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Research Article

Nonunique Fixed Point Results via Kannan F-Contraction on Quasi-Partial b-Metric Space

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This paper is aimed at acquainting with a new Kannan F-expanding type mapping by the approach of Wardowski in the complete metric space. We establish some fixed point results for Kannan F-expanding type mapping and F-contractive type mappings which satisfy F-contraction conditions. Additionally, some new results are given which generalize several results present in the literature. Moreover, some applications and examples are provided to show the practicality of our results.

1. Introduction and Preliminaries

In 1922, Banach [1] commenced one of the most essential and notable results called the Banach contraction principle, i.e., let P be a self-mapping on a nonempty set X and d be a complete metric, if there exists a constant $k \in [0, 1)$ such that

$$d(Pu, Pv) \leq k\, d(u, v), \tag{1}$$

for all $u, v \in X$. Then, it has a unique fixed point in X. Due to its significance, in 1968, Kannan [2] introduced a different intuition of the Banach contraction principle which removes the condition of continuity, i.e., for all $u, v \in [0, 1/2]$, there exists a constant $\rho \in [0, 1)$ such that

$$d(Pu, Pv) \leq \rho[d(u, Pu) + d(v, Pv)]. \tag{2}$$

On the other hand, the notion of metric space has been generalized in several directions, and the abovementioned contraction principle has been enhanced in the new settings by considering the concept of convergence of functions. In 1989, Bakhtin [3] introduced the notion of b-metric space which was revaluated by Czerwik [4] in 1993.

Definition 1. A b-metric space on a nonempty set X is a function $d : X \times X \longrightarrow [0,\infty)$ such that for all $u, v, w \in X$ and for some real number $s \geq 1$, it satisfies the following:

($M1$) If $d(u, v) = 0$, then $u = v$
($M2$) $d(u, v) = d(v, u)$
($M3$) $d(u, w) \leq s[d(u, v) + d(v, w)]$

Then, the pair (X, d, s) is called the b-metric space. Motivated by this, many researchers [5–8] generalized the concept of metric spaces and established on the existence of fixed points in the setting of b-metric space keeping in mind that, unlike standard metric, b-metric is not necessarily continuous due to the modified triangle inequality. In general, a b-metric does not induce a topology on X.

Partial metric space is one of the attempts to generalize the notion of the metric space. In 1994, Matthews [9] introduced the notion of a partial metric space in which $d(u, u)$ are no longer necessarily zero.

Definition 2. A partial metric on a nonempty set X is a function $p : X \times X \longrightarrow [0,\infty)$ such that for all $u, v, w \in X$, it satisfies the following:

($PM1$) If $p(u, u) = p(u, v) = p(v, v)$, then $u = v$
($PM2$) $p(u, u) \leq p(u, v)$

(PM3) $p(u,v) = p(v,u)$
(PM4) $p(u,v) \leq p(u,w) + p(w,v) - p(w,w)$

Then, the pair (X,p) is called the partial metric space.

Definition 3. Let (X,p) be a partial metric space. Then, several topological concepts for partial metric space can be easily defined as follows:

(1) A sequence $\{u_n\}$ in the partial metric space (X,p) converges to the limit u if $p(u,u) = \lim_{n\longrightarrow\infty} p(u,u_n)$

(2) It is said to be a Cauchy sequence if $\lim_{n\longrightarrow\infty} p(u_n, u_m)$ exists and is finite

(3) A partial metric space (X,p) is called complete if every Cauchy sequence $\{u_n\}$ in X converges with respect to τ_p, to a point $u \in X$ such that $p(u,u) = \lim_{n\longrightarrow\infty} p(u_n,u_m)$

For more details, see, for example, [10–12], and the related references therein. The following definition gives room for the lack of symmetry in the spaces under study. In 2013, Karapinar et al. [13] introduced quasi-partial metric space that satisfies the same axioms as metric spaces.

Definition 4. A quasi-partial metric on a nonempty set X is a function $q : X \times X \longrightarrow \mathbb{R}^+$ that satisfies the following:
(QPM1) If $q(u,u) = q(u,v) = q(v,v)$, then $u = v$
(QPM2) $q(u,u) \leq q(u,v)$
(QPM3) $q(u,u) \leq q(v,u)$
(QPM4) $q(u,v) + q(w,w) \leq q(u,w) + q(w,v)$for all $u, v, w \in X$. Then, the pair (X,q) is called quasi-partial metric space.

Later on, Gupta and Gautam [14, 15] introduced quasi-partial b-metric space.

Definition 5. A quasi-partial b-metric on a nonempty set X is a function $qp_b : X \times X \longrightarrow [0,\infty)$ such that for some real number $\rho \geq 1$, it satisfies the following:
(QPb_1) If $qp_b(u,u) = qp_b(u,v) = qp_b(v,v)$, then $u = v$ (indistancy implies equality)
(QPb_2) $qp_b(u,u) \leq qp_b(u,v)$ (small self-distances)
(QPb_3) $qp_b(u,u) \leq qp_b(v,u)$ (small self-distances)
(QPb_4) $qp_b(u,v) + qp_b(w,w) \leq \rho\{qp_b(u,w) + qp_b(w,v)\}$ (triangularity)
for all $u, v, w \in X$. The infimum over all reals $\rho \geq 1$ satisfying (QPb_4) is called the coefficient of (X, qp_b) and represented by $R(X, qp_b)$.

Lemma 6. *Let (X, qp_b) be a quasi-partial b-metric space. Then, the following hold*:

(i) If $qp_b(u,v) = 0$, then $u = v$

(ii) If $u \neq v$, then $qp_b(u,v) > 0$ and $qp_b(v,u) > 0$

Definition 7. Let (X, qp_b) be a quasi-partial b-metric. Then,

(i) a sequence $\{u_n\} \subset X$ converges to $u \in X$ if and only if

$$qp_b(u,u) = \lim_{n\longrightarrow\infty} qp_b(u,u_n) = \lim_{n\longrightarrow\infty} qp_b(u_n,u) \tag{3}$$

(ii) a sequence $\{u_n\} \subset X$ is called a Cauchy sequence if and only if

$$\lim_{n,m\longrightarrow\infty} qp_b(u_n,u_m) \& \lim_{m,n\longrightarrow\infty} qp_b(u_m,u_n) \text{ exist} \tag{4}$$

(iii) the quasi-partial b-metric space (X, qp_b) is said to be complete if every Cauchy sequence $\{u_n\} \subset X$ converges with respect to τqp_b to a point $u \in X$ such that

$$qp_b(u,u) = \lim_{n,m\longrightarrow\infty} qp_b(u_n,u_m) = \lim_{m,n\longrightarrow\infty} qp_b(u_m,u_n) \tag{5}$$

(iv) a mapping $f : X \longrightarrow X$ is said to be continuous at $u_0 \in X$, if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $f(B(u_0,\delta)) \subset B(f(u_0),\varepsilon)$

The extensive application of the Banach contraction principle has motivated many researchers to study the possibility of its generalization. A great number of generalizations of this famous result have appeared in the literature. In 2012, Wardowski [16] established a new notion of F-contraction and proved the fixed point theorem which generalized the Banach contraction principle.

Definition 8 (see [16]). Let (X,d) be a metric space, and there exists a mapping $F : (0,\infty) \longrightarrow \mathbb{R}$ which satisfies the following condition:
(F_1) F is strictly increasing
(F_2) For any sequence $\{x_n\}_{n\in N}$, $\lim_{n\longrightarrow\infty} x_n = 0$ if and only if $\lim_{n\longrightarrow\infty} F(x_n) = -\infty$
(F_3) $\lim_{x\longrightarrow 0^+} x^k F(x) = 0$ for some $k \in (0,1)$
Then, a mapping $P : X \longrightarrow X$ is said to be Wardowski F-contraction if $d(Pu,Pv) > 0$ implies

$$\delta + F(d(Pu,Pv)) \leq F(d(u,v)) \tag{6}$$

for all $u, v \in X$

Theorem 9 (see [16]). *Let (X,d) be a complete metric space and $T : X \longrightarrow X$ an F-contraction. Then, T has a unique fixed point $x^* \in X$, and for every $x \in X$, the sequence $\{T^n x\}_{n\in N}$ converges to x^*.*

In 2012, Samet et al. [17] established the class of α-admissible mappings as follows.

Definition 10 (see [17]). Let $\alpha : X \times X \longrightarrow [0,\infty)$ be given mapping where X is a nonempty set. A self-mapping T is called α-admissible if for all $x, y \in X$, we have

$$\alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, Ty) \geq 1. \tag{7}$$

Motivated by this, Aydi et al. [18] extended the notion of F-contraction and prove the following result.

Theorem 11 (see [18]). *Let (X, d) be a metric space. A self-mapping $T : X \longrightarrow X$ is said to be a modified F-contraction via α-admissible mappings. Suppose that*

(i) T is α-admissible

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$

(iii) T is continuous

Then, T has a fixed point. In 2015, Kumam et al. [19] generalized the contraction condition by adding four new values $d(T^2x, x), d(T^2x, Tx), d(T^2x, y), d(T^2x, Ty)$ and introduced F-Suzuki contraction mappings in complete metric space. The Suzuki-type generalization can be said to have many applications, as in computer science, game theory, biosciences, and in other areas of mathematical sciences such as in dynamic programming, integral equations, and data dependence. Recently, Wardowski [20] proposed the replacement of the positive constant δ in equation (6) by a function ϕ and relaxed the conditions on F.

Definition 12 (see [20]). Let (X, d) be a metric space, $F : (0,\infty) \longrightarrow \mathbb{R}$ and $\phi : (0,\infty) \longrightarrow (0,\infty)$ satisfy the following:

(1) F is strictly increasing, i.e., $x < y$ implies $F(x) < F(y)$ for all $x, y \in (0,\infty)$

(2) $\lim_{\alpha \longrightarrow 0^+} F(\alpha) = -\infty$

(3) $\liminf_{\alpha \longrightarrow s^+} \phi(\alpha) > 0$ for all $s > 0$

A mapping $T : X \longrightarrow X$ is called an (ϕ, F)-contraction on (X, d) if

$$\phi(d(x, y)) + F(d(Tx, Ty)) \leq F(d(x, y)), \tag{8}$$

for all $x, y \in X$ for which $Tx \neq Ty$.

Consider a function $F_B : \mathbb{R}^+ \longrightarrow \mathbb{R}$ by $F_B(u) = \ln u$. Note that with $F = F_B$, the F-contraction reduces to a Banach contraction. Therefore, the Banach contractions are a particular case of F-contractions. Meanwhile, there exist F-contractions which are not Banach contractions.

The concept of an F-contraction has been generalized in many directions (see, e.g., [21–24]), and as an extension, engaging work was done by many authors [25–34], which enhanced this field. In 2015, Cosentino et al. [35] extended the concept of F-contraction in metric space to F-contraction in b-metric space by introducing the following condition with continuation of Definition 7.

(F_4) For some $\delta > 0$ and any sequence $\{x_n\}$, we have

$$\begin{aligned} \delta + F(sx_n) &\leq F(x_{n-1}), \\ \delta + F(s^n x_n) &\leq F\left(s^{n-1} x_{n-1}\right) \end{aligned} \tag{9}$$

for all $n \in N, s \in \mathbb{R}$

In 2017, Gornicki [36] established F-expanding type mappings.

Definition 13. Let (X, d) be a metric space. A mapping $P : X \longrightarrow X$ is called F-expanding if for all $u, v \in X$ and $\delta > 0$, we have

$$d(u, v) > 0 \Rightarrow F(d(Pu, Pv)) \geq F(d(u, v)) + \delta. \tag{10}$$

The concept of F-expanding type mappings was redefined as Kannan F-expanding type mappings by Goswami et al. [37].

Definition 14 (see [37]). A mapping $P : X \longrightarrow X$ is said to be Kannan F-expanding type mapping if there exists $\Delta > 0$ such that $d(u, Pu)d(v, Pv) \neq 0$ implies

$$\Delta + F(sd(u, v)) \leq \frac{1}{2}\{F(d(u, Pu)) + F(d(v, Pv))\}, \tag{11}$$

and $d(u, Pu)d(v, Pv) = 0$ implies

$$\Delta + F(sd(u, v)) \leq \frac{1}{2}\{F(d(u, Pv)) + F(d(v, Pu))\} \tag{12}$$

for all $u, v \in X$. Following this direction, we have established a new type of mapping, i.e., Kannan F-expanding type mapping, and proved some fixed point results for F-contractive type mappings as well as Kannan F-expanding type mappings in the setting of quasi-partial b-metric space without using the continuity of mapping. Also, we attain the non-unique fixed point in quasi-partial b-metric space which lacks symmetry property.

The main motive behind this study is that today, this field of research has vast literature. The significance of the Kannan type mapping is that it characterizes completeness which the Banach contraction does not; also, it does not require continuous mapping. In this paper, some examples and applications for the solution of a certain integral equation and the existence of a bounded solution of the functional equation are also given to represent the practicality of the results obtained. The application shows the role of fixed point theorems in dynamic programming, which is used in computer programming and optimization.

The future aspect of this study is to prove the existence of a unique fixed point in Kannan F-expanding type mapping. Another field of research can be the existence of a common fixed point for the same. The notion of interpolative F

-contraction as well as interpolation for Kannan F-expanding type mapping can also be future studies concerning the present manuscript.

2. Fixed Point for F-Contractive Type Mappings

In this section, the existence of a fixed point for F-contractive type mappings in a quasi-partial b-metric space is obtained.

Definition 15. For a quasi-partial b-metric space (X, qp_b), a mapping $P : X \longrightarrow X$ is said to be an F-contractive type mapping if there exists $\delta > 0$ such that, if $qp_b(u, Pu)qp_b(v, Pv) \neq 0$, then

$$\delta + F(\rho qp_b(Pu, Pv)) \leq \frac{1}{3}[F(qp_b(u, v)) + F(qp_b(u, Pv)) + F(qp_b(v, Pu))] - F(qp_b(w, Pw)), \tag{13}$$

and if $qp_b(u, Pu)qp_b(v, Pv) = 0$, then

$$\delta + F(\rho qp_b(Pu, Pv)) \leq \frac{1}{3}[F(qp_b(u, v)) + F(qp_b(u, Pu)) + F(qp_b(v, Pv))] - F(qp_b(w, Pw)) \tag{14}$$

for all $u, v, w \in X$ and $\rho \geq 1$.

Definition 16. Let (X, qp_b) be a quasi-partial b-metric space. A self-mapping P on X is called an F-contraction if there exist $\tau \in \mathbb{R}^+$ such that

$$\tau + F(\rho qp_b(Pu, Pv)) \leq F(qp_b(u, v)), \tag{15}$$

for all $u, v \in X$ with $qp_b(Pu, Pv) > 0$.

Example 17. Let $F : \mathbb{R}^+ \longrightarrow \mathbb{R}$ be given by $F(u) = \log u$. Here, F satisfies (F1)-(F3) for any $k \in (0, 1)$. Each mapping $P : X \longrightarrow X$ satisfying Definition 16 is an F-contraction such that

$$qp_b(Pu, Pv) \leq \rho e^{-\tau} qp_b(u, v) \tag{16}$$

for all $u, v \in X, Pu \neq Pv$.

It is clear that for $u, v \in X$ such that $Pu = Pv$, the previous inequality also holds, and hence, P is a contraction as shown in Figure 1.

Example 18. Consider a function $F(u) = -1/\sqrt{u}, u > 0$ where F satisfies (F1)-(F3) for any $k \in (1/2, 1)$. In this case, a mapping $P : X \longrightarrow X$ satisfies

$$\rho qp_b(Pu, Pv) \leq \frac{1}{\left(1 + \tau\sqrt{qp_b(u, v)}\right)^2} qp_b(u, v) \tag{17}$$

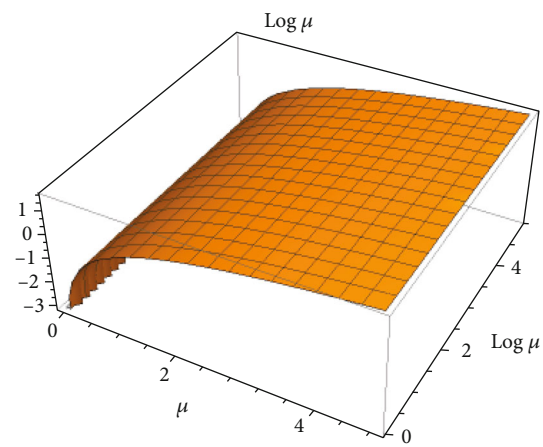

Figure 1: The graphical surface represents a 3-D view of F-contractive function $F(u) = \log u$ where $u \in (0, 5), v \in (0, 5)$ and $w \in \mathbb{R}^+$.

for all $u, v \in X, Pu \neq Pv$. Hence, P is a contraction as shown in Figure 2.

Theorem 19. *Let (X, qp_b) be a quasi-partial b-metric space and $P : X \longrightarrow X$ be an F-contractive type mapping. Then, P has a unique fixed point $u^* \in X$, and for every $u_0 \in X$, a sequence $\{P^n x_0\}_{n\in\mathbb{N}}$ is convergent to u^*.*

Proof. Let u_0 be an arbitrary and fixed point in X, and we assume a sequence $\{u_n\}_{n\in N} \subset X$ such that $u_{n+1} = Pu_n, n = 0, 1, \cdots$. To prove P has a fixed point, we need to show that if $u_{n_0+1} = u_{n_0}$, then $Pu_{n_0} = u_{n_0}$ for all $n_0 \in \mathbb{N}$. Suppose that $u_{n+1} \neq u_n$ for every $n \in \mathbb{N}$, then $qp_b(u_{n+1}, u_n) > 0$, and using equation (6), we have

$$F(qp_b(u_{n+1}, u_n)) \leq \rho F(qp_b(u_n, u_{n-1})) - \delta \leq \rho F(qp_b(u_{n-1}, u_{n-2})) - 2\delta \leq \vdots \leq \rho F(qp_b(u, u_0)) - n\delta, \tag{18}$$

which implies

$$\lim_{n\longrightarrow\infty} \rho F(qp_b(u_{n+1}, u_n)) = -\infty. \tag{19}$$

Using (F_2), we get

$$\lim_{n\longrightarrow\infty} \rho\, qp_b(u_{n+1}, u_n) = 0. \tag{20}$$

Also, using (F_3), there exists $k \in (0, 1)$ such that

$$\lim_{n\longrightarrow\infty} qp_b(u_{n+1}, u_n)^k F(qp_b(u_{n+1}, u_n)) = 0. \tag{21}$$

Let us denote $qp_b(u_{n+1}, u_n)$ by α_n. From inequality (18), the following holds

$$F(\alpha_n) - \alpha_n^k F(\alpha_0)) \leq \rho\left(\alpha_n^k(F(\alpha_0) - n\delta) - \alpha_n^k F(\alpha_0)\right) = -\rho\, \alpha_n^k\, n\delta \leq 0, \tag{22}$$

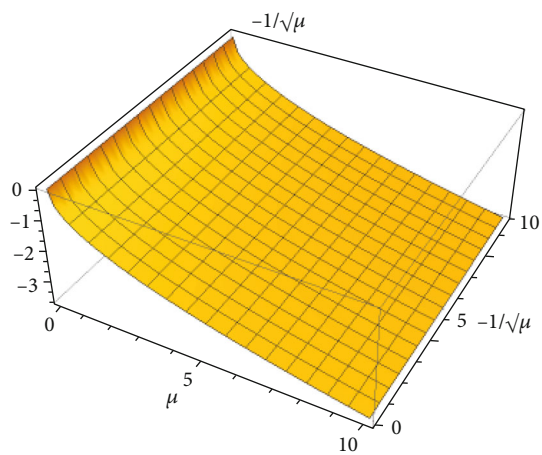

Figure 2: The graphical surface represents a 3-D view of F-contractive function $F(u) = -1/\sqrt{u}$ where $u \in (0, 10), v \in (0, 10)$ and $w \in \mathbb{R}^+$.

which implies

$$\lim_{n \longrightarrow \infty} n\alpha_n^k = 0. \tag{23}$$

Also, if there exists $n_1 < n \in \mathbb{N}$ such that $n\alpha_n^k \leq 1$, we have

$$\rho\, \alpha_n \leq n^{-1/k}. \tag{24}$$

To prove $\{u_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence, let us consider $m, n \in \mathbb{N}$ such that $m > n \geq n_1$. From the definition of quasi-partial b-metric space and equation (24), we have

$$qp_b(u_m, u_n) \leq \rho(\alpha_{m-1} + \alpha_{m-2} + \cdots + \alpha_n) \leq \rho \sum_{i=n}^{\infty} \alpha_i \leq \rho \sum_{i=n}^{\infty} i^{-1/k}. \tag{25}$$

Using the convergence of series, we get that $\{u_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence. Since X is complete, there exists $u^* \in X$ such that $\lim_{n\longrightarrow\infty} u_n = u^*$, and the continuity of P implies

$$qp_b(Pu^*, u^*) = \rho \lim_{n\longrightarrow\infty} qp_b(Pu_n, u_n) = \rho qp_b(u_{n+1}, u_n) = 0. \tag{26}$$

Hence, P has a unique fixed point.□ □

Theorem 20. *For a quasi-partial b-metric space (X, qp_b), we say X is complete if for every closed subset Y of X, $P : Y \longrightarrow Y$ is an F-contractive type mapping having a fixed point.*

Proof. Suppose that there does not exist any Cauchy sequence in X which has a convergent subsequence and we have a sequence

$$\theta(u_n) = \inf\, \{qp_b(u_n, u_m) \colon m > n\} > 0 \tag{27}$$

for all $n \in \mathbb{N}$ where $\theta(u_n) \leq \theta(u_m)$ for $m \geq n$. Also, we consider a subsequence $\{u_{n_k}\}$ such that

$$\rho qp_b(u_i, u_j) < a\theta(u_{n_{k-1}}), \tag{28}$$

for any a with $0 < a < 1$ and for all $i, j \geq n_k$. Then, $Y = \{u_{n_k} : k \in \mathbb{N}\}$ is a closed subset of X. Define $P : X \longrightarrow X$ by

$$Pu_{n_k} = u_{n_k+1} \tag{29}$$

for all $k \in \mathbb{N}$, which implies P has no fixed point. Now,

$$\rho qp_b(Pu, Pv) = \rho qp_b(Pu_{n_k}, Pu_{n_k+1}) = \rho qp_b(u_{n_k+1}, Pu_{n_{k+i+1}}) < a\theta(u_{n_k}). \tag{30}$$

By definition,

$$\begin{aligned}\theta(u_{n_k}) &\leq \rho qp_b(u_{n_k}, u_{n_{k+i}}) = \rho qp_b(u, v) \leq \rho qp_b(u_{n_k}, u_{n_k+1}) \\ &= \rho qp_b(u, Pu) \leq \theta(u_{n_{k+i}}) \leq \rho qp_B(u_{n_{k+i}}, u_{n_{k+i+1}}) \\ &= \rho qp_b(v, Pv),\end{aligned} \tag{31}$$

which implies

$$\begin{aligned}\delta + F(\rho qp_b(Pu, Pv)) \leq \frac{1}{3}\{&F(qp_b(u, v)) + F(qp_b(u, Pu)) \\ &+ F(qp_b(v, Pv))\} - (qp_b(w, Pw))\end{aligned} \tag{32}$$

for some $\delta > 0$. Hence, it proves that P is an F-contractive type mapping on a closed subset of X which has no fixed point. Thus, this is a contradiction and X is complete.□ □

Theorem 21. *Let (X, qp_b) be a quasi-partial b-metric space and $P : X \times C(X)$ be a closed F-contraction. Then, P has a fixed point.*

Proof. Let $u_0 \in X$ be an arbitrary point of X, and we have $u_1 \in Pu_0$. If $u_1 = u_0$, then u_1 is a fixed point of P, and hence, the proof is completed. Now, assume that $u_1 \neq u_0$. Since P is a F-contraction, there exists $u_2 \in Pu_1$ such that

$$\tau + F(\rho qp_b(u_1, u_2)) \leq F(M(u_0, u_1)), \tag{33}$$

where

$$M(u_0, u_1) = \max\left\{qp_b(u_0, u_1), qp_b(u_0, Pu_0), qp_b(u_1, Pu_1), \frac{1}{2}[qp_b(u_0, Pu_1) + qp_b(u_1, Pu_0)]\right\}, \tag{34}$$

and $u_2 \neq u_1$. Also, there exists $u_3 \in Pu_2$ such that

$$\tau + F(\rho qp_b(u_2, u_3)) \leq F(M(u_1, u_2)), \tag{35}$$

and $u_3 \neq u_2$. With the recurrence of the same process, we get

$$\tau + F(\rho qp_b(u_n, u_{n+1})) \leq F(M(u_{n-1}, u_n)) \tag{36}$$

for all $n \in \mathbb{N}$. It implies

$$\tau + F(\rho qp_b(u_n, u_{n+1})) \leq F(M(u_{n-1}, u_n)). \tag{37}$$

Assume that $qp_{b_n} = qp_b(x_n, x_{n+1}) > 0$ for all $n \in \mathbb{N} \cup \{0\}$. By equation (37), we have

$$F\left(\rho qp_{b_n}\right) \leq F\left(\rho qp_{b_{n-1}}\right) \leq \cdots < \leq F\left(\rho qp_{b_0}\right) - n\tau \tag{38}$$

for all $n \in \mathbb{N}$. Letting $n \longrightarrow \infty$, property (F_2) implies

$$\lim_{n \longrightarrow \infty} F\left(\rho qp_{b_n}\right) = -\infty. \tag{39}$$

Let $k \in (0, 1)$ such that

$$\lim_{n \longrightarrow \infty} qp_{b_n}^k F\left(\rho qp_{b_n}\right) = 0. \tag{40}$$

By equation (38), the following holds

$$\begin{aligned} &qp_{b_n}^k F\left(\rho qp_{b_n}\right) - qp_{b_n}^k F\left(\rho qp_{b_0}\right) \\ &\quad \leq qp_{b_n}^k \left(F\left(\rho qp_{b_0}\right) - n\tau\right) - qp_{b_n}^k F\left(\rho qp_{b_0}\right) = -n\tau qp_{b_n}^k \leq 0 \end{aligned} \tag{41}$$

for all $n \in \mathbb{N}$. Letting $n \longrightarrow \infty$, we get

$$\lim_{n \longrightarrow \infty} nqp_{b_n}^k = 0. \tag{42}$$

This implies $\lim_{n \longrightarrow \infty} n^{1/k} qp_{b_n} = 0$ and $\sum_{n=1}^{+\infty} qp_{b_n}$ is convergent. Hence, $\{u_n\}$ is Cauchy sequence. Since X is complete, there exists $x \in X$ such that $u_n = x$ as $n \longrightarrow +\infty$. Since P is closed, $(u_n, u_{n+1}) \longrightarrow (x, x)$, we get $x \in Px$, and hence, x is the fixed point of P.□ □

Corollary 22. *Let (X, qp_b) be a quasi-partial b-metric space and $P : X \longrightarrow C(X)$ be an upper semicontinuous F-contraction. Then, P has a fixed point.*

Example 23. Consider the quasi-partial b-metric space (X, qp_b) where $X = \{0, 2, 4, \cdots\}$ and $qp_b : X \times X \longrightarrow [0, \infty)$ is given by

$$qp_b(u, v) = \begin{cases} u + v, & u \neq v, \\ 0, & u = v, \end{cases} \tag{43}$$

which is also shown in Figure 3, and $P : X \longrightarrow C(X)$ is defined by

$$P(u) = \begin{cases} \{0\}, & u \in [0, 1], \\ \{0, 2, \cdots, 2u - 2\}, & u \geq 4. \end{cases} \tag{44}$$

Now, we show that P satisfies Definition 16, where $\rho = 2, \tau = 2$ and $F(u) = \log u + u$ for each $u \in \mathbb{R}^+$. Let for all $u, v \in X$ with $v \in Pu$, we have $w = 0 \in Pv$. Here, $qp_b(v, w) > 0$ iff

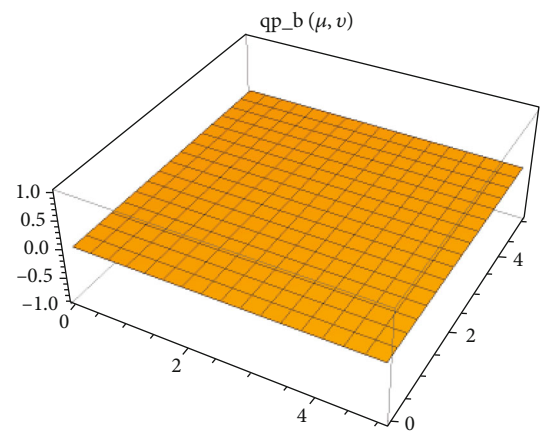

Figure 3: 3-D view. The plane in yellow represents the quasi-partial b-metric space defined by $qp_b : X \times X \longrightarrow [0, \infty)$ and $qp_b(u, v) = u + v$ when $u \neq v$ for all $u, v \in X$.

$x \geq 4$ and $w > 0$. If it is true, then

$$qp_b(v, w) = v < u + v = qp_b(u, v). \tag{45}$$

This implies

$$\begin{aligned} &qp_b(v, w) - M(u, v) \leq qp_b(v, w) - qp_b(u, v) \leq -4, \\ &\frac{qp_b(v, w)}{M(u, v)} e^{qp_b(u,v) - M(u,v)} \leq e^{-2}. \end{aligned} \tag{46}$$

Hence,

$$2 + F(\rho qp_b(v, w)) \leq F(M(u, v)) \tag{47}$$

for all $u, v \in X$ and $qp_b(v, w) > 0$. Then, by Theorem 21, P has a fixed point.

3. Fixed Point for Kannan F-Expanding Type Mapping

In this section, we prove the fixed point results for Kannan F-expanding type mappings in a quasi-partial b-metric space.

Definition 24. Let us consider a mapping $P : X \longrightarrow X$; it is said to be Kannan F-expanding type mapping if there exists $\Delta > 0$ such that $qp_b(u, Pu) qp_b(v, Pv) \neq 0$ implies

$$\begin{aligned} \Delta + F(\rho qp_b(u, v)) \leq \frac{1}{2} &[F(qp_b(u, Pu)) + F(qp_b(v, Pv))] \\ &- F(qp_b(Pw, Pw)), \end{aligned} \tag{48}$$

and $qp_b(u, Pu)qp_b(v, Pv) = 0$ implies

$$\Delta + F(\rho qp_b(u, v)) \leq \frac{1}{2}[F(qp_b(u, Pv)) + F(qp_b(Pv, Pu))] - F(qp_b(Pw, Pw)) \tag{49}$$

for all $u, v, w \in X$.

Lemma 25. *Let (X, qp_b) be a quasi-partial b-metric space and $P : X \longrightarrow X$ be surjective. Then, there exists a mapping $P^* : X \longrightarrow X$ such that $P \circ P^*$ is the identity map on X.*

Proof. For any point $u \in X$, let $v_u \in X$ be any point such that $Pv_u = u$. Let $P^*u = v_u$ for all $u \in X$. Then, $(P \circ P^*)(u) = P(P^*u) = Pv_u = u$ for all $u \in X$.□ □

Theorem 26. *Let (X, qp_b) be a quasi-partial b-metric space and $P : X \longrightarrow X$ be surjective and a Kannan F-expanding type mapping. Then, P has a unique fixed point $\gamma \in X$.*

Proof. Assume that there exists a mapping $P^* : X \longrightarrow X$ such that $P \circ P^*$ is the identity map on X. Let u, v be arbitrary points of X such that $u \neq v$ and $x = P^*u, y = P^*v$ which also implies that $x \neq y$. Applying equation (48) on x, y, we have

$$\Delta + F(\rho qp_b(x, y)) \leq \frac{1}{2}[F(qp_b(x, Px)) + F(qp_b(y, Py))] - F(qp_b(z, Pz)) \tag{50}$$

for $qp_b(x, Px)qp_b(y, Py) \neq 0$ and

$$\Delta + F(\rho qp_b(x, y)) \leq \frac{1}{2}[F(qp_b(x, Py)) + F(qp_b(y, Px))] - F(qp_b(z, Pz)) \tag{51}$$

for $qp_b(x, Px)qp_b(y, Py) = 0$. Since $Px = P(P^*(u)) = u$ and $Py = P(P^*(v)) = v$, we get

$$\Delta + F(\rho qp_b(P^*u, P^*v)) \leq \frac{1}{2}[F(qp_b(u, P^*u)) + F(qp_b(v, P^*v))] - F(qp_b(w, P^*w)) \tag{52}$$

for $qp_b(u, Pu)qp_b(v, Pv) \neq 0$ and

$$\Delta + F(\rho qp_b(P^*u, P^*v)) \leq \frac{1}{2}[F(qp_b(u, P^*v)) + F(qp_b(v, P^*u))] - F(qp_b(w, P^*w)) \tag{53}$$

for $qp_b(u, Pu)qp_b(v, Pv) = 0$, which implies P^* is Kannan F-contractive type mapping. Also, we know that P^* has a unique fixed point $\gamma \in X$, and for every $u_0 \in X$, the sequence $\{P^{*n}u_0\}$ converges to γ. In particular, γ is also a fixed point of P since $P^*\gamma = \gamma$ implies that

$$P\gamma = P(P^*\gamma) = \gamma. \tag{54}$$

Finally, if $\gamma_0 = P\gamma_0$ is another fixed point, then from equation (49),

$$\Delta + F(\rho qp_B(\gamma, \gamma_0)) \leq \frac{1}{2}[F(qp_b(\gamma, P\gamma_0)) + F(qp_b(\gamma_0, P\gamma))] - F(qp_b(v, Pv)), \tag{55}$$

which is not possible, and hence, P has a unique fixed point.□ □

4. Applications of F-Contraction

In this section, we discuss the applications of the results obtained to prove the existence of the solution of an integral equation and a functional equation.

4.1. Existence of Solution of Integral Equation. Now, we study the existence of solution of the following Volterra type integral equation

$$u(x) = \int_0^x f(x, y, u(y))dy + g(x), \tag{56}$$

$x \in [0, \sigma]$ where $\sigma > 0$. Let $C([0, \sigma], \mathbb{R})$ denote space of all continuous functions on $[0, \sigma]$, and for an arbitrary $u \in C([0, \sigma], \mathbb{R})$, we define

$$||u||_\tau = \sup_{x\in[0,\sigma]}\{|u(x)|e^{-\tau x}\}, \tag{57}$$

where $\tau > 0$ is taken arbitrary. Clearly, $(C([0, \sigma], \mathbb{R}), \|\cdot\|_\tau)$ is endowed with quasi-partial b-metric defined by

$$qp_b(u, v) = \sup_{x\in[0,\sigma]}\{|u(x) - v(x) \mid e^{-\tau x}\} \tag{58}$$

for all $u, v \in C([0, \sigma], \mathbb{R})$ is a Banach space and

$$u \leq v \Leftrightarrow u(x) \leq v(x) \tag{59}$$

for all $x \in [0, \sigma]$.

Theorem 27. *Let us consider that for the integral equation (56), the following conditions are satisfied:*

(i) f and g are continuous where $f : [0, \sigma] \times [0, \sigma] \times \mathbb{R} \longrightarrow \mathbb{R}$ and $g : [0, \sigma] \longrightarrow \mathbb{R}$

(ii) $f(x, y, .)$: $\mathbb{R} \longrightarrow \mathbb{R}$ is increasing

(iii) $u_0(x) \leq \int_0^x f(x, y, u_0(y))dy + g(x)$ for some $u_0 \in C([0, \sigma], \mathbb{R})$

(iv) There exists $\tau \in [1,\infty)$ such that

$$|f(x, y, u) - f(x, y, v)| \leq \tau e^{-2\tau}\rho|u - v| \tag{60}$$

for all $x, y \in [0, \sigma]$ and $u, v \in \mathbb{R}$. Then, integral equation (56) has a solution.

Proof. Here, $(C([0,\sigma], \mathbb{R}), qp_b)$ is a quasi-partial b-metric space, where qp_b is the quasi-partial b-metric given by equation (58). Let us define $P : C([0,\sigma], \mathbb{R}) \longrightarrow C([0,\sigma], \mathbb{R})$ by

$$P(u)(x) = \int_0^x f(x, y, u(y))dy + g(x), x \in [0, \sigma]. \tag{61}$$

From (iv) we have,

$$\begin{aligned} |P(u)(x) - P(v)(x)| &\leq \int_0^x |f(x, y, u(y)) - f(x, y, v(y))|dy \\ &\leq \int_0^x \tau e^{-2\tau} \rho |u(y) - v(y)|dy \\ &= \int_0^x \tau e^{-2\tau} \rho |u(y) - v(y)| e^{-\tau y} e^{\tau y} dy \\ &\leq \int_0^x \tau e^{-2\tau} e^{\tau y} \rho |u(y) - v(y)| e^{-\tau y} dy \\ &\leq \tau e^{-2\tau} \rho \|u - v\|_\tau \int_0^x e^{\tau y} dy \\ &\leq \tau e^{-2\tau} \rho \frac{1}{\tau} \|u - v\|_\tau e^{\tau x}. \end{aligned} \tag{62}$$

It implies

$$|P(u)(x) - P(v)(x)| e^{-\tau x} \leq e^{-2\tau} \rho \|u - v\|_\tau \tag{63}$$

or

$$qp_b(P(u), P(v)) \leq e^{-2\tau} \rho qp_b(u, v). \tag{64}$$

Taking logarithm in both sides, we get

$$\ln (qp_b(P(u), P(v))) \leq \ln \left(e^{-2\tau} \rho qp_b(u, v))\right), \tag{65}$$

which on solving reduces to

$$2\tau + \ln (qp_b(P(u), P(v))) \leq \ln (\rho qp_b(u, v))). \tag{66}$$

Now, we observe that the function $F : \mathbb{R}^+ \longrightarrow \mathbb{R}$ defined by $F(u) = \log u$ for each $u \in C([0,\sigma], \mathbb{R})$ is F-contraction. Clearly, from (iii), we have

$$u_0 \leq P(u_0), \tag{67}$$

and hence, Theorem 19 applies to P, which has a fixed point $u^* \in C([0,\sigma], \mathbb{R})$. Hence, u^* is a solution of integral equation (56).□ □

4.2. Existence of Bounded Solutions of Functional Equations. Fixed point theory is widely used in the field of dynamic programming which is the most commonly used tool for mathematical optimization. With this approach, the problem of the dynamic programming process reduces to solving the functional equations.

Let us consider that U and V are Banach spaces, $W \subset U$ is a state space, i.e., the set of the initial state of process, and $D \subset V$ is a decision space, i.e., the set of possible actions that are allowed for the process.

Here, we will prove the existence of the bounded solution of the following functional equation:

$$\phi(u) = \sup_{x \in D} \{f(u, v) + g(u, v + \phi(\tau(u, v)))\}, \tag{68}$$

where $\tau : W \times D, f : W \times D \longrightarrow \mathbb{R}, g : W \times D \times \mathbb{R} \longrightarrow \mathbb{R}$. Let $B(W)$ denote the set of all bounded real valued functions W and for an arbitrary $\alpha \in B(W)$, define $\|\alpha\| = \sup_{x \in W} |\alpha(x)|$. Clearly, $(B(w), \|\cdot\|)$ endowed with quasi-partial b-metric defined by

$$qp_b(\alpha, \beta) = \sup_{x \in W} |\alpha(x) - \beta(x)| \tag{69}$$

for all $\alpha, \beta \in B(W)$ is a Banach space. Thus, if we consider a Cauchy sequence $\{\alpha_n\}$ in $B(W)$, then $\{\alpha_n\}$ converges uniformly to a function, let α^* that is bounded and so $\alpha^* \in B(w)$. Also, we have $P : B(W) \times B(W)$ defined by

$$P(\alpha)(x) = \sup_{y \in D} \{f(x, y) + g(x, y, \alpha(\tau(x, y)))\} \tag{70}$$

for all $\alpha \in B(W)$ and $x \in W$. Hence, P is well defined if f and g are bounded.

Theorem 28. *Let $P : B(W) \longrightarrow B(W)$ be an upper semicontinuous operator defined by (70), and we assume that the following conditions are satisfied:*

(i) f and g are bounded and continuous where $f : W \times D \longrightarrow \mathbb{R}$ and $g : W \times D \times \mathbb{R} \longrightarrow \mathbb{R}$

(ii) There exists $\tau \in \mathbb{R}^+$ such that

$$|g(x, y, \alpha(x)) - g(x, y, \beta(x))| \leq \frac{\rho \,|\, \alpha - \beta \,|}{\left(1 + \tau \sqrt{|\alpha - \beta|}\right)^2} \tag{71}$$

for all $\alpha, \beta \in B(W), x \in W, y \in D, \rho \geq 1$. Then, the functional equation (68) has a bounded solution.

Proof. Clearly, $(B(W), qp_b)$ is a quasi-partial b-metric given by equation (69). Let σ be an arbitrary positive number, $x \in W$, $\alpha_1, \alpha_2 \in B(W)$, then there exist $y_1, y_2 \in D$ such that

$$P(\alpha_1)(x) < f(x, y_1) + g(x, y_1, \alpha_1(\tau(x, y_1))) + \sigma, \tag{72}$$

$$P(\alpha_2)(x) < f(x, y_2) + g(x, y_2, \alpha_2(\tau(x, y_2))) + \sigma, \tag{73}$$

$$P(\alpha_1)(x) < f(x, y_2) + g(x, y_2, \alpha_1(\tau(x, y_2))) + \sigma, \tag{74}$$

$$P(\alpha_2)(x) < f(x, y_1) + g(x, y_1, \alpha_2(\tau(x, y_1))) + \sigma. \tag{75}$$

From equations (72) and (75),

$$\begin{aligned} P(\alpha_1)(x) - P(\alpha_2)(x) &< g(x, y_1, \alpha_1(\tau(x, y_1))) - g(x, y_1, \alpha_2(\tau(x, y_1))) + \sigma \\ &\leq |g(x, y_1, \alpha_1(\tau(x, y_1))) - g(x, y_1, \alpha_2(\tau(x, y_1)))| + \sigma \\ &\leq \frac{\rho \mid \alpha_1 - \alpha_2 \mid}{\left(1 + \tau\sqrt{|\alpha_1 - \alpha_2 \mid}\right)^2} + \sigma. \end{aligned} \tag{76}$$

It implies,

$$P(\alpha_1)(x) - P(\alpha_2)(x) \leq \frac{\rho \mid \alpha_1 - \alpha_2 \mid}{\left(1 + \tau\sqrt{|\alpha_1 - \alpha_2 \mid}\right)^2} + \sigma. \tag{77}$$

Similarly, from equations (73) and (74),

$$P(\alpha_2)(x) - P(\alpha_1)(x) \leq \frac{\rho \mid \alpha_1 - \alpha_2 \mid}{\left(1 + \tau\sqrt{|\alpha_1 - \alpha_2 \mid}\right)^2} + \sigma. \tag{78}$$

From equations (77) and (78), we get

$$|P(\alpha_1)(x) - P(\alpha_2)(x)| \leq \frac{\rho|\alpha_1 - \alpha_2|}{\left(1 + \tau\sqrt{|\alpha_1 - \alpha_2|}\right)^2} + \sigma, \tag{79}$$

i.e.,

$$qp_b(P(\alpha_1), P(\alpha_2)) \leq \frac{\rho \mid \alpha_1 - \alpha_2 \mid}{\left(1 + \tau\sqrt{|\alpha_1 - \alpha_2 \mid}\right)^2} + \sigma. \tag{80}$$

Hence, we conclude that

$$qp_b(P(\alpha_1), P(\alpha_2)) \leq \frac{\rho \mid \alpha_1 - \alpha_2 \mid}{\left(1 + \tau\sqrt{|\alpha_1 - \alpha_2 \mid}\right)^2}. \tag{81}$$

Now, we observe that the function $F : \mathbb{R}^+ \longrightarrow \mathbb{R}$ defined by $F(\alpha) = -1/\sqrt{\alpha}$ for each $\alpha \in W$ is F-contractive function, and hence, operator P is F-contractive.

Since any upper semicontinuous F-contractive function has a fixed point $\alpha^* \in B(W)$, it implies that there exists a bounded solution of functional equation (68).□ □

5. Conclusion

In this manuscript, we established a new type of mappings that is Kannan F-expanding mappings and obtained fixed point theorems for contractive mappings in the framework of quasi-partial b-metric spaces. Moreover, we provided examples that demonstrate the usability of our results. As an application of our result, we also studied a system of integral and functional inclusions. It would be more engaging to work on the obtained results to prove the uniqueness of the fixed point in the future.

Data Availability

This clause is not applicable to this paper.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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