Research Article

S^{*P} - b -Partial Metric Spaces with some Results in Common Fixed Point Theorems

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In this paper, we introduce the notion of S^{*P} - b -partial metric spaces which is a generalization each of S - b -metric spaces and partial-metric space. Also, we study and prove some topological properties, to know the convergence of the sequences and Cauchy sequence. Finally, we study a new common fixed point theorem in these spaces.

1. Introduction

There are a large number of generalizations of the Banach contraction principle with different use forms of contractual terms in a variety of generalized metric spaces. For some of these, the circulars are obtained through contractual terms expressed in reasonable terms. Latif et al. [1] introduced the notion of G -rational Geraghty contractive mappings in the setup of ordered generalized b -metric spaces and investigated the existence of fixed points for such mappings. They also provided an example to illustrate the presented results and show that they are more general than some existing ones.

One such generalization is a partial-metric space introduced by Matthews [2] in 1994. Since in this space, a self-distance of an arbitrary point need not to be equal zero. Shahkoobi and Razani [3] introduced new classes of rational Geraghty contractive mappings in the setup of b -metric spaces and the existence of some fixed point for such mappings in ordered b -metric spaces. Bakhtin [4] and Bourbaki [5] introduced a notion of b -metric spaces. Later on, Czerwik [6] generalized the Banach contraction mapping theorem by formally defining a b -metric space and giving a postulate which was weaker than the condition of triangular inequality. After that, Fagin and Stockmeyer [7] defined some kind of relaxation in triangular inequality and called this new distance measure as the nonlinear elastic mathing. All these notions and applications pushed us to introduce the concept

of extended b -metric space by S -metric space and partial metric space.

In 2007, Sedghi et al. [8] introduced D^* -metric spaces which are the generalization of the notion of D -metric spaces introduced by Dhage [9]. Also, Sedghi et al. proved some of the basic properties in D^* -metric spaces. In 2012, Sedghi et al. [10] introduced S -metric spaces and give some properties and fixed point theorem for a complete S -metric space. Soliman and Zidan [11] introduced a new coupled fixed point theorem in a generalized metric space, and they utilized the same to study the stability for a system of set-valued functional equations. In 2015, Gupta and Deep [12] proved fixed point theorems for nonlinear contractive mappings in S -metric spaces. On the other way, Chauhan and Gupta [13] introduced the notion of fuzzy cone b -metric space and defined fuzzy cone b -contractive mapping and proved Banach contraction theorem for a single mapping in the setting of fuzzy cone b -metric space.

Recently, in 2019, Mustafa et al. [14] introduced the structure of S_b -metric spaces which is a generalization of S -metric space and gave some properties and fixed point theorem for this spaces.

2. Preliminaries and Definitions

We begin the section with some basic definitions and concepts.

Definition 1. Let Y be a nonempty set and $\zeta \geq 1$ be a given real number. A function $d_b : Y \times Y \longrightarrow [0, \infty)$ is called b -metric, if it satisfies the following properties for each $w, t, r \in Y$:

- (b₁) $d_b(w, t) = 0 \Leftrightarrow w = t$
  - (b₂) $d_b(w, t) = d_b(t, w)$
  - (b₃) $d_b(w, t) \leq \zeta[d_b(w, r) + d_b(r, t)]$
- The pair (Y, d_b) is called a b -metric space.

Example 2. Let $Y = l_{\zeta}\mathbb{R}$ with $0 < \zeta < 1$ where $l_{\zeta}\mathbb{R} : \{\{w_n\} \subset \mathbb{R} : \sum_{n=1}^{\infty} |w_n|^{\zeta} < \infty\}$. Define $d_b : Y \times Y \longrightarrow [0, \infty)$ as

$$d_b(w, t) = \left(\sum_{n=1}^{\infty} |w_n - t_n|^{\zeta} \right)^{1/\zeta}, \quad (1)$$

where $w = \{w_n\}, t = \{t_n\}$. Then, d_b is a b -metric space.

Definition 3 (see [2]). Let Y be a set. A function $P : Y \times Y \longrightarrow [0, \infty)$ is said to be a partial metric spaces on a nonempty set Y , if for all $w, t, r \in Y$, the following conditions hold:

- (P₁) $0 \leq P(w, w) \leq P(w, t)$ (nonnegativity and small self-distances)
- (P₂) If $P(w, w) = P(w, t) = P(t, t)$ then $w = t$ (indistancy implies equality)
- (P₃) If $P(w, t) = P(t, w)$ (symmetry)
- (P₄) $P(w, t) \leq P(w, r) + P(r, t) - P(r, r)$ (triangularity)

Hence, the function P is called a P -metric on Y and the pair (Y, P) is called a partial-metric space.

Mustafa and Sims [15] extended the Banach principle by introducing the notation of generalized metric spaces the so-called G -metric spaces as follows.

Definition 4. Let Y be a set. A function $G : Y \times Y \times Y \longrightarrow [0, \infty)$ is said to be generalized metric spaces on a nonempty set Y , if for all $w, t, r, \eta \in Y$, the following conditions hold:

- (G₁) $G(w, t, r) = 0 \Leftrightarrow w = t = r$
- (G₂) $G(w, t, r) > 0$ and $w \neq t$
- (G₃) $G(w, w, t) \leq G(w, t, r)$ and $t \neq r$
- (G₄) $G(w, t, r) = G(w, r, t) = G(t, r, w) = \dots$, (symmetry in all three variables)
- (G₅) $G(w, t, r) \leq G(w, \eta, \eta) + G(\eta, r, w)$

Hence, the function G is called a G -metric on Y and the pair (Y, G) is called a generalized metric space.

Mustafa and Sims [15] found some basic properties and examples of G -metric spaces.

After that, Zand and Nezhad [16] introduced the definition of G^p -metric space by generalization and unification of both the partial-metric space and G -metric space as follows:

Definition 5. Let Y be a set. A function $G^p : Y \times Y \times Y \longrightarrow [0, \infty)$ is said to be a generalized partial metric spaces on a nonempty set Y , if for all $w, t, r, \eta \in Y$, the following conditions hold:

- (G₁^p) $w = t = r$ if $G^p(w, w, w) = G^p(t, t, t) = G^p(r, r, r)$
- (G₂^p) $0 \leq G^p(w, w, w) \leq G^p(w, w, t) \leq G^p(w, t, y)$
- (G₃^p) $G^p(w, t, r) = G^p(w, r, t) = G^p(t, r, w) = \dots$, (symmetry in all three variables)
- (G₄^p) $G^p(w, t, r) \leq G^p(w, \eta, \eta) + G^p(\eta, r, w) - G^p(\eta, \eta, \eta)$

Hence, the function G^p is called a G^p -metric on Y and the pair (Y, G^p) is called a generalized partial metric space.

Example 6 (see [16]). Let $Y = [0, \infty)$ and $G^p(w, t, r) = \max\{w, t, r\}$ for all $w, t, r \in Y$, then (Y, G^p) is a G^p -metric space. Also, (Y, G^p) is not a G -metric space.

Proposition 7. Let (Y, G^p) is a G^p -metric space, then for any $w, t, r \in Y$ and $\eta \in Y$, it follows that

- (i) $G^p(w, t, r) \leq G^p(w, w, t) + G^p(w, w, r) - G^p(w, w, w)$
- (ii) $G^p(w, t, t) \leq 2G^p(w, w, t) - G^p(w, w, w)$
- (iii) $G^p(w, t, r) \leq G^p(w, \eta, \eta) + G^p(t, \eta, \eta) + G^p(r, \eta, \eta) - 2G^p(\eta, \eta, \eta)$

Sedghi et al. [8] introduced the notion of D^* -metric which is a modification of the definition of D -metric introduced by Dhage [9, 17], and they proved some basic properties in D^* -metric spaces.

Definition 8 (see [8]). Let Y be a set. A function $D^* : Y \times Y \times Y \longrightarrow [0, \infty)$ is said to be a D^* -metric spaces on a nonempty set Y , if for all $w, t, r \in Y$, the following conditions hold:

- (D₁^{*}) $D^*(w, t, r) \geq 0$
- (D₂^{*}) $D^*(w, t, r) \Leftrightarrow w = t = r$
- (D₃^{*}) $D^*(w, t, r) = D^*(\aleph\{w, t, r\})$, (where $\aleph$ is a permutation function)

Hence, the function D^* is called a D^* -metric on Y and the pair (Y, D^*) is called a generalized partial metric space.

Example 9 (see [8]). Let $Y = \mathbb{R}$. Denote $D^*(w, t, r) = |w - t| + |t - r| + |r - w|$, for all $w, t, r \in \mathbb{R}$. Since

$$B_{D^*}(w, \lambda) = \{t \in Y : D^*(w, t, t) < \lambda\}, \quad (2)$$

Hence,

$$\begin{aligned} B_{D^*}(1, 2) &= \{t \in \mathbb{R} : D^*(1, t, t) < 2\} \\ &= \{t \in \mathbb{R} : |t - 1| + |t - 1| < 2\} \\ &= \{t \in \mathbb{R} : |t - 1| < 1\} = (0, 2). \end{aligned} \quad (3)$$

On the other way, Sedghi et al. introduced S -metric spaces as follows.

Definition 10 (see [10]). Let Y be a set. A function $S : Y^3 \longrightarrow [0, \infty)$ is said to be S -metric spaces on a nonempty set Y , if for all $w, t, r, \eta \in Y$ the following conditions hold:

- (S₁) $S(w, t, r) > 0$
- (S₂) $S(w, t, r) \Leftrightarrow w = t = r$

(S_3) $S(w, t, r) \leq S(w, w, \eta) + S(t, t, \eta) + S(r, r, \eta)$ for all $w, t, r, \eta \in Y$ (rectangle inequality)

Hence, the function S is called a S -metric on Y and the pair (Y, S) is called an S -metric space.

Example 11 (see [10]). Let $Y = \mathbb{R}^n$ and $\|\cdot\|$ a norm on Y , then

$$S(w, t, r) = \|t + r - 2w\| + \|t - r\|, \quad (4)$$

is a S -metric on Y .

Remark 12. Every D^* -metric is S -metric, but in general, the converse is not true, see the following example.

Example 1. Let $Y = \mathbb{R}^n$ and $\|\cdot\|$ a norm on Y , then $S(w, t, r) = \|t + r - 2w\| + \|t - r\|$ is S -metric on Y , but it is not D^* -metric because it is not symmetric.

Example 2. Let $Y = \mathbb{R}^2$ and d is an ordinary metric on Y . Therefore, $S(w, t, r) = d(w, t) + d(w, r) + d(t, r)$ is an S -metric on Y . Then, if the points w, t, r connected by a line. Hence, the triangle when choose a point a mediating this triangle then the inequality

$$S(w, t, r) \leq s(w, w, \eta) + s(t, t, \eta) + (r, r, \eta), \quad (5)$$

for all $w, t, r, \eta \in Y$ holds.

Definition 13 (see [10]). Let (Y, S) be a S -metric space. Then, for $w_0 \in Y$ and $\lambda > 0$, the S -open ball and S -closed ball of radius λ with centered at w_0 is

$$\begin{aligned} B_S(w_0, \lambda) &= \{t \in Y : S(w_0, w_0, t) < \lambda\}, \\ \bar{B}_S(w_0, \lambda) &= \{t \in Y : S(w_0, w_0, t) \leq \lambda\}. \end{aligned} \quad (6)$$

Proposition 14 (see [10]). Let (Y, S) be a S -metric space and $A \subset Y$:

- (1) If for every $w_0 \in A$ there exists $\lambda > 0$ such that $B_S(w_0, \lambda)$, then the subset A is called an open subset of Y
- (2) A subset A of Y is said to be S -bounded if there exists $\lambda > 0$ such that $S(w, w, t) < \lambda$ for all $w, t \in A$
- (3) A sequence $\{w_n\}$ in Y converges to w if and only if $S(w_n, w_n, w) \rightarrow 0$ as $n \rightarrow \infty$. That is, for each $\lambda > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0$, $S(w_n, w_n, w) < \lambda$ and we denote this by $\lim_{n \rightarrow \infty} w_n = w$

Lemma 15 (see [10]). Let (Y, S) be an S -metric space, then the following is satisfied:

- (1) If $\lambda > 0$ and $w \in Y$, then the ball $B_S(w_0, \lambda)$ is an open subset of Y
- (2) If the sequence $\{w_n\}$ in Y converges to w , then w is unique

(3) If the sequence $\{w_n\}$ in Y converges to w , then $\{w_n\}$ is a Cauchy sequence

Definition 16 (see [15]). Two classes of the following mappings are

- (1) $\Pi = \{\pi : \pi : [0, \infty) \rightarrow [0, \infty)\}$ is nondecreasing, continuous and $\pi^{-1}(0) = 0$
- (2) $\Gamma = \{\gamma : \gamma : [0, \infty) \rightarrow [0, \infty)\}$ is nondecreasing, lower semi-continuous and $\pi^{-1}(0) = 0$

Definition 17. Let $(Y, \leq)$ be a partially ordered set. Two maps $\mathfrak{f}, \mathfrak{g} : Y \times Y$ are said to be weak increasing if $\mathfrak{f}w \leq \mathfrak{g}w$ and $\mathfrak{g}w^\circ \mathfrak{f}w$ for all $w \in Y$.

Barakat and Zidan [18] proved a common fixed point theorem for weak contractive maps by using the concept of G^p -metric spaces.

Theorem 18. Let $(Y, \leq)$ be a partially ordered set with $\mathfrak{f}$ and $\mathfrak{g}$ be weakly increasing self mapping on a complete G^p -partial metric space. Suppose that there exist $\pi \in \Pi$ and $\gamma \in \Gamma$ such that

$$\pi(G^p(\mathfrak{f}(w), \mathfrak{f}(w), \mathfrak{g}(t))) \leq \pi(M(w, w, t)) - \gamma(M(t, t, w)), \quad (7)$$

for all $w, t \in Y$, where

$$\begin{aligned} M(w, w, t) &= \zeta_1 G^p(w, w, t) + \zeta_2 G^p(w, w, \mathfrak{f}(w)) + \zeta_3 G^p(t, t, \mathfrak{g}(t)) \\ &\quad + \zeta_4 [G^p(w, w, \mathfrak{g}(t)) + G^p(t, t, \mathfrak{f}(w))], \end{aligned} \quad (8)$$

where $\zeta_i > 0$ for $i = \{1, 2, 3, 4\}$ with $\zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 \geq 1$.

Then, of the following two cases, assume that one of the following cases is satisfied:

- (a) If a nondecreasing sequence $\{w_n\}$ converges to $r \in Y$ implies $\{w_n\}^\circ r$ for all $n \in \mathbb{N}$
- (b) $\mathfrak{f}$ or $\mathfrak{g}$ is continuous

Therefore, the maps $\mathfrak{f}$ or $\mathfrak{g}$ have a common fixed point.

The present paper is aimed at introducing the notion of S_b^* -partial metric spaces which is a generalization each of S -metric spaces and partial-metric space. Also, we give some of the topological properties that are important in knowing the convergence of the sequences and Cauchy sequence. Finally, we study a new fixed point theory in this spaces.

3. S_b^* -Partial Metric Spaces and some Properties

We first introduce the concept of a S_b^{*P} -partial metric space or (S_b^{*P}) .

Definition 19. Let Y be a set and $\zeta \geq 1$. A function $S_b^{*P} : Y^3 \rightarrow [0, \infty)$ is said to be a S_b^{*P} -partial metric spaces on a non-empty set Y , if for all $w, t, r, \eta \in Y$, the following conditions hold:

$$\begin{aligned} (S_b^{*P} 1) \quad S_b^{*P}(w, t, r) &= S_b^{*P}(w, w, w) = S_b^{*P}(t, t, t) = S_b^{*P}(r, r, r) \text{ then, } w = t = r \\ (S_2^{*P}) \end{aligned}$$

$S_b^{*P}(w, t, r) \leq \zeta [S_b^{*P}(w, w, \eta) + S_b^{*P}(t, t, \eta) + S_b^{*P}(r, r, \eta) - S_b^{*P}(\eta, \eta, \eta)]$ for all $w, t, r, \eta \in Y$ (rectangle inequality)

Hence, the function S_b^{*P} is called an S_b^{*P} -partial metric on Y and the pair (Y, S_b^{*P}) is called an S_b^{*P} -partial metric space.

Example 20. Let $Y = \mathbb{R}$ and $\|\cdot\|$ a norm on Y , then we have

$$S_b^{*P}(w, t, r) = \max \{w, t, r\} + \|w - r\| + \|t - w\|, \quad (9)$$

is a S_b^{*P} -partial metric on Y .

Remark 21. From Example 20, we get every S -metric is S_b^{*P} -metric, but the converse is not true at all.

Definition 22. Let (Y, S_b^{*P}) be a S_b^{*P} -partial metric space. Then, for $w_0 \in Y$ and $\lambda > 0$, the S_b^{*P} -open ball and S_b^{*P} -open closed of radius λ with centered at w_0 is

$$B_{S_b^{*P}}(w_0, \lambda) = \{t \in Y : S_b^{*P}(w_0, w_0, t) < \lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(t, t, w_0)\},$$

$$\bar{B}_{S_b^{*P}}(w_0, \lambda) = \{t \in Y : S_b^{*P}(w_0, w_0, t) \leq \lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(t, t, w_0)\}.$$

(10)

Proposition 23. Let (Y, S_b^{*P}) be a S_b^{*P} -partial metric space. Then for $w_0 \in Y$ and $\lambda > 0$, the following statements are satisfying:

- (1) If $S_b^{*P}(w_0, w_0, w) \leq \lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(w, w, w_0)$ and $S_b^{*P}(w_0, w_0, t) \leq \lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(t, t, w_0)$, then $w, t \in B_{S_b^{*P}}(w_0, \lambda)$
- (2) If $t \in B_{S_b^{*P}}(w_0, \lambda)$, then there exist $\rho > 0$, such that $B_{S_b^{*P}}(t, \rho) \subseteq B_{S_b^{*P}}(w_0, \lambda)$

Proof.

- (1) The proof is straightforward.
- (2) Let $t \in B_{S_b^{*P}}(w_0, \lambda)$, then we have

$$S_b^{*P}(w_0, w_0, t) \leq \lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(t, t, w_0) \quad (11)$$

Also, we suppose

$$\rho = \lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(r, r, w_0) - 2S_b^{*P}(w_0, w_0, t), \quad (12)$$

and $r \in B_{S_b^{*P}}(t, \rho)$, then we get

$$S_b^{*P}(t, t, r) \leq \rho + S_b^{*P}(t, t, t) - S_b^{*P}(r, r, t). \quad (13)$$

Therefore, $t \in B_{S_b^{*P}}(w_0, \lambda)$, and so (2) holds.

$$\begin{aligned} S_b^{*P}(w_0, w_0, r) &\leq \zeta [S_b^{*P}(w_0, w_0, t) + S_b^{*P}(w_0, w_0, t) + S_b^{*P}(r, r, t) - S_b^{*P}(t, t, t)] \\ &\leq \zeta [2S_b^{*P}(w_0, w_0, t) + S_b^{*P}(r, r, t) + \lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(r, r, w_0) - 2S_b^{*P}(w_0, w_0, t) - S_b^{*P}(t, t, t)] \\ &\leq \zeta [\lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(r, r, t)] \\ &\leq \lambda + S_b^{*P}(w_0, w_0, w_0) - S_b^{*P}(r, r, t). \end{aligned} \quad (14)$$

Hence, $r \in B_{S_b^{*P}}(w_0, \lambda)$.

Definition 24. Let (Y, S_b^{*P}) be a S_b^{*P} -partial metric space and $\{w_n\}$ a sequence in Y . A point $w \in Y$ is said to be the limit of the sequence if

$$\lim_{n, m \rightarrow \infty} S_b^{*P}(w, w_n, w_m) = S_b^{*P}(w, w, w). \quad (15)$$

Hence, the sequence $\{w_n\}$ is S_b^{*P} -convergent to w .

Therefore, if $\{w_n\} \rightarrow w$ in a S_b^{*P} -partial metric space (Y, S_b^{*P}) . Then, for any $\hbar > 0$, there exists $\mathfrak{N} \in \mathbb{N}$ such that, for all $n, m > \mathfrak{N}$, we have

$$|S_b^{*P}(w, w_n, w_m) - S_b^{*P}(w, w, w)| < \hbar. \quad (16)$$

Definition 25. A S_b^{*P} -partial metric space (Y, S_b^{*P}) is called a S_b^{*P} -partial asymmetric space if

$$S_b^{*P}(w, w, t) = S_b^{*P}(t, t, w). \quad (17)$$

Lemma 26. (Y, S_b^{*P}) be a S_b^{*P} -partial metric space. If the sequence $\{w_n\}$ in Y converges to w . Therefore, we get w is unique.

Proof. Let $\{w_n\}$ converges to w and t . Therefore, for each $\lambda > 0$, there exist $n_1, n_2 \in \mathbb{N}$, then we have

$$\begin{aligned} n \geq n_1 &\Rightarrow S_b^{*P}(w_n, w_n, w) < \frac{\lambda}{3}, \\ n \geq n_2 &\Rightarrow S_b^{*P}(w_n, w_n, t) < \frac{\lambda}{3}. \end{aligned} \quad (18)$$

If set $n_0 = \max \{n_1, n_2\}$, then for every $n \geq n_0$. Also, we have a third condition of S_b^{*P} -partial metric

$$\begin{aligned}
S_b^{*P}(w, w, t) &\leq \zeta [S_b^{*P}(w, w, w_n) + S_b^{*P}(w, w, w_n) \\
&\quad + S_b^{*P}(t, t, w_n) - S_b^{*P}(w_n, w_n, w_n)] \\
&\leq \zeta [2S_b^{*P}(w, w, w_n) + S_b^{*P}(t, t, w_n) - S_b^{*P}(w_n, w_n, w_n)] \\
&\leq \zeta [2S_b^{*P}(w, w, w_n) + S_b^{*P}(t, t, w_n)] \\
&\leq 2S_b^{*P}(w, w, w_n) + S_b^{*P}(t, t, w_n) < 2\frac{\lambda}{3} + \frac{\lambda}{3} = \lambda.
\end{aligned} \tag{19}$$

Hence, $S_b^{*P}(w, w, t) = 0 \Rightarrow w = t$, but the converse is not necessarily true.

Definition 27. Let (Y, S_b^{*P}) be a S_b^{*P} -partial metric space. Then, for a s' sequence $\{w_n\} \subseteq Y$ and a point $w \in Y$, the following are equivalent:

- (1) $\{w_n\}$ is a S_b^{*P} convergent to w
- (2) $\lim_{n,m \rightarrow \infty} S_b^{*P}(w_n, w_n, w) = S_b^{*P}(w, w, w)$

Proposition 28. Let (Y, S_b^{*P}) be a S_b^{*P} -partial metric space. If then, we get

- (1) A sequence $\{w_n\} \in Y$ is called a Cauchy sequence if for each $\lambda > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$S_b^{*P}(w_n, w_n, w_m) < \lambda + S_b^{*P}(w, w, w), \tag{20}$$

for each $n, m \geq 0$.

Definition 29. Let (Y, S_b^{*P}) be a S_b^{*P} -partial metric space. Then, (Y, S_b^{*P}) is said to be complete if every Cauchy sequence is convergent.

Definition 30. Let (Y, S_b^{*P}) and let (Y_2, S_{b2}^{*P}) be two S_b^{*P} -partial metric space. Also, we suppose a function

$$\mathfrak{F} : (Y_1, S_{b1}^{*P}) \longrightarrow (Y_2, S_{b2}^{*P}), \tag{21}$$

then $\mathfrak{F}$ is said to be S_b^{*P} -continuous at a point $x \in Y_1$ if and only if, for given $\hbar > 0$, there exists $\lambda > 0$ such that $w, t \in Y_1$ and the inequality

$$S_{b1}^{*P}(x, w, t) < \hbar + S_{b1}^{*P}(w, w, w). \tag{22}$$

This is an indication

$$S_{b2}^{*P}(\mathfrak{F}(x), \mathfrak{F}(w), \mathfrak{F}(t)) < \lambda + S_{b2}^{*P}(\mathfrak{F}(w), \mathfrak{F}(w), \mathfrak{F}(w)). \tag{23}$$

Hence, a function $\mathfrak{F}$ is S_b^{*P} -continuous on Y_1 if and only if it is S_b^{*P} -continuous at all $x \in Y_1$.

Definition 31. The two classes of following mappings are defined $\Pi = \{\pi : [0, \infty) \rightarrow [0, \infty)\}$ is continuous, nonde-

creasing, and $\pi^{-1}(0) = 0$. $\Gamma = \{\gamma : [0, \infty) \rightarrow [0, \infty)\}$ is lower semicontinuous, nondecreasing, and $\gamma^{-1}(0) = 0$.

4. A Generalization of Common Point Theorems in S_b^{*P} -Partial Metric Spaces

Theorem 32. Let $(Y, \leq)$ be a partially ordered set, $\mathfrak{f}$ and $\mathfrak{g}$ be weakly increasing self-mapping on a complete S_b^{*P} -metric space with $\zeta \geq 1$. Suppose that there exist $\pi \in \Pi$ and $\gamma \in \Gamma$ such that

$$\pi(S_b^{*P}(\mathfrak{f}(w), \mathfrak{f}(w), \mathfrak{g}(t))) \leq \pi(M(w, w, t)) - \gamma(M(t, t, w)), \tag{24}$$

for all $w, t \in Y$, where

$$\begin{aligned}
M(w, w, t) &= \max \{S_b^{*P}(w, w, t), S_b^{*P}(w, w, \mathfrak{f}(w)), S_b^{*P}(t, t, \mathfrak{g}(t)), \\
&\quad \cdot [S_b^{*P}(w, w, \mathfrak{g}(t)) + S_b^{*P}(t, t, \mathfrak{f}(w))]\}.
\end{aligned} \tag{25}$$

Assume that one of the following cases is satisfied:

- (a) if a nondecreasing sequence $\{w_n\}$ converges to $r \in Y$ implies $\{w_n\}^\circ r$ for all $n \in \mathbb{N}$
- (b) $\mathfrak{f}$ or $\mathfrak{g}$ is continuous

Therefore, the maps $\mathfrak{f}$ or $\mathfrak{g}$ have a common fixed point.

Proof. Suppose that $\hbar$ is a fixed point of $\mathfrak{f}$ and $S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar)) > 0$. From 4.1 with $w = t = \hbar$, we have

$$\begin{aligned}
\pi(S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar))) &= \pi(S_b^{*P}(\mathfrak{f}(\hbar), \mathfrak{f}(\hbar), \mathfrak{g}(\hbar))) \\
&\leq \pi(M(\hbar, \hbar, \hbar)) - \gamma(M(\hbar, \hbar, \hbar)),
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
M(\hbar, \hbar, \hbar) &= \max \{S_b^{*P}(\hbar, \hbar, \hbar), S_b^{*P}(\hbar, \hbar, \mathfrak{f}(\hbar)), S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar)), \\
&\quad \cdot [S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar)) + S_b^{*P}(\hbar, \hbar, \mathfrak{f}(\hbar))]\} \\
&= \max \{S_b^{*P}(\hbar, \hbar, \hbar), S_b^{*P}(\hbar, \hbar, \hbar), S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar)), \\
&\quad \cdot [S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar)) + S_b^{*P}(\hbar, \hbar, \hbar)]\} \\
&= \max \{S_b^{*P}(\hbar, \hbar, \hbar), S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar))\} \\
&= S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar)).
\end{aligned} \tag{27}$$

Hence, we have

$$\begin{aligned}
\pi(S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar))) &= \pi(S_b^{*P}(\mathfrak{f}(\hbar), \mathfrak{f}(\hbar), \mathfrak{g}(\hbar))) \\
&\leq \pi(S_b^{*P}(\mathfrak{f}(\hbar), \mathfrak{f}(\hbar), \mathfrak{g}(\hbar))) \\
&\quad - \gamma(S_b^{*P}(\mathfrak{f}(\hbar), \mathfrak{f}(\hbar), \mathfrak{g}(\hbar))) \\
&\Rightarrow \gamma(S_b^{*P}(\hbar, \hbar, \mathfrak{g}(\hbar))) \leq 0.
\end{aligned} \tag{28}$$

A contradiction. Therefore, $S_b^{*P}(\mathfrak{f}(\bar{h}), \mathfrak{f}(\bar{h}), \mathfrak{g}(\bar{h})) = 0$. So, $\bar{h}$ is common fixed point of $\mathfrak{f}$ and $\mathfrak{g}$. Similarly, if $\bar{h}$ is a fixed point of $\mathfrak{g}$, then one can deduce that $\bar{h}$ is also fixed point of $\mathfrak{f}$.

If, we let w_0 be an arbitrary point of Y with $\mathfrak{f}w_0 = w_0$, then the proof is finished, so we assume that $\mathfrak{f}w_0 \neq w_0$.

Now, one can construct a sequence $\{w_n\} \in Y$ as follows:

$$\begin{aligned} w_1 &= \mathfrak{f}w_0 \leq \mathfrak{g}\mathfrak{f}w_0 = \mathfrak{g}w_1 = w_2, \\ w_2 &= \mathfrak{f}w_1 \leq \mathfrak{g}\mathfrak{f}w_1 = \mathfrak{g}w_2 = w_3, \\ &\vdots \\ w_n &\leq w_{n+1}. \end{aligned} \quad (29)$$

Since w_{2n} and w_{2n+1} are comparable, then we may assume that $S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}) > 0$, for every $n \in N$. If not, then $w_{2n} = w_{2n+1}$ for some n . For all those n , using (24), we obtain

$$\begin{aligned} \pi(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})) &= \pi(S_b^{*P}(\mathfrak{f}w_{2n}, \mathfrak{f}w_{2n}, \mathfrak{g}w_{2n+1})) \\ &\leq \pi(M(w_{2n}, w_{2n}, w_{2n+1})) \\ &\quad - \gamma(M(w_{2n}, w_{2n}, \mathfrak{g}w_{2n+1})), \end{aligned}$$

$$\begin{aligned} M(w_{2n}, w_{2n}, w_{2n+1}) &= \max \left\{ \frac{S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}), S_b^{*P}(w_{2n}, w_{2n}, \mathfrak{f}w_{2n}), S_b^{*P}(w_{2n+1}, w_{2n+1}, \mathfrak{g}w_{2n+1}), [S_b^{*P}(w_{2n}, w_{2n}, \mathfrak{g}w_{2n+1}), S_b^{*P}(w_{2n+1}, w_{2n+1}, \mathfrak{f}w_{2n})]}{2} \right\} \\ &= \max \left\{ S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}), S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}), S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+1}), \left[\frac{S_b^{*P}(w_{2n}, w_{2n}, w_{2n+2}), S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+1})}{2} \right] \right\} \\ &\leq \max \left\{ S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2}), \frac{1}{2} [\zeta [S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}) + S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}) + S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+1}) - S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+1})] + S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+1})] \right\} \\ &= S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2}). \end{aligned} \quad (30)$$

Therefore

$$\begin{aligned} \pi(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})) &\leq \pi(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})) \\ &\quad - \gamma(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})). \end{aligned} \quad (31)$$

It means that $\gamma(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})) = 0$ and $w_{2n+1} = w_{2n+2}$. Following the similar arguments, we obtain $w_{2n+2} = w_{2n+3}$ and hence w_{2n} becomes a common fixed point of $\mathfrak{f}$ and $\mathfrak{g}$.

By taking $S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}) > 0$ for $n = 1, 2, 3, \dots$, now, we consider

$$\begin{aligned} \pi(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})) &= \pi(S_b^{*P}(\mathfrak{f}w_{2n}, \mathfrak{f}w_{2n}, \mathfrak{g}w_{2n+1})) \\ &\leq \pi(M(w_{2n}, w_{2n}, w_{2n+1})) \\ &\quad - \gamma(M(w_{2n}, w_{2n}, w_{2n+1})). \end{aligned}$$

$$\begin{aligned} M(w_{2n}, w_{2n}, w_{2n+1}) &= \max \{ S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}), S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2}) \}. \end{aligned} \quad (32)$$

Now if $S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2}) \geq S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1})$ for some $n = 0, 1, 2, \dots$, then $M(w_{2n}, w_{2n}, w_{2n+1}) = S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})$ and from (32), we have

$$\begin{aligned} \pi(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})) &\leq \pi(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})), \\ &\quad - \gamma(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})), \end{aligned} \quad (33)$$

implying that $\gamma(S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2})) = 0$ is a contradiction. Therefore, for all $n \geq 0$,

$$S_b^{*P}(w_{2n+1}, w_{2n+1}, w_{2n+2}) \leq S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}). \quad (34)$$

Similarly, we have

$$S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}) \leq S_b^{*P}(w_{2n-1}, w_{2n-1}, w_{2n}), \quad (35)$$

for all $n \geq 0$. Hence, we get

$$S_b^{*P}(w_{n+1}, w_{n+1}, w_{n+2}) \leq S_b^{*P}(w_n, w_n, w_{n+1}). \quad (36)$$

Also, $\{S_b^{*P}(w_{n+1}, w_{n+1}, w_{n+2})\}$ is a nonincreasing sequence, then there exists $\mathfrak{L}$, such that

$$\lim_{n \rightarrow \infty} S_b^{*P}(w_{n+1}, w_{n+1}, w_{n+2}) = \mathfrak{L}. \quad (37)$$

Hence, by the lower semicontinuity of γ ,

$$\gamma(\mathfrak{L}) \leq \liminf_{n \rightarrow \infty} \gamma(S_b^{*P}(w_n, w_n, w_{n+1})) \quad (38)$$

Now, we claim that $\mathfrak{L} = 0$. By lower semicontinuity of γ , taking the upper limit as $n \rightarrow \infty$ on either side of

$$\begin{aligned} \pi(S_b^{*P}(w_{n+1}, w_{n+1}, w_{n+2})) &\leq \pi(M(w_n, w_n, w_{n+1})) \\ &\quad - \gamma(M(w_n, w_n, w_{n+1})), \end{aligned} \quad (39)$$

then, we get

$$\pi(\mathfrak{L}) \leq \pi(\mathfrak{L}) - \liminf_{n \rightarrow \infty} \gamma(M(w_n, w_n, w_{n+1})) \leq \pi(\mathfrak{L}) - \gamma(\mathfrak{L}), \quad (40)$$

this implies that $\gamma(\mathfrak{L}) \leq 0 \Rightarrow \gamma(\mathfrak{L}) = 0$, then we have

$$\lim_{n \rightarrow \infty} S_b^{*P}(w_{n+1}, w_{n+1}, w_{n+2}) = 0. \quad (41)$$

To show that $\{w_{n+1}\}$ is a S^{*P} -Cauchy sequence for each n, m , and $n, m > N$, we have

$$\begin{aligned}
S_b^{*P}(w_n, w_n, w_m) &\leq \zeta [2S_b^{*P}(w_n, w_n, w_{n+1}) + 2\zeta S_b^{*P}(w_{n+1}, w_{n+1}, w_{n+2}) \\
&\quad + 2\zeta^2 S_b^{*P}(w_{n+2}, w_{n+2}, w_{n+3}) + \dots + 2\zeta^m S_b^{*P}(w_m, w_m, w_{m+1})] \\
&\quad - \zeta [\{2S_b^{*P}(w_{n+1}, w_{n+1}, w_{n+1}) + \dots + 2\zeta^m S_b^{*P} \\
&\quad \cdot (w_{m+1}, w_{m+1}, w_{m+1})\}].
\end{aligned} \tag{42}$$

By taking the limit as $n, m \rightarrow \infty$ to both sides of the above inequality and from (41), we have

$$\lim_{n, m \rightarrow \infty} S_b^{*P}(w_n, w_n, w_m) = 0. \tag{43}$$

It follows that $\{w_n\}$ is a S_b^{*P} -Cauchy sequence and by S_b^{*P} -completeness of Y , so there exist $r \in Y$ such that $\{w_n\}$ converges to r as $n \rightarrow \infty$. Now, we will distinguish the cases (a) and (b) of this theorem.

(a) Suppose g is continuous, since $w_{2n+1} \rightarrow r$, we obtain that

$$w_{2n+1} = g(w_{2n+1}) = g(r). \tag{44}$$

But $w_{2n+1} \rightarrow r$, as a subsequence of $\{w_n\}$. It follows that $g(r) = r$ and from the beginning of the prove we get $g(r) = r = f(r)$. The proof, assuming that f is continuous, is similar to the above.

(b) Suppose that $S_b^{*P}(r, r, g(r))$ and for $\{w_n\}$ and a non-decreasing sequence with $w_n \rightarrow r$ in Y indicate that $w_{2n+1} \leq r, \forall n \in \mathbb{N}$. Therefore, from (24), we have

$$\begin{aligned}
\pi(S_b^{*P}(w_{2n+1}, w_{2n+1}, g(r))) &= \pi(S_b^{*P}(fw_{2n}, fw_{2n}, g(r))), \\
&\leq \pi(M(w_{2n}, w_{2n}, r)) - \gamma(M(w_{2n}, w_{2n}, r)),
\end{aligned} \tag{45}$$

where

$$\begin{aligned}
M(w_{2n}, w_{2n}, r) &= \max \left\{ S_b^{*P}(w_{2n}, w_{2n}, r), S_b^{*P}(w_{2n}, w_{2n}, fw_{2n}), S_b^{*P}(r, r, gr), \frac{1}{2} \right. \\
&\quad \cdot [S_b^{*P}(w_{2n}, w_{2n}, gr) + S_b^{*P}(r, r, fw_{2n})] \Big\} \\
&= \max \left\{ S_b^{*P}(w_{2n}, w_{2n}, r), S_b^{*P}(w_{2n}, w_{2n}, w_{2n+1}), S_b^{*P}(r, r, gr), \frac{1}{2} \right. \\
&\quad \cdot [S_b^{*P}(w_{2n}, w_{2n}, gr) + S_b^{*P}(r, r, w_{2n+1})] \Big\}.
\end{aligned} \tag{46}$$

By taking limit as $n \rightarrow \infty$, this implies

$$\lim_{n \rightarrow \infty} M(w_{2n}, w_{2n}, r) = S_b^{*P}(r, r, gr). \tag{47}$$

Hence,

$$\begin{aligned}
\pi(S_b^{*P}(r, r, gr)) &= \lim_{n \rightarrow \infty} \sup \pi(S_b^{*P}(fw_{2n}, fw_{2n}, gr)) \\
&\leq \lim_{n \rightarrow \infty} \sup [\pi(M(w_{2n}, w_{2n}, r)) \\
&\quad - \gamma(M(w_{2n}, w_{2n}, r))].
\end{aligned} \tag{48}$$

This is a contradiction. Thus, we have

$$S_b^{*P}(r, r, gr) = 0 \Rightarrow r = fr = gr. \tag{49}$$

Corollary 33. Let $(Y, \leq)$ be a partially ordered set, f and g be weakly increasing self-mapping on a complete S_b^{*P} -metric space with $\zeta \geq 1$. Suppose that there exist $\gamma \in \Gamma$ such that

$$S_b^{*P}(f(w), f(w), g(t)) \leq M(w, w, t) - \gamma(M(t, t, w)), \tag{50}$$

for all $w, t \in Y$, where

$$\begin{aligned}
M(w, w, t) &= \max \{ S_b^{*P}(w, w, t), S_b^{*P}(w, w, f(w)), S_b^{*P}(t, t, g(t)) \\
&\quad + \frac{1}{2} [S_b^{*P}(w, w, g(t)) + S_b^{*P}(t, t, f(w))] \}.
\end{aligned} \tag{51}$$

Assume that one of the following cases is satisfied:

(a) if a nondecreasing sequence $\{w_n\}$ converges to $r \in Y$ implies $\{w_n\} \leq r$ for all $n \in \mathbb{N}$

(b) f or g is continuous

Therefore, the maps f or g have a common fixed point.

Proof. Put $\pi(t) = t$ in Theorem 32.

Corollary 34. Let $(Y, \leq)$ be a partially ordered set, f and g be weakly increasing self-mapping on a complete S_b^{*P} -metric space. Suppose that there exist $\pi \in \Pi$ and $\gamma \in \Gamma$ such that

$$\pi(S_b^{*P}(f(w), f(w), g(t))) \leq \pi(M(w, w, t)) - \gamma(M(t, t, w)), \tag{52}$$

for all $w, t \in Y$, where

$$\begin{aligned}
M(w, w, t) &= \mu_1 S_b^{*P}(w, w, t) + \mu_2 S_b^{*P}(w, w, f(w)) \\
&\quad + \mu_3 S_b^{*P}(t, t, g(t)) + \mu_4 \\
&\quad \cdot [S_b^{*P}(w, w, g(t)) + S_b^{*P}(t, t, f(w))],
\end{aligned} \tag{53}$$

where $\mu_i > 0$ for $i = \{1, 2, 3, 4\}$ with $\mu_1 + \mu_2 + \mu_3 + \mu_4 \geq 1$.

Then, of the following two cases, assume that one of the following cases is satisfied:

(a) if a nondecreasing sequence $\{w_n\}$ converges to $r \in Y$ implies $\{w_n\} \leq r$ for all $n \in \mathbb{N}$

(b) f or g is continuous

Therefore, the maps f or g have a common fixed point.

Proof. A corollary is S^{*P} -partial metric spaces version of Theorem 18.

Corollary 35. Let $(Y, \leq)$ be a partially ordered set, $\mathbf{f}$ and $\mathbf{g}$ be weakly increasing self-mapping on a complete S_b^{*P} -metric space. Suppose that there exist $\gamma \in \Gamma$ such that

$$S_b^{*P}(\mathbf{f}(w), \mathbf{f}(w), \mathbf{g}(t)) \leq M(w, w, t) - \gamma(M(t, t, w)), \quad (54)$$

for all $w, t \in Y$, where

$$\begin{aligned} M(w, w, t) = & \mu_1 S_b^{*P}(w, w, t) + \mu_2 S_b^{*P}(w, w, \mathbf{f}(w)) \\ & + \mu_3 S_b^{*P}(t, t, \mathbf{g}(t)) + \mu_4 \\ & \cdot [S_b^{*P}(w, w, \mathbf{g}(t)) + S_b^{*P}(t, t, \mathbf{f}(w))], \end{aligned} \quad (55)$$

where $\zeta_i > 0$ for $i = \{1, 2, 3, 4\}$ with $\mu_1 + \mu_2 + \mu_3 + \mu_4 \geq 1$.

Then, of the following two cases, assume that one of the following cases is satisfied:

- (a) if a nondecreasing sequence $\{w_n\}$ converges to $r \in Y$ implies $\{w_n\} \leq r$ for all $n \in \mathbb{N}$
- (b) $\mathbf{f}$ or $\mathbf{g}$ is continuous

Therefore, the maps $\mathbf{f}$ or $\mathbf{g}$ have a common fixed point.

Proof. Put $\pi(t) = t$ in Corollary 34.

Remark 36. It will be interesting to find more applications to our current paper in other fields, see [11, 19–29].

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

- [1] A. Latif, Z. Kadelburg, V. Parvaneh, and J. R. Roshan, "Some fixed point theorems for G-rational Geraghty contractive mappings in ordered generalized b-metric spaces," *Journal of Nonlinear Sciences and Applications*, vol. 8, no. 6, pp. 1212–1227, 2015.
- [2] S. G. Matthews, "Partial metric topology," *Annals of the New York Academy of Sciences-Paper Edition*, vol. 728, pp. 183–197, 1994.
- [3] R. J. Shahkoobi and A. Razani, "Some fixed point theorems for rational Geraghty contractive mappings in ordered b-metric spaces," *Journal of Inequalities and Applications*, vol. 2014, no. 1, 2014.
- [4] I. A. Bakhtin, "The contraction mapping principle in almost metric spaces," *Journal of Functional Analysis*, vol. 30, pp. 26–37, 1989.
- [5] N. Bourbaki, *Topologie Generale*, Herman, Paris, France, 1974.
- [6] S. Czerwik, "Contraction mappings in b-metric spaces," *Acta Mathematica et Informatica Universitatis Ostraviensis*, vol. 1, pp. 5–11, 1993.
- [7] R. Fagin and L. Stockmeyer, "Relaxing the triangle inequality in pattern matching," *International Journal of Computer Vision*, vol. 28, no. 3, pp. 219–231, 1998.
- [8] S. Sedghi, N. Shobe, and H. Zhou, "A common fixed point theorem in $D\hat{A}^\alpha$ -metric spaces," *Fixed point theory and Applications*, vol. 2007, Article ID 27906, 13 pages, 2007.
- [9] B. C. Dhage, "Generalized metric spaces mappings with fixed point," *Bulletin of the Calcutta Mathematical Society*, vol. 84, pp. 329–336, 1992.
- [10] S. Sedghi, N. Shobe, and A. Aliouche, "A generalization of fixed point theorems in S-metric spaces," *Matematiski Vesnik*, vol. 64, no. 5, pp. 258–266, 2012.
- [11] A. H. Soliman and A. M. Zidan, "A new coupled fixed point result in extended metric spaces with an application to study the stability of set-valued functional equations," *Journal of Function Spaces*, vol. 2019, Article ID 4146328, 6 pages, 2019.
- [12] V. Gupta and R. Deep, "Some coupled fixed point theorems in partially ordered S-metric spaces," *Miskolc Mathematical Notes*, vol. 16, no. 1, pp. 181–194, 2015.
- [13] S. S. Chauhan and V. Gupta, "Banach contraction theorem on fuzzy cone b-metric space," *Journal of Applied Research and Technology*, vol. 18, no. 4, pp. 154–160, 2020.
- [14] Z. Mustafa, R. J. Shahkoobi, V. Parvaneh, Z. Kadelburg, and M. M. M. Jaradat, "Ordered S_p -metric spaces and some fixed point theorems for contractive mappings with application to periodic boundary value problems," *Fixed Point Theory and Applications*, vol. 2019, Article ID 16, 2019.
- [15] Z. Mustafa and S. Sims, "A new approach to generalized metric spaces," *Journal of Nonlinear and Convex Analysis*, vol. 7, no. 2, pp. 289–2663, 2006.
- [16] M. R. A. Zand and A. D. Nezhad, "A generalization of partial metric spaces," *Journal of Contemporary Applied Mathematics-ISSN*, vol. 24, pp. 86–93, 2011.
- [17] B. C. Dhage, "A common fixed point principle in D-metric spaces," *Bulletin of the Calcutta Mathematical Society*, vol. 91, no. 6, pp. 475–480, 1999.
- [18] M. A. Barakat and A. M. Zidan, "A common fixed point theorem for weak contractive maps in G^p -metric spaces," *Journal of the Egyptian Mathematical Society*, vol. 23, pp. 309–314, 2015.
- [19] A.-H. A-Aty, M. M. A. Khater, A. M. Zidan, and R. A. M. Attia, "New analytical solutions of wick-type stochastic Schamel KdV equation via modified Khater method," *Journal of Information Science and Engineering*, vol. 36, pp. 1279–1291, 2020.
- [20] H. Aydi, H. Lakzian, Z. D. Mitrovic, and S. Radenovic, "Best proximity points of MT-cyclic contractions with property UC," *Numerical Functional Analysis and Optimization*, vol. 41, no. 7, pp. 871–882, 2020.
- [21] E. Karapinar, R. P. Agarwal, and H. Aydi, "Interpolative Reich–Rus–Ćirić type contractions on partial metric spaces," *Mathematics*, vol. 6, no. 11, p. 256, 2018.

- [22] E. Karapinar, S. Czerwik, and H. Aydi, "Meir-Keeler contraction mappings in generalized b -metric spaces," *Journal of Function spaces*, vol. 2018, 3264624 pages, 2018.
- [23] T. Abdeljawad, N. Mlaiki, H. Aydi, and N. Souayah, "Double controlled metric type spaces and some fixed point results," *Mathematics*, vol. 6, no. 12, p. 320, 2018.
- [24] E. Karapinar, S. I. Moustafa, A. Shehata, and R. P. Agarwal, "Fractional hybrid differential equations and coupled fixed-point results for ϕ -admissible contractions in metric spaces," *Discrete Dynamics in Nature and Society*, vol. 2020, Article ID 7126045, 13 pages, 2020.
- [25] P. Patle, D. Patel, H. Aydi, and S. Radenovic, "On H^+ -type multivalued contractions and applications in symmetric and probabilistic spaces," *Mathematics*, vol. 7, no. 2, p. 144, 2019.
- [26] A. H. Soliman, T. Nabil, and A. M. Zidan, "On quasi-partial generalized type of metric spaces and an application to complexity analysis of computer algorithms," *Alexandria Engineering Journal*, vol. 59, no. 3, pp. 1233–1238, 2020.
- [27] A. H. Soliman and A. M. Zidan, "Existential examination of the coupled fixed point in generalized b -metric spaces and an application," *Journal of Intelligent & Fuzzy Systems*, vol. 38, no. 3, pp. 2801–2807, 2020.
- [28] A. M. Zidan, A. H. Soliman, T. Nabil, and M. A. Barakat, "An investigation of new quicker implicit iterations in hyperbolic spaces," *Thermal Science*, vol. 24, pp. 199–207, 2020.
- [29] A. M. Zidan and A. Al Rwaily, "On new type of ϕ -contractive mapping for quasipartial b -metric spaces and some results of fixed-point theorem and application," *Journal of Mathematics*, vol. 2020, Article ID 8825805, 8 pages, 2020.