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Research Article

On a Couple of Nonlocal Singular Viscoelastic Equations with Damping and General Source Terms: Blow-Up of Solutions

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Under some given conditions, we prove the explosion result of the solution of the system of nonlocal singular viscoelastic with damping and source terms on general case. This current study is a general case of the previous work of Boulaaras.

1. Introduction

During the last decades, many nonlocal problems of deterministic and parabolic partial differential equations have been studied. These equations and their systems represent the modeling of many physical phenomena related to time. These constraints can be data measured directly at the boundary or give integral boundary conditions (for instance, see [1–25]).

In this work, we investigate the blow-up of the following system of nonlinear damping term:

$$\begin{cases} u_{tt} - \dfrac{1}{x}(xu_x)_x + \displaystyle\int_0^t g_1(t-s)\dfrac{1}{x}(xu_x(x,s))_x \mathrm{d}s + |u_t|^{m-1}u_t = f_1(u,v), Q, \\ v_{tt} - \dfrac{1}{x}(xv_x)_x + \displaystyle\int_0^t g_2(t-s)\dfrac{1}{x}(xv_x(x,s))_x \mathrm{d}s + |v_t|^{m-1}v_t = f_2(u,v), Q, \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), \quad x \in (0,\alpha), \\ v(x,0) = v_0(x), v_t(x,0) = v_1(x), \quad x \in (0,\alpha), \\ u(\alpha,t) = v(\alpha,t) = 0, \displaystyle\int_0^\alpha xu(x,t)\mathrm{d}x = \int_0^\alpha xv(x,t)\mathrm{d}x = 0, \end{cases} \tag{1}$$

where$f_1(u,v), f_2(u,v)$: $R^2 \longrightarrow R$given by

$$\begin{aligned} f_1(u,v) &= a|u+v|^{2(r+1)}(u+v) + b|u|^r u|v|^{r+2}, \\ f_2(u,v) &= a|u+v|^{2(r+1)}(u+v) + b|v|^r v|u|^{r+2}, \end{aligned} \tag{2}$$

with $a, b \in R$, $r \geq -1$ (we get $a = b = 1$), $Q = (0,\alpha) \times (0,T)$, $\alpha < \infty$, $T < \infty$, and

$$g_1(.), g_2(.) : R^+ \longrightarrow R^+ \tag{3}$$

are given functions which will be specified later. The motivation of our work is because of some results regarding the following research paper: in [12], under some conditions suitable for the relaxation function, the author explained that solutions with initial negative energy explode in a finite time if $p > m$ and continue to find if $m \geq p$, for the following studied problem:

$$\begin{cases} u_{tt} - \Delta u + \displaystyle\int_0^t g(t-s)\Delta u + u_t|u_t|^{m-2} = |u|^{p-2}u, \quad \text{in } \Omega \times (0,\infty), \\ u = 0 x \in \partial\Omega, \quad t \in (0,\infty), \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), \quad x \in \Omega. \end{cases} \tag{4}$$

In [4], the author studied a model describing the movement of a flexible two-dimensional viscous body on the unit disk (i.e., radial solutions) and by using some density arguments and some prior estimates, the authors demonstrated the existence and uniqueness of a generalized solution to the following problem:

$$\begin{cases} u_{tt} - \frac{1}{x}(xu_x)_x + \int_0^t g(t-s)\frac{1}{x}(xu_x(x,s))_x \mathrm{d}s = f(x,t,u,u_x), & \text{in } Q, u_x(1,t)=0, \\ \int_0^1 xu(x,t)\mathrm{d}x = 0, \quad t \in (0,T), \\ u(x,0) = \varphi(x), \\ u_t(x,0) = \psi(x), \quad x \in (0,1), \end{cases} \tag{5}$$

where

$$Q = (0,1) \times (0,T) \tag{6}$$

and f is the right-hand side that satisfied the Lipschitzian condition. Recently, in [3], the authors demonstrated the decay result of energy for a small enough initial data together with the explosion result of large initial data of the following singular problem:

$$\begin{cases} u_{tt} - \frac{1}{x}(xu_x)_x + \int_0^t g(t-s)\frac{1}{x}(xu_x(x,s))_x \mathrm{d}s = |u|^{p-2}u, \\ u(a,t) = 0, \int_0^a xu(x,t)\mathrm{d}x = 0, \\ u(x,0) = \varphi(x), u_t(x,0) = \psi(x). \end{cases} \tag{7}$$

That is, they obtained the blow-up properties of local solution by Georgiev-Todorova method with nonpositive initial energy. More work followed up on similar nonlocal singular viscoelastic equations and systems in [8, 9].

In this work, we continue the study on system (1). According to some given conditions, we prove the explosion result of the solution of the system of nonlocal singular viscoelastic with damping and source terms on general case, where we begin by giving basic definitions and theories about the function spaces we need, and then, we mention the theorem of local existence. Finally, we announce and prove the main result of our studied problem in (1).

1.1. Preliminaries. In this section, we introduce some functional spaces and give some lemma's need for the remaining of this paper. Let $L_x^p = L_x^p((0,\alpha))$ be the weighed Banach space equipped with the norm

$$\|u\|_{L_x^p} = \left(\int_0^\alpha x|u|^p \mathrm{d}x\right)^{1/p}. \tag{8}$$

$H = L_x^2((0,\alpha))$is, in particular, the Hilbert space of square integral functions having the finite norm

$$\|u\|_H = \left(\int_0^\alpha xu^2 \mathrm{d}x\right)^{1/2}. \tag{9}$$

$V = V_x^1((0,\alpha))$is the Hilbert space equipped with the norm

$$\|u\|_V = \left(\|u\|_H^2 + \|u_x\|_H^2\right)^{1/2}, \quad V_0 = \{u \in V : u(\alpha) = 0\}. \tag{10}$$

Lemma 1 (Poincare-type inequality). *For any $u \in V_0$,*

$$\int_0^\alpha xu^2 dx \le C_P \int_0^\alpha xu_x^2 dx, \tag{11}$$

where C_P is some positive constant.

Remark 2. It is clear that $\|u\|_{V_0} = \|u_x\|_H$ defines an equivalent norm on V_0.

Lemma 3. *For any $u \in V_0$ and $2 < p < 4$, we have*

$$\|u\|_{L_x^p(0,\alpha)}^p \le C_* \|u_x\|_{H=L_x^2(0,\alpha)}^p, \tag{12}$$

where C_* is a constant depending on α and p only. For the g_1 and g_2 functions, assumptions are as follows:(G1): $g_1(.)$, $g_2(.)$: $R^+ \longrightarrow R^+$ are two differentiable and nonincreasing functions with

$$g_1(t) \ge 0, 1 - \int_0^\infty g_1(s)ds = I_1 \ge 0, \quad g_2(t) \ge 0, 1 - \int_0^\infty g_2(s)ds = I_2 \ge 0, \tag{13}$$

(G2): For all $t \ge 0$,

$$g_1(t) \ge 0, \quad g_1'(t) \le 0, \quad g_2(t) \ge 0, \quad g_2'(t) \le 0. \tag{14}$$

(G3): $r \ge -1$.

Theorem 4. *Suppose that (G1), (G2), and (G3) hold. Then, for all $(u_0, v_0) \in V_0^2$ and all $(u_1, v_1) \in H^2$, problem ((1)) admits a unique local solution (u, v):*

$$u, v \in C((0,T); V_0) \cap C^1((0,T); H), \tag{15}$$

for $T > 0$ small enough.

Lemma 5. *Assume that (G1), (G2), and (G3) hold and (u, v) is a solution of problem((1)); then, the energy functional*

$$E(t) = \frac{1}{2}\int_0^\alpha xu_t^2\,dx + \frac{1}{2}\int_0^\alpha xv_t^2\,dx + \frac{1}{2}\left(1-\int_0^t g_1(s)ds\right)\int_0^\alpha xu_x^2\,dx + \frac{1}{2}\left(1-\int_0^t g_2(s)ds\right)\int_0^\alpha xv_x^2dx + \frac{1}{2}(g_1\circ u_x)(t) + \frac{1}{2}(g_2\circ v_x)(t) - \int_0^\alpha xF(u,v)dx, \tag{16}$$

where

$$\begin{aligned}(g_1\circ u_x)(t) &= \int_0^\alpha\int_0^t xg_1(t-s)|u_x(x,s)-u_x(x,t)|^2dsdx,\\ (g_2\circ v_x)(t) &= \int_0^\alpha\int_0^t xg_2(t-s)|v_x(x,s)-v_x(x,t)|^2dsdx,\\ F(u,v) &= \frac{1}{2(r+2)}\left[\left|u+v\right|^{2(r+2)} + 2\left|uv\right|^{r+2}\right].\end{aligned} \tag{17}$$

Remark 6. Multiplying the first equation in((1))by xu_t and the second equation in((1))by xv_t integrating over $(0,\alpha)$, we obtain the following equation:

$$\begin{aligned}\frac{d}{dt}[E(t)] &= -\int_0^\alpha x|u_t|^{m+1}\,dx - \int_0^\alpha x|v_t|^{m+1}\,dx,\\ &= -\left[\|u_t\|_{L_x^{m+1}}^{m+1} + \|v_t\|_{L_x^{m+1}}^{m+1}\right].\end{aligned} \tag{18}$$

The definition of the norm is as follows:

$$\left[\|u_t\|_{L_x^{m+1}}^{m+1} + \|v_t\|_{L_x^{m+1}}^{m+1}\right] \ge 0. \tag{19}$$

From here,

$$-\left[\|u_t\|_{L_x^{m+1}}^{m+1} + \|v_t\|_{L_x^{m+1}}^{m+1}\right] \le 0. \tag{20}$$

Thus,

$$\frac{d}{dt}[E(t)] = -\left[\|u_t\|_{L_x^{m+1}}^{m+1} + \|v_t\|_{L_x^{m+1}}^{m+1}\right] \le 0. \tag{21}$$

Lemma 7. *There exist c_0 and c_1 positive constants such that*

$$\frac{c_0}{2(r+2)}\left(|u|^{2(r+2)} + |v|^{2(r+2)}\right) \le F(u,v) \le \frac{c_1}{2(r+2)}\left(|u|^{2(r+2)} + |v|^{2(r+2)}\right). \tag{22}$$

Lemma 8. *If $2\le s\le p$,*

$$\|u\|_{L_x^p}^s \le C\left(\|u_x\|_H^2 + \|u\|_{L_x^p}^p\right). \tag{23}$$

2. Blow-Up of Solution

In this section, we shall deal with the blow-up behavior of solutions for problem (1). We derive the blow-up properties of solutions of problem (1) with nonpositive initial energy by the method given in [1].

Theorem 9. *Assume that (G1), (G2), and (G3) hold. $E(0)<0$ and*

$$\int_0^\infty g_i(s)ds < \frac{r+1}{r+1+1/(4(r+2))},\ i=1,2. \tag{24}$$

Then, the solution of problem (1) blows up in finite time.

Proof. Since $(d/dt)[E(t)] = E'(t) \le 0$,

$$E(t) \le E(0) < 0, \quad \forall t\ge 0. \tag{25}$$

We define $H(t) = -E(t)$; then,

$$0 < H(0) \le H(t) = -E(t), \quad \forall t \ge 0. \tag{26}$$

□

We obviously substitute $E(t)$ in (26); then,

$$\begin{aligned}0<H(0)\le H(t) = &-\frac{1}{2}\int_0^\alpha xu_t^2\,dx - \frac{1}{2}\int_0^\alpha xv_t^2\,dx\\ &-\frac{1}{2}\left(1-\int_0^t g_1(s)ds\right)\int_0^\alpha xu_x^2dx\\ &-\frac{1}{2}\left(1-\int_0^t g_2(s)ds\right)\int_0^\alpha xv_x^2dx\\ &-\frac{1}{2}(g_1\circ u_x)(t) - \frac{1}{2}(g_2\circ v_x)(t)\\ &+\int_0^\alpha xF(u,v)dx.\end{aligned} \tag{27}$$

From (22) and (27),

$$\begin{aligned}0<H(0)\le H(t) &\le \int_0^\alpha xF(u,v)\,dx\\ &\le \frac{c_1}{2(r+2)}\left[\int_0^\alpha x|u|^{2(r+2)}dx + \int_0^\alpha x|v|^{2(r+2)}dx\right]\\ &= \frac{c_1}{2(r+2)}\left[\|u\|_{L_x^{2(r+2)}}^{2(r+2)} + \|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right].\end{aligned} \tag{28}$$

Thus,

$$H(t) \le \frac{c_1}{2(r+2)}\left[\|u\|_{L_x^{2(r+2)}}^{2(r+2)} + \|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right]. \tag{29}$$

Equation (29) will then be used as an important data for proof of the theorem. Now, we define

$$L(t)=H^{1-\sigma}(t)+\varepsilon\left(\int_0^\alpha xuu_t\mathrm{d}x+\int_0^\alpha xvv_t\mathrm{d}x\right) \quad (30)$$

for ε small enough and

$$0<\sigma\le\min\left\{\frac{2(r+2)-\mathrm{m}}{2m(r+2)},\frac{2(r+2)-m}{2m(r+2)},\frac{2r+2}{4(r+2)}\right\}. \quad (31)$$

By differentiating (30), using (1) and $H'(t)=\|u_t\|_{L_x^{m+1}}^{m+1}+\|v_t\|_{L_x^{m+1}}^{m+1}$, we obtain

$$\begin{aligned}L'(t)&=(1-\sigma)H^{-\sigma}(t)\left[\|u_t\|_{L_x^{m+1}}^{m+1}+\|v_t\|_{L_x^{m+1}}^{m+1}\right]+\varepsilon\int_0^\alpha xu_t^2\mathrm{d}x\\&+\varepsilon\int_0^\alpha xv_t^2\mathrm{d}x-\varepsilon\left(1-\int_0^t g_1(s)\mathrm{d}s\right)\int_0^\alpha xu_x^2\mathrm{d}x\\&-\varepsilon\left(1-\int_0^t g_2(s)\mathrm{d}s\right)\int_0^\alpha xv_x^2\mathrm{d}x+\varepsilon\int_0^\alpha\int_0^t xg_1\\&\cdot(t-s)u_x(x,t)[u_x(x,s)-u_x(x,t)]\mathrm{d}s\mathrm{d}x+\varepsilon\int_0^\alpha\int_0^t xg_2\\&\cdot(t-s)v_x(x,t)[v_x(x,s)-v_x(x,t)]\mathrm{d}s\mathrm{d}x\\&-\varepsilon\int_0^\alpha xuu_t|u_t|^{m-1}\mathrm{d}x-\varepsilon\int_0^\alpha xvv_t|v_t|^{m-1}\mathrm{d}x\\&+\varepsilon 2(r+2)\int_0^\alpha xF(u,v)\mathrm{d}x.\end{aligned} \quad (32)$$

By using Young inequality and from $H(t)=-E(t)$,

$$\begin{aligned}\int_0^\alpha xF(u,v)\,\mathrm{d}x&=H(t)+\frac{1}{2}\int_0^\alpha xu_t^2\,\mathrm{d}x+\frac{1}{2}\int_0^\alpha xv_t^2\,\mathrm{d}x\\&+\frac{1}{2}\left(1-\int_0^t g_1(s)\mathrm{d}s\right)\int_0^\alpha xu_x^2\mathrm{d}x\\&+\frac{1}{2}\left(1-\int_0^t g_2(s)\mathrm{d}s\right)\int_0^\alpha xv_x^2\mathrm{d}x\\&+\frac{1}{2}(g_1\circ u_x)(t)+\frac{1}{2}(g_2\circ v_x)(t),\end{aligned} \quad (33)$$

we obtain

$$\begin{aligned}L'(t)&\ge(1-\sigma)H^{-\sigma}(t)\left[\|u_t\|_{L_x^{m+1}}^{m+1}+\|v_t\|_{L_x^{m+1}}^{m+1}\right]\\&+\varepsilon(r+3)\int_0^\alpha xu_t^2\mathrm{d}x+\varepsilon(r+3)\int_0^\alpha xv_t^2\mathrm{d}x\\&+\varepsilon\left[(r+1)-\left((r+1)+\frac{1}{4\theta}\right)\int_0^t g_1(s)\mathrm{d}s\right]\int_0^\alpha xu_x^2\mathrm{d}x\\&+\varepsilon\left[(r+1)-\left((r+1)+\frac{1}{4\theta}\right)\int_0^t g_2(s)\mathrm{d}s\right]\int_0^\alpha xv_x^2\mathrm{d}x\\&+\varepsilon(r-\theta+2)(g_1\circ u_x)(t)+\varepsilon(r-\theta+2)(g_2\circ v_x)(t)+\varepsilon 2(r+2)H(t)\\&-\varepsilon\int_0^\alpha xuu_t|u_t|^{m-1}\mathrm{d}x-\varepsilon\int_0^\alpha xvv_t|v_t|^{m-1}\mathrm{d}x.\end{aligned} \quad (34)$$

where

$$\begin{aligned}\alpha_3&=r-\theta+2>0\Rightarrow r+2>\theta>0,\\\alpha_4&=r-\theta+2>0\Rightarrow r+2>\theta>0,\\\alpha_1&=\left[(r+1)-\left((r+1)+\frac{1}{4\theta}\right)\int_0^t g_1(s)\mathrm{d}s\right]>0,\\\alpha_2&=\left[(r+1)-\left((r+1)+\frac{1}{4\theta}\right)\int_0^t g_2(s)\mathrm{d}s\right]>0.\end{aligned} \quad (35)$$

From (34),

$$\begin{aligned}L'(t)&\ge(1-\sigma)H^{-\sigma}(t)\left[\|u_t\|_{L_x^{m+1}}^{m+1}+\|v_t\|_{L_x^{m+1}}^{m+1}\right]\\&+\varepsilon(r+3)\int_0^\alpha xu_t^2\mathrm{d}x+\varepsilon(r+3)\int_0^\alpha xv_t^2\mathrm{d}x+\varepsilon\alpha_1\int_0^\alpha xu_x^2\mathrm{d}x\\&+\varepsilon\alpha_2\int_0^\alpha xv_x^2\mathrm{d}x+\varepsilon\alpha_3(g_1\circ u_x)(t)+\varepsilon\alpha_4(g_2\circ v_x)(t)\\&+\varepsilon 2(r+2)H(t)-\varepsilon\int_0^\alpha xuu_t\left|u_t\right|^{m-1}\mathrm{d}x-\varepsilon\int_0^\alpha xvv_t\left|v_t\right|^{m-1}\mathrm{d}x,\end{aligned} \quad (36)$$

$$\begin{aligned}H(t)=-E(t)&=-\frac{1}{2}\int_0^\alpha xu_t^2\,\mathrm{d}x-\frac{1}{2}\int_0^\alpha xv_t^2\,\mathrm{d}x\\&-\frac{1}{2}\left(1-\int_0^t g_1(s)\mathrm{d}s\right)\int_0^\alpha xu_x^2\mathrm{d}x-\frac{1}{2}\left(1-\int_0^t g_2(s)\mathrm{d}s\right)\int_0^\alpha xv_x^2\mathrm{d}x\\&-\frac{1}{2}(g_1\circ u_x)(t)-\frac{1}{2}(g_2\circ v_x)(t)+\int_0^\alpha xF(u,v)\mathrm{d}x,\end{aligned} \quad (37)$$

and for $a_5<\min\{\alpha_1,\alpha_2,\alpha_3,\alpha_4,2(r+2)\}$,

$$\begin{aligned}\varepsilon 2(r+2)H(t)&=\varepsilon(a_5+(2(r+2)-a_5))H(t)\\&=\varepsilon a_5H(t)+\varepsilon(2(r+2)-a_5)H(t)\\&=-\frac{\varepsilon}{2}a_5\int_0^\alpha xu_t^2\mathrm{d}x-\frac{\varepsilon}{2}a_5\int_0^\alpha xv_t^2\mathrm{d}x\\&-\frac{\varepsilon}{2}a_5\left(1-\int_0^t g_1(s)\mathrm{d}s\right)\int_0^\alpha xu_x^2\mathrm{d}x\\&-\frac{\varepsilon}{2}a_5\left(1-\int_0^t g_2(s)\mathrm{d}s\right)\int_0^\alpha xv_x^2\mathrm{d}x\\&-\frac{\varepsilon}{2}a_5(g_1\circ u_x)(t)-\frac{\varepsilon}{2}a_5(g_2\circ v_x)(t)\\&+\varepsilon a_5\int_0^\alpha xF(u,v)\mathrm{d}x+\varepsilon((2(r+2)-a_5))H(t).\end{aligned} \quad (38)$$

To estimate the last term in (36), we apply the three-parameter Young inequality: $a,b\ge 0,(1/r)+(1/q)=1$ $,ab\le(\delta^r/r)a^r+(\delta^{-q}b^q/q),\forall\delta>0$. We take

$$r=m+1\ v\,eq=\frac{m+1}{m}, \quad (39)$$

in this case:

$$-\varepsilon\int_0^\alpha xuu_t|u_t|^{m-1}\mathrm{d}x\ge-\varepsilon\frac{\delta_1^{m+1}}{m+1}\|u\|_{L_x^{m+1}}^{m+1}-\varepsilon\frac{m}{m+1}\delta_1^{-((m+1)/m)}\|u_t\|_{L_x^{m+1}}^{m+1}. \quad (40)$$

Similarly

$$-\varepsilon\int_0^\alpha xvv_t|v_t|^{m-1}\mathrm{d}x \geq -\varepsilon\frac{\delta_2^{m+1}}{m+1}\|v\|_{L_x^{m+1}}^{m+1} - \varepsilon\frac{m}{m+1}\delta_2^{-((m+1)/m)}\|v_t\|_{L_x^{m+1}}^{m+1}. \tag{41}$$

Substituting (38), (40), and (41) into (36), by organizing, we obtain

$$\begin{aligned}L'(t) \geq& \left[(1-\sigma)H^{-\sigma}(t) - \frac{m}{m+1}\varepsilon\delta_1^{-((m+1)/m)}\right]\|u_t\|_{L_x^{m+1}}^{m+1} \\ &+ \left[(1-\sigma)H^{-\sigma}(t) - \frac{m}{m+1}\varepsilon\delta_2^{-((m+1)/m)}\right]\|v_t\|_{L_x^{m+1}}^{m+1} \\ &+ \varepsilon\left((r+3) - \frac{a_5}{2}\right)\int_0^\alpha xu_t^2\mathrm{d}x + \varepsilon\left((r+3) - \frac{a_5}{2}\right)\int_0^\alpha xv_t^2\mathrm{d}x \\ &+ \varepsilon\left(\alpha_1 - \frac{a_5}{2}\left(1-\int_0^t g_1(s)\mathrm{d}s\right)\right)\int_0^\alpha xu_x^2\mathrm{d}x \\ &+ \varepsilon\left(\alpha_2 - \frac{a_5}{2}\left(1-\int_0^t g_2(s)\mathrm{d}s\right)\right)\int_0^\alpha xv_x^2\mathrm{d}x \\ &+ \varepsilon\left(\alpha_3 - \frac{a_5}{2}\right)(g_1\circ u_x)(t) + \varepsilon\left(\alpha_4 - \frac{a_5}{2}\right)(g_2\circ v_x)(t) \\ &+ \varepsilon a_5\int_0^\alpha xF(u,v)\mathrm{d}x + \varepsilon((2(r+2)-a_5))H(t) \\ &- \varepsilon\frac{\delta_1^{m+1}}{m+1}\|u\|_{L_x^{m+1}}^{m+1} - \varepsilon\frac{\delta_2^{m+1}}{m+1}\|v\|_{L_x^{m+1}}^{m+1}.\end{aligned} \tag{42}$$

Since integration in estimate (40) and (41) is performed over the space, the parameter δ_1 and δ_2 can be a function of time; we get them as follows:

$$\delta_1^{-((m+1)/m)} = k_1H^{-\sigma}(t) \Rightarrow \delta_1^{m+1} = k_1^{-m}H^{\sigma m}(t), \tag{43}$$

$$\delta_2^{-((m+1)/m)} = k_2H^{-\sigma}(t) \Rightarrow \delta_2^{m+1} = k_2^{-m}H^{\sigma m}(t), \tag{44}$$

where $k_1 > 0$ and $k_2 > 0$ are sufficiently large constants to be specified further. By using (43) and (44) in (42), we have

$$\begin{aligned}L'(t) \geq& \left((1-\sigma) - \frac{m}{m+1}\varepsilon k_1\right)H^{-\sigma}(t)\|u_t\|_{L_x^{m+1}}^{m+1} \\ &+ \left((1-\sigma) - \frac{m}{m+1}\varepsilon k_2\right)H^{-\sigma}(t)\|v_t\|_{L_x^{m+1}}^{m+1} \\ &+ \varepsilon\left((r+3) - \frac{a_5}{2}\right)\int_0^\alpha xu_t^2\mathrm{d}x + \varepsilon\left((r+3) - \frac{a_5}{2}\right)\int_0^\alpha xv_t^2\mathrm{d}x \\ &+ \varepsilon\left(\alpha_1 - \frac{a_5}{2}\left(1-\int_0^t g_1(s)\mathrm{d}s\right)\right)\int_0^\alpha xu_x^2\mathrm{d}x \\ &+ \varepsilon\left(\alpha_2 - \frac{a_5}{2}\left(1-\int_0^t g_2(s)\mathrm{d}s\right)\right)\int_0^\alpha xv_x^2\mathrm{d}x \\ &+ \varepsilon\left(\alpha_3 - \frac{a_5}{2}\right)(g_1\circ u_x)(t) + \varepsilon\left(\alpha_4 - \frac{a_5}{2}\right)(g_2\circ v_x)(t) \\ &+ \varepsilon a_5\int_0^\alpha xF(u,v)\mathrm{d}x + \varepsilon((2(r+2)-a_5))H(t) \\ &- \frac{\varepsilon k_1^{-m}}{m+1}H^{\sigma m}(t)\|u\|_{L_x^{m+1}}^{m+1} - \frac{\varepsilon k_2^{-m}}{m+1}H^{\sigma m}(t)\|v\|_{L_x^{m+1}}^{m+1}.\end{aligned} \tag{45}$$

To estimate the last two terms in (45), we use (29); then,

$$\frac{\varepsilon k_1^{-m}}{m+1}(H(t))^{\sigma m}\|u\|_{L_x^{m+1}}^{m+1} \leq \frac{\varepsilon k_1^{-m}c_2^{\sigma m}}{m+1}\left[\|u\|_{L_x^{2(r+2)}}^{2(r+2)} + \|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right]^{\sigma m}\|u\|_{L_x^{m+1}}^{m+1}. \tag{46}$$

On the other hand, since $r > \max\{m, m\}$ from $L_x^{2(r+2)^\circ} L_x^{m+1}$,

$$\|u\|_{L_x^{m+1}}^{m+1} \leq C\|u\|_{L_x^{2(r+2)}}^{m+1} \leq C\left[\|u\|_{L_x^{2(r+2)}} + \|v\|_{L_x^{2(r+2)}}\right]^{m+1}. \tag{47}$$

Substituting (47) into (46),

$$\frac{\varepsilon k_1^{-m}}{m+1}(H(t))^{\sigma m}\|u\|_{L_x^{m+1}}^{m+1} \leq \frac{\varepsilon k_1^{-m}c_2^{\sigma m}C}{m+1}\left[\|u\|_{L_x^{2(r+2)}}^{2(r+2)} + \|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right]^{\sigma m} \cdot \left[\|u\|_{L_x^{2(r+2)}} + \|v\|_{L_x^{2(r+2)}}\right]^{m+1}. \tag{48}$$

By using

$$\begin{gathered}a, b \geq 0, \\ 1 \leq p < \infty, \\ a^p + b^p \leq (a+b)^p,\end{gathered} \tag{49}$$

we can estimate the following:

$$\left[\|u\|_{L_x^{2(r+2)}}^{2(r+2)} + \|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right] \leq \left[\|u\|_{L_x^{2(r+2)}} + \|v\|_{L_x^{2(r+2)}}\right]^{2(r+2)}. \tag{50}$$

Consequently, we have

$$\frac{\varepsilon k_1^{-m}}{m+1}(H(t))^{\sigma m}\|u\|_{L_x^{m+1}}^{m+1} \leq \frac{\varepsilon k_1^{-m}c_2^{\sigma m}C}{m+1}\left[\|u\|_{L_x^{2(r+2)}} + \|v\|_{L_x^{2(r+2)}}\right]^{2(r+2)\sigma m+m+1}. \tag{51}$$

Similarly

$$\frac{\varepsilon k_2^{-m}}{m+1}(H(t))^{\sigma m}\|v\|_{L_x^{m+1}}^{m+1} \leq \frac{\varepsilon k_2^{-m}c_2^{\sigma m}C}{m+1}\left[\|u\|_{L_x^{2(r+2)}} + \|v\|_{L_x^{2(r+2)}}\right]^{2(r+2)\sigma m+m+1}. \tag{52}$$

By using (51) and (52),

$$a, b \geq 0, \quad 1 \leq p < \infty, (a+b)^p \leq c\left(a^p + b^p\right), \quad \left(c = 2^{p-1}\right), \tag{53}$$

for$c \cdot C = C'$; we have

$$\frac{\varepsilon k_1^{-m}}{m+1}(H(t))^{\sigma m}\|u\|_{L_x^{m+1}}^{m+1} \leq \frac{\varepsilon k_1^{-m}c_2^{\sigma m}C'}{m+1}\left[\|u\|_{L_x^{2(r+2)}}^{2(r+2)\sigma m+m+1} + \|v\|_{L_x^{2(r+2)}}^{2(r+2)\sigma m+m+1}\right], \tag{54}$$

$$\frac{\varepsilon k_2^{-m}}{m+1}(H(t))^{\sigma m}\|v\|_{L_x^{m+1}}^{m+1} \leq \frac{\varepsilon k_2^{-m}c_2^{\sigma m}C'}{m+1}\left[\|u\|_{L_x^{2(r+2)}}^{2(r+2)\sigma m+m+1} + \|v\|_{L_x^{2(r+2)}}^{2(r+2)\sigma m+m+1}\right]. \tag{55}$$

From (31),

$$\begin{gathered}2(r+2)\sigma m+m+1\le 2(r+2)\left(\frac{2(r+2)-m}{2m(r+2)}\right)m+m+1=2(r+2)+1,\\ r\ge -1,\\ 2(r+2)\ge 2,\\ 2(r+2)\sigma m+m+1\ge 2.\end{gathered}\tag{56}$$

From here,

$$2\le 2(r+2)\sigma m+m+1\le 2(r+2)+1.\tag{57}$$

Thus, by applying (23), we obtain

$$\begin{aligned}\|u\|_{L_x^{2(r+2)}}^{2(r+2)\sigma m+m+1}&\le \|u_x\|_H^2+\|u\|_{L_x^{2(r+2)}}^{2(r+2)},\\ \|v\|_{L_x^{2(r+2)}}^{2(r+2)\sigma m+m+1}&\le \|v_x\|_H^2+\|v\|_{L_x^{2(r+2)}}^{2(r+2)}.\end{aligned}\tag{58}$$

Substituting these inequalities in (54) and (55), in this case,

$$\frac{\varepsilon k_1^{-m}}{m+1}(H(t))^{\sigma m}\|u\|_{L_x^{m+1}}^{m+1}\le \frac{\varepsilon k_1^{-m}c_2^{\sigma m}C'}{m+1}\left[\|u_x\|_H^2+\|u\|_{L_x^{2(r+2)}}^{2(r+2)}+\|v_x\|_H^2+\|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right],\tag{59}$$

$$\frac{\varepsilon k_2^{-m}}{m+1}(H(t))^{\sigma m}\|v\|_{L_x^{m+1}}^{m+1}\le \frac{\varepsilon k_2^{-m}c_2^{\sigma m}C'}{m+1}\left[\|u_x\|_H^2+\|u\|_{L_x^{2(r+2)}}^{2(r+2)}+\|v_x\|_H^2+\|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right].\tag{60}$$

With the combination of (59) and (60), we obtain

$$\begin{aligned}-\frac{\varepsilon k_1^{-m}}{m+1}H^{\sigma m}(t)\|u\|_{L_x^{m+1}}^{m+1}-\frac{\varepsilon k_2^{-m}}{m+1}H^{\sigma m}(t)\|v\|_{L_x^{m+1}}^{m+1}\\ \ge\left[-\frac{\varepsilon k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{\varepsilon k_2^{-m}c_2^{\sigma m}C'}{m+1}\right]\left(\|u\|_{L_x^{2(r+2)}}^{2(r+2)}+\|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right)\\ +\left[-\frac{\varepsilon k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{\varepsilon k_2^{-m}c_2^{\sigma m}C'}{m+1}\right]\left(\|u_x\|_H^2+\|v_x\|_H^2\right).\end{aligned}\tag{61}$$

Finally,

$$\begin{gathered}\|u_x\|_H^2=\int_0^\alpha xu_x^2\mathrm{d}x,\ \|v_x\|_H^2=\int_0^\alpha xv_x^2\mathrm{d}x,\\ \|u\|_{L_x^{2(r+2)}}^{2(r+2)}=\int_0^\alpha x|u|^{2(r+2)}\mathrm{d}x,\ \|v\|_{L_x^{2(r+2)}}^{2(r+2)}=\int_0^\alpha x|v|^{2(r+2)}\mathrm{d}x,\\ c'\cdot\left(\int_0^\alpha x|u|^{2(r+2)}\mathrm{d}x+\int_0^\alpha x|v|^{2(r+2)}\mathrm{d}x\right)\le\int_0^\alpha xF(u,v)\mathrm{d}x,\ \left(\frac{c_0}{2(r+2)}=c'\right),\end{gathered}\tag{62}$$

and by considering (61), thus by organizing (45), we have

$$\begin{aligned}L'(t)\ge&\left((1-\sigma)-\frac{m}{m+1}\varepsilon k_1\right)H^{-\sigma}(t)\|u_t\|_{L_x^{m+1}}^{m+1}\\ &+\left((1-\sigma)-\frac{m}{m+1}\varepsilon k_2\right)H^{-\sigma}(t)\|v_t\|_{L_x^{m+1}}^{m+1}\\ &+\varepsilon\left((r+3)-\frac{a_5}{2}\right)\int_0^\alpha xu_t^2\mathrm{d}x+\varepsilon\left((r+3)-\frac{a_5}{2}\right)\int_0^\alpha xv_t^2\mathrm{d}x\\ &+\varepsilon\left[\left(\alpha_1-\frac{a_5}{2}\left(1-\int_0^t g_1(s)\mathrm{d}s\right)\right)-\frac{k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{k_2^{-m}c_2^{\sigma m}C'}{m+1}\right]\\ &\cdot\int_0^\alpha xu_x^2\mathrm{d}x+\varepsilon\left[\left(\alpha_2-\frac{a_5}{2}\left(1-\int_0^t g_2(s)\mathrm{d}s\right)\right)-\frac{k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{k_2^{-m}c_2^{\sigma m}C'}{m+1}\right]\\ &\cdot\int_0^\alpha xv_x^2\mathrm{d}x+\varepsilon\left[c'a_5-\frac{k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{k_2^{-m}c_2^{\sigma m}C'}{m+1}\right]\int_0^\alpha x|u|^{2(r+2)}\mathrm{d}x\\ &+\varepsilon\left[c'a_5-\frac{k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{k_2^{-m}c_2^{\sigma m}C'}{m+1}\right]\int_0^\alpha x|v|^{2(r+2)}\mathrm{d}x\\ &+\varepsilon\left[\alpha_3-\frac{a_5}{2}\right](g_1\circ u_x)(t)+\varepsilon\left[\alpha_4-\frac{a_5}{2}\right](g_2\circ u_x)(t)+\varepsilon[2(r+2)-a_5]H(t),\end{aligned}\tag{63}$$

which introduce the constant

$$\begin{aligned}\gamma=\varepsilon\cdot\min\Bigg\{&(r+3)-\frac{a_5}{2},\left[\left(\alpha_1-\frac{a_5}{2}\left(1-\int_0^t g_1(s)\mathrm{d}s\right)\right)-\frac{k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{k_2^{-m}c_2^{\sigma m}C'}{m+1}\right],\\ &\left[\left(\alpha_2-\frac{a_5}{2}\left(1-\int_0^t g_2(s)\mathrm{d}s\right)\right)-\frac{k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{k_2^{-m}c_2^{\sigma m}C'}{m+1}\right],\\ &\left[c'a_5-\frac{k_1^{-m}c_2^{\sigma m}C'}{m+1}-\frac{k_2^{-m}c_2^{\sigma m}C'}{m+1}\right],\left[\alpha_3-\frac{a_5}{2}\right],\left[\alpha_4-\frac{a_5}{2}\right],[2(r+2)-a_5]\Bigg\}.\end{aligned}\tag{64}$$

Taking sufficiently large $k_1>0$ and $k_2>0$ for the positive constant γ, we simplify (63)

$$\begin{aligned}L'(t)\ge&\left((1-\sigma)-\frac{m}{m+1}\varepsilon k_1\right)H^{-\sigma}(t)\|u_t\|_{L_x^{m+1}}^{m+1}\\ &+\left((1-\sigma)-\frac{m}{m+1}\varepsilon k_2\right)H^{-\sigma}(t)\|v_t\|_{L_x^{m+1}}^{m+1}\\ &+\varepsilon\gamma\left[\int_0^\alpha xu_t^2\mathrm{d}x+\int_0^\alpha xv_t^2\mathrm{d}x+\int_0^\alpha xu_x^2\mathrm{d}x+\int_0^\alpha xv_x^2\mathrm{d}x\right.\\ &+\int_0^\alpha x|u|^{2(r+2)}\mathrm{d}x+\int_0^\alpha x|v|^{2(r+2)}\mathrm{d}x+(g_1\circ u_x)(t)\\ &\left.+(g_2\circ v_x)(t)+H(t)\right].\end{aligned}\tag{65}$$

For fixed $k_1>0$, $k_2>0$, and $\gamma>0$, we choose $\varepsilon>0$ so small that the following inequality holds:

$$\begin{aligned}\left((1-\sigma)-\frac{m}{m+1}\varepsilon k_1\right)\ge 0,\\ \left((1-\sigma)-\frac{m}{m+1}\varepsilon k_2\right)\ge 0.\end{aligned}\tag{66}$$

Moreover, we assume that the initial data satisfy the estimate

$$L(0)=H^{1-\sigma}(0)+\varepsilon\left(\int_0^\alpha xu_0u_1\mathrm{d}x+\int_0^\alpha xv_0v_1\mathrm{d}x\right)>0.\tag{67}$$

Then, from (65), we obtain the following inequality:

$$L'(t) \geq \varepsilon\gamma\left[\int_0^\alpha xu_t^2\,dx + \int_0^\alpha xv_t^2\,dx + \int_0^\alpha xu_x^2\,dx + \int_0^\alpha xv_x^2\,dx + \int_0^\alpha x|u|^{2(r+2)}dx + \int_0^\alpha x|v|^{2(r+2)}dx + (g_1\circ u_x)(t) + (g_2\circ v_x)(t) + H(t)\right]. \quad (68)$$

On the other hand, in Equation (30), we take the1/$(1-\sigma)\cdot$ powerof each side

$$[L(t)]^{1/(1-\sigma)} = \left[H^{1-\sigma}(t) + \varepsilon\left(\int_0^\alpha xuu_t dx + \int_0^\alpha xvv_t dx\right)\right]^{1/(1-\sigma)}. \quad (69)$$

Twice by applying the following inequality to (69)

$$a, b \geq 0, 1 \leq p < \infty, (a+b)^p \leq 2^{p-1}\left(a^p + b^p\right), \quad (70)$$

we have

$$\begin{aligned}[L(t)]^{1/(1-\sigma)} &\leq 2^{\sigma/(1-\sigma)}\left[H(t) + \varepsilon^{1/(1-\sigma)}\left|\int_0^\alpha xuu_t\,dx + \int_0^\alpha xvv_t\,dx\right|^{1/(1-\sigma)}\right] \\ &\leq 2^{\sigma/(1-\sigma)}\left[H(t) + \varepsilon^{1/(1-\sigma)}2^{\sigma/(1-\sigma)}\left(\left|\int_0^\alpha xuu_t dx\right|^{1/(1-\sigma)} + \left|\int_0^\alpha xvv_t dx\right|^{1/(1-\sigma)}\right)\right] \\ &\leq C\left[H(t) + \left|\int_0^\alpha xuu_t dx\right|^{1/(1-\sigma)} + \left|\int_0^\alpha xvv_t dx\right|^{1/(1-\sigma)}\right],\end{aligned} \quad (71)$$

where $C > 0$. Now, to estimate the last two terms in (71), we, respectively, apply Holder inequality, $L_x^{2(r+2)\circ}L_x^H$, and Young inequality; thus,

$$\begin{aligned}\left|\int_0^\alpha xuu_t\,dx\right|^{1/(1-\sigma)} &\leq \|u\|_H^{1/(1-\sigma)}\|u_t\|_H^{1/(1-\sigma)} \leq C\|u\|_{L_x^{2(r+2)}}^{1/(1-\sigma)}\|u_t\|_H^{1/(1-\sigma)} \\ &\leq C\left(\|u\|_{L_x^{2(r+2)}}^{\theta/(1-\sigma)} + \|u_t\|_H^{\mu/(1-\sigma)}\right).\end{aligned} \quad (72)$$

Similarly,

$$\left|\int_0^\alpha xvv_t dx\right|^{1/(1-\sigma)} \leq C\left(\|v\|_{L_x^{2(r+2)}}^{\theta/(1-\sigma)} + \|v_t\|_H^{\mu/(1-\sigma)}\right), \quad (73)$$

where $(1/\theta) + (1/\mu) = 1$. In these inequalities by collecting side by side, we obtain

$$\left|\int_0^\alpha xuu_t\,dx\right|^{1/(1-\sigma)} + \left|\int_0^\alpha xvv_t\,dx\right|^{1/(1-\sigma)} \leq C\left(\|u\|_{L_x^{2(r+2)}}^{\theta/(1-\sigma)} + \|u_t\|_H^{\mu/(1-\sigma)} + \|v\|_{L_x^{2(r+2)}}^{\theta/(1-\sigma)} + \|v_t\|_H^{\mu/(1-\sigma)}\right). \quad (74)$$

We choose $\mu = 2(1-\sigma)$, to get

$$\theta = \frac{2(1-\sigma)}{1-2\sigma} \leq 2(r+2), \quad (75)$$

then

$$\left|\int_0^\alpha xuu_t\,dx\right|^{1/(1-\sigma)} + \left|\int_0^\alpha xvv_t\,dx\right|^{1/(1-\sigma)} \leq C\left(\|u\|_{L_x^{2(r+2)}}^{2/(1-2\sigma)} + \|u_t\|_H^2 + \|v\|_{L_x^{2(r+2)}}^{2/(1-2\sigma)} + \|v_t\|_H^2\right). \quad (76)$$

By applying (23), we can write

$$\begin{aligned}\|u\|_{L_x^{2(r+2)}}^{2/(1-2\sigma)} &\leq C\left(\|u_x\|_H^2 + \|u\|_{L_x^{2(r+2)}}^{2(r+2)}\right), \\ \|v\|_{L_x^{2(r+2)}}^{2/(1-2\sigma)} &\leq C\left(\|v_x\|_H^2 + \|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right).\end{aligned} \quad (77)$$

From here, we obtain

$$\left|\int_0^\alpha xuu_t\,dx\right|^{1/(1-\sigma)} + \left|\int_0^\alpha xvv_t\,dx\right|^{1/(1-\sigma)} \leq C\left(\|u_t\|_H^2 + \|v_t\|_H^2 + \|u_x\|_H^2 + \|v_x\|_H^2 + \|u\|_{L_x^{2(r+2)}}^{2(r+2)} + \|v\|_{L_x^{2(r+2)}}^{2(r+2)}\right). \quad (78)$$

Thus, by considering (78) and the following in (71),

$$\begin{aligned}&\|u_t\|_H^2 = \int_0^\alpha xu_t^2 dx, \|v_t\|_H^2 = \int_0^\alpha xv_t^2 dx, \\ &\|u_x\|_H^2 = \int_0^\alpha xu_x^2 dx, \|v_x\|_H^2 = \int_0^\alpha xv_x^2 dx, \\ &\|u\|_{L_x^{2(r+2)}}^{2(r+2)} = \int_0^\alpha x|u|^{2(r+2)}dx, \|v\|_{L_x^{2(r+2)}}^{2(r+2)} = \int_0^\alpha x|v|^{2(r+2)}dx, \\ &(g_1\circ u_x)(t) \geq 0, (g_2\circ v_x)(t) \geq 0,\end{aligned} \quad (79)$$

we obtain

$$[L(t)]^{1/(1-\sigma)} \leq C\left[H(t) + \int_0^\alpha xu_t^2\,dx + \int_0^\alpha xv_t^2\,dx + \int_0^\alpha xu_x^2\,dx + \int_0^\alpha xv_x^2 dx + \int_0^\alpha x|u|^{2(r+2)}dx + \int_0^\alpha x|v|^{2(r+2)}dx + (g_1\circ u_x)(t) + (g_2\circ v_x)(t)\right]. \quad (80)$$

Finally, by combining (68) and (80), we obtain the following ordinary differential inequality:

$$L'(t) \geq \lambda L^{1/(1-\sigma)}(t)\forall t \geq 0, \quad (81)$$

obviously, where $\lambda > 0$ is a constant depending only C, ε, and

γ. This differential inequality integration over $(0, t)$ gives

$$L^{1/(1-\sigma)}(t) \geq \frac{1}{L^{-\sigma/(1-\sigma)}(0) - \lambda(\sigma/(1-\sigma))t}, \quad (82)$$

where we choose

$$t \leq T^* = \frac{1-\sigma}{\lambda\sigma L^{\sigma/(1-\sigma)}(0)}. \quad (83)$$

Hence,

$$\lim_{t \longrightarrow T^{*-}} L(t) \longrightarrow \infty. \quad (84)$$

3. Conclusions

The purpose of this paper is to study the explosion result of the solution of the system of nonlocal singular viscoelastic with damping and source terms on general case. This current study is a general case of the previous work of Boulaaras in ([5]). In the next work, we will try to obtain the same result for the two-dimensional problem that allows a reasonable description of the phenomenon occurring in a three-dimensional domain. Then, we will try to prove uniqueness results of the weak solution.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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