic Fuzzy Automata with Application in Fault Diagnosis of an Aircraft Twin-Spool Turbofan Engine

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The twin-spool turbofan engine is an important component of almost every modern aircraft. Fault detection at an early stage can improve engine performance and health. The current research is based on the construction of an inference system for fault diagnosis in a generalized fuzzy environment. For such an inference system, finite-state deterministic intuitionistic fuzzy automata (FDIFA) are established. A semigroup of FDIFA and its algebraic properties including substructures and structure-preserving maps are studied. The FDIFA semigroups are used as variables for the inference system, and FDIFA semigroup homomorphisms are used to indicate the relation between variables. The newly established model is then applied to diagnose the possible fault and their nature in aircraft twin-spool turbofan engines by modelling the performance of the supercharger and air cooler.

1. Introduction

During the last several decades, significant investigation and research have been conducted to increase aircraft safety by enhancing the performance of diagnostic systems for aircraft components, especially the gas turbine engines. Improved safety [1], consistency, and operational costs are just a few of the advantages that such systems can give to aircraft operators. With the considerable expansion in air traffic forecast in the coming years, the demand for improved diagnostic methods is expected to continue to rise. In general, aircraft engine performance diagnostics are achieved by predicting the performance, health, and strength parameters of certain components of the engine based on the accessible sensor measurements [2–4]. Productivity, efficiency, and flow capability of engine components are some of the health parameters that indicate the performance effectiveness or decline of

the component. The sensor readings comprise gas path pressures and temperatures, fuel flows, and spool speeds and provides essential data on the engine's health. The semigroup is an important algebraic structure, serving as the theoretical foundation for a variety of scientific fields with several applications [5]. The semigroup's role in theoretical computer science is inevitable in particular; the semigroup and automata are widely studied and applied in artificial intelligence, game theory, dynamical system, system biology, and fault diagnosis. A finite-state machine (FSM) or finite-state automata (FSA) are a mathematical model of computation that can be changed from one state to another state in connection to suitable inputs. There are two types of FSMs: deterministic (DFSM) the one that accepts or rejects a given string of inputs, following a state sequence uniquely obtained from the string, and nondeterministic (NDFSM) which does not obey these restrictions. Moreover, for each NDFSM, an

equivalent DFSM can be constructed. The relation between the semigroup and FSA was established by Krohn and Rhodes [6], and they discussed semigroup decomposition employing decomposition of FSA.

During the investigation, it was discovered that to diagnose a machine's problem situation, it is necessary to look at its operating state, the fault degree, the accuracy degree, and expected changes between states at different stages. That can be done by establishing an appropriate FSA. The uncertainty and ambiguity in the diagnosis process enforce to carry out mathematical computation in a fuzzy environment. Fuzzy automata are used to handle system uncertainties more accurately where classical automata fail to cater to the circumstances. Fuzzy automata have been frequently employed since the introduction of fuzzy technology and neural networks [7–13]. Furthermore, there were a variety of problems to be resolved, for example, a car anticrash radar, freeway management, urban road traffic control, and obstacle recognition in front of a vehicle which required flexible, quick, and accurate decisions. Fuzzy neural network automata (FNNA) [14, 15] are an excellent choice. FNNA had an increasingly prominent role, particularly in data communications. In FNNA, fuzzy technology is used to compare with ordinary identification and control devices utilizing several features and techniques of the neural network including fast and accurate numerical calculation of large numbers and self-organization learning ability. As a result, FNNA was able to solve a variety of problems in a flexible, quick, and precise manner and it had several advantages such as the compact size, being lightweight, and stability. The research findings also revealed that it has a significant ability for self-organization learning and the experimental results demonstrated fuzzy automata's superiority. Lvzhou and Daowen [16] introduced the technique of minimizing fuzzy automata and constructed a new automaton system that dealt accurately with many states at a time. They used an ordered lattice to reduce a fuzzy automaton to another fuzzy automaton with m states that were functionally correspondent to the original fuzzy automata. Several language-preserving methods for minimizing deterministic fuzzy automata were established. The Moghari and Zahedi [17] method preserves both language and behavior in the minimization process. De Mendivil and Garitagoitia [18] described factorization of fuzzy states that is used in the determination of fuzzy automata. Stamenkovic et al. [19] studied fuzzy automata and the reduction of fuzzy automata states and relational equations of the fuzzy system by considering the solution of the fuzzy equivalent equation. These solutions are then used to reduce the number of acceptable states. The solution of the fuzzy system of the quasiorder number was also used to reduce the states of the fuzzy automaton.

Fuzzy inference is a method of formulating a rule or a mapping from a given set of inputs to a given set of outputs utilizing the fuzzy logic. The fuzzy inference system is widely used in control systems [20], artificial intelligence [21], and game theory [22]. The fuzzy set over a nonempty set X as defined by Zadeh [23] is a function (membership function) from X to the closed interval $[0, 1]$ so the representation of

uncertain data by using closed interval is equivalent to the representation by using fuzzy sets [24, 25]. The fuzzy inference of fuzzy automata is introduced and utilized in machine defect diagnosis as the impact of the combination of fuzzy automaton rules and the method of the fuzzy set is greater than the classical automata. For the derivation of automata, some researchers used neural networks [24, 26, 27]. Although neural networks and fuzzy systems are not the same, fuzzy inference has been widely applied in the disciplines of control and intelligence [28]. The fuzzy set proposed by Zadeh is based on the formulation of the membership function from X to $[0, 1]$, where the images are termed as membership grades or degrees of membership of elements of X. Atansassov [29] proposed the notion of an intuitionistic fuzzy set (IFS) which is an extension of the perception of the fuzzy set where the degree of nonmembership is also considered along with the degree of membership. Fuzzy inference is a method of formulating a rule or a mapping from a given set of inputs to a given set of outputs utilizing the fuzzy logic. The fuzzy inference system and fuzzy automata and algebraic structures on fuzzy automata are investigated and successfully applied in computer science and engineering [30]. IFS provides a more effective framework to handle imprecision and uncertainties more accurately than the fuzzy set and logic. The work is based on the development of inference systems and automata in an intuitionistic fuzzy environment. Intuitionistic fuzzy automata have several states at a certain time or stage, each equipped with certain grade of reliability and nonreliability. Thus, the intuitionistic fuzzy automata are more realistic and close to the objective. To achieve the goal firstly, finite-state deterministic intuitionistic fuzzy automata (FDIFA) are defined. Secondly, a semigroup on intuitionistic fuzzy automata is designed and investigated its algebraic properties and intuitionistic fuzzy inference rules on the FDIFA semigroup are established. The mathematical computations are supported and justified by conduction fault diagnosis in the aircraft twin-spool turbofan engine.

2. Finite-State Deterministic Intuitionistic Fuzzy Automata (FDIFA)

The septuple $\mathrm{MT} = (R, Z, \zeta, r_0, H, P, E)$ is termed as finite-state deterministic intuitionistic fuzzy automata (FDIFA), where R is a finite collection of states, Z is a finite collection of input characters, $r_0 \in R$ is the initial state, H is an intuitionistic fuzzy set of final states, P, E are sets of fuzzy grades of the membership and nonmembership at the state transition and $P, E \subseteq [0, 1]$, and $\zeta : R \times Z \times P \times E \longrightarrow R$ is the transition mapping with $\zeta(r_i, z, \mu, \nu) = \{r_j\}$, where $r_i, r_j \in R, z \in Z, \mu \in P$ and $\nu \in E$. These states $r_j \in R$ exist in some accepted states. The extended transition function $\zeta^* : R \times Z^* \times P \times E \longrightarrow R$ is defined as follows:

$$\zeta^*(r, \varepsilon, \mu, \nu) = r,$$
$$\zeta^*(r, \omega z, \mu, \nu) = \zeta\left(\zeta^*\left(r, \omega, \mu', \nu'\right), z, \mu, \nu\right), \tag{1}$$

where $\varepsilon, \varpi \in Z^*$ (set of finite strings/words/sequences of input characters), $\forall z \in Z, \mu, \mu' \in P$, and $\nu, \nu' \in E$. Let H be a intuitionistic fuzzy set of final states; the cut set $H_{\alpha,\gamma}$ of H is given as follows:

$$H_{\alpha,\gamma} = \{x \in R : \mu_H(x) \geq \alpha, \nu_H(x) \leq \gamma\}, \tag{2}$$

where $\alpha, \gamma \in [0, 1]$ are fixed numbers, such that $\alpha + \gamma \leq 1$. The set

$$H_\alpha = \{\langle x, \mu_H(x), \nu_H(x)\rangle : x \in R \text{ and } \mu_H(x) \geq \alpha\} \tag{3}$$

is called the set of the level of membership α generated by H. Also,

$$H^\gamma = \{\langle x, \mu_H(x), \nu_H(x)\rangle : x \in R \text{ and } \nu_H(x) \leq \gamma\} \tag{4}$$

is called the set of the level of nonmembership γ generated by H. Clearly,

$$\begin{aligned} H_{\alpha,\gamma} &\subset H_\alpha, \\ H^\gamma &\subset H. \end{aligned} \tag{5}$$

Example 1. Consider

(1) $R = \{r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8\}$ the set of input states with r_0 as the initial state

(2) $Z = \{z_1, z_2, z_3, z_4, z_5\}$ the set of input characters

(3) $P = \{1, 0.9, 0.7, 0.5, 0.4, 0.3, 0.2\}$ the set of state transition membership grades

(4) $E = \{0, 0.1, 0.2, 0.3, 0.5, 0.7\}$ the set of state transition nonmembership grades

Define the transition mapping $\zeta : R \times Z \times P \times E \longrightarrow R$ for the accepting states $r_1, r_2, r_3, r_4, r_5, r_6, r_7$ as follows:

$$\begin{aligned} \zeta(r_0, z_4, 0.4, 0.2) &= \{r_1\}, \\ \zeta(r_0, z_2, 0.2, 0.5) &= \{r_4\}, \\ \zeta(r_1, z_1, 0.7, 0.1) &= \{r_2\}, \\ \zeta(r_2, z_5, 0.9, 0) &= \{r_3\}, \\ \zeta(r_4, z_3, 0.3, 0.7) &= \{r_5\}, \\ \zeta(r_5, z_5, 0.5, 0.3) &= \{r_6\}, \\ \zeta(r_6, z_2, 1, 0) &= \{r_7\}, \end{aligned} \tag{6}$$

with the transition diagram in Figure 1.

Then, $H = \{(r_1, 0.4, 0.2), (r_4, 0.2, 0.5), (r_2, 0.7, 0.1), (r_3, 0.9, 0), (r_5, 0.3, 0.7), (r_6, 0.5, 0.3), (r_7, 1, 0)\}$ is an intuitionistic fuzzy set of final states. Thus, $MT = (R, Z, \zeta, r_0, H, P, E)$ is an FDIFA.
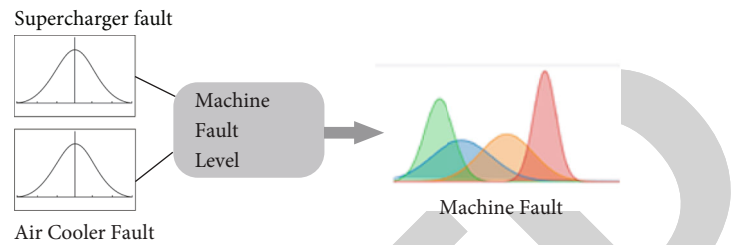

Figure 1: State transition diagram.

3. Semigroup of the Finite-State Deterministic Intuitionistic Fuzzy Automata

To define the semigroup structure of FDIFA, first, we need a binary operation that involves state transition membership and nonmembership grades.

Definition 1. Let $MT = (R, Z, \zeta, r_0, H, P, E)$ be an FDIFA. For any input string or sequence $\varpi \in Z^*$, each state $q \in R$ is transited to another state $r \in H$ with membership μ and nonmembership ν. As a result, the triplet (ϖ, μ, ν) induces a state transition $\zeta(r_i, \varpi, \mu, \nu) = \{r_j\}$ from R to H. If r_i and r_k are two states in R such that $\zeta(r_i, \varpi, \mu, \nu) = \{r_j\}$ and $\zeta(r_k, \varpi, \mu, \nu) = \{r_l\}$, respectively, the overall transition through states r_i and r_k is denoted by $\{r_j\} \circ \{r_l\} = \{r_n\}$. Clearly, $\circ$ is a binary operation on FDIFA such that for any mt_1, mt_2 and $mt_3 \in MT$ with

$$\begin{aligned} mt_1 &= (R_1, Z_1, \zeta_1, r_{01}, H_1, P_1, E_1), & \zeta_1(r_{i1}, a_1, \mu_1, \nu_1) &= \{r_{j1}\}, \\ mt_2 &= (R_2, Z_2, \zeta_2, r_{02}, H_2, P_2, E_2), & \zeta_2(r_{i2}, a_2, \mu_2, \nu_2) &= \{r_{j2}\}, \\ mt_3 &= (R_3, Z_3, \zeta_3, r_{03}, H_3, P_3, E_3), & \zeta_3(r_{i3}, a_3, \mu_3, \nu_3) &= \{r_{j3}\}, \end{aligned} \tag{7}$$

where for $i = 1, 2, 3$, the set R_i is the collection of states, Z_i is the collection input symbols, H_i is the intuitionistic fuzzy set of final states, and r_{0i} is the initial state. We have

$$mt_1 \circ (mt_2 \circ mt_3) = (mt_1 \circ mt_2) \circ mt_3, \tag{8}$$

which turns FDIFA MT into a semigroup, called an FDIFA semigroup. The expression $mt_1 \circ mt_2$ represents the overall state transition through ζ_1 and ζ_2 in the intuitionistic fuzzy automata mt_1 and mt_2, respectively.

The FDIFA $(MT, \circ)$ is a called a monoid (semigroup with identity) if there exist a fuzzy automata $e \in MT$ such that $mt_1 \circ e = e \circ mt_1 = mt_1$ for all $mt_1 \in MT$. Clearly, the identity element is unique in MT. Being a semigroup, the elements of MT must satisfy the power axioms of semigroups that are as follows:

(1) $mt^n = mt \circ mt \circ, \cdots, \circ mt$ (n times)

(2) $mt^l \circ mt^n = mt^{l+n}$

(3) $(mt^l)^n = mt^{ln}$

(4) $mt^0 = e$

for all $l, n \in \mathbb{Z}^+$. FDIFA semigroup MT is termed to be *commutative* if the binary operation $\circ$ is commutative. FDIFA semigroup MT is called *Boolean* if all of its elements are idempotent, that is, $mt \circ mt = mt$ for all $mt \in MT$. If MT is a monoid, then, the identity element is an essential idempotent in MT. An FDIFA monoid $(MT, \circ)$ is called a *cyclic* if there is an element $mt \in MT$ such that $MT = \{mt^n : n \in \mathbb{Z}^+\}$. That is, all the elements of MT are expressed as nonnegative powers of mt.

Proposition 2. *Every FDIFA cyclic monoid automaton MT is commutative.*

Proof. Let $(MT, \circ)$ be a FDIFA cyclic monoid automaton generated by mt. Now, for $mt_1, mt_2 \in MT$, there exist $l, k \in \mathbb{Z}^+$ such that $mt_1 = mt^l$ and $mt_2 = mt^k$. Thus, $mt_1 \circ mt_2 = mt^l \circ mt^k = mt^{l+k} = mt^{k+l} = mt_2 \circ mt_1$ imply that MT is commutative. □

Definition 3. Let $(MT, \circ)$ be an FDIFA semigroup, where

$$MT = \{(R_n, Z_n, \zeta_n, r_{0n}, H_n, P_n, E_n): n \in N\}, \quad \zeta_n(r_i, a, \mu, \nu) = \{r_j\}. \tag{9}$$

Consider that

$$MT_i = \{(R_i, Z_i, \zeta_i, r_{0i}, H_i, P_i, E_i): i \in N\}, \quad \zeta_i\left(r_k, \acute{a}, \acute{\mu}, \acute{\nu}\right) = \{r_l\}, \tag{10}$$

where $r_j \in H_n$ and $r_l \in H_i$, $R_i \subseteq R_n$, $Z_i \subseteq Z_n$, $\zeta_i(r_k, \acute{a}, \acute{\mu}, \acute{\nu}) \subseteq \zeta_n(r_i, a, \mu, \nu)$, $r_{0i} = r_{0n}$, $H_i \subseteq H_n$, and $P_i \subseteq P_n$. If MT_i is closed under the binary operation $\circ$; symbolically, $mt_1 \circ mt_2 \in MT_i$ $\forall mt_1$ and $mt_2 \in MT_i$. Equivalently, $\{r_j\} \circ \{r_l\} = \{r_n\} \subseteq H_i$, then, $(MT_i, \circ)$ is known as a *subsemigroup automaton* of $(MT, \circ)$. If MT is a monoid with identity e and e belongs to the submonoid MT_1, then, MT_1 is known as a *submonoid automaton* of MT.

Proposition 4. *For any FDIFA commutative monoid $(MT, \circ)$, the set of idempotent MT_I is a submonoid automaton.*

Proof. Assume that MT_I is set of all idempotents of MT. Clearly, the identity element $e \in MT_I$. As MT_I is a set of idempotents, so, $mt_{k_1} \circ mt_{k_1} = mt_{k_1}, mt_{k_2} \circ mt_{k_2} = mt_{k_2}, \forall mt_{k_1}, mt_{k_2} \in MT_I$. The commutativity of MT imply that $\left(mt_{k_1} \circ mt_{k_2}\right)^2 = \left(mt_{k_1} \circ mt_{k_2}\right) \circ \left(mt_{k_1} \circ mt_{k_2}\right) = \left(mt_{k_1} \circ mt_{k_1}\right) \circ \left(mt_{k_2} \circ mt_{k_2}\right) = mt_{k_1} \circ mt_{k_2}$. Thus, $mt_{k_1} \circ mt_{k_2} \in K$ and $(MT_I, \circ)$ is a submonoid. □

Definition 5. Any two FDIFA semigroups can be related to each other employing of structure-preserving maps, commonly known as homomorphisms. Such maps not only preserve the binary operations used to design semigroups on FDIFA but also the components involved in the formulation

of FDIFAs. Formally, if $(MT, \circ)$ and $(MT', \circ')$ are any two FDIFA semigroups, then, a map $\theta : MT \longrightarrow MT'$ is called an FDIFA semigroup homomorphism, if $\forall mt_1, mt_2 \in MT$, $\theta(mt_1 \circ mt_2) = \theta(mt_1) \circ' \theta(mt_2)$. If θ is a homomorphism that also establishes a one-to-one correspondence between MT and MT', then, it is termed to be an isomorphism and the FDIFA semigroups are denoted by $MT \simeq MT'$.

Theorem 6. *Let $(MT, \circ)$, $(MT', \circ')$, and $(MT'', \circ'')$ be any three FDIFA semigroups and $\theta : MT \longrightarrow MT'$ and $\theta' : MT' \longrightarrow MT''$ be an FDIFA semigroup homomorphism.*

(1) *If $mt \in MT_I$, then, $\theta(mt) \in MT_I'$*

(2) *If e is the identity element of MT, then, $\theta(e)$ is the identity element of $\theta(MT)$. If θ is an isomorphism, then, $\theta(e)$ is the identity element of MT'*

(3) *If MT_1 is a subsemigroup automaton of MT, the $\theta(MT_1)$ is a subsemigroup automaton of MT'*

(4) *$\theta' \circ \theta : MT \longrightarrow MT''$ is an FDIFA semigroup homomorphism*

Proof.

(1) $mt \circ mt = mt$ imply that $\theta(mt) = \theta(mt \circ mt) = \theta(mt) \circ' \theta(mt)$. Thus, $\theta(mt)$ is idempotent

(2) $mt \circ e = mt = e \circ mt$ imply that $\theta(mt) \circ' \theta(e) = \theta(mt \circ e) = \theta(mt) = \theta(e \circ mt) = \theta(e) \circ' \theta(mt)$. Thus, $\theta(e)$ is the identity element of $\theta(MT)$

(3) $mt_1 \circ mt_2 \in MT_1$ imply that $\theta(mt_1) \circ' \theta(mt_2) \in \theta(MT_1)$. Thus, $\theta(MT_1)$ is an FDIFA subsemigroup of MT'

(4) As $\theta' \circ \theta(mt_1 \circ mt_2) = \theta'(\theta(mt_1) \circ' \theta(mt_2)) = \theta'(\theta(mt_1)) \circ'' \theta'(\theta(mt_2)) = \theta' \circ \theta((mt_1)) \circ'' \theta' \circ \theta(mt_2))$, we get that $\theta' \circ \theta$ is an FDIFA semigroup homomorphism

□

4. Intuitionistic Fuzzy Inference Rule on the FDIFA Semigroup

On the semigroup of FDIFA, intuitionistic fuzzy inference is a cognitive procedure that proceeds to a new decision or statement based on one or more previous decisions or propositions. In general, the FDIFA inference is divided into two components. The premise is a well-known decision used as a starting point for inference. The second is the conclusion, which is a fresh decision generated by the proposition. The following inference rules are offered based on the characteristics of homomorphic mapping:

(Premise1) If $a \in X$ semigroup, then, $b \in Y$ must be a semigroup

(Premise2) If $a \in X'$ is a semigroup

The conclusion is that $b \in Y' = X' \circ (X \longrightarrow Y)$, where $\circ$ is a binary operation and $X \longrightarrow Y$ is a homomorphism.

The intuitionistic fuzzy inference system is made up of certain inference rules that must obey certain arithmetic requirements. The relation between X and Y for premise1 is $X \longrightarrow Y$, which is a homomorphism, grades of membership, and nonmembership corresponding to an intuitionistic fuzzy relation matrix G, which is defined as follows:

$$
\begin{aligned}
G(p,q) &= (\mu_{X \longrightarrow Y}(p,q), \nu_{X \longrightarrow Y}(p,q)) \\
&= [(\mu_X(p) \wedge \mu_Y(q)) \vee (1 - \mu_X(p)), (\nu_X(p) \vee \nu_Y(q)) \wedge (1 - \nu_X(p) - \varepsilon)],
\end{aligned}
\tag{11}
$$

where $\mu_{X \longrightarrow Y}(p,q), \nu_{X \longrightarrow Y}(p,q)$ are the grades of membership and nonmembership for the intuitionistic inference rule, ε is a error of hesitancy, $\mu_Y(q), \nu_Y(q)$ are the grades of membership and nonmembership for the assumption that $q \in Y$ is a semigroup, and $\mu_X(p), \nu_X(p)$ are the grades of membership and nonmembership for the assumption that $p \in X$ is a semigroup and $\vee$ is an "or" operation and $\wedge$ is an "and" operation:

$$
Y' = X' \circ (X \longrightarrow Y).
\tag{12}
$$

The inference relationship between X' and from X to Y can be used to synthesize the conclusion Y'. G can be used to obtain the conclusion's membership and nonmembership functions:

$$
(\mu_{Y'}(q), \nu_{Y'}(q)) = (\mu_{X'}(p), \nu_{X'}(p)) \circ G(p,q).
\tag{13}
$$

The following is a description of $\circ$, the synthetic binary operator:

$$
\begin{aligned}
(\mu_{Y'}(q), \nu_{Y'}(q)) &= (\mu_{X'}(p), \nu_{X'}(p)) \circ G(p,q) \\
&= [\vee_{p \in X}\{\mu_{X'}(p) \wedge \mu_G(p,q)\}, \wedge_{p \in X}\{\nu_{X'}(p) \vee \nu_G(p,q)\}],
\end{aligned}
\tag{14}
$$

such that $0 \le \mu_{Y'}(q) + \nu_{Y'}(q) \le 1$. In the classical fuzzy inference system, the grade on the membership of the fuzzy subset is a variable and variables are related through the operations defined on membership grades. However, in the inference system of the FDIFA semigroup, the FDIFA semigroup serves as a variable and the relation between variables is exhibited by utilizing the FDIFA semigroup homomorphism. In this case, state transitions are carried out with different membership and nonmembership grades that are further combined to get the overall state transition.

5. Application

The search for the fastest traveling and trading routes and mechanisms turns aviation into a multimillion dollar indus-

try. The first aircraft was launched in 1939; since then, several aircraft models are introduced with numerous modifications in the original designs. Air traffic accidents caused either by mechanical fault or human error are the biggest threats to the aviation industry. Proper investigation of such accidents will be beneficial in the improvement of aircraft models and mechanisms. Fault detection at an early level can reduce the risk of air crashes saving the lives of hundreds on board. Engines are the most important components of any aircraft, not only that they give enough lift to fly the plane but also that major accidents are caused by engine failure, fuel leakage, or sudden power shut down. The first aircraft Heinkel He 178 engines use a gas turbine to keep the plane in the air. With a slight modification of the design, the first jetliner de Havilland DH.106 Comet was launched in 1952 where the turbojet engines were placed near the wing roots. Four types of gas turbine engines are used in the aircraft, a turbojet engine that uses its exhaust, a turbofan engine that uses its fan, and turboprop that uses the propulsion system and converts heat energy into shaft power to create thrust. A twin-spool turbofan engine contains three compressors, two turbines, a burner, and two nozzles interconnected through various duct channels. A two-spool engine is designed to maintain high pressure and low temperature by two concentric shafts rotating at two different speeds, connecting the high-pressure turbine with the high-pressure compressor stages and the low-pressure turbine to the low-pressure compressor and fan. The structural characteristics of the twin-spool engine make it suitable for engine fuel efficiency and the ultimate choice of jetliners [31]. Here, we will create an intuitionist fuzzy inference system for fault detection in a twin-spool turbofan engine by considering the working principle of the supercharger and air cooler [32].

5.1. Fault Detection Rules. The fault detection problem is stated as follows: assign a value between 0 and 1 that indicates the supercharger fault case, with 1 indicating a serious fault. The air cooler's fault scenario is represented by a number between 0 and 1. In the same way, the number 1 signals a major flaw. Then, we use three rules to determine the state of the machine fault. The following are the three rules:

(1) The engine has a light fault if the supercharger has a weak fault or the air cooler is faulty

(2) The engine is faulty if the supercharger has a major issue

(3) The engine defect is serious if the supercharger and the air cooler are both seriously faulty

The degree of the FDIFA semigroup membership and nonmembership can show the engine's degree of fault and accuracy. As a result, the degree of membership is utilized to indicate the severity of the defect. Allow 5% to represent a minor issue with the engine and 10% to denote a weak malfunction. 15% denotes an evident problem with the engine, while 15% denotes a medium-level defect. If the percentage is greater than 25%, the engine has a major problem

and it should be considered a serious flaw. As indicated in Figure 2, the dual-input and dual-output systems will be built in this example. The dual inputs are a decrease in supercharger efficiency and a decrease in air cooler heat transfer, while the outputs are the engine fault level and engine accuracy level.

To put the first rule in the aforementioned inference system's rule editor, various selections are updated as follows:

(1) Select "weak" for the supercharger fault variable

(2) Select "fault" for the air cooler fault variable

(3) In the connection box, select the wireless button "or"

(4) Under the output variable of the engine fault, select "weak"

The following are the rules that are generated:

(1) Weak fault: the supercharger fault is weak or the air cooler is faulty

(2) Medium fault: the supercharger fault is medium

(3) Serious fault: the supercharger fault is serious or the air cooler fault is serious

Figure 3 depicts these three rules. The FDIFA semigroup's inference system has now been completely specified, comprising variables, membership functions, nonmembership functions, and the essential rules for diagnosing engine faults. The FDIFA semigroup rule viewer permits us to totally comprehend the entire intuitionistic fuzzy inference process at once. It also demonstrates the influences of the membership and nonmembership functions on the overall outcomes of the intuitionistic fuzzy inference. If the problem being examined is complex in nature subject to several conflicting factors, then, FDIFA semigroup inference is an appropriate tool for its solution.

5.2. FDIFA Semigroup. Now, we construct an FDIFA semigroup $(MT, \circ)$ fault diagnosis model for the twin-spool turbofan engine. Let $MT = (R, Z, \zeta, r_0, H, P, E)$ be an FDIFA, where R is a collection of states that the twin-spool turbofan engine can be in, such as normal, medium, or serious faults;

Z is an input character signal set; $r_0 \in R$ is the initial state of the processing signal; H is a fault state set and a subset of R; P is a fuzzy degree of the membership set; E is a intuitionistic fuzzy degree of the nonmembership set; $\zeta : R \times Z \times P \times E \longrightarrow R$ is the transition function, that is, $\zeta(r_i, a, \mu, \nu) = r_j$, where $r_i, r_j \in R$, $a \in Z$, $\mu \in P$, and $\nu \in E$. Because $r_0 \in R$, R denotes the entire states, $H \subseteq R$, and ζ is a transition procedure from one state to another; H, Z, P, and E are mostly treated in the following order:

(i) Fault-state or output variable set **H**: the normal working condition is represented by W_1, fan efficiency drop is represented by W_2, fan flow reduction is denoted by W_3, booster (BST) flow reduction is denoted by W_4, high-pressure turbine (HPT) efficiency drop is denoted by W_5, HPT flow reduction is denoted by W_6, high-pressure compressor (HPC) efficiency drop is denoted by W_7, HPC flow reduction is denoted by W_8, low-pressure turbine (LPT) efficiency drop is denoted by W_9, and LPT flow reduction is denoted by W_{10}. The range of values from W_1 to W_{10} is included inside the $[0, 1]$ interval. The 0 in the interval denotes the absence of such a flaw. 1 denotes a major flaw [33]

(ii) Processing signal or input variable set **Z**: the following 12 felt parameters are chosen as input variables: z_1 is the low-pressure spool speed, z_2 is the high-pressure spool speed, z_3 is the HPC inlet temperature, z_4 is the combustor inlet temperature, z_5 is the bypass duct static pressure, z_6 is the fan exit static pressure, z_7 is the booster tip pressure, z_8 is the combustor inlet static pressure, z_9 is the interturbine pressure, z_{10} is the fuel flow, z_{11} is the nozzle area, and z_{12} is the variable bypass duct area [33]

(iii) Fault degree P and accuracy degree E: the degrees to which each fault and accuracy parameter correspond to the categories of "normal," "serious," and "medium" are presented in terms of the membership and the intuitionistic degree of nonmembership grades as follows:

$$(\mu_{normal}, \nu_{normal}) = ((1, 0) (0.2, 0.4)(0.1, 0.6)(0.2, 0.5)(0.1, 0.8)(0, 0.9)(0, 0.9)(0.1, 0.7)(0.1, 0.9)(0, 0.6)), \tag{15}$$

$$(\mu_{serious}, \nu_{serious}) = ((0, 0.8)(0.2, 0.7)(0.3, 0.7)(0.4, 0.5)(0.6, 0.2)(0.8, 0.1)(0.6, 0.2)(0.7, 0.1)(0.1, 0.9)(0.3, 0.5)), \tag{16}$$

$$(\mu_{medium}, \nu_{medium}) = ((1, 0)(0.3, 0.6)(0.2, 0.8)(0.2, 0.7)(0.4, 0.5)(0.3, 0.6)(0.3, 0.7)(0.2, 0.8)(0.1, 0.9)(0, 0.7)). \tag{17}$$

5.2.1. Intuitionistic Fuzzy Inference Model. Faults are divided into three categories to make diagnosis easier: serious fault, medium fault, and no fault. The result is processed as follows, based on the output state of the intuitionistic fuzzy inference model of the FDIFA semigroup:

If $0.60 \leq \mu(W_i) \leq 1$, $0 \leq \nu(W_i) \leq 0.40$, W_i is level 1, recognized as the serious fault.

When $0.25 < \mu(W_i) < 0.60$, $0.40 < \nu(W_i) < 0.75$, W_i is level 2, recognized as the medium defect.

If $\mu(W_i) \leq 0.25$, $\nu(W_i) \leq 0.75$, W_i is normal, recognized as no fault.
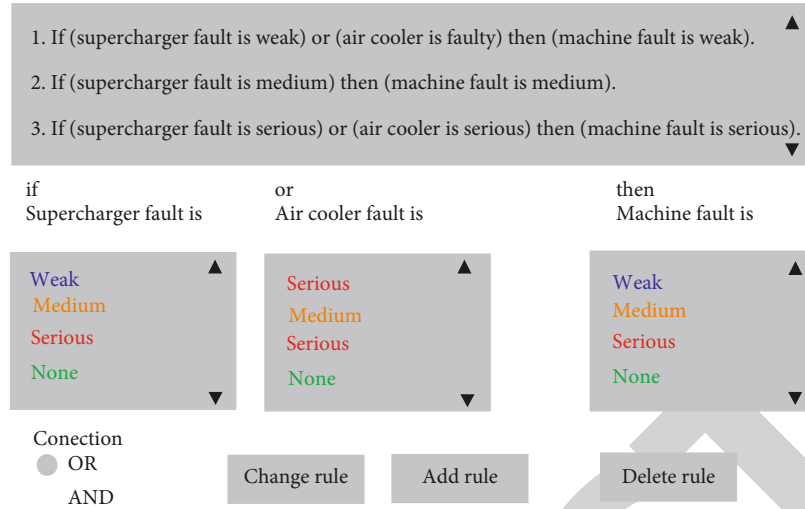
Figure 2: Intuitionistic inference system of the FDIFA semigroup for the engine fault.
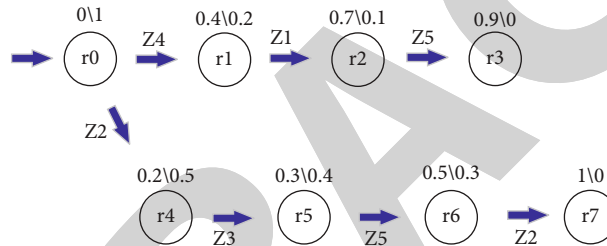

Figure 3: Fuzzy rules.

Table 1: Test data.

Input vector	Cause of fault
$((0.2,0.7)(0.1,0.7)(0.2,0.7)(0.6,0.3)(0,0.8)(0,0.9)(0.1,0.7)(0.1,0.8)(0,0.9))$	W_5: serious fault
$((0.2,0.6)(0.1,0.7)(0.2,0.6)(0.1,0.7)(0.4,0.5)(0,0.8)(0.1,0.7)(0.1,0.8)(0,0.9))$	W_6: medium fault
$((0.2,0.6)(0.1,0.8)(0.2,0.7)(0.1,0.8)(0,0.8)(0.6,0.3)(0.1,0.8)(0.1,0.7)(0,0.8))$	W_7: serious fault
$((0.2,0.7)(0.1,0.8)(0.2,0.7)(0.1,0.8)(0,0.8)(0,0.8)(0.7,0.2)(0.1,0.7)(0,0.9))$	W_8: serious fault

The seriousness of the defect can be obtained by utilizing this information. The semigroup's intuitionistic fuzzy inference model is put to the test with a collection of real data to see if it can correctly identify a twin-spool turbofan engine issue. The intuitionistic fuzzy inference model can be employed if the fault can be appropriately diagnosed. The corresponding parameters are monitored and their values are acquired when W_5, W_6, W_7, and W_8 are defective. The data is utilized as input vectors in simulation functions to compute the output of an intuitionistic fuzzy inference model. The fault criterion is used to determine whether or not a flaw exists. Table 1 shows the test results.

5, 6, 7, and 8 in W_i – out signify distinct types of faults, as illustrated in Table 1.

Suppose that MT_1, MT_2 belongs to the FDIFA semigroup $(MT, \circ)$ constructed above and $h : MT_1 \longrightarrow MT_2$ is

an inference rule in terms of FDIFA homomorphism. In the aforesaid application, the inference model for fault diagnostics can be built as follows:

(Premise1) If $MT_1{}'s$ parameter a is normal, $MT_2{}'s$ output b is serious

(Premise2) If $MT_1{}'s$ parameter a' is normal

The conclusion is to try to figure out what the fault level of the output b' in MT.

If $MT_3 = MT_1 \circ (MT_1 \longrightarrow MT_2)$, then, $MT_3 \in MT$. The degree of membership and nonmembership that corresponds to each inference step of the model can be determined using the intuitionistic fuzzy inference system, starting with the known condition.

$$G(a, b) = \left(\mu_{MT_1 \to MT_2}(a, b), \nu_{MT_1 \to MT_2}(a, b) \right) = \left[\left(\mu_{MT_1}(a) \wedge \mu_{MT_2}(b) \right) \vee \left(1 - \mu_{MT_1}(a) \right), \left(\nu_{MT_1}(a) \vee \nu_{MT_2}(b) \right) \wedge \left(1 - \nu_{MT_1}(a) - \varepsilon \right) \right],$$

$$A = \left(\left(\mu_{MT_1}(a), \nu_{MT_1}(a) \right) (\wedge, \vee) \left(\mu_{MT_2}(b), \nu_{MT_2}(b) \right) \right)$$

$$= \begin{pmatrix}
(0,0.8) & (0.2,0.7) & (0.3,0.7) & (0.4,0.5) & (0.6,0.2) & (0.8,0.1) & (0.6,0.2) & (0.7,0.1) & (0.1,0.9) & (0.3,0.5) \\
(0,0.8) & (0.2,0.7) & (0.2,0.7) & (0.2,0.5) & (0.2,0.4) & (0.2,0.4) & (0.2,0.4) & (0.2,0.4) & (0.1,0.9) & (0.2,0.5) \\
(0,0.8) & (0.1,0.7) & (0.1,0.7) & (0.1,0.6) & (0.1,0.6) & (0.1,0.6) & (0.1,0.6) & (0.1,0.6) & (0.1,0.9) & (0.1,0.6) \\
(0,0.8) & (0.2,0.7) & (0.2,0.7) & (0.2,0.5) & (0.2,0.5) & (0.2,0.5) & (0.2,0.5) & (0.2,0.5) & (0.1,0.9) & (0.2,0.5) \\
(0,0.8) & (0.1,0.8) & (0.1,0.8) & (0.1,0.8) & (0.1,0.8) & (0.1,0.8) & (0.1,0.8) & (0.1,0.8) & (0.1,0.9) & (0.1,0.8) \\
(0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) \\
(0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) & (0,0.9) \\
(0,0.8) & (0.1,0.7) & (0.1,0.7) & (0.1,0.7) & (0.1,0.7) & (0.1,0.7) & (0.1,0.7) & (0.1,0.7) & (0.1,0.9) & (0.1,0.7) \\
(0,0.9) & (0.1,0.9) & (0.1,0.9) & (0.1,0.9) & (0.1,0.9) & (0.1,0.9) & (0.1,0.9) & (0.1,0.9) & (0.1,0.9) & (0.1,0.9) \\
(0,0.8) & (0,0.7) & (0,0.7) & (0,0.6) & (0,0.6) & (0,0.6) & (0,0.6) & (0,0.6) & (0,0.9) & (0,0.6)
\end{pmatrix},$$

$$B = \left(\left(1 - \mu_{MT_1}(a) \right), \left(1 - \nu_{MT_1}(a) \right) \right) = ((0,0.9)(0.8,0.5)(0.9,0.3)(0.8,0.4)(0.9,0.1)(1,0)(1,0)(0.9,0.2)(0.9,0)(1,0.3)),$$

$$G(a, b) = G_{(\mu,\nu)}(a, b) = A(\vee, \wedge)B = \begin{pmatrix}
(0,0.8) & (0.2,0.7) & (0.3,0.7) & (0.4,0.5) & (0.6,0.2) & (0.8,0.1) & (0.6,0.2) & (0.7,0.1) & (0.1,0.9) & (0.3,0.5) \\
(0.8,0.5) & (0.8,0.5) & (0.8,0.5) & (0.8,0.5) & (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0.8,0.5) & (0.8,0.5) \\
(0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0.9,0.3) & (0.9,0.3) \\
(0.8,0.4) & (0.80.4) & (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0.8,0.4) & (0.8,0.4) \\
(0.9,0.1) & (0.9,0.1) & (0.9,0.1) & (0.9,0.1) & (0.9,0.1) & (0.9,0.1) & (0.9,0.1) & (0.9,0.1) & (0.9,0.1) & (0.9,0.1) \\
(1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) \\
(1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) & (1,0) \\
(0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) & (0.9,0.2) \\
(0.9,0) & (0.9,0) & (0.9,0) & (0.9,0) & (0.9,0) & (0.9,0) & (0.9,0) & (0.9,0) & (0.9,0) & (0.9,0) \\
(1,0.3) & (1,0.3) & (1,0.3) & (1,0.3) & (1,0.3) & (1,0.3) & (1,0.3) & (1,0.3) & (1,0.3) & (1,0.3)
\end{pmatrix},$$

(18)

where $\wedge$ is an "and" operation that can be used in the "min" operation and $\vee$ is an "or" operation that can be used in the "max" operation.

Step 1. The intuitionistic fuzzy relation matrix G can be calculated using premise1 and formula (11), as well as the intuitionistic fuzzy degree of membership and nonmembership given by the aforementioned equations (13) and (14).

Step 2. The defect level of output b' for the conclusion is as follows, based on premise2 and formula (14), as well as the above expression (17). The Fuzzy degree membership and intuitionistic degree nonmembership of $b' = [a'$ is medium] $\circ [a$ is normal, then b is serious] are as follows:

$$\left(\mu_{MT_3}(b'), \nu_{MT_3}(b') \right) = \left(\mu_{MT_1}(a'), \nu_{MT_1}(a') \right) \circ G(a, b)$$
$$= ((1, 0)(0.3,0.6)(0.2,0.8)(0.2,0.7)(0.4,0.5)(0.3,0.6)$$
$$\cdot (0.3,0.7)(0.2,0.8)(0.1,0.9)(0,0.7)) \circ G(a, b)$$
$$= ((0.4,0.5)(0.4,0.5)(0.4,0.5)(0.4,0.5)(0.6,0.2)$$
$$\cdot (0.8,0.1)(0.6,0.2)(0.7,0.1)(0.4,0.5)(0.4,0.5)).$$

(19)

When compared to the degree $(\mu_{\text{serious}}, \nu_{\text{serious}})((0,0.8)$ $(0.2,0.7)(0.3,0.7)(0.4,0.5)(0.6,0.2)(0.8,0.1)(0.6,0.2)(0.7,0.1)$ $(0.1,0.9)(0.3,0.5))$ of "serious" in MT_2, which corresponds to the fault parameters $W_1, W_2, \cdots, W_{10}$, the degree of b' in MT_3 is $((0.4,0.5)(0.4,0.5)(0.4,0.5)(0.4,0.5)(0.6,0.2)($ $0.8,0.1)(0.6,0.2)(0.7,0.1)(0.4,0.5)(0.4,0.5))$ which signifies "more serious." Since $MT_3 \in MT$, if a' is a medium fault,

the inference result for the output $b' \in MT$ is a more serious fault, which corresponds to objective reality.

6. Conclusion

The intuitionistic fuzzy set (IFS) is a well-known generalization of fuzzy sets widely studied, and significant research has been carried out to investigate IFS set theocratic properties and application in other fields. Due to the involvement of nonmembership grades, IFS can handle uncertainties better than fuzzy sets. In this article, effectiveness of IFS is utilized for fault detection in a turbofan engine. Finite deterministic intuitionistic fuzzy automata FDIFA is defined, and then, the semigroup over FDIFA is designed. The algebraic properties of the FDIFA semigroup are discussed and employed in the formulation of inference systems over the FDIFA semigroup. The proposed method is superior to the previously defined fuzzy inference method. The fuzzy inference can be derived from intuitionistic fuzzy inference by considering only the membership values. Moreover, the maximum value of the membership and nonmembership grades can be used to diagnose the maximum default and current running state of any machine. The present approaches must build equations or expressions to deal with the target based on the discussed target properties. The approach described, on the other hand, can perform characteristic processing using its state transition mapping and just requires the selection of system parameters. There are several practically useful generalizations of fuzzy sets [29, 34, 35]; the same methodology can be employed to set inference rules for these generalizations. The real-life example indicates the supremacy of the proposed methodology as it detects the fault accurately and efficiently. The proposed methodology can be used for the detection of default in any machine by identifying its crucial components, the parameters that can cause any fault in the crucial components, and the fault caused by these parameters. Membership and nonmembership grades can be assigned based on the nature and seriousness of the fault.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there were no conflicts of interest regarding the publication of this article.

Authors' Contributions

All authors contributed equally to the preparation of this manuscript.

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