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Research Article

New Results on Perov-Interpolative Contractions of Suzuki Type Mappings

Erdal Karapınar,[1,2,3] **Andreea Fulga**,[4] **and Seher Sultan Yesilkaya**[1]

[1]*Division of Applied Mathematics, Thu Dau Mot University, Binh Duong Province, Vietnam*
[2]*Department of Mathematics, Cankaya University, 06790 Etimesgut, Ankara, Turkey*
[3]*Department of Medical Research, China Medical University Hospital, China Medical University, 40402 Taichung, Taiwan*
[4]*Department of Mathematics and Computer Sciences, Transilvania University of Brasov, Brasov, Romania*

Correspondence should be addressed to Andreea Fulga; afulga@unitbv.ro

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In this paper, we introduce some common fixed point theorems for interpolative contraction operators using Perov operator which satisfy Suzuki type mappings. Further, some results are given. These results generalize several new results present in the literature.

1. Introduction

Banach [1] introduced the Banach contraction principle that generalized in various wide directions by many researchers. One of the generalizations was supposed by Kannan [2] in 1968 and later with other researchers such as C′iric′ Reich Rus. In 2018, Karapınar [3] adopted the interpolative approach to define the generalized Kannan type contraction on a complete metric space and proved the following.

A mapping $F : \mathscr{P} \longrightarrow \mathscr{P}$ on $(\mathscr{P}, d)$ a complete metric space such that

$$d(Fx, Fz) \le k[d(x, Fx)]^{\alpha} \cdot [d(z, Fz)]^{1-\alpha}, \tag{1}$$

where $k \in [0, 1)$ and $\alpha \in (0, 1)$, for each $x, z \in \mathscr{P} \setminus Fix(\mathrm{F})$. Then, F has a unique fixed point in $\mathscr{P}$. Afterward, this concept has been extended in different aspects for example [4–14] and also see e.g. [15–19].

On the other hand, Perov [20, 21] gave a characterization of Banach contraction principle in the framework vector-valued metric space.

For a nonempty set $\mathscr{P}$, a function $\mathrm{d} : \mathscr{P} \times \mathscr{P} \longrightarrow \mathbb{R}^k$ is called a vector-valued metric on $\mathscr{P}$ if the followings are fulfilled:

(1) $d(x, z) \ge 0$ for all $x, z \in \mathscr{P}$; if $d(x, z) = 0$, then $x = z$

(2) $d(x, z) = d(z, x)$ for all $x, z \in \mathscr{P}$

(3) $d(x, z) \le d(x, t) + d(t, z)$ for all $x, z \in \mathscr{P}$

(4) where $0 \coloneqq \underbrace{(0, 0, \cdots, 0)}_{k\text{-times}}$. We mention that, for $x, y \in \mathbb{R}^k$, that is, $x = (x_i)_{i=1}^k$ and $y = (y_i)_{i=1}^k$

$$x \le y \Leftrightarrow x_i \le y_i \quad \text{for each} \quad i \in \{1, 2, 3, \cdots, k\}. \tag{2}$$

The notations $M_{mm}(\mathbb{R})$ (respectively, $M_{mm}(\mathbb{R}_0^+)$) denote the collection of all square matrices of real numbers (respectively, nonnegative real numbers) where $\mathbb{R}$ (respectively, $\mathbb{R}_0^+ \coloneqq [0,\infty)$) is the set of real numbers (respectively, nonnegative real numbers). Furthermore, $\mathbb{C}$ denotes the complex numbers, as usual.

A matrix $A \in M_{mm}(\mathbb{R})$ converges to zero if and only if its spectral radius is strictly less than 1, that is, $\rho(A) < 1$, see, e.g., [22]. It is equivalent to saying that all the eigenvalues of A are in the open unit disc, that is, $|\lambda| < 1$, for every $\lambda \in \mathbb{C}$ with $\det(A - \lambda I) = 0$, where I denotes the unit matrix of $M_{mm}(\mathbb{R})$.

Theorem 1 (see, e.g., [22, 23]). *Let $A \in M_{mm}(\mathbb{R}_0^+)$. Then, the following assertions are equivalent:*

(i) A converges to zero

$$\rho(A) < 1; \tag{3}$$

(ii) $A^n \longrightarrow 0$ as $n \longrightarrow \infty$

(iii) The matrix $(I - A)$ is nonsingular and

$$(I - A)^{-1} = I + A + \cdots + A^n + \cdots; \tag{4}$$

(iv) The matrix $(I - A)$ is nonsingular and $(I - A)^{-1}$ has nonnegative elements

(v) $A^n q$ and qA^n are convergent towards zero as $n \longrightarrow \infty$, for each $q \in \mathbb{R}^m$

Note also that if $A, B \in M_{mm}(\mathbb{R}_0^+)$ with $A \leq B$ (in the component-wise meaning), then, $\rho(B) < 1$ implies $\rho(A) < 1$.

Theorem 2 (Perov Cauchy, perov1966certain). *Let $(\mathscr{P}, d)$ be a complete vector-valued metric space and the operator $F : \mathscr{P} \longrightarrow \mathscr{P}$ with the property that there exists a matrix $A \in M_{mm}(\mathbb{R}_0^+)$ convergent towards zero such that*

$$d(F(x), F(z)) \leq A d(x, z), \quad \text{for all} \quad x, z \in \mathscr{P}. \tag{5}$$

Then, F possesses a unique fixed point.

Ali et al. [24] defined Λ admissible that a generalized of α-admissible given by Samet et al. [25].

Definition 3 (see [24]). Let $\mathscr{P} \neq \varnothing$, $\Lambda : \mathscr{P} \times \mathscr{P} \longrightarrow M_{mm}(\mathbb{R}_+)$. A mapping $F : \mathscr{P} \longrightarrow \mathscr{P}$
is said to be Λ-admissible if

$$\Lambda(x, z) \geq I \quad \text{implies} \quad \Lambda(Fx, Fz) \geq I, \quad \text{for all} \quad x, z \in \mathscr{P}, \tag{6}$$

where I is the $m \times m$ identity matrix and the inequality between matrices mans entrywise inequality.

We define some related to Λ-admissible the following concept of admissibility using the above definition and given by some authors [25–30].

Definition 4. Let $\mathscr{P} \neq \varnothing$, $\Lambda : \mathscr{P} \times \mathscr{P} \longrightarrow M_{mm}(\mathbb{R}_+)$, and $F : \mathscr{P} \longrightarrow \mathscr{P}$ be a mapping. We say that F is an Λ-orbital admissible mapping if

$$\Lambda(x, Fx) \geq I \quad \text{implies} \quad \Lambda\left(Fx, F^2x\right) \geq I, \quad \text{for all} \quad x, z \in \mathscr{P}. \tag{7}$$

Moreover, an Λ-orbital admissible mapping F is said to be triangular Λ-orbital admissible if for all $x, z \in \mathscr{P}$, we have

$$\Lambda(x, z) \geq I \quad \text{and} \quad \Lambda(z, Fz) \geq I, \quad \text{implies} \quad \Lambda(x, Fz) \geq I. \tag{8}$$

Definition 5. For a nonempty set $\mathscr{P}$, let $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$ and $\Lambda : \mathscr{P} \times \mathscr{P} \longrightarrow M_{mm}(\mathbb{R}_+)$ be mappings. We say that $(F, \mathscr{G})$ is a generalized Λ -admissible pair if for all $x, z \in \mathscr{P}$, we have

$$\Lambda(x, z) \geq I \Rightarrow \Lambda(Fx, \mathscr{G}z) \geq I \quad \text{and} \quad \Lambda(\mathscr{G}z, Fx) \geq I. \tag{9}$$

Lemma 6. *Let $\mathscr{P} \neq \varnothing$ and $F : \mathscr{P} \longrightarrow \mathscr{P}$ be a triangular Λ -admissible map. Suppose that there exists $x_0 \in \mathscr{P}$ such that $\Lambda(x_0, Fx_0) \geq I$. Identify a sequence $\{x_n\}$ using $x_{n+1} = Fx_n$. Thus, we have $\Lambda(x_n, x_m) \geq I$ for all $m, n \in \mathbb{N} \cup \{0\}$ with $n < m$.*

Lemma 7. *Let $\mathscr{P} \neq \varnothing$ and $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$ be a triangular Λ -admissible mapping. Assume that there exists $x_0 \in \mathscr{P}$ such that $\Lambda(x_0, Fx_0) \geq I$. Identify sequence $x_{2r+1} = Fx_{2r}$, and $x_{2r+2} = \mathscr{G}x_{2r+1}$, where $r = 0, 1, 2, \cdots$. So, we have $\Lambda(x_n, x_m) \geq I$ for all $m, n \in \mathbb{N} \cup \{0\}$ with $n < m$.*

Recently, one of the interesting generalizations was given by Suzuki [31, 32] which characterizes the completeness of underlying metric spaces. Suzuki introduced and generalized versions of Banach's and Edelstein's basic results. In addition, Popescu [33] has modified the nonexpansiveness situation with the weaker C-condition presented by Suzuki. As stated, the existence of fixed points of maps satisfying the C-condition has been extensively studied; see [34–38]. Karapnar [39] investigated the definition of a nonexpansive mapping satisfying the C-condition.

Definition 8. A mapping F on a metric space $(\mathscr{P}, d)$ satisfies the C-condition if

$$\frac{1}{2} d(x, Fx) \leq d(x, z) \Rightarrow d(Fx, Fz) \leq d(x, z), \tag{10}$$

for each $x, z \in \mathscr{P}$.

Theorem 9. *Let $(\mathscr{P}, d)$ be a compact metric space and $F : \mathscr{P} \longrightarrow \mathscr{P}$ be a mapping satisfying condition (C) for all $x, z \in \mathscr{P}$. Then, F has a unique fixed point.*

2. Main Results

For the rest of the paper, we use the following notation: $\mathscr{P}_{\mathrm{F}} = \mathscr{P} \setminus Fix(\mathrm{F})$, where $Fix(\mathrm{F}) = \{x \in \mathscr{P} \mid \mathrm{F}x = x\}$.

Definition 10. Let $(\mathscr{P}, d)$ be a complete vector-valued metric space and $\Lambda : \mathscr{P} \times \mathscr{P} \longrightarrow M_{mm}(\mathbb{R}_+)$, $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$ be mappings. We say that $(F, \mathscr{G})$ forms a pair of Perov-interpolative C'iric$'$-Reich-Rus contractions of Suzuki type, if there exist $A, B \in M_{mm}(\mathbb{R}_+)$ converges towards zero, (where $B = A^q$, $q > 1$) such that

$$\begin{aligned} &\frac{1}{2} \min \{d(x, Fx), d(z, \mathscr{G}z)\} \le d(x, z) \Rightarrow \Lambda(x, z) d(Fx, \mathscr{G}z) \\ &\quad \le A\Big([d(x, z)]^{\beta} \cdot [d(x, Fx)]^{\alpha} \cdot [d(z, \mathscr{G}z)]^{1-\alpha-\beta}\Big), \end{aligned} \tag{11}$$

for each $(x, z) \in \mathscr{P}_{\mathrm{F}} \times \mathscr{P}_{\mathscr{G}}$, where $\beta \ge 0$, $\alpha > 0$ are such that $\beta + \alpha < 1$.

Theorem 11. *Let $(\mathscr{P}, d)$ be a complete vector-valued metric space and $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$ be two mappings such that $(F, \mathscr{G})$ is a pair of Perov-interpolative C'iric$'$-Reich-Rus contractions of Suzuki type. Assume that*

(i) $(F, \mathscr{G})$ is a generalized Λ-admissible pair

(ii) There exists $x_0 \in \mathscr{P}$ such that $\Lambda(x_0, Fx_0) \ge I$ and $\Lambda(Fx_0, x_0) \ge I$

(iii) F and $\mathscr{G}$ are continuous mappings

Then, F and $\mathscr{G}$ have a common fixed point.

Proof. Let $x_0 \in \mathscr{P}$ such that $\Lambda(x_0, \mathrm{F}x_0) \ge I$. We define the sequence $\{x_r\}$ in $\mathscr{P}$ as following

$$x_{2r+1} = Fx_{2r} \text{ and } x_{2r+2} = \mathscr{G}x_{2r+1}, \tag{12}$$

for every $\mathrm{r} \in \mathbb{N}$. From (i) and (ii), $\Lambda(x_0, Fx_0) \ge I$ and $\Lambda(Fx_0, x_0) \ge I$, and then $\Lambda(x_1, x_2) = \Lambda(Fx_0, \mathscr{G}x_1) \ge I$ and $\Lambda(x_2, x_1) = \Lambda(\mathscr{G}x_1, Fx_0) \ge I$. Similarly, we get $\Lambda(x_2, x_3) = \Lambda(Fx_1, \mathscr{G}x_2) \ge I$ and $\Lambda(x_3, x_2) = \Lambda(\mathscr{G}x_2, Fx_1) \ge I$. Repeating this process, we write

$$\Lambda(x_m, x_{m+1}) \ge I \quad \text{and} \quad \Lambda(x_{m+1}, x_m) \ge I, \tag{13}$$

for every $\mathrm{m} \in \mathbb{N}$.

On the other hand, we have

$$\begin{aligned} &\frac{1}{2} \min \{d(x_{2r}, Fx_{2r}), d(x_{2r+1}, \mathscr{G}x_{2r+1})\} \\ &\quad = \frac{1}{2} \min \{d(x_{2r}, x_{2r+1}), d(x_{2r+1}, x_{2r+2})\} \le d(x_{2r}, x_{2r+1}). \end{aligned} \tag{14}$$

So, since the mappings $\mathrm{F}, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$ forms a pair of Perov-interpolative contractions of Suzuki type, we get

$$\begin{aligned} d(x_{2r+1}, x_{2r+2}) &= Id(Fx_{2r}, \mathscr{G}x_{2r+1}) \le \Lambda(x_{2r}, x_{2r+1}) d(Fx_{2r}, \mathscr{G}x_{2r+1}) \\ &\le A\Big([d(x_{2r}, x_{2r+1})]^{\beta} \cdot [d(x_{2r}, Fx_{2r})]^{\alpha} \\ &\quad \cdot [d(x_{2r+1}, \mathscr{G}x_{2r+1})]^{1-\alpha-\beta}\Big) \\ &= A\Big([d(x_{2r}, x_{2r+1})]^{\beta+\alpha} \cdot [d(x_{2r+1}, x_{2r+2})]^{1-\alpha-\beta}\Big). \end{aligned} \tag{15}$$

Therefore, it follows that

$$[d(x_{2r+1}, x_{2r+2})]^{\alpha+\beta} \le A[d(x_{2r}, x_{2r+1})]^{\beta+\alpha}, \tag{16}$$

or equivalent

$$d(x_{2r+1}, x_{2r+2}) \le A^q d(x_{2r}, x_{2r+1}), \tag{17}$$

where $q = 1/\beta + \alpha > 1$. Then,

$$d(x_{2r+1}, x_{2r+2}) \le B d(x_{2r}, x_{2r+1}) \quad \text{where} \quad A^q = B. \tag{18}$$

Letting $x = x_{2\mathrm{r}}$ and $z = x_{2\mathrm{r}-1}$, since

$$\begin{aligned} &\frac{1}{2} \min \{d(x_{2r}, Fx_{2r}), d(x_{2r-1}, \mathscr{G}x_{2r-1})\} \\ &\quad = \frac{1}{2} \min \{d(x_{2r}, x_{2r+1}), d(x_{2r-1}, x_{2r})\} \le d(x_{2r}, x_{2r-1}), \end{aligned} \tag{19}$$

similarly, we get

$$d(x_{2r}, x_{2r+1}) \le B d(x_{2r-1}, x_{2r}). \tag{20}$$

Thus, combining (18) and (20), we have that

$$d(x_{2r+1}, x_{2r+2}) \le B^{2r} d(x_1, x_2), \tag{21}$$

$$d(x_{2r}, x_{2r+1}) \le B^{2r} d(x_0, x_1). \tag{22}$$

Take into account (21) and (22), we obtain that for each $m \in \mathbb{N}$

$$d(x_m, x_{m+1}) \le B^m w(x), \tag{23}$$

where $w(x) = \max \{d(x_0, x_1), d(x_1, x_2)\}$. By this way, using triangular inequality and (23), for $p \ge 0$, we get

$$\begin{aligned} d(x_m, x_{m+p}) &\le d(x_m, x_{m+1}) + d(x_{m+1}, x_{m+2}) + \cdots + d(x_{m+p-1}, x_{m+p}) \\ &= \sum_{m=l}^{m+p-1} d(x_i, x_{i+1}) \le B^m \left(\sum_{i=0}^{\infty} B^i \right) w(x) \\ &= B^m (I + B + \cdots + B^m + \cdots) w(x). \end{aligned} \tag{24}$$

Because B is convergent to zero, we attain that $(I - B)$ is nonsingular and

$$(I-B)^{-1} = I + B + \cdots + B^m + \cdots. \tag{25}$$

Therefore,

$$d\left(x_m, x_{m+p}\right) B^m (I-B)^{-1} w(x), \tag{26}$$

$$d\left(x_m, x_{m+p}\right) \longrightarrow 0 asm \longrightarrow \infty. \tag{27}$$

So, the sequence (x_m) is a fundamental (Cauchy), and using the completeness of the space $(\mathscr{P}, d)$, there exists $t \in \mathscr{P}$ such that

$$\lim_{m \longrightarrow \infty} d(x_m, t) = 0. \tag{28}$$

We claim that the point t is a common fixed point of F and $\mathscr{G}$. If (iii.) is provide, that is, the mapping F and $\mathscr{G}$ are continuous, we have

$$\lim_{r \longrightarrow \infty} d(x_{2r+1}, \mathrm{Ft}) = \lim_{r \longrightarrow \infty} d(Fx_{2r}, \mathrm{Ft}) = 0, \tag{29}$$

then, $\mathrm{F}t = t$. Also, similarly, we get

$$\lim_{r \longrightarrow \infty} d(x_{2r+2}, \mathscr{G}t) = \lim_{r \longrightarrow \infty} d(\mathscr{G}x_{2r+1}, \mathscr{G}t) = 0. \tag{30}$$

$\mathscr{G}t = t$. Therefore, t is a common fixed point of F and $\mathscr{G}$. The proof is complete.□ □

In the following theorem, we remove the assumption of the continuity of the mappings F and $\mathscr{G}$.

Theorem 12. *Besides the hypothesis (i) and (ii) of Theorem 11, if we assume that the condition:*

(i) If $\{x_r\}$ is a sequence in $\mathscr{P}$ such that $x_r \longrightarrow x \in \mathscr{P}$ as $r \longrightarrow \infty$ and, there exists a subsequence $\{x_{r_k}\}$ of $\{x_r\}$ such that $\Lambda(x_{r_k}, x) \geq I$ and $\Lambda(x, x_{r_k}) \geq I$, for all k

holds, then, the mappings F and $\mathscr{G}$ have a common fixed point.

Proof. From Theorem 11, the sequence $\{x_r\}$ defined by (12) is a Cauchy sequence and converges to some $t \in \mathscr{P}$. Similarly, using (13) and the condition (H_Λ), there exists a subsequence $\{x_{r_k}\}$ of $\{x_r\}$ such that $\Lambda(x_{2r_k}, t) \geq I$ and $\Lambda(t, x_{2r_{k-1}}) \geq I$ for all k. We claim that for all $k \geq 0$

$$\frac{1}{2} \min \left\{ d\left(x_{2r_k}, Fx_{2r_k}\right), d(t, \mathscr{G}t) \right\} \leq d\left(x_{2r_k}, t\right). \tag{31}$$

Supposing on the contrary,

$$\frac{1}{2} \min \left\{ d\left(x_{2r_k}, Fx_{2r_k}\right), d(t, \mathscr{G}t) \right\} > d\left(x_{2r_k}, t\right), \tag{32}$$

we get

$$\frac{1}{2} \min \left\{ d\left(x_{2r_k}, x_{2r_{k+1}}\right), d(t, \mathscr{G}t) \right\} > d\left(x_{2r_k}, t\right), \tag{33}$$

and letting $k \longrightarrow \infty$, we acquire that a contradiction. Therefore, the condition (31) holds, and from (11), we obtain

$$\begin{aligned} d\left(x_{2r_k+1}, \mathscr{G}t\right) = Id\left(Fx_{2r_k}, \mathscr{G}t\right) &\leq \Lambda\left(x_{2r_k}, t\right) d\left(Fx_{2r_k}, \mathscr{G}t\right) \\ &\leq A\left(\left[d\left(x_{2r_k}, t\right)\right]^\beta \cdot \left[d\left(x_{2r_k}, Fx_{2r_k}\right)\right]^\alpha \cdot [d(t, \mathscr{G}t)]^{1-\alpha-\beta}\right) \\ &= A\left(\left[d\left(x_{2r_k}, t\right)\right]^\beta \cdot \left[d\left(x_{2r_k}, x_{2r_{k+1}}\right)\right]^\alpha \cdot [d(t, \mathscr{G}t)]^{1-\alpha-\beta}\right). \end{aligned} \tag{34}$$

On the taking k tend to infinity, it follows that we get $\mathscr{G}t = t$. Similarly, we assert that, for all $k \geq 0$

$$\frac{1}{2} \min \left\{ d(t, \mathrm{Ft}), d\left(x_{2r_{k-1}}, \mathscr{G}x_{2r_{k-1}}\right) \right\} \leq d\left(t, x_{2r_{k-1}}\right), \tag{35}$$

Supposing on the contrary,

$$\frac{1}{2} \min \left\{ d(t, \mathrm{Ft}), d\left(x_{2r_{k-1}}, \mathscr{G}x_{2r_{k-1}}\right) \right\} > d\left(t, x_{2r_{k-1}}\right), \tag{36}$$

and, so

$$\frac{1}{2} \min \left\{ d(t, \mathrm{Ft}), d\left(x_{2r_{k-1}}, x_{2r_k}\right) \right\} > d\left(t, x_{2r_{k-1}}\right), \tag{37}$$

taking $k \longrightarrow \infty$, we obtain that a contradiction. Hence, condition (35) is true and from (11), we obtain

$$\begin{aligned} d\left(\mathrm{Ft}, x_{2r_k}\right) = Id\left(\mathrm{Ft}, \mathscr{G}x_{2r_{k-1}}\right) &\leq \Lambda\left(t, x_{2r_{k-1}}\right) d\left(\mathrm{Ft}, \mathscr{G}x_{2r_{k-1}}\right) \\ &\leq A\left(\left[d\left(t, x_{2r_{k-1}}\right)\right]^\beta \cdot [d(t, \mathrm{Ft})]^\alpha \cdot \left[d\left(x_{2r_{k-1}}, \mathscr{G}x_{2r_{k-1}}\right)\right]^{1-\alpha-\beta}\right) \\ &= A\left(\left[d\left(t, x_{2r_{k-1}}\right)\right]^\beta \cdot [d(t, \mathrm{Ft})]^\alpha \cdot \left[d\left(x_{2r_{k-1}}, x_{2r_k}\right)\right]^{1-\alpha-\beta}\right). \end{aligned} \tag{38}$$

Letting k tend to infinity, it follows that we acquire $\mathrm{Ft} = t$. Thus, t is a common fixed point of F and $\mathscr{G}$.□ □

Corollary 13. *Let $(\mathscr{P}, d)$ be a complete vector-valued metric space and $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$ be two continuous mappings such that*

$$\begin{aligned} \frac{1}{2} \min \{d(x, Fx), d(z, \mathscr{G}z)\} \leq d(x, z) \Rightarrow d(Fx, \mathscr{G}z) \\ \leq A\left([d(x, z)]^\beta \cdot [d(x, Fx)]^\alpha \cdot [d(z, \mathscr{G}z)]^{1-\alpha-\beta}\right), \end{aligned} \tag{39}$$

for each $(x, z) \in \mathscr{P}_F \times \mathscr{P}_\mathscr{G}$, where $A, A^q \in M_{mm}(\mathbb{R}_+)$, $q > 1$, converges towards zero and $\beta \geq 0, \alpha > 0$, are such that $\beta + \alpha < 1$. Then, F and $\mathscr{G}$ have a common fixed point.

Corollary 14. *Let $(\mathscr{P}, d)$ be a complete vector-valued metric space and the mappings $\Lambda : \mathscr{P} \times \mathscr{P} \longrightarrow M_{mm}(\mathbb{R}_+)$ and $F, \mathscr{G}$*

: $\mathscr{P} \longrightarrow \mathscr{P}$ such that

$$\Lambda(x,z)d(Fx,\mathscr{G}z) \le A\left([d(x,z)]^{\beta}\cdot[d(x,Fx)]^{\alpha}\cdot[d(z,\mathscr{G}z)]^{1-\alpha-\beta}\right), \tag{40}$$

for each $(x,z) \in \mathscr{P}_F \times \mathscr{P}_{\mathscr{G}}$, where $A, A^q \in M_{mm}(\mathbb{R}_+)$, $q > 1$, converges towards zero and the constants $\beta \ge 0$, $\alpha > 0$, are such that $\beta + \alpha < 1$. Assume that

(i) $(F,\mathscr{G})$ is a generalized Λ-admissible pair

(ii) There exists $x_0 \in \mathscr{P}$ such that $\Lambda(x_0, Fx_0) \ge I$ and $\Lambda(Fx_0, x_0) \ge I$

(iii) The condition (H_Λ) holds or F and $\mathscr{G}$ are continuous mappings

Then, F and $\mathscr{G}$ have a common fixed point.

Letting $F = \mathscr{G}$ in Theorem 11, we obtain the next results.

Corollary 15. *Let $(\mathscr{P}, d)$ be a generalized metric spaces and $\Lambda : \mathscr{P}\times\mathscr{P} \longrightarrow M_{mm}(\mathbb{R}_+)$. Let $F : \mathscr{P} \longrightarrow \mathscr{P}$ be a Λ-orbital admissible mapping such that*

$$\begin{aligned}\frac{1}{2}d(x,Fx) \le d(x,z) \Rightarrow \Lambda(x,z)d(Fx,Fz)\\ \le A[d(x,z)]^{\beta}\cdot[d(x,Fx)]^{\alpha}\cdot[d(z,Fz)]^{1-\alpha-\beta},\end{aligned} \tag{41}$$

for each $x, z \in \mathscr{P}_F$, where $A, A^q \in M_{mm}(\mathbb{R}_+)$, $q > 1$, converges towards zero, and β, α are constants, such that $\beta \ge 0$, $\alpha > 0$, and $\beta + \alpha < 1$. Assume that

(i) F is a triangular Λ-orbital admissible

(ii) There exists $x_0 \in \mathscr{P}$ such that $\Lambda(x_0, Fx_0) \ge I$ and $\Lambda(Fx_0, x_0) \ge I$

(iii) Either F is continuous, or the condition (H_Λ) holds

Then, F has a fixed point.

Definition 16. Let $(\mathscr{P}, d)$ be a vector-valued metric space and $\Lambda : \mathscr{P}\times\mathscr{P} \longrightarrow M_{mm}(\mathbb{R}_+)$, $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$, be mappings. We say that $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$ are Perov-interpolative Kannan contractions of Suzuki type, if there exist a real number $\alpha \in (0,1)$ and $A, C \in M_{mm}(\mathbb{R}_+)$ converges towards zero, where $A^{1/a} = C$, such that

$$\begin{aligned}\frac{1}{2}\min\{d(x,Fx), d(z,\mathscr{G}z)\} \le d(x,z) \Rightarrow \Lambda(x,z)d(Fx,\mathscr{G}z)\\ \le A\left([d(x,Fx)]^{\alpha}\cdot[d(z,\mathscr{G}z)]^{1-\alpha}\right),\end{aligned} \tag{42}$$

for each $(x,z) \in \mathscr{P}_F \times \mathscr{P}_{\mathscr{G}}$.

Theorem 17. *Let $(\mathscr{P}, d)$ be a complete vector-valued metric space and $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$ be Perov-interpolative Kannan contractions of Suzuki type. Assume that*

(i) $(F,\mathscr{G})$ is a generalized Λ-admissible pair

(ii) There exists $x_0 \in \mathscr{P}$ such that $\Lambda(x_0, Fx_0) \ge I$ and $\Lambda(Fx_0, x_0) \ge I$

(iii) Either, F and $\mathscr{G}$ are continuous mappings or, the condition (H_Λ) holds

Then, F and $\mathscr{G}$ have a common fixed point.

Proof. Taking $\beta = 0$ in Theorem 11.□ □

Remark 18. If $m = 1$ and $A = \kappa \in (0,1)$ in the above Theorems, then, we find the concept of the usual metric spaces and interpolative Kannan contraction of Suzuki type and interpolative Ćirić–Reich–Rus contraction of Suzuki type.

Example 19. Let $\mathscr{P} = [0,2]$, $d : \mathscr{P}\times\mathscr{P} \longrightarrow [0,+\infty)$, where $d(x,z) = \begin{pmatrix}|x-z|\\ |x-z|\end{pmatrix}$, and two mappings $F, \mathscr{G} : \mathscr{P} \longrightarrow \mathscr{P}$, defined as

$$\mathrm{F}x = \begin{cases}\dfrac{1}{3}, & if\, x \in [0,1],\\ \dfrac{x}{4}, & if\, x \in (1,2],\end{cases} \tag{43}$$

respectively,

$$\mathscr{G}x = \begin{cases}\dfrac{1}{3}, & if\, x \in [0,1],\\ \dfrac{x}{8}, & if\, x \in (1,2].\end{cases} \tag{44}$$

We choose $\beta = 1/3$, $\alpha = 1/3$ and $A = \begin{pmatrix}1/2 & 0\\ 0 & 1/2\end{pmatrix}$. Let also $\Lambda : \mathscr{P}\times\mathscr{P} \longrightarrow M_{22}(\mathbb{R}_+)$, where

$$\Lambda(x,z) = \begin{cases}\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix} & \text{if} \quad x, z \in [0,1,]\\ \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} & \text{if} \quad x = 0, z = 2,\\ \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} & \text{otherwise.}\end{cases} \tag{45}$$

Then, we have to check that (11) holds. We have to examine the following cases:

(1) $x, z \in [0,1]$. Let $U_1 = \{1/2\} \times \{1/2n : n \in \{2,3,4,\cdots\}\}$ and $U_2 = \{1/2n : n \in \{2,3,4,\cdots\}\} \times \{1/2\}$. For $(x,z) \in A_1 \cup A_2$

$$\frac{1}{2}\min\{d(x,Fx),d(z,\mathcal{G}z)\} \le \begin{pmatrix}\frac{1}{12}\\ \frac{1}{12}\end{pmatrix} < \begin{pmatrix}\frac{n-1}{2n}\\ \frac{n-1}{2n}\end{pmatrix} = d(x,z)\Lambda(x,z)d(Fx,\mathcal{G}z) \le A[d(x,z)]^{1/3}[d(x,Fx]^{1/3}[d(z,\mathcal{G}z)]^{1/3}, \tag{46}$$

and since $d(Fx,\mathcal{G}z)=0$, the inequality (11) holds.

(2) $x,z \in (1,2]$. Similarly, since $\Lambda(x,z)=0$, the relation (11) holds

(3) For $x=0$ and $z=2$

$$\begin{aligned}\frac{1}{2}\min\left\{\left(d\left(0,\frac{1}{3}\right),d\left(2,\frac{1}{4}\right)\right\} &= \frac{1}{2}\min\left\{\begin{pmatrix}\frac{1}{3}\\ \frac{1}{3}\end{pmatrix},\begin{pmatrix}\frac{7}{4}\\ \frac{7}{4}\end{pmatrix}\right\}\\ &= \begin{pmatrix}\frac{1}{6}\\ \frac{1}{6}\end{pmatrix} < \begin{pmatrix}2\\ 2\end{pmatrix} = d(0,2) \Rightarrow \Lambda(0,2)d(F0,\mathcal{G}2)\\ &= \begin{pmatrix}\frac{1}{12}\\ \frac{1}{12}\end{pmatrix} < \begin{pmatrix}\left(\frac{7}{48}\right)^{1/3}\\ \left(\frac{7}{48}\right)^{1/3}\end{pmatrix}\\ &= A[d(0,2)]^{1/3}\cdot[d(0,\mathrm{F}0)]^{1/3}\cdot[d(2,\mathcal{G}2)]^{1/3}.\end{aligned} \tag{47}$$

Then, (11) holds. Consequently, the assumptions of Theorem 11 being satisfied, it follows that the mappings F and $\mathcal{G}$ have a fixed point, which is $x=1/3$.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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