Retraction

Retracted: Study of Fuzzy Fractional Nonlinear Equal Width Equation in the Sense of Novel Operator

Journal of Function Spaces

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] M. Naeem, A. A. Khammash, I. Mahariq, G. Laouini, and J. Kafle, "Study of Fuzzy Fractional Nonlinear Equal Width Equation in the Sense of Novel Operator," *Journal of Function Spaces*, vol. 2021, Article ID 8387044, 12 pages, 2021.

Research Article

Study of Fuzzy Fractional Nonlinear Equal Width Equation in the Sense of Novel Operator

Muhammad Naeem [iD],[1] **Ahmed A. Khammash,**[2] **Ibrahim Mahariq** [iD],[3] **Ghaylen Laouini,**[3] **and Jeevan Kafle** [iD][4]

[1]*Deanship of Joint First Year Umm Al-Qura University, Makkah, Saudi Arabia*
[2]*Department of Mathematics, Deanship of Applied Sciences Umm Al-Qura University, Makkah, Saudi Arabia*
[3]*College of Engineering and Technology, American University of the Middle East, Kuwait*
[4]*Central Department of Mathematics, Tribhuvan University, Kritipur, Kathmandu, Nepal*

Correspondence should be addressed to Muhammad Naeem; naeemtazkeer@yahoo.com
and Jeevan Kafle; jeevan.kafle@cdmath.tu.edu.np

In this paper, we designed an algorithm by applying the Laplace transform to calculate some approximate solutions for fuzzy fractional-order nonlinear equal width equations in the sense of Atangana-Baleanu-Caputo derivatives. By analyzing the fuzzy theory, the suggested technique helps the solution of the fuzzy nonlinear equal width equations be investigated as a series of expressions in which the components can be effectively recognised and produce a pair of numerical results with the uncertainty parameters. Several numerical examples are analyzed to validate convergence outcomes for the given problem to show the proposed method's utility and capability. The simulation outcomes reveal that the fuzzy iterative transform method is an effective method for accurately and precisely studying the behavior of suggested problems. We test the developed algorithm by three different problems. The analytical analysis provided that the results of the models converge to their actual solutions at the integer-order. Furthermore, we find that the fractional derivative produces a wide range of fuzzy results.

1. Introduction

Modern calculus has been applied to a wide range of topics in applied sciences where data is uncertain. Zadeh [1] proposed the fuzzy set to deal with similar issues. Fuzzy relations and fuzzy control were described further by Klir and Yuan [2]. Fixed point theory, control systems, algebra, topology, and fuzzy logic, among other fields, use fuzzy set theory. For the basic set of fuzzy, the scholars suggested a simple fuzzy calculus (FC) [3–5]. Dobius established the fundamental idea of fuzzy integral equations [6]. Fractional integral and differential equations have gained appeal among scholars in recent years. As a result, FC was expanded to fractional fuzzy integral and fractional fuzzy differential problems, which have several implementations in biology and physics. It is preferable to define fuzzy parameter notion rather than a crisp number when studying problems where pieces of infor-

mation are uncertain. As a result, numerous scholars have focused on analyzing fractional fuzzy differential models in diverse directions. In [7–9], several fundamental difficulties are investigated. The Atangana-Baleanu-Caputo derivative [10], a fractional nonsingular derivative proposed by Atangana, Baleanu, and Caputo, has recently gained prominence among scholars [11, 12]. Using fuzzy derivatives to analyze fractional differential equations under the Atangana-Baleanu-Caputo derivative can yield a lot of exciting outcomes and open up new avenues for younger scholars [13–16].

Fuzzy differential equations and fuzzy analysis have recently been proposed to resolve uncertainty caused by insufficient information found in numerous computer models or mathematical representations of many physical models. This concept has been more advance, and [17–19] and the literature therein discuss a wide range of implementations of this premise. In [20, 21], the authors introduced

the idea of fuzzy Rieamann-Liouville derivative with the help of Hukuhara differentiability. They demonstrated specific fuzzy integral equations that exist with sufficient compactness criteria using the Hausdorff measure of noncompactness. Various generalized Hukuhara differentiability-based techniques and methodologies were then introduced to many research papers in literature (see, for instance, [22–24]), and we will now briefly summarise some of these conclusions. The authors proved several solutions on the uniqueness and existence of fractional fuzzy differential equation solutions in the sense of Hukuhara fractional derivative. In [25, 26], they mentioned fuzzy solutions with the help of the Caputo derivative and the generalized Hukuhara fractional Riemann-Liouville. In [27], the authors used a modified fractional Euler method to solve a fuzzy fractional initial value problem under Caputo generalized Hukuhara differentiability [28], and they established the existence and uniqueness of a solution to a fuzzy fractional differential equation under a Caputo type-2 fuzzy fractional derivative, as well as a definition of the Laplace transform of type-2 fuzzy fractional derivative. The authors offer mathematical basics for investigation of random fuzzy fractional integral equations involving a fuzzy integral of fractional order in [29, 30], and they created the uniqueness and existence of the result to a fractional fuzzy starting value models under generalized Caputo Hukuhara differentiability, focusing on approximation solutions employing product rectangle techniques.

The fractional-order nonlinear equal width models are very significant partial differential equations that identify the numerous complex nonlinear occurrences in science's research area, especially in chemical physics, thermal waves, plasma physics, solid physics, fluid mechanics, etc. [31, 32]. The equal width equations study the behavior of nonlinear dispersive ocean in a broad class of nonlinear schemes as shallow water, ion-acoustic waves in plasma, hydromagnetic waves in a cold plasma, surface wave incompressible fluids, acoustic waves in enharmonic crystal, etc. [33, 34]. This work introduces a coupling of Laplace transform and iterative method identified as iterative transform method [35, 36]. We demonstrate the validity of this technique by solving fractional-order equal width equation, modified equal width equation, and variant modified equal width equation.

The rest of the article is divided as follows. In Section 2, we give basic definitions of fractional calculus, fractional fuzzy derivative, and fuzzy set. The general methodology of the present method is in Section 3. In Section 4, to detect the validity and effectiveness of the suggested algorithm, we present some numeric problems. Meanwhile, we present the results in figures to see the effect of the Atangana-Baleanu-Caputo operator to the considered model. Finally, the conclusion will be drawn in the last section.

2. Basic Definitions

Definition 1. Let a continuous fuzzy function of $\tilde{\Phi}(\mathfrak{I})$ on $[0, \wp] \subset R$ in the sense of Atangana-Baleanu-Caputo operator with respect to $\mathfrak{I}$ as the following [16].

The Atangana-Baleanu-Caputo derivative of $\tilde{\Phi}(\mathfrak{I}) \in \mathscr{H}^1(0, \mathfrak{I})$ is expressed as

$$D_{\mathfrak{I}}^{\varrho} \tilde{\Phi}(\mathfrak{I}) = \frac{ABC(\varrho)}{1-\varrho} \int_0^{\mathfrak{I}} \frac{d}{d\varepsilon} \tilde{\Phi}(\varepsilon) M_{\varrho}\left[\frac{-\varrho}{1-\varrho}(\mathfrak{I}-\varepsilon)^{\varrho}\right] d\varepsilon. \quad (1)$$

Replacing $E_{\varrho}[-\varrho/1-\varrho(\mathfrak{I}-\varepsilon)^{\varrho}]$ by $E_1[-\varrho/1-\varrho(\mathfrak{I}-\varepsilon)]$, we have "Caputo Fabrizo differential operator". Further, if $\tilde{\Phi}(\mathfrak{I}) \in C^F[0, \wp] \cap L^F[0, \wp]$, such that $\tilde{\Phi}(\mathfrak{I}) = [\underline{\Phi}_{\varrho}, \bar{\Phi}_{\varrho}(\mathfrak{I})]$, $\varrho \in [0, 1]$, and $\mathfrak{I}_0 \in (0, \wp)$. Then, the fuzzy fractional Atangana-Baleanu-Caputo derivative is defined by

$$\left[D_{\mathfrak{I}}^{\varrho} \tilde{\Phi}(\mathfrak{I})\right]_{\delta} = \left[D_{\mathfrak{I}}^{\varrho} \underline{\Phi}_{\varrho}(\mathfrak{I}), D_{\mathfrak{I}}^{\varrho} \bar{\Phi}_{\delta}(\mathfrak{I})\right], 0 \leq \delta \leq 1, \quad (2)$$

such that

$$D_{\mathfrak{I}}^{\varrho} \underline{\Phi}_{\rho}(\mathfrak{I}) = \frac{ABC(\varrho)}{1-\varrho} \int_0^{\mathfrak{I}} \frac{d}{d\varepsilon} \underline{\Phi}(\varepsilon) E_{\varrho}\left[\frac{-\varrho}{1-\varrho}(\mathfrak{I}-\varepsilon)^{\varrho}\right] d\varepsilon,$$

$$D_{\mathfrak{I}}^{\varrho} \tilde{\Phi}_{\varrho}(\mathfrak{I}) = \frac{ABC(\varrho)}{1-\varrho} \int_0^{\mathfrak{I}} \frac{d}{d\varepsilon} \tilde{\Phi}(\varepsilon) E_{\varrho}\left[\frac{-\varrho}{1-\varrho}(\mathfrak{I}-\varepsilon)^{\varrho}\right] d\varepsilon,$$

$$D_{\mathfrak{I}}^{\varrho}[\text{constant}] = 0. \quad (3)$$

Here, $ABC(\varrho)$ show "normalization function" and expressed by $\kappa(0) = \kappa(1) = 1$, and E_{ρ} is named as "Mittag-Leffler" function.

Definition 2. Let $W \in L[0, T]$. Then, the Atangana-Baleanu-Caputo integral is given by [16]

$$\mathbb{I}_{\mathfrak{I}}^{\varrho} \tilde{\Phi}(\mathfrak{I}) = \frac{(1-\varrho)\tilde{\Phi}(\mathfrak{I})}{ABC(\varrho)} + \frac{\varrho}{ABC(\varrho)} \int_0^{\mathfrak{I}} \frac{(\mathfrak{I}-\varepsilon)^{\varrho-1}}{\Gamma(\varrho)} \tilde{\Phi}(\varepsilon) d\varepsilon. \quad (4)$$

Further, if $\tilde{\Phi}(\mathfrak{I}) \in C^F[0, \wp] \cap L^F[0, \wp]$, where $C^F[0, \wp]$ and $L^F[0, \wp]$, define the "fuzzy space continuous function 1" is the space of "integrable Lebesgue fuzzy functions", respectively. Then, fractional fuzzy Atangana-Baleanu-Caputo integral is expressed as follows: $[\mathbb{I}_0^{\varrho} \tilde{\Phi}(\mathfrak{I})]_{\delta} = [\mathbb{I}_0^{\delta} \underline{\Phi}_{\delta}(\mathfrak{I}), \mathbb{I}_0^{\delta} \bar{\Phi}_{\delta}(\mathfrak{I})]$ $0 \leq \delta \leq 1$, such that

$$\mathbb{I}_0^{\delta} \underline{\Phi}_{\delta}(\mathfrak{I}) = \frac{1-\varrho}{ABC(\varrho)} \underline{\Phi}(\mathfrak{I}) + \frac{\varrho}{ABC(\varrho)\Gamma(\varrho)} \int_0^{\mathfrak{I}} (\mathfrak{I}-\varepsilon)^{\varrho-1} \underline{\Phi}(\varepsilon) d\varepsilon,$$

$$\mathbb{I}_0^{\delta} \bar{\Phi}_{\delta}(\mathfrak{I}) = \frac{1-\varrho}{ABC(\varrho)} \bar{\Phi}(\mathfrak{I}) + \frac{\varrho}{ABC(\varrho)\Gamma(\varrho)} \int_0^{\mathfrak{I}} (\mathfrak{I}-\varepsilon)^{\varrho-1} \bar{\Phi}(\varepsilon) d\varepsilon. \quad (5)$$

Definition 3. The "Laplace Fuzzy transformation" of Atangana-Baleanu-Caputo derivative of $\tilde{\Phi}(\mathfrak{I})$ is defined by [16]

$$\mathscr{L}\left[D_0^\varrho \tilde{\Phi}(\mathfrak{I})\right] = \frac{ABC(\varrho)}{[s^\varrho(1-\varrho)+\varrho]}\left[s^\varrho \mathscr{L}\left[\tilde{\Phi}(\mathfrak{I})\right] - s^{\varrho-1}\tilde{\Phi}(0)\right]. \tag{6}$$

Definition 4. The "Mittag-Leffler" function $E_\beta(\mathfrak{I})$ is given by [16]

$$E_\beta(\mathfrak{I}) = \sum_{n=0}^\infty \frac{\mathfrak{I}^n}{\Gamma(n\beta+1)}, \beta > 0. \tag{7}$$

Definition 5. A mapping $\kappa : R \longrightarrow [0,1]$. If holds, it is considered to be a fuzzy number [16].

(i) κ is upper semicontinuous

(ii) $\kappa\{\mu(\varepsilon_1)+\mu(\varepsilon_2)\} \geq \min\{\kappa(\varepsilon_1),\kappa(\varepsilon_2)\}$

(iii) $\exists \varepsilon_0 \in R$ such that $\kappa(\varepsilon_0) = 1$

(iv) $\mathrm{cl}\{r \in R, \kappa(r) > 0\}$ is compact

Definition 6. The parametric form of a fuzzy number is $(\underline{\kappa}(\delta), \bar{\kappa}(\delta))$ such that $0 \leq \delta \leq 1$; the conditions are as follows [16]:

(i) $\underline{k}(\delta)$ increasing, left-continuous over $(0,1]$ and right continues at 0

(ii) $\bar{k}(\delta)$ decreasing, left-continuous over $(0,1]$ and right continues at 0

(iii) $k(\delta) \leq \bar{k}(\delta)$.

3. Methodology

In this article, we use Laplace transformation to investigate general result of partial differential equation. Applying on both sides of Laplace transformation, we get

$$\mathscr{L}\left[D_{\mathfrak{I}}^\varrho(\tilde{\Phi}(\varphi,\mathfrak{I}))\right] = \mathscr{L}\left[A\frac{\partial^2}{\partial\varphi^2}(\tilde{\Phi}(\varphi,\mathfrak{I})) + \frac{\partial}{\partial x}(h(\varphi)\tilde{\Phi}(\varphi,\mathfrak{I}))\right]. \tag{8}$$

Evaluating the Laplace transform, Equation (8) implies that

$$\frac{ABC(\varrho)}{[s^\varrho(1-\varrho)+\varrho]}\left[s^\varrho \mathscr{L}\left[\tilde{\Phi}(\varphi,\mathfrak{I})\right] - s^{\varrho-1}\tilde{\Phi}(\varphi,0)\right] \\ = \mathscr{L}\left[A\frac{\partial^2}{\partial\varphi^2}(\tilde{\Phi}(\varphi,\mathfrak{I})) + \frac{\partial}{\partial x}(h(\varphi)\tilde{\Phi}(\varphi,\mathfrak{I}))\right]. \tag{9}$$

By using initial condition, we get

$$s^\varrho \mathscr{L}\left[\tilde{\Phi}(\varphi,\mathfrak{I})\right] = s^{\varrho-1}\tilde{g}(\varphi,\mathfrak{I}) + \frac{[s^\varrho(1-\varrho)+\varrho]}{ABC(\varrho)}\mathscr{L} \\ \cdot \left[A\frac{\partial^2}{\partial\varphi^2}(\tilde{\Phi}(\varphi,\mathfrak{I})) + \frac{\partial}{\partial x}(h(\varphi)\tilde{\Phi}(\varphi,\mathfrak{I}))\right], \tag{10}$$

or

$$\mathscr{L}\left[\tilde{\Phi}(\varphi,\mathfrak{I})\right] = \frac{1}{s}\tilde{g}(\varphi,\mathfrak{I}) + \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho ABC(\varrho)}\mathscr{L} \\ \cdot \left[A\frac{\partial^2}{\partial\varphi^2}(\tilde{\Phi}(\varphi,\mathfrak{I})) + \frac{\partial}{\partial x}(h(\varphi)\tilde{\Phi}(\varphi,\mathfrak{I}))\right]. \tag{11}$$

To investigate the series type result, we can write the unknown functions as $\tilde{\Phi}(\varphi,\mathfrak{I}) = \sum_{n=0}^\infty \tilde{\Phi}_n(\varphi,\mathfrak{I})$. In this type of representations, Equation (8) becomes

$$\mathscr{L}\left[\sum_{n=0}^\infty \tilde{\Phi}_n(\varphi,\mathfrak{I})\right] = \frac{1}{s}\tilde{g}(\varphi,\mathfrak{I}) + \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho ABC(\varrho)}\mathscr{L} \\ \cdot \left[A\frac{\partial^2}{\partial\varphi^2}\left(\sum_{n=0}^\infty \tilde{\Phi}_n(\varphi,\mathfrak{I})\right) + \frac{\partial}{\partial x} \\ \cdot \left(h(\varphi)\sum_{n=0}^\infty \tilde{\Phi}_n(\varphi,\mathfrak{I})\right)\right]. \tag{12}$$

Comparing terms by terms of Equation (12), we get

$$\mathscr{L}\left[\tilde{\Phi}_0(\varphi,\mathfrak{I})\right] = \frac{1}{s}\tilde{g}(\varphi,\mathfrak{I}),$$

$$\mathscr{L}\left[\tilde{\Phi}_1(\varphi,\mathfrak{I})\right] = \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho ABC(\varrho)}\mathscr{L} \\ \cdot \left[A\frac{\partial^2}{\partial\varphi^2}(\tilde{\Phi}_0(\varphi,\mathfrak{I})) + \frac{\partial}{\partial x}(h(\varphi)\tilde{\Phi}_0(\varphi,\mathfrak{I}))\right],$$

$$\mathscr{L}\left[\tilde{\Phi}_2(\varphi,\mathfrak{I})\right] = \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho ABC(\varrho)}\mathscr{L} \\ \cdot \left[A\frac{\partial^2}{\partial\varphi^2}(\tilde{\Phi}_1(\varphi,\mathfrak{I})) + \frac{\partial}{\partial x}(h(\varphi)\tilde{\Phi}_1(\varphi,\mathfrak{I}))\right],$$

$$\vdots$$

$$\mathscr{L}\left[\tilde{\Phi}_{n+1}(\varphi,\mathfrak{I})\right] = \frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho ABC(\varrho)}\mathscr{L} \\ \cdot \left[A\frac{\partial^2}{\partial\varphi^2}(\tilde{\Phi}_n(\varphi,\mathfrak{I})) + \frac{\partial}{\partial x}(h(\varphi)\tilde{\Phi}_n(\varphi,\mathfrak{I}))\right], n \geq 0. \tag{13}$$

Applying inverse Laplace transform in Equation (13), we have

$$\tilde{\Phi}_0(\varphi, \mathfrak{F}) = \mathscr{L}^{-1}\left[\frac{1}{s}\tilde{g}(\varphi, \mathfrak{F})\right],$$

$$\tilde{\Phi}_1(\varphi, \mathfrak{F}) = \mathscr{L}^{-1}\left[\frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho ABC(\varrho)}\mathscr{L}\left[A\frac{\partial^2}{\partial\varphi^2}\left(\tilde{\Phi}_0(\varphi, \mathfrak{F})\right)\right.\right.$$
$$\left.\left. + \frac{\partial}{\partial x}\left(h(\varphi)\tilde{\Phi}_0(\varphi, \mathfrak{F})\right)\right]\right],$$

$$\tilde{\Phi}_2(\varphi, \mathfrak{F}) = \mathscr{L}^{-1}\left[\frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho ABC(\varrho)}\mathscr{L}\left[A\frac{\partial^2}{\partial\varphi^2}\left(\tilde{\Phi}_1(\varphi, \mathfrak{F})\right)\right.\right.$$
$$\left.\left. + \frac{\partial}{\partial x}\left(h(\varphi)\tilde{\Phi}_1(\varphi, \mathfrak{F})\right)\right]\right],$$

$$\vdots$$

$$\tilde{\Phi}_{n+1}(\varphi, \mathfrak{F}) = \mathscr{L}^{-1}\left[\frac{[s^\varrho(1-\varrho)+\varrho]}{s^\varrho ABC(\varrho)}\mathscr{L}\left[A\frac{\partial^2}{\partial\varphi^2}\left(\tilde{\Phi}_n(\varphi, \mathfrak{F})\right)\right.\right.$$
$$\left.\left. + \frac{\partial}{\partial x}\left(h(\varphi)\tilde{\Phi}_n(\varphi, \mathfrak{F})\right)\right]\right], n \geq 0. \tag{14}$$

Thus, the fuzzy results are achieved as

$$\underline{\Phi}(\varphi, \mathfrak{F}) = \sum_{n=0}^{\infty}\underline{\Phi}_n(\varphi, \mathfrak{F}), \bar{\Phi}(\varphi, \mathfrak{F}) = \sum_{n=0}^{\infty}\bar{\Phi}_n(\varphi, \mathfrak{F}). \tag{15}$$

4. Numerical Results

In this section, we compute the following fractional fuzzy nonlinear equal width problems via the developed algorithm.

4.1. Case-I. Consider fractional nonlinear equal width equation under the fuzzy initial conditions as

$$^{ABC}D_\mathfrak{F}^\varrho\tilde{\Phi} + \tilde{\Phi}\tilde{\Phi}_\varphi - \tilde{\Phi}_{\varphi\varphi\mathfrak{F}} = 0, \mathfrak{F} > 0, \varphi \in R, 0 < \rho \leq 1, \tag{16}$$

and the initial condition is

$$\tilde{\Phi}(\varphi, 0) = \tilde{\kappa}3 \sec h^2\left(\frac{\varphi - 15}{2}\right), \tilde{\kappa} = (\underline{\kappa}(\delta)\bar{\kappa}(\delta)) = (\delta - 1, 1 - \delta). \tag{17}$$

Applying the suggested technique, we get

$$\underline{\Phi}_0(\varphi, \mathfrak{F}) = \underline{\kappa}(\delta)3 \sec h^2\left(\frac{\varphi - 15}{2}\right), \tag{18}$$

$$\bar{\Phi}_0(\varphi, \mathfrak{F}) = \bar{\kappa}(\delta)3 \sec h^2\left(\frac{\varphi - 15}{2}\right), \tag{19}$$

$$\underline{\Phi}_1(\varphi, \mathfrak{F}) = \frac{\underline{\kappa}(\delta)9 \sec h^4(\varphi - 15/2)\tanh(\varphi - 15/2)}{ABC(\varrho)}\left[1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right], \tag{20}$$

$$\bar{\Phi}_1(\varphi, \mathfrak{F}) = \frac{\bar{\kappa}(\delta)9 \sec h^4(\varphi - 15/2)\tanh(\varphi - 15/2)}{ABC(\varrho)}\left[1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right], \tag{21}$$

$$\underline{\Phi}_2(\varphi, \mathfrak{F}) = \frac{9}{4}\frac{\underline{\kappa}(\delta)}{\cosh^1 2(1/2\varphi - 15/2)}$$
$$\cdot\left[\sinh\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\left\{-24\frac{1}{(ABC(\varrho))^2}\right.\right.$$
$$\cdot\left((1-\varrho)^2 + \frac{2\varrho(1-\varrho)\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)} + \frac{\varrho^2\mathfrak{F}^{2\varrho}}{\Gamma(2\varrho + 1)}\right)\cosh^3$$
$$\cdot\left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 30\frac{1}{(ABC(\varrho))^2}$$
$$\cdot\left((1-\varrho)^2 + \frac{2\varrho(1-\varrho)\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)} + \frac{\varrho^2\mathfrak{F}^{2\varrho}}{\Gamma(2\varrho + 1)}\right)\cosh$$
$$\cdot\left(\frac{1}{2}\varphi - \frac{15}{2}\right) - 72\frac{1}{(ABC(\varrho))}$$
$$\cdot\left(1 - \rho + \frac{\rho\mathfrak{F}^\rho}{\Gamma(\rho + 1)}\right)\sinh\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\cosh$$
$$\cdot\left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 135\frac{1}{ABC(\rho)}\left(1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right)\sinh$$
$$\cdot\left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 4\cosh^7\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\right\}\frac{1}{ABC(\varrho)}$$
$$\left.\cdot\left(1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right)\right], \tag{22}$$

$$\bar{\Phi}_2(\varphi, \mathfrak{F}) = \frac{9}{4}\frac{\bar{\kappa}(\delta)}{\cosh^1 2(1/2\varphi - 15/2)}$$
$$\cdot\left[\sinh\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\left\{-24\frac{1}{(ABC(\varrho))^2}\right.\right.$$
$$\cdot\left((1-\varrho)^2 + \frac{2\varrho(1-\varrho)\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)} + \frac{\varrho^2\mathfrak{F}^{2\varrho}}{\Gamma(2\varrho + 1)}\right)\&\cosh^3$$
$$\cdot\left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 30\frac{1}{(ABC(\varrho))^2}$$
$$\cdot\left((1-\varrho)^2 + \frac{2\varrho(1-\varrho)\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)} + \frac{\varrho^2\mathfrak{F}^{2\varrho}}{\Gamma(2\varrho + 1)}\right)\cosh$$
$$\cdot\left(\frac{1}{2}\varphi - \frac{15}{2}\right) - 72\frac{1}{(ABC(\varrho))}\left(1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right)\sinh$$
$$\cdot\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\cosh\left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 135\frac{1}{ABC(\varrho)}$$
$$\cdot\left(1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right)\sinh\left(\frac{1}{2}\varphi - \frac{15}{2}\right)$$
$$\left.+ 4\cosh^7\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\right\}\frac{1}{ABC(\varrho)}\left(1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right)\right]. \tag{23}$$

In a similar fashion, we can obtain the higher terms. The series solution is obtained using Equation (16); therefore, we write
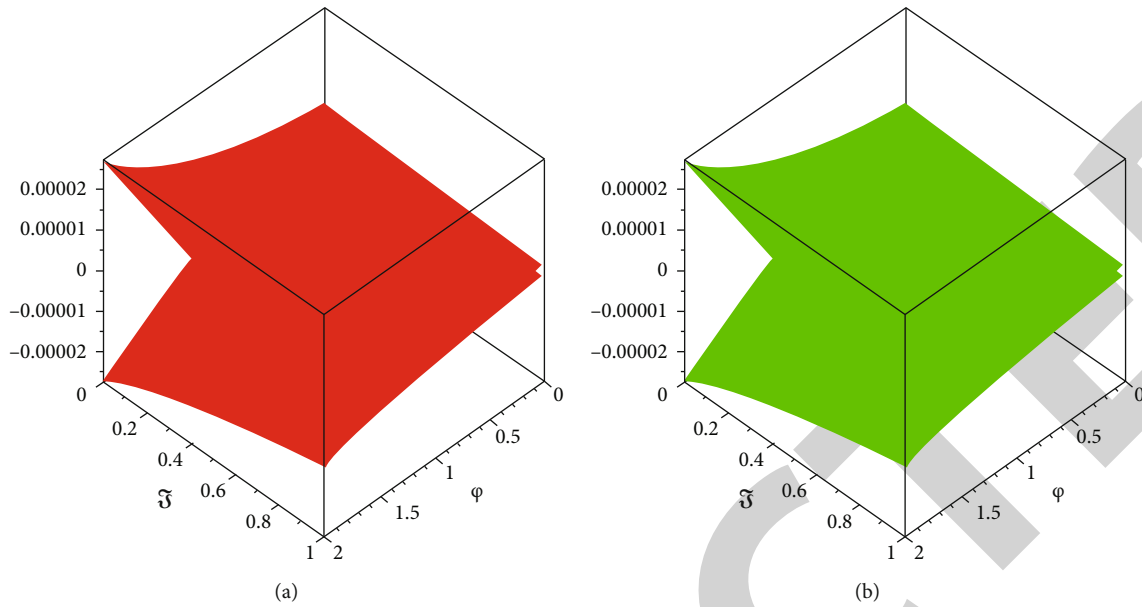
(a)

(b)

Figure 1: (a) The three-dimensional fuzzy upper and lower branch graph of approximate series result at $\varrho = 1$ and (b) at fractional-order of $\varrho = 0.8$.

$$\tilde{\Phi}(\varphi, \mathfrak{I}) = \tilde{\Phi}_0(\varphi, \mathfrak{I}) + \tilde{\Phi}_1(\varphi, \mathfrak{I}) + \tilde{\Phi}_2(\varphi, \mathfrak{I}) + \tilde{\Phi}_3(\varphi, \mathfrak{I}) + \tilde{\Phi}_4(\varphi, \mathfrak{I}) + \cdots. \tag{24}$$

The lower and upper portion form is

$$\underline{\Phi}(\varphi, \mathfrak{I}) = \underline{\Phi}_0(\varphi, \mathfrak{I}) + \underline{\Phi}_1(\varphi, \mathfrak{I}) + \underline{\Phi}_2(\varphi, \mathfrak{I}) + \underline{\Phi}_3(\varphi, \mathfrak{I}) + \underline{\Phi}_4(\varphi, \mathfrak{I}) + \cdots,$$

$$\bar{\Phi}(\varphi, \mathfrak{I}) = \bar{\Phi}_0(\varphi, \mathfrak{I}) + \bar{\Phi}_1(\varphi, \mathfrak{I}) + \bar{\Phi}_2(\varphi, \mathfrak{I}) + \bar{\Phi}_3(\varphi, \mathfrak{I}) + \bar{\Phi}_4(\varphi, \mathfrak{I}) + \cdots, \tag{25}$$

$$\underline{\Phi}(\varphi, \mathfrak{I}) = \underline{\kappa}(\delta)3 \sec h^2\left(\frac{\varphi - 15}{2}\right)$$
$$+ \frac{\underline{\kappa}(\delta)9 \sec h^4(\varphi - 15/2) \tanh (\varphi - 15/2)}{ABC(\varrho)}\left[1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right]$$
$$+ \frac{9}{4}\frac{\underline{\kappa}(\delta)}{\cosh^1 2(1/2\varphi - 15/2)}$$
$$\cdot \left[\sinh\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\left\{-24\frac{1}{(ABC(\varrho))^2}\right.\right.$$
$$\cdot \left((1 - \varrho)^2 + \frac{2\varrho(1 - \varrho)\mathfrak{I}^\varrho}{\Gamma(\varrho + 1)} + \frac{\varrho^2 \mathfrak{I}^{2\varrho}}{\Gamma(2\varrho + 1)}\right) \cosh^3\left(\frac{1}{2}\varphi - \frac{15}{2}\right)$$
$$+ 30\frac{1}{(ABC(\varrho))^2}\left((1 - \varrho)^2 + \frac{2\varrho(1 - \varrho)\mathfrak{I}^\varrho}{\Gamma(\varrho + 1)} + \frac{\varrho^2 \mathfrak{I}^{2\varrho}}{\Gamma(2\varrho + 1)}\right) \cosh$$
$$\cdot \left(\frac{1}{2}\varphi - \frac{15}{2}\right) - 72\frac{1}{(ABC(\varrho))}\left(1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right) \sinh$$
$$\cdot \left(\frac{1}{2}\varphi - \frac{15}{2}\right) \cosh\left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 135\frac{1}{ABC(\varrho)}$$
$$\cdot \left(1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right) \sinh\left(\frac{1}{2}\varphi - \frac{15}{2}\right)$$
$$+ 4 \cosh^7\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\left.\left.\right\}\frac{1}{ABC(\varrho)}\left(1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right)\right] \cdots, \tag{26}$$

$$\bar{\Phi}(\varphi, \mathfrak{I}) = \bar{\kappa}(\delta)3 \sec h^2\left(\frac{\varphi - 15}{2}\right)$$
$$+ \frac{\bar{\kappa}(\delta)9 \sec h^4(\varphi - 15/2) \tanh (\varphi - 15/2)}{ABC(\varrho)}$$
$$\cdot \left[1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right] + \frac{9}{4}\frac{\bar{\kappa}(\delta)}{\cosh^1 2(1/2\varphi - 15/2)}$$
$$\cdot \left[\sinh\left(\frac{1}{2}\varphi - \frac{15}{2}\right)\left\{-24\frac{1}{(ABC(\varrho))^2}\right.\right.$$
$$\cdot \left((1 - \varrho)^2 + \frac{2\varrho(1 - \varrho)\mathfrak{I}^\varrho}{\Gamma(\varrho + 1)} + \frac{\varrho^2 \mathfrak{I}^{2\varrho}}{\Gamma(2\varrho + 1)}\right) \cosh^3$$
$$\cdot \left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 30\frac{1}{(ABC(\varrho))^2}$$
$$\cdot \left((1 - \varrho)^2 + \frac{2\varrho(1 - \varrho)\mathfrak{I}^\varrho}{\Gamma(\varrho + 1)} + \frac{\varrho^2 \mathfrak{I}^{2\varrho}}{\Gamma(2\varrho + 1)}\right) \cosh$$
$$\cdot \left(\frac{1}{2}\varphi - \frac{15}{2}\right) - 72\frac{1}{(ABC(\varrho))}\left(1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right) \sinh$$
$$\cdot \left(\frac{1}{2}\varphi - \frac{15}{2}\right) \cosh\left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 135\frac{1}{ABC(\varrho)}$$
$$\cdot \left(1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right) \sinh\left(\frac{1}{2}\varphi - \frac{15}{2}\right) + 4 \cosh^7$$
$$\cdot \left(\frac{1}{2}\varphi - \frac{15}{2}\right)\left.\left.\right\}\frac{1}{ABC(\varrho)}\left(1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right)\right] \cdots, \tag{27}$$

which is the classical result of upper and lower fuzzy of the given model. The exact solution is as follows:

$$\tilde{\Phi}(\varphi, \mathfrak{I}) = \tilde{\kappa}3 \sec h^2\left(\frac{\varphi - 15 - \mathfrak{I}}{2}\right). \tag{28}$$

Figure 1(a) shows the three-dimensional fuzzy upper and lower branch graph of approximate series result at $\varrho = 1$ and Figure 1(b) at fractional-order of $\varrho = 0.8$. Figure 2(a) shows the three-dimensional fuzzy lower and
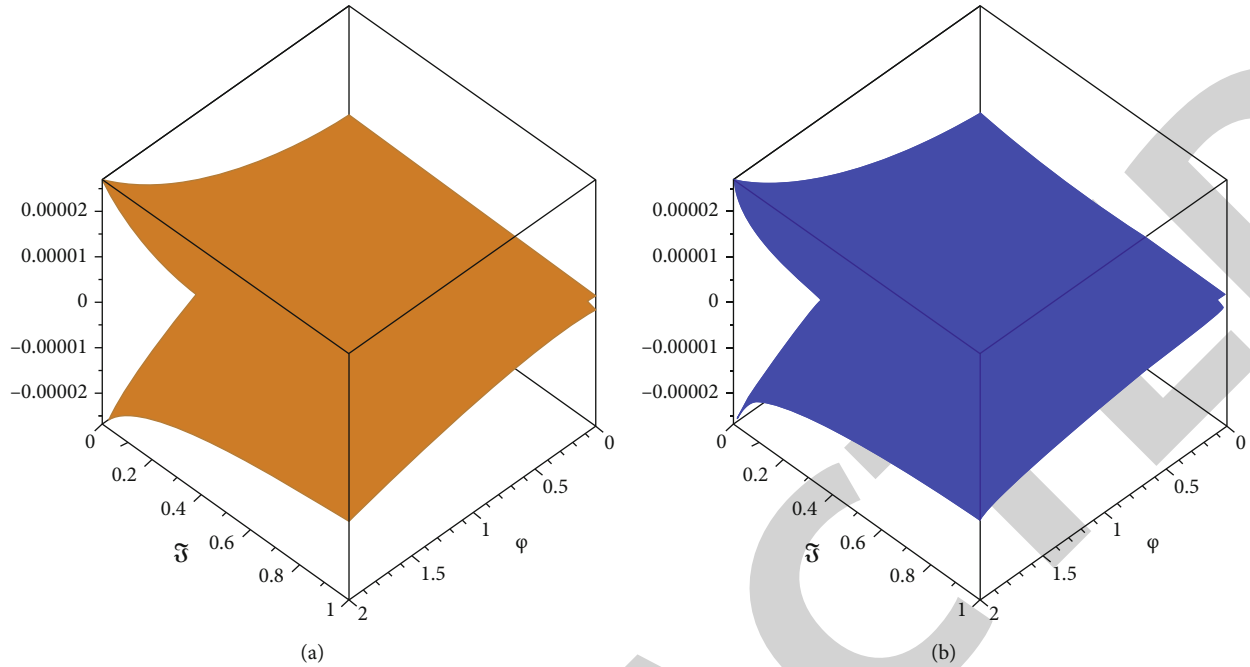
(a)　　　　　　　　　　　　　　　　　　　　　　　(b)

Figure 2: (a) The three-dimensional fuzzy lower and upper branch graph of approximate series result at $\varrho = 0.6$ and (b) at fractional-order of $\varrho = 0.4$.

upper branch graph of approximate series result at $\varrho = 0.6$ and Figure 2(b) at fractional-order of $\varrho = 0.4$. In Figure 3, the graph shows the three-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ. In Figure 4, it shows the two-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ.

4.2. Case-II.
Consider fractional nonlinear modified equal width equation under the fuzzy initial conditions as

$$^{ABC}D_{\mathfrak{I}}^{\varrho}\tilde{\Phi} + 3\tilde{\Phi}^2\tilde{\Phi}_{\varphi} - \tilde{\Phi}_{\varphi\varphi\mathfrak{I}} = 0, \mathfrak{I} > 0, \varphi \in R, 0 < \rho \le 1, \quad (29)$$

and the initial condition is

$$\tilde{\Phi}(\varphi, 0) = \tilde{\kappa}\frac{1}{4}\ \text{sech}\ (\varphi - 30), \tilde{\kappa} = (\underline{\kappa}(\delta)\bar{\kappa}(\delta)) = (\delta - 1, 1 - \delta). \quad (30)$$

Applying the suggested technique, we get

$$\underline{\Phi}_0(\varphi, \mathfrak{I}) = \underline{\kappa}(\delta)\frac{1}{4}\ \text{sec}\ h(\varphi - 30),$$

$$\bar{\Phi}_0(\varphi, \mathfrak{I}) = \bar{\kappa}(\delta)\frac{1}{4}\ \text{sec}\ h(\varphi - 30),$$

$$\underline{\Phi}_1(\varphi, \mathfrak{I}) = \frac{\underline{\kappa}(\delta)\left(3/64\ \text{sec}\ h^3(\varphi - 30)\ \tanh\ (\varphi - 30)\right)}{ABC(\varrho)}\left[1 - \varrho + \frac{\varrho\mathfrak{I}^{\varrho}}{\Gamma(\varrho + 1)}\right],$$

$$\bar{\Phi}_1(\varphi, \mathfrak{I}) = \frac{\bar{\kappa}(\delta)\left(3/64\ \text{sec}\ h^3(\varphi - 30)\ \tanh\ (\varphi - 30)\right)}{ABC(\varrho)}\left[1 - \varrho + \frac{\varrho\mathfrak{I}^{\varrho}}{\Gamma(\varrho + 1)}\right]. \quad (31)$$

In a similar fashion, we can obtain the higher terms. The series solution is obtained using Equation (29); therefore, we write

$$\tilde{\Phi}(\varphi, \mathfrak{I}) = \tilde{\Phi}_0(\varphi, \mathfrak{I}) + \tilde{\Phi}_1(\varphi, \mathfrak{I}) + \tilde{\Phi}_2(\varphi, \mathfrak{I}) + \tilde{\Phi}_3(\varphi, \mathfrak{I}) + \cdots. \quad (32)$$

The lower and upper portion form is

$$\underline{\Phi}(\varphi, \mathfrak{I}) = \underline{\Phi}_0(\varphi, \mathfrak{I}) + \underline{\Phi}_1(\varphi, \mathfrak{I}) + \underline{\Phi}_2(\varphi, \mathfrak{I}) + \underline{\Phi}_3(\varphi, \mathfrak{I}) + \cdots,$$

$$\bar{\Phi}(\varphi, \mathfrak{I}) = \bar{\Phi}_0(\varphi, \mathfrak{I}) + \bar{\Phi}_1(\varphi, \mathfrak{I}) + \bar{\Phi}_2(\varphi, \mathfrak{I}) + \bar{\Phi}_3(\varphi, \mathfrak{I}) + \cdots, \quad (33)$$

$$\bar{\Phi}(\varphi, \mathfrak{I}) = \underline{\kappa}(\delta)\frac{1}{4}\ \text{sech}\ (\varphi - 30)$$
$$+ \frac{\underline{\kappa}(\delta)\left(3/64\ \text{sech}^3(\varphi - 30)\ \tanh\ (\varphi - 30)\right)}{ABC(\varrho)}$$
$$\cdot \left[1 - \varrho + \frac{\varrho\mathfrak{I}^{\varrho}}{\Gamma(\varrho + 1)}\right] + \cdots,$$

$$\bar{\Phi}(\varphi, \mathfrak{I}) = \bar{\kappa}(\delta)\frac{1}{4}\ \text{sech}\ (\varphi - 30)$$
$$+ \frac{\bar{\kappa}(\delta)\left(3/64\ \text{sech}^3(\varphi - 30)\ \tanh\ (\varphi - 30)\right)}{ABC(\varrho)}$$
$$\left[1 - \varrho + \frac{\varrho\mathfrak{I}^{\varrho}}{\Gamma(\varrho + 1)}\right] + \cdots,$$
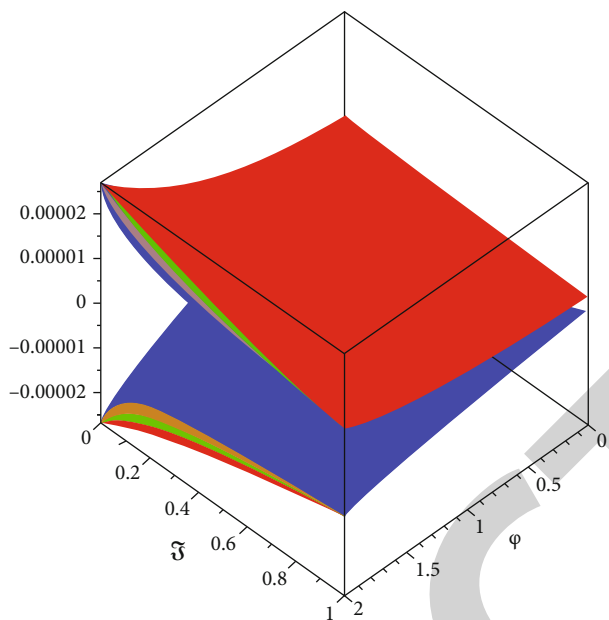
$$(34)$$

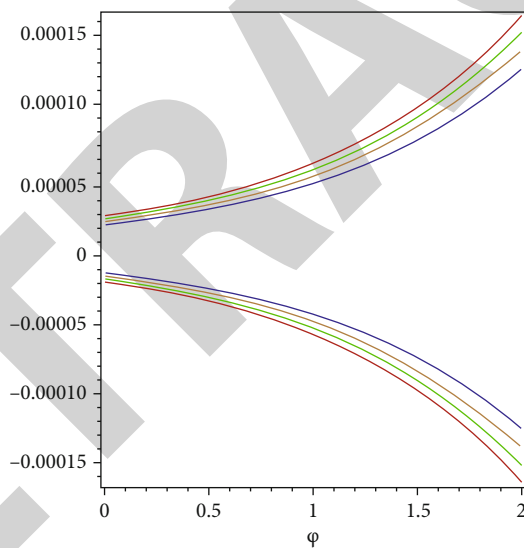FIGURE 3: The three-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ.

FIGURE 4: The two-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ.

which is the classical result of lower and upper fuzzy of the given model. The exact solution is as follows:

$$\tilde{\Phi}(\varphi, \mathfrak{I}) = \tilde{\kappa} \frac{1}{4} \sec h\left(x - 30 - \frac{\mathfrak{I}}{4}\right). \tag{35}$$

Figure 5(a) shows the three-dimensional fuzzy upper and lower branch graph of approximate series result at ϱ = 1 and Figure 5(b) at fractional-order of ϱ = 0.8. Figure 6(a)shows the three-dimensional fuzzy lower and upper branch graph of approximate series result at ϱ = 0.6 and Figure 6(b) at fractional-order of ϱ = 0.4. In Figure 7, the graph shows the three-dimensional fuzzy lower and upper branch graph of the different fractional-

order of ϱ. In Figure 8, it shows the two-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ.

4.3. Case-III. Consider fractional-order variant modified nonlinear equal width equation under the fuzzy initial conditions as

$$^{ABC}D_{\mathfrak{I}}^{\varrho}\tilde{\Phi} + \frac{12}{7}\left(\tilde{\Phi}^6\right)_{\varphi} - \frac{3}{7}\left(\tilde{\Phi}^6\right)_{\varphi\varphi\mathfrak{I}} = 0, \mathfrak{I} > 0, \varphi \in R, 0 < \rho \leq 1,$$
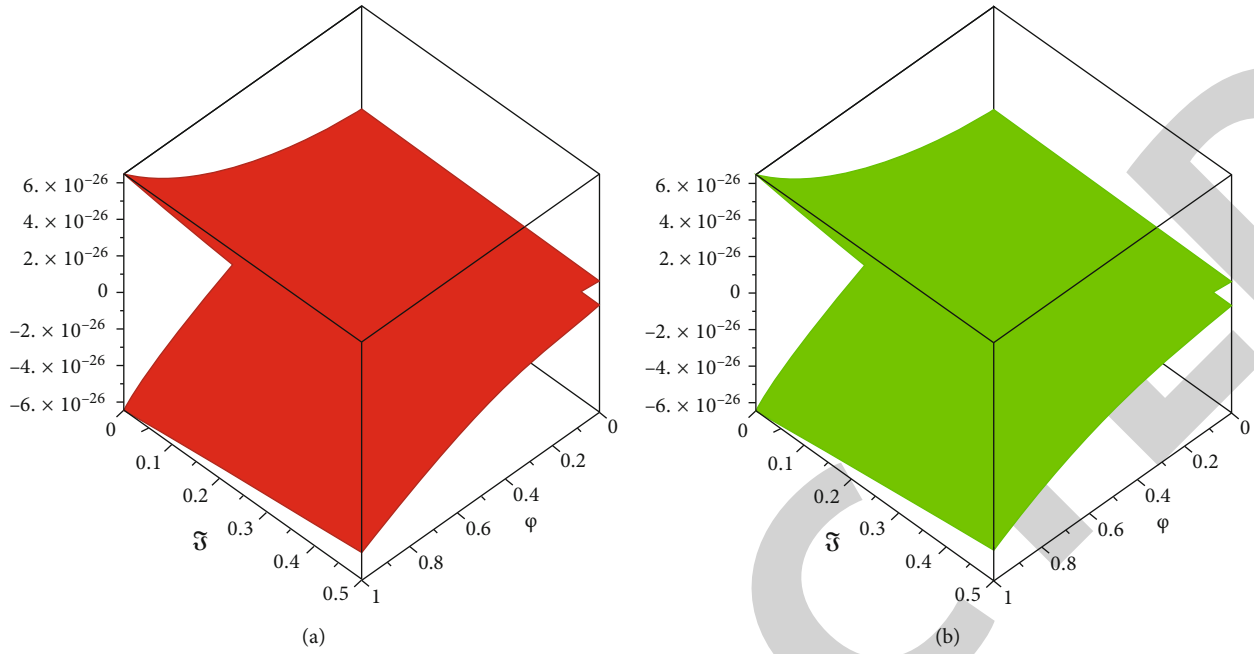
$$(36)$$

with initial condition

(a)

(b)

FIGURE 5: (a) The three-dimensional fuzzy lower and upper branch graph of approximate series result at $\varrho = 1$ and (b) at fractional-order of $\varrho = 0.8$.
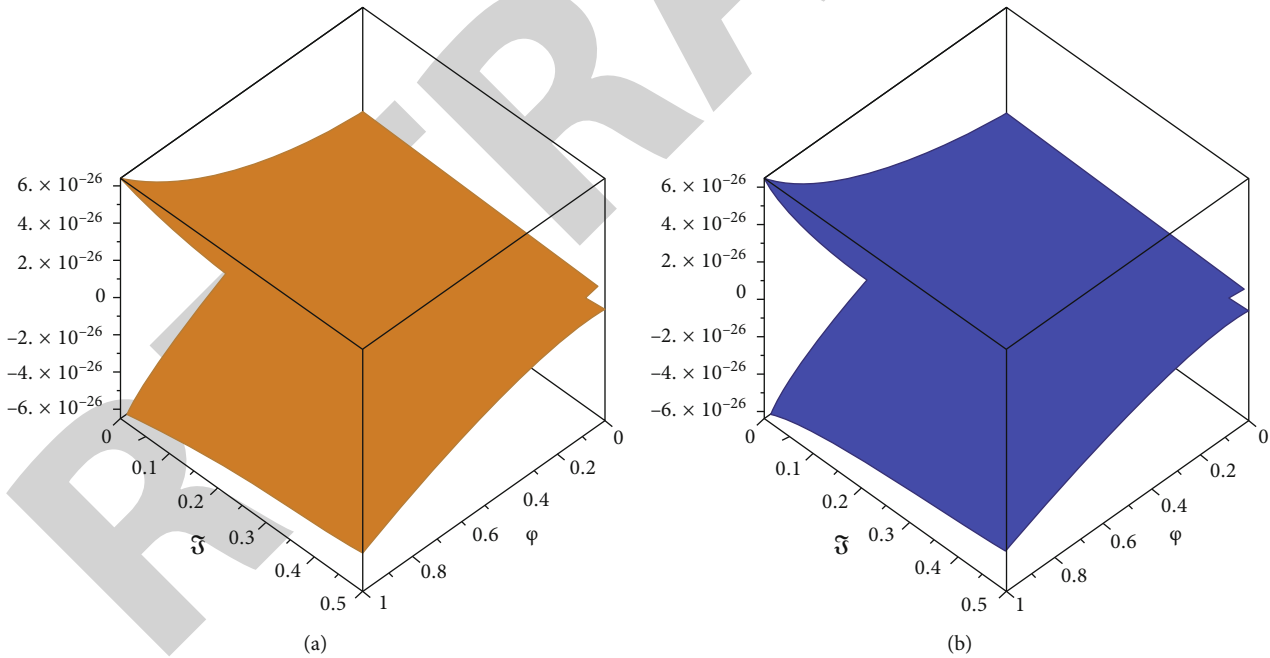

(a)

(b)

FIGURE 6: (a) The three-dimensional fuzzy lower and upper branch graph of approximate series result at $\varrho = 0.6$ and (b) at fractional-order of $\varrho = 0.4$.
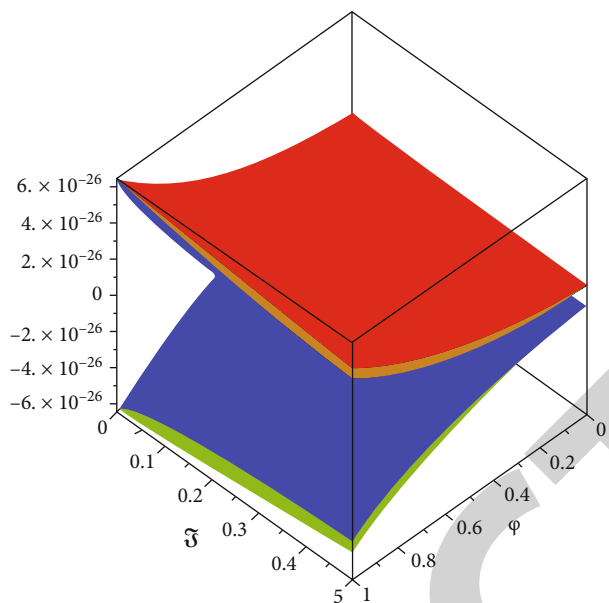
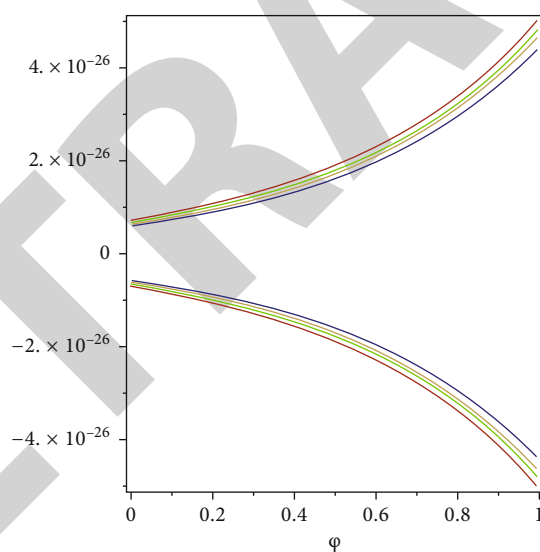FIGURE 7: The three-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ.

FIGURE 8: The two-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ.

$$\tilde{\Phi}(\varphi, 0) = \cosh^{2/5}\left(\frac{5\varphi}{6}\right). \tag{37}$$

Applying the suggested technique, we get

$$\underline{\Phi}_0(\varphi, \mathfrak{F}) = \underline{\kappa}(\delta) \cosh^{2/5}\left(\frac{5\varphi}{6}\right),$$

$$\bar{\Phi}_0(\varphi, \mathfrak{F}) = \bar{\kappa}(\delta) \cosh^{2/5}\left(\frac{5\varphi}{6}\right),$$

$$\underline{\Phi}_1(\varphi, \mathfrak{F}) = \frac{\underline{\kappa}(\delta)\left(-24/7 \cosh^{7/5}(5\varphi/6) \sinh (5\varphi/6)\right)}{ABC(\varrho)}\left[1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right],$$

$$\bar{\Phi}_1(\varphi, \mathfrak{F}) = \frac{\bar{\kappa}(\delta)\left(-24/7 \cosh^{7/5}(5\varphi/6) \sinh (5\varphi/6)\right)}{ABC(\varrho)}\left[1 - \varrho + \frac{\varrho\mathfrak{F}^\varrho}{\Gamma(\varrho + 1)}\right]. \tag{38}$$

In a similar fashion, we can obtain the higher terms. The series solution is obtained using Equation (36); therefore, we write

$$\tilde{\Phi}(\varphi, \mathfrak{F}) = \tilde{\Phi}_0(\varphi, \mathfrak{F}) + \tilde{\Phi}_1(\varphi, \mathfrak{F}) + \tilde{\Phi}_2(\varphi, \mathfrak{F}) + \tilde{\Phi}_3(\varphi, \mathfrak{F}) + \cdots. \tag{39}$$
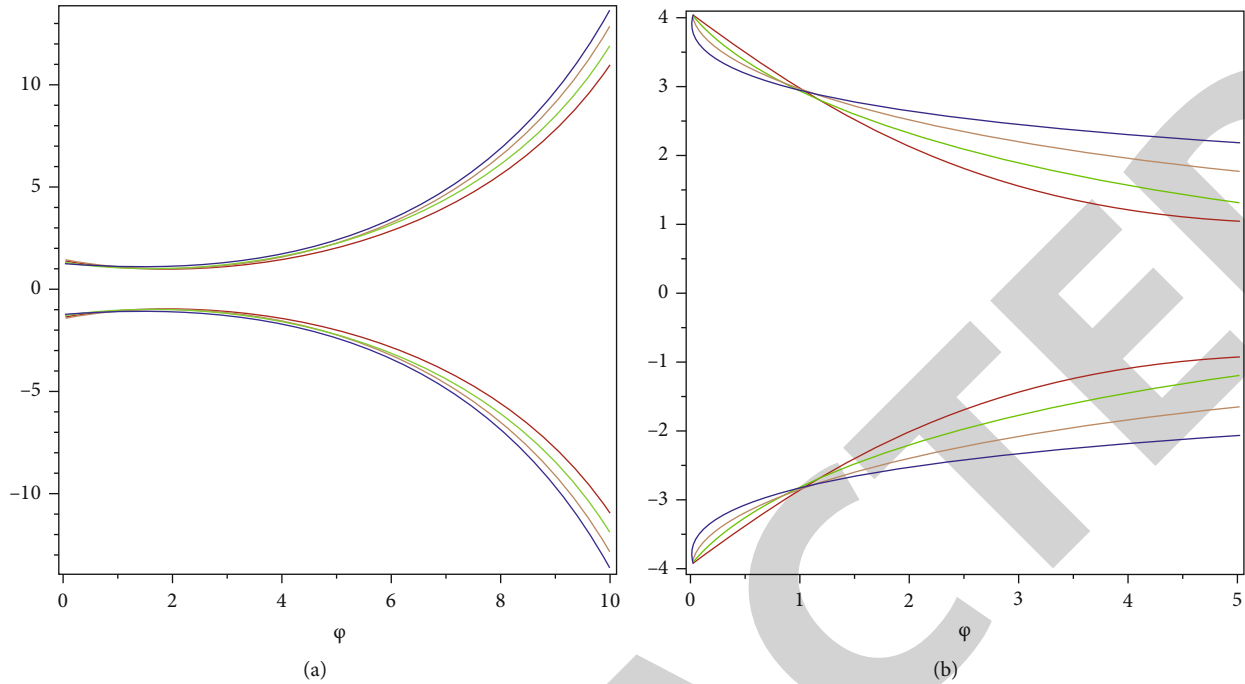
Figure 9: (a) The two-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ with respect to φ and (b) with respect to $\mathfrak{I}$.

The lower and upper portion form is

$$
\underline{\Phi}(\varphi, \mathfrak{I}) = \underline{\Phi}_0(\varphi, \mathfrak{I}) + \underline{\Phi}_1(\varphi, \mathfrak{I}) + \underline{\Phi}_2(\varphi, \mathfrak{I}) + \underline{\Phi}_3(\varphi, \mathfrak{I}) + \cdots,
$$
$$
\bar{\Phi}(\varphi, \mathfrak{I}) = \bar{\Phi}_0(\varphi, \mathfrak{I}) + \bar{\Phi}_1(\varphi, \mathfrak{I}) + \bar{\Phi}_2(\varphi, \mathfrak{I}) + \bar{\Phi}_3(\varphi, \mathfrak{I}) + \cdots,
$$

$$(40)$$

$$
\underline{\Phi}(\varphi, \mathfrak{I}) = \underline{\kappa} \cosh^{2/5}\left(\frac{5\varphi}{6}\right)
$$
$$
+ \frac{\underline{\kappa}(\delta)\left(-24/7 \cosh^{7/5}(5\varphi/6) \sinh(5\varphi/6)\right)}{ABC(\varrho)}
$$
$$
\cdot \left[1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right] + \cdots,
$$
$$
\bar{\Phi}(\varphi, \mathfrak{I}) = \bar{\kappa}(\delta) \cosh^{2/5}\left(\frac{5\varphi}{6}\right)
$$
$$
+ \frac{\bar{\kappa}(\delta)\left(-24/7 \cosh^{7/5}(5\varphi/6) \sinh(5\varphi/6)\right)}{ABC(\varrho)}
$$
$$
\cdot \left[1 - \varrho + \frac{\varrho \mathfrak{I}^\varrho}{\Gamma(\varrho + 1)}\right] + \cdots,
$$

$$(41)$$

which is the classical result of upper and lower fuzzy of the given model. The exact solution is as follows:

$$
\tilde{\Phi}(\varphi, \mathfrak{I}) = \tilde{\kappa} \cosh^{2/5}\left\{\frac{5}{6}(\varphi - \mathfrak{I})\right\}.
$$

$$(42)$$

Figure 9(a) shows the two-dimensional fuzzy lower and upper branch graph of the different fractional-order of ϱ with respect to φ and Figure 9(b) with respect to $\mathfrak{I}$.

5. Conclusion

We have successfully introduced a Laplace transform method to calculate several numerical solutions for fractional nonlinear equal width equations under fuzzy concepts. We have investigated the proposed problem. Some significant findings have been produced. Also, for the analytical results, we have provided the graphic representation by using Maple 13. Further, we noted that the solutions converged to their actual results at the integer-order in all three models. Since we have provided the analytical results for the first few terms corresponding to various fractional-order and at values obtained of uncertainty and space variables, we noted that fractional derivative provides a comprehensive spectrum of fuzzy results to the evaluated models. In future research, this technique can be implemented to obtain analytical and approximate results of perturbed fractional differential equations under the uncertainty equipped with nonclassical and integral boundary conditions in the sense of the Atangana-Baleanu operator.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

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