Research Article

General Solution and Stability of Additive-Quadratic Functional Equation in IRN-Space

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The investigation of the stabilities of various types of equations is an interesting and evolving research area in the field of mathematical analysis. Recently, there are many research papers published on this topic, especially additive, quadratic, cubic, and mixed type functional equations. We propose a new functional equation in this study which is quite different from the functional equations already dealt in the literature. The main feature of the equation dealt in this study is that it has three different solutions, namely, additive, quadratic, and mixed type functions. We also prove that the stability results hold good for this equation in intuitionistic random normed space (briefly, IRN-space).

1. Introduction

The theory of random normed spaces (RN-spaces) is important as a generalization of the deterministic result of linear normed spaces and also in the study of random operator equations. The RN-spaces may also provide us with the tools to study the geometry of nuclear physics and have important applications in quantum particle physics.

The concept of stability of a functional equation arises when one replaces a functional equation by an inequality which acts as a perturbation of the equation. The first stability problem concerning group homomorphisms was raised by Ulam [1] in 1940 and affirmatively solved by Hyers [2]. Aoki generalized the result of Hyers [3] for approximate additive mappings and by Rassias [4] for approximate linear mappings by allowing the difference Cauchy equation $\|f(x+y) - f(x) - f(y)\|$ to be controlled by $\varepsilon(\|x\|^p + \|y\|^p)$. In 1994, a generalization of the Th.M. Rassias' theorem was got by Gavruta [5], who replaced $\varepsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$. For additional information regard-

ing the outcomes about such issues, the related background in [6–12] can be examined. Absorbing new outcomes concerning mixed-type functional equations has as of late been acquired by Najati et al. [13–15], Jun and Kim [16, 17], and Park [18–22].

The functional equations

$$f(x+y) = f(x) + f(y), \quad (1)$$

and

$$f(x+y) + f(x-y) = 2f(x) + 2f(y), \quad (2)$$

are called the additive and quadratic functional equations, respectively. Every solution of the additive and quadratic functional equations is said to be additive mapping and quadratic mapping, respectively.

As of late, Zhang [23] examined the cubic functional equation in intuitionistic random space. The stability of various equations in RN-spaces has been as of late concentrated

in Alsina [24], Eshaghi Gordji et al. [25, 26], Mihet and Radu [27–29], and Saadati et al. [30]. Xu et al. [31–33] presented the various mixed types of functional equations investigated in Intuitionistic fuzzy normed spaces, quasi Banach spaces, and random normed spaces. Also, Shu et al. [33–35] discussed various differential equations to study the Hyers-Ulam stability, which provides a wide view of this stability problem.

In this present work, we introduce a new mixed type additive-quadratic functional equation

$$\begin{aligned} & \varphi\left(\sum_{i=1}^m a^i v_i\right) + \sum_{i=1}^m \varphi\left(-a^i v_i + \sum_{j=1, i \neq j}^m a^j v_j\right) \\ &= (m-3) \sum_{i=1}^m \varphi(a^i v_i + a^i v_j) - (m^2 - 5m + 2) \\ & \quad \cdot \sum_{i=1}^m a^{2i} \left[\frac{\varphi(v_i) + \varphi(-v_i)}{2} \right] - (m^2 - 5m + 4) \\ & \quad \cdot \sum_{i=1}^m a^i \left[\frac{\varphi(v_i) - \varphi(-v_i)}{2} \right], \end{aligned} \quad (3)$$

where a is a fixed integer and $m \geq 5$ and investigate the Ulam-Hyers stability results of this mixed type additive-quadratic functional equation in an intuitionistic random normed space.

So far various forms of additive and quadratic functional equations are considered in this research field to obtain their stability results through different methods. For the first time, a new mixed additive-quadratic functional equation is proposed in this paper, and its stability results are proved in an intuitionistic random normed space.

This type of functional equation can be of use in solving many physical problems and also has significant relevance in various scientific fields of research and study. In particular, additive-quadratic functional equations have applications in electric circuit theory, physics, and relations connecting the harmonic mean and arithmetic mean of several values. Providing a wealth of essential insights and new concepts in the field of functional equations.

2. Preliminaries

We recall the following ideas and conceptions of IRN-spaces in [36–41].

Definition 1 (see [42]). A mapping $\mu : \mathbb{R} \longrightarrow [0, 1]$ is said to be a measure distribution function, if μ is left continuous on $\mathbb{R}$, non-decreasing, $\inf_{t \in \mathbb{R}} \mu(t) = 0$, and $\sup_{t \in \mathbb{R}} \mu(t) = 1$.

Definition 2 (see [42]). A mapping $\nu : \mathbb{R} \longrightarrow [0, 1]$ is said to be a non-measure distribution function, if ν is right continuous on $\mathbb{R}$, non-increasing, $\sup_{t \in \mathbb{R}} \nu(t) = 1$, and $\inf_{t \in \mathbb{R}} \nu(t) = 0$.

Lemma 3 (see [43, 44]). Let L^* be a set with an operator $\leq_{L^*}$ is defined by

$$\begin{aligned} L^* &= \{(v_1, v_2) : (v_1, v_2) \in [0, 1]^2 \text{ and } v_1 + v_2 \leq 1\}, \\ (v_1, v_2) \leq_{L^*} (w_1, w_2) &\Leftrightarrow v_1 \leq w_1, v_2 \geq w_2, (v_1, v_2), (w_1, w_2) \in L^*. \end{aligned} \quad (4)$$

Then, the pair $(L^*, \leq_{L^*})$ is a complete lattice.

We denote its units by $0_{L^*} = (0, 1)$ and $1_{L^*} = (1, 0)$. Typically, a triangular norm (t-norm) $* = \Phi$ on $[0, 1]$ is defined as an increasing, commutative, associative mapping $\Phi : [0, 1]^2 \longrightarrow [0, 1]$ satisfying $\Phi(1, p) = 1 * p = p$ for every $p \in [0, 1]$, and a triangular conorm (t-conorm) $Y = \diamond$ is defined as an increasing, commutative, associative mapping $Y : [0, 1]^2 \longrightarrow [0, 1]$ satisfying $Y(0, p) = 0 \diamond p = p$ for all $p \in [0, 1]$.

By using the lattice $(L^*, \leq_{L^*})$, these definitions can be straightforwardly extended.

Definition 4 (see [44]). A triangular norm (t-norm) on L^* is a mapping $\Phi : (L^*)^2 \longrightarrow L^*$ satisfying the following conditions:

(i) Boundary condition

$$\text{i.e., } \Phi(p, 1_{L^*}) = p, \forall p \in L^*;$$

(ii) Commutativity

$$\text{i.e., } \Phi(p, q) = \Phi(q, p), \forall (p, q) \in (L^*)^2;$$

(iii) Associativity

$$\text{i.e., } \Phi(p, \Phi(q, r)) = \Phi(\Phi(p, q), r), \forall (p, q, r) \in (L^*)^3;$$

(iv) Monotonicity

$$\text{i.e., } p \leq_{L^*} p' \text{ and } q \leq_{L^*} q' \Rightarrow \Phi(p, q) \leq_{L^*} \Phi(p', q') \text{ for all } (p, p', q, q') \in (L^*)^4.$$

If $(L^*, \leq_{L^*}, \Phi)$ is an Abelian topological monoid with unit 1_{L^*} , then Φ is called a continuous t-norm.

Definition 5 (see [42]). A negator on L^* is any decreasing mapping N from L^* to L^* satisfying $N(1_{L^*}) = 0_{L^*}$ and $N(0_{L^*}) = 1_{L^*}$. If $N(N(p)) = p$ for all $p \in L^*$, then N is called an involutive negator. A negator on $[0, 1]$ is a decreasing mapping $N : [0, 1] \longrightarrow [0, 1]$ satisfying $N(0) = 1$ and $N(1) = 0$.

N_s denotes the standard negator on $[0, 1]$ defined by

$$N_s(p) = 1 - p, \quad (5)$$

for all $p \in [0, 1]$.

Definition 6 (see [23]). Let μ and ν be measure and nonmeasure distribution functions from $V \times (0, +\infty)$ to $[0, 1]$ such that

$$\mu_p(t) + \nu_p(t) \leq 1, p \in V, t > 0. \quad (6)$$

The triple $(V, I_{\mu, \nu}, \Phi)$ is said to be an intuitionistic random normed space if a vector space V , continuous t -representable Φ , and a mapping $I_{\mu, \nu} : V \times (0, +\infty) \longrightarrow L^*$ holds the following conditions: for all $p, q \in V$ and $t_1, t_2 > 0$

$$\begin{aligned} I_{\mu, \nu}(p, 0) &= 0_{L^*}, \\ I_{\mu, \nu}(p, t_1) &= 1_{L^*} \Leftrightarrow p = 0, \\ I_{\mu, \nu}(\alpha p, t_1) &= I_{\mu, \nu}\left(p, \frac{t_1}{|\alpha|}\right), \forall \alpha \neq 0, \\ I_{\mu, \nu}(p + q, t_1 + t_2) &\geq_{L^*} \Phi(I_{\mu, \nu}(p, t_1), I_{\mu, \nu}(q, t_2)). \end{aligned} \quad (7)$$

Thus, $I_{\mu, \nu}$ is called an intuitionistic random norm. Hence,

$$I_{\mu, \nu}(p, t_1) = (\mu_p(t_1), \nu_p(t_1)). \quad (8)$$

Example 1 (see [42]). Let $(V, \|\cdot\|)$ be a normed space. Let $\Phi(p, q) = (p_1 q_1, \min(p_2 + q_2, 1))$ for all $p = (p_1, p_2)$, $q = (q_1, q_2) \in L^*$ and let μ, ν be measure and non-measure distribution functions defined by

$$I_{\mu, \nu}(v, \varepsilon) = (\mu_v(\varepsilon), \nu_v(\varepsilon)) = \left(\frac{\varepsilon}{\varepsilon + \|v\|}, \frac{\|v\|}{\varepsilon + \|v\|} \right), \forall \varepsilon \in \mathbb{R}^+. \quad (9)$$

Then, $(V, I_{\mu, \nu}, \Phi)$ is an IRN-space.

Definition 7 (see [42]). Let $(V, I_{\mu, \nu}, \Phi)$ be an IRN-space.

- (i) A sequence $\{p_m\}$ in $(V, I_{\mu, \nu}, \Phi)$ is known as a Cauchy sequence if, for some $\delta > 0$ and $t > 0$, there is an $m_0 \in \mathbb{N}$ satisfies

$$I_{\mu, \nu}(p_m - p_n, t) \geq_{L^*} (N_s(\delta), \delta), m, n \geq m_0. \quad (10)$$

- (ii) The sequence $\{p_m\}$ is convergent to any point $p \in V$ if $I_{\mu, \nu}(p_m - p, t) \longrightarrow 1_{L^*}$ as $m \longrightarrow \infty$ for all $t > 0$
- (iii) An intuitionistic random normed space $(V, I_{\mu, \nu}, \Phi)$ is known as complete if every Cauchy sequence in V is convergent to a point $p \in V$

3. Solution of the Functional Equation (3)

In this section, let us consider V and W are two real vector spaces.

Theorem 8. *If an odd mapping $\varphi : V \longrightarrow W$ satisfies the functional equation (3) for all $v_1, v_2, \dots, v_m \in V$, then the function φ is additive.*

Proof. In the view of the oddness of φ , we have $\varphi(-v) = -\varphi(v)$ for all $v \in V$. Using the oddness property, the functional equation (3) reduces as

$$\begin{aligned} &\varphi\left(\sum_{i=1}^m a^i v_i\right) + \sum_{i=1}^m \varphi\left(-a^i v_i + \sum_{j=1; i \neq j}^m a^j v_j\right) \\ &= (m-3) \sum_{i=1}^m \varphi(a^i v_i + a^i v_j) - (m^2 - 5m + 4) \sum_{i=1}^m a^i \varphi(v_i), \end{aligned} \quad (11)$$

for all $v_1, v_2, \dots, v_m \in V$. Now, replacing $(v_1, v_2, \dots, v_m)$ by $(0, 0, \dots, 0)$ in (11), we get $\varphi(0) = 0$. Interchanging $(v_1, v_2, \dots, v_m)$ with $(v, 0, 0, \dots, 0)$ in (11), we get

$$\varphi(av) = a\varphi(v), v \in V. \quad (12)$$

Again interchanging v with av in (12), we have

$$\varphi(a^2 v) = a^2 \varphi(v), \quad (13)$$

for all $v \in V$. Replacing v by av in (13), we obtain

$$\varphi(a^3 v) = a^3 \varphi(v), v \in V. \quad (14)$$

From the equalities (12)–(14), we can generalize the results for any nonnegative integer m as

$$\varphi(a^m v) = a^m \varphi(v), v \in V. \quad (15)$$

Similarly, we have

$$\varphi\left(\frac{v}{a^m}\right) = \frac{1}{a^m} \varphi(v), v \in V. \quad (16)$$

Replacing $(v_1, v_2, \dots, v_m)$ by $((x/a), (y/a^2), 0, \dots, 0)$ in (11), we have

$$\varphi(x + y) = \varphi(x) + \varphi(y), x, y \in V. \quad (17)$$

Hence, the function φ is additive. $\square$

Theorem 9. *If an even mapping $\varphi : V \longrightarrow W$ satisfies the functional equation (3) for all $v_1, v_2, \dots, v_m \in V$, then the function φ is quadratic.*

Proof. Since, in the view of evenness of φ , we have $\varphi(-v) = \varphi(v)$ for all $v \in V$. Now, the functional equation (3) reduces as

$$\begin{aligned} &\varphi\left(\sum_{i=1}^m a^i v_i\right) + \sum_{i=1}^m \varphi\left(-a^i v_i + \sum_{j=1; i \neq j}^m a^j v_j\right) \\ &= (m-3) \sum_{i=1}^m \varphi(a^i v_i + a^i v_j) - (m^2 - 5m + 2) \sum_{i=1}^m a^{2i} \varphi(v_i). \end{aligned} \quad (18)$$

for all $v_1, v_2, \dots, v_m \in V$. Now, replacing $(v_1, v_2, \dots, v_m)$ by $(0, 0, \dots, 0)$ in (18), we obtain $\varphi(0) = 0$. Interchanging $(v_1, v_2, \dots, v_m)$ with $(v, 0, 0, \dots, 0)$ in (18), we obtain

$$\varphi(av) = a^2 \varphi(v), v \in V. \quad (19)$$

Replacing v by av in (19), we reach

$$\varphi(a^2v) = a^4\varphi(v), v \in V. \quad (20)$$

Switching v by av in (20), we get

$$\varphi(a^3v) = a^6\varphi(v), v \in V. \quad (21)$$

From (19)–(21), we can generalize the results for any nonnegative integer m as

$$\varphi(a^mv) = a^{2m}\varphi(v), v \in V. \quad (22)$$

Similarly, we have

$$\varphi\left(\frac{v}{a^m}\right) = \frac{1}{a^{2m}}\varphi(v), v \in V. \quad (23)$$

Replacing $(v_1, v_2, \dots, v_m)$ by $((x/a), (y/a^2), 0, \dots, 0)$ in (18), we obtain

$$\varphi(x+y) + \varphi(x-y) = 2\varphi(x) + 2\varphi(y), x, y \in V. \quad (24)$$

Hence, the function φ is quadratic. $\square$

Theorem 10. If a mapping $\varphi : V \longrightarrow W$ satisfies $\varphi(0) = 0$ and satisfies the functional equation (3) for all $v_1, v_2, \dots, v_m \in V$ if and only if there exists a symmetric biadditive mapping $Q : V \times V \longrightarrow W$ and an additive mapping $A : V \longrightarrow W$ satisfies $\varphi(v) = Q(v, v) + A(v)$ for all $v \in V$.

Proof. Let a mapping $\varphi : V \longrightarrow W$ with $\varphi(0) = 0$ satisfies the functional equation (3). We divide the function φ into the odd part and even part as

$$\varphi_o(v) = \frac{\varphi(v) - \varphi(-v)}{2}, \varphi_e(v) = \frac{\varphi(v) + \varphi(-v)}{2}, v \in V, \quad (25)$$

respectively. Clearly, $\varphi(v) = \varphi_e(v) + \varphi_o(v)$ for all $v \in V$. $\square$

It is easy to prove that φ_o and φ_e satisfies the functional equation (3). By Theorems 8 and 9, we conclude that φ_o and φ_e are additive and quadratic, respectively. Then, there exist a symmetric biadditive mapping $Q : V \times V \longrightarrow W$ which satisfies $\varphi_e(v) = Q(v, v)$ and an additive mapping $A : V \longrightarrow W$ which satisfies $\varphi_o(v) = A(v)$ for all $v \in V$. Hence, $\varphi(v) = Q(v, v) + A(v)$ for all $v \in V$.

Conversely, suppose that there exists a symmetric biadditive mapping $Q : V \times V \longrightarrow W$ and an additive mapping $A : V \longrightarrow W$ and satisfies $\varphi(v) = Q(v, v) + A(v)$ for all $v \in V$. It is easy to prove that the mappings $v \mapsto Q(v, v)$ and $A : V \longrightarrow W$ satisfy the functional equation (3). Hence, the mapping $\varphi : V \longrightarrow W$ satisfies the functional equation (3).

For our notational convenience, we can define a mapping $\varphi : V \longrightarrow W$ by

$$\begin{aligned} D\varphi(v_1, v_2, \dots, v_m) &= \varphi\left(\sum_{i=1}^m a^i v_i\right) + \sum_{i=1}^m \varphi\left(-a^i v_i + \sum_{j=1; j \neq i}^m a^j v_j\right) \\ &\quad - (m-3) \sum_{i=1}^m \varphi(a^i v_i + a^j v_j) \\ &\quad + (m^2 - 5m + 2) \sum_{i=1}^m a^{2i} \left[\frac{\varphi(v_i) + \varphi(-v_i)}{2} \right] \\ &\quad + (m^2 - 5m + 4) \sum_{i=1}^m a^i \left[\frac{\varphi(v_i) - \varphi(-v_i)}{2} \right], \end{aligned} \quad (26)$$

for all $v_1, v_2, \dots, v_m \in V$.

In the following sections, we consider V is a linear space, $(V, I_{\mu', \nu'}, Y)$ is an intuitionistic random normed space and $(W, I_{\mu, \nu}, Y)$ is a complete intuitionistic random normed space.

4. Stability Results for Even Case

Theorem 11. Let $\alpha, \beta : V^m \longrightarrow D^+$, where $\alpha(v_1, v_2, \dots, v_m)$ is denoted by $\alpha_{v_1, v_2, \dots, v_m}$, $\beta(v_1, v_2, \dots, v_m)$ is denoted by $\beta_{v_1, v_2, \dots, v_m}$ and $\alpha_{v_1, v_2, \dots, v_m}(\varepsilon), \beta_{v_1, v_2, \dots, v_m}(\varepsilon)$ is denoted by $\Theta_{\alpha, \beta}(v_1, v_2, \dots, v_m, \varepsilon)$, be a mapping such that

$$\lim_{w \longrightarrow \infty} \Theta_{\alpha, \beta}(a^w v_1, a^w v_2, \dots, a^w v_m, a^{2w} \varepsilon) = I_{L^*}, \quad (27)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, and

$$\lim_{w \longrightarrow \infty} \Phi_{i=1}^{\infty}(\Theta_{\alpha, \beta}(a^{w+i-1} v, 0, \dots, 0, a^{2w+i} \varepsilon)) = I_{L^*}, \quad (28)$$

for all $v \in V$ and all $\varepsilon > 0$. If an even mapping $\varphi : V \longrightarrow W$ with $\varphi(0) = 0$ satisfies

$$I_{\mu, \nu}(D\varphi(v_1, v_2, \dots, v_m), \varepsilon) \geq_{L^*} \Theta_{\alpha, \beta}(v_1, v_2, \dots, v_m, \varepsilon), \quad (29)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, then there exists a unique quadratic mapping $Q_2 : V \longrightarrow W$ such that

$$\begin{aligned} I_{\mu, \nu}(\varphi(v) - Q_2(v), \varepsilon) \\ \geq_{L^*} \Phi_{i=1}^{\infty}(\Theta_{\alpha, \beta}(a^{i-1} v, 0, \dots, 0, a^i(m^2 - 5m + 2)\varepsilon)), \end{aligned} \quad (30)$$

for all $v \in V$ and all $\varepsilon > 0$.

Proof. Replacing $(v_1, v_2, \dots, v_m)$ by $(v, 0, \dots, 0)$ in (29), we have

$$\begin{aligned} I_{\mu, \nu}((m^2 - 5m + 2)\varphi(av) - (m^2 - 5m + 2)a^2\varphi(v), \varepsilon) \\ \geq_{L^*} \Theta_{\alpha, \beta}(v, 0, \dots, 0, \varepsilon), \end{aligned} \quad (31)$$

for all $v \in V$ and all $\varepsilon > 0$. From inequality (31), we get

$$I_{\mu,v} \left(\frac{\varphi(av)}{a^2} - \varphi(v), \varepsilon \right) \geq_{L^*} \Theta_{\alpha,\beta}(v, 0, \dots, 0, a^2(m^2 - 5m + 2)\varepsilon),$$

$$v \in V, \varepsilon > 0. \quad (32)$$

Interchanging v with av in (32), we obtain

$$I_{\mu,v} \left(\frac{\varphi(a^2v)}{a^4} - \frac{\varphi(av)}{a^2}, \varepsilon \right) \geq_{L^*} \Theta_{\alpha,\beta}(av, 0, \dots, 0, a^{2(2)}(m^2 - 5m + 2)\varepsilon), v \in V, \varepsilon > 0. \quad (33)$$

Replacing v by $a^{l-1}v$ and divide by a^{2l} in (33), we conclude that

$$I_{\mu,v} \left(\frac{\varphi(a^{l+1}v)}{a^{2(l+1)}} - \frac{\varphi(a^lv)}{a^{2l}}, \varepsilon \right) \geq_{L^*} \Theta_{\alpha,\beta}(a^lv, 0, \dots, 0, a^{2(l+1)}(m^2 - 5m + 2)\varepsilon), \quad (34)$$

for all $v \in V$ and all $\varepsilon > 0$. Thus,

$$I_{\mu,v} \left(\frac{\varphi(a^wv)}{a^{2w}} - \varphi(v), \varepsilon \right) \geq_{L^*} \Phi_{l=0}^{w-1} \left(I_{\mu,v} \left(\frac{\varphi(a^{l+1}v)}{a^{2(l+1)}} - \frac{\varphi(a^lv)}{a^{2l}}, \sum_{l=0}^{w-1} \frac{\varepsilon}{a^{l+1}} \right) \right), \quad (35)$$

for all $v \in V$ and all $\varepsilon > 0$. To prove the convergence of the sequence $\{\varphi(a^wv)/a^{2w}\}$, replacing v by a^kv in (35), we obtain

$$I_{\mu,v} \left(\frac{\varphi(a^{w+k}v)}{a^{2(w+k)}} - \frac{\varphi(a^kv)}{a^{2k}}, \varepsilon \right) \geq_{L^*} \Phi_{l=k}^{w+k-1} \left(I_{\mu,v} \left(\frac{\varphi(a^{l+1}v)}{a^{2(l+1)}} - \frac{\varphi(a^lv)}{a^{2l}}, \sum_{l=k}^{w+k-1} \frac{\varepsilon}{a^{l+1}} \right) \right), \quad (36)$$

for all $v \in V$ and all $\varepsilon > 0$ and all $k, w \geq 0$. Since the R.H.S of the inequality (36) tends to 1_{L^*} as $w, k \rightarrow \infty$, the sequence $\{\varphi(a^wv)/a^{2w}\}$ is a Cauchy sequence in $(W, I_{\mu,v}, Y)$. Since $(W, I_{\mu,v}, Y)$ is a complete IRN-space, this sequence converges to some point $Q_2(v) \in W$. So one can define the mapping $Q_2 : V \rightarrow W$ by

$$Q_2(v) = \lim_{w \rightarrow \infty} \frac{\varphi(a^wv)}{a^{2w}}, \quad (37)$$

for all $v \in V$. Letting $k = 0$ in (36), we obtain

$$I_{\mu,v} \left(\frac{\varphi(a^wv)}{a^{2w}} - \varphi(v), \varepsilon \right) \geq_{L^*} \Phi_{l=0}^{w-1} \left(I_{\mu,v} \left(\frac{\varphi(a^{l+1}v)}{a^{2(l+1)}} - \frac{\varphi(a^lv)}{a^{2l}}, \sum_{l=0}^{w-1} \frac{\varepsilon}{a^{l+1}} \right) \right), \quad (38)$$

for all $v \in V$ and all $\varepsilon > 0$. Taking the limit $w \rightarrow \infty$ in (38), we get

$$I_{\mu,v}(\varphi(v) - Q_2(v), \varepsilon) \geq_{L^*} \Phi_{l=1}^{\infty} \left(\Theta_{\alpha,\beta}(a^{l-1}v, 0, \dots, 0, a^l(m^2 - 5m + 2)\varepsilon) \right), \quad (39)$$

for all $v \in V$ and all $\varepsilon > 0$.

Next, we prove that the function Q_2 is quadratic. Replacing $(v_1, v_2, \dots, v_m)$ by $(a^wv_1, a^wv_2, \dots, a^wv_m)$ in (29), we obtain

$$I_{\mu,v} \left(\frac{1}{a^{2w}} D\varphi(a^wv_1, a^wv_2, \dots, a^wv_m), \varepsilon \right) \geq_{L^*} \Theta_{\alpha,\beta}(a^wv_1, a^wv_2, \dots, a^wv_m, a^{2w}\varepsilon), \quad (40)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$. Taking the limit as $w \rightarrow \infty$, we find that $I_{\mu,v}(DQ_2(v_1, v_2, \dots, v_m), \varepsilon) = 1_{L^*}$ for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, which implies $DQ_2(v_1, v_2, \dots, v_m) = 0$. Thus, the function Q_2 satisfies the functional equation (3). Hence, $Q_2 : V \rightarrow W$ is a quadratic mapping. Passing to the limit as $w \rightarrow \infty$ in (35), we have (30).

Finally, to show the uniqueness of Q_2 subject to (30), consider that there exists another quadratic function Q'_2 which satisfies the inequality (30). Clearly, $Q_2(a^wv) = a^{2w}Q_2(v)$ and $Q'_2(a^wv) = a^{2w}Q'_2(v)$ for all $v \in V$ and $w \in \mathbb{N}$, from (30) and (28) that

$$I_{\mu,v}(Q_2(v) - Q'_2(v), \varepsilon) \geq_{L^*} I_{\mu,v}(Q_2(a^wv) - Q'_2(a^wv), a^{2w}\varepsilon) \geq_{L^*} \Phi \left(I_{\mu,v} \left(Q_2(a^wv) - \varphi(a^wv), \frac{a^{2w}\varepsilon}{2} \right), I_{\mu,v} \left(\varphi(a^wv) - Q'_2(a^wv), \frac{a^{2w}\varepsilon}{2} \right) \right) \geq_{L^*} \Phi \left(\Phi_{i=1}^{\infty} \left(\Theta_{\alpha,\beta}(a^{w+i-1}v, 0, \dots, 0, a^{2w+i}(m^2 - 5m + 2)\frac{\varepsilon}{2}) \right), \Phi_{i=1}^{\infty} \left(\Theta_{\alpha,\beta}(a^{w+i-1}v, 0, \dots, 0, a^{2w+i}(m^2 - 5m + 2)\frac{\varepsilon}{2}) \right) \right), \quad (41)$$

for all $v \in V$ and all $\varepsilon > 0$. By taking $w \rightarrow \infty$ in (41), we show the uniqueness of Q_2 . This ends the proof of the uniqueness, as desired. $\square$

Corollary 12. *If an even mapping $\varphi : V \longrightarrow W$ satisfies*

$$I_{\mu,v}(D\varphi(v_1, v_2, \dots, v_m, \varepsilon)) \geq_{L^*} I_{\mu',v'}\left(\sum_{i=1}^m v_i, \varepsilon\right), \quad (42)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, and

$$\lim_{w \rightarrow \infty} \Phi_{i=1}^{\infty} \left(I_{\mu',v'}(a^{w+i-1}v, a^{2w+i}(m^2 - 5m + 2)\varepsilon) \right) = I_{L^*}, \quad (43)$$

for all $v \in V$ and all $\varepsilon > 0$, then there exists a unique quadratic mapping $Q_2 : V \longrightarrow W$ such that

$$I_{\mu,v}(\varphi(v) - Q_2(v), \varepsilon) \geq_{L^*} \Phi_{i=1}^{\infty} \left(I_{\mu',v'}(a^{i-1}v, a^i(m^2 - 5m + 2)\varepsilon) \right), \quad (44)$$

for all $v \in V$ and all $\varepsilon > 0$.

Proof. By taking $\Theta_{\alpha,\beta}(v_1, v_2, \dots, v_m, \varepsilon) = I_{\mu',v'}(\sum_{i=1}^m v_i, \varepsilon)$ in Theorem 11, we obtain our desired result. $\square$

5. Stability Results for Odd Case

Theorem 13. *Let $\alpha, \beta : V^m \longrightarrow D^+$, where $\alpha(v_1, v_2, \dots, v_m)$ is denoted by $\alpha_{v_1, v_2, \dots, v_m}$, $\beta(v_1, v_2, \dots, v_m)$ is denoted by $\beta_{v_1, v_2, \dots, v_m}$ and $\alpha_{v_1, v_2, \dots, v_m}(\varepsilon), \beta_{v_1, v_2, \dots, v_m}(\varepsilon)$ is denoted by $\Theta_{\alpha,\beta}(v_1, v_2, \dots, v_m, \varepsilon)$, be a mapping such that*

$$\lim_{w \rightarrow \infty} \Theta_{\alpha,\beta}(a^w v_1, a^w v_2, \dots, a^w v_m, a^w \varepsilon) = I_{L^*}, \quad (45)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, and

$$\lim_{w \rightarrow \infty} \Phi_{i=1}^{\infty} (\Theta_{\alpha,\beta}(a^{w+i-1}v, 0, \dots, 0, a^w \varepsilon)) = I_{L^*}, \quad (46)$$

for all $v \in V$ and all $\varepsilon > 0$. If an odd mapping $\varphi : V \longrightarrow W$ with $\varphi(0) = 0$ satisfies

$$I_{\mu,v}(D\varphi(v_1, v_2, \dots, v_m), \varepsilon) \geq_{L^*} \Theta_{\alpha,\beta}(v_1, v_2, \dots, v_m, \varepsilon), \quad (47)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, then there exist a unique additive mapping $A_1 : V \longrightarrow W$ such that

$$I_{\mu,v}(\varphi(v) - A_1(v), \varepsilon) \geq_{L^*} \Phi_{i=1}^{\infty} (\Theta_{\alpha,\beta}(a^{i-1}v, 0, \dots, 0, (m^2 - 5m + 4)\varepsilon)), \quad (48)$$

for all $v \in V$ and all $\varepsilon > 0$.

Proof. Replacing $(v_1, v_2, \dots, v_m)$ by $(v, 0, \dots, 0)$ in (47), we obtain

$$I_{\mu,v}((m^2 - 5m + 4)\varphi(av) - (m^2 - 5m + 4)a\varphi(v), \varepsilon) \geq_{L^*} \Theta_{\alpha,\beta}(v, 0, \dots, 0, \varepsilon), \quad (49)$$

for all $v \in V$ and all $\varepsilon > 0$. From inequality (49), we get

$$I_{\mu,v}\left(\frac{\varphi(av)}{a} - \varphi(v), \varepsilon\right) \geq_{L^*} \Theta_{\alpha,\beta}(v, 0, \dots, 0, a(m^2 - 5m + 4)\varepsilon), \quad (50)$$

for all $v \in V$ and all $\varepsilon > 0$. Replacing v by av in the above inequality (50), we have

$$I_{\mu,v}\left(\frac{\varphi(a^2v)}{a^2} - \frac{\varphi(av)}{a}, \varepsilon\right) \geq_{L^*} \Theta_{\alpha,\beta}(av, 0, \dots, 0, a^2(m^2 - 5m + 4)\varepsilon), \quad (51)$$

for all $v \in V$ and all $\varepsilon > 0$. Replacing v by $a^{l-1}v$ in (51), we conclude that

$$I_{\mu,v}\left(\frac{\varphi(a^{l+1}v)}{a^{l+1}} - \frac{\varphi(a^l v)}{a^l}, \varepsilon\right) \geq_{L^*} \Theta_{\alpha,\beta}(a^l v, 0, \dots, 0, a^{l+1}(m^2 - 5m + 4)\varepsilon), \quad (52)$$

for all $v \in V$ and all $\varepsilon > 0$. Thus,

$$I_{\mu,v}\left(\frac{\varphi(a^w v)}{a^w} - \varphi(v), \varepsilon\right) \geq_{L^*} \Phi_{l=0}^{w-1} \left(I_{\mu,v}\left(\frac{\varphi(a^{l+1}v)}{a^{l+1}} - \frac{\varphi(a^l v)}{a^l}, \sum_{l=0}^{w-1} \frac{\varepsilon}{a^{l+1}}\right) \right), \quad (53)$$

for all $v \in V$ and all $\varepsilon > 0$. To prove the convergence of the sequence $\{\varphi(a^w v)/a^w\}$, replacing v by $a^k v$ in (53), we obtain

$$I_{\mu,v}\left(\frac{\varphi(a^{w+k}v)}{a^{w+k}} - \frac{\varphi(a^k v)}{a^k}, \varepsilon\right) \geq_{L^*} \Phi_{l=k}^{w+k-1} \left(I_{\mu,v}\left(\frac{\varphi(a^{l+1}v)}{a^{l+1}} - \frac{\varphi(a^l v)}{a^l}, \sum_{l=k}^{w+k-1} \frac{\varepsilon}{a^{l+1}}\right) \right), \quad (54)$$

for all $v \in V$ and all $\varepsilon > 0$ and all $k, w \geq 0$. Since the R.H.S of the inequality (54) tends to I_{L^*} as $w, k \longrightarrow \infty$, the sequence $\{\varphi(a^w v)/a^w\}$ is a Cauchy sequence in $(W, I_{\mu,v}, Y)$. Since $(W, I_{\mu,v}, Y)$ is a complete IRN-space, this sequence converges to some point $A_1(v) \in W$. So one can define the mapping $A_1 : V \longrightarrow W$ by

$$A_1(v) = \lim_{w \rightarrow \infty} \frac{\varphi(a^w v)}{a^w}, \quad (55)$$

for all $v \in V$. Letting $k = 0$ in (54), we obtain

$$\begin{aligned} & I_{\mu,v} \left(\frac{\varphi(a^w v)}{a^w} - \varphi(v), \varepsilon \right) \\ & \geq_{L^*} \Phi_{l=0}^{w-1} \left(I_{\mu,v} \left(\frac{\varphi(a^{l+1} v)}{a^{(l+1)}} - \frac{\varphi(a^l v)}{a^{(l)}}, \sum_{l=0}^{w-1} \frac{\varepsilon}{a^{l+1}} \right) \right), \end{aligned} \quad (56)$$

for all $v \in V$ and all $\varepsilon > 0$. Taking the limit as $w \rightarrow \infty$ in (56), we get

$$\begin{aligned} & I_{\mu,v}(\varphi(v) - A_1(v), \varepsilon) \\ & \geq_{L^*} \Phi_{l=1}^{\infty} \left(\Theta_{\alpha,\beta} \left(a^{l-1} v, 0, \dots, 0, (m^2 - 5m + 4)\varepsilon \right) \right), \end{aligned} \quad (57)$$

for all $v \in V$ and all $\varepsilon > 0$.

Next, we want to prove that the function A_1 is additive. Replacing $(v_1, v_2, \dots, v_m)$ by $(a^w v_1, a^w v_2, \dots, a^w v_m)$ in (47), we obtain

$$\begin{aligned} & I_{\mu,v} \left(\frac{1}{a^w} D\varphi(a^w v_1, a^w v_2, \dots, a^w v_m), \varepsilon \right) \\ & \geq_{L^*} \Theta_{\alpha,\beta}(a^w v_1, a^w v_2, \dots, a^w v_m, a^w \varepsilon), \end{aligned} \quad (58)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$. Taking the limit as $w \rightarrow \infty$, we find that $I_{\mu,v}(DA_1(v_1, v_2, \dots, v_m), \varepsilon) = 1_{L^*}$ for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, which implies $DA_1(v_1, v_2, \dots, v_m) = 0$. Thus, A_1 satisfies the functional equation (3). Hence, the function $A_1 : V \rightarrow W$ is additive. Passing to the limit as $w \rightarrow \infty$ in (53), we have (48).

Finally, to show the uniqueness of the additive function A_1 subject to (48), consider that there exists another additive function A'_1 which satisfies the inequality (48). Evidently, $A_1(a^w v) = a^w A_1(v)$ and $A'_1(a^w v) = a^w A'_1(v)$ for all $v \in V$ and $w \in \mathbb{N}$, from (48) and (46) that

$$\begin{aligned} & I_{\mu,v} \left(A_1(v) - A'_1(v), \varepsilon \right) \geq_{L^*} I_{\mu,v} \left(A_1(a^w v) - A'_1(a^w v), a^w \varepsilon \right) \\ & \geq_{L^*} \Phi \left(I_{\mu,v} \left(A_1(a^w v) - \varphi(a^w v), \frac{a^w \varepsilon}{2} \right), \right. \\ & \quad \left. I_{\mu,v} \left(\varphi(a^w v) - A'_1(a^w v), \frac{a^w \varepsilon}{2} \right) \right) \\ & \geq_{L^*} \Phi \left(\Phi_{i=1}^{\infty} \left(\Theta_{\alpha,\beta} \left(a^{w+i-1} v, 0, \dots, 0, a^w (m^2 - 5m + 4) \frac{\varepsilon}{2} \right) \right), \right. \\ & \quad \left. \Phi_{i=1}^{\infty} \left(\Theta_{\alpha,\beta} \left(a^{w+i-1} v, 0, \dots, 0, a^w (m^2 - 5m + 4) \frac{\varepsilon}{2} \right) \right) \right), \end{aligned} \quad (59)$$

for all $v \in V$ and all $\varepsilon > 0$. By taking the limit $w \rightarrow \infty$ in (59), we show the uniqueness of A_1 . $\square$

Corollary 14. *If an odd mapping $\varphi : V \rightarrow W$ satisfies*

$$I_{\mu,v}(D\varphi(v_1, v_2, \dots, v_m, \varepsilon)) \geq_{L^*} I_{\mu',v'} \left(\sum_{i=1}^m v_i, \varepsilon \right), \quad (60)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, and

$$\lim_{w \rightarrow \infty} \Phi_{i=1}^{\infty} \left(I_{\mu',v'}(a^{w+i-1} v, a^w (m^2 - 5m + 4)\varepsilon) \right) = 1_{L^*}, \quad (61)$$

for all $v \in V$ and all $\varepsilon > 0$. Then, there exists a unique additive mapping $A_1 : V \rightarrow W$ such that

$$I_{\mu,v}(\varphi(v) - A_1(v), \varepsilon) \geq_{L^*} \Phi_{i=1}^{\infty} \left(I_{\mu',v'}(a^{i-1} v, (m^2 - 5m + 4)\varepsilon) \right), \quad (62)$$

for all $v \in V$ and all $\varepsilon > 0$.

Proof. By taking $\Theta_{\alpha,\beta}(v_1, v_2, \dots, v_m, \varepsilon) = I_{\mu',v'}(\sum_{i=1}^m v_i, \varepsilon)$ in Theorem 13, we obtain our desired result. $\square$

6. Stability Results for Mixed Case

Theorem 15. *Let $\alpha, \beta : V^m \rightarrow D^+$ be mappings satisfying (27), (28), (45), and (46) for all $v_1, v_2, \dots, v_m, v \in V$ and all $\varepsilon > 0$. If a mapping $\varphi : V \rightarrow W$ with $\varphi(0) = 0$ satisfies (29) for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, then there exist a unique quadratic mapping $Q_2 : V \rightarrow W$ and a unique additive mapping $A_1 : V \rightarrow W$ satisfying (3) and*

$$\begin{aligned} & I_{\mu,v}(\varphi(v) - Q_2(v) - A_1(v), \varepsilon) \\ & \geq_{L^*} \Phi \left(\Phi_{i=1}^{\infty} (a^{i-1} v, 0, \dots, 0, a^i (m^2 - 5m + 2)\varepsilon), \right. \\ & \quad \Phi_{i=1}^{\infty} (-a^{i-1} v, 0, \dots, 0, a^i (m^2 - 5m + 2)\varepsilon)) \\ & \quad \cdot \left(\Phi_{i=1}^{\infty} (a^{i-1} v, 0, \dots, 0, (m^2 - 5m + 4)\varepsilon), \right. \\ & \quad \left. \Phi_{i=1}^{\infty} (-a^{i-1} v, 0, \dots, 0, (m^2 - 5m + 4)\varepsilon) \right), \end{aligned} \quad (63)$$

for all $v \in V$.

Proof. Let $\varphi_e(v) = \varphi(v) + \varphi(-v)/2$ for all $v \in V$. Thus, $\varphi_e(0) = 0$, $\varphi_e(-v) = \varphi_e(v)$ and for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$,

$$\begin{aligned} & I_{\mu,v}(D\varphi_e(v_1, v_2, \dots, v_m), \varepsilon) \\ & = I_{\mu,v} \left(\frac{D\varphi(v_1, v_2, \dots, v_m) + D\varphi(-v_1, -v_2, \dots, -v_m)}{2}, \varepsilon \right), \\ & \geq_{L^*} \Phi \left(I_{\mu,v}(D\varphi(v_1, v_2, \dots, v_m), \varepsilon), I_{\mu,v}(D\varphi(-v_1, -v_2, \dots, -v_m), \varepsilon) \right), \\ & \geq_{L^*} \Phi \left(\Theta_{\alpha,\beta}(v_1, v_2, \dots, v_m, \varepsilon), \Theta_{\alpha,\beta}(-v_1, -v_2, \dots, -v_m, \varepsilon) \right), \end{aligned} \quad (64)$$

By Theorem 11, there exists a quadratic mapping $Q_2 : V \rightarrow W$ such that

$$\begin{aligned} & I_{\mu,v}(\varphi_e(v) - Q_2(v), \varepsilon) \\ & \geq_{L^*} \Phi \left(\Phi_{i=1}^{\infty} (a^{i-1} v, 0, \dots, 0, a^i (m^2 - 5m + 2)\varepsilon), \right. \\ & \quad \left. \Phi_{i=1}^{\infty} (-a^{i-1} v, 0, \dots, 0, a^i (m^2 - 5m + 2)\varepsilon) \right), \end{aligned} \quad (65)$$

for all $v \in V$ and all $\varepsilon > 0$. $\square$

On the other hand, let $\varphi_o(v) = \varphi(v) - \varphi(-v)/2$ for all $v \in V$. Then $\varphi_o(0) = 0$, $\varphi_o(-v) = -\varphi_o(v)$. By Theorem 13, there exists a additive mapping $A_1 : V \longrightarrow W$ satisfies

$$\begin{aligned} & I_{\mu,v}(\varphi_o(v) - A_1(v), \varepsilon) \\ & \geq_{L^*} \Phi(\Phi_{i=1}^\infty(a^{i-1}v, 0, \dots, 0, (m^2 - 5m + 4)\varepsilon), \\ & \quad \Phi_{i=1}^\infty(-a^{i-1}v, 0, \dots, 0, (m^2 - 5m + 4)\varepsilon)), \end{aligned} \quad (66)$$

for all $v \in V$ and all $\varepsilon > 0$. From inequalities (65) and (66), we obtain our desired result (64).

Corollary 16. *If a mapping $\varphi : V \longrightarrow W$ satisfies*

$$I_{\mu,v}(D\varphi(v_1, v_2, \dots, v_m, \varepsilon) \geq_{L^*} I_{\mu',v'}\left(\sum_{i=1}^m v_i, \varepsilon\right), \quad (67)$$

for all $v_1, v_2, \dots, v_m \in V$ and all $\varepsilon > 0$, and

$$\begin{aligned} & \lim_{w \rightarrow \infty} \Phi_{i=1}^\infty(I_{\mu',v'}(a^{w+i-1}v, a^{2w+i}(m^2 - 5m + 2)\varepsilon)) = I_{L^*}, \\ & \lim_{w \rightarrow \infty} \Phi_{i=1}^\infty(I_{\mu',v'}(a^{w+i-1}v, a^w(m^2 - 5m + 4)\varepsilon)) = I_{L^*}, \end{aligned} \quad (68)$$

for all $v \in V$ and all $\varepsilon > 0$. Then, there exists a unique quadratic mapping $Q_2 : V \longrightarrow W$ and a unique additive mapping $A_1 : V \longrightarrow W$ such that

$$\begin{aligned} & I_{\mu,v}(\varphi(v) - Q_2(v) - A_1(v), \varepsilon) \\ & \geq_{L^*} \Phi(\Phi(\Phi_{i=1}^\infty(a^{i-1}v, a^i(m^2 - 5m + 2)\varepsilon)), \\ & \quad \Phi_{i=1}^\infty(-a^{i-1}v, a^i(m^2 - 5m + 2)\varepsilon))) \\ & \quad \cdot (\Phi(\Phi_{i=1}^\infty(a^{i-1}v, (m^2 - 5m + 4)\varepsilon)), \\ & \quad \Phi_{i=1}^\infty(-a^{i-1}v, (m^2 - 5m + 4)\varepsilon))), \end{aligned} \quad (69)$$

for all $v \in V$ and all $\varepsilon > 0$.

7. Conclusion

In this paper, we introduced a new mixed type of additive-quadratic functional equation, and we applied the Hyers direct technique to investigate the Hyers-Ulam stability of the mixed type of additive-quadratic functional equation. Moreover, we have derived its general solution. The main objective of this work has been discussed: In Section 4, we have proved its Ulam-Hyers stability for the even case; in Section 5, examined Ulam-Hyers stability for odd case, and in Section 6, investigated Ulam-Hyers stability for the mixed cases, respectively, in intuitionistic random normed space.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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