Research Article

Existence and Numerical Analysis of Imperfect Testing Infectious Disease Model in the Sense of Fractional-Order Operator

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In the present paper, we study a mathematical model of an imperfect testing infectious disease model in the sense of the Mittag-Leffler kernel. The Banach contraction principle has been used for the existence and uniqueness of solutions of the suggested model. Furthermore, a numerical method equipped with Lagrangian polynomial interpolation has been utilized for the numerical outcomes. Diagramming and discussion are used to clarify the effects of related parameters in the fractional-order imperfect testing infectious disease model.

1. Introduction

The aggregate of human microbiota is called human microbiome, including viruses, bacteria, protists, archaea, and fungi. These microbiomes live in or on our body including the skin, placenta, mammary glands, seminal fluid, ovarian follicles, uterus, lung, oral mucosa, conjunctiva, saliva, biliary, and gastrointestinal tract [1]. A number of infectious maladies are caused by these microbiome such as influenza, malaria, dengue, Ebola, COVID-19, HIV/AIDS, rabies, syphilis, yellow fever, hepatitis, Zika virus infection, and tuberculosis [2]. Yearly, 9.2 million people died due to infectious diseases [3, 4]. Due to this life-threatening situation, health departments spend more money to reduce the outbreak of infectious maladies. Several techniques were applied to minimize the exposure of infectious diseases, such as prevention measures, screening, testing diagnostics, education, and counseling. Among all of these techniques, testing diagnostics is a very useful tool to identify the infected individuals and susceptible. For the laboratory test, a sample is required such as a stool, tissue, cerebrospinal fluid, genital area, mucus

from the nose, blood, sputum, urine, stool, and throat. There are two main types after testing, germ-negative and germ-positive. If the individual can identify as germ-positive then a proper treatment begins for the disease. Sometimes, the test results suffer due to test imperfections. These effects come from sensitivity and specificity, which might be useful when trying to mitigate an epidemic.

Mathematics plays an effective rule in modeling to understand the dynamical behavior of complex phenomena in the life sciences. By mathematical models, one can easily know the basic properties of the complex system such as severity, clinical features, structures, risk analysis, evaluated interventions, and various transmission forms of viruses have been studied along with different dimensions, see for more details, [5–7]. Bernoulli [8] for the first time formulated a mathematical model for infectious diseases and analyzed the impact of prevention smallpox vaccines and life tables. After that, numbers of models have been systemized for infectious maladies such as control strategies for tuberculosis [9], sexually transmitted infections [10], human immunodeficiency virus [11], control of foot and mouth disease epidemic in the UK in

2001 [12], Middle East respiratory syndrome corona virus (MERS-CoV) [13], and severe acute respiratory syndrome (SARS) [14].

Fractional calculus is a generalization of classical calculus. Fractional calculus has lately gained popularity due to its remarkable properties, nonsingular, nonlocal, and memory and filter effects. Researchers of various disciplines are applying the fractional-order operators to real-life phenomena to study the behavior of the models theoretically and numerically. Atangana [15] studied Markovian and non-Markovian, Gaussian and non-Gaussian, and random and nonrandom properties of the fractional derivative, providing numerical examples. Bonyah et al. [16] formulated a human African trypanosomiasis model consisting of an AB-fractional operator and provide numerical solutions. Khan et al. [17] provided an HIV-TB model including AB-fractional derivative and analyzed the model for well-posedness, stability analysis, and numerical solutions. Koca in [18] studied the AB-fractional spread Ebola virus model for the existence of solutions and illustrated the results numerically. Khan et al. [19] considered the AB-fractional-order HIV/AIDS model and applied the fixed point theorem for the existence results and studied the stability analysis. Atangana [20] analyzed the numerical approximations for fractional differentiation based on the Riemann-Liouville definition, from the power law kernel to the generalized Mittag-Leffler law via the exponential decay law.

In this paper, we investigate the dynamical behavior of the fractional-order ITI disease model. The integer-order derivative of the model is replaced by fractal fractional operator in the sense of the Mittag-Leffler kernel. To study the existence of solutions and numerical simulations for the fractional-order ITI disease model. The paper is organized as follows: in Section 2, the definition of fractal fractional operators is shown. In Section 3, the model formulation is discussed. In Section 4, existence and uniqueness of solutions are shown. In Section 5, the numerical scheme is discussed. In Section 6, the numerical discussion and data fitting is discussed. In Section 7, the conclusion is discussed.

2. Preliminaries

Here, we will discuss some definitions which are utilized in the main proof of this study [21–24].

Definition 1 (see [22]). Let $F(t)$ be a continuous and fractal differential in the open interval (a, b) with $0 < n - 1 < \sigma \leq n$; then, the fractal fractional operator $0 < n - 1 < \epsilon \leq n$ in the sense of Caputo with power law is characterized as

$${}^{FFP}_a D_t^{\epsilon, \sigma} F(t) = \frac{1}{\Gamma[n - \epsilon]} \int_a^t \frac{dF(z)}{dz^\sigma} (t - z)^{n - \epsilon - 1} dz, \quad (1)$$

where

$$\frac{dF(z)}{dz^\sigma} = \lim_{t \rightarrow z} \frac{F(t) - F(z)}{t^\sigma - z^\sigma}.$$

The generalized form is given as

$${}^{FFP}_a D_t^{\epsilon, \sigma} F(t) = \frac{1}{\Gamma[n - \epsilon]} \int_a^t \frac{d^\theta F(z)}{dz^\sigma} (t - z)^{n - \epsilon - 1} dz, \quad (2)$$

where

$$\frac{d^\theta F(z)}{dz^\sigma} = \lim_{t \rightarrow z} \frac{F^\theta(t) - F^\theta(z)}{t^\sigma - z^\sigma}, \text{ where } 0 < \theta \leq 1.$$

Definition 2 (see [22]). Let $F(t)$ be a continuous and fractal differential in the open interval (a, b) with $0 < \sigma \leq 1$; then, the fractal fractional operator $0 < \epsilon \leq 1$, in the sense of Caputo with the exponential decay kernel, is characterized as

$${}^{FFP}_a D_t^{\epsilon, \sigma} F(t) = \frac{\wp(\epsilon)}{[1 - \epsilon]} \int_a^t \frac{dF(z)}{dz^\sigma} \exp \left[-\frac{\epsilon}{1 - \epsilon} (t - z) \right] dz. \quad (3)$$

The generalized form given as

$${}^{FFP}_a D_t^{\epsilon, \sigma, \theta} F(t) = \frac{\wp(\epsilon)}{[1 - \epsilon]} \int_a^t \frac{d^\theta F(z)}{dz^\sigma} \exp \left[-\frac{\epsilon}{1 - \epsilon} (t - z) \right] dz, \quad 0 < \theta \leq 1, \quad (4)$$

where $\wp(t)$ is the normalization function such that $\wp(0) = 1 = \wp(1)$.

Definition 3 (see [22, 23]). Let $F(t)$ be a continuous and fractal differential in the open interval (a, b) with $0 < \sigma \leq 1$; then, the fractal fractional operator $0 < \epsilon \leq 1$, in the sense of Caputo with the generalized Mittag-Leffler kernel, is characterized as

$${}^{FFM}_a D_t^{\epsilon, \sigma} F(t) = \frac{AB(\epsilon)}{[1 - \epsilon]} \int_a^t \frac{dF(z)}{dz^\sigma} E_\epsilon \left[-\frac{\epsilon}{1 - \epsilon} (t - z)^\epsilon \right] dz, \quad (5)$$

$$AB(\epsilon) = 1 - \epsilon + \frac{\epsilon}{\Gamma(\epsilon)}.$$

The generalized form is given as

$${}^{FFM}_a D_t^{\epsilon, \sigma, \theta} F(t) = \frac{AB(\epsilon)}{[1 - \epsilon]} \int_a^t \frac{d^\theta F(z)}{dz^\sigma} E_\epsilon \left[-\frac{\epsilon}{1 - \epsilon} (t - z)^\epsilon \right] dz, \quad 0 < \theta \leq 1, \quad (6)$$

where $\wp(t)$ is normalization function such that $\wp(0) = 1 = \wp(1)$.

Definition 4 (see [21, 22]). Assume that $F(t)$ is a continuous and fractal differential in the open interval (a, b) with then the fractal fractional integral $0 < \epsilon \leq 1$, in the sense of the power law kernel, is characterized as

$${}^{FFP}_{a, t} I_{0, t}^{\epsilon, \sigma} F(t) = \frac{1}{\Gamma(\epsilon)} \int_0^t (t - z)^{\epsilon - 1} z^{1 - \sigma} F(z) dz. \quad (7)$$

Definition 5 (see [21, 22]). Let $F(t)$ be a continuous and

fractal differential in the open interval (a, b) with then the fractal fractional integral $0 < \epsilon \leq 1$, in the sense of the exponential kernel, is characterized as

$${}_a^{FFE} \mathbf{I}_{0,t}^{\epsilon,\sigma} F(t) = \frac{\sigma(1-\epsilon)t^{\sigma-1}F(t)}{\wp(\epsilon)} + \frac{\epsilon\sigma}{\wp(\epsilon)} \int_0^t z^{\epsilon-1} F(z) dz. \quad (8)$$

Definition 6 (see [21, 22]). Let $F(t)$ be a continuous and fractal differential in the open interval (a, b) with then the fractal fractional integral $0 < \epsilon \leq 1$, in the sense of the Mittag-Leffler kernel, is characterized as

$${}_a^{FFM} \mathbf{I}_{0,t}^{\epsilon,\sigma} F(t) = \frac{\sigma(1-\epsilon)t^{\sigma-1}F(t)}{AB(\epsilon)} + \frac{\epsilon\sigma}{AB(\epsilon)} \int_0^t (t-z)^{\epsilon-1} z^{\epsilon-1} F(z) dz. \quad (9)$$

3. Model Formulation

In this section, we will study the dynamics of the ordinary differential equations of the infectious disease model formulated in the reference therein [25–27], which is leveraged from the SIR system. This model has two main components, $\mathbf{S}_m(t)$, a stand for rate of susceptible individual tested, $\mathbf{I}_m(t)$, infected cases which is tested positive and started treatment. While $\mathbf{S}(t)$ denotes susceptible, $\mathbf{I}(t)$ denotes infected and $\mathbf{R}(t)$ denotes the class of recovered individuals. The following SSIIR model:

$$\begin{aligned} \dot{\mathbf{S}}(t) &= \eta \mathbf{S}_m(t) + \mu - \theta \mathbf{S}(t) - \beta \mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \dot{\mathbf{S}}_m(t) &= \theta \mathbf{S}(t) - (\eta + \mu) \mathbf{S}_m(t) - \beta_m \mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \dot{\mathbf{I}}(t) &= \beta \mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)) - (\gamma + \alpha + \mu) \mathbf{I}(t) + \beta_m \mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \dot{\mathbf{I}}_m(t) &= \alpha \mathbf{I}(t) - \mu \mathbf{I}_m(t) - \gamma_m \mathbf{I}_m(t), \\ \dot{\mathbf{R}}(t) &= \gamma \mathbf{I}(t) - \mu \mathbf{R}(t) + \gamma_m \mathbf{I}_m(t), \end{aligned} \quad (10)$$

where β denotes the rate of infected susceptible individuals, $\theta = \kappa(1 - \lambda)$ denotes the rate of susceptible individuals that are tested and deemed incorrectly, and β_m denotes the rate of effectively infected individuals. For this model, $\beta_m < \beta$ is assumed, μ denotes total population, κ denotes the rate of converging from susceptible to θ susceptible-infected-deemed, ψ is the sensitivity, $\alpha = \kappa\psi$ rate of treatment, γ rate of recovered individuals, and γ_m denotes the rate of recovered infected-under-treatment that $\gamma < \gamma_m$ assumed in the model.

The predominant of this paper is to study the existence of results and numerical analysis of fractal fractional-order ITI disease model. In the upcoming section, we are going to produce existence of solution for the model (10) and later on the uniqueness of solution is our interest. For these, we need to define the following Banach's space. Consider $\mathcal{Y} = \mathcal{S} \times \mathbb{R}^5 \rightarrow \mathbb{R}$, where $\mathcal{S} = [0, \tau]$, for $0 < t < \tau < \infty$, with a norm defined by $\|(\mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R})\| = \max_{t \in \mathcal{S}} \{|\mathbf{S}| + |\mathbf{S}_m| + |\mathbf{I}| + |\mathbf{I}_m| + |\mathbf{R}|\}$.

Then, clearly $(\mathcal{Y}, \|\cdot\|)$ is a Banach space.

4. Existence and Uniqueness of Solutions

In this section, the fixed point theorem is used to investigate the existence and uniqueness of the results for the fractional-order ITI disease model. The integer operator of the model (10) is replaced by a fractal fractional operator

$$\begin{aligned} {}^{ABR} \mathbb{D}_{0,t}^{\epsilon}(\mathbf{S}(t)) &= \eta \mathbf{S}_m(t) + \mu - \theta \mathbf{S}(t) - \beta \mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ {}^{ABR} \mathbb{D}_{0,t}^{\epsilon}(\mathbf{S}_m(t)) &= \theta \mathbf{S}(t) - (\eta + \mu) \mathbf{S}_m(t) - \beta_m \mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ {}^{ABR} \mathbb{D}_{0,t}^{\epsilon}(\mathbf{I}(t)) &= \beta \mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)) - (\gamma + \alpha + \mu) \mathbf{I}(t) \\ &\quad + \beta_m \mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ {}^{ABR} \mathbb{D}_{0,t}^{\epsilon}(\mathbf{I}_m(t)) &= \alpha \mathbf{I}(t) - \mu \mathbf{I}_m(t) - \gamma_m \mathbf{I}_m(t), \\ {}^{ABR} \mathbb{D}_{0,t}^{\epsilon}(\mathbf{R}(t)) &= \gamma \mathbf{I}(t) - \mu \mathbf{R}(t) + \gamma_m \mathbf{I}_m(t). \end{aligned} \quad (11)$$

with initial boundary value conditions

$$\mathbf{S}(0) = \mathbf{S}_0, \mathbf{S}_m(0) = \mathbf{S}_{m0}, \mathbf{I}(0) = \mathbf{I}_0, \mathbf{I}_m(0) = \mathbf{I}_{m0}, \mathbf{R}(0) = \mathbf{R}_0. \quad (12)$$

For simplicity, we write the system:

$$\begin{aligned} \Lambda_1(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \eta \mathbf{S}_m(t) + \mu - \theta \mathbf{S}(t) - \beta \mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \Lambda_2(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \theta \mathbf{S}(t) - (\eta + \mu) \mathbf{S}_m(t) \\ &\quad - \beta_m \mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \Lambda_3(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \beta \mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)) - (\gamma + \alpha + \mu) \mathbf{I}(t) \\ &\quad + \beta_m \mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \Lambda_4(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \alpha \mathbf{I}(t) - \mu \mathbf{I}_m(t) - \gamma_m \mathbf{I}_m(t), \\ \Lambda_5(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \gamma \mathbf{I}(t) - \mu \mathbf{R}(t) + \gamma_m \mathbf{I}_m(t). \end{aligned} \quad (13)$$

Furthermore, the above system (11) can be written as

$${}^{ABR} D_{0,t}(\Phi(t)) = \Psi(t, \Phi(t)), \quad (14)$$

where $\Phi(t)$ and Ψ stand for

$$\Phi(t) = \begin{cases} \mathbf{S}(t) \\ \mathbf{S}_m(t) \\ \mathbf{I}(t) \\ \mathbf{I}_m(t) \\ \mathbf{R}(t) \end{cases} \quad \text{and } \psi\{t, \Phi(t)\} = \begin{cases} \Lambda_1(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}), \\ \Lambda_2(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}), \\ \Lambda_3(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}), \\ \Lambda_4(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}), \\ \Lambda_5(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}). \end{cases} \quad (15)$$

By applying Definition (3), to (14), we get the following form:

$$\frac{AB(\epsilon)}{1-\epsilon} \frac{d}{dt} \int_0^t \Psi(\omega, \Phi(\omega)) E_{\epsilon} \left[-\frac{\epsilon}{1-\epsilon} (t-z)^{\epsilon} \right] dz = z t^{z-1} \Psi(t, \Phi(t)). \quad (16)$$

Now, by employing Definition (6), with (16), we get the following form:

$$\begin{aligned}\Psi(t) &= \Psi(0) + \frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \Psi(t, \Phi(t)) \\ &\quad + \frac{z\epsilon}{AB(\epsilon)\Gamma(\epsilon)} \int_0^t (t-z)^{\epsilon-1} \Psi(\omega, \Phi(\omega)) \omega^{z-1} dz.\end{aligned}\quad (17)$$

Let us consider here:

$$\mathcal{B}_a^q = \mathcal{H}_n(t_n) \times \overline{C_0(\Phi_0)}, \quad (18)$$

where $\mathcal{H}_n = [t_{n-a}, t_{n+a}]$ and $\overline{C_0(\Phi_0)} = [t_0 - b, t_0 + b]$. Now, assume $\sup_{t \in \mathcal{B}_a^q} \|\Psi\| = \mathcal{P}$.

Let us define a norm:

$$\|\Omega\|_\infty = \sup_{t \in \mathcal{B}_a^q} |\Omega(t)|. \quad (19)$$

Now, consider operator $\Xi : \mathcal{G}[\mathcal{H}_n(t_n), C_b(t_n)] \longrightarrow \mathcal{G}[\mathcal{H}_n(b), C_b(t_n)]$ such that

$$\begin{aligned}\Xi\Phi(t) &= \Phi_0 + \frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \Psi(t, \Phi(t)) \\ &\quad + \frac{z\epsilon}{AB(\epsilon)\Gamma(\epsilon)} \int_0^t (t-z)^{\epsilon-1} \Psi(\omega, \Phi(\omega)) \omega^{z-1} dz.\end{aligned}\quad (20)$$

First, we will show $\|\Xi\Phi(t) - \Phi_0\| < q$. For this, we have

$$\begin{aligned}\|\Xi\Phi(t) - \Phi_0\| &\leq \frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \|\Psi(t, \Phi(t))\| \\ &\quad + \frac{z\epsilon}{AB(\epsilon)\Gamma(\epsilon)} \int_0^t (t-z)^{\epsilon-1} \|\Psi(\omega, \Phi(\omega))\| \omega^{z-1} dz, \\ &\leq \frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \mathcal{P} + \frac{z\epsilon}{AB(\epsilon)\Gamma(\epsilon)} \int_0^t (t-z)^{\epsilon-1} \omega^{z-1} dz.\end{aligned}\quad (21)$$

Consider $\omega = tv$ and putting in Equation (21), then we get the following:

$$\|\Xi\Phi(t) - \Phi_0\| \leq \frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \mathcal{P} + \frac{z\epsilon\mathcal{P}}{AB(\epsilon)\Gamma(\epsilon)} \omega^{z+\epsilon-u} \mathcal{Q}(z, \epsilon), \quad (22)$$

which yields

$$\|\Xi\Phi(t) - \Phi_0\| \leq q \longrightarrow \mathcal{P} < \frac{q\mathcal{Q}(z, \epsilon)AB(\epsilon)\Gamma(\epsilon)}{(1-\epsilon)\Gamma(\alpha)zt^{z-1} + 1 - \epsilon z t^{z+\epsilon-u}}. \quad (23)$$

Now, consider for any $\Phi_1, \Phi_2 \in \mathcal{G}[\mathcal{H}_n(t_n), C_b(t_n)]$, then, we have

$$\begin{aligned}\|\Xi\Phi_1 - \Xi\Phi_2\| &\leq \frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \|\Psi(t, \Phi_1(t)) - \Psi(t, \Phi_2(t))\| \\ &\quad + \frac{z\epsilon}{AB(\epsilon)\Gamma(\epsilon)} \int_0^t (t-z)^{\epsilon-1} \|\Psi(t, \Phi_1(t)) \\ &\quad - \Psi(t, \Phi_2(t))\| \omega^{z-1} dz.\end{aligned}\quad (24)$$

As Ξ is a contraction, then we have

$$\begin{aligned}\|\Xi\Phi_1 - \Xi\Phi_2\| &\leq \frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \mathcal{X} \|\Phi_1 - \Phi_2\|_\infty \\ &\quad + \frac{z\epsilon\mathcal{X}}{AB(\epsilon)\Gamma(\epsilon)} \int_0^t (t-z)^{\epsilon-1} \|\Phi_1 - \Phi_2\|_\infty \omega^{z-1} dz. \\ &\leq \frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \mathcal{X} \|\Phi_1 - \Phi_2\|_\infty \\ &\quad + \frac{z\epsilon\mathcal{X}}{AB(\epsilon)\Gamma(\epsilon)} \|\Phi_1 - \Phi_2\|_\infty t^{\epsilon+z-3} \mathcal{Q}(z, \epsilon), \\ \|\Xi\Phi_1 - \Xi\Phi_2\| &\leq \left[\frac{1-\epsilon}{AB(\epsilon)} z t^{z-1} \mathcal{X} + \frac{z\epsilon\mathcal{X}}{AB(\epsilon)\Gamma(\epsilon)} t^{\epsilon+z-3} \mathcal{Q}(z, \epsilon) \right] \|\Phi_1 - \Phi_2\|_\infty\end{aligned}\quad (25)$$

Therefore, Ξ is a contraction if

$$\|\Xi\Phi_1 - \Xi\Phi_2\|_\infty \leq \|\Phi_1 - \Phi_2\|. \quad (26)$$

Then, we have

$$\mathcal{X} < \frac{1}{((1-\epsilon)/(AB(\epsilon)))zt^{z-1} + (z\epsilon/AB(\epsilon)\Gamma(\epsilon))t^{\epsilon+z-3}\mathcal{Q}(z, \epsilon)}, \quad (27)$$

such that

$$\mathcal{P} < \frac{1}{((1-\epsilon)/(AB(\epsilon)))zt^{z-1} + (z\epsilon/AB(\epsilon)\Gamma(\epsilon))t^{\epsilon+z-3}\mathcal{Q}(z, \epsilon)}. \quad (28)$$

Hence, by necessary condition, the proposed fractional-order ITI disease model (11) has a unique solution.

5. Numerical Scheme

We consider the ITI disease model (10), in the sense of the fractal fractional Mittag-Leffler Kernel

$${}^{\text{ABR}}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{S}(t)) = \eta\mathbf{S}_m(t) + \mu - \theta\mathbf{S}(t) - \beta\mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \quad (29)$$

$${}^{\text{ABR}}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{S}_m(t)) = \theta\mathbf{S}(t) + (\eta + \mu)\mathbf{S}_m(t) - \beta_m\mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \quad (30)$$

$$\begin{aligned}{}^{\text{ABR}}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{I}(t)) &= \beta\mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)) - (\gamma + \alpha + \mu)\mathbf{I}(t) \\ &\quad + \beta_m\mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)),\end{aligned}\quad (31)$$

$${}^{\text{ABR}}\mathbb{D}_{0,t}^{\epsilon,\sigma}(\mathbf{I}_m(t)) = \alpha\mathbf{I}(t) - \mu\mathbf{I}_m(t) - \gamma_m\mathbf{I}_m(t), \quad (32)$$

$${}^{\text{ABR}}\mathbb{D}_{0,t}^{\varepsilon,\sigma}(\mathbf{R}(t)) = \gamma\mathbf{I}(t) - \mu\mathbf{R}(t) - \gamma_m\mathbf{I}_m(t). \quad (33)$$

For simplicity,

$$\begin{aligned} \Lambda_1(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \eta\mathbf{S}_m(t) + \mu - \theta\mathbf{S}(t) \\ &\quad - \beta\mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \Lambda_2(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \theta\mathbf{S}(t) - (\eta + \mu)\mathbf{S}_m(t) \\ &\quad - \beta_m\mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \Lambda_3(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \beta\mathbf{S}(t)(\mathbf{I}(t) + \mathbf{I}_m(t)) - (\gamma + \alpha + \mu)\mathbf{I}(t) \\ &\quad + \beta_m\mathbf{S}_m(t)(\mathbf{I}(t) + \mathbf{I}_m(t)), \\ \Lambda_4(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \alpha\mathbf{I}(t) - \mu\mathbf{I}_m(t) - \gamma_m\mathbf{I}_m(t), \\ \Lambda_5(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) &= \gamma\mathbf{I}(t) - \mu\mathbf{R}(t) - \gamma_m\mathbf{I}_m(t). \end{aligned} \quad (34)$$

By applying the Atangan-Baleanu integral operator to Equation (29), which deduced to the following form:

$$\begin{aligned} \mathbf{S}(t) &= \mathbf{S}(0) + \frac{\sigma t^{\sigma-1}(1-\varepsilon)\Lambda_1(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R})}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^t (t-z)^{\varepsilon-1} \Lambda_1(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \end{aligned} \quad (35)$$

$$\begin{aligned} \mathbf{S}_m(t) &= \mathbf{S}_m(0) + \frac{\sigma t^{\sigma-1}(1-\varepsilon)\Lambda_2(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R})}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^t (t-z)^{\varepsilon-1} \Lambda_2(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \end{aligned} \quad (36)$$

$$\begin{aligned} \mathbf{I}(t) &= \mathbf{I}(0) + \frac{\sigma t^{\sigma-1}(1-\varepsilon)\Lambda_3(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R})}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^t (t-z)^{\varepsilon-1} \Lambda_3(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \end{aligned} \quad (37)$$

$$\begin{aligned} \mathbf{I}_m(t) &= \mathbf{I}_m(0) + \frac{\sigma t^{\sigma-1}(1-\varepsilon)\Lambda_4(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R})}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^t (t-z)^{\varepsilon-1} \Lambda_4(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \end{aligned} \quad (38)$$

$$\begin{aligned} \mathbf{R}(t) &= \mathbf{R}(0) + \frac{\sigma t^{\sigma-1}(1-\varepsilon)\Lambda_5(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R})}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^t (t-z)^{\varepsilon-1} \Lambda_5(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz. \end{aligned} \quad (39)$$

For the numerical scheme fitting $t = t_{n+1}$ in Equation (35), which deduced to the below form:

$$\begin{aligned} \mathbf{S}^{n+1}(t) &= \mathbf{S}^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_1(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^{t_{n+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_1(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \end{aligned} \quad (40)$$

$$\begin{aligned} \mathbf{S}_m^{n+1}(t) &= \mathbf{S}_m^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_2(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^{t_{n+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_2(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \end{aligned} \quad (41)$$

$$\begin{aligned} \mathbf{I}^{n+1}(t) &= \mathbf{I}^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_3(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^{t_{n+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_3(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \end{aligned} \quad (42)$$

$$\begin{aligned} \mathbf{I}_m^{n+1}(t) &= \mathbf{I}_m^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_4(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^{t_{n+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_4(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \end{aligned} \quad (43)$$

$$\begin{aligned} \mathbf{R}^{n+1}(t) &= \mathbf{R}^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_5(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \int_0^{t_{n+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_5(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz. \end{aligned} \quad (44)$$

By approximating the integral in the above system (40), then we get

$$\begin{aligned} \mathbf{S}^{n+1}(t) &= \mathbf{S}^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_1(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \sum_{r=0}^q \int_{t_r}^{t_{r+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_1(t, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \\ \mathbf{S}_m^{n+1}(t) &= \mathbf{S}_m^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_2(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \sum_{r=0}^q \int_{t_r}^{t_{r+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_2(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \\ \mathbf{I}^{n+1}(t) &= \mathbf{I}^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_3(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ \mathbf{I}_m^{n+1}(t) &= \mathbf{I}_m^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_4(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \sum_{r=0}^q \int_{t_r}^{t_{r+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_4(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz, \\ \mathbf{R}^{n+1}(t) &= \mathbf{R}^0 + \frac{\sigma t_n^{\sigma-1}(1-\varepsilon)\Lambda_5(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\varepsilon)} \\ &\quad + \frac{\varepsilon\sigma}{AB(\varepsilon)\Gamma(\varepsilon)} \sum_{r=0}^q \int_{t_r}^{t_{r+1}} (t_{n+1}-z)^{\varepsilon-1} \Lambda_5(z, \mathbf{S}, \mathbf{S}_m, \mathbf{I}, \mathbf{I}_m, \mathbf{R}) z^{\sigma-1} dz. \end{aligned} \quad (45)$$

FIGURE 1: Numerical solution of (10) for $\kappa = 0.1$ and different values of ϵ fractional order.

FIGURE 2: Numerical solution of (10) for $\kappa = 0.3$ and different values of ϵ fractional order.

FIGURE 3: Numerical solution of system (10) for $\kappa = 0.5$ and different values of ϵ fractional order.

Now, by applying the Lagrangian interpolation polynomial piecewise which yields the following form:

$$\begin{aligned}
\mathbf{S}^{n+1}(t) &= \mathbf{S}^0 \\
&+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Lambda_1(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\epsilon)} \\
&+ \frac{\sigma(\blacktriangle t)^\epsilon}{AB(\epsilon)\Gamma(\epsilon+2)} \\
&\cdot \sum_{r=0}^q [t_r^{\sigma-1}\Lambda_1(t_r, \mathbf{S}^r, \mathbf{S}_m^r, \mathbf{I}^r, \mathbf{I}_m^r, \mathbf{R}^r) \\
&\times ((q+1-r)^\epsilon(q-r+2+\epsilon) - (q-r)^\epsilon(p-r+2+2\epsilon)) \\
&- t_{r-1}^{\sigma-1}\Lambda_1(t_r, \mathbf{S}^{r-1}, \mathbf{S}_m^{r-1}, \mathbf{I}^{r-1}, \mathbf{I}_m^{r-1}, \mathbf{R}^{r-1}) \\
&\times ((q-r+1)^{\epsilon+1} - (p-r)^\epsilon(p-r+1+\epsilon))], \\
\mathbf{S}_m^{n+1}(t) &= \mathbf{S}_m^0 \\
&+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Lambda_2(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\epsilon)} \\
&+ \frac{\sigma(\blacktriangle t)^\epsilon}{AB(\epsilon)\Gamma(\epsilon+2)} \\
&\cdot \sum_{r=0}^q [t_r^{\sigma-1}\Lambda_2(t_r, \mathbf{S}^r, \mathbf{S}_m^r, \mathbf{I}^r, \mathbf{I}_m^r, \mathbf{R}^r) \\
&\times ((q+1-r)^\epsilon(q-r+2+\epsilon) - (q-r)^\epsilon(p-r+2+2\epsilon)) \\
&- t_{r-1}^{\sigma-1}\Lambda_2(t_r, \mathbf{S}^{r-1}, \mathbf{S}_m^{r-1}, \mathbf{I}^{r-1}, \mathbf{I}_m^{r-1}, \mathbf{R}^{r-1}) \\
&\times ((q-r+1)^{\epsilon+1} - (p-r)^\epsilon(p-r+1+\epsilon))], \\
\mathbf{I}^{n+1}(t) &= \mathbf{I}^0 \\
&+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Lambda_3(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\epsilon)} \\
&+ \frac{\sigma(\blacktriangle t)^\epsilon}{AB(\epsilon)\Gamma(\epsilon+2)} \\
&\cdot \sum_{r=0}^q [t_r^{\sigma-1}\Lambda_3(t_r, \mathbf{S}^r, \mathbf{S}_m^r, \mathbf{I}^r, \mathbf{I}_m^r, \mathbf{R}^r) \\
&\times ((q+1-r)^\epsilon(q-r+2+\epsilon) - (q-r)^\epsilon(p-r+2+2\epsilon)) \\
&- t_{r-1}^{\sigma-1}\Lambda_3(t_r, \mathbf{S}^{r-1}, \mathbf{S}_m^{r-1}, \mathbf{I}^{r-1}, \mathbf{I}_m^{r-1}, \mathbf{R}^{r-1}) \\
&\times ((q-r+1)^{\epsilon+1} - (p-r)^\epsilon(p-r+1+\epsilon))], \\
\mathbf{I}_m^{n+1}(t) &= \mathbf{I}_m^0 \\
&+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Lambda_4(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\epsilon)}
\end{aligned}$$

$$\begin{aligned}
&+ \frac{\sigma(\blacktriangle t)^\epsilon}{AB(\epsilon)\Gamma(\epsilon+2)} \\
&\cdot \sum_{r=0}^q [t_r^{\sigma-1}\Lambda_4(t_r, \mathbf{S}^r, \mathbf{S}_m^r, \mathbf{I}^r, \mathbf{I}_m^r, \mathbf{R}^r) \\
&\times ((q+1-r)^\epsilon(q-r+2+\epsilon) - (q-r)^\epsilon(p-r+2+2\epsilon)) \\
&- t_{r-1}^{\sigma-1}\Lambda_4(t_r, \mathbf{S}^{r-1}, \mathbf{S}_m^{r-1}, \mathbf{I}^{r-1}, \mathbf{I}_m^{r-1}, \mathbf{R}^{r-1}) \\
&\times ((q-r+1)^{\epsilon+1} - (p-r)^\epsilon(p-r+1+\epsilon))], \\
\mathbf{R}^{n+1}(t) &= \mathbf{R}^0 \\
&+ \frac{\sigma t_n^{\sigma-1}(1-\epsilon)\Lambda_5(t_n, \mathbf{S}^n, \mathbf{S}_m^n, \mathbf{I}^n, \mathbf{I}_m^n, \mathbf{R}^n)}{AB(\epsilon)} \\
&+ \frac{\sigma(\blacktriangle t)^\epsilon}{AB(\epsilon)\Gamma(\epsilon+2)} \\
&\cdot \sum_{r=0}^q [t_r^{\sigma-1}\Lambda_5(t_r, \mathbf{S}^r, \mathbf{S}_m^r, \mathbf{I}^r, \mathbf{I}_m^r, \mathbf{R}^r) \\
&\times ((q+1-r)^\epsilon(q-r+2+\epsilon) - (q-r)^\epsilon(p-r+2+2\epsilon)) \\
&- t_{r-1}^{\sigma-1}\Lambda_5(t_r, \mathbf{S}^{r-1}, \mathbf{S}_m^{r-1}, \mathbf{I}^{r-1}, \mathbf{I}_m^{r-1}, \mathbf{R}^{r-1}) \\
&\times ((q-r+1)^{\epsilon+1} - (p-r)^\epsilon(p-r+1+\epsilon))]. \quad (46)
\end{aligned}$$

6. Discussion and Numerical Results

A numerical scheme utilized to obtain the approximate solutions for the fractal fractional-order ITI disease model. Different scenarios have been discussed for the fractal fractional-order ITI disease model by choosing different parametric values and testing rates for the model. We observed that as the testing rate κ increasing; then, the incidence decreased effectively. We apply the aforementioned iterative scheme for the numerical analysis to demonstrate graphically the fractal fractional ITI disease model. To examine the dynamical behavior of the model, we choose suitable constant values for the parameters used in the model.

Figure 1 shows the effect of testing rate κ increasing and different order values of fractal fractional operator. By choosing the parametric values involved in the fractional-order ITI disease model such that $\kappa = 0.1$, $\beta = 0.15$, $\beta_m = 0.1$, $\gamma = 0.1$, $\gamma_m = 0.15$, $\mu = 0.003$, and $\eta = 0.1$ and assuming initial conditions for $\mathbf{S}$, $\mathbf{S}_m$, $\mathbf{I}$, $\mathbf{I}_m$ and $\mathbf{R}$. (c) and (d) show the infected class decreasing as the κ value increases.

Figure 2 shows the effect of testing rate κ increasing and different order values of fractal fractional operator. By choosing the parametric values involved in the fractional order ITI disease model such that $\kappa = 0.3$, $\beta = 0.15$, $\beta_m = 0.1$, $\gamma = 0.1$, $\gamma_m = 0.15$, $\mu = 0.003$, and $\eta = 0.1$ and assuming initial conditions for $\mathbf{S}$, $\mathbf{S}_m$, $\mathbf{I}$, $\mathbf{I}_m$, and $\mathbf{R}$. (c) and (d) show the infected class decreasing as the κ value increasing.

Figure 3 shows the effect of testing rate κ increasing and different order values of fractal fractional operator. By

choosing the parametric values involved in the fractional order ITI disease model such that $\kappa = 0.5$, $\beta = 0.15$, $\beta_m = 0.1$, $\gamma = 0.1$, $\gamma_m = 0.15$, $\mu = 0.003$, and $\eta = 0.1$ and assuming initial conditions for S , S_m , I , I_m , and R . (c) and (d) shows the infected class decreasing as the κ value increases.

7. Conclusion

In this article, we study the theoretical and numerical aspects of the fractal fractional ITI disease model in the sense of the Mittag-leffler kernel. The existence of a solution is derived with the help of a fixed point theorem for the proposed model. Different scenarios have been investigated for fractal fractional-order ITI disease models by choosing different parametric values and testing rates for the model. We observed that as different fractional orders for and testing rate κ increase, then the infected class decreased effectively. The numerical approximate solutions achieved by Lagrangian polynomial piecewise interpolation iterative method. Furthermore, one can study the stability analysis fractal fractional-order ITI disease model by using various types of approaches.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

All authors contributed equally in writing this article. All authors read and approved the final manuscript.

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