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Research Article

Analytical Solution to 1D Compressible Navier-Stokes Equations

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There exist complex behavior of the solution to the 1D compressible Navier-Stokes equations in half space. We find an interesting phenomenon on the solution to 1D compressible isentropic Navier-Stokes equations with constant viscosity coefficient on $(x, t) \in [0,+\infty) \times R_+$, that is, the solutions to the initial boundary value problem to 1D compressible Navier-Stokes equations in half space can be transformed to the solution to the Riccati differential equation under some suitable conditions.

1. Introduction

We consider the 1D compressible Navier-Stokes equations in half space in the following:

$$\rho_t + (\rho u)_x = 0, (x, t) \in [0,+\infty) \times R_+, \tag{1}$$

$$\rho(u_t + u u_x) + p_x = \mu u_{xx}, (x, t) \in [0,+\infty) \times R_+, \tag{2}$$

where $\rho(x, t)$, $u(x, t)$ stand for the density and velocity of compressible flow. μ is the constant viscosity coefficient. $p = p(\rho)$ means the pressure of the flow. We assume the initial data:

$$(\rho, u)(x, 0) = (\rho_0, u_0)(x), \tag{3}$$

and the boundary condition:

$$u(0, t) = g(t), \tag{4}$$

And let

$$\rho_0(x) > 0 \text{ and } \rho_0(x) \in C^2(0,+\infty), g(t) \in C^1|(0,+\infty), p(\cdot) \in C^1(0,+\infty). \tag{5}$$

There is huge literature on the studies of the global existence and large time behavior of solutions to the 1D compressible Navier-Stokes equations. As the viscosity μ is a positive constant and the initial density away from vacuum, Kanel [1] addressed the problems for sufficiently smooth data, and Serre [2, 3] and Hoff [4] considered the problems for discontinuous initial data. As viscosity μ depends on density and has a positive constant lower bound, [5–8] gave the global well-posedness and large time behavior of solutions to the system without initial vacuum. However, when the initial data admits the presence of vacuum, many papers are related to compressible fluid dynamics [9–17]. When we get the well-posedness of solutions to the compressible Navier-Stokes equations, the existence of vacuum is a major difficulty. Ding et al. [18] got the global existence of classical solutions to 1D compressible Navier-Stokes equations in bounded domains, provided that $\mu \in C^2[0,+\infty)$ satisfies $0 < \bar{\mu} \le \mu(\rho) \le C(1 + P(\rho))$. Ye [19] obtained the global classical large solutions to the Cauchy problem (1) and (2) with the restriction $\mu(\rho) = 1 + \rho^{\beta}$; $0 \le \beta < \gamma$. Zhang and Zhu [20] derived the global existence of classical solution to the initial boundary value problem for the one-dimensional Navier-Stokes equations for viscous compressible and heat-conducting fluids in a bounded domain with the Robin boundary condition on temperature. Li et al. [21] derive the uniform upper bound of density and the global well-posedness of strong (classical) large solutions to the Cauchy problem with the external force. For two-dimensional case, global well-posedness of classical solutions to the Cauchy problem or periodic domain problem of compressible Navier-Stokes equations with vacuum was obtained in [22–24] when the first and second viscosity coefficients are μ and $\lambda(\rho)$, respectively. Li and Xin [25] derived the global well-posedness and large time asymptotic behavior of strong and classical solutions to the Cauchy problem of the Navier-Stokes equations for viscous compressible barotropic flows in two or three spatial dimensions with vacuum as far field density, provided the smooth initial data are of small total energy and the viscosity coefficients are two constants.

In this paper, we find an interesting phenomenon on the solution to 1D compressible Navier-Stokes equations (1) and (2) with constant viscosity coefficient, that is, the solutions to the problem (1) and (2) in half space can be transformed to the solution to the Riccati differential equation under some suitable conditions. Before stating the main results, we first denote

$$
\begin{aligned}
U(t) &= 1 + g(t),\\
A_1(x,t) &= \mu\left(\frac{\rho_0''(x+t)}{\rho_0^2(x+t)} - \frac{2\left(\rho_0'(x+t)\right)^2}{\rho_0^3(x+t)}\right),\\
B_1(x,t) &= \frac{\rho_0'(x+t)}{\rho_0^2(x+t)},\\
C_1(x,t) &= p'(\rho_0(x+t))\rho_0'(x+t),\\
A(x,t) &= \frac{\rho_0'(t)}{\rho_0(t)} + A_1(x,t),\\
B(x,t) &= -\rho_0(t)B_1(x,t),\\
C(x,t) &= -\frac{C_1(x,t)}{\rho_0(t)}.
\end{aligned} \tag{6}
$$

Theorem 1. *The function* $(\rho,u)(t,x) = (\rho_0(x+t), (1+g(t))((\rho_0(t))/(\rho_0(x+t))) - 1)$ *is the solution to compressible Navier-Stokes equations (1) and (2) with the initial data (3) and boundary condition (4), if and only if* $g(t) = U(t) - 1$ *and* $U(t)$ *satisfies the Riccati differential equation:*

$$U'(t) + Q(t)U(t) + P(t)U^2(t) = R(t), \tag{7}$$

where

(1)

$$
\begin{aligned}
&\rho_0(x) > 0 \text{ and } \rho_0(x) \in C^2(0,+\infty), g(t)\\
&\in C^1(0,+\infty), p(\cdot) \in C^1(0,+\infty),
\end{aligned} \tag{8}
$$

(2) There exist three functions $Q(t)$, $P(t)$, *and* $R(t)$ *which only depend on* t *such that*

$$Q(t) = \frac{\rho_0'(t)}{\rho(t)} + A_1(x,t), P(t) = -\rho_0(t)B_1(x,t), R(t) = -\frac{C_1(x,t)}{\rho_0(t)}, \tag{9}$$

for $(x,t) \in [0,+\infty) \times [0,+\infty)$.

Theorem 2. *If*

$$2Q' + Q^2 - 2Q\frac{P'}{P} - 2\frac{P''}{P} + 3\left(\frac{P'}{P}\right)^2 + 4R(t) > 0, \tag{10}$$

then we have the global existence of (7).

Remark 3. In Theorem 1, we do not know whether the solution $U(T)$ of Ricatti equation exist globally, because the existence of general solution to Riccati equation is an open problem. If we add the condition of $P(t)$, $Q(t)$, and $R(t)$, such as (25) and (26), the global existence for (7) can be obtained, which can be seen in Theorem 2.

Remark 4. In Theorem 1, the initial value $\rho_0(x)$, $u_0(x)$ can be bounded in Sobolev space or not bounded in Sobolev space, which is determined by the given specific function for the initial value.

(1) Bounded case. Furthermore, if assumed that

$$\|\rho_0(x)\|_{L^1([0,+\infty))} \le C, \rho_0(0) = 0, \tag{11}$$

and $g(t) = 0$, then we have the initial energy $\rho(x)u_0^2(x) \in L^1([0,+\infty))$. In fact,

$$
\begin{aligned}
\int_0^\infty \rho_0(x)u_0^2(x)dx &= \int_0^\infty \rho_0(x)\left(\frac{(1+g(0))\rho_0(0)}{\rho_0(x)} - 1\right)^2(x)dx\\
&= \int_0^\infty \rho_0(x)dx \le C.
\end{aligned} \tag{12}
$$

As a result of basic energy estimate, we easily get

$$\int_0^\infty \rho(x,t)u(x,t)^2 dx \le C. \tag{13}$$

(2) Unbounded case. We can see the boundary condition of velocity $g(t) \ne 0$.

2. The Proof of Main Result

Proof of Theorem 1. If we have the analytical function:

$$\rho(t,x) = \rho_0(x+t), \tag{14}$$

we will get

$$
\begin{aligned}
u(x,t) &= (1+u(0,t))\exp\left\{-\int_0^x \frac{\rho_0'(y+t)}{\rho_0(y+t)}dy\right\} - 1\\
&= (1+g(t))\frac{\rho_0(t)}{\rho_0(x,t)} - 1,
\end{aligned} \tag{15}
$$

though the equation

$$\rho_0'(x+t) + u\rho_0'(x+t) + \rho_0(x+t)u_x = 0, \tag{16}$$

and the boundary condition (4).

So, the derivatives of $u(x,t)$ are

$$u_t(x,t) = g'(t)\frac{\rho_0(t)}{\rho_0(x+t)} + (1+g(t))\frac{\rho_0'(t)\rho_0(x+t)}{\rho_0^2(x+t)}, \tag{17}$$

$$u_x(x,t) = -(1+g(t))\frac{\rho_0(t)\rho_0'(x+t)}{\rho_0^2(x+t)}, \tag{18}$$

$$u_{xx}(x,t) = -(1+g(t))\frac{\rho_0(t)\rho_0^2(x+t)\rho_0''(x+t) - 2\rho_0(t)\rho_0(x+t)\left(\rho_0'(x+t)\right)^2}{\rho_0^3(x+t)}, \tag{19}$$

Substituting (18) and (19) into the moment equation, we obtain

$$\begin{aligned}\rho_0(x+t)\Bigg(g'(t)\frac{\rho_0(t)}{\rho_0(x+t)} &+ (1+g(t))\frac{\rho_0'\rho_0(x+t) - \rho_0(t)\rho_0'(x+t)}{\rho_0^2(x+t)}\\ &- \left\{(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right\}(1+g(t))\frac{\rho_0(t)\rho_0'(x+t)}{\rho_0^2(x+t)}\Bigg)\\ &+ p'(\rho_0(x+t))\rho_0'(x+t) = -\mu(1+g(t))\\ &\times\frac{\rho_0(t)\rho_0(x+t)\rho_0''(x+t) - 2\rho_0(t)\left(\rho_0'(x+t)\right)^2}{\rho_0^3(x+t)},\end{aligned} \tag{20}$$

i.e.,

$$\begin{aligned}g'(t) + (1+g(t))&\left[\frac{\rho_0'(t)}{\rho_0(t)} + \mu\left(\frac{\rho_0''(x+t)}{\rho_0^2(x+t)} - \frac{2\left(\rho_0'(x+t)\right)^2}{\rho_0^3(x+t)}\right)\right]\\ &- (1+g(t))^2\frac{\rho_0(t)}{\rho_0(x+t)}\frac{\rho_0'(x+t)}{\rho_0(x+t)}\\ &= -\frac{p'(\rho_0(x+t))}{\rho_0(t)}\rho_0'(x+t).\end{aligned} \tag{21}$$

If $U(t) = 1 + g(t)$, then $U(t)$ satisfies

$$\begin{aligned}U'(t) + U(t)&\left[\frac{\rho_0'(t)}{\rho_0(t)} + \mu\left(\frac{\rho_0''(x+t)}{\rho_0^2(x+t)} - \frac{2\left(\rho_0'(x+t)\right)^2}{\rho_0^3(x+t)}\right)\right]\\ &- U^2\frac{\rho_0(t)}{\rho_0(x+t)}\frac{\rho_0'(x+t)}{\rho_0(x+t)} = -\frac{p'(\rho_0(x+t))}{\rho_0(t)}\rho_0'(x+t).\end{aligned} \tag{22}$$

and $((\rho_0''(x+t))/(\rho_0^2(x+t))) - ((2(\rho_0'(x+t))^2)/(\rho_0^3(x+t)))$, $((\rho_0'(x+t))/(\rho_0^2(x+t)))$ and $p'(\rho_0(x+t))\rho_0'(x+t)$ do not depend on the spatial variable x.

Therefore, $U(t)$ is the solution to the Riccati differential equation (7) with the conditions (8) and (9).

If $g(t) = U(t) - 1$, $U(t)$ satisfies (7), and the conditions (8) and (9) hold; it is easy to get

$$[\rho(x+t)]_t + \left(\rho(x+t)\left[(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right]\right)_x = 0,$$

$$\begin{aligned}\rho(x+t)\Bigg(\left[1+g(t)\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right]_t &+ \left[(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right]\\ \times\left[(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)} - 1\right]_x\Bigg) &+ p'(p(x+t))\rho'((x+t))\\ = \mu\left[(1+g(t))\frac{\rho_0(t)}{\rho_0(x+t)}\right]_{xx}&.\end{aligned} \tag{23}$$

So, $(\rho,u)(t,x) = (\rho_0(x+t), (1+g(t))((\rho_0(t))/(\rho_0(x+t))) - 1)$ is the solution to compressible Navier-Stokes equations (1) and (2) with the initial data (3) and boundary condition (4).

Since Riccati put forward the Riccati equation in the seventeenth century, there has been no general solution for it for more than 300 years. Although there are many special solutions, none of them can fundamentally solve this equation. Here, we give the global existence for Riccati equation (7) under some condition of $P(t)$, $Q(t)$, and $R(t)$, motivated by the results in the reference [26, 27].

Proof of Theorem 2. By taking $W(t) = P(t)U(t)$, equation (7) becomes

$$W'(t) = -W^2(t) - f(t)W + R(t), \tag{24}$$

where $f(t) = Q(t) - ((P'(t))/(P(t)))$. Due to $\rho_0 \in C^2(0,+\infty)$ and (10), we have

$$f(t) \in C^1(0,+\infty), R(t) \in C^1(0,+\infty), \tag{25}$$

$$\frac{1}{2}f'(t) + \frac{1}{4}f^2(t) + R(t) > 0. \tag{26}$$

With (25) and (26), we can obtain the global existence of the Riccati equation (7), according to [26, 27].

3. Example

In this section, we give some examples. First of all, it is easy to check that.

Example 1. Suppose the initial data $\rho0$; u0 are both constants and the pressure $p(\rho) = \rho^\gamma$ (for any $\gamma > 0$), we can get that the solution to compressible Navier-Stokes equations (1) and (2) satisfy the result of Theorem 1.

Specially, we can deduce the following interesting example if some nonphysical condition is given.

Example 2. Assume $p(\rho) = -\rho^{-1}$, and suppose that

$$\rho_0(x) = \frac{1}{x+1}, u_0(x) = \frac{2c_0+1}{2c_0-1}x + \frac{2}{2c_0-1}\left(c_0 \neq \frac{1}{2}\right). \quad (27)$$

Then, we can get the nontrivial analytical solution to the compressible Navier-Stokes equations (1) and (2)

$$\rho(x,t) = \frac{1}{x+t+1}, u(x,t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(x+t+1) - 1. \quad (28)$$

Moreover, we can get the particle path of compressible flow

$$x(t) = \frac{x(0)+1}{|2c_0-1|}e^{-t}\left|2c_0e^{2t}-1\right| - t - 1, \quad (29)$$

where $x(0)$ stands for the initial position of the particle.

Proof. From (27), we have the initial data

$$\rho(x,0) = \rho_0(x) = \frac{1}{x+1}, u(x,0) = u_0(x) = \frac{2c_0+1}{2c_0-1}x + \frac{2}{2c_0-1}, \quad (30)$$

and the compatibility condition

$$g(0) = u(0,t)|_{t=0} = u(0,0) = u_0(0) = \frac{2}{2c_0-1}. \quad (31)$$

By the initial data, we have

$$\begin{aligned} A_1(x,t) &= \mu\left(\frac{\rho_0''(x+t)}{\rho_0^2(x+t)} - \frac{2\left(\rho_0'(x+t)\right)^2}{\rho_0^3(x+t)}\right) = 0, \\ B_1(x,t) &= \frac{\rho_0'(x+t)}{\rho_0^2(x+t)} = -1, \\ C_1(x,t) &= p'(\rho_0(x+t))\rho_0'(x+t) = -1. \end{aligned} \quad (32)$$

Consequently,

$$Q(t) = -\frac{1}{1+t}, P(t)\frac{1}{1+t}, R(t) = 1+t. \quad (33)$$

Due to the variable substitution

$$U(t) = 1 + g(t), \quad (34)$$

we obtain

$$U'(t) - \frac{1}{1+t}U(t) + \frac{1}{1+t}U^2(t) = 1+t. \quad (35)$$

Let

$$V(t) = U(t) - (t+1), \quad (36)$$

then we have that $V(t)$ satisfies

$$V'(t) = \left(\frac{1}{1+t} - 2\right)V(t) - \frac{1}{1+t}V^2(t). \quad (37)$$

The above equation (37) divided by $-(1/V^2)$, we get

$$\left(\frac{1}{V(t)}\right)' = \left(2 - \frac{1}{1+t}\right)\frac{1}{V(t)} + \frac{1}{1+t}\left(\frac{1}{V(t)}\right)^2. \quad (38)$$

By the method of constant variation and the compatibility condition $g(0) = u_0(0) = (2/(2c_0-1))$, we arrive at

$$V(t) = \frac{2(1+\mathrm{t})}{2c_0e^{2t}-1}. \quad (39)$$

Combining the variation substitutions (34) and (36), we get

$$g(t) = \frac{2(1+\mathrm{t})}{2c_0e^{2t}-1} + t. \quad (40)$$

From the result of Theorem 1, we finally obtain (28).

Due to the particle path satisfies $x'(t) = u(x,t)$, we have, from (28),

$$x'(t) = \frac{2c_0e^{2t}+1}{2c_0e^{2t}-1}(x+t+1) - 1. \quad (41)$$

Direct calculation gives

$$x(t) = (x(0)+1)\exp\left\{t + \int_0^{2t}\frac{1}{2c_0e^s-1}ds\right\} - t - 1, \quad (42)$$

that is the function (29). □

Remark 5. In the theorem 1, we can get $\forall_p > 1$,

$$\begin{aligned} \|\rho_0(x)\|_{L^p([0,+\infty))} &= \int_0^\infty \frac{1}{(x+1)^p}dx \le 1, \\ \|\rho_0(x,t)\|_{L^p([0,+\infty))} &= \int_0^\infty \frac{1}{(x+t+1)^p}dx \le \frac{1}{(1+t)^{(p-1)/p}}. \end{aligned} \quad (43)$$

However, $\|\rho_0(x)\|_{L^1([0,+\infty))}$ is not bounded, and it is hard to get the boundedness of $\|\rho_0u_0^2\|_{L^1([0,+\infty))}$ and $\|\rho u^2\|_{L^1([0,+\infty))}$ for the given initial data condition (27).

If $c_0 = -(1/2)$, then the initial data of velocity $u_0(x) = -1$. Then, the solution of compressible Navier-Stokes equation (1) and (2) with the pressure $p(\rho) = -\rho^{-1}$ can be expressed as

$$\rho(x,t) = \frac{1}{x+t+1}, u(x,t) = \frac{e^{2t}-1}{e^{2t}+1}(x+t+1) - 1. \quad (44)$$

And the particle path of compressible flow is

$$x(t) = \frac{x(0)+1}{2} e^{-t}\left(e^{2t}+1\right) - t - 1. \quad (45)$$

Remark 6. The functions $\rho(x,t) = (1/(x+t+1))$, $u(x,t) = ((2c_0e^{2t}+1)/(2c_0e^{2t}-1))(x+t+1) - 1$ are also the solution to the following Euler equations:

$$\begin{aligned} &\rho_t + (\rho u)_x = 0 \ (x,t) \in [0,+\infty) \times R_+, \\ &\rho(u_t + uu_x) + p_x = 0, (x,t) \in [0,+\infty) \times R_+, \end{aligned} \quad (46)$$

with the assumption of the pressure and initial data.

$$\begin{aligned} &p(\rho) = -\rho^{-1}, \\ &\rho_0(x) = \frac{1}{x+1}, u_0(x) = \frac{2c_0+1}{2c_0-1}x + \frac{2}{2c_0-1} \left(c_0 \neq \frac{1}{2}\right), \end{aligned} \quad (47)$$

for the second order derivative of $u(x,t)$ with the spatial variable x is 0.

Data Availability

All references in this paper can be found on the Web of Science; this manuscript mainly gets some interesting theorems which need serious proof, and there is no data analysis in this paper.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed equally and significantly in writing this article. All the authors read and approved the final manuscript.

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