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Research Article

On the Chebyshev Polynomial for a Certain Class of Analytic Univalent Functions

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In this work, by considering the Chebyshev polynomial of the first and second kind, a new subclass of univalent functions is defined. We obtain the coefficient estimate, extreme points, and convolution preserving property. Also, we discuss the radii of starlikeness, convexity, and close-to-convexity.

1. Introduction

Let Δ be the open unit disk $\{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{A}$ be the class of analytic functions in Δ, satisfying the normalized conditions:

$$\begin{aligned} f(0) &= 0, \\ f'(0) &= 1. \end{aligned} \tag{1}$$

Thus, each $f \in \mathcal{A}$ has the following Taylor expansion:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \tag{2}$$

Furthermore, by $\mathcal{S}$, we shall denote the family of all functions in $\mathcal{A}$ that are univalent in Δ. Denote by $\mathcal{N}$ the subclass of $\mathcal{A}$ consisting of functions with negative coefficients of the type:

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k, (a_k \geq 0, z \in \Delta), \tag{3}$$

see [1].

Many researchers deal with orthogonal polynomials of Chebyshev, see [2, 3] and [4]. The Chebyshev polynomials of first kind and the second kind are defined by

$$\begin{aligned} T_k(t) &= \cos k\theta, \\ U_k(t) &= \frac{\sin (k+1)\theta}{\sin \theta}, \end{aligned} \tag{4}$$

respectively, where $-1 < t < 1$, $t = \cos \theta$, and k is the degree of polynomial.

The polynomial in (1) is connected by the following relations:

$$\frac{dT_k(t)}{dt} = kU_{k-1}(t), T_k(t) = U_k(t) - kU_{k-1}(t), \tag{5}$$

$$2T_k(t) = U_k(t) - U_{k-2}(t). \tag{6}$$

We note that if $t = \cos \theta$, $(-\pi/3 < \theta < \pi/3)$, then

$$H(z,t) = \frac{1}{1 - 2z\cos\theta + z2} = 1 + \sum_{k=1}^{\infty} \frac{\sin (k+1)\theta}{\sin\theta} z^k, (z \in \Delta). \tag{7}$$

Also, we have

$$H(z,t) = 1 + U_1(t)z + U_2(t)z^2 + \cdots, (z \in \Delta, -1 < t < 1), \quad (8)$$

where

$$Uk-1(t) = \frac{\sin (k \arccos t)}{\sqrt{1-t^2}}, (k \in \mathbb{N}), \quad (9)$$

are the Chebyshev polynomials of the second kind, see [5, 6] and [7].

The generating function of the first kind of Chebyshev polynomial $T_k(t)$, $t \in [-1,1]$ is given by

$$\sum_{k=0}^{\infty} T_k(t)z^k = \frac{1-tz}{1-2tz+z^2}. \quad (10)$$

For more details, see [8, 9] and [10].

For two functions f and g, analytic in Δ, we say that f is subordinate to g in Δ, written

$$f(z) \prec g(z), (z \in \Delta), \quad (11)$$

if there exists a Schwarz function w which is analytic in Δ with

$$\begin{gathered} w(0) = 0, \\ |w(z)| < 1, (z \in \Delta), \end{gathered} \quad (12)$$

such that $f(z) = g(w(z))$, $(z \in \Delta)$, see [11].

Also, if g is univalent in Δ, then

$$f(z) \prec g(z), (z \in \Delta) \Longleftrightarrow f(0) = g(0), f(\Delta) \subset g(\Delta). \quad (13)$$

Furthermore, if $f(z) = z - \sum_{k=2}^{\infty} a_k z^k$ and $g(z) = z - \sum_{k=2}^{\infty} b_k z^k$, then the Hadamard product (or covolution) of f and g is defined by

$$(f * g)(z) = z - \sum_{k=2}^{\infty} a_k b_k z^k = (g * f)(z). \quad (14)$$

Now, we consider the following functions which are connected with the Chebyshev polynomial of the first and second kind:

$$\mathscr{C}_1(z) = 1 + (1+\cos\theta)z - \frac{1-tz}{1-2tz+z2}, \quad (15)$$

$$\mathscr{C}_2(z) = 1 + (2\cos\theta + 1)z - H(z,t), \quad (16)$$

$$Q(z) = [(\mathscr{C}_1 * \mathscr{C}_1) * (\mathscr{C}_2 * \mathscr{C}_2) * f](z), \quad (17)$$

where $f(z) = z - \sum_{k=2}^{\infty} a_k z^k \in \mathcal{N}$ and "$*$" denotes the Hadamard product.

With a simple calculation we conclude that $Q(z)$ belongs to $\mathcal{N}$ and it is of the form:

$$Q(z) = z - \sum_{k=2}^{\infty} \left[\frac{\sin (k+1)\theta}{\sin\theta} T_k(t) \right]^2 a_k z^k, \quad (18)$$

where $-\pi/3 < \theta < \pi/3$ and $t = \cos\theta$.

Definition. For $M = \alpha + (\beta - \alpha)(1-\gamma)$, $-1 \le \beta < \alpha \le 1$, $0 < \gamma < 1$, and $0 \le \lambda \le 1$, we say that $Q(z)$ of the form (18) is a member of $\mathscr{E}_\gamma^\lambda(\alpha,\beta)$ if the following subordination relation holds

$$\frac{zQ'(z)}{f_\lambda(z)} \prec \frac{1+Mz}{1+\alpha z}, \quad (19)$$

where $f_\lambda(z) = (1-\lambda)z + \lambda f(z)$, $f(z) \in \mathcal{N}$.

Equation (19) is equivalent to the following inequality:

$$\left| \frac{\left(zQ'(z)/f_\lambda(z) \right) - 1}{M - \alpha z \left(Q'(z)/f_\lambda(z) \right)} \right| < 1. \quad (20)$$

2. Main Results

In this section, we introduce a sharp coefficient bound for the class $\mathscr{E}_\gamma^\lambda(\alpha,\beta)$. Also, the convolution preserving property is investigated.

Theorem 1. *The function $Q(z)$ of form (18) belongs to $\mathscr{E}_\gamma^\lambda(\alpha,\beta)$ if and only if*

$$\begin{aligned} &\sum_{k=2}^{\infty} \left[\left(\left(\sqrt{k} \frac{\sin (k+1)\theta}{\sin\theta} T_k(t) \right)^2 - \lambda \right) (1-\alpha) + \lambda(\beta - \alpha)(1-\gamma) \right] a_k \\ &\quad \le (\beta - \alpha)(1-\gamma). \end{aligned} \quad (21)$$

Proof. Let the inequality (21) holds and $z \in \partial\Delta = \{z \in \mathbb{C} : |z| = 1\}$. We have to prove that (19) or equivalently (20) holds true. But we have

$$
\begin{aligned}
Y &= \left|zQ'(z) - f_\lambda(z)\right| - \left|Mf_\lambda(z) - \alpha zQ'(z)\right| \\
&= \left|z - \sum_{k=2}^{\infty}\left[\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right]^2 ka_k z^k - (1-\lambda)z - \lambda \right. \\
&\quad \cdot \left.\left(z - \sum_{k=2}^{\infty} a_k z^k\right)\right| - \left|M\left((1-\lambda)z + \lambda\left(z - \sum_{k=2}^{\infty} a_k z^k\right)\right)\right. \\
&\quad \left. - \alpha z + \sum_{k=2}^{\infty}\left[\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right]^2 \alpha k a_k z^k\right| \\
&= \left|-\sum_{k=2}^{\infty}\left[\left(\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right)2k - \lambda\right]a_k z^k\right| \\
&\quad - \left|(M-\alpha)z - \sum_{k=2}^{\infty}\left[\lambda M - \alpha k\left(\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right)^2\right]a_k z^k\right|.
\end{aligned} \tag{22}
$$

By putting $z \in \partial\Delta$ and

$$
\lambda M - \alpha k\left(\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right)^2 = \lambda(M-\alpha) - \left[k\left(\frac{\sin(k+1)\theta}{\sin\theta}T_k(\theta)\right)^2 - \lambda\right]\alpha, \tag{23}
$$

the above expression reduces to

$$
Y \le \left|\sum_{k=2}^{\infty}\left[\left(k\left(\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right)^2 - \lambda\right)(1-\alpha) + \lambda(M-\alpha)\right]a_k - (M-\alpha)\right|. \tag{24}
$$

Since $H - \alpha = (\beta-\alpha)(1-\gamma)$, by using inequality (21), we get $Y \le 0$, so $Q \in \mathscr{E}_\gamma^\lambda(\alpha,\beta)$.

To prove the converse, let $Q \in \mathscr{E}_\gamma^\lambda(\alpha,\beta)$, thus

$$
\left|\frac{\left(zQ'(z)/f_\lambda(z)\right) - 1}{M - \alpha z\left(Q'(z)/f_\lambda(z)\right)}\right| = \frac{\left|z - \sum_{k=2}^{\infty}\left(\sqrt{k}(\sin(k+1)\theta/\sin\theta)T_k(t)\right)^2 a_k z^k - (1-\lambda)z + \lambda f(z)\right|}{\left|M((1-\lambda)z + \lambda f(z)) - \alpha z\left(1 - \sum_{k=2}^{\infty}\left(\sqrt{k}(\sin(k+1)\theta/\sin\theta)T_k(t)\right)^2 a_k z^k - 1\right)\right|} < 1, \tag{25}
$$

for all $z \in \Delta$. By $\operatorname{Re}(z) \le |z|$ for all $z \in \Delta$, we have

$$
\operatorname{Re}\left\{\frac{\sum_{k=2}^{\infty}\left[\left(\sqrt{k}(\sin(k+1)\theta/\sin\theta)T_k(t)\right)^2 - \lambda\right]a_k z^k}{(M-\alpha)z - \sum_{k=2}^{\infty}\left[\lambda M - \alpha\left(\sqrt{k}(\sin(k+1)\theta/\sin\theta)T_k(t)\right)^2\right]a_k z^k}\right\} < 1. \tag{26}
$$

By letting $z \longrightarrow 1$, through positive values and choose the values of z such that $zQ'(z)/f_\lambda(z)$ is real, we have

$$
\sum_{k=2}^{\infty}\left[\left(\left(\sqrt{k}\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right)^2 - \lambda\right)(1-\alpha) + t(M-\alpha)\right]a_k \le M - \alpha, \tag{27}
$$

and this completes the proof. □

Remark. We note that the function:

$$
V(z) = z - \frac{(\beta-\alpha)(1-\gamma)}{\left[\left(\sqrt{2}(\sin 3\theta/\sin\theta)\cos 2\theta\right)^2 - \lambda\right](1-\alpha) + \lambda(\beta-\alpha)(1-\gamma)}z^2, \tag{28}
$$

shows that the inequality (21) is sharp.

Theorem 2. *Let*

$$
Q_1(z) = z - \sum_{k=2}^{\infty}\left[\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right]^2 a_k z^k, \tag{29}
$$

$$
Q_2(z) = z - \sum_{k=2}^{\infty}\left[\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right]^2 b_k z^k, \tag{30}
$$

be in the class $\mathscr{E}_\gamma^\lambda(\alpha,\beta)$, then $(Q_1 * Q_2)(z)$ belongs to $\mathscr{E}_\gamma^\lambda(\alpha,\tilde{\beta})$, where

$$
\tilde{\beta} \le \alpha + \frac{(\beta-\alpha)2(1-\gamma)X(1-\alpha)}{(X(1-\alpha) + \lambda(\beta-\alpha)(1-\gamma))^2 - \lambda(1-\gamma)^2(\beta-\alpha)^2}, \tag{31}
$$

$$
X = \left(\sqrt{k}\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right)^2 - \lambda. \tag{32}
$$

Proof. It is sufficient to show that

$$
\sum_{k=2}^{\infty}\left[\left(\left(\sqrt{k}\frac{\sin(k+1)\theta}{\sin\theta}T_k(t)\right)^2 - \lambda\right)\left(\frac{1-\alpha}{(\beta-\alpha)(1-\gamma)}\right) + \lambda\right]a_k b_k \le 1. \tag{33}
$$

By using the Cauchy-Schwarz inequality, from (21), we obtain

$$\sum_{k=2}^{\infty}\left[\left(\left(\sqrt{k}\frac{\sin (k+1)\theta}{\sin\theta}T_k(t)\right)^2-\lambda\right)\left(\frac{1-\alpha}{(\beta-\alpha)(1-\gamma)}\right)+\lambda\right]\sqrt{a_k b_k}\le 1. \tag{34}$$

Here, we find the largest k such that

$$\sum_{k=2}^{\infty}\left[\left(\left(\sqrt{k}\frac{\sin (k+1)\theta}{\sin\theta}T_k(t)\right)^2-\lambda\right)\left(\frac{1-\alpha}{(\beta-\alpha)(1-\gamma)}\right)-\lambda\right]\sqrt{a_k b_k}\le 1, \tag{35}$$

or equivalently for $k\ge 2$,

$$\sqrt{a_k b_k}\le\frac{[X(1-\alpha)+\lambda(\beta-\alpha)(1-\gamma)]\left(\tilde{\beta}-\alpha\right)}{[X(1-\alpha)+\lambda(\beta-\alpha)(1-\gamma)](\beta-\alpha)}, \tag{36}$$

where X is given by (32).
This inequality holds if

$$\frac{(\beta-\alpha)(1-\gamma)}{X(1-\alpha)+\lambda(\beta-\alpha)(1-\gamma)}\le\frac{[X(1-\alpha)+\lambda(\beta-\alpha)(1-\gamma)]\left(\tilde{\beta}-\alpha\right)}{\left[X(1-\alpha)+\lambda\left(\tilde{\beta}-\alpha\right)(1-\gamma)\right](\beta-\alpha)}, \tag{37}$$

or equivalently

$$\tilde{\beta}\le\alpha+\frac{(\beta-\alpha)^2(1-\gamma)X(1-\alpha)}{(X(1-\alpha)+\lambda(\beta-\alpha)(1-\gamma))^2-\lambda(1-\gamma)^2(\beta-\alpha)^2}, \tag{38}$$

where X is given by (32), so the proof is complete. □

3. Geometric Properties of $\mathscr{E}_{\gamma}^{\lambda}(\alpha,\beta)$

In this section, we show that the class $\mathscr{E}_{\gamma}^{\lambda}(\alpha,\beta)$ is a convex set. Also, the radii of starlikeness, convexity, and close-to-convexity are obtained.

Theorem 3. *The class $\mathscr{E}_{\gamma}^{\lambda}(\alpha,\beta)$ is a convex set.*

Proof. It is enough to prove that if for $j=1,2,\cdots,m$,

$$Q_j(z)=z-\sum_{k=2}^{\infty}\left[\frac{\sin (k+1)\theta}{\sin\theta}\right]^k a_{k,j}z^k, \tag{39}$$

be in $\mathscr{E}_{\gamma}^{\lambda}(\alpha,\beta)$, then the function

$$F(z)=\sum_{j=1}^{m}d_jQ_j(z), \tag{40}$$

is also in $\mathscr{E}_{\gamma}^{\lambda}(\alpha,\beta)$, where $\sum_{j=1}^{m}d_j=1$. But, we have

$$\begin{aligned}F(z)&=z-\sum_{k=2}^{\infty}\left(\sum_{j=1}^{m}\left(\frac{\sin (k+1)\theta}{\sin\theta}T_k(t)\right)^2 d_ja_{k,j}\right)z^k,\\&=z-\sum_{k=2}^{\infty}\left(\frac{\sin (k+1)\theta}{\sin\theta}T_k(t)\right)^2\left(\sum_{j=1}^{m}d_ja_{k,j}\right)z^k.\end{aligned} \tag{41}$$

Since by Theorem 1,

$$\begin{aligned}&\sum_{k=2}^{\infty}\left[\left(\left(\sqrt{k}\frac{\sin (k+1)\theta}{\sin\theta}T_k(t)\right)^2-\lambda\right)(1-\alpha)+\lambda(\beta-\alpha)(1-\gamma)\right]\left(\sum_{j=1}^{m}d_ja_{k,j}\right)\\&=\sum_{j=1}^{m}\left(\sum_{k=2}^{\infty}\left[\left(\left(\sqrt{k}\frac{\sin (k+1)\theta}{\sin\theta}T_k(t)\right)^2-\lambda\right)\right.\right.\\&\left.\left.\cdot(1-\alpha)+\lambda(\beta-\alpha)(1-\gamma)\right]a_{k,j}\right)d_j\\&<\sum_{j=1}^{m}(\beta-\alpha)(1-\gamma)d_j=(\beta-\alpha)(1-\gamma)\left(\sum_{j=1}^{m}d_j\right)\\&=(\beta-\alpha)(1-\gamma),\end{aligned} \tag{42}$$

so, $F(z)\in\mathscr{E}_{\gamma}^{\lambda}(\alpha,\beta)$. Hence, the proof is complete. □

Theorem 4. *Let $f\in\mathscr{E}_{\gamma}^{\lambda}(\alpha,\beta)$, then*

(i) f is a starlike of order θ_1 ($\cos\theta_1<1$) in $|z|<R_1$ where

$$\begin{aligned}R_1=\inf_k\left\{\left[\left(\left(\sqrt{k}\frac{\sin (k+1)\theta}{\sin\theta}T_k(t)\right)^2-\lambda\right)\right.\right.\\\left.\left.\cdot\left(\frac{1-\alpha}{(\beta-\alpha)(1-\gamma)}\right)+\lambda\right]\left(\frac{1-\theta_1}{k-\theta_1}\right)\right\}^{1/k-1}\end{aligned} \tag{43}$$

(ii) f is convex of order θ_2 ($0\le\theta_2<1$) in $|z|<R_2$ where

$$\begin{aligned}R_2=\inf_k\left\{\left[\left(\left(\sqrt{k}\frac{\sin (k+1)\theta}{\sin\theta}T_k(t)\right)^2-\lambda\right)\right.\right.\\\left.\left.\cdot\left(\frac{1-\alpha}{(\beta-\alpha)(1-\gamma)}\right)+\lambda\right]\left(\frac{1-\theta_2}{k(k-\theta_2)}\right)\right\}^{1/k-1}\end{aligned} \tag{44}$$

(iii) f is close-to-convex of order θ_3 ($0\theta_3<1$) in $|z|<R_3$, where

$$R_3 = \inf_k \left\{ \left[\left(\left(\sqrt{k} \frac{\sin (k+1)\theta}{\sin \theta} T_k(t) \right)^2 - \lambda \right) \cdot \left(\frac{1-\alpha}{(\beta-\alpha)(1-\gamma)} \right) + \lambda \right] \left(\frac{1-\theta_3}{k} \right) \right\}^{1/k-1} \tag{45}$$

Proof.

(i) For $0 \leq \theta_1 < 1$, we need to show that

$$\left| \frac{zf'}{f} - 1 \right| < 1 - \theta_1 \tag{46}$$

In other words, it is sufficient to show that

$$\left| \frac{zf'(z)}{f(z)} - 1 \right| = \left| \frac{\sum_{k=2}^{\infty} (k-1) a_k z^{k-1}}{1 - \sum_{k=2}^{\infty} a_k z^{k-1}} \right| \leq \frac{\sum_{k=2}^{\infty} (k-1) a_k |z|^{k-1}}{1 - \sum_{k=2}^{\infty} a_k |z|^{k-1}} < 1 - \theta_1, \tag{47}$$

$$\sum_{k=2}^{\infty} \left(\frac{k-\theta_1}{1-\theta_1} \right) a_k |z|^{k-1} < 1. \tag{48}$$

By (21), it is easy to see that above inequality holds if

$$|z|^{k-1} \leq \left[\left(\left(\sqrt{k} \frac{\sin (k+1)\theta}{\sin \theta} T_k(t) \right)^2 - \lambda \right) \left(\frac{1-\alpha}{(\beta-\alpha)(1-\gamma)} \right) + \lambda \right] \left(\frac{1-\theta_1}{k-\theta_1} \right). \tag{49}$$

This completes the proof of (i).

(ii) Since f is convex if and only if zf' is starlike, we get the required result (ii)

(iii) We must show that $|f'(z) - 1| \leq 1 - \theta_3$. But

$$|f'(z) - 1| = \left| \sum_{k=2}^{\infty} k a_k z^{k-1} \right| \leq \sum_{k=2}^{\infty} k a_k |z|^{k-1} \tag{50}$$

Thus, $|f'(z) - 1| < 1 - \theta_3$ if $\sum_{k=2}^{\infty} (k/1 - \theta_3) a_k |z|^{k-1} \leq 1$. But by Theorem 1, the above inequality holds true, if

$$|z|^{k-1} \leq \left[\left(\left(\sqrt{k} \frac{\sin (k+1)\theta}{\sin \theta} T_k(t) \right)^2 - \lambda \right) \left(\frac{1-\alpha}{(\beta-\alpha)(1-\gamma)} \right) + \lambda \right] \left(\frac{1-\theta_3}{k} \right). \tag{51}$$

Hence, the proof is complete. □

4. Conclusions

Univalent functions have always been the main interests of many researchers in geometric function theory. Many studies recently related to Chebyshev polynomials revolved around classes of analytic normalized univalent functions. In this particular work, the geometric properties are obtained for functions in more general class using the Chebyshev polynomials associated with a convolution structure. In this paper, when the parameters being complex numbers could be subject to further investigation. Also, by changing the operator and extending, it may be for future studies.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Authors' Contributions

All authors contributed equally to and approved the final manuscript.

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