Research Article

On Fuzzy b-Metric-Like Spaces

Khalil Javed,¹ Fahim Uddin,¹ Hassen Aydi ,^{2,3} Muhammad Arshad,⁴ Umar Ishtiaq,¹ and Habes Alsamir ⁵

¹Department of Math & Stats, International Islamic University Islamabad, Pakistan

²Université de Sousse, Institut Supérieur d'Informatique et des Techniques de Communication, H. Sousse 4000, Tunisia

³China Medical University Hospital, China Medical University, Taichung 40402, Taiwan

⁴Department of Mathematics & Statistics, International Islamic University, H-10, Islamabad 44000, Pakistan

⁵College of Business Administration-Finance Department, Dar Al Uloom University Saudi Arabia, Saudi Arabia

Correspondence should be addressed to Hassen Aydi; hassen.aydi@isima.rnu.tn and Habes Alsamir; habes@dau.edu.sa

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The aim of this manuscript is to introduce the concept of fuzzy b-metric-like spaces and discuss some related fixed point results. Some examples are imparted to illustrate the feasibility of the proposed methods. Finally, to validate the superiority of the obtained results, an application is provided to solve a first kind of Fredholm type integral equations.

1. Introduction and Preliminaries

The concept of fuzzy sets was initiated by Zadeh [1], which gave a new aspect to research activity leading with the improvement of fuzzy systems. Afterwards, several researchers contributed towards some basic significant results in fuzzy sets.

Kramosil and Michlek [2] introduced the concept of fuzzy metric spaces by generalizing the concepts of probabilistic metric spaces to fuzzy metric spaces. George and Veeramani [3] derived a Hausdorff topology which was initiated by a fuzzy metric to modify the concept of fuzzy metric spaces. Later on, the fixed point theory dealing with a fuzzy metric has been enriched with a number of different generalizations. Garbiec displayed the fuzzy version of Banach contraction principle in fuzzy metric spaces. For some necessary definitions, examples, and basic results, we refer to [4–6] and the references herein.

As we know, fixed point theory plays a crucial role in proving the existence of solutions for different mathematical models and has a wide range of applications in different fields related to mathematics. This theory has intrigued many researchers. Recently, Harandi [7] initiated the concept of metric-like spaces, which generalizes the notion of metric

spaces in a nice way. Alghamdi et al. [8] used the concept metric-like spaces to introduce the notion of b-metric-like spaces. Since then, a number of authors contributed in same. For more details, we refer to [7, 9–16]. In this sequel, Shukla and Abbas [8] generalized the concept of metric-like spaces and introduced fuzzy metric-like spaces. For more details on this topic, please see [17–29].

In this article, our aim is to generalize the concept of b-metric-like spaces by introducing the concept of fuzzy b-metric-like spaces and prove some related fixed point results. We also support this work by some examples and an application to solve an integral equation.

First, we write some notations used throughout this paper, as C-t-norm for a continuous triangular norm, b-metric-l for b-metric-like, F-metric-l for fuzzy metric-like, F-b-metric for fuzzy b-metric, F-b-metric-l for fuzzy b-metric-like, and s.t. for such that.

Definition 1 [17]. A binary operation $*$: $[0, 1] \times [0, 1] \longrightarrow [0, 1]$ is called a C-t-norm if it satisfies the following assertions:

- (1) $a * b = b * a$, $(\forall) a, b \in [0, 1]$
- (2) $a * 1 = a$, $(\forall) a \in [0, 1]$

- (3) $(a * b) * c = a * (b * c)$, $(\forall) a, b, c \in [0, 1]$
 (4) If $a \leq c$ and $b \leq d$, with $a, b, c, d \in [0, 1]$, then $a * b \leq c * d$

Some fundamental examples of a t-norm are $a * b = a \cdot b$, $a * b = \min \{a, b\}$ and $a * b = \max \{a + b - 1, 0\}$.

Definition 2 [8]. A b-metric-1 on a set $\mathcal{G} \neq \emptyset$ is a function $\mu : \mathcal{G} \times \mathcal{G} \longrightarrow [0, +\infty)$ such that for all $w, k, z \in \mathcal{G}$ and $b \geq 1$, it satisfies the following conditions:

- (1) If $\mu(w, k) = 0 \Rightarrow w = k$
 (2) $\mu(w, k) = \mu(k, w)$
 (3) $\mu(w, k) \leq b[\mu(w, z) + \mu(z, k)]$

The pair $(\mathcal{G}, \mu)$ is called a b-metric-1 space.

Example 1 [8]. Let $\mathcal{G} = [0, \infty)$. Define the function $\mu : \mathcal{G} \times \mathcal{G} \longrightarrow [0, +\infty)$ by

$$\mu(w, k) = (w + k)^2. \quad (1)$$

Then, $(\mathcal{G}, \mu)$ is called a b-metric-1 space with $b = 2$.

Example 2 [8]. Let $\mathcal{G} = [0, \infty)$. Define the function $\mu : \mathcal{G} \times \mathcal{G} \longrightarrow [0, +\infty)$ by

$$\mu(w, k) = (\max \{w, k\})^2. \quad (2)$$

Then, $(\mathcal{G}, \mu)$ is named as a b-metric-1 space with $b = 2$.

Definition 3 [18]. A 3-tuple $(\mathcal{G}, \Omega, *)$ is said an F-metric-1 space if $\mathcal{G} \neq \emptyset$ is a random set, $*$ is a C-t-norm, and Ω is a fuzzy set on $\mathcal{G} \times \mathcal{G} \times (0, \infty)$ meeting the conditions below for all $w, k, z \in \mathcal{G}$, $\tau, s > 0$:

- FL1) $\Omega(w, k, \tau) > 0$
 FL2) If $\Omega(w, k, \tau) = 1$, then $w = k$
 FL3) $\Omega(w, k, \tau) = \Omega(k, w, \tau)$
 FL4) $\Omega(w, z, \tau + s) \geq \Omega(w, k, \tau) * \Omega(k, z, s)$
 FL5) $\Omega(w, k, \bullet) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Example 3 [18]. Let $\mathcal{G} = \mathbb{R}^+$, $q \in \mathbb{R}^+$, and $m > 0$. Define a t-norm by $a * b = ab$ and the fuzzy set Ω on $\mathcal{G} \times \mathcal{G} \times (0, \infty)$ by

$$\Omega(w, k, \tau) = \frac{q\tau}{q\tau + m(\max \{w, k\})}, \quad \forall w, k \in \mathcal{G}, \tau > 0. \quad (3)$$

Then, $(\mathcal{G}, \Omega, *)$ is an F-metric-1 space.

Definition 4 [5]. A 3-tuple $(\mathcal{G}, \Omega, *)$ is said an F-b-metric space if $\mathcal{G}$ is a random (nonempty) set, $*$ is a C-t-norm, and Ω is a fuzzy set on $\mathcal{G} \times \mathcal{G} \times (0, \infty)$ meeting the conditions below for all $w, k, z \in \mathcal{G}$, $\tau, s > 0$ and a provided real number $b \geq 1$:

- FB1) $\Omega(w, k, \tau) > 0$
 FB2) $\Omega(w, k, \tau) = 1$ iff $w = k$
 FB3) $\Omega(w, k, \tau) = \Omega(k, w, \tau)$
 FB4) $\Omega(w, z, \tau + s) \geq \Omega(w, k, (\tau/b)) * \Omega(k, z, (s/b))$
 FB5) $\Omega(w, k, \bullet) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Example 4 [9]. Let $\Omega(w, k, \tau) = e^{-|w-k|^p/\tau}$, where $p > 1$ is a real number. It is then simple to show that Ω is an F-b-metric with $b = 2^{p-1}$.

2. Main Results

We start this section with the introduction of F-b-metric-1 spaces and prove some related fixed point results.

Definition 5. A 4-tuple $(\mathcal{G}, \Omega, *, b)$ is named an F-b-metric-1 space if $\mathcal{G} \neq \emptyset$ is a random set, $*$ is a C-t-norm, and Ω is a fuzzy set on $\mathcal{G} \times \mathcal{G} \times (0, \infty)$ meeting the following conditions below for all $w, k, z \in \mathcal{G}$, $\tau, s > 0$:

- B1) $\Omega(w, k, \tau) > 0$
 B2) If $\Omega(w, k, \tau) = 1$, then $w = k$
 B3) $\Omega(w, k, \tau) = \Omega(k, w, \tau)$
 B4) $\Omega(w, z, b(\tau + s)) \geq \Omega(w, k, \tau) * \Omega(k, z, s)$, for $b \in \mathbb{N}$
 B5) $\Omega(w, k, \bullet) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Remark 6. In the above definition, a set $\mathcal{G}$ is endowed by an F-b-metric-1 with a t-norm $(*)$. An F-b-metric-1 space does not satisfy the (FB2) condition of F-b-metric spaces, that is, the self-distance may not be equal to 1, i.e., $\Omega(w, w, \tau) \neq 1$ for all $\tau > 0$, for some or may be for all $w \in \mathcal{G}$. But all other conditions are the same. Hence, the F-b-metric-1 may not be a F-b-metric, but the converse is true.

Definition 7. Let $(\mathcal{G}, \Omega, *, b)$ be an F-b-metric-1 space. For $x \in X, \Theta \in (0, 1), \tau > 0$, we define the open ball as

$$B(x, \Theta, \tau) = \{k \in \mathcal{G} : \Omega(w, k, \tau) > 1 - \Theta\}. \quad (4)$$

Then,

$$T_M = \{T \subset \mathcal{G} : w \in T \iff \text{there exist } \tau > 0 \text{ and } \Theta \in (0, 1) : B(x, \Theta, \tau) \subseteq T\}, \quad (5)$$

is a topology on $\mathcal{G}$.

The following simplest example shows that an F-b-metric-1 need not to be an F-b-metric.

Example 5. Take $\mathcal{G} = (0, \infty)$. Consider the t-norm defined by $a * b = ab$, then

$$\Omega(w, k, \tau) = \left[e^{(w+k)^2/\tau} \right]^{-1}, \quad \forall w, k \in \mathcal{G}, \tau > 0. \quad (6)$$

is an F-b-metric-1. But it is not an F-b-metric.

Proof. (B1), (B2), (B3), and (B5) are obvious. Here, we prove (B4). We have

$$|w + z| \leq \left(\frac{b(\tau + s)}{\tau} \right) |w + k| + \left(\frac{b(\tau + s)}{s} \right) |k + z|. \quad (7)$$

Here, b is an arbitrary integer. We have

$$\frac{|w + z|}{b(\tau + s)} \leq \frac{|w + k|}{\tau} + \frac{|k + z|}{s}. \quad (8)$$

Hence,

$$e^{|w+z|/(b(\tau+s))} \leq e^{|w+k|/\tau} + e^{|k+z|/s}. \quad (9)$$

Since e^w is an increasing function for $w > 0$, one writes

$$\left[e^{(w+z)^2/(b(\tau+s))} \right]^{-1} \geq \left[e^{(w+k)^2/\tau} \right]^{-1} + \left[e^{(k+z)^2/s} \right]^{-1}. \quad (10)$$

That is,

$$\Omega(w, z, b(\tau + s)) \geq \Omega(w, k, \tau) * \Omega(k, z, s), \quad \forall w, k, z \in \mathcal{G}, \forall \tau, s > 0 \quad (11)$$

Hence, (B4) satisfied.

Now, we have to prove that $(\mathcal{G}, \Omega, *, b)$ is not an F-b-metric space. For this purpose, we investigate the self-distance. Indeed,

$$\Omega(w, w, \tau) = \left[e^{(w+w)^2/\tau} \right]^{-1} = \frac{1}{e^{(w+w)^2/\tau}} \neq 1, \quad \forall \tau > 0, \forall w \in \mathcal{G}. \quad (12)$$

Hence, $(\mathcal{G}, \Omega, *, b)$ is not an F-b-metric space.

The following example shows that an F-b-metric-l space need not be continuous.

Example 6. Let $\mathcal{G} = [0, \infty)$, $\Omega(w, k, \tau) = (e^{-(d(w,k)/\tau)})$, $\forall w, k \in \mathcal{G}$, $\tau > 0$ and

$$d(w, k) = \begin{cases} 0, & \text{if } w = k, \\ 2(k + w)^2, & \text{if } k, w \in [0, 1], \\ \frac{1}{2}(k + w)^2, & \text{otherwise.} \end{cases} \quad (13)$$

If we consider the t-norm defined by $a * b = ab$, then $(\mathcal{G}, \Omega, *, b)$ is an F-b-metric-l space with a coefficient $b = 4$. To illustrate the discontinuity, we have

$$\lim_{n \rightarrow \infty} \Omega\left(0, 1 - \frac{1}{n}, \tau\right) = \lim_{n \rightarrow \infty} e^{-2(1-(1/n))^2} = e^{-2} = \Omega(0, 1, \tau). \quad (14)$$

However, since

$$\lim_{n \rightarrow \infty} \Omega\left(1, 1 - \frac{1}{n}, \tau\right) = \lim_{n \rightarrow \infty} e^{-2(2-(1/n))^2} = e^{-8} \neq 1 = \Omega(1, 1, \tau). \quad (15)$$

One can assert that $\Omega(w, k, \tau)$ is not continuous.

Proposition 8. Let $(\mathcal{G}, \mu)$ be a b-metric-l space. Then, $(\mathcal{G}, \Omega, *, b)$ is an F-b-metric-l space defined as

$$\Omega(w, k, \tau) = e^{-\mu(w,k)/\tau^n}, \quad \forall \tau > 0, \forall w, k \in \mathcal{G}, n \in \mathbb{N}. \quad (16)$$

Proof. (B1), (B2), (B3), and (B5) are obvious. Here, we prove (B4). We have

$$\mu(w, z) \leq b[\mu(w, k) + \mu(k, z)]. \quad (17)$$

Therefore,

$$\frac{\mu(w, z)}{(\tau + s)^n} \leq \frac{b[\mu(w, k) + \mu(k, z)]}{(\tau + s)^n}. \quad (18)$$

That is,

$$\frac{\mu(w, z)}{b(\tau + s)^n} \leq \frac{\mu(w, k)}{\tau^n} + \frac{\mu(k, z)}{s^n}. \quad (19)$$

Hence,

$$e^{\mu(w,z)/(b(\tau+s)^n)} \geq e^{\mu(w,k)/\tau^n} + e^{\mu(k,z)/s^n}. \quad (20)$$

That is, $\Omega(w, z, b(\tau + s)) \geq \Omega(w, k, \tau) * \Omega(k, z, s)$. Hence, (B4) is satisfied and $(\mathcal{G}, \Omega, *, b)$ is an F-b-metric-l space.

Definition 9. A sequence $\{w_n\}$ in a F-b-metric-l space $(\mathcal{G}, \Omega, *, b)$ is said to be convergent to $w \in \mathcal{G}$, if

$$\lim_{n \rightarrow \infty} \Omega(w_n, w, \tau) = \Omega(w, w, \tau), \quad \forall \tau > 0. \quad (21)$$

Definition 10. A sequence $\{w_n\}$ in an F-b-metric-l space $(\mathcal{G}, \Omega, *, b)$ is said to be Cauchy if

$$\lim_{n \rightarrow \infty} \Omega(w_n, w_{n+p}, \tau), \quad \forall \tau > 0, p \geq 1, \quad (22)$$

exists and is finite.

Definition 11. An F-b-metric-l space $(\mathcal{G}, \Omega, *, b)$ is said to be complete if every Cauchy sequence $\{w_n\}$ in $\mathcal{G}$ converges to some $w \in \mathcal{G}$ such that

$$\lim_{n \rightarrow \infty} \Omega(w_n, w, \tau) = \Omega(w, w, \tau) = \lim_{n \rightarrow \infty} \Omega(w_n, w_{n+p}, \tau), \quad \forall \tau > 0, p \geq 1. \quad (23)$$

Definition 12. Let $(\mathcal{G}, \Omega, *, b)$ be an F-b-metric-l space. A mapping $\mathcal{F} : \mathcal{G} \rightarrow \mathcal{G}$ is said to be fuzzy contractive if there exists $q \in (0, 1)$ such that

$$\frac{1}{\Omega(\mathcal{F}w, \mathcal{F}k, \tau)} - 1 \leq q \left[\frac{1}{\Omega(w, k, \tau)} - 1 \right], \quad (24)$$

for all $w, k \in \mathcal{G}$ and $\tau > 0$. Here, q is called the fuzzy contractive constant of $\mathcal{F}$.

Theorem 13. Let $(\mathcal{G}, \Omega, *, b)$ be a complete F - b -metric- l space and $\mathcal{F} : \mathcal{G} \rightarrow \mathcal{G}$ be a fuzzy contractive mapping with a fuzzy contractive constant q , then $\mathcal{F}$ has a unique fixed point $v \in \mathcal{G}$ so that $\Omega(v, v, \tau) = 1$ for all $\tau > 0$.

Proof. For an arbitrary $w_0 \in \mathcal{G}$, define a sequence $\{w_n\}$ in $\mathcal{G}$ by

$$w_1 = \mathcal{F}w_0, w_2 = \mathcal{F}^2w_0 = \mathcal{F}w_1, \dots, w_n = \mathcal{F}^nw_0 = \mathcal{F}w_{n-1} \text{ for all } n \in \mathbb{N}. \quad (25)$$

If $w_n = w_{n-1}$ for some $n \in \mathbb{N}$, then w_n is a fixed point of $\mathcal{F}$. We assume that $w_n \neq w_{n-1}$ for all $n \in \mathbb{N}$. For $\tau > 0$ and $n \in \mathbb{N}$, we get from (24)

$$\begin{aligned} \frac{1}{\Omega(w_n, w_{n+1}, \tau)} - 1 &= \frac{1}{\Omega(\mathcal{F}w_{n-1}, \mathcal{F}w_n, \tau)} - 1 \\ &\leq q \left[\frac{1}{\Omega(w_{n-1}, w_n, \tau)} - 1 \right]. \end{aligned} \quad (26)$$

We have

$$\frac{1}{\Omega(w_n, w_{n+1}, \tau)} \leq \frac{q}{\Omega(w_{n-1}, w_n, \tau)} + (1 - q), \quad \forall \tau > 0. \quad (27)$$

Therefore,

$$\begin{aligned} \frac{q}{\Omega(\mathcal{F}w_{n-2}, \mathcal{F}w_{n-1}, \tau)} + (1 - q) &\leq \frac{q^2}{\Omega(w_{n-2}, w_{n-1}, \tau)} \\ &+ q(1 - q) + (1 - q). \end{aligned} \quad (28)$$

Continuing in this way, we get

$$\begin{aligned} \frac{1}{\Omega(w_n, w_{n+1}, \tau)} &\leq \frac{q^n}{\Omega(w_0, w_1, \tau)} + q^{n-1}(1 - q) + q^{n-2}(1 - q) \\ &+ \dots + q(1 - q) + (1 - q) \leq \frac{q^n}{\Omega(w_0, w_1, \tau)} \\ &+ (q^{n-1} + q^{n-2} + \dots + 1)(1 - q) \\ &\leq \frac{q^n}{\Omega(w_0, w_1, \tau)} + (1 - q^n). \end{aligned} \quad (29)$$

We have

$$\frac{1}{(q^n / (\Omega(w_0, w_1, \tau))) + (1 - q^n)} \leq \Omega(w_n, w_{n+1}, \tau), \quad \forall \tau > 0, n \in \mathbb{N}. \quad (30)$$

Now, for $p \geq 1$ and $n \in \mathbb{N}$, we have

$$\begin{aligned} \Omega(w_n, w_{n+p}, \tau) &\geq \Omega\left(w_n, w_{n+1}, \frac{\tau}{b}\right) * \Omega\left(w_{n+1}, w_{n+p}, \frac{\tau}{b}\right) \\ &\geq \Omega\left(w_n, w_{n+1}, \frac{\tau}{b}\right) * \Omega\left(w_{n+1}, w_{n+2}, \frac{\tau}{b^2}\right) * \Omega \\ &\quad \cdot \left(w_{n+2}, w_{n+p}, \frac{\tau}{b^2}\right). \end{aligned} \quad (31)$$

Continuing in this way, we get

$$\begin{aligned} \Omega(w_n, w_{n+p}, \tau) &\geq \Omega\left(w_n, w_{n+1}, \frac{\tau}{b}\right) * \Omega\left(w_{n+1}, w_{n+2}, \frac{\tau}{b^2}\right) * \dots * \Omega \\ &\quad \cdot \left(w_{n+p-1}, w_{n+p}, \frac{\tau}{b^{p-1}}\right). \end{aligned} \quad (32)$$

By using (30) in the above inequality, we have

$$\begin{aligned} \Omega(w_n, w_{n+p}, \tau) &\geq \frac{1}{(q^n / (\Omega(w_0, w_1, (\tau/b))) + (1 - q^n))} * \\ &\quad \cdot \frac{1}{(q^{n+1} / (\Omega(w_0, w_1, (\tau/b^2)))) + (1 - q^{n+1})} * \dots * \\ &\quad \cdot \frac{1}{(q^{n+p-1} / (\Omega(w_0, w_1, (\tau/b^{p-1})))) + (1 - q^{n+p-1})} \\ &\geq \frac{1}{(q^n / (\Omega(w_0, w_1, (\tau/b))) + 1)} * \\ &\quad \cdot \frac{1}{(q^{n+1} / (\Omega(w_0, w_1, (\tau/b^2)))) + 1} * \dots * \\ &\quad \cdot \frac{1}{(q^{n+p-1} / (\Omega(w_0, w_1, (\tau/b^{p-1})))) + 1}. \end{aligned} \quad (33)$$

Here, b is an arbitrary positive integer, and as $q \in (0, 1)$, we deduce from the above expression that

$$\lim_{n \rightarrow \infty} \Omega(w_n, w_{n+p}, \tau) = 1 \quad \text{for all } \tau > 0, p \geq 1. \quad (34)$$

Therefore, $\{w_n\}$ is a Cauchy sequence in $(\mathcal{G}, \Omega, *, b)$. By the completeness of $(\mathcal{G}, \Omega, *, b)$, there is $v \in \mathcal{G}$ such that

$$\begin{aligned} \lim_{n \rightarrow \infty} \Omega(w_n, v, \tau) &= \lim_{n \rightarrow \infty} \Omega(w_n, w_{n+p}, \tau) = \lim_{n \rightarrow \infty} \Omega(v, v, \tau) \\ &= 1, \quad \forall \tau > 0, p \geq 1. \end{aligned} \quad (35)$$

Now, we prove that v is a fixed point for $\mathcal{F}$. For this, we obtain from (24) that

$$\begin{aligned} \frac{1}{\Omega(\mathcal{F}w_n, \mathcal{F}v, \tau)} - 1 &\leq q \left[\frac{1}{\Omega(w_n, v, \tau)} - 1 \right] = \frac{q}{\Omega(w_n, v, \tau)} - q, \\ \frac{1}{(q/(\Omega(w_n, v, \tau))) + 1 - q} &\leq \Omega(\mathcal{F}w_n, \mathcal{F}v, \tau) \end{aligned} \quad (36)$$

Using the above inequality, we obtain

$$\begin{aligned} \Omega(v, \mathcal{F}v, \tau) &\geq \Omega\left(v, w_{n+1}, \frac{\tau}{2b}\right) * \Omega\left(w_{n+1}, \mathcal{F}v, \frac{\tau}{2b}\right) \\ &= \Omega\left(v, w_{n+1}, \frac{\tau}{2b}\right) * \Omega\left(\mathcal{F}w_n, \mathcal{F}v, \frac{\tau}{2b}\right) \\ &\geq \Omega\left(v, w_{n+1}, \frac{\tau}{2b}\right) * \\ &\quad \cdot \frac{1}{(q/(\Omega(w_n, v, (\tau/2b)))) + 1 - q}. \end{aligned} \quad (37)$$

Taking limit as $n \rightarrow \infty$ and using (35) in the above expression, we get that $\Omega(v, \mathcal{F}v, \tau) = 1$, that is, $\mathcal{F}v = v$. Therefore, v is a fixed point of $\mathcal{F}$ and $\Omega(v, v, \tau) = 1$ for all $\tau > 0$.

Now, we investigate the uniqueness of the fixed point v of $\mathcal{F}$. Let v be another fixed point of $\mathcal{F}$, s.t. $\Omega(v, v, t) < 1$ for some $\tau > 0$. It follows from (24) that

$$\begin{aligned} \frac{1}{\Omega(v, v, \tau)} - 1 &= \frac{1}{\Omega(\mathcal{F}v, \mathcal{F}v, \tau)} - 1 \leq q \left[\frac{1}{\Omega(v, v, \tau)} - 1 \right] \\ &< \frac{1}{\Omega(v, v, \tau)} - 1, \end{aligned} \quad (38)$$

a contradiction. Therefore, we must have $\Omega(v, v, \tau) = 1$, for all $\tau > 0$, and hence $v = v$.

Corollary 14. Let $(\mathcal{G}, \Omega, *, b)$ be a complete F-b-metric-l space and $\mathcal{F} : \mathcal{G} \rightarrow \mathcal{G}$ be a mapping satisfying

$$\frac{1}{\Omega(\mathcal{F}^n w, \mathcal{F}^n k, \tau)} - 1 \leq q \left[\frac{1}{\Omega(w, k, \tau)} - 1 \right], \quad (39)$$

for some $n \in \mathbb{N}$, $\forall w, k \in \mathcal{G}$, $\tau > 0$, where $0 < q < 1$. Then, $\mathcal{F}$ has a unique fixed point $v \in \mathcal{G}$ and $\Omega(v, v, \tau) = 1, \forall \tau > 0$.

Proof. $v \in \mathcal{G}$ is the unique fixed point of $\mathcal{F}^n$ by using Theorem 13, and $\Omega(v, v, \tau) = 1, \forall \tau > 0$. $\mathcal{F}v$ is also a fixed point of $\mathcal{F}^n$ as $\mathcal{F}^n(\mathcal{F}v) = \mathcal{F}v$ and from Theorem 13, $\mathcal{F}v = v$, v is the unique fixed point, since the unique fixed point of $\mathcal{F}$ is also the unique fixed point of $\mathcal{F}^n$.

Example 7. Let $\mathcal{G} = [0, 2]$ and the t-norm be defined as $a * b = ab$. Given Ω as

$$\Omega(w, k, \tau) = e^{-(\max\{w, k\})^2/\tau}, \text{ for all } w, k \in \mathcal{G} \text{ and } \tau > 0. \quad (40)$$

(From Example 2 and Proposition 8, Ω is an F-b-metric-l).

Then, $(\mathcal{G}, \Omega, *, b)$ is a complete F-b-metric-l space. Define $\mathcal{F} : \mathcal{G} \rightarrow \mathcal{G}$ as

$$\mathcal{F}w = \begin{cases} 0, & w = 1, \\ \frac{w}{2}, & w \in [0, 1), \\ \frac{w}{4}, & w \in (1, 2]. \end{cases} \quad (41)$$

Then, we have 8 cases:

Case 1. If $w = k = 1$, then $\mathcal{F}w = \mathcal{F}k = 0$.

Case 2. If $w = 1$ and $k \in [0, 1)$, then $\mathcal{F}w = 0$ and $\mathcal{F}k = k/2$.

Case 3. If $w = 1$ and $k \in (1, 2]$, then $\mathcal{F}w = 0$ and $\mathcal{F}k = k/4$.

Case 4. If $w \in [0, 1)$ and $k \in (1, 2]$, then $\mathcal{F}w = w/2$ and $\mathcal{F}k = k/4$.

Case 5. If $w \in [0, 1)$ and $k \in [0, 1)$, then $\mathcal{F}w = w/2$ and $\mathcal{F}k = k/2$.

Case 6. If $w \in [0, 1)$ and $k = 1$, then $\mathcal{F}w = w/2$ and $\mathcal{F}k = 0$.

Case 7. If $w \in (1, 2]$ and $k = 1$, then $\mathcal{F}w = w/4$ and $\mathcal{F}k = 0$.

Case 8. If $w \in (1, 2]$ and $k \in (1, 2]$, then $\mathcal{F}w = w/4$ and $\mathcal{F}k = k/4$.

All above cases satisfy the fuzzy contraction:

$$\frac{1}{\Omega(\mathcal{F}w, \mathcal{F}k, \tau)} - 1 \leq q \left[\frac{1}{\Omega(w, k, \tau)} - 1 \right], \quad (42)$$

with $q \in [1/2, 1)$ the fuzzy contractive constant. Hence $\mathcal{F}$ is a fuzzy contractive mapping with $q \in [1/2, 1)$. All conditions of Theorem 13 are satisfied. Also, 0 is the unique fixed point of $\mathcal{F}$ and $\Omega(0, 0, \tau) = 1, \forall \tau > 0$.

Theorem 15. Let $(\mathcal{G}, \Omega, *, b)$ be a complete F-b-metric-l space such that

$$\lim_{\tau \rightarrow \infty} \Omega(w, k, \tau) = 1, \quad (43)$$

for all $w, k \in \mathcal{G}$, $\tau > 0$, and $\mathcal{F} : \mathcal{G} \rightarrow \mathcal{G}$ be a mapping satisfying the condition

$$\Omega(\mathcal{F}w, \mathcal{F}k, \alpha\tau) \geq \Omega(w, k, \tau), \quad (44)$$

for all $w, k \in \mathcal{G}$, $\tau > 0$, where $\alpha \in (0, 1)$. Then $\mathcal{F}$ has a unique fixed point $v \in \mathcal{G}$ and

$$\Omega(v, v, \tau) = 1, \quad \forall \tau > 0. \quad (45)$$

Proof. For an arbitrary $w_0 \in \mathcal{G}$, define a sequence $\{w_n\}$ in $\mathcal{G}$

by

$$w_1 = \mathcal{F}w_0, w_2 = \mathcal{F}^2w_0 = \mathcal{F}w_1, \dots, w_n = \mathcal{F}^nw_0 = \mathcal{F}w_{n-1} \quad \text{for all } n \in \mathbb{N}. \quad (46)$$

If $w_n = w_{n-1}$ for some $n \in \mathbb{N}$, then w_n is a fixed point of $\mathcal{F}$. We assume that $w_n \neq w_{n-1}$ for all $n \in \mathbb{N}$. For $\tau > 0$ and $n \in \mathbb{N}$, we get from (44) that

$$\begin{aligned} \Omega(w_n, w_{n+1}, \tau) &\geq \Omega(w_{n+1}, w_n, \alpha\tau) = \Omega(\mathcal{F}w_n, \mathcal{F}w_{n-1}, \alpha\tau) \\ &\geq \Omega(w_n, w_{n-1}, \tau), \end{aligned} \quad (47)$$

for all $n \in \mathbb{N}$ and $\tau > 0$. Therefore, by applying the above expression, we can deduce that

$$\begin{aligned} \Omega(w_{n+1}, w_n, \tau) &\geq \Omega(w_{n+1}, w_n, \alpha\tau) = \Omega(\mathcal{F}w_n, \mathcal{F}w_{n-1}, \alpha\tau) \\ &\geq \Omega(w_n, w_{n-1}, \tau) = \Omega(\mathcal{F}w_{n-1}, \mathcal{F}w_{n-2}, \tau) \\ &\geq \Omega(w_{n-1}, w_{n-2}, \frac{\tau}{\alpha}) \geq \dots \geq \Omega(w_1, w_0, \frac{\tau}{\alpha^n}), \end{aligned} \quad (48)$$

for all $n \in \mathbb{N}$, $p \geq 1$, and $\tau > 0$. Thus, we have

$$\Omega(w_n, w_{n+p}, \tau) \geq \Omega(w_n, w_{n+1}, \frac{\tau}{b}) * \Omega(w_{n+1}, w_{n+p}, \frac{\tau}{b}). \quad (49)$$

Continuing in this way, we get

$$\begin{aligned} \Omega(w_n, w_{n+p}, \tau) &\geq \Omega(w_n, w_{n+1}, \frac{\tau}{b}) * \Omega(w_{n+1}, w_{n+2}, \frac{\tau}{b^2}) * \dots * \Omega \\ &\quad \cdot \left(w_{n+p-1}, w_{n+p}, \frac{\tau}{b^{p-1}} \right). \end{aligned} \quad (50)$$

Using (48) in the above inequality, we deduce

$$\begin{aligned} \Omega(w_n, w_{n+p}, \tau) &\geq \Omega(w_0, w_1, \frac{\tau}{b\alpha^n}) * \Omega(w_0, w_1, \frac{\tau}{b^2\alpha^{n+1}}) * \dots * \Omega \\ &\quad \cdot \left(w_0, w_1, \frac{\tau}{b^{p-1}\alpha^{n+p-1}} \right). \end{aligned} \quad (51)$$

Here, b is an arbitrary positive integer. We know that $\lim_{n \rightarrow \infty} \Omega(w, k, \tau) = 1$, $\forall w, k \in \mathcal{G}$ and $\tau > 0$, $\alpha \in (0, 1)$. So, from (51), we deduce that

$$\lim_{n \rightarrow \infty} \Omega(w_n, w_{n+p}, \tau) = 1 * 1 * \dots * 1 = 1, \quad \forall \tau > 0, p \geq 1. \quad (52)$$

Hence, $\{w_n\}$ is a Cauchy sequence. The hypothesis of completeness of the F-b-metric-1 space $(\mathcal{G}, \Omega, *, b)$ ensures that there exists $v \in \mathcal{G}$ such that

$$\begin{aligned} \lim_{n \rightarrow \infty} \Omega(w_n, v, \tau) &= \lim_{n \rightarrow \infty} \Omega(w_n, w_{n+p}, \tau) = \Omega(v, v, \tau) \\ &= 1, \quad \forall \tau > 0, p \geq 1. \end{aligned} \quad (53)$$

Now, we derive that $v \in \mathcal{G}$ is a fixed point of $\mathcal{F}$. We have

$$\begin{aligned} \Omega(v, \mathcal{F}v, \tau) &\geq \Omega\left(v, w_{n+1}, \frac{\tau}{2b}\right) * \Omega\left(w_{n+1}, \mathcal{F}v, \frac{\tau}{2b}\right), \forall \tau > 0 \\ &= \Omega\left(v, w_{n+1}, \frac{\tau}{2b}\right) * \Omega\left(\mathcal{F}w_n, \mathcal{F}v, \frac{\tau}{2b}\right) \\ &\geq \Omega\left(v, w_{n+1}, \frac{\tau}{2b}\right) * \Omega\left(w_n, v, \frac{\tau}{2b\alpha}\right). \end{aligned} \quad (54)$$

Taking limit as $n \rightarrow +\infty$, and by (53), we get

$$\Omega(v, \mathcal{F}v, \tau) = 1 * 1 = 1. \quad (55)$$

Therefore, v is a fixed point of $\mathcal{F}$ and $F(v, v, \tau) = 1$, $\forall \tau > 0$.

Now, we investigate the uniqueness of fixed point. For this, assume that v and v are two fixed points of $\mathcal{F}$. Then, by (44), we have

$$\begin{aligned} \Omega(v, v, \tau) &= \Omega(\mathcal{F}v, \mathcal{F}v, \tau) \geq \Omega\left(v, v, \frac{\tau}{\alpha}\right), \\ \Omega(v, v, \tau) &\geq \Omega\left(v, v, \frac{\tau}{\alpha}\right), \quad \forall \tau > 0 \end{aligned} \quad (56)$$

We obtain

$$\Omega(v, v, \tau) \geq \Omega\left(v, v, \frac{\tau}{\alpha^n}\right), \quad \forall n \in \mathbb{N}. \quad (57)$$

Taking limit as $n \rightarrow +\infty$ and using the fact $\lim_{\tau \rightarrow \infty} \Omega(w, k, \tau) = 1$, so $v = v$; hence the fixed point is unique.

Example 8. Let $\mathcal{G} = [0, 1]$ and the t-norm be defined as $a * b = ab$. Also, Ω is defined as

$$\Omega(w, k, \tau) = e^{-(w+k)^2/\tau}, \quad \forall w, k \in \mathcal{G}, \tau > 0. \quad (58)$$

Then, $(\mathcal{G}, \Omega, *, b)$ is a complete F-b-metric-1 space. Define $\mathcal{F} : \mathcal{G} \rightarrow \mathcal{G}$ by

$$\mathcal{F}w = \begin{cases} 0, & w \in \left[0, \frac{1}{2}\right], \\ \frac{w}{6}, & w \in \left(\frac{1}{2}, 1\right]. \end{cases} \quad (59)$$

Now, $\lim_{\tau \rightarrow \infty} \Omega(w, k, \tau) = \lim_{\tau \rightarrow \infty} e^{-(w+k)^2/\tau} = 1$. For $\alpha \in [1/2, 1)$, we have four cases:

Case 1. If $w, k \in [0, 1/2]$, then $\mathcal{F}w = \mathcal{F}k = 0$. Here,

$$\Omega(\mathcal{F}w, \mathcal{F}k, \alpha\tau) = \Omega(0, 0, \alpha\tau) = e^0 \geq e^{-(w+k)^2/\tau} = \Omega(w, k, \tau). \quad (60)$$

Case 2. If $w \in [0, 1/2]$ and $k \in (1/2, 1]$, then $\mathcal{F}w = 0$ and $\mathcal{F}k = k/6$. We have

$$\begin{aligned} \Omega(\mathcal{F}w, \mathcal{F}k, \alpha\tau) &= \Omega\left(0, \frac{w}{6}, \alpha\tau\right) = e^{-(0+(k/6))^2/\alpha\tau} = e^{-(k^2/36\alpha\tau)} \\ &\geq e^{-(w+k)^2/\tau} = \Omega(w, k, \tau). \end{aligned} \quad (61)$$

Case 3. If $w, k \in (1/2, 1]$, then $\mathcal{F}w = w/6$ and $\mathcal{F}k = k/6$. Here,

$$\begin{aligned} \Omega(\mathcal{F}w, \mathcal{F}k, \alpha\tau) &= \Omega\left(\frac{w}{6}, \frac{k}{6}, \alpha\tau\right) = e^{-(w+k)^2/36\alpha\tau} \\ &\geq e^{-(w+k)^2/\tau} = \Omega(w, k, \tau). \end{aligned} \quad (62)$$

Case 4. If $w \in (1/2, 1]$ and $k \in [0, 1/2]$, then $\mathcal{F}w = w/6$ and $\mathcal{F}k = 0$. Then,

$$\Omega(\mathcal{F}w, \mathcal{F}k, \alpha\tau) = \Omega\left(\frac{w}{6}, 0, \alpha\tau\right) = e^{-w^2/36\alpha\tau} \geq e^{-(w+k)^2/\tau} = \Omega(w, k, \tau). \quad (63)$$

From all 4 cases, we obtain that

$$\Omega(\mathcal{F}w, \mathcal{F}k, \alpha\tau) \geq \Omega(w, k, \tau). \quad (64)$$

Hence, all conditions of Theorem 13 are satisfied, and 0 is the unique fixed point of $\mathcal{F}$. Also,

$$\Omega(v, v, \tau) = \Omega(0, 0, \tau) = e^0 = 1, \quad \forall \tau > 0. \quad (65)$$

3. An Application to an Integral Equation

Consider the following integral equation:

$$w(l) = \int_0^J \partial(l, r, w(r)) dr, \quad (66)$$

where $J > 0$ and $\partial : [0, J] \times [0, J] \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$.

Let $\mathcal{G} = C[0, J]$ be the set of all continuous real valued functions defined on $[0, J]$. Consider the b-metric-1 given as

$$\mu(w, k) = (|w| + |k|)^2. \quad (67)$$

Then, by Proposition 8,

$$\Omega(w(l), k(l), \tau) = \max_{l \in [0, J]} e^{-(|w(l)| + |k(l)|)^2/\tau}, \quad \forall w, k \in \mathcal{G}. \quad (68)$$

Clearly, $(\mathcal{G}, \Omega, *, b)$ is a complete F-b-metric-1 space. Let

$$\mathcal{F}w(l) = \int_0^J \partial(l, r, w(r)) dr, \quad \forall w \in \mathcal{G}, \quad (69)$$

for $l \in [0, J]$. Observe that the existence of a solution of (66) is equivalent to the existence of a fixed point of $\mathcal{F}$.

Theorem 16. Assume that the following hypotheses hold:

- (1) $\partial : [0, J] \times [0, J] \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is continuous
- (2) $\forall l, r \in [0, J]$, there is a continuous function $\beta : [0, J] \times [0, J] \longrightarrow \mathbb{R}^+$ such that

$$|\partial(l, r, w(r))| + |\partial(l, r, k(r))| \leq \alpha^{1/2} \beta(l, r) (|w(l)| + |k(l)|), \quad (70)$$

where $\alpha \in (0, 1)$ and $b > 1$.

$$(3) \lim_{\tau \rightarrow \infty} \Omega(w(l), k(l), \tau) = 1 \text{ and } \sup_{l \in [0, J]} \int_0^J \beta(l, r) dr \leq 1$$

Then, the integral Equation (66) has a unique solution.

Proof. For all $l \in [0, J]$, we have

$$\begin{aligned} \Omega(\mathcal{F}w(l), \mathcal{F}k(l), \alpha\tau) &= e^{-(|\mathcal{F}w(l)| + |\mathcal{F}k(l)|)^2/\alpha\tau} \\ &= e^{-\left(\left|\int_0^J \partial(l, r, w(r)) dr\right| + \left|\int_0^J \partial(l, r, k(r)) dr\right|\right)^2/\alpha\tau} \\ &\geq e^{-\left(\left(\int_0^J |\partial(l, r, w(r))| dr + \int_0^J |\partial(l, r, k(r))| dr\right)\right)^2/\alpha\tau} \\ &= e^{-\left(\left(\int_0^J (|\partial(l, r, w(r))| + |\partial(l, r, k(r))|) dr\right)\right)^2/\alpha\tau} \\ &\geq e^{-\left(\left(\int_0^J (\alpha^{1/2} \beta(l, r) (|w(l)| + |k(l)|)) dr\right)\right)^2/\alpha\tau} \\ &= e^{-\left(\left(\int_0^J (\alpha^{1/2} \beta(l, r) (|w(l)| + |k(l)|)^2)^{1/2} dr\right)\right)^2/\alpha\tau} \\ &= \sup_{l \in [0, J]} e^{-\left(\alpha (|w(l)| + |k(l)|)^2 \left(\int_0^J \beta(l, r) dr\right)^2\right)/\alpha\tau} \\ &\geq e^{-(|w(l)| + |k(l)|)^2/\tau} = \Omega(w(l), k(l), \tau). \end{aligned} \quad (71)$$

Therefore, $\Omega(\mathcal{F}w(l), \mathcal{F}k(l), \alpha\tau) \geq \Omega(w(l), k(l), \tau)$. Also, observe that all conditions of Theorem 15 are satisfied. Hence, the operator $\mathcal{F}$ has a unique fixed point. This means that the integral Equation (66) has a unique solution.

Example 9. Consider the nonlinear integral equation below

$$w(l) = \frac{1}{8} \int_0^1 r w(r) dr. \quad (72)$$

Then, it has a solution in $\mathcal{G}$.

Proof. Let $\mathcal{F} : \mathcal{G} \longrightarrow \mathcal{G}$ be defined by $\mathcal{F}w(l) = (1/8) \int_0^1 r w(r) dr$. Set $\partial(l, r, w(r)) = (1/8) r w(r)$ in Theorem 16, we get

- (i) $\partial : [0, J] \times [0, J] \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is continuous

- (ii) $\forall l, r \in [0, J]$, there is a continuous function $\beta(l, r) = r$ such that

$$\begin{aligned} |\partial(l, r, w(r))| + |\partial(l, r, k(r))| &= \frac{1}{4} \times \frac{r}{2} [|w(r)| + k(r)] \\ &\leq \frac{1}{4} r [|w(r)| + k(r)] \\ &= \left(\frac{1}{16}\right)^{1/2} r [|w(r)| + k(r)] \\ &= \alpha^{1/2} \beta(l, r) (|w(l)| + |k(l)|), \end{aligned} \quad (73)$$

where $\alpha = (1/16) \in (0, (1/b))$

$$\begin{aligned} \text{(iii)} \quad \lim_{\tau \rightarrow \infty} \Omega(w(l), k(l), \tau) &= 1 \\ \text{and } \sup_{l \in [0, J]} \int_0^1 \beta(l, r) &= \sup_{l \in [0, J]} \int_0^1 r dr = (1/2) < 1 \end{aligned}$$

Hence, all hypotheses of Theorem 16 are fulfilled. Therefore, the problem (72) has a solution on $\mathcal{G}$.

4. Conclusion

Fixed point techniques are used to solve many mathematical problems, as differential and integral equations, integro-differential equations, game theory, and economics. The intent of this manuscript is to present a new space, so-called a fuzzy b-metric-like space. Topological properties and related examples are addressed, so our fixed point results are new. Ultimately, to illustrate the practical side of the theoretical results, a solution of a nonlinear integral equation is given.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

All authors contributed equally in writing this article. All authors read and approved the final manuscript.

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