

## Research Article

# Numerical Solution of Fractional Model of HIV-1 Infection in Framework of Different Fractional Derivatives

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In this paper, we have extended the model of HIV-1 infection to the fractional mathematical model using Caputo-Fabrizio and Atangana-Baleanu fractional derivative operators. A detailed proof for the existence and the uniqueness of the solution of fractional mathematical model of HIV-1 infection in Atangana-Baleanu sense is presented. Numerical approach is used to find and study the behavior of the solution of the stated model using different derivative operators, and the graphical comparison between the solutions obtained for the Caputo-Fabrizio and the Atangana-Baleanu operator is presented to see which fractional derivative operator is more efficient.

## 1. Introduction

HIV stands for the Human Immunodeficiency Virus. This virus attacks a person's immune system. The immune system of a person's body works as a defence mechanism against infections such as bacteria and viruses.

All viruses proliferate by infecting the living cells of the body. HIV targets such immune system cells which are meant to defend the body. These cells are known as CD4<sup>+</sup> T cells. CD4<sup>+</sup> T cells are basically white blood cells that fight infection. CD4<sup>+</sup> T cell count is a measure of immune function in a patient with HIV. It is an imperative determinant for the need for opportunistic infection (OI) prophylaxis. CD4<sup>+</sup> T cell count is determined from blood as a part of laboratory monitoring for HIV infection.

When HIV takes over these cells, it transforms a cell into a virus factory. It makes the cell to produce thousands of copies of the virus, and these copies then infect the other CD4<sup>+</sup> T cells. The infected cells do not function desirably and die early. The paucity of CD4<sup>+</sup> T cells deteriorates the immune system and makes it challenging for the body to stay healthy.

CD4<sup>+</sup> T cell count is generally measured after being diagnosed with HIV (at baseline). It is measured in every three to six months during the first two years until these cell counts in a patient's body increase above 300 cells/mm<sup>3</sup>; else, it is measured in every twelve months. Most HIV patients can expect an average increase of about 50-100 cells/mm<sup>3</sup>. Patients who indulge in therapy with low CD4<sup>+</sup> T cell count or at an older age may not experience the same increase in their CD4<sup>+</sup> T cell count despite virologic suppression.

There are a lot of factors that affect these cell counts. Medication is one of the options to keep these T cell counts high.

The characteristics of Acquired Immunodeficiency Syndrome (AIDS) pathogenesis are progressive depletion of CD4<sup>+</sup> T cell population in close association with progressive impairment of cellular immunity and increased susceptibility to opportunistic infections (OI). HIV was originally hypothesized to be a consequence of sluggish CD4<sup>+</sup> T cell destruction. However, massive CD4<sup>+</sup> memory T cell destruction now occurs at a primitive level of infection. In most individuals, the initial destruction is countered by CD4<sup>+</sup> memory T cell regeneration that protects these T cell numbers and

functions above the threshold associated with overt immunodeficiency (see [1, 2]).

The mathematical model of HIV-1 infection of CD4<sup>+</sup> T cells is given as follows

$$\begin{aligned}\frac{dU}{dt} &= \varepsilon - dU - \alpha UV, U(0) = \gamma_1, \\ \frac{dI}{dt} &= \alpha UV - sI, I(0) = \gamma_2, \\ \frac{dV}{dt} &= cI - \delta V, V(0) = \gamma_3,\end{aligned}\quad (1)$$

here,  $U(t)$  represents the population with uninfected CD4<sup>+</sup> T cells,  $I(t)$  represents the population with infected CD4<sup>+</sup> T cells, and  $V(t)$  represents the density of virions in plasma at time “ $t$ ”, respectively. Further,  $\varepsilon$  is the rate of generation of CD4<sup>+</sup> T cells,  $d$  is the natural death rate,  $\alpha$  is the infected death rate of CD4<sup>+</sup> T cells,  $s$  is the rate by which viruses make the cells dead,  $c$  is the rate of formation of virions viruses due to infected CD4<sup>+</sup> T cells, and  $\delta$  gives the death rate of viruses (see [3]). In the current work, we will be concerned with the fractional mathematical model of HIV-1 infection. Most biological systems have aftereffects or memory, so the modeling of such biological systems using fractional order derivatives have many advantages in which the effects like memory are neglected. It has been concluded that there is fractional order electrical conductance in the cell membrane of many biological organisms, and they are classified in groups of noninteger models. Thus, fractional derivatives, see also ([4–12]) study the behavior of these biological models more efficiently. Fractional calculus have been successfully growing and giving fruitful and countless developments in fields of chemistry, physics, biochemistry, medicine, biology, etc. (see [13–16]). In this work, we will consider the fractional mathematical model of HIV-1 infection in sense of Caputo-Fabrizio derivative operator [17] and Atangana-Baleanu derivative operator [18]. A detailed proof for the existence and the uniqueness of the solution is presented. The numerical solutions are presented for the fractional mathematical model in sense of the Caputo-Fabrizio and the Atangana-Baleanu derivative operator. Further, the results obtained are compared graphically to see which fractional derivative operator gives the better result.

*Definition 1* (see [17]). Let  $f$  be an integrable function on  $\mathbb{R}$ ,  $t > 0$ ,  $0 < \beta < 1$ , the Caputo-Fabrizio fractional derivative of order  $\beta$  is defined as

$${}_{0}^{CF}\zeta_t^\beta(f(t)) = \frac{N(\beta)}{1-\beta} \int_0^t \exp\left(\frac{-\beta(t-\tau)}{1-\beta}\right) f'(\tau) d\tau, \quad (2)$$

where  ${}_{0}^{CF}\zeta_t^\beta$  represents the Caputo-Fabrizio fractional derivative of order  $\beta$  and  $N(\beta)$  is a normalization function and the following holds  $N(0) = N(1) = 1$ .

*Definition 2* (see [19]). Let  $f$  be an integrable function on  $\mathbb{R}$ ,  $t > 0$ ,  $0 < \beta < 1$ , the Caputo-Fabrizio time fractional integral of order  $\beta$  is given as

$${}_{0}^{CF}\mathcal{I}_t^\beta(f(t)) = \frac{2(1-\beta)}{(2-\beta)N(\beta)} f(t) + \frac{2\beta}{(2-\beta)N(\beta)} \int_0^t f(\tau) d\tau, \quad (3)$$

where  $N(\beta)$  is the normalization function and the following holds  $N(0) = N(1) = 1$ .

*Definition 3* (see [18]). Let  $f$  be an integrable function on  $\mathbb{R}$ , let  $0 < \beta < 1$ , then the Atangana-Baleanu fractional derivative is given as

$${}_{0}^{ABC}\zeta_t^\beta(f(t)) = \frac{N(\beta)}{1-\beta} \int_0^t f'(\tau) E_\beta\left[-\beta \frac{(t-\tau)^\beta}{1-\beta}\right] d\tau, \quad (4)$$

where  ${}_{0}^{ABC}\zeta_t^\beta$  is the Atangana-Baleanu fractional derivative of order  $\beta$  in Caputo sense,  $E_\beta$  is the Mittag-Leffler function, and  $N(\beta)$  is the normalization function such that  $N(0) = N(1) = 1$ .

*Definition 4* (see [18]). Let  $f$  be an integrable function on  $\mathbb{R}$ , the fractional integral of Atangana-Baleanu fractional derivative of order  $\beta$  is given as

$${}_{0}^{AI}\mathcal{I}_t^\beta(f(t)) = \frac{1-\beta}{N(\beta)} f(t) + \frac{\beta}{N(\beta)\Gamma(\beta)} \int_0^t f(\tau) (t-\tau)^{\beta-1} d\tau. \quad (5)$$

**Theorem 5** (see [18]). *The fractional differential equation*

$${}_{0}^{AI}\mathcal{I}_t^\beta(f(t)) = v(t), \quad (6)$$

*possesses a solution which is unique given as*

$$f(t) = \frac{1-\beta}{M(\beta)} v(t) + \frac{\beta}{M(\beta)\Gamma(\beta)} \int_0^t v(\tau) (t-\tau)^{\beta-1} d\tau. \quad (7)$$

## 2. Fractional HIV-1 Model in Caputo-Fabrizio Sense

The fractional mathematical model of HIV-1 infection in Caputo-Fabrizio sense is given as follows.

$$\begin{aligned}{}_{0}^{CF}\zeta_t^\beta U(t) &= \varepsilon - dU - \alpha UV, U(0) = \gamma_1, \\ {}_{0}^{CF}\zeta_t^\beta I(t) &= \alpha UV - sI, I(0) = \gamma_2, \\ {}_{0}^{CF}\zeta_t^\beta V(t) &= cI - \delta V, V(0) = \gamma_3,\end{aligned}\quad (8)$$

here,  ${}_{0}^{CF}\zeta_t^\beta$  denotes the Caputo-Fabrizio fractional derivative of order  $\beta$ .

2.1. *Derivation of Numerical Scheme in Sense of Caputo-Fabrizio Derivative.* To illustrate the method ([20, 21]), we contemplate the following general equation:

$${}_0^{CF}\zeta_t^\beta \eta(t) = g(t, \eta(t)), t \geq 0, \eta(0) = \eta_0, \quad (9)$$

where  $\eta(t)$  stands for  $U(t), I(t), V(t)$  and  $g(t, \eta(t))$  is an integrable function that stands for  $\varepsilon - dU - \alpha UV, \alpha UV - sI$ , and  $cI - \delta V$ , respectively. Using the fundamental theorem, the above equation can be written as

$$\eta(t) - \eta(0) = \frac{1 - \beta}{N(\beta)} g(t, \eta(t)) + \frac{\beta}{N(\beta)} \int_0^t g(\tau, \eta(\tau)) d\tau. \quad (10)$$

At point  $t = t_{k+1}$ , for  $k = 0, 1, 2 \dots$ , equation (10) becomes

$$\eta(t_{k+1}) - \eta(0) = \frac{1 - \beta}{N(\beta)} g(t_k, \eta(t_k)) + \frac{\beta}{N(\beta)} \int_0^{t_{k+1}} g(\tau, \eta(\tau)) d\tau. \quad (11)$$

At point  $t = t_k$ , for  $k = 0, 1, 2 \dots$ , equation (10) becomes

$$\eta(t_k) - \eta(0) = \frac{1 - \beta}{N(\beta)} g(t_{k-1}, \eta(t_{k-1})) + \frac{\beta}{N(\beta)} \int_0^{t_k} g(\tau, \eta(\tau)) d\tau. \quad (12)$$

From the above two equations, we get

$$\begin{aligned} \eta(t_{k+1}) - \eta(t_k) &= \frac{1 - \beta}{N(\beta)} [g(t_k, \eta(t_k)) - g(t_{k-1}, \eta(t_{k-1}))] \\ &+ \frac{\beta}{N(\beta)} \int_{t_k}^{t_{k+1}} g(\tau, \eta(\tau)) d\tau. \end{aligned} \quad (13)$$

Considering  $g(\tau, \eta(\tau))$  through Lagrange polynomial interpolation,

$$q_n = g(\tau, \eta(\tau)) = \frac{\tau - t_{i-1}}{t_i - t_{i-1}} g(t_i, \eta_{t_i}) + \frac{\tau - t_i}{t_{i-1} - t_i} g(t_{i-1}, \eta_{t_{i-1}}), \quad (14)$$

where  $\eta(t_i)$  is a function at time  $t_i$  and  $\eta(t_{i-1})$  is a function at time  $t_{i-1}$ . Substituting the value of  $g(\tau, \eta(\tau))$  in equation (13), we get

$$\begin{aligned} \eta_{k+1} - \eta_k &= \frac{1 - \beta}{N(\beta)} [g(t_k, \eta(t_k)) - g(t_{k-1}, \eta(t_{k-1}))] \\ &+ \frac{\beta}{N(\beta)} \int_{t_k}^{t_{k+1}} \left( \frac{g(t_k, \eta_k)}{h} (\tau - t_{k-1}) - \frac{g(t_{k-1}, \eta_{k-1})}{h} (\tau - t_k) \right) d\tau. \end{aligned} \quad (15)$$

Substituting  $h = t_i - t_{i-1}$  and on solving, we get

$$\begin{aligned} \eta_{k+1} &= \eta_0 + \left( \frac{1 - \beta}{N(\beta)} + \frac{3h}{2N(\beta)} \right) g(t_k, \eta(t_k)) \\ &- \left( \frac{1 - \beta}{N(\beta)} + \frac{\beta h}{2N(\beta)} \right) g(t_{k-1}, \eta(t_{k-1})). \end{aligned} \quad (16)$$

Using the aforementioned concept of numerical scheme, the numerical scheme for the fractional model of HIV-1 infection in framework of Caputo-Fabrizio derivative operator is given as

$$\begin{aligned} U_{k+1} &= U_0 + \left( \frac{1 - \beta}{N(\beta)} + \frac{3h}{2N(\beta)} \right) L_1(t_k, \eta(t_k)) \\ &- \left( \frac{1 - \beta}{N(\beta)} + \frac{\beta h}{2N(\beta)} \right) L_1(t_{k-1}, \eta(t_{k-1})), \end{aligned} \quad (17)$$

where

$$L_1(t, U) = \varepsilon - dU - \alpha UV,$$

$$\begin{aligned} I_{k+1} &= I_0 + \left( \frac{1 - \beta}{N(\beta)} + \frac{3h}{2N(\beta)} \right) L_2(t_k, \eta(t_k)) \\ &- \left( \frac{1 - \beta}{N(\beta)} + \frac{\beta h}{2N(\beta)} \right) L_2(t_{k-1}, \eta(t_{k-1})), \end{aligned} \quad (18)$$

where

$$L_2(t, I) = \alpha UV - sI,$$

$$\begin{aligned} V_{k+1} &= V_0 + \left( \frac{1 - \beta}{N(\beta)} + \frac{3h}{2N(\beta)} \right) L_3(t_k, \eta(t_k)) \\ &- \left( \frac{1 - \beta}{N(\beta)} + \frac{\beta h}{2N(\beta)} \right) L_3(t_{k-1}, \eta(t_{k-1})), \end{aligned} \quad (19)$$

where

$$L_3(t, V) = cI - \delta V. \quad (20)$$

### 3. Fractional HIV-1 Model in Atangana-Baleanu Sense

The fractional mathematical model of HIV-1 infection in Atangana-Baleanu sense is given as follows.

$$\begin{aligned} {}_0^{ABC}\zeta_t^\beta U(t) &= \varepsilon - dU - \alpha UV, U(0) = \gamma_1, \\ {}_0^{ABC}\zeta_t^\beta I(t) &= \alpha UV - sI, I(0) = \gamma_2, \\ {}_0^{ABC}\zeta_t^\beta V(t) &= cI - \delta V, V(0) = \gamma_3, \end{aligned} \quad (21)$$

here,  ${}_0^{ABC}\zeta_t^\beta$  is the Atangana-Baleanu fractional derivative of order  $\beta$ .

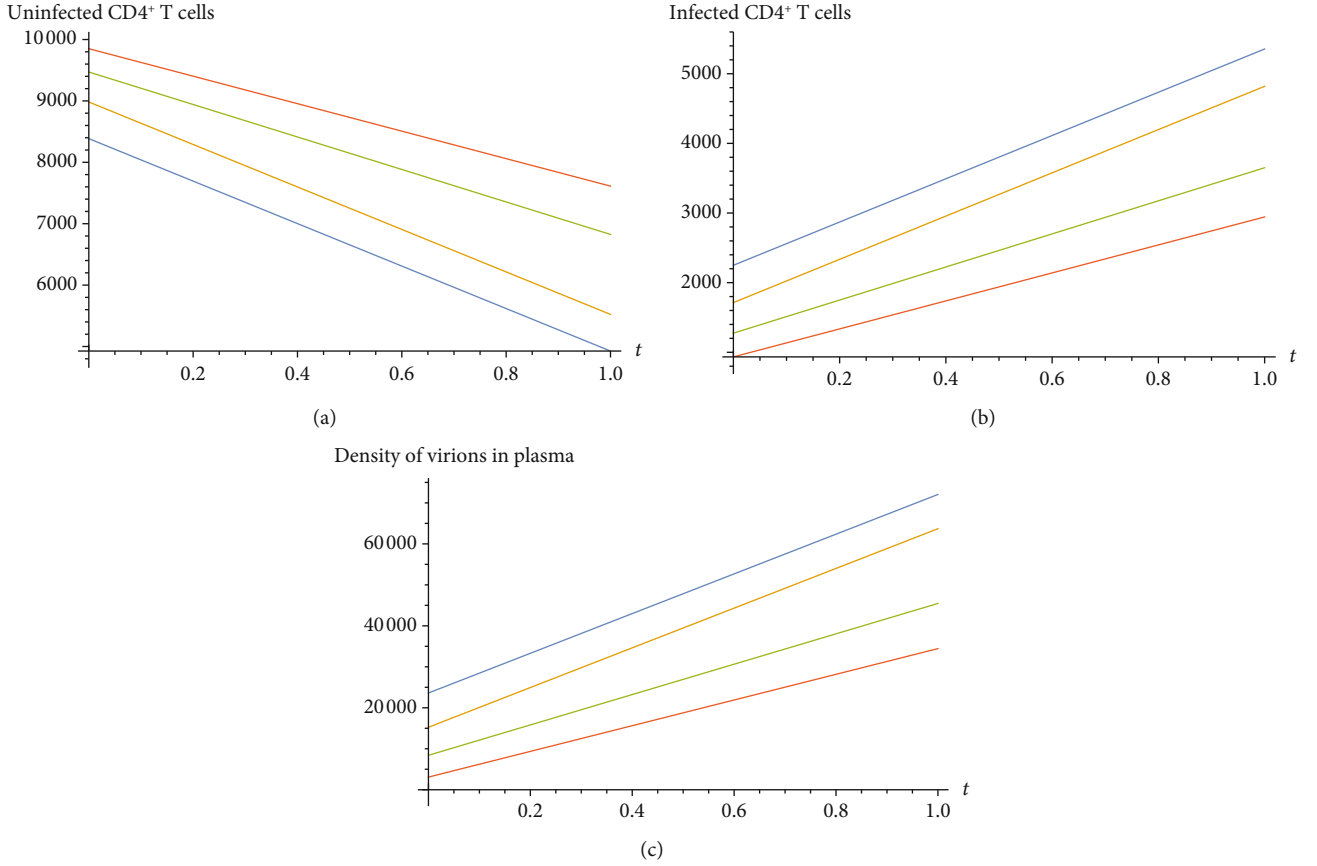


FIGURE 1: The graphical results to show the (a) uninfected  $CD4^+$  T cells, (b) infected  $CD4^+$  T cells, and (c) density of virions in plasma for different order fractional derivatives in sense of Caputo-Fabrizio operator, where blue represents graph for  $\beta = 0.3$ , yellow for  $\beta = 0.5$ , green for  $\beta = 0.7$ , and red for  $\beta = 0.9$ , respectively.

### 3.1. Existence and Uniqueness of the Solution

**Theorem 6.** *The kernels*

$$\begin{aligned} L_1(t, U) &= \varepsilon - dU - \alpha UV, \\ L_2(t, I) &= \alpha UV - sI, \\ L_3(t, V) &= cI - \delta V, \end{aligned} \quad (22)$$

satisfy the Lipschitz condition and contractions if following hold:

$$\begin{aligned} 0 < k_1 < 1, \\ 0 < k_2 < 1, \\ 0 < k_3 < 1, \end{aligned} \quad (23)$$

where  $k_1 = d + n_1$ ,  $k_2 = s$ , and  $k_3 = \delta$ .

*Proof.* Consider the kernel

$$L_1(t, U) = \varepsilon - dU - \alpha UV. \quad (24)$$

Let  $U$  and  $U_1$  be two functions, then

$$\begin{aligned} \|L_1(t, U) - L_1(t, U_1)\| &= \|(\varepsilon - dU - \alpha UV) \\ &\quad - (\varepsilon - dU_1 - \alpha U_1 V)\| \\ &\leq d\|U - U_1\| + \alpha\|V\|\|U - U_1\|. \end{aligned} \quad (25)$$

Let  $n_1 = \max_t \|V(t)\|$ , then

$$\|L_1(t, U) - L_1(t, U_1)\| \leq k_1 \|U(t) - U_1(t)\|, \quad (26)$$

where

$$k_1 = d + n_1 \alpha. \quad (27)$$

Hence, this proves Lipschitz's condition for  $L_1(t, U)$ , and if  $0 < k_1 < 1$ , then this proves contraction for  $L_1(t, U)$ . Similarly, we can prove the result for the kernels  $L_2(t, I)$  and  $L_3(t, V)$ .

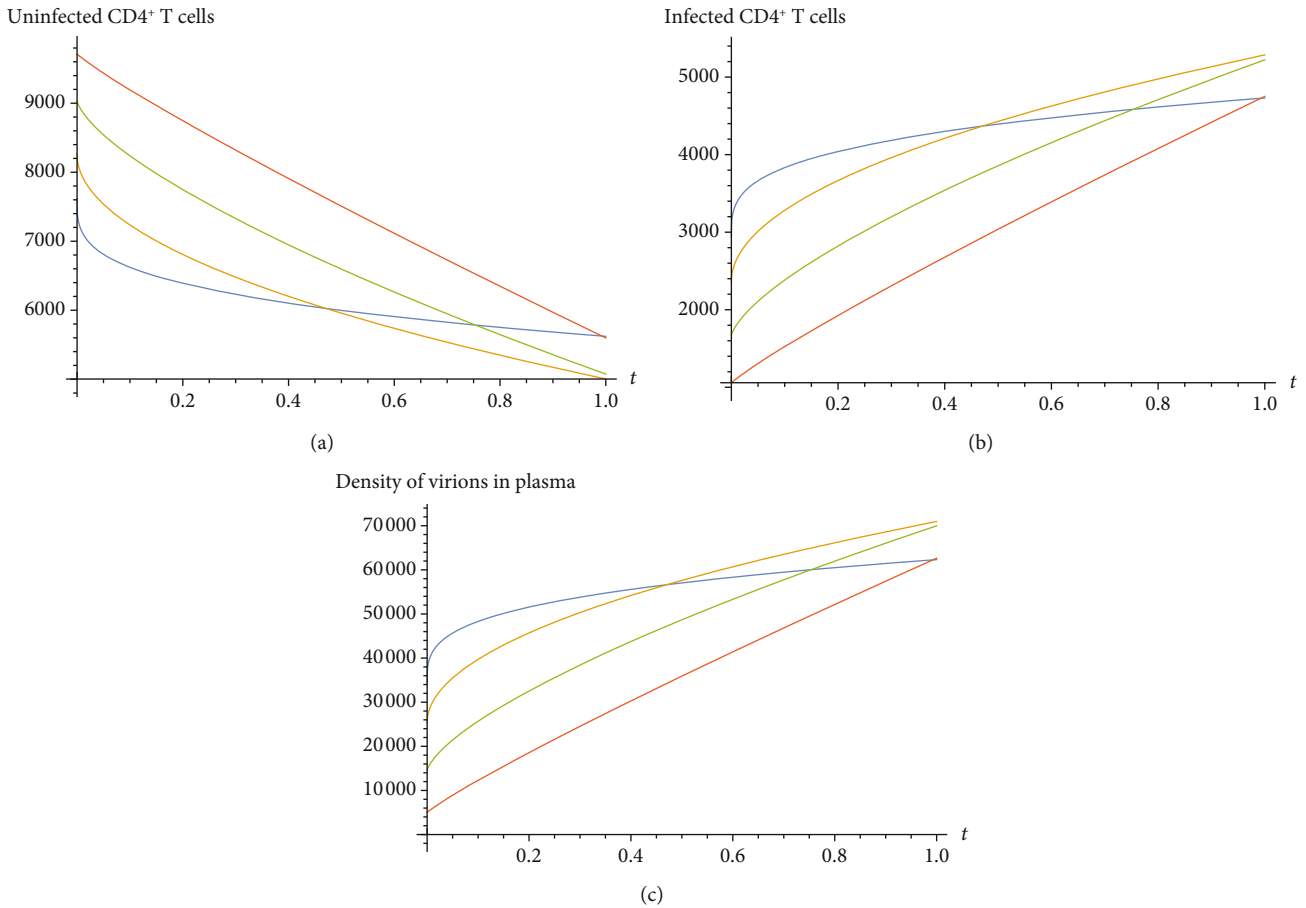


FIGURE 2: The graphical results to show the (a) uninfected CD4<sup>+</sup> T cells, (b) infected CD4<sup>+</sup> T cells, and (c) density of virions in plasma for different order fractional derivatives in sense of ABC operator, where blue represents graph for  $\beta = 0.3$ , yellow for  $\beta = 0.5$ , green for  $\beta = 0.7$ , and red for  $\beta = 0.9$ , respectively.

**Theorem 7.** *The time fractional HIV-1 infection model*

$${}_0^{ABC}\zeta_t^\beta U(t) = \varepsilon - dU - \alpha UV, U(0) = \gamma_1, \quad (28)$$

$${}_0^{ABC}\zeta_t^\beta I(t) = \alpha UV - sI, I(0) = \gamma_2, \quad (29)$$

$${}_0^{ABC}\zeta_t^\beta V(t) = cI - \delta V, V(0) = \gamma_3, \quad (30)$$

possesses a unique solution under the conditions that we are able to search  $t_{\max}$  which satisfies

$$\frac{1 - \beta}{N(\beta)} k_i + \frac{t_{\max}^\beta}{N(\beta)\Gamma(\beta)} k_i < 1, \text{ for } i = 1, 2, 3, \quad (31)$$

where  $k_1 = d + n_1$ ,  $k_2 = s$ , and  $k_3 = \delta$ .

*Proof.* We prove the result for equation (28).

Consider equation (28)

$${}_0^{ABC}\zeta_t^\beta U(t) = \varepsilon - dU - \alpha UV, U(0) = \gamma_1. \quad (32)$$

Let  $L_1(t, U) = \varepsilon - dU - \alpha UV$ .

Then, equation (32) can be written as

$${}_0^{ABC}\zeta_t^\beta U(t) = L_1(t, U). \quad (33)$$

Using Theorem 5, we get

$$U(t) = U_0 + \frac{1 - \beta}{N(\beta)} L_1(t, U(t)) + \frac{\beta}{N(\beta)\Gamma(\beta)} \int_0^t (t - \tau)^{\beta-1} L_1(\tau, U(\tau)) d\tau. \quad (34)$$

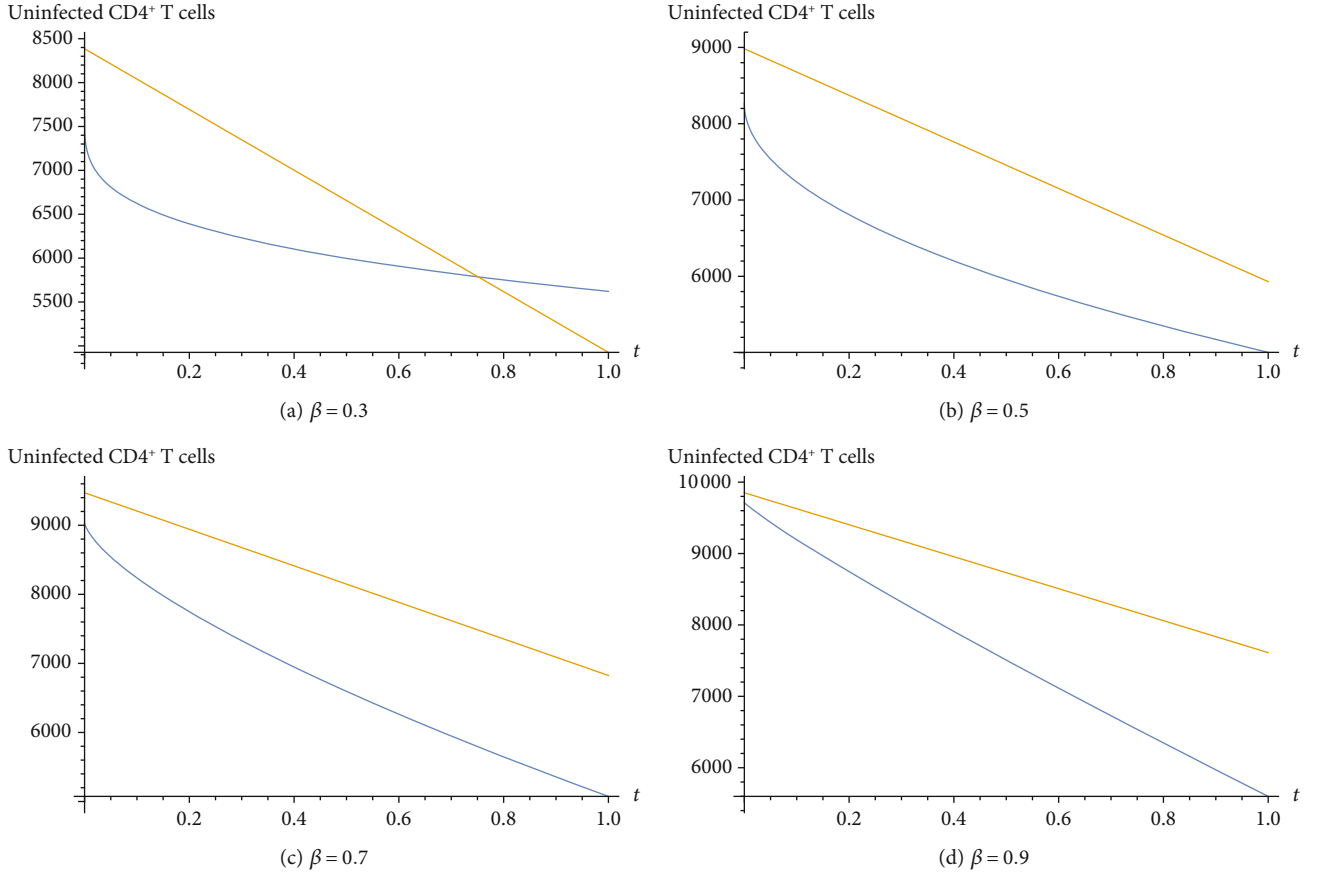


FIGURE 3: The comparison of the behavior of uninfected CD4<sup>+</sup> T cells in Caputo-Fabrizio and Atangana Baleanu sense for different order fractional derivatives, where blue colour represents graph for ABC operator and orange represents graph for CF operator.

Let  $J = (0, T)$  and define an operator  $Z : C(J, \mathbb{R}^3) \rightarrow C(J, \mathbb{R}^3)$  such that

$$\begin{aligned} Z[U(t)] &= U_0 + \frac{1-\beta}{N(\beta)} L_1(t, U(t)) \\ &+ \frac{\beta}{N(\beta)\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} L_1(\tau, U(\tau)) d\tau. \end{aligned} \quad (35)$$

So equation (34) can be seen as  $Z[U(t)] = U(t)$ . Define the supremum norm on  $J$  as  $\|U\| = \sup_{t \in J} |U(t)|$ . Then,  $C(J, \mathbb{R}^3)$  and  $\|\cdot\|$  define a Banach Space. Finally, consider

$$\begin{aligned} Z[U_1(t)] - Z[U_2(t)] &= \frac{1-\beta}{N(\beta)} (L_1(t, U_1(t)) - L_2(t, U_2(t))) \\ &+ \frac{\beta}{N(\beta)\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} (L_1(\tau, U_1(\tau)) \\ &- L_1(\tau, U_2(\tau))) d\tau. \end{aligned} \quad (36)$$

Taking modulus on both sides of equation (36) and using triangle inequality, we get

$$\begin{aligned} &|Z[U_1(t)] - Z[U_2(t)]| \\ &\leq \frac{1-\beta}{N(\beta)} |(L_1(t, U_1(t)) - L_2(t, U_2(t)))| \\ &+ \frac{\beta}{N(\beta)\Gamma(\beta)} \int_0^t |(t-\tau)^{\beta-1} (L_1(\tau, U_1(\tau)) \\ &- L_1(\tau, U_2(\tau)))| d\tau. \end{aligned} \quad (37)$$

Lastly, using the fact that the kernel  $L_1(t, U(t))$  satisfies Lipschitz condition, we get

$$|Z(U_1) - Z(U_2)| \leq \left( \frac{1-\beta}{N(\beta)} k_1 + \frac{t_{\max}^\beta}{N(\beta)\Gamma(\beta)} k_1 \right) |U_1 - U_2|. \quad (38)$$

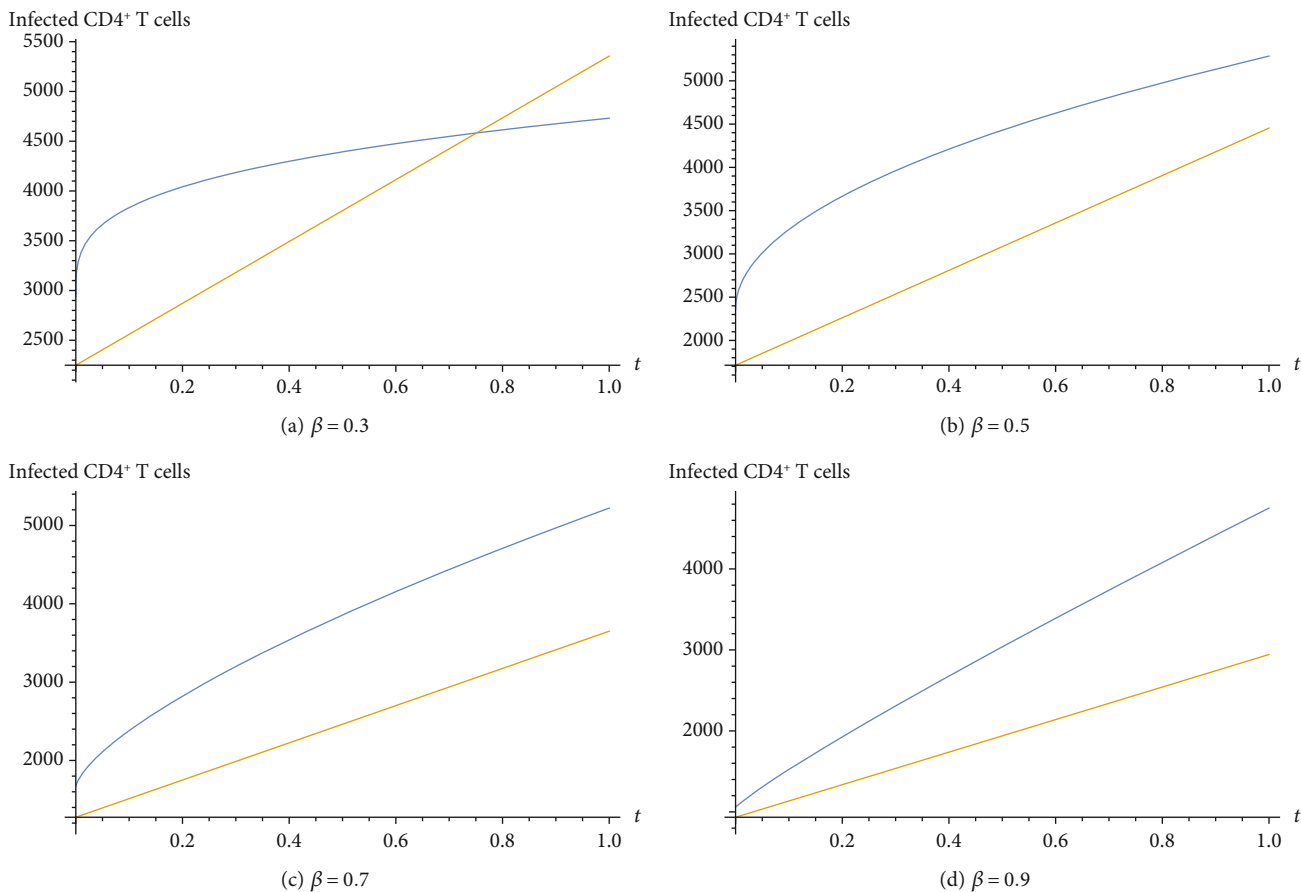


FIGURE 4: The comparison of the behavior of infected CD4+ T cells in Caputo-Fabrizio and Atangana Baleanu sense for different order fractional derivatives, where blue colour represents graph for ABC operator and orange represents graph for CF operator.

Equation (38) is a contraction if

$$\frac{1 - \beta}{N(\beta)} k_1 + \frac{t_{\max}^\beta}{N(\beta)\Gamma(\beta)} k_1 < 1. \tag{39}$$

Hence, using the Banach Fixed Point theorem, we govern the existence of a unique solution for the fractional model of HIV-1 infection in framework of Atangana-Baleanu derivative operator.

3.2. Derivation of Numerical Scheme in Sense of ABC Derivative. Toufik and Atangana ([21, 22]) introduced a numerical scheme for solving fractional derivatives having nonsingular and nonlocal kernel. To illustrate the method, contemplate the following fractional differential equation:

$${}_0^{ABC}\zeta_t^\beta \eta(t) = g(t, \eta(t)), t \geq 0, \eta(0) = \eta_0. \tag{40}$$

Using Theorem 5, the above equation can be written as

$$\eta(t) - \eta(0) = \frac{1 - \beta}{N(\beta)} g(t, \eta(t)) + \frac{\beta}{N(\beta)\Gamma(\beta)} \cdot \int_0^t (t - \tau)^{\beta-1} g(\tau, \eta(\tau)) d\tau. \tag{41}$$

At point  $t = t_{k+1}$ , for  $k = 0, 1, 2 \dots$ , equation (41) becomes

$$\eta(t_{n+1}) - \eta(0) = \frac{1 - \beta}{N(\beta)} g(t_k, \eta(t_k)) + \frac{\beta}{N(\beta)\Gamma(\beta)} \int_0^{t_{k+1}} (t_{k+1} - \tau)^{\beta-1} g(\tau, \eta(\tau)) d\tau, \tag{42}$$

$$\eta_{k+1} = \eta(t_{k+1}) = \eta(0) + \frac{1 - \beta}{N(\beta)} g(t_k, \eta(t_k)) + \frac{\beta}{N(\beta)\Gamma(\beta)} \sum_{j=0}^k \int_{t_j}^{t_{j+1}} (t_{k+1} - \tau)^{\beta-1} g(\tau, \eta(\tau)) d\tau. \tag{43}$$

Considering  $g(\tau, \eta(\tau))$  through Lagrange polynomial interpolation,

$$g_n = g(\tau, \eta(\tau)) = \frac{\tau - t_{i-1}}{t_i - t_{i-1}} g(t_i, \eta_{t_i}) + \frac{\tau - t_i}{t_{i-1} - t_i} g(t_{i-1}, \eta_{t_{i-1}}). \tag{44}$$

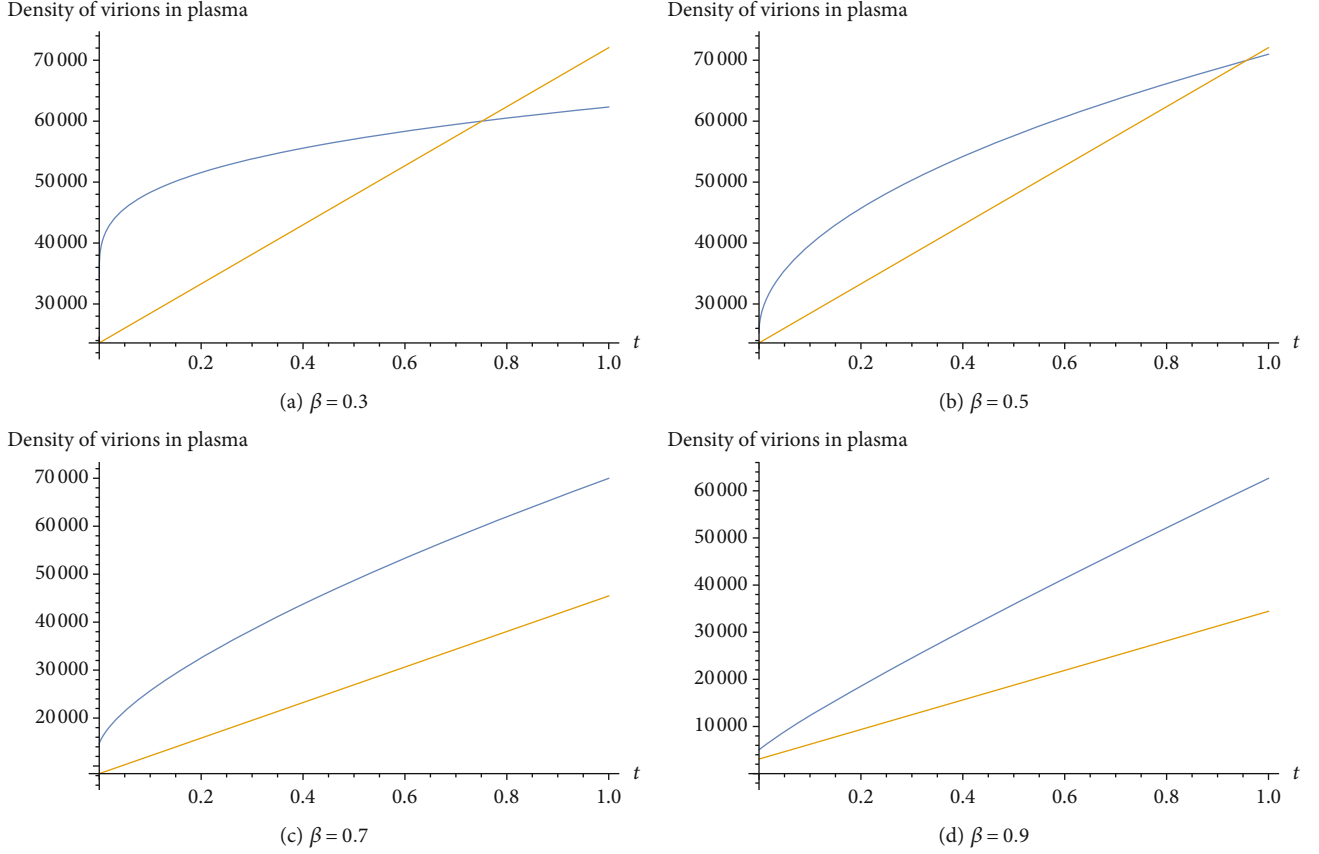


FIGURE 5: The comparison of the behavior of density of virions in plasma in Caputo-Fabrizio and Atangana Baleanu sense for different order fractional derivatives, where blue colour represents graph for ABC operator and orange represents graph for CF operator.

Substituting the value of  $g(\tau, \eta(\tau))$  in equation (43), we get

$$\begin{aligned} \eta_{k+1} = & \eta(0) + \frac{1-\beta}{N(\beta)} g(t_k, \eta(t_k)) + \frac{\beta}{N(\beta)\Gamma(\beta)} \sum_{i=0}^k \\ & \cdot \left( \frac{g(t_i, \eta(t_i))}{h} \int_{t_i}^{t_{i+1}} (t-t_{i-1})(t_{k+1}-t)^{\beta-1} dt \right. \\ & \left. - \frac{g(t_{i-1}, \eta(t_{i-1}))}{h} \int_{t_i}^{t_{i+1}} (t-t_{i-1})(t_{k+1}-t)^{\beta-1} dt \right). \end{aligned} \quad (45)$$

Substituting  $h = t_i - t_{i-1}$  and on solving, we get

$$\begin{aligned} \eta_{k+1} = & \eta_0 + \frac{1-\beta}{N(\beta)} g(t_k, \eta(t_k)) + \frac{\beta}{N(\beta)} \sum_{i=0}^k \\ & \cdot \left[ \frac{h^\beta g(t_i, \eta(t_i))}{\Gamma(\beta+2)} \left( (k+1-i)^\beta (k-i+2+\beta) \right. \right. \\ & \left. \left. - (k-i)^\beta (k-i+2+2\beta) \right) - \frac{h^\beta g(t_{i-1}, \eta(t_{i-1}))}{\Gamma(\beta+2)} \right. \\ & \left. \cdot \left( (k+1-i)^{\beta+1} - (k-i)^\beta (k-i+1+\beta) \right) \right]. \end{aligned} \quad (46)$$

Using the aforementioned concept of numerical scheme, the numerical scheme for the fractional model of HIV-1 infection in framework of Atangana-Baleanu derivative operator is given as

$$\begin{aligned} U_{k+1} = & U_0 + \frac{1-\beta}{N(\beta)} L_1(t_k, U(t_k)) \\ & + \frac{\beta}{N(\beta)} \sum_{i=0}^k \left[ \frac{h^\beta L_1(t_i, U(t_i))}{\Gamma(\beta+2)} \right. \\ & \cdot \left( (k+1-i)^\beta (k-i+2+\beta) \right. \\ & \left. \left. - (k-i)^\beta (k-i+2+2\beta) \right) \right. \\ & \left. - \frac{h^\beta L_1(t_{i-1}, U(t_{i-1}))}{\Gamma(\beta+2)} \right. \\ & \left. \cdot \left( (k+1-i)^{\beta+1} - (k-i)^\beta (k-i+1+\beta) \right) \right], \end{aligned} \quad (47)$$

where

$$L_1(t, U) = \varepsilon - dU - \alpha UV,$$



$$\begin{aligned}
I_{k+1} = & I_0 + \frac{1-\beta}{N(\beta)} L_2(t_k, I(t_k)) + \frac{\beta}{N(\beta)} \sum_{i=0}^k \\
& \cdot \left[ \frac{h^\beta L_2(t_i, I_i)}{\Gamma(\beta+2)} \left( (k+1-i)^\beta (k-i+2+\beta) \right. \right. \\
& \left. \left. - (k-i)^\beta (k-i+2+2\beta) \right) - \frac{h^\beta L_2(t_{i-1}, I(t_{i-1}))}{\Gamma(\beta+2)} \right. \\
& \left. \cdot \left( (k+1-i)^{\beta+1} - (k-i)^\beta (k-i+1+\beta) \right) \right], \quad (48)
\end{aligned}$$

where,

$$L_2(t, I) = \alpha UV - sI,$$

$$\begin{aligned}
V_{k+1} = & V_0 + \frac{1-\beta}{N(\beta)} L_3(t_k, V(t_k)) + \frac{\beta}{N(\beta)} \sum_{i=0}^k \\
& \cdot \left[ \frac{h^\beta L_3(t_i, V_i)}{\Gamma(\beta+2)} \left( (k+1-i)^\beta (k-i+2+\beta) \right. \right. \\
& \left. \left. - (k-i)^\beta (k-i+2+2\beta) \right) - \frac{h^\beta L_3(t_{i-1}, V(t_{i-1}))}{\Gamma(\beta+2)} \right. \\
& \left. \cdot \left( (k+1-i)^{\beta+1} - (k-i)^\beta (k-i+1+\beta) \right) \right], \quad (49)
\end{aligned}$$

where,

$$L_3(t, V) = cI - \delta V. \quad (50)$$

#### 4. Conclusion

The graphical representation shows the change in the number of uninfected CD4<sup>+</sup> T cells, infected CD4<sup>+</sup> T cells and the density of virions in plasma with respect to time. We have also inferred the effect of different fractional order derivatives to see the change in the number of uninfected CD4<sup>+</sup> T cells, infected CD4<sup>+</sup> T cells and the density of virions in plasma with respect to time. From Figures 1 and 2, we see that on increasing the value of  $\beta$  ( $\beta$  close to 1), the number of uninfected CD4<sup>+</sup> T cells increases well while the number of infected CD4<sup>+</sup> T cells and the density of virions in plasma decreases. In Figures 3–5, we have shown the graphical comparison between the Caputo-Fabrizio operator and the Atanagana-Baleanu derivative operator. We have seen that on decreasing the value of  $\beta$ , the number of uninfected CD4<sup>+</sup> T cells decreases well for the Atangana-Baleanu (ABC) operator as compared to the Caputo-Fabrizio (CF) operator while the number of infected CD4<sup>+</sup> T cells and the density of virions in plasma increases well for the ABC operator than the CF operator. This shows that ABC derivative operator provides better results than the CF derivative operator.

#### Data Availability

No data were used to support this study.

#### Conflicts of Interest

There is no conflict of interest.

#### Authors' Contributions

All authors contributed in writing the draft and the software, and all reviewed and approved the final version of the manuscript.

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