Research Article

A Fractional Order Hepatitis C Mathematical Model with Mittag-Leffler Kernel

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In this paper, a mathematical fractional order Hepatitis C virus (HCV) spread model is presented for an analytical and numerical study. The model is a fractional order extension of the classical model. The paper includes the existence, singularity, Hyers-Ulam stability, and numerical solutions. Our numerical results are based on the Lagrange polynomial interpolation. We observe that the model of fractional order has the same behavior of the solutions as the integer order existing model.

1. Introduction

Hepatitis C is a kind of viral maladies caused by the Hepatitis C virus (HCV), which mostly damages the liver. People generally have minor or no symptoms when they first become infected. Black urine, Fever, yellow-tinged skin, and abdominal pain are all symptoms that might occur. The virus remains in the liver, in roughly 75 percent to 85 percent of patients who are infected. Initially, in the period of a chronic infection, there are usually no symptoms. However, it frequently develops to cirrhosis over time. Hepatitis C, on the other hand, can sometimes lead to liver cancer, blood cancer, and liver failure [1]. The most common way for HCV to spread is by blood-to-blood contact, which is related to injectable usage of drugs, improperly cleaned equipment for medical care, needle stick injuries in health care, and transfusions. The most typical reason for liver transplantation is Hepatitis C, even though virus generally returns after the procedure. Hepatitis C infects an estimated 71 million people (1 percent of the global population) in 2015. Low- and middle-income nations bear the brunt of the health burden, with Africa and Central and East Asia having the greatest rates of prevalence. In 2015, Hepatitis C caused around 167,000 liver cancer deaths and 326,000 cirrhosis deaths. 15th Hepatitis C is a disease that can be transmitted from

one person to another. Hepatitis C was first identified in the 1970s, it was thought to be a kind of non-A non-B Hepatitis, and its presence was confirmed in 1989. Only humans and chimps are infected with Hepatitis C; for more details, see [2, 3].

In natural and physical sciences, mathematical and computational tools have been used to investigate phenomena at many scales, ranging from the global human population to individual atoms within a biomolecule. The relevant modeling methodologies span time spans ranging from years to picoseconds, region to region of interest (impacts ranging from evolutionary to atomic), and importance. This exploration will go over some of the most common and useful approaches in mathematics and computing. Differential equations, statistical models, dynamical systems, and game theoretic models are all examples of mathematical models; we refer to [4–10]. These and other types of models can be mixed and matched, resulting in a single model that has a diverse set of abstract structures. Logic models can be used in mathematical models in general. In many instances, the quality of a scientific topic is determined by how well theoretical mathematical models accord with the results of repeated experiments. As better theories are discovered, a lack of concordance between mathematical models that are theoretical and experimental findings frequently leads to

significant advancements. Many mathematical models were formulated for the Hepatitis C diseases to understand the dynamics of the diseases and control the spreading of diseases; we refer to [11–14].

Fractional calculus is a discipline of mathematics that investigates the various ways in which the differentiation operator can be defined in terms of real or complex number powers. Because of their ability to include notion of nonlocal operators used to incorporate more complicated natural phenomena into mathematical equations, differential equations have attracted many scholars from practically all disciplines of science, technology, and engineering in recent years. The exponential decay law, the power law, and the extended Mittag-Leffler law were recommended as three dominants in fractional calculus. The kernel Mittag-Leffler function was shown to be more broadly applicable than the power law and exponential decay functions; both Riemann-Liouville and Caputo-Fabrizio are special examples of the Atangana-Baleanu fractional operator; we refer to [15–18]. Many biological models have been studied on fractional operators; Atangana and Alqahtani [19] considered a mathematical model river blindness as in Caputo sense and beta operators. Stability and numerical solutions were obtained for the fractional order model. Gómez-Aguilar et al. [20] examined a cancer model in three dimensions in the sense of Caputo-Fabrizio-Caputo and the novel fractional derivative with Mittag-Leffler kernel. Special solutions were obtained by an iterative process that used the Laplace transform rule, Sumudu-Picard integration approach, and the Adams-Moulton approach. Shah and Bushnaq [21] evaluated an endemic infection model in the fractional sense. Numerical solutions were obtained for the proposed model by combining the Laplace transform with the Adomian decomposition approach. Arfan et al. [22] studied semianalytical solutions for a fractional order COVID-19 model under Caputo derivative; for more details, see [23–26].

Inspired from the above literature, in this paper, we consider a Hepatitis C model in the sense of a fractional order derivative. Furthermore, we investigate the existence and uniqueness of the fractional order model with the help of a fixed point theorem and stability analysis of the fractional order Hepatitis C model. Finally, numerical simulations of the solutions are demonstrated and compared with classical derivatives by using different values of fractional order and parameters. This paper is organized as follows; Section 1: introduction of the paper; Section 2: framework of the model; Section 3: preliminaries; Section 4: existence of solutions; Section 5: numerical data fitting; Section 6: conclusion.

2. Framework of the Model

In this section, we discuss an integer order Hepatitis C model. The population is split into four classes based on their size. $\mathbb{S}$ denotes class of susceptible, $\mathbb{I}$ class of acutely infected, $\mathbb{P}$ class of persistently infected, and $\mathbb{T}$ class of treatment for infection:

$$\begin{cases} \mathbb{D}_t' \mathbb{S}(t) = \mu - \lambda \mathbb{S}(t) - \mu \mathbb{S}(t), \\ \mathbb{D}_t' \mathbb{I}(t) = (1 - \psi) \lambda \mathbb{S}(t) + \psi \lambda (1 - \mathbb{I} - \mathbb{P} - \mathbb{T}) - (\mu + \sigma + \varepsilon + d) \mathbb{I}, \\ \mathbb{D}_t' \mathbb{P}(t) = \varepsilon \mathbb{I} + \rho \mathbb{T} - (\mu + \delta + \wp) \mathbb{P}, \\ \mathbb{D}_t' \mathbb{T}(t) = \wp \mathbb{P} - (\mu + \rho + \theta) \mathbb{T}. \end{cases} \quad (1)$$

With the initial conditions

$$\begin{aligned} \mathbb{S}(0) &\geq 0, \\ \mathbb{I}(0) &\geq 0, \\ \mathbb{P}(0) &\geq 0, \\ \mathbb{T}(0) &\geq 0, \end{aligned} \quad (2)$$

where μ is the rate of natural death, λ is force of infection, ψ is the rate of susceptibility of recovered, σ is the rate of recovery from acute infection, ε is the rate of progression to chronic infection, d is the rate of death due to acute infection, ρ is the rate of treatment failure of chronically infected, δ is the rate of recovery from chronic infection, $\wp$ is the rate of treatment of chronically infected, and θ is the rate of treatment cure. For more details on the existence of infection, endemic equilibrium, reproduction numbers, and stability of endemic equilibrium, see [11–13].

Definition 1 (see [17, 27]). On the basis of Mittag-Leffler kernel and $\psi \in H^*(a, b)$, $b > a$, for $q \in [0, 1]$, the ABC-fractional differential operator is given as

$${}_{a}^{ABC} \mathbb{D}_{\wp}^q \psi(\wp) = \frac{\beta(q)}{1-q} \int_a^{\wp} \psi'(s) E_q \left[\frac{-q(\wp-s)^q}{1-q} \right] ds, \quad (3)$$

where $B(q)$ denote a weighted function which satisfied the main property $\beta(0) = \beta(1) = 1$:

$$E_{\rho}(h) = \sum_{r=0}^{\infty} \frac{h^r}{(1+\rho r)}, \quad \rho > 0. \quad (4)$$

Definition 2 (see [17, 27]). On the basis of Mittag-Leffler kernel and $\psi \in H^*(a, b)$, $b > a$, $q \in [0, 1]$, the ABR-fractional derivative is defined as

$${}_{a}^{ABR} \mathbb{D}_{\wp}^q \psi(\wp) = \frac{\beta(q)}{1-q} \frac{d}{d\wp} \int_a^{\wp} \psi(s) E_q \left[\frac{-q(\wp-s)^q}{1-q} \right] ds. \quad (5)$$

Definition 3 (see [17, 27]). Let $\psi \in H^*(a, b)$, $b > a$, $0 < \rho < 1$; the AB-integral is given

$${}_{a}^{AB} \mathbb{I}_{\wp}^q \psi(\wp) = \frac{1-q}{\beta(q)} \psi(\wp) + \frac{q}{\beta(q)\Gamma(q)} \int_a^{\wp} \psi(s) (\wp-s)^{q-1} ds. \quad (6)$$

Lemma 4 (see [17]). Let function ψ , then the AB fractional integral and derivative satisfy the following special character of Newton-Leibniz formula:

$${}^{AB}\mathbb{I}_{\wp}^{\mathfrak{Q}}\left({}^{ABC}\mathbb{D}_{\wp}^{\mathfrak{Q}}\psi(\wp)\right)=\psi(\wp)-\psi(a). \quad (7)$$

3. Existence Criteria

Let $\mathcal{B} = \mathcal{J} \times \mathbb{R}^4 \longrightarrow \mathbb{R}$, where $\mathcal{J} = [0, T]$, for $0 < t < T < \infty$, with a norm defined by $\|(\mathbb{S}, \mathbb{I}, \mathbb{P}, \mathbb{T})\| = \max_{t \in \mathcal{J}}\{|\mathbb{S}| + |\mathbb{I}| + |\mathbb{P}| + |\mathbb{T}|\}$. Then, clearly, $(\mathcal{B}, \|\cdot\|)$ is Banach's space. Let us consider system (1) in the sense of fractional order operator:

$$\begin{cases} {}^{ABC}\mathbb{D}_t^{\rho}\mathbb{S}(t) = \mu - \lambda\mathbb{S}(t) - \mu\mathbb{S}(t), \\ {}^{ABC}\mathbb{D}_t^{\rho}\mathbb{I}(t) = (1 - \psi)\lambda\mathbb{S}(t) + \psi\lambda(1 - \mathbb{I} - \mathbb{P} - \mathbb{T}) - (\mu + \sigma + \varepsilon + d)\mathbb{I}, \\ {}^{ABC}\mathbb{D}_t^{\rho}\mathbb{P}(t) = \varepsilon\mathbb{I} + \rho\mathbb{T} - (\mu + \delta + \wp)\mathbb{P}, \\ {}^{ABC}\mathbb{D}_t^{\rho}\mathbb{T}(t) = \wp\mathbb{P} - (\mu + \rho + \theta)\mathbb{T}. \end{cases} \quad (8)$$

By employing Definition (3) to (8), we have

$$\begin{aligned} \mathbb{S}(t) - \mathbb{S}(0) &= \frac{1 - \mathfrak{Q}}{\beta(\mathfrak{Q})}(\mu - \lambda\mathbb{S} - \mu\mathbb{S}(t)) \\ &+ \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1}(\mu - \lambda\mathbb{S} - \mu\mathbb{S})ds, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathbb{I}(t) - \mathbb{I}(0) &= \frac{1 - \mathfrak{Q}}{\beta(\mathfrak{Q})}((1 - \psi)\lambda\mathbb{S}(t) + \psi\lambda(1 - \mathbb{I} - \mathbb{P} - \mathbb{T}) \\ &- (\mu + \sigma + \varepsilon + d)\mathbb{I}) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1}((1 - \psi)\lambda\mathbb{S}(t) \\ &+ \psi\lambda(1 - \mathbb{I} - \mathbb{P} - \mathbb{T}) - (\mu + \sigma + \varepsilon + d)\mathbb{I})ds, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathbb{P}(t) - \mathbb{P}(0) &= \frac{1 - \mathfrak{Q}}{\beta(\mathfrak{Q})}(\varepsilon\mathbb{I} + \rho\mathbb{T} - (\mu + \delta + \wp)\mathbb{P}) \\ &+ \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1}(\varepsilon\mathbb{I} + \rho\mathbb{T} - (\mu + \delta + \wp)\mathbb{P})ds, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathbb{T}(t) - \mathbb{T}(0) &= \frac{1 - \mathfrak{Q}}{\beta(\mathfrak{Q})}(\wp\mathbb{P} - (\mu + \rho + \theta)\mathbb{T}) \\ &+ \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1}(\wp\mathbb{P} - (\mu + \rho + \theta)\mathbb{T})ds. \end{aligned} \quad (12)$$

For simplicity in the above Equation (9), we introduce $\mathbb{H}_i$ for $i = 1, 2, 3, 4$, given below:

$$\mathbb{H}_1(t, \mathbb{S}) = \mu - \lambda\mathbb{S} - \mu\mathbb{S}, \quad (13)$$

$$\mathbb{H}_2(t, \mathbb{I}) = (1 - \psi)\lambda\mathbb{S} + \psi\lambda(1 - \mathbb{I} - \mathbb{P} - \mathbb{T}) - (\mu + \sigma + \varepsilon + d)\mathbb{I}, \quad (14)$$

$$\mathbb{H}_3(t, \mathbb{P}) = \varepsilon\mathbb{I} + \rho\mathbb{T} - (\mu + \delta + \wp)\mathbb{P}, \quad (15)$$

$$\mathbb{H}_4(t, \mathbb{T}) = \wp\mathbb{P} - (\mu + \rho + \theta)\mathbb{T}, \quad (16)$$

$$\begin{cases} \chi_1 = \lambda - \mu, \\ \chi_2 = \mu + \sigma + \varepsilon + d + \psi\lambda, \\ \chi_3 = (\mu + \delta + \wp), \\ \chi_4 = (\mu + \rho + \theta). \end{cases} \quad (17)$$

For proving our results, we consider the following assumption $\mathbb{B}$. For the below continuous functions $\mathbb{S}(t), \mathbb{S}^*(t), \mathbb{I}(t), \mathbb{I}^*(t), \mathbb{P}(t), \mathbb{P}^*(t), \mathbb{T}(t), \mathbb{T}^*(t), \in L[0, 1]$ such that $\|\mathbb{S}(t)\| \leq \xi_1, \|\mathbb{I}(t)\| \leq \xi_2, \|\mathbb{P}(t)\| \leq \xi_3, 0 < \|\mathbb{T}(t)\| \leq \xi_4$, there exist three constant $\kappa_i > 0, i \in \mathbb{N}_1^3$, such that the below hold:

$$\begin{aligned} \|\mathbb{S}(t)\| &\leq \kappa_1, \\ \|\mathbb{P}(t) - \mathbb{T}(t)\| &\leq \kappa_2, \\ \|\mathbb{P}\| &\leq \kappa_3. \end{aligned} \quad (18)$$

Theorem 5. $\mathbb{H}_i$, for $i \in \mathbb{N}_1^5$, satisfy Lipschitz condition if $\max\{\chi_i, \text{ for } i = 1, 2, 3, 4\} < 1$ for χ_i defined in (17).

Consider for $\mathbb{H}_1$, below

$$\begin{aligned} \|\mathbb{H}_1(t, \mathbb{I}) - \mathbb{H}_1(t, \mathbb{I}^*)\| &= \|\mu - \lambda\mathbb{S}(t) - \mu\mathbb{S}(t) - \mu + \lambda\mathbb{S}^*(t) \\ &+ \mu\mathbb{S}^*(t)\| \leq [\lambda - \mu]\|\mathbb{S} - \mathbb{S}^*\| = \chi_1\|\mathbb{S} - \mathbb{S}^*\|. \end{aligned} \quad (19)$$

For $\mathbb{H}_2(t, \mathbb{I})$, we have

$$\begin{aligned} \|\mathbb{H}_2(t, \mathbb{I}) - \mathbb{H}_2(t, \mathbb{I}^*)\| &= \|(1 - \psi)\lambda\mathbb{S}(t) + \psi\lambda(1 - \mathbb{I} - \mathbb{P} - \mathbb{T}) \\ &- (\mu + \sigma + \varepsilon + d)\mathbb{I} - ((1 - \psi)\lambda\mathbb{S}(t) + \psi\lambda(1 - \mathbb{I}^* - \mathbb{P} - \mathbb{T}) \\ &- (\mu + \sigma + \varepsilon + d)\mathbb{I}^*)\| \leq \psi\lambda\|\mathbb{I} - \mathbb{I}^*\| + (\mu + \sigma + \varepsilon + d)\|\mathbb{I} - \mathbb{I}^*\| \\ &\leq (\mu + \sigma + \varepsilon + d + \psi\lambda)\|\mathbb{I} - \mathbb{I}^*\| = \chi_2\|\mathbb{I} - \mathbb{I}^*\|. \end{aligned} \quad (20)$$

$\mathbb{H}_3(t, \mathbb{P})$ implies

$$\begin{aligned} \|\mathbb{H}_3(t, \mathbb{P}) - \mathbb{H}_3(t, \mathbb{P}^*)\| &= \|\varepsilon\mathbb{I} + \rho\mathbb{T} \\ &- (\mu + \delta + \wp)\mathbb{P} - (\varepsilon\mathbb{I} + \rho\mathbb{T} - (\mu + \delta + \wp)\mathbb{P}^*)\| \\ &\leq (\mu + \delta + \wp)\|\mathbb{P} - \mathbb{P}^*\| = \chi_3\|\mathbb{P} - \mathbb{P}^*\|. \end{aligned} \quad (21)$$

$\mathbb{H}_4(t, \mathbb{T})$ implies

$$\begin{aligned} \|\mathbb{H}_4(t, \mathbb{T}) - \mathbb{H}_4(t, \mathbb{T}^*)\| &= \|\wp\mathbb{P} - (\mu + \rho + \theta)\mathbb{T} \\ &- (\wp\mathbb{P} - (\mu + \rho + \theta)\mathbb{T}^*)\| \leq (\mu + \rho + \theta)\|\mathbb{T} - \mathbb{T}^*\| \\ &= \chi_4\|\mathbb{T} - \mathbb{T}^*\|. \end{aligned} \quad (22)$$

Thus, from (19)-(22), we have that $\mathbb{Q}_i$ for $i = 1, 2, 3, 4$, satisfying the Lipschitz condition. And this completes the proof. Assuming that $\mathbb{S}(0) = \mathbb{I}(0) = \mathbb{P}(0) = \mathbb{T}(0) = 0$, then we have

$$\mathbb{S}(t) = \frac{1 - \mathfrak{Q}}{\beta(\mathfrak{Q})}\mathbb{H}_1(t, \mathbb{S}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1}\mathbb{H}_1(s, \mathbb{S})(s)ds, \quad (23)$$

$$\mathbb{I}(t) = \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_2(t, \mathbb{I}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_2(s, \mathbb{I})(s) ds, \quad (24)$$

$$\mathbb{P}(t) = \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_3(t, \mathbb{P}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_3(s, \mathbb{P})(s) ds, \quad (25)$$

$$\mathbb{T}(t) = \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_4(t, \mathbb{T}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_4(s, \mathbb{T})(s) ds. \quad (26)$$

For the iterative scheme of the ABC-fractional order HCV model (8), define

$$\begin{aligned} \mathbb{S}_n(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_1(t, \mathbb{S}_{n-1}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_1(s, \mathbb{S}_{n-1})(s) ds, \\ \mathbb{I}_n(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_2(t, \mathbb{I}_{n-1}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_2(s, \mathbb{I}_{n-1})(s) ds, \\ \mathbb{P}_n(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_3(t, \mathbb{P}_{n-1}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_3(s, \mathbb{P}_{n-1})(s) ds, \\ \mathbb{T}_n(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_4(t, \mathbb{T}_{n-1}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_4(s, \mathbb{T}_{n-1})(s) ds. \end{aligned} \quad (27)$$

Theorem 6. The ABC-fractional order HCV model (8) has a solution if

$$\Delta = \max \{ \chi_i \} < 1, \quad i \in \mathbb{N}_I^4. \quad (28)$$

We define the function

$$\begin{aligned} \mathbb{G}_1(t) &= \mathbb{S}_{n+1}(t) - \mathbb{S}(t), \\ \mathbb{G}_2(t) &= \mathbb{I}_{n+1}(t) - \mathbb{I}(t), \end{aligned} \quad (29)$$

$$\begin{aligned} \mathbb{G}_3(t) &= \mathbb{P}_{n+1}(t) - \mathbb{P}(t), \\ \mathbb{G}_4(t) &= \mathbb{T}_{n+1}(t) - \mathbb{T}(t). \end{aligned} \quad (30)$$

By the help of (29) and (30), we have

$$\begin{aligned} \|\mathbb{G}_1\| &\leq \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \|\mathbb{H}_1(t, \mathbb{S}_n(t)) - \mathbb{H}_1(t, \mathbb{S}_{n-1}(t))\| \\ &\quad + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \|\mathbb{H}_1(s, \mathbb{S}_n(s)) \\ &\quad - \mathbb{H}_1(s, \mathbb{S}_{n-1}(s))\| ds \leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right] \chi_1 \|\mathbb{S}_n - \mathbb{S}_{n-1}\| \\ &\leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right]^n \Delta^n \|\mathbb{S}_n - \mathbb{S}_{n-1}\|, \end{aligned}$$

$$\begin{aligned} \|\mathbb{G}_2\| &\leq \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \|\mathbb{H}_2(t, \mathbb{I}_n(t)) - \mathbb{H}_2(t, \mathbb{I}_{n-1}(t))\| \\ &\quad + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \|\mathbb{H}_2(s, \mathbb{I}_n(s)) \\ &\quad - \mathbb{H}_2(s, \mathbb{I}_{n-1}(s))\| ds \leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right] \chi_2 \|\mathbb{I}_n - \mathbb{I}_{n-1}\| \\ &\leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right]^n \Delta^n \|\mathbb{I}_n - \mathbb{I}_{n-1}\|. \end{aligned} \quad (31)$$

Similarly,

$$\begin{aligned} \|\mathbb{G}_3\| &\leq \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \|\mathbb{H}_3(t, \mathbb{P}_n(t)) - \mathbb{H}_3(t, \mathbb{P}_{n-1}(t))\| \\ &\quad + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \|\mathbb{H}_3(s, \mathbb{P}_n(s)) \\ &\quad - \mathbb{H}_3(s, \mathbb{P}_{n-1}(s))\| ds \leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right] \chi_3 \|\mathbb{P}_n - \mathbb{P}_{n-1}\| \\ &\leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right]^n \Delta^n \|\mathbb{P}_n - \mathbb{P}_{n-1}\|, \\ \|\mathbb{G}_4\| &\leq \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \|\mathbb{H}_4(t, \mathbb{T}_n(t)) - \mathbb{H}_4(t, \mathbb{T}_{n-1}(t))\| \\ &\quad + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \|\mathbb{H}_4(s, \mathbb{T}_n(s)) \\ &\quad - \mathbb{H}_4(s, \mathbb{T}_{n-1}(s))\| ds \leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right] \chi_4 \|\mathbb{T}_n - \mathbb{T}_{n-1}\| \\ &\leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right]^n \Delta^n \|\mathbb{T}_n - \mathbb{T}_{n-1}\|, \end{aligned} \quad (32)$$

which ensure that $\mathbb{G}(t)_n \longrightarrow 0$, $i \in \mathbb{N}_I^4$, as $n \longrightarrow \infty$ for $\Delta < 1$, which completes the proof.

4. Uniqueness Solution

For our suggested model (8), we study the analysis of the uniqueness of solution.

Theorem 7. The ABC-fractional order Hepatitis C model (8) has a unique solution provided that

$$\left[\frac{1-\mathfrak{Q}_i}{\beta(\mathfrak{Q}_i)} + \frac{1}{\beta(\mathfrak{Q}_i)\Gamma(\mathfrak{Q}_i)} \right] \chi_i \leq 1, \quad i \in \mathbb{N}_I^4. \quad (33)$$

Assume another solution exist $\bar{\mathbb{S}}(t), \bar{\mathbb{I}}(t), \bar{\mathbb{P}}(t), \bar{\mathbb{T}}(t)$, and $\bar{\mathbb{R}}(t)$ such that

$$\begin{aligned}
\mathbb{S}(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_1(t, \mathbb{H}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_1(s, \mathbb{S}(s)) ds, \\
\mathbb{I}(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_2(t, \mathbb{I}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_2(s, \mathbb{I}(s)) ds, \\
\mathbb{P}(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_3(t, \mathbb{P}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_3(s, \mathbb{P}(s)) ds, \\
\mathbb{T}(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_4(t, \mathbb{T}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_4(s, \mathbb{T}(s)) ds.
\end{aligned} \tag{34}$$

Then,

$$\begin{aligned}
\|\mathbb{S}(t) - \mathbb{S}(t)\| &\leq \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \|\mathbb{H}_1(t, \mathbb{S}(t)) - \mathbb{H}_1(t, \mathbb{S}(t))\| \\
&\quad + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \|\mathbb{H}_1(s, \mathbb{S}(s)) \\
&\quad - \mathbb{H}_1(s, \mathbb{S}(s))\| ds \leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right] \chi_1 \|\mathbb{S} - \mathbb{S}\|,
\end{aligned} \tag{35}$$

which implies

$$\left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \chi_1 + \frac{\chi_1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} - 1 \right] \|\mathbb{S} - \mathbb{S}\| \geq 0. \tag{36}$$

By (33), (36) holds if $\|\mathbb{S} - \mathbb{S}\| = 0$; this implies $\mathbb{S}(t) = \mathbb{S}(t)$. With the same procedure, for $\mathbb{I}$, we have

$$\begin{aligned}
\|\mathbb{I}(t) - \mathbb{I}(t)\| &\leq \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \|\mathbb{H}_2(t, \mathbb{I}(t)) - \mathbb{H}_2(t, \mathbb{I}(t))\| \\
&\quad + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \|\mathbb{H}_2(s, \mathbb{I}(s)) \\
&\quad - \mathbb{H}_2(s, \mathbb{I}(s))\| ds \leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right] \chi_2 \|\mathbb{I} - \mathbb{I}\|,
\end{aligned} \tag{37}$$

which implies

$$\left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \chi_1 + \frac{\chi_1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} - 1 \right] \|\mathbb{P} - \mathbb{P}\| \geq 0. \tag{38}$$

By (33), (38) is true if $\|\mathbb{P} - \mathbb{P}(t)\| = 0$; this implies $\mathbb{P}(t) = \mathbb{P}(t)$. With the same procedure, for $\mathbb{T}$, we have

$$\begin{aligned}
\|\mathbb{T}(t) - \mathbb{T}(t)\| &\leq \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \|\mathbb{H}_3(t, \mathbb{T}(t)) - \mathbb{H}_3(t, \mathbb{T}(t))\| \\
&\quad + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \|\mathbb{H}_3(s, \mathbb{T}(s)) \\
&\quad - \mathbb{H}_3(s, \mathbb{T}(s))\| ds \leq \left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} + \frac{1}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \right] \chi_3 \|\mathbb{T} - \mathbb{T}\|,
\end{aligned} \tag{39}$$

which implies

$$\left[\frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \chi_4 + \frac{\chi_4}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} - 1 \right] \|\mathbb{T} - \mathbb{T}\| \geq 0. \tag{40}$$

By (33), (40) holds if $\|\mathbb{T} - \mathbb{T}\| = 0$, which implies $\mathbb{T}(t) = \mathbb{T}(t)$. Similarly, $\mathbb{T}(t) = \mathbb{T}(t)$. Thus, the ABC-fractional order (8) has a unique solution.

5. Hyers-Ulam Stability

Definition 8. The integral system (23)–(26) is Hyers-Ulam stable if for $\Delta_i > 0$, $i \in \mathbb{N}_1^4$, and $\gamma_i > 0$, $i \in \mathbb{N}_1^5$, such that

$$\begin{aligned}
\left| \mathbb{S}(t) - \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_1(t, \mathbb{S}(t)) - \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_1(s, \mathbb{S}(s)) ds \right| &\leq \gamma_1, \\
\left| \mathbb{I}(t) - \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_2(t, \mathbb{I}(t)) - \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_2(s, \mathbb{I}(s)) ds \right| &\leq \gamma_2, \\
\left| \mathbb{P}(t) - \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_3(t, \mathbb{P}(t)) - \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_3(s, \mathbb{P}(s)) ds \right| &\leq \gamma_3, \\
\left| \mathbb{T}(t) - \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_4(t, \mathbb{T}(t)) - \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_4(s, \mathbb{T}(s)) ds \right| &\leq \gamma_4.
\end{aligned} \tag{41}$$

We have $\dot{\mathbb{S}}(t), \dot{\mathbb{I}}(t), \dot{\mathbb{P}}(t), \dot{\mathbb{T}}(t)$ which implies

$$\begin{aligned}
\dot{\mathbb{S}}(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_1(t, \dot{\mathbb{S}}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_1(s, \dot{\mathbb{S}}(s)) ds, \\
\dot{\mathbb{I}}(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_2(t, \dot{\mathbb{I}}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_2(s, \dot{\mathbb{I}}(s)) ds, \\
\dot{\mathbb{P}}(t) &= \frac{1-\rho}{\beta(\rho)} \mathbb{H}_3(t, \dot{\mathbb{P}}(t)) + \frac{\rho}{\beta(\rho)\Gamma(\rho)} \int_0^t (t-s)^{\rho-1} \mathbb{H}_3(s, \dot{\mathbb{P}}(s)) ds, \\
\dot{\mathbb{T}}(t) &= \frac{1-\mathfrak{Q}}{\beta(\mathfrak{Q})} \mathbb{H}_4(t, \dot{\mathbb{T}}(t)) + \frac{\mathfrak{Q}}{\beta(\mathfrak{Q})\Gamma(\mathfrak{Q})} \int_0^t (t-s)^{\mathfrak{Q}-1} \mathbb{H}_4(s, \dot{\mathbb{T}}(s)) ds,
\end{aligned} \tag{42}$$

such that

$$\begin{aligned}
|\mathbb{S}(t) - \dot{\mathbb{S}}(t)| &\leq \delta_1 \gamma_1, \\
|\mathbb{I}(t) - \dot{\mathbb{I}}(t)| &\leq \delta_2 \gamma_2, \\
|\mathbb{P}(t) - \dot{\mathbb{P}}(t)| &\leq \delta_3 \gamma_3, \\
|\mathbb{T}(t) - \dot{\mathbb{T}}(t)| &\leq \delta_4 \gamma_4.
\end{aligned} \tag{43}$$

Theorem 9. Let $(\mathbb{B})$ be satisfied. Then, ABC-fractional order HCV model (8) is Hyers-Ulam stable.

Proof. By Theorem 7, the ABC-fractional order HCV model (8) has a unique solution, say $\mathbb{S}(t), \mathbb{I}(t), \mathbb{P}(t), \mathbb{T}(t)$. Let us consider $\dot{\mathbb{S}}(t), \dot{\mathbb{I}}(t), \dot{\mathbb{P}}(t), \dot{\mathbb{T}}(t)$ to be another solution of (8) satisfying (23)–(26). Then, we have

FIGURE 1: The joint solution for $q = 1$. FIGURE 2: The joint solution for $q = 0.98$.

$$\begin{aligned} \|\mathbb{S}(t) - \dot{\mathbb{S}}(t)\| &\leq \frac{1-q}{\beta(q)} \|\mathbb{H}_1(t, \mathbb{S}(t)) - \mathbb{H}_1(t, \dot{\mathbb{S}}(t))\| \\ &\quad + \frac{q}{\beta(q)\Gamma(q)} \int_0^t (t-s)^{q-1} \|\mathbb{H}_1(s, \mathbb{S}(s)) - \mathbb{H}_1(s, \dot{\mathbb{S}}(s))\| ds \\ &\leq \left[\frac{1-q}{\beta(q)} + \frac{1}{\beta(q)\Gamma(q)} \right] \chi_1 \|\mathbb{S} - \dot{\mathbb{S}}\|. \end{aligned} \quad (44)$$

Taking $\gamma_1 = \chi_1$, $\Delta = (1-q)/\beta(q) + q/\beta(q)\Gamma(q)$, this implies

$$\|\mathbb{S}(t) - \dot{\mathbb{S}}(t)\| \leq \gamma_1 \Delta. \quad (45)$$

Similarly, for $\mathbb{I}(t), \dot{\mathbb{I}}(t), \mathbb{P}(t), \dot{\mathbb{P}}(t), \mathbb{T}(t), \dot{\mathbb{T}}(t)$, we have

$$\left\{ \begin{aligned} \|\mathbb{I}(t) - \dot{\mathbb{I}}(t)\| &\leq \gamma_2 \Delta, \|\mathbb{P}(t) - \dot{\mathbb{P}}(t)\| \leq \gamma_3 \Delta, \|\mathbb{T}(t) - \dot{\mathbb{T}}(t)\| \leq \gamma_4 \Delta. \end{aligned} \right. \quad (46)$$

This implies that system (8) is Hyers-Ulam stable which ultimately ensures the stability of (8). This completes the proof. $\square$

6. Numerical Scheme

We provide the following numerical scheme by the Caputo fractional derivative:

$$\begin{cases} {}^{ABC}_0 \mathbb{D}_t^q \mathbb{S}(t) = \mathbb{M}_1(t, \mathbb{S}), \\ {}^{ABC}_0 \mathbb{D}_t^q \mathbb{I}_T(t) = \mathbb{M}_2(t, \mathbb{I}_T), \\ {}^{ABC}_0 \mathbb{D}_t^q \mathbb{I}_H(t) = \mathbb{M}_3(t, \mathbb{I}_H), \\ {}^{ABC}_0 \mathbb{D}_t^q \mathbb{I}_{TH}(t) = \mathbb{M}_4(t, \mathbb{I}_{TH}). \end{cases} \quad (47)$$

FIGURE 3: The joint solution for $q = 0.96$.

With the help of fractional AB-integral operator, (48) gets the following form:

$$\mathbb{S}(t) - \mathbb{S}(0) = \frac{1-q}{\beta(q)} \mathbb{M}_1(t, \mathbb{S}) + \frac{q}{\beta(q)\Gamma(q)} \int_0^t (t-s)^{q-1} \mathbb{M}_1(s, \mathbb{S}) ds, \quad (48)$$

$$\mathbb{I}(t) - \mathbb{I}(0) = \frac{1-q}{\beta(q)} \mathbb{M}_2(t, \mathbb{I}) + \frac{q}{\beta(q)\Gamma(q)} \int_0^t (t-s)^{q-1} \mathbb{M}_2(s, \mathbb{I}) ds, \quad (49)$$

FIGURE 4: The joint solution for $q = 0.94$.

$$\mathbb{P}(t) - \mathbb{P}(0) = \frac{1-q}{\beta(q)} \mathbb{M}_3(t, \mathbb{P}) + \frac{q}{\beta(q)\Gamma(q)} \int_0^t (t-s)^{q-1} \mathbb{M}_3(s, \mathbb{P}) ds, \quad (50)$$

$$\mathbb{T}(t) - \mathbb{T}(0) = \frac{1-q}{\beta(q)} \mathbb{M}_4(t, \mathbb{P}) + \frac{q}{\beta(q)\Gamma(q)} \int_0^t (t-s)^{q-1} \mathbb{M}_4(s, \mathbb{T}) ds. \quad (51)$$

By dividing the assumed interval $[0, t]$ into subintervals by the help of point t_{n+1} , for $n = 0, 1, 2, \dots$, we have

$$\begin{aligned} \mathbb{S}(t_{n+1}) - \mathbb{S}(0) &= \frac{1-q}{\beta(q)} \mathbb{M}_1(t_n, \mathbb{S}) + \frac{q}{\beta(q)\Gamma(q)} \sum_{k=0}^n \int_{t_k}^{t_{k+1}} \\ &\quad \cdot (t_{n+1} - s)^{q-1} \mathbb{M}_1(s, \mathbb{S}) ds, \end{aligned}$$

$$\begin{aligned} \mathbb{I}(t_{n+1}) - \mathbb{I}(0) &= \frac{1-q}{\beta(q)} \mathbb{M}_2(t_n, \mathbb{I}) + \frac{q}{\beta(q)\Gamma(q)} \sum_{k=0}^n \int_{t_k}^{t_{k+1}} \\ &\quad \cdot (t_{n+1} - s)^{q-1} \mathbb{M}_2(s, \mathbb{I}) ds, \end{aligned}$$

$$\begin{aligned} \mathbb{P}(t_{n+1}) - \mathbb{P}(0) &= \frac{1-q}{\beta(q)} \mathbb{M}_3(t_n, \mathbb{P}) + \frac{q}{\beta(q)\Gamma(q)} \sum_{k=0}^n \int_{t_k}^{t_{k+1}} \\ &\quad \cdot (t_{n+1} - s)^{q-1} \mathbb{M}_3(s, \mathbb{P}) ds, \end{aligned}$$

$$\begin{aligned} \mathbb{T}(t_{n+1}) - \mathbb{T}(0) &= \frac{1-q}{\beta(q)} \mathbb{M}_4(t_n, \mathbb{T}) + \frac{q}{\beta(q)\Gamma(q)} \sum_{k=0}^n \int_{t_k}^{t_{k+1}} \\ &\quad \cdot (t_{n+1} - s)^{q-1} \mathbb{M}_4(s, \mathbb{T}) ds. \end{aligned} \quad (52)$$

Now, using Lagrange's interpolation, we have

$$\begin{aligned} \mathbb{S}(t_{n+1}) &= \mathbb{S}(0) + \frac{1-q}{\beta(q)} \mathbb{M}_1(t_k, \mathbb{S}) + \frac{q}{\beta(q)} \times \sum_{k=0}^n \\ &\quad \cdot \left[\frac{h^q \mathbb{M}_1(t_k, \mathbb{S})}{\Gamma(q+2)} ((n+1-k)^q (n-k+2+q) \right. \\ &\quad \left. - (n-k)^q (n-k+2+2q)) - \frac{h^q \mathbb{Z}_1(t_{k-1}, \mathbb{S})}{\Gamma(q+2)} \right. \\ &\quad \left. \times ((n+1-k)^q - (n-k)^q (n+1-k+q)) \right], \end{aligned}$$

$$\begin{aligned} \mathbb{I}(t_{n+1}) &= \mathbb{I}(0) + \frac{1-q}{\beta(q)} \mathbb{M}_2(t_k, \mathbb{I}) + \frac{q}{\beta(q)} \times \sum_{k=0}^n \\ &\quad \cdot \left[\frac{h^q \mathbb{M}_2(t_k, \mathbb{I})}{\Gamma(q+2)} ((n+1-k)^q (n-k+2+q) \right. \\ &\quad \left. - (n-k)^q (n-k+2+2q)) - \frac{h^q \mathbb{M}_2(t_{k-1}, \mathbb{I})}{\Gamma(q+2)} \right. \\ &\quad \left. \times ((n-k+1)^q - (n-k)^q (n+1-k+q)) \right], \end{aligned}$$

$$\begin{aligned} \mathbb{P}(t_{n+1}) &= \mathbb{P}(0) + \frac{1-q}{\beta(q)} \mathbb{M}_3(t_k, \mathbb{P}) + \frac{q}{\beta(q)} \times \sum_{k=0}^n \\ &\quad \cdot \left[\frac{h^q \mathbb{M}_3(t_k, \mathbb{P})}{\Gamma(q+2)} ((n+1-k)^q (n-k+2+q) \right. \\ &\quad \left. - (n-k)^q (n-k+2+2q)) - \frac{h^q \mathbb{Z}_3(t_{k-1}, \mathbb{P}_H)}{\Gamma(q+2)} \right. \\ &\quad \left. \times ((n+1-k)^q - (n-k)^q (n+1-k+q)) \right], \end{aligned}$$

TABLE 1: Shows the parameter values given in Equation (8). The time units are taken in years.

Parameter	Description	Value	Reference
μ	Rate of natural death	0.09	[12]
λ	Force of infection	Assumed	
ψ	Rate of susceptibility of recovered	0.5	[11–13]
σ	Rate of recovery from acute infection	0.02	[12]
ε	Rate of progression to chronic infection	0.7	[12]
d	Rate of death due to acute infection	0.035	[12]
ρ	Rate of treatment failure of chronically infected	0.50	[12]
δ	Rate of recovery from chronic infection	0.012	[12]
$\wp$	Rate of treatment of chronically infected	0.34	[12]
θ	Rate of treatment cure	0.67	[12]

FIGURE 5: The susceptible class $\mathbb{S}(t)$ for $q = 1.0, 0.98, 0.96, 0.94$. FIGURE 7: The chronically infected $P(t)$ for $q = 1.0, 0.98, 0.96, 0.94$. FIGURE 6: The infected class $\mathbb{I}(t)$ for $q = 1.0, 0.98, 0.96, 0.94$.

$$\begin{aligned}
 \mathbb{T}(t_{n+1}) = & \mathbb{T}(0) + \frac{1-q}{\beta(q)} \mathbb{M}_4(t_k, \mathbb{T}) + \frac{q}{\mathbb{B}(q)} \times \sum_{k=0}^n \\
 & \cdot \left[\frac{h^q \mathbb{M}_4(t_k, \mathbb{T})}{\Gamma(q+2)} ((n+1-k)^q (n-k+2+q) \right. \\
 & - (n-k)^q (n-k+2+2q)) - \frac{h^q \mathbb{M}_4(t_{k-1}, \mathbb{I}_{TH})}{\Gamma(q+2)} \\
 & \left. \times ((n+1-k)^{q+1} - (n-k)^q (n+1-k+q)) \right]. \quad (53)
 \end{aligned}$$

7. Computational Results

Here, we present some computational results based on the parametric values defined in [12]. We consider the initial values $\mathbb{S}_0 = 0.90, \mathbb{I}_0 = 0.70, \mathbb{P}_0 = 0.66, \mathbb{T}_0 = 0.60$, and the parametric values given by $\beta = 1.30, \mu = 0.09, \chi = 0.5, \psi = 0.5, \sigma = 0.5, \varepsilon = 0.7, d = 0.035, \rho = 0.50, \delta = 0.012, \wp = 0.34, \theta = 0.67$. We get the same behavior of the fractional order model as for the integer order.

FIGURE 8: Chronically infected treated class $T(t)$ for $q = 1.0, 0.98, 0.96, 0.94$.

Figures 1–4 represent the numerical solution of the model for the orders $q = 1.0, 0.98, 0.96, 0.94$ and using Table 1 values, respectively. Figure 5 represents the susceptible class for the orders $q = 1.0, 0.98, 0.96, 0.94$ and using Table 1 values, which are increasing with the passage of time and get stability after 60 days of the treatment. The infection class is given in Figure 6, for the fractional orders $q = 1.0, 0.98, 0.96, 0.94$ and using Table 1 values, it has been observed that with the passage of time, the infection is decreased to a certain limit. Figure 7 shows the chronic infection class for $q = 1.0, 0.98, 0.96, 0.94$, and using Table 1 values, finally the chronically infected treated class for $q = 1.0, 0.98, 0.96, 0.94$ is given in Figure 8. This numerical analysis of the fractional order model ensures that the fractional order model is more informative and has the same behavior as the classical model.

8. Conclusions

In this article, we have given a mathematical fractional order Hepatitis C virus (HCV) spread model for an analytical and numerical study. The model is a fractional order extension of the classical model. The paper includes the existence, uniqueness, Hyers-Ulam stability, and a numerical scheme for the computational results. Our numerical results are based on the Lagrange polynomial interpolation. On the basis of the numerical scheme, we have given graphical explanation of the model and its subclasses. For details, Figures 1–4 represent the numerical solution of the model with the fractional orders $q = 1.0, 0.98, 0.96, 0.94$, respectively. Figure 5 represents the susceptible class for the orders $q = 1.0, 0.98, 0.96, 0.94$ which are increasing with the passage of time and get stability after 60 days of treatment. The infection class is given in Figure 6 for the fractional orders $q = 1.0, 0.98, 0.96, 0.94$, and it has been observed

that with the passage of time the infection is decreased to a certain limit. Figure 7 shows the chronic infection class for $q = 1.0, 0.98, 0.96, 0.94$, and finally, the chronically infected treated class for $q = 1.0, 0.98, 0.96, 0.94$ is given in Figure 8. Our numerical analysis of the fractional order model ensures that the fractional order model is more informative and has the same behavior as the classical model.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

All authors contributed equally in writing this article. All authors read and approved the final manuscript.

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