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Research Article

A New Family of Degenerate Poly-Genocchi Polynomials with Its Certain Properties

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In this paper, we introduce a new type of degenerate Genocchi polynomials and numbers, which are called degenerate poly-Genocchi polynomials and numbers, by using the degenerate polylogarithm function, and we derive several properties of these polynomials systematically. Then, we also consider the degenerate unipoly-Genocchi polynomials attached to an arithmetic function, by using the degenerate polylogarithm function, and investigate some identities of those polynomials. In particular, we give some new explicit expressions and identities of degenerate unipoly polynomials related to special numbers and polynomials.

1. Introduction

In [1, 2], Carlitz initiated a study of degenerate versions of some special polynomials and numbers, namely, the degenerate Bernoulli and Euler polynomials and numbers. Kim et al. [3–5] have studied the degenerate versions of special numbers and polynomials actively. These ideas provide a powerful tool in order to define special numbers and polynomials of their degenerate versions. The notion of degenerate version forms a special class of polynomials because of their great applicability. Despite the applicability of special functions in classical analysis and statistics, they also arise in communication systems, quantum mechanics, nonlinear wave propagation, electric circuit theory, electromagnetic theory, etc. In particular, Genocchi numbers have been extensively studied in many different contexts in such branches of mathematics as, for instance, elementary number theory, complex analytic number theory, differential topology (differential structures on spheres), theory of modular forms (Eisenstein series), p-adic analytic number theory (p-adic L-functions), and quantum physics (quantum groups). The works of Genocchi numbers and their combinatorial relations have received much attention [6–11]. In the paper, we focus on a new type of degenerate poly-Genocchi polynomial and numbers.

The aim of this paper is to introduce a degenerate version of the poly-Genocchi polynomials and numbers, the so-called new type of degenerate poly-Genocchi polynomials and numbers, constructing from the degenerate polylogarithm function. We derive some explicit expressions and identities for those numbers and polynomials.

The classical Euler polynomials $E_n(x)$ and the classical Genocchi polynomials $G_n(x)$ are, respectively, defined by the following generating functions (see [12–22]):

$$\frac{2}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}E_n(x)\frac{t^n}{n!},\quad |t|<\pi, \tag{1}$$

$$\frac{2t}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}G_n(x)\frac{t^n}{n!},\quad |t|<\pi. \tag{2}$$

In the case when $x=0$, $E_n(0):=E_n$ and $G_n(0):=G_n$ are, respectively, called the Euler numbers and Genocchi numbers.

The degenerate exponential function [23, 24] is defined by

$$\begin{aligned} e_\lambda^x(t)&=(1+\lambda t)^{x/\lambda},\\ e_\lambda^1(t)&=e_\lambda(t)\ (\lambda\in\mathbb{R}). \end{aligned} \tag{3}$$

Note that

$$\lim_{\lambda\longrightarrow 0}(1+\lambda t)^{x/\lambda}=\sum_{n=0}^{\infty}\frac{x^n t^n}{n!}=e^{xt}. \tag{4}$$

In [1, 2], Carlitz introduced the degenerate Bernoulli and degenerate Euler polynomials defined by

$$\frac{t}{e_\lambda(t)-1}e_\lambda^x(t)=\frac{t}{(1+\lambda t)^{1/\lambda}-1}(1+\lambda t)^{x/\lambda}=\sum_{n=0}^{\infty}\beta_n(x;\lambda)\frac{t^n}{n!}, \tag{5}$$

$$\frac{2}{e_\lambda(t)+1}e_\lambda^x(t)=\frac{2}{(1+\lambda t)^{1/\lambda}-1}(1+\lambda t)^{x/\lambda}=\sum_{n=0}^{\infty}\mathfrak{E}_n(x;\lambda)\frac{t^n}{n!}. \tag{6}$$

In the case when $x=0$, $B_{n,\lambda}(0):=B_{n,\lambda}$ are called the degenerate Bernoulli numbers and $E_{n,\lambda}(0):=E_{n,\lambda}$ are called the degenerate Euler numbers.

Let $(x)_{n,\lambda}$ be the degenerate falling factorial sequence given by

$$(x)_{n,\lambda}:=x(x-\lambda)\cdots(x-(n-1)\lambda)\ (n\geq 1), \tag{7}$$

with the assumption $(x)_{0,\lambda}=1$.

In [5], Kim et al. considered the degenerate Genocchi polynomials given by

$$\frac{2t}{e_\lambda(t)+1}e_\lambda^x(t)=\sum_{n=0}^{\infty}G_{n,\lambda}(x)\frac{t^n}{n!}. \tag{8}$$

In the case when $x=0$, $G_{n,\lambda}:=G_{n,\lambda}(0)$ are called the degenerate Genocchi numbers.

For $k\in\mathbb{Z}$, the polylogarithm function is defined by a power series in t, which is also a Dirichlet series in k (see [25, 26]):

$$\mathrm{Li}_k(t)=\sum_{n=1}^{\infty}\frac{t^n}{n^k}=t+\frac{t^2}{2^k}+\frac{t^3}{3^k}+\cdots(|t|<1). \tag{9}$$

This definition is valid for arbitrary complex order k and for all complex arguments t with $|t|<1$: it can be extended to $|t|\geq 1$ by analytic continuation.

It is noticed that

$$\mathrm{Li}_1(t)=\sum_{n=1}^{\infty}\frac{t^n}{n}=-\log(1-t). \tag{10}$$

For $\lambda\in\mathbb{R}$, Kim and Kim [3] defined the degenerate version of the logarithm function, denoted by $\log_\lambda(1+t)$, as follows (see [4]):

$$\log_\lambda(1+t)=\sum_{n=1}^{\infty}\lambda^{n-1}(1)_{n,1/\lambda}\frac{t^n}{n!}, \tag{11}$$

being the inverse of the degenerate version of the exponential function $e_\lambda(t)$ as has been shown below:

$$e_\lambda(\log_\lambda(t))=\log_\lambda(e_\lambda(t))=t. \tag{12}$$

It is noteworthy to mention that

$$\lim_{\lambda\longrightarrow 0}\log_\lambda(1+t)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{t^n}{n!}=\log(1+t). \tag{13}$$

The degenerate polylogarithm function [3] is defined by Kim and Kim to be

$$l_{k,\lambda}(x)=\sum_{n=1}^{\infty}\frac{(-\lambda)^{n-1}(1)_{n,1/\lambda}}{(n-1)!n^k}x^n\ (k\in\mathbb{Z},|x|<1). \tag{14}$$

It is clear that (see [27, 28])

$$\lim_{\lambda\longrightarrow 0}l_{k,\lambda}(x)=\sum_{n=1}^{\infty}\frac{x^n}{n^k}=\mathrm{Li}_k(x). \tag{15}$$

From (11) and (14), we get

$$l_{1,\lambda}(x)=\sum_{n=1}^{\infty}\frac{(-\lambda)^{n-1}(1)_{n,1/\lambda}}{n!}x^n=-\log_\lambda(1-x). \tag{16}$$

Very recently, Kim and Kim [3] introduced the new type of degenerate version of the Bernoulli polynomials and numbers, by using the degenerate polylogarithm function as follows:

$$\frac{l_{k,\lambda}(1-e_\lambda(-t))}{1-e_\lambda(-t)}e_\lambda^x(t)=\sum_{n=0}^{\infty}\beta_{n,\lambda}^{(k)}(x)\frac{t^n}{n!}. \tag{17}$$

When $x=0$, $\beta_{j,\lambda}^{(k)}:=\beta_{j,\lambda}^{(k)}(0)$ are called the new type of degenerate poly-Bernoulli numbers.

The degenerate Stirling numbers of the first kind [24] are defined by

$$\frac{1}{k!}(\log_\lambda(1+t))^k=\sum_{n=k}^{\infty}S_{1,\lambda}(n,k)\frac{t^n}{n!}\ (k\geq 0). \tag{18}$$

It is clear that

$$\lim_{\lambda \longrightarrow 0} S_{1,\lambda}(n,k) \coloneqq S_1(n,k), \tag{19}$$

calling the Stirling numbers of the first kind given by (see [29, 30])

$$\frac{1}{k!}(\log(1+t))^k = \sum_{n=k}^{\infty} S_1(n,k)\frac{t^n}{n!}\ (k \geq 0). \tag{20}$$

The degenerate Stirling numbers of the second kind [31] are given by (see [2, 13–22, 25–32])

$$\frac{1}{k!}(e_\lambda(t)-1)^k = \sum_{n=k}^{\infty} S_{2,\lambda}(j,k)\frac{t^n}{n!}\ (k \geq 0). \tag{21}$$

Note here that

$$\lim_{\lambda \longrightarrow 0} S_{2,\lambda}(n,k) \coloneqq S_2(n,k), \tag{22}$$

standing for the Stirling numbers of the second kind given by means of the following generating function (see [1–8, 12–38]):

$$\frac{1}{k!}\left(e^t-1\right)^k = \sum_{n=k}^{\infty} S_2(n,k)\frac{t^n}{n!}\ (k \geq 0). \tag{23}$$

This paper is organized as follows. In Section 1, we recall some necessary stuffs that are needed throughout this paper. These include the degenerate exponential functions, the degenerate Genocchi polynomials, the degenerate Euler polynomials, and the degenerate Stirling numbers of the first and second kinds. In Section 2, we introduce the new type of degenerate poly-Genocchi polynomials by making use of the degenerate polylogarithm function. We express those polynomials in terms of the degenerate Genocchi polynomials and the degenerate Stirling numbers of the first kind and also of the degenerate Euler polynomials and the Stirling numbers of the first kind. We represent the generating function of the degenerate poly-Genocchi numbers by iterated integrals from which we obtain an expression of those numbers in terms of the degenerate Bernoulli numbers of the second kind. In Section 3, we introduce the new type of degenerate unipoly-Genocchi polynomials by making use of the degenerate polylogarithm function. We express those polynomials in terms of the degenerate Genocchi polynomials and the degenerate Stirling numbers of the first kind and also of the degenerate Euler polynomials and the Stirling numbers of the first kind and second kind.

2. New Type of Degenerate Genocchi Numbers and Polynomials

In this section, we define the new type of degenerate Genocchi numbers and polynomials by using the degenerate polylogarithm function which is called the degenerate poly-Genocchi polynomials as follows.

For $k \in \mathbb{Z}$, we define the new type of degenerate Genocchi numbers, which are called the degenerate poly-Genocchi numbers, as

$$\frac{2}{e_\lambda(t)+1} l_{k,\lambda}(1-e_\lambda(-t)) = \sum_{n=0}^{\infty} G_{n,\lambda}^{(k)}\frac{t^n}{n!}. \tag{24}$$

Note that

$$\sum_{n=0}^{\infty} G_{n,\lambda}^{(1)}\frac{t^n}{n!} = \frac{2}{e_\lambda(t)+1} l_{1,\lambda}(1-e_\lambda(-t)) = \frac{2t}{e_\lambda(t)+1} = \sum_{n=0}^{\infty} G_{n,\lambda}\frac{t^n}{n!}. \tag{25}$$

Thus, we have (see [6])

$$G_{n,\lambda}^{(1)} = G_{n,\lambda}\ (n \geq 0). \tag{26}$$

Now, we consider the new type of degenerate Genocchi polynomials which are called the degenerate poly-Genocchi polynomials defined by

$$\frac{2l_{k,\lambda}(1-e_\lambda(-t))}{e_\lambda(t)+1} e_\lambda^x(t) = \sum_{n=0}^{\infty} G_{n,\lambda}^{(k)}(x)\frac{t^n}{n!}. \tag{27}$$

In the case when $x=0$, $G_{n,\lambda}^{(k)} \coloneqq G_{n,\lambda}^{(k)}(0)$. Using equation (27), we see

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,\lambda}^{(k)}(x)\frac{t^n}{n!} &= \frac{2l_{k,\lambda}(1-e_\lambda(-t))}{e_\lambda(t)+1} e_\lambda^x(t)\\ &= \sum_{m=0}^{\infty} G_{m,\lambda}^{(k)}\frac{t^m}{m!}\sum_{n=0}^{\infty}(x)_{n,\lambda}\frac{t^n}{n!}\\ &= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}(n/m)G_{m,\lambda}^{(k)}(x)_{n-m,\lambda}\right)\frac{t^n}{n!}.\end{aligned} \tag{28}$$

Therefore, by equation (28), we obtain the following theorem.

Theorem 1. *Let n be a nonnegative integer. Then,*

$$G_{n,\lambda}^{(k)}(x) = \sum_{m=0}^{n}(n/m)G_{m,\lambda}^{(k)}(x)_{n-m,\lambda}. \tag{29}$$

From (27), we note that

$$\sum_{n=0}^{\infty} G_{n,\lambda}^{(k)}(x)\frac{t^n}{n!} = \frac{2t}{e_\lambda(t)+1} e_\lambda^x(t)\frac{1}{t} l_{k,\lambda}(1-e_\lambda(-t)), \tag{30}$$

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,\lambda}^{(k)}(x)\frac{t^n}{n!} &= \frac{2t}{e_\lambda(t)+1}e_\lambda^x(t)\frac{1}{t}\sum_{m=1}^{\infty}\frac{(-\lambda)^{m-1}(1)_{m,1/\lambda}}{(m-1)!m^k}(1-e_\lambda(-t))^m \\ &= \frac{2t}{e_\lambda(t)+1}e_\lambda^x(t)\frac{1}{t}\sum_{m=1}^{\infty}\frac{(-\lambda)^{m-1}(1)_{m,1/\lambda}}{m^{k-1}} \\ &\quad \cdot \sum_{l=m}^{\infty}(-1)^{l-m}S_{2,\lambda}(l,m)\frac{t^l}{l!} = \frac{2t}{e_\lambda(t)+1}e_\lambda^x(t)\frac{1}{t} \\ &\quad \cdot \sum_{l=1}^{\infty}\left(\sum_{m=1}^{l}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}}{m^{k-1}}S_{2,\lambda}(l,m)\right)\frac{t^l}{l!} \\ &= \left(\sum_{n=0}^{\infty}G_{n,\lambda}(x)\frac{t^n}{n!}\right)\frac{1}{t}\left(\sum_{l=0}^{\infty}\left(\sum_{m=1}^{l+1}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l}{m^{k-1}}S_{2,\lambda}(l+1,m)\right)\frac{t^{l+1}}{(l+1)!}\right) \\ &= \left(\sum_{n=0}^{\infty}G_{n,\lambda}(x)\frac{t^n}{n!}\right)\left(\sum_{l=0}^{\infty}\left(\sum_{m=1}^{l+1}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l}{m^{k-1}}\frac{S_{2,\lambda}(l+1,m)}{l+1}\right)\frac{t^l}{l!}\right) \\ &= \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}\binom{n}{l}\sum_{m=1}^{l+1}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l}{m^{k-1}}\frac{S_{2,\lambda}(l+1,m)}{l+1}G_{n-l,\lambda}(x)\right)\frac{t^n}{n!}.\end{aligned} \tag{31}$$

Therefore, by equations (30) and (31), we get the following theorem.

Theorem 2. *Let n be a nonnegative integer. Then,*

$$G_{n,\lambda}^{(k)}(x) = \sum_{l=0}^{n}\binom{n}{l}\sum_{m=1}^{l+1}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l}{m^{k-1}}\frac{S_{2,\lambda}(l+1,m)}{l+1}G_{n-l,\lambda}(x). \tag{32}$$

Using equations (27) and (6), we see

$$\sum_{n=0}^{\infty}G_{n,\lambda}^{(k)}(x)\frac{t^n}{n!} = \frac{2}{e_\lambda(t)+1}e_\lambda^x(t)l_{k,\lambda}(1-e_\lambda(-t)), \tag{33}$$

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,\lambda}^{(k)}(x)\frac{t^n}{n!} &= \frac{2}{e_\lambda(t)+1}e_\lambda^x(t)\sum_{m=1}^{\infty}\frac{(-\lambda)^{m-1}(1)_{m,1/\lambda}}{(m-1)!m^k}(1-e_\lambda(-t))^m \\ &= \frac{2}{e_\lambda(t)+1}e_\lambda^x(t)\sum_{m=1}^{\infty}\frac{(-\lambda)^{m-1}(1)_{m,1/\lambda}}{m^{k-1}} \\ &\quad \cdot \sum_{l=m}^{\infty}(-1)^{l-m}S_{2,\lambda}(l,m)\frac{t^l}{l!} = \frac{2}{e_\lambda(t)+1}e_\lambda^x(t) \\ &\quad \cdot \sum_{l=1}^{\infty}\left(\sum_{m=1}^{l}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}}{m^{k-1}}S_{2,\lambda}(l,m)\right)\frac{t^l}{l!} \\ &= \left(\sum_{n=0}^{\infty}E_{n,\lambda}(x)\frac{t^n}{n!}\right)\left(\sum_{l=1}^{\infty}\left(\sum_{m=1}^{l}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}}{m^{k-1}}S_{2,\lambda}(l,m)\right)\frac{t^l}{l!}\right) \\ &= \sum_{n=1}^{\infty}\left(\sum_{l=1}^{n}\binom{n}{l}\sum_{m=1}^{l}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}}{m^{k-1}}S_{2,\lambda}(l,m)E_{n-l,\lambda}(x)\right)\frac{t^n}{n!}.\end{aligned} \tag{34}$$

By equations (33) and (34), we obtain the following theorem.

Theorem 3. *Let n be a nonnegative integer. Then,*

$$G_{n,\lambda}^{(k)}(x) = \sum_{l=1}^{n}\binom{n}{l}\sum_{m=1}^{l}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}}{m^{k-1}}S_{2,\lambda}(l,m)E_{n-l,\lambda}(x). \tag{35}$$

From (27), we have

$$\begin{aligned}\sum_{n=0}^{\infty}G_{n,\lambda}^{(k)}\frac{x^n}{n!} &= \frac{2}{e_\lambda(x)+1}l_{k,\lambda}(1-e_\lambda(-x)) = \frac{2}{e_\lambda(x)+1} \\ &\quad \cdot \underbrace{\int_0^x\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}\int_0^t\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}\cdots\int_0^t\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}tdtdt\cdots dt}_{(k-2)-times}\sum_{n=0}^{\infty}G_{n,\lambda}^{(k)}\frac{x^n}{n!} \\ &= \frac{2}{e_\lambda(x)+1}\underbrace{\int_0^x\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}\int_0^t\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}\cdots\int_0^t\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}tdtdt\cdots dt}_{(k-2)-times}.\end{aligned} \tag{36}$$

For $k=2$ in (36) and using [3] (Eq. (27)), we get

$$\begin{aligned}\sum_{n=0}^{\infty}G_{n,\lambda}^{(2)}\frac{x^n}{n!} &= \frac{2}{e_\lambda(x)+1}\int_0^x\frac{t}{1-e_\lambda(-t)}e_\lambda^{1-\lambda}(-t)dt \\ &= \frac{2}{e_\lambda(x)+1}\int_0^x\sum_{j=0}^{\infty}\beta_{j,\lambda}(1-\lambda)(-1)^j\frac{t^j}{j!}dt \\ &= \frac{2x}{e_\lambda(x)+1}\sum_{j=0}^{\infty}\frac{\beta_{j,\lambda}(1-\lambda)}{j+1}(-1)^j\frac{x^j}{j!} \\ &= \sum_{n=0}^{\infty}G_{n,\lambda}\frac{x^n}{n!}\sum_{j=0}^{\infty}\frac{\beta_{j,\lambda}(1-\lambda)}{j+1}(-1)^j\frac{x^j}{j!} \\ &= \sum_{n=0}^{\infty}\left(\sum_{j=0}^{n}(n/j)(-1)^jG_{n-j,\lambda}\frac{\beta_{j,\lambda}(1-\lambda)}{j+1}\right)\frac{x^n}{n!}.\end{aligned} \tag{37}$$

Therefore, by equation (37), we get the following theorem.

Theorem 4. *Let n be a nonnegative integer. Then,*

$$G_{n,\lambda}^{(k)}(x) = \sum_{m=0}^{n}(n/m)G_{m,\lambda}^{(k)}(x)_{n-m,\lambda}. \tag{38}$$

In general, by equation (37), we see

$$\begin{aligned}\sum_{n=0}^{\infty}G_{n,\lambda}^{(k)}\frac{x^n}{n!} &= \frac{2}{e_\lambda(x)+1}\int_0^x\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}\int_0^t\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}\cdots \\ &\quad \cdot\int_0^t\frac{e_\lambda^{1-\lambda}(-t)}{1-e_\lambda(-t)}tdtdt\cdots dt \\ &= \sum_{n_1,n_2,\cdots,n_{k-1}=n}^{\infty}\frac{1}{n_1!n_2!\cdots n_{k-1}!}\frac{\beta_{n_1,\lambda}(1-\lambda)}{n_1+1}\frac{\beta_{n_2,\lambda}(1-\lambda)}{n_1+n_2+1} \\ &\quad \times\cdots\frac{\beta_{n_{k-1},\lambda}(1-\lambda)}{n_1+\cdots+n_{k-1}+1}(-x)^{n_1,n_2,\cdots,n_{k-1}}\frac{2x}{e_\lambda(x)+1} \\ &= \sum_{n=0}^{\infty}(-1)^n\sum_{n_1,n_2,\cdots,n_k=n}\binom{n}{n_1,n_2,\cdots,n_k}\frac{\beta_{n_1,\lambda}(1-\lambda)}{n_1+1} \\ &\quad \cdot\frac{\beta_{n_2,\lambda}(1-\lambda)}{n_1+n_2+1}\cdots\frac{\beta_{n_{k-1},\lambda}(1-\lambda)}{n_1+\cdots+n_{k-1}+1}G_{n,\lambda}\frac{x^n}{n!}.\end{aligned} \tag{39}$$

By equation (39), we obtain the following theorem.

Theorem 5. *Let $k \in \mathbb{Z}$ and $n \geq 0$, we have*

$$G_{n,\lambda}^{(k)} = (-1)^n \sum_{n_1,n_2,\cdots,n_k=n} \binom{n}{n_1, n_2, \cdots, n_k} \frac{\beta_{n_1,\lambda}(1-\lambda)}{n_1+1} \cdot \frac{\beta_{n_2,\lambda}(1-\lambda)}{n_1+n_2+1} \cdots \frac{\beta_{n_{k-1},\lambda}(1-\lambda)}{n_1+\cdots+n_{k-1}+1} G_{n,\lambda}. \tag{40}$$

From (27), we observe that

$$\begin{aligned} 2l_{k,l}(1-e_\lambda(-t)) &= (1+e_\lambda(t)) \sum_{m=0}^{\infty} G_{m,\lambda}^{(k)} \frac{t^m}{m!} \\ &= \sum_{j=1}^{\infty} \left(G_{j,\lambda}^{(k)} + G_{j,\lambda}^{(k)}(1) \right) \frac{t^j}{j!}. \end{aligned} \tag{41}$$

On the other hand,

$$\begin{aligned} 2l_{k,l}(1-e_\lambda(-t)) &= 2\sum_{r=1}^{\infty} \frac{(-\lambda)^{r-1}(1)_{r,1/\lambda}}{(r-1)!r^k} (1-e_\lambda(-t))^r \\ &= 2\sum_{m=1}^{\infty} \frac{(-\lambda)^{m-1}(1)_{m,1/\lambda}}{m^{k-1}} \frac{1}{m!} (1-e_\lambda(-t))^m \\ &= 2\sum_{r=1}^{\infty} \frac{(-\lambda)^{r-1}(1)_{r,1/\lambda}}{r^{k-1}} \sum_{j=r}^{\infty} S_{2,\lambda}(j,r)(-1)^{j-r} \frac{t^j}{j!} \\ &= 2\sum_{j=1}^{\infty} \left(\sum_{r=1}^{j} \frac{(-1)^{j-1}(1)_{r,1/\lambda}}{r^{k-1}} \lambda^{r-1} S_{2,\lambda}(j,r) \right) \frac{t^j}{j!}. \end{aligned} \tag{42}$$

Therefore, by equations (41) and (42), we get the following theorem.

Theorem 6. *Let $k \in \mathbb{Z}$ and $j \geq 1$. Then,*

$$\frac{1}{2}\left[G_{j,\lambda}^{(k)} + G_{j,\lambda}^{(k)}(1) \right] = (-1)^{j-1} \sum_{r=1}^{j} \frac{(1)_{r,1/\lambda}}{r^{k-1}} \lambda^{r-1} S_{2,\lambda}(j,r). \tag{43}$$

From equations (27) and (14), we see

$$\begin{aligned} 2t = 2l_{1,l}(1-e_\lambda(-t)) &= 2\sum_{m=1}^{\infty} \frac{(-\lambda)^{m-1}(1)_{m,1/\lambda}}{(m-1)!m^k} (1-e_\lambda(-t))^m \\ &= 2\sum_{m=1}^{\infty} \frac{(-\lambda)^{m-1}(1)_{m,1/\lambda}}{m!} (1-e_\lambda(-t))^m \\ &= 2\sum_{m=1}^{\infty} (-\lambda)^{m-1}(1)_{m,1/\lambda} \sum_{n=m}^{\infty} S_{2,\lambda}(n,m)(-1)^{n-m} \frac{t^n}{n!} \\ &= 2\sum_{n=1}^{\infty} \left(\sum_{m=1}^{n} (-1)^{n-1}(1)_{m,1/\lambda} \lambda^{m-1} S_{2,\lambda}(n,m) \right) \frac{t^n}{n!}. \end{aligned} \tag{44}$$

By comparing the coefficients on both sides of (44), we obtain the following theorem.

Theorem 7. *For $n \in \mathbb{N}$, we have*

$$\sum_{m=1}^{n} (-1)^{n-1}(1)_{m,\frac{1}{\lambda}} \lambda^{m-1} S_{2,\lambda}(n,m) = \delta_{n,1}, \tag{45}$$

where $\delta_{n,k}$ is Kronecker's symbol.

Note that

$$\lim_{\lambda \longrightarrow 0} G_{n,\lambda}^{(1)} = G_n, \quad \lim_{\lambda \longrightarrow 0} G_{n,\lambda}^{(1)}(x) = G_n(x). \tag{46}$$

3. Degenerate Unipoly-Genocchi Numbers and Polynomials

Let p be any arithmetic function which is a real or complex valued function defined on the set of positive integers $\mathbb{N}$. Kim and Kim [29] defined the unipoly function attached to polynomials $p(x)$ by

$$u_k(x \mid p) = \sum_{n=1}^{\infty} \frac{p(n)}{n^k} x^n \ (k \in \mathbb{Z}). \tag{47}$$

Moreover (see [25]),

$$u_k(x \mid 1) = \sum_{n=1}^{\infty} \frac{x^n}{n^k} = \mathrm{Li}_k(x) \tag{48}$$

is the ordinary polylogarithm function.

In [8], Lee and Kim defined the degenerate unipoly function attached to polynomials $p(x)$ as follows:

$$u_{k,\lambda}(x \mid p) = \sum_{i=1}^{\infty} p(i) \frac{(-\lambda)^{i-1}(1)_{i,1/\lambda}}{i^k} x^i. \tag{49}$$

It is worthy to note that

$$u_{k,\lambda}\left(x \mid \frac{1}{\Gamma} \right) = l_{k,\lambda}(x) \tag{50}$$

is the degenerate polylogarithm function.

Now, we define the degenerate unipoly-Genocchi polynomials attached to polynomials $p(x)$ by

$$\frac{2u_{k,\lambda}(1-e_\lambda(-t) \mid p)}{e_\lambda(t)+1} e_\lambda^x(t) = \sum_{n=0}^{\infty} G_{n,\lambda,p}^{(k)}(x) \frac{t^n}{n!}. \tag{51}$$

In the case when $x = 0$, $G_{n,\lambda,p}^{(k)} \coloneqq G_{n,\lambda,p}^{(k)}(0)$ are called the degenerate unipoly-Genocchi numbers attached to p.

From (51), we see

$$\begin{aligned}\sum_{n=0}^{\infty} G^{(k)}_{n,\lambda,1/\Gamma}\frac{t^n}{n!} &= \frac{2}{e_\lambda(t)+1}u_{k,\lambda}\left(1-e_\lambda(-t)\mid\frac{1}{\Gamma}\right)\\ &= \frac{2}{e_\lambda(t)+1}\sum_{r=1}^{\infty}\frac{(-\lambda)^{r-1}(1)_{r,1/\lambda}(1-e_\lambda(-t))^r}{r^k(r-1)!}\\ &= \frac{2}{e_\lambda(t)+1}l_{k,\lambda}(1-e_\lambda(-t)) = \sum_{n=0}^{\infty}G^{(k)}_{n,\lambda}\frac{t^n}{n!}.\end{aligned} \tag{52}$$

Thus, by (52), we have

$$G^{(k)}_{n,\lambda,\frac{1}{\Gamma}} = G^{(k)}_{n,\lambda}. \tag{53}$$

From (51), we have

$$\begin{aligned}\sum_{n=0}^{\infty} G^{(k)}_{n,\lambda,p}(x)\frac{t^n}{n!} &= \frac{2e^x_\lambda(t)}{e_\lambda(t)+1}u_{k,\lambda}(1-e_\lambda(-t)\mid p)\\ &= \frac{2e^x_\lambda(t)}{e_\lambda(t)+1}\frac{1}{t}\sum_{m=1}^{\infty}\frac{p(m)(-\lambda)^{m-1}(1)_{m,1/\lambda}}{m^k}(1-e_\lambda(-t))^m\\ &= \frac{2t}{e_\lambda(t)+1}e^x_\lambda(t)\frac{1}{t}\sum_{m=1}^{\infty}\frac{p(m)(-\lambda)^{m-1}(1)_{m,1/\lambda}m!}{m^k}\\ &\quad\cdot\sum_{l=m}^{\infty}(-1)^{l-m}S_{2,\lambda}(l,m)\frac{t^l}{l!} = \frac{2t}{e_\lambda(t)+1}e^x_\lambda(t)\frac{1}{t}\sum_{l=1}^{\infty}\\ &\quad\cdot\left(\sum_{m=1}^{l}\frac{\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}m!}{m^k}S_{2,\lambda}(l,m)\right)\frac{t^l}{l!} = \left(\sum_{n=0}^{\infty}G_{n,\lambda}(x)\frac{t^n}{n!}\right)\frac{1}{t}\\ &\quad\cdot\left(\sum_{l=0}^{\infty}\left(\sum_{m=1}^{l+1}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{m^k}S_{2,\lambda}(l+1,m)\right)\frac{t^{l+1}}{(l+1)!}\right)\\ &= \left(\sum_{n=0}^{\infty}G_{n,\lambda}(x)\frac{t^n}{n!}\right)\left(\sum_{l=0}^{\infty}\left(\sum_{m=1}^{l+1}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{m^k}\frac{S_{2,\lambda}(l+1,m)}{l+1}\right)\frac{t^l}{l!}\right)\\ &= \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}\binom{n}{l}\sum_{m=1}^{l+1}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{m^k}\frac{S_{2,\lambda}(l+1,m)}{l+1}G_{n-l,\lambda}(x)\right)\frac{t^n}{n!}.\end{aligned} \tag{54}$$

Therefore, by equation (54), we get the following theorem.

Theorem 8. *Let n be a nonnegative integer. Then,*

$$G^{(k)}_{n,\lambda,p}(x) = \sum_{l=0}^{n}\binom{n}{l}\sum_{m=1}^{l+1}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{m^k}\frac{S_{2,\lambda}(l+1,m)}{l+1}G_{n-l,\lambda}(x). \tag{55}$$

Using equations (49) and (51), we see

$$\begin{aligned}\sum_{n=0}^{\infty} G^{(k)}_{n,\lambda}(x)\frac{t^n}{n!} &= \frac{2}{e_\lambda(t)+1}e^x_\lambda(t)\sum_{m=1}^{\infty}\frac{p(m)(-\lambda)^{m-1}(1)_{m,1/\lambda}}{m^k}(1-e_\lambda(-t))^m\\ &= \frac{2}{e_\lambda(t)+1}e^x_\lambda(t)\sum_{m=1}^{\infty}\frac{p(m)(-\lambda)^{m-1}(1)_{m,1/\lambda}m!}{m^k}\\ &\quad\cdot\sum_{l=m}^{\infty}(-1)^{l-m}S_{2,\lambda}(l,m)\frac{t^l}{l!} = \frac{2}{e_\lambda(t)+1}e^x_\lambda(t)\\ &\quad\cdot\sum_{l=1}^{\infty}\left(\sum_{m=1}^{l}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}m!}{m^k}S_{2,\lambda}(l,m)\right)\frac{t^l}{l!}\\ &= \left(\sum_{n=0}^{\infty}E_{n,\lambda}(x)\frac{t^n}{n!}\right)\left(\sum_{l=1}^{\infty}\left(\sum_{m=1}^{l}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}m!}{m^k}S_{2,\lambda}(l,m)\right)\frac{t^l}{l!}\right)\\ &= \sum_{n=1}^{\infty}\left(\sum_{l=1}^{n}\binom{j}{l}\sum_{m=1}^{l}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}m!}{m^k}S_{2,\lambda}(l,m)E_{n-l,\lambda}(x)\right)\frac{t^n}{n!}.\end{aligned} \tag{56}$$

By, equations (51) and (56), we obtain the following theorem.

Theorem 9. *Let n be a nonnegative integer. Then,*

$$G^{(k)}_{n,\lambda,p}(x) = \sum_{l=1}^{n}\binom{n}{l}\sum_{m=1}^{l}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^{l-1}m!}{m^k}S_{2,\lambda}(l,m)E_{n-l,\lambda}(x). \tag{57}$$

From (6), (49), and (51), we get

$$\begin{aligned}\sum_{n=0}^{\infty} G^{(k)}_{n,\lambda,p}(x)\frac{t^n}{n!} &= \frac{2e^x_\lambda(t)}{e_\lambda(t)+1}u_{k,\lambda}(1-e_\lambda(-t)\mid p) = \frac{2t}{e_\lambda(t)+1}\frac{e_\lambda(t)-1}{e_\lambda(t)-1}e^x_\lambda(t)\frac{1}{t}\\ &\quad\cdot\sum_{m=1}^{\infty}\frac{p(m)(-\lambda)^{m-1}(1)_{m,1/\lambda}}{m^k}(1-e_\lambda(-t))^m\\ &= \frac{2te^x_\lambda(t)}{e^2_\lambda(t)-1}(e_\lambda(t)-1)\frac{1}{t}\sum_{m=1}^{\infty}\frac{p(m)(-\lambda)^{m-1}(1)_{m,1/\lambda}}{m^k}(1-e_\lambda(-t))^m\\ &= \frac{2te^{x/2}_{\lambda/2}(2t)}{e_{\lambda/2}(2t)-1}(e_\lambda(t)-1)\sum_{l=0}^{\infty}\sum_{m=1}^{l+1}\\ &\quad\cdot\frac{p(m)(-1)^l(\lambda)^{m-1}(1)_{m,1/\lambda}m!}{m^k}S_{2,\lambda}(l+1,m)\\ &\quad\cdot\frac{t^l}{(l+1)!} = \left(\sum_{n=0}^{\infty}\beta_{n,\lambda/2}\left(\frac{x}{2}\right)\frac{2^nt^n}{n!}\right)\left(\sum_{i=1}^{\infty}(1)_{i,\lambda}\frac{t^i}{i!}\right)\\ &\quad\cdot\left(\sum_{l=0}^{\infty}\left(\sum_{m=1}^{l+1}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{m^k}\frac{S_{2,\lambda}(l+1,m)}{l+1}\right)\frac{t^l}{l!}\right)\\ &= \left(\sum_{n=0}^{\infty}\beta_{n,\lambda/2}\left(\frac{x}{2}\right)\frac{2^nt^n}{n!}\right)\left(\sum_{i=0}^{\infty}\frac{(1)_{i+1,\lambda}}{i+1}\frac{t^i}{i!}\right)\\ &\quad\cdot\left(\sum_{l=0}^{\infty}\left(\sum_{m=1}^{l+1}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{m^k}\frac{S_{2,\lambda}(l+1,m)}{l+1}\right)\frac{t^l}{l!}\right)\\ &\quad\cdot\left(\sum_{n=0}^{\infty}\beta_{n,\lambda/2}\left(\frac{x}{2}\right)\frac{2^nt^n}{n!}\right)\left(\sum_{i=0}^{\infty}\left(\sum_{l=0}^{i}\binom{i}{l}\frac{(1)_{i-l+1,\lambda}}{i-l+1}\right.\right.\\ &\quad\left.\left.\cdot\sum_{m=1}^{l+1}\frac{p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{m^k}\frac{S_{2,\lambda}(l+1,m)}{l+1}\right)\frac{t^i}{i!}\right)\\ &= \sum_{n=0}^{\infty}\left(\sum_{i=0}^{n}\sum_{l=0}^{i}\binom{n}{i}\binom{i}{l}\sum_{m=1}^{l+1}\right.\\ &\quad\cdot\frac{(1)_{i-l+1,\lambda}p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{(i-l+1)m^k}\frac{S_{2,\lambda}(l+1,m)}{l+1}\\ &\quad\left.\cdot 2^{n-i}\beta_{n-i,\lambda/2}\left(\frac{x}{2}\right)\right)\frac{t^n}{n!}.\end{aligned} \tag{58}$$

Therefore, by (58), we obtain the following theorem.

Theorem 10. *Let n be a nonnegative integer and* $k\in\mathbb{Z}$. *Then,*

$$\begin{aligned}G^{(k)}_{n,\lambda,p}(x) &= \sum_{i=0}^{n}\sum_{l=0}^{i}\binom{n}{i}\binom{i}{l}\sum_{m=1}^{l+1}\frac{(1)_{i-l+1,\lambda}p(m)\lambda^{m-1}(1)_{m,1/\lambda}(-1)^l m!}{(i-l+1)m^k}\\ &\quad\cdot\frac{S_{2,\lambda}(l+1,m)}{l+1}\times 2^{n-i}\beta_{n-i,\lambda/2}\left(\frac{x}{2}\right).\end{aligned} \tag{59}$$

From (51), we have

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,\lambda,p}^{(k)}(x)\frac{t^n}{n!} &= \frac{2}{e_\lambda(t)+1}u_{k,\lambda}(1-e_\lambda(-t)\mid p)(e_\lambda(t)-1+1)^x \\ &= \frac{2u_{k,\lambda}(1-e_\lambda(-t)\mid p)}{e_\lambda(t)+1}\sum_{i=0}^{\infty}(x)_i\frac{(e_\lambda(t)-1)^i}{i!} \\ &= \sum_{n=0}^{\infty} G_{n,\lambda,p}^{(k)}\frac{t^n}{n!}\sum_{i=0}^{\infty}(x)_i\sum_{l=i}^{\infty}S_{2,\lambda}(l,i)\frac{t^l}{l!} \\ &= \sum_{n=0}^{\infty} G_{n,\lambda,p}^{(k)}\frac{t^n}{n!}\sum_{i=0}^{\infty}\sum_{i=0}^{l}(x)_i S_{2,\lambda}(l,i)\frac{t^l}{l!} \\ &= \sum_{n=0}^{\infty}\left(\sum_{l=0}^{n}\sum_{i=0}^{l}\binom{n}{l}(x)_i S_{2,\lambda}(l,i)G_{n-l,\lambda,p}^{(k)}\right)\frac{t^n}{n!}.\end{aligned} \tag{60}$$

By equation (60), we get the following theorem.

Theorem 11. *Let n be a nonnegative integer and $k \in \mathbb{Z}$. Then,*

$$G_{n,\lambda,p}^{(k)}(x) = \sum_{l=0}^{n}\sum_{i=0}^{l}\binom{n}{l}(x)_i S_{2,\lambda}(l,i)G_{n-l,\lambda,p}^{(k)}. \tag{61}$$

4. Conclusion

In this article, we introduced degenerate poly-Genocchi polynomials and numbers by using the degenerate polylogarithm function and derived several properties on the degenerate poly-Genocchi numbers. We represented the generating function of the degenerate poly-Genocchi numbers by iterated integrals in Theorems 4–6 and explicit degenerate poly-Genocchi polynomials in terms of the Euler polynomials and degenerate Stirling numbers of the second kind in Theorem 3. We also represented those numbers in terms of the degenerate Stirling numbers of the second kind in Theorem 7. In the last section, we defined the degenerate unipoly-Genocchi polynomials by using degenerate polylogarithm function and obtained the identity degenerate unipoly-Genocchi polynomials in terms of the degenerate Genocchi polynomials and degenerate Stirling numbers of the second kind in Theorem 8, the degenerate Euler polynomials and the degenerate Stirling numbers of the second kind in Theorem 9, the degenerate Bernoulli and degenerate Stirling numbers of the second kind in Theorem 10, and the degenerate unipoly-Genocchi numbers and Stirling numbers of the second kind in Theorem 11. It is important that the study of the degenerate version is widely applied not only to numerical theory and combinatorial theory but also to symmetric identity, differential equations, and probability theory. In particular, many symmetric identities have been studied for degenerate versions of many special polynomials [1, 3, 12, 23, 29–32]. Genocchi numbers have been also extensively studied in many different branches of mathematics. The works of Genocchi numbers and their combinatorial relations have received much attention [6–9]. With this in mind, as a future project, we would like to continue to study degenerate versions of certain special polynomials and numbers and their applications to physics, economics, and engineering as well as mathematics.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

All authors contributed equally to the manuscript and typed, read, and approved the final manuscript.

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