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Research Article

Study of a Fractal-Fractional Smoking Models with Relapse and Harmonic Mean Type Incidence Rate

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This manuscript investigates fractal-fractional order smoking models with relapse and harmonic mean type incidence rate under the Caputo derivative. We derive the existence and unique results about the solution for the considered model via fixed point theory. For the stability of the considered system, Ulam-Hyers (UH) approach is used. We compute the numerical solution by using fractional Adams-Bashforth method. For the simulation of the model, we consider different values of fractional order δ and fractal dimension θ by using some real values of the parameters. The proposed scheme is used to simulate the available data for some smoking community including potential, light, and quit smokers. Various graphical presentations are given to understand the dynamics of the model at various fractional orders.

1. Introduction

The first biological model that describes the dynamics of infectious disease was presented in 1927. Later on, scientists and researchers started to investigate different properties of the models such as the spreading behavior and trends of the diseases by studying the various aspects [1–4]. They have formulated several models for different diseases like pine wilt, HIV, viral disease including leishmania, TB, and COVID-19 [5–12].

Smoking is also similar to infectious diseases by spreading its behavior in the population. The ratio of diseases due to smoking is increasing day by day. Castillo-Garsow et al. [13] formulated for the first time a simple giving up smoking model with known spreading behavior of smoking in the community. The same authors modified and extended the work by adding another class of light smoking. The authors [14] focused on the control strategy of smoking epidemic by choosing optimal campaigns. Furthermore, some of the smokers may relapse because they may have frequent contacts with smokers, whereas some of them may cease smoking permanently. Rahman et al. [15] have been worked on a smoking model and included the relapse terms for the quit smokers.

The abovementioned models have been investigated under ordinary derivatives. During the last twenty years, fractional calculus (FC) has gained more interest from the researcher and been used in different fields of sciences. Mathematical models along with fractional differential equations (FDEs) have been proved for several smoking models. Compared with integer-order model, fractional-order models have better fitting degree with different experimental results in signal processing, mathematical biology and engineering [16, 17–19]. In this regard, Mahdy et al. [20] found the approximate solution for a smoking model by utilizing the Sumudu transform with Caputo derivative. Sing et al. [21] has been introduced a giving up smoking dynamic fractional model with nonsingular kernel. Khan et al. [22] have been studied a biological model of smoking type with some iterative method. Mohamed et al. [23] used reduced differential transform method to solve the nonlinear smoking fractional-order model. Alrabaiah et al. [24] have been applied Adams-Bashforth-Moulton method to investigate the tobacco smoking fractional model order containing snuffing class. Therefore, for the past periods, to develop the real phenomena for a better degree of precision and accuracy, FDEs have been utilized very well. Many researchers have utilized several methods for studying the theoretical investigation of fractional-order mathematical models, (for instance see [25–29]). For further detail, see [30–35]. Adomian in 1980 introduced a useful decomposition method for the solution of nonlinear systems analytically. Later on, the abovementioned method has been slowly enforced as an actual tool for consideration semianalytical or estimated results to several systems of applied sciences. Mathematical models have been examined widely using the Homotopy method, decomposition method along with integral transforms, and difference methods, for details, see [30, 31]. Recently, many methods have been utilized to handle problems of fractional order (see details in [36–38]).

Keeping in mind that derivative of noninteger can be defined in several ways. The first definition of fractional derivative was given by Riemann-Liouville. Later on in 1967, Caputo gave his own definition which has been increasingly used. The mentioned both definitions include singular kernels which often cause problem in numerical investigations. To overcome these difficulties, recently, Caputo and Fabrizio [39] have introduced a new definition. The said definition contains exponential function instead of singular kernel. In subsequent years, the said definition has been further generalized by Atangana and Baleanu [40] by replacing exponential function on Mittag-Leffler one. This fact has been proved that the concerned derivative also has interesting features (see [41–46]).

Recently, the area involves fractal-fractional derivative has got much attention (see [47, 49–51]). Motivated from the above work and from, we consider the model presented in [48] to fractal-fractional (FF) order in sense of Caputo operator which has various advantages. This model consists of four compartments, namely, people vulnerable to smoking $P(t)$, light smokers $L(t)$, smoker class $S(t)$, and quit smokers $Q(t)$. This work also includes theoretical, practical analytical, and numerical results of smoking models with relapse and harmonic mean type incidence rate. Our considered model under Caputo operator for fractional-order δ and fractal dimension θ is as follows:

$$\begin{aligned}
{}^{FF}\mathbf{D}^{\delta,\theta}P &= \prod -2\beta\frac{P(t)L(t)}{P(t)+L(t)} - (b+\alpha)P(t) + \tau Q(t),\\
{}^{FF}\mathbf{D}^{\delta,\theta}L &= 2\beta\frac{P(t)L(t)}{P(t)+L(t)} - (b+\alpha+\zeta)L(t),\\
{}^{FF}\mathbf{D}^{\delta,\theta}S &= \zeta L(t) - (b+\alpha+\phi)S(t),\\
{}^{FF}\mathbf{D}^{\delta,\theta}Q &= \phi S(t) - (b+\alpha+\tau)Q(t),
\end{aligned} \tag{1}$$

with initial conditions

$$P(0)=P_0, L(0)=L_0, S(0)=S_0, Q(0)=Q_0, \tag{2}$$

where β is the transmission rate that the potential smoker contact with the chain smoker, τ is the relapse rate, Π is the recruitment rate, α is the natural death rate, and b is the death rate induced by smoking. Also, ζ is the conversion rate from light to chain smoker class. In same line, φ is the chain smokers rate when they quit smoking. We also discuss some stability results devoted to UH type. The mentioned stability has been recently investigated for various problems of FDEs (see [55–57]).

The rest of the paper we organized is as follows: Section 2 is related to basic definitions and theorems. By using fixed point theorems, we show some suitable results for the uniqueness and existence in Section 3. With the help of famous AB technique, we find the numerical solution of the considered system in Section 4. Using the AB technique, we also perform the numerical simulation by using Matlab for getting the graphical representation for our analytic and briefly discuss the obtained results. Finally, we conclude our work in Section 5.

2. Basic Results

Definition 1 (see [47]). Let $\boldsymbol{\mathcal{U}}(t)$ on $a<t<b$ be a continuous and differentiable function with order θ, then the *FF* order derivative can be defined as

$${}^{FF}\mathbf{D}_t^{\delta,\theta}(\boldsymbol{\mathcal{U}}(t)) = \frac{1}{(p-\delta)}\frac{d}{dt^\theta}\int_0^t (t-x)^{p-\delta-1}\boldsymbol{\mathcal{U}}(x)dx, \tag{3}$$

along-with $p-1<\delta$, $\theta\le p$, where $p\in N$ and $d\boldsymbol{\mathcal{U}}(x)/dx^\theta = \lim_{t\longrightarrow 0}\boldsymbol{\mathcal{U}}(t)-\boldsymbol{\mathcal{U}}(x)/t^\theta - x^\theta$.

Definition 2 (see [47]). Let $\boldsymbol{\mathcal{U}}(t)$ be continuous on $a<t<b$ then the *FF* order integral of $\boldsymbol{\mathcal{U}}(t)$ with order δ is defined as

$${}^{FF}\mathbf{I}^{\delta}\boldsymbol{\mathcal{U}}(t) = \frac{\theta}{\Gamma(\delta)}\int_0^t (t-x)^{\delta-1}x^{\theta-1}\boldsymbol{\mathcal{U}}(x)dx.. \tag{4}$$

Definition 3. The system (1) is *UH* stable if $\exists$ any real number $C_{\delta,\theta}\ge 0$ such that for every $\epsilon>0$ and all the solutions $\mathscr{W}\in C^1([0,T],R)$, where $0<t<T<\infty$, the inequality can be defined as

$$\left|{}^{FF}\mathbf{D}^{\delta,\theta}\mathscr{W}(t) - \Psi(t,\mathscr{W}(t))\right| \le \epsilon, t\in[0,T], \tag{5}$$

$\boldsymbol{\mathscr{Y}}\in C^1([0,T],R)$ is the unique solution for the considered model (1) such that

$$|\mathscr{W}(t)-\mathscr{Y}(t)| \le C_{\delta,\theta}, t\in[0,T]. \tag{6}$$

Note: let us define a Banach space For the qualitative analysis $U=X\times X\times X\times X$, where $X=\mathbf{C}([0,T])$ with norm: $\|\boldsymbol{\mathscr{W}}\| = \|\mathrm{P,L,S,Q}\| = \max_{t\in[0,T]}\{|P(t)|+|L(t)|+|S(t)|+|Q(t)|\}$.

3. Theoretical Results of Model (1)

Here in this section, we will investigate the model (1) for existence. Since the given integral is differentiable, so we can express the RHS of the model (1) as

$$
\begin{aligned}
&{}^{\mathscr{RL}}\mathbf{D}^{\delta}P(t)=\theta t^{\theta-1}G_1(P,L,S,Q,t)=\prod -2\beta\frac{P(t)L(t)}{P(t)+L(t)}-(b+\alpha)P(t)+\tau Q(t),\\
&{}^{\mathscr{RL}}\mathbf{D}^{\delta}L(t)=\theta t^{\theta-1}G_2(P,L,S,Q,t)=2\beta\frac{P(t)L(t)}{P(t)+L(t)}-(b+\alpha+\zeta)L(t),\\
&{}^{\mathscr{RL}}\mathbf{D}^{\delta}S(t)=\theta t^{\theta-1}G_3(P,L,S,Q,t)=\zeta L(t)-(b+\alpha+\varphi)S(t),\\
&{}^{\mathscr{RL}}\mathbf{D}^{\delta}Q(t)=\theta t^{\theta-1}G_4(P,L,S,Q,t)=\varphi S(t)-(b+\alpha+\tau)Q(t).
\end{aligned}
\tag{7}
$$

In view of (7) and for $t\in[0,T]$, the proposed model may be written in the following form

$$
{}^{\mathscr{RL}}\mathbf{D}^{\delta}\mathscr{W}(t)=\theta t^{\theta-1}\Psi(t,\mathscr{W}(t)),\ 0<\delta,\theta\le 1, \tag{8}
$$

$$
\mathscr{W}(0)=\mathscr{W}_0, \tag{9}
$$

by changing ${}^{\mathscr{RL}}\mathbf{D}^{\delta,\theta}$ with ${}^{C}\mathbf{D}^{\delta,\theta}$ and using the integral of Riemann-Liouville, the solution of (8) will be

$$
\mathscr{W}(t)=\mathscr{W}_0+\frac{\theta}{\Gamma(\delta)}\int_0^t x^{\theta-1}(t-x)^{\delta-1}\Psi(x,\mathscr{W}(x))dx, \tag{10}
$$

where

$$
\mathscr{W}(t)=\begin{cases}\mathrm{P}(t)\\ \mathrm{L}(t)\\ \mathrm{S}(t)\\ \mathrm{Q}(t),\end{cases},\ \mathscr{W}_0(t)=\begin{cases}\mathrm{P}_0\\ \mathrm{L}_0\\ \mathrm{S}_0\\ \mathrm{Q}_0,\end{cases},\ \Psi(t,\mathscr{W}(t))=\begin{cases}\mathrm{G}_4(\mathrm{P},\mathrm{L},\mathrm{S},\mathrm{Q},t),\\ \mathrm{G}_4(\mathrm{P},\mathrm{L},\mathrm{S},\mathrm{Q},t),\\ \mathrm{G}_4(\mathrm{P},\mathrm{L},\mathrm{S},\mathrm{Q},t),\\ \mathrm{G}_4(\mathrm{P},\mathrm{L},\mathrm{S},\mathrm{Q},t).\end{cases} \tag{11}
$$

Now, if we transform (1) to fixed point problem and let the operator $\mathscr{T}:\mathbf{V}\longrightarrow\mathbf{V}$ can be defined by

$$
\mathscr{T}(\mathscr{W})(\mathrm{t})=\mathscr{W}_0(\mathrm{t})+\frac{\theta}{\Gamma(\delta)}\int_0^t x^{\theta-1}(t-x)^{\delta-1}\Psi(x,\mathscr{W}(x))dx. \tag{12}
$$

To find the existence results of the considered model, we use the following theorem [54].

Theorem 4. *If the operator $\mathscr{T}:\mathbf{V}\longrightarrow\mathbf{V}$ be completely continuous and the set*

$$
\mathscr{J}(\mathscr{T})=\{\mathscr{W}\in\mathbf{V}:\mathscr{W}=\nu\mathscr{T}(\mathscr{W}),\nu\in[0,1]\}, \tag{13}
$$

is bounded, then, the operator $\mathscr{T}$ has at least one fixed point in $\mathbf{V}$.

Theorem 5. *Suppose the operator $\Psi:\mathscr{D}\times V\longrightarrow R$ is a continuous operator. Then, $\mathscr{T}$ is compact.*

Proof. First, we will show that $\mathscr{T}:\mathbf{V}\longrightarrow\mathbf{V}$ which is defined in (12) is continuous. Consider $\mathscr{B}$ is a bounded set in $\mathbf{V}$, then $\exists C_\Psi>0$ with $|\Psi(t,\mathscr{W}(t))|\le C_\Psi$, for all $\mathscr{W}\in B$. Any $\mathscr{W}\in B$, we have

$$
\begin{aligned}
\|\mathscr{T}(\mathscr{W})\|&\le\frac{\theta\mathscr{C}_\Psi}{\Gamma(\delta)}\max_{t\in[0,T]}\left|\int_0^t(\tau-x)^{\delta-1}x^{\theta-1}dx\right|\\
&\le\frac{\theta\mathscr{C}_\Psi}{\Gamma(\delta)}\max_{t\in[0,T]}\int_0^t(1-z)^{\theta-1}z^{\delta-1}t^{\delta+\theta-1}dz\\
&\le\frac{\theta\mathscr{C}_\Psi T^{\delta+\theta-1}}{\Gamma(\delta)}\mathrm{B}(\delta,\theta).
\end{aligned}
\tag{14}
$$

Hence, (14) implies that $\mathscr{T}$ is uniformly bounded, where Beta function can be written as $B(\delta,\theta)$. Further, for equi-continuity of the operator $\mathscr{T}$, for any $t_1,t_2\in D$ and $\mathscr{W}\in B$, we obtain

$$
\begin{aligned}
\|\mathscr{T}(\mathscr{W}(t_1)+)-\mathscr{T}(\mathscr{W}(t_2))\|&\le\frac{\theta\mathscr{C}_\Psi}{\Gamma(\delta)}\max_{t\in[0,T]}\left|\int_0^{t_1}(t_1-x)^{\delta-1}x^{\theta-1}dx\right.\\
&\quad\left.-\int_0^{t_2}(t_2-x)^{\delta-1}x^{\theta-1}dx\right|\\
&\le\frac{\theta\mathscr{C}_\Psi\mathrm{B}(\delta,\theta)}{\Gamma(\delta)}\left(t_1^{\delta+\theta-1}-t_2^{\delta+\theta-1}\right)\\
&\longrightarrow 0\ as\ t_1\longrightarrow t_2.
\end{aligned}
\tag{15}
$$

Hence, $\mathscr{T}$ is equi-continuous and then the operator $\mathscr{T}$ is bounded and continuous as well, therefore, by Arzel*á*-Ascoli theorem, the operator $\mathscr{T}$ is relatively compact and so completely continuous. Furthermore, we use the following hypothesis:

(C) There exist constants $L_\Psi>0$ such that, for each $\mathscr{W}$, $\bar{\mathscr{W}}\in f$, we have

$$
\left|\Psi(t,\mathscr{W})-\Psi\left(t,\bar{\mathscr{W}}\right)\right|\le\mathrm{L}_\Psi|\mathscr{W}|-\bar{\mathscr{W}}|. \tag{16}
$$

For existence uniqueness, we use fixed point approach as given in [54]. □

Theorem 6. *Applying the hypothesis (C) and if $\Theta<1$, then, the model (1) has a unique solution if*

$$
\Theta=\frac{\theta L_\Psi T^{\delta+\theta-1}}{\Gamma(\delta)}B(\delta,\theta)<1. \tag{17}
$$

Proof. Assume $\max_{t\in[0,T]}|\Psi(t,0)|=\mathscr{K}_\Psi<\infty$, such that

$$
r\ge\frac{\theta\mathrm{T}^{\delta+\theta-1}\mathrm{B}(\delta,\theta)\mathscr{K}_\Psi}{\Gamma(\delta)-\theta\mathrm{T}^{\delta+\theta-1}\mathrm{B}(\delta,\theta)\mathrm{L}_\Psi}. \tag{18}
$$

We prove that $\mathcal{T}(\mathcal{B}r) \subset \mathcal{B}r$, where $B_r = \{W \in f : \|W\| \le r\}$ and $\mathcal{W} \in B_r$, we have

$$\begin{aligned}\|\mathcal{T}(\mathcal{W})\| &\le \frac{\theta}{\Gamma(\delta)} \max_{t\in[0,T]} \int_0^t x^{\theta-1}(t-x)^{\delta-1}(|\Psi(t,\mathcal{W}(t)) - \Psi(t,0)| + |\Psi(t,0)|)dx \\ &\le \frac{\theta \mathrm{T}^{\delta+\theta-1}\mathrm{B}(\delta,\theta)(\mathrm{L}_\Psi\|\mathcal{W}\| + \mathcal{K}_\Psi)}{\Gamma(\delta)}, \\ &\le \frac{\theta \mathrm{T}^{\delta+\theta-1}\mathrm{B}(\delta,\theta)(\mathrm{L}_\Psi r + \mathcal{K}_\Psi)}{\Gamma(\delta)}, \le r.\end{aligned} \tag{19}$$

Suppose the operator $\mathcal{T} : \mathbf{V} \longrightarrow \mathbf{V}$ is defined in (12). Using the assumption $\mathbf{C}$ and for every $t \in \mathcal{D}$, $\mathcal{W}, \bar{\mathcal{W}} \in \mathcal{D}$, we obtain

$$\begin{aligned}\left\|\mathcal{T}(\mathcal{W}) - \mathcal{T}(\bar{\mathcal{W}})\right\| &\le \frac{\theta}{\Gamma(\delta)} \max_{t\in[0,T]} \left| \int_0^t x^{\theta-1}(t-x)^{\delta-1}\Psi(x,\mathcal{W}(x))dx \right. \\ &\quad \left. - \int_0^t x^{\theta-1}(t-x)^{\delta-1}\Psi\left(x,\bar{\mathcal{W}}(x)\right)dx \right|, \\ &\le \Theta\left\|\mathcal{W} - \bar{\mathcal{W}}\right\|.\end{aligned} \tag{20}$$

By this, $\mathcal{T}$ is contraction by using (20). Therefore, equation (10) has one solution and so our model (1) has unique solution.

Now, we have to develop UH stability for the considered system (1), taking $\psi \in C(D)$ depending on the solution with $\psi(0) = 0$. Then

(i) $|\psi(t)| \le \varepsilon$, for $\varepsilon > 0$

(ii) ${}^{FF}D_t^{\delta,\theta}\mathcal{W}(t) = \Psi(t,\mathcal{W}(t)) + \psi(t)$

□

Lemma 7. *The solution of perturbed equation*

$$\begin{aligned}{}^{FF}\mathbf{D}^{\delta,\theta}\mathcal{W}(t) &= \Psi(t,\mathcal{W}(t)) + \psi(t), \\ \mathcal{W}(0) &= \mathcal{W}_0,\end{aligned} \tag{21}$$

satisfies the given relation

$$\begin{aligned}&\left|\mathcal{W}(t) - \left(\mathcal{W}_0(t) + \frac{\theta}{\Gamma(\delta)}\int_0^t x^{\theta-1}(t-s)^{\delta-1}\Psi(x,\mathcal{W}(x))\right)\right| \\ &\quad \le \left(\frac{\theta T^{\delta+\theta-1}B(\delta,\theta)}{\Gamma(\delta)}\right)\varepsilon = C_{\delta,\theta}\varepsilon.\end{aligned} \tag{22}$$

Theorem 8. *With the assumption (***C***) and (22), the solution of the integral equation (1) is UH stable. Hence, the analytical results of the considered system are UH stable if $\Theta < 1$, where Θ is given in (17).*

Table 1: Parametric values for our model (1).

Parameter	Value	Source
Π	10.25	Assumed
β	0.038	[52]
b	0.0019	[52, 53]
α	0.0111	[52]
ζ	0.021	[15, 52]
φ	0.000274	[15]
τ	0.006	Assumed

Proof. Suppose that $\mathcal{Z} \in \mathbf{V}$ be a unique solution and $\mathcal{W} \in \mathbf{V}$ be any solution of (10), then using fractalfractional integration as in an equation (2), we have

$$\begin{aligned}|\mathcal{W}(t) - \mathcal{Z}(t)| &= \left|\mathcal{W}(t) - \left(\mathcal{Z}_0(t) + \frac{\theta}{\Gamma(\delta)}\int_0^t (t-x)^{\delta-1}x^{\theta-1}\Psi(x,\mathcal{Z}(x))dx\right)\right|, \\ &\le \left|\mathcal{W}(t) - \left(\mathcal{W}_0(t) + \frac{\theta}{\Gamma(\delta)}\int_0^t (t-x)^{\delta-1}x^{\theta-1}\Psi(x,\mathcal{W}(x))dx\right)\right|, \\ &\quad + \left|\left(\mathcal{W}_0(t) + \frac{\theta}{\Gamma(\delta)}\int_0^t (t-x)^{\delta-1}x^{\theta-1}\Psi(x,\mathcal{W}(x))dx\right)\right. \\ &\quad \left. - \left(\mathcal{Z}_0(t) + \frac{\theta}{\Gamma(\delta)}\int_0^t (t-x)^{\delta-1}x^{\theta-1}\Psi(x,\mathcal{Z}(x))dx\right)\right|, \\ &\le \mathrm{C}_{\delta,\theta}\varepsilon + \frac{\theta \mathrm{T}^{\delta+\theta-1}\mathrm{L}_{\mathcal{W}}}{\Gamma(\delta)}\mathrm{B}(\delta,\theta)\|\mathcal{W} - \mathcal{Z}\|, \\ &\le \mathrm{C}_{\delta,\theta} + \Theta\|\mathcal{W} - \mathcal{Z}\|.\end{aligned} \tag{23}$$

Which we have

$$\|\mathcal{W} - \mathcal{Z}\| \le \mathrm{C}_{\delta,\theta} + \Theta\|\mathcal{W} - \mathcal{Z}\|. \tag{24}$$

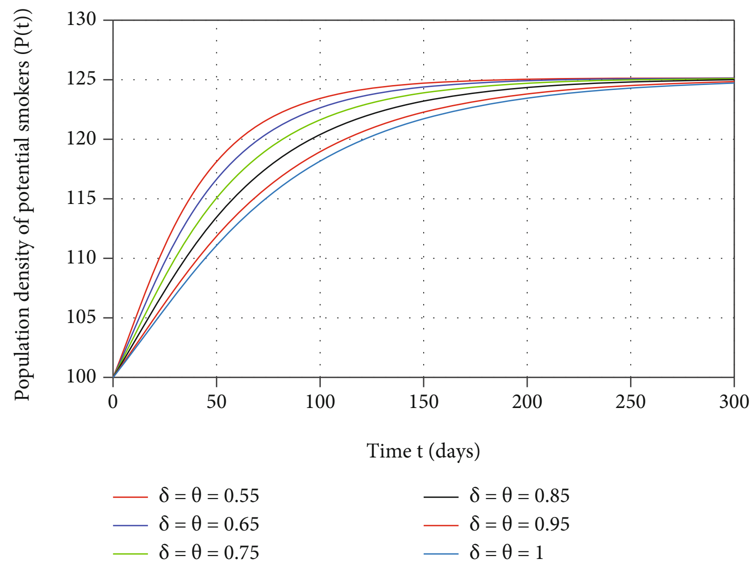
From (24), we can write as

$$\|\mathcal{W} - \mathcal{Z}\| \le \left(\frac{\mathrm{C}_{\delta,\theta}}{1-\Theta}\right)\varepsilon. \tag{25}$$

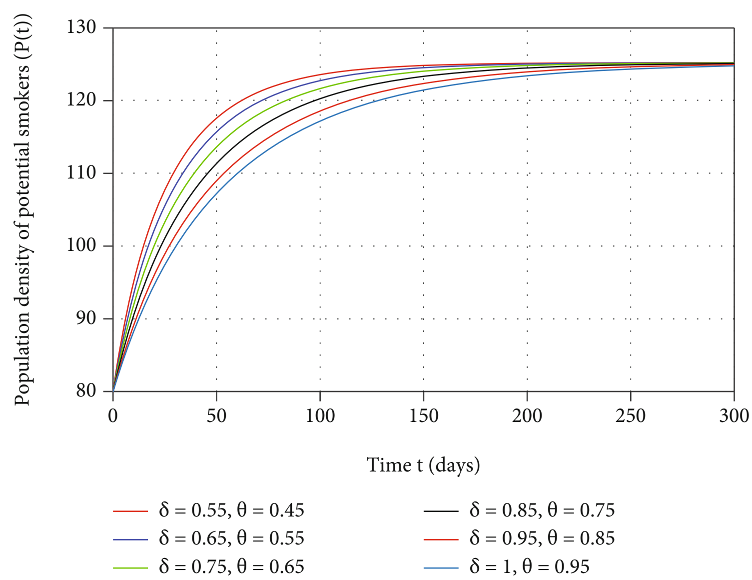
Thus, from the (25), we conclude that the solution of (10) is UH stable and therefore the proposed model (1) solution is UH stable. □

4. Numerical Scheme

In this part of the paper, we are constructing the numerical algorithm for the considered model to perform numerical simulation. Here, for numerical method, the construction of equation (10) of the considered model goes to the following form



(a)



(b)

Figure 1: Graphical representation of potential smokers $(P(t))$ having two different initial values of $P_0 = 100$, 80 in the model under investigation (1) at different arbitrary fractal dimension and fractional orders.

$$\begin{aligned}
P(t) &= P(0) + \frac{\theta}{\Gamma(\delta)} \int_0^t x^{\theta-1} (t-x)^{\delta-1} G_1(P, L, S, Q, x) dx, \\
L(t) &= L(0) + \frac{\theta}{\Gamma(\delta)} \int_0^t x^{\theta-1} (t-x)^{\delta-1} G_2(P, L, S, Q, x) dx, \\
S(t) &= S(0) + \frac{\theta}{\Gamma(\delta)} \int_0^t x^{\theta-1} (t-x)^{\delta-1} G_3(P, L, S, Q, x) dx, \\
Q(t) &= Q(0) + \frac{\theta}{\Gamma(\delta)} \int_0^t x^{\theta-1} (t-x)^{\delta-1} G_4(P, L, S, Q, x) dx.
\end{aligned} \tag{26}$$

Now, we are presenting the numerical solution to the (26) and using the new approach t_{k+1}. The first equation of the above system becomes

$$P_{k+1} = P_0 + \frac{\theta}{\Gamma(\delta)} \int_0^{t_{k+1}} x^{\theta-1} (t_{k+1} - x)^{\delta-1} G_1(P, L, S, Q, x) dx. \tag{27}$$

We obtained the approximate integral from the above equation as

$$P_{k+1} = P_0 + \frac{\theta}{\Gamma(\delta)} \sum_{j=0}^{k} \int_{t_j}^{t_{j+1}} y^{\theta-1} (t_{k+1} - x)^{\delta-1} G_1(P, L, S, Q, x) dx. \tag{28}$$

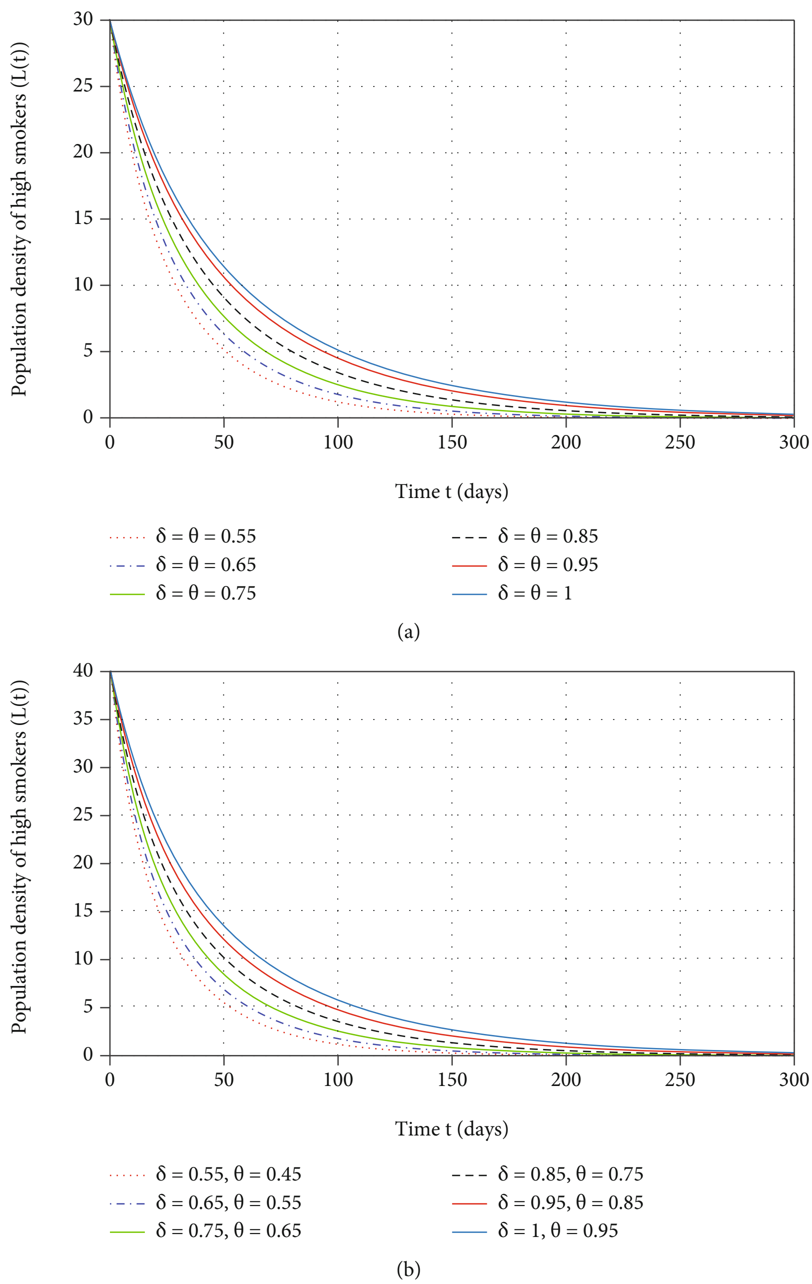


Figure 2: Graphical representation of light smokers $(L(t))$ having two different initial values of $L_0 = 30, 40$ in the model under investigation (1) at different arbitrary fractal dimension and fractional orders.

Within the infinite interval $[t_j, t_{j+1}]$ in term of Lagrange interpolation polynomials the function $G_1(P, L, S, Q, t)$ along with $\sim = [t_j - t_{j-1}]$, such that

$$P_k^\star \approx \frac{1}{\hbar}\Big[\big(t - t_{j-1}\big)t_j^{\theta-1}G_1\big(P_j, L_j, S_j, Q_j, t_j\big) - \big(t - t_j\big)t_{j-1}^{\theta-1}G_1\big(P_{j-1}, L_{j-1}, S_{j-1}, Q_{j-1}, t_{j-1}\big)\Big], \tag{29}$$

putting (29) into (28), then, we can write (28) as

$$P_{k+1} = P_0 + \frac{\theta}{\Gamma(\delta)}\sum_{j=0}^{k}\int_{t_j}^{t_{j+1}} x^{\theta-1}\left(t_{k+1} - x\right)^{\delta-1}P_k^\star dx. \tag{30}$$

Simplifying the right side integrals of (30), we obtain the numerical iterative results for the P_h class in (1) by using the FF derivatives in the Caputo form as:

$$\begin{aligned} P_{k+1} = P_0 + \frac{\theta\hbar^\delta}{\Gamma(\delta+2)}\sum_{j=0}^{k}\Big[& t_j^{\theta-1}G_1\big(P_j, L_j, S_j, Q_j, t_j\big) \\ & \times\Big(\big((k+1-j)^\theta(k-j+2+\theta) - (k-j)^\theta(k-j+2+2\theta)\big)\Big) \\ & - t_{j-1}^{\theta-1}G_1\big(P_{j-1}, L_{j-1}, S_{j-1}, Q_{j-1}, t_{j-1}\big) \\ & \times\Big(\big((k+1-j)^\theta + 1 - (k-j)^\theta(k-j+1+\theta)\big)\Big)\Big]. \end{aligned} \tag{31}$$

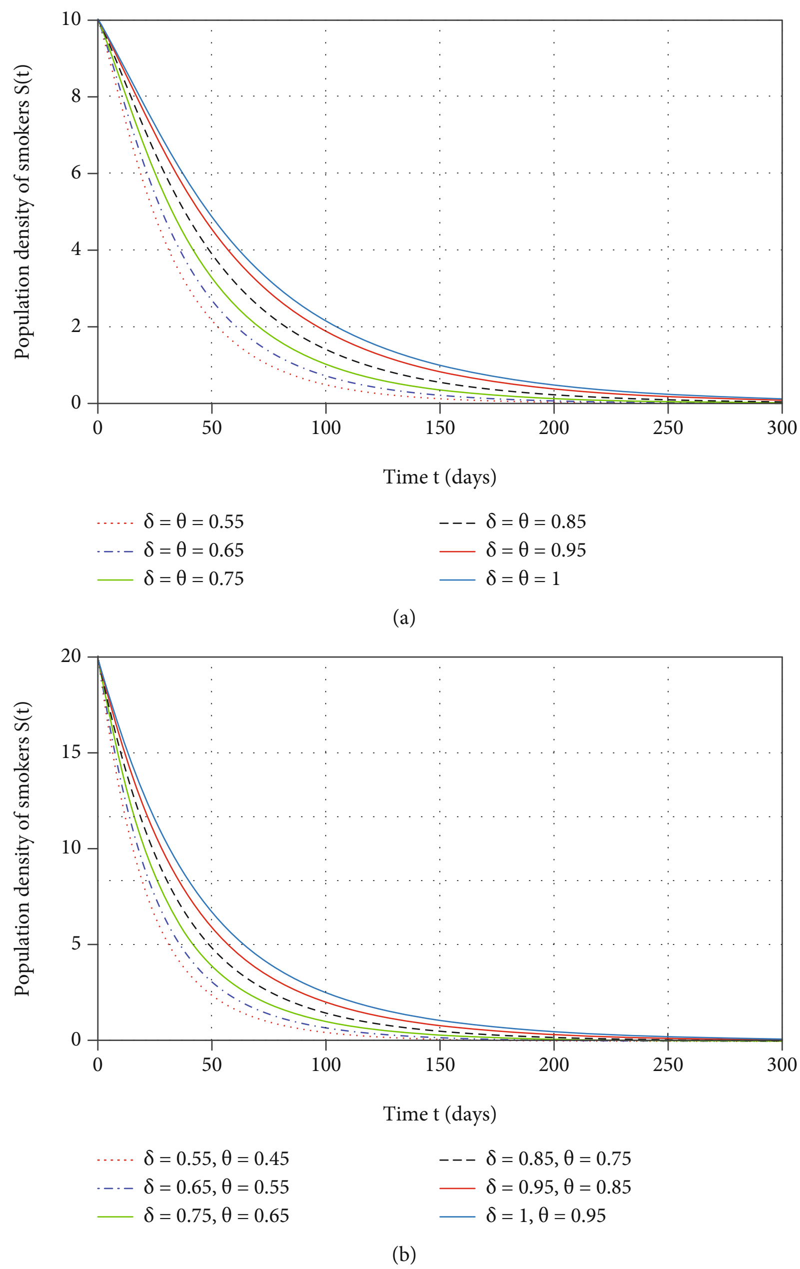


Figure 3: Graphical representation of smokers ($S(t)$) having two different initial values of $L_0 = 10, 20$ in the model under investigation (1) at different arbitrary fractal dimension and fractional orders.

Similarly, the remaining terms can be written as

$$\begin{aligned}
L_{k+1} &= L_0 + \frac{\theta \hbar^{\delta}}{\Gamma(\delta+2)} \sum_{j=0}^{k} \Big[t_j^{\theta-1} G_2\left(P_j, L_j, S_j, Q_j, t_j\right) \\
&\quad \times \left((k+1-j)^{\theta}(k-j+2+\theta) - (k-j)^{\theta}(k-j+2+2\theta) \right) \\
&\quad - t_{j-1}^{\theta-1} G_2\left(P_{j-1}, L_{j-1}, S_{j-1}, Q_{j-1}, t_{j-1}\right) \\
&\quad \times \left((k+1-j)^{\theta} + 1 - (k-j)^{\theta}(k-j+1+\theta) \right) \Big], \\
S_{k+1} &= S_0 + \frac{\theta \hbar^{\delta}}{\Gamma(\delta+2)} \sum_{j=0}^{k} \Big[t_j^{\theta-1} G_3\left(P_j, L_j, S_j, Q_j, t_j\right) \\
&\quad \times \left((k+1-j)^{\theta}(k-j+2+\theta) - (k-j)^{\theta}(k-j+2+2\theta) \right) \\
&\quad - t_{j-1}^{\theta-1} G_3\left(P_{j-1}, L_{j-1}, S_{j-1}, Q_{j-1}, t_{j-1}\right) \\
&\quad \times \left((k+1-j)^{\theta} + 1 - (k-j)^{\theta}(k-j+1+\theta) \right) \Big], \\
Q_{k+1} &= Q_0 + \frac{\theta \hbar^{\delta}}{\Gamma(\delta+2)} \sum_{j=0}^{k} \Big[t_j^{\theta-1} G_4\left(P_j, L_j, S_j, Q_j, t_j\right) \\
&\quad \times \left((k+1-j)^{\theta}(k-j+2+\theta) - (k-j)^{\theta}(k-j+2+2\theta) \right) \\
&\quad - t_{j-1}^{\theta-1} G_4\left(P_{j-1}, L_{j-1}, S_{j-1}, Q_{j-1}, t_{j-1}\right) \\
&\quad \times \left((k+1-j)^{\theta} + 1 - (k-j)^{\theta}(k-j+1+\theta) \right) \Big].
\end{aligned} \tag{32}$$

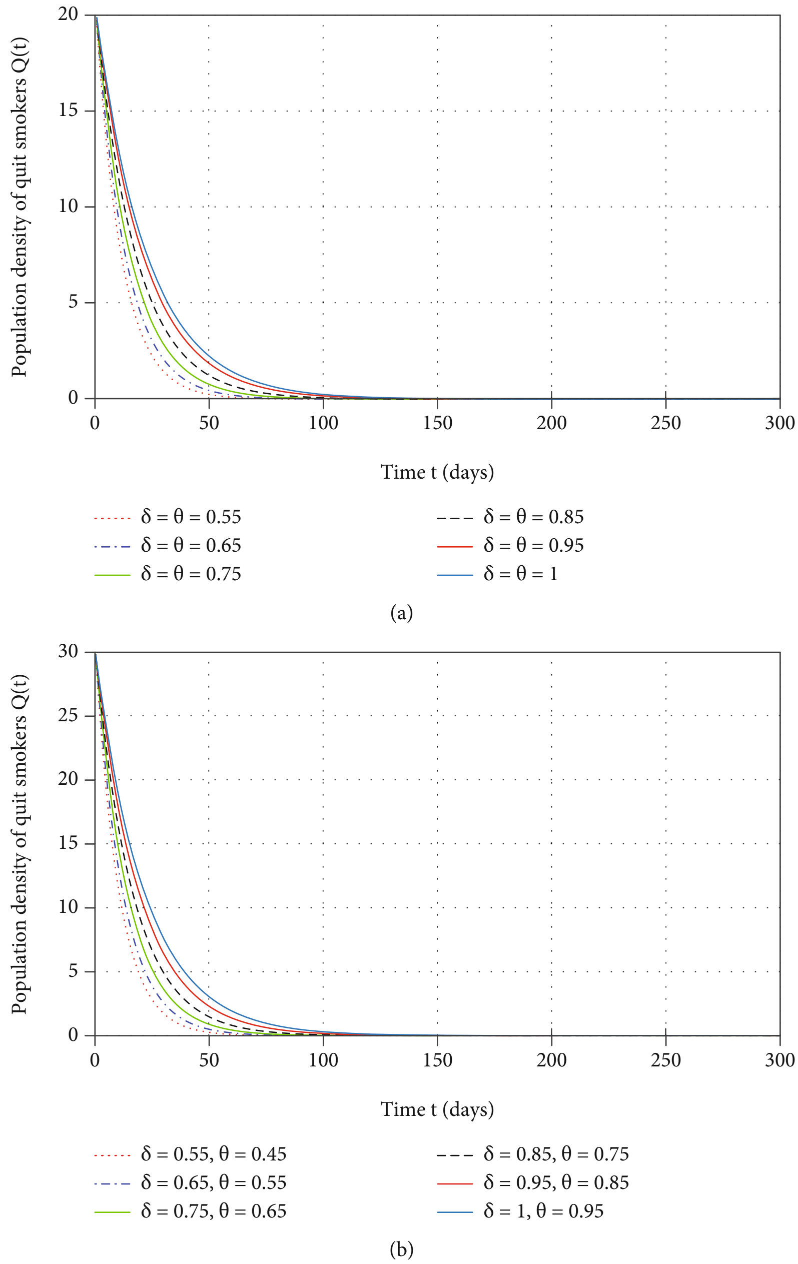


Figure 4: Graphical representation of quit smokers ($Q(t)$) having two different initial values of $L_0 = 20, 30$ in the model under investigation (1) at different arbitrary fractal dimension and fractional orders.

4.1. Graphical Representations. In this section, we provide the numerical solution of our proposed model (1) using different values of parameters given in Table 1 for verification of the obtained scheme. We have taken two different sets of initial values of all the compartments in problem (1) for two different fractal dimension θ and fractional order δ.

Figures 1(a) and 1(b) show the dynamical behavior of potential smoker population $P(t)$ at various fractal dimension θ and fractional order δ at two different initial values. On different six fractal-fractional values, the class increases and becomes stable which converges to the same point having two initial values. The increase occurs quickly at high order and slowly at low order and converges to the integer order as we increase the fractional order.

Figures 2(a) and 2(b) are the representation of the dynamical behavior of light smokers $L(t)$ at different fractal dimension θ and fractional order of δ at two different initial values. On different six fractal-fractional values, the potential smoker class becomes stable and converges to the same point having two initial values.

Figures 3(a) and 3(b) show the dynamical behavior of smokers $S(t)$ at various fractal dimension θ and fractional order δ at two different initial values. At six different fractional values, the class decreases and becomes stable which converges to the same converging point for two different initial approximations.

Figures 4(a) and 4(b) show the dynamical behavior of quit smoker $Q(t)$ at various fractal dimension θ and fractional order δ at two different starting values. On six different fractional values, the class declines quickly nearly at all fractional orders but then becomes stable which converges to the same point having two initial guesses.

5. Conclusion

In this manuscript, we have analyzed a giving up smoking model under the concept of fractal-fractional order derivative in Caputo sense. The considered model has been investigated for some theatrical analysis including existence theory and stability results. In this regard, sufficient results have been established for existence and uniqueness of solution by using Banach-contraction and Schauder's theorems of nonlinear functional analysis. The Ulam-Hyers stability analysis has been developed by using the usual nonlinear analysis tools. Further, we have used fractional Adam Bashforth method and developed an algorithm to compute numerical results. We have used various values of fractal dimensions and fractional orders to present the results graphically. From graphical presentation, one can observe that fractal and fractional calculus have the ability to present the dynamics of real-world problems more comprehensively.

Data Availability

Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All authors read and approved the final manuscript.

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