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Retraction

Retracted: Solving Nonlinear Fractional Differential Equations for Contractive and Weakly Compatible Mappings in Neutrosophic Metric Spaces

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] U. Ali, H. A. Alyousef, U. Ishtiaq, K. Ahmad, and S. Ali, "Solving Nonlinear Fractional Differential Equations for Contractive and Weakly Compatible Mappings in Neutrosophic Metric Spaces," *Journal of Function Spaces*, vol. 2022, Article ID 1491683, 19 pages, 2022.

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Research Article

Solving Nonlinear Fractional Differential Equations for Contractive and Weakly Compatible Mappings in Neutrosophic Metric Spaces

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In this article, we aim to prove various unique fixed point results for contractive and weakly compatible mappings in the sense of neutrosophic metric spaces. Several nontrivial examples are also imparted. To support main result, uniqueness of solution of nonlinear fractional differential equations is examined.

1. Introduction and Preliminaries

Uncertainty and fuzziness are prevalent in many applications in today's complicated world. For capturing the ambiguity and fuzziness of information, Zadeh [1] first developed the concept of fuzzy sets (FSs). Many extensions of FSs, such as intuitionistic fuzzy sets (IFSs), picture FSs, q-rung orthopair FSs, and neutrosophic sets (NSs), have been proposed since its inception to better convey complicated information. Researchers and scholars have gathered a large number of research findings related to their decision-making theories and approaches.

As a generalization of FSs, Atanassove [2] proposed and explored IFSs. With the use of continuous t-norms (CTNs) and continuous t-conorms (CTCNs), Park [3] established the concept of intuitionistic fuzzy metric space (IFMS) in 2004. Alaca et al. [4] proposed IFMS with the help of CTNs and CTCNs, a generalization of fuzzy metric space according to Kramosil and Michalek [5] in 2006, using the idea of IFSs and proved various fixed point theorems for contraction mappings. Hu [6] did nice work for different type contractions. Smarandache [7] proposed NSs, which is a generalization of IFSs. Kirişci and Simşek [8] proposed the concept of neutrosophic metric space (NMS), based on the concept of NSs and proved several theorems in the proposed space. Ishtiaq et al. [9] proposed the notion of orthogonal NMS and proved some fixed point results in the sense of complete orthogonal NMS. Several fixed point results for generalized contractions in NMS were demonstrated by Sowndrara et al. [10]. Gulzar et al. [11] used the notion of FSs in subgroups and proved numerous nice results. Gulzar et al. [12] used the notion of FSs in the structure of field and did an amazing work. Several fixed point results for weakly compatible mappings were proved by Sharma et al. [13] in the structure of IFMS. Davvaz et al. [14] did exquisite work by using IFSs. Simsek and Kirişci [15] used the notion of NMS and proved various fixed point theorems.

We aim to establish a number of unique fixed point results for contractive and weakly compatible mappings in the context of NMS in this paper. A number of nontrivial examples are also presented. The uniqueness of solutions to nonlinear fractional differential equations is investigated to support the main result.

First, we present definitions of CTN, CTCN, IFMS, NSs, and NMS, contraction mapping, and weekly contractive mapping that are helpful for this study.

$I = [0, 1]$ is used in this study.

Definition 1 (see [2]). A binary operation $^*: I \times I \longrightarrow I$ is called a CTN if

S1. $Þ * \varpi = \varpi * Þ$, for all $Þ, \varpi \in I$;

S2. * is continuous;

S3. $Þ * 1 = Þ$, for all $Þ \in I$;

S4. $(Þ * \varpi) * z = Þ * (\varpi * z)$, for all $Þ, \varpi, z \in I$;

S5. $Þ \leq c$ and $\varpi \leq d$, with $Þ, \varpi, c, d \in I$, then $Þ * \varpi \leq c * d$.

Definition 2 (see [2]). A binary operation $\bigcirc: I \times I \longrightarrow I$ is called a CTCN if

C1. $Þ \bigcirc \varpi = \varpi \bigcirc Þ$, for all $Þ, \varpi \in I$;

C2. $\bigcirc$ is continuous;

C3. $Þ \bigcirc 0 = 0$, for all $Þ \in I$;

C4. $(Þ \bigcirc \varpi) \bigcirc z = Þ \bigcirc (\varpi \bigcirc z)$, for all $Þ, \varpi, z \in I$;

C5. $Þ \leq c$ and $\varpi \leq d$, with $Þ, \varpi, c, d \in I$, then $Þ \bigcirc \varpi \leq c \bigcirc d$.

Definition 3 (see [4]). Let X be nonempty and * be a CTN and $\bigcirc$ be a CTCN. Let M and N be FSs on $X^2 \times (0, +\infty)$, if the following conditions are satisfied:

FS1. $M(Þ, \varpi, \Theta) > 0$;

FS2. $M(Þ, \varpi, \Theta) + N(Þ, \varpi, \Theta) \leq 1$;

FS3. $M(Þ, \varpi, \Theta) = 1$ for all $\Theta > 0$, if and only if $Þ = \varpi$;

FS4. $M(Þ, \varpi, \Theta) = M(\varpi, Þ, \Theta)$;

FS5. $M(Þ, z, \Theta + \partial) \geq M(Þ, \varpi, \Theta) * M(\varpi, z, \partial)$;

FS6. $M(Þ, \varpi, \cdot): (0, +\infty) \longrightarrow [0, 1]$ is continuous and $\lim_{\Theta \longrightarrow +\infty} M(Þ, \varpi, \Theta) = 1$;

FS7. $N(Þ, \varpi, \Theta) > 0$;

FS8. $N(Þ, \varpi, \Theta) = 0$ for all $\Theta > 0$, if and only if $Þ = \varpi$;

FS9. $N(Þ, \varpi, \Theta) = N(\varpi, Þ, \Theta)$;

FS10. $N(Þ, z, \Theta + \partial) \leq N(Þ, \varpi, \Theta) \bigcirc N(\varpi, z, \partial)$;

FS11. $N(Þ, \varpi, \cdot): (0, +\infty) \longrightarrow [0, 1]$ is continuous and $\lim_{\Theta \longrightarrow +\infty} N(Þ, \varpi, \Theta) = 0$.

Then, $(X, M, N, *, \bigcirc)$ is called an IFMS.

Definition 4 (see [7]). Let a set X be nonempty and $Þ \in E$. A NS G in X is classified by a truth-membership function, $M_G(Þ)$, an indeterminacy-membership function $N_G(Þ)$, and a falsity-membership function $O_G(Þ)$. The functions $M_G(Þ)$, $N_G(Þ)$, and $O_G(Þ)$ are subsets of $]0^-, 1^+[$, that is, $M_G(Þ): X \longrightarrow]0^-, 1^+[$, $N_G(Þ): X \longrightarrow]0^-, 1^+[$, and $O_G(Þ): X \longrightarrow]0^-, 1^+[$. So,

$$0^- \leq \sup M_G(Þ) + \sup N_G(Þ) + \sup O_G(Þ) \leq 3^+. \quad (1)$$

Definition 5 (see [8]). Let X be nonempty and * be a CTN and $\bigcirc$ be a CTCN. M, N, and O are NSs on $X \times X \times (0, +\infty)$, then (X, M, N, O) is named neutrosophic metric on X, if for all $Þ, \varpi, z \in X$, the below circumstances are satisfied:

(nm1). $M(Þ, \varpi, \Theta) + N(Þ, \varpi, \Theta) + O(Þ, \varpi, \Theta) \leq 3$;

(nm2). $M(Þ, \varpi, \Theta) > 0$;

(nm3). $M(Þ, \varpi, \Theta) = 1$ for all $\Theta > 0$, if and only if $Þ = \varpi$;

(nm4). $M(Þ, \varpi, \Theta) = M(\varpi, Þ, \Theta)$;

(nm5). $M(Þ, z, \Theta + \partial) \geq M(Þ, \varpi, \Theta) * M(\varpi, z, \partial)$;

(nm6). $M(Þ, \varpi, \cdot): (0, +\infty) \longrightarrow [0, 1]$ is continuous and $\lim_{\Theta \longrightarrow +\infty} M(Þ, \varpi, \Theta) = 1$;

(nm7). $N(Þ, \varpi, \Theta) < 1$;

(nm8). $N(Þ, \varpi, \Theta) = 0$ for all $\Theta > 0$, if and only if $Þ = \varpi$;

(nm9). $N(Þ, \varpi, \Theta) = N(\varpi, Þ, \Theta)$;

(nm10). $N(Þ, z, \Theta + \partial) \leq N(Þ, \varpi, \Theta) \bigcirc N(\varpi, z, \partial)$;

(nm11). $N(Þ, \varpi, \cdot): (0, +\infty) \longrightarrow [0, 1]$ is continuous and $\lim_{\Theta \longrightarrow +\infty} N(Þ, \varpi, \Theta) = 0$;

(nm12). $O(Þ, \varpi, \Theta) < 1$;

(nm13). $O(Þ, \varpi, \Theta) = 0$ for all $\Theta > 0$, if and only if $Þ = \varpi$;

(nm14). $O(Þ, \varpi, \Theta) = O(\varpi, Þ, \Theta)$;

(nm15). $O(Þ, z, \Theta + \partial) \leq O(Þ, \varpi, \Theta) \bigcirc O(\varpi, z, \partial)$;

(nm16). $O(Þ, \varpi, \cdot): (0, +\infty) \longrightarrow [0, 1]$ is continuous and $\lim_{\Theta \longrightarrow +\infty} O(Þ, \varpi, \Theta) = 0$;

(nm17). If $\Theta \leq 0$, then $M(Þ, \varpi, \Theta) = 0$, $N(Þ, \varpi, \Theta) = 1$ and $O(Þ, \varpi, \Theta) = 1$.

Then, $(X, M, N, O, *, \bigcirc)$ is called an NMS.

Here, functions $M(Þ, \varpi, \Theta), N(Þ, \varpi, \Theta)$, and $O(Þ, \varpi, \Theta)$, respectively, denote the degrees of nearness, nonnearness, and indeterminacy. In $(X, M, N, O, *, \bigcirc)$, the following conditions hold for all $Þ, \varpi \in X$, and $\Theta > 0$.

(i) $\lim_{\Theta \longrightarrow +\infty} M(Þ, \varpi, \Theta) = 1$, $\lim_{\Theta \longrightarrow +\infty} N(Þ, \varpi, \Theta) = 0$, and $\lim_{\Theta \longrightarrow +\infty} O(Þ, \varpi, \Theta) = 0$.

(ii) If $Þ_n \longrightarrow Þ$, $\varpi_n \longrightarrow \varpi$ and $\Theta_n \longrightarrow \Theta$, then $N(Þ_n, \varpi_n, \Theta_n) = N(Þ, \varpi, \varpi)$.

(iii) A sequence $\{Þ_n\}$ converges to $Þ \in X$ if the following limits exist:

$$\lim_{\Theta \longrightarrow +\infty} M(Þ_n, \varpi, \Theta) = 1, \lim_{\Theta \longrightarrow +\infty} N(Þ_n, \varpi, \Theta) = 0 \, an\, d \lim_{\Theta \longrightarrow +\infty} O(Þ_n, \varpi, \Theta) = 0. \quad (2)$$

(iv) A sequence $\{Þ_n\} \in X$ is a Cauchy sequence if and only if for each $\varepsilon \in (0, 1)$, there exists $n_0 \in \mathbb{N}$ such that

$$\lim_{\Theta \longrightarrow +\infty} M(Þ_{n+\sigma}, Þ_n, \Theta) = 1, \quad \lim_{\Theta \longrightarrow +\infty} N(Þ_{n+\sigma}, Þ_n, \Theta) = 0, \quad (3)$$

(v) and

$$\lim_{\Theta\longrightarrow+\infty} O(\text{Þ}_{n+\sigma}, \text{Þ}_n, \Theta) = 0 \text{ for all } \sigma > 0 \text{ and } \Theta > 0. \quad (4)$$

(vi) Every Cauchy sequence is convergent in X if and only if NMS is complete.

(vii) Every sequence contains a convergent subsequence if and only if NMS is compact.

(viii) A mapping $T: X \longrightarrow X$ is a neutrosophic contractive mapping if there exists $k \in (0,1)$, for all $\text{Þ}, \varpi \in X$, and $\Theta > 0$, such that

$$\left[\frac{1}{M(T(\text{Þ}), T(\varpi), \Theta)}\right] \leq k\left[\frac{1}{M(\text{Þ}, \varpi, \Theta)} - 1\right], \quad N(T(\text{Þ}), T(\varpi), \Theta) \leq kN(\text{Þ}, \varpi, \Theta), \quad (5)$$

(ix) and

$$O(T(\text{Þ}), T(\varpi), \Theta) \leq kO(\text{Þ}, \varpi, \Theta). \quad (6)$$

Now, we state several useful definitions from [13].

(i) Let $\sup_{0<\Theta<1}\Delta(\Theta, \Theta) = 1$. A CTN Δ is said to be H-type if $\{\Delta^m(\Theta)\}_{m=1}^{+\infty}$ is the family of functions and is equicontinuous at $\Theta = 1$, where

$$\Delta^1(\Theta) = \Theta\Delta\Theta, \ \Delta^{m+1}(\Theta) = \Theta\Delta(\Delta^m(\Theta)), \quad m = 1, 2, \ldots, \Theta \in [0,1]. \quad (7)$$

(ii) For any $\lambda \in (0,1)$, clearly that Δ is a CTN of H-type if and only if, there exists $\delta(\lambda) \in (0,1)$ such that $\Delta^m(\Theta) < \lambda$ for all $m \in \mathbb{N}$, $\Theta < \delta$.

(iii) Consider $\Phi = \{\varnothing | \varnothing: \mathbb{R}^+ \longrightarrow \mathbb{R}^+\}$ each $\varnothing \in \Phi$ fulfill the below assertions:

(iv) $\varnothing$ is nondecreasing;

(v) $\varnothing$ is an upper semicontinuous from the right;

(vi) $\sum_{n=0}^{+\infty} \varnothing^n(\Theta) < +\infty$, for all $\Theta > 0$ where $\varnothing^{n+1}(\Theta) = \varnothing(\varnothing^n(\Theta)$, $n \in \mathbb{N}$, and also $\varnothing(\Theta) < \Theta$ for all $\Theta > 0$.

(vii) A coupled fixed point is an element $(\text{Þ}, \varpi) \in X \times X$ of the mapping $T: X \times X \longrightarrow X$ if $T(\text{Þ}, \varpi) = \text{Þ}$ and $T(\varpi, \text{Þ}) = \varpi$.

(viii) A coupled coincidence point is an element $(\text{Þ}, \varpi) \in X \times X$ of the mapping $T: X \times X \longrightarrow X$ and $G: X \longrightarrow X$ if $T(\text{Þ}, \varpi) = G(\text{Þ})$ and $T(\varpi, \text{Þ}) = G(\varpi)$.

(ix) A common coupled fixed point is an element $(\text{Þ}, \varpi) \in X \times X$ of the mapping $T: X \times X \longrightarrow X$ and $G: X \longrightarrow X$ if $T(\text{Þ}, \varpi) = G(\text{Þ}) = \text{Þ}$ and $T(\varpi, \text{Þ}) = G(\varpi) = \varpi$.

(x) A common fixed point is an element $\text{Þ} \in X$ of the mapping $T: X \times X \longrightarrow X$ and $G: X \longrightarrow X$ if $T(\text{Þ}, \text{Þ}) = G(\text{Þ}) = \text{Þ}$.

(xi) The mappings $T: X \times X \longrightarrow X$ and $G: X \times X \longrightarrow X$ are said to be weakly compatible mapping if $T(\text{Þ}, \varpi) = G(\text{Þ}), T(\varpi, \text{Þ}) = G(\varpi)$ implies that

$$G(T(\text{Þ}, \varpi)) = T(G(\text{Þ}), G(\varpi)), G(T(\varpi, \text{Þ})) = T(G(\varpi), G(\text{Þ})) \text{ for all } \text{Þ}, \varpi \in X. \quad (8)$$

(xii) If T and G are weakly compatible, then maybe they are not compatible, but the converse is true.

2. Main Results

This section contains several results for contraction mappings.

Definition 6. Let $(X, M, N, O, *, \bigcirc)$ be a NMS. The mapping $T: X \times X \longrightarrow X$ and $G: X \longrightarrow X$ are called compatible if the following limits exist:

$$\begin{aligned}
&\lim_{n\longrightarrow+\infty} M(G(T(\text{Þ}_n, \varpi_n))), T(G(\text{Þ}_n), G(\varpi_n), \Theta) = 1,\\
&\lim_{n\longrightarrow+\infty} M(G(T(\varpi_n, \text{Þ}_n))), T(G(\varpi_n), G(\text{Þ}_n), \Theta) = 1,\\
&\lim_{n\longrightarrow+\infty} N(G(T(\text{Þ}_n, \varpi_n))), T(G(\text{Þ}_n), G(\varpi_n), \Theta) = 0,\\
&\lim_{n\longrightarrow+\infty} N(G(T(\varpi_n, \text{Þ}_n))), T(G(\varpi_n), G(\text{Þ}_n), \Theta) = 0.
\end{aligned} \quad (9)$$

For all $\Theta > 0$ and

$$\begin{aligned}
&\lim_{n\longrightarrow+\infty} O(G(T(\text{Þ}_n, \varpi_n))), T(G(\text{Þ}_n), G(\varpi_n), \Theta) = 0,\\
&\lim_{n\longrightarrow+\infty} O(G(T(\varpi_n, \text{Þ}_n))), T(G(\varpi_n), G(\text{Þ}_n), \Theta) = 0.
\end{aligned} \quad (10)$$

Whenever $\{\text{Þ}_n\}$ and $\{\varpi_n\}$ are sequence in X, such that

$$\lim_{n\longrightarrow+\infty} T(\text{Þ}_n, \varpi_n) = \lim_{n\longrightarrow+\infty} G(\text{Þ}_n) = \text{Þ} \text{ and } \lim_{n\longrightarrow+\infty} T(\varpi_n, \text{Þ}_n) = \lim_{n\longrightarrow+\infty} G(\varpi_n) = \varpi \text{ for all } \text{Þ}, \varpi \in X. \quad (11)$$

Lemma 1. *Let $(X, M, N, O, *, \bigcirc)$ be a NMS, where $*$ and $\bigcirc$ are CTN and CTCN of H-type, if there exists $\varnothing \in \Phi$, such that*

$$M(\text{Þ}, \varpi, \varnothing(\Theta)) \geq M(\text{Þ}, \varpi, \Theta), \ N(\text{Þ}, \varpi, \varnothing(\Theta)) \leq N(\text{Þ}, \varpi, \Theta), \quad (12)$$

and

$$O(\text{Þ}, \varpi, \varnothing(\Theta)) \leq O(\text{Þ}, \varpi, \Theta) \text{ for all } \Theta > 0, \text{ then } \text{Þ} = \varpi. \quad (13)$$

By the help of [13], we have the following definition.

Definition 7. Let $\varnothing: [0,1] \times [0,1] \longrightarrow [-1/2, 1/2]$ be a mapping fulfilling the below assertions:

(i) $\varnothing(1, 1) = 0$, $\varnothing(0, 0) = 0$,

(ii) $\varnothing(\Theta, \partial) < 1/\partial + 1 - 1/\Theta + 1$,

(iii) If $\{\Theta_n\}, \{\partial_n\}$ are any two sequences in $[0, 1]$ such that $\lim_{n\longrightarrow+\infty} \Theta_n = \lim_{n\longrightarrow+\infty} \partial_n < 1$, then $\lim_{n\longrightarrow+\infty} \varnothing(\Theta_n, \partial_n) < 0$.

Definition 8. Let $(X, M, N, O, *, \bigcirc)$ be a NMS. A mapping $T\colon X \longrightarrow X$ is said to be a contractive mapping (CM) with respect to $\varnothing$ if it fulfills the below assertions:

$$\begin{aligned}&\varnothing(M(T(Þ), T(\varpi), \Theta), M(Þ, \varpi, \Theta)) \geq 0 \text{ for all } Þ, \varpi \in X,\\&\varnothing(N(T(Þ), T(\varpi), \Theta), N(Þ, \varpi, \Theta)) \leq 0 \text{ for all } Þ, \varpi \in X,\\&\varnothing(O(T(Þ), T(\varpi), \Theta), O(Þ, \varpi, \Theta)) \leq 0. \text{ for all } Þ, \varpi \in X.\end{aligned} \tag{14}$$

Remark 1. From the definition of $\varnothing$, it is clear that for all $\partial > \Theta$, $\varnothing(\Theta, \partial) \leq 0$, also $\varnothing(\Theta, \partial) > 0$ for all $\partial < \Theta$. If it is CM w.r.t. $\varnothing$, then

$$\begin{aligned}M(T(Þ), T(\varpi), \Theta) &> M(Þ, \varpi, \Theta),\\N(T(Þ), T(\varpi), \Theta) &< N(Þ, \varpi, \Theta),\\O(T(Þ), T(\varpi), \Theta) &< O(Þ, \varpi, \Theta).\end{aligned} \tag{15}$$

Lemma 2. *Suppose $(X, M, N, O, *, \bigcirc)$ is a NMS and T is a contraction with respect to $\varnothing$, then the fixed point of T in X is unique, if exists.*

Proof. Suppose $Þ \in X$ and $T(Þ) = Þ$. Let $\varpi \in X$ be another fixed point of T distinct from $Þ$, i.e., $T(Þ) = Þ$ and $Þ \neq \varpi$, then by the definition, we have

$$\begin{aligned}&0 \leq \varnothing(M(T(Þ), T(\varpi), \Theta) = M(Þ, \varpi, \Theta)), \text{ for all } Þ, \varpi \in X,\\&0 \geq \varnothing(N(T(Þ), T(\varpi), \Theta) = N(Þ, \varpi, \Theta)), \text{ for all } Þ, \varpi \in X,\\&0 \geq \varnothing(O(T(Þ), T(\varpi), \Theta) = O(Þ, \varpi, \Theta)), \text{ for all } Þ, \varpi \in X.\end{aligned} \tag{16}$$

This is contradiction, hence $Þ = \varpi$. □

Theorem 1. *Suppose $(X, M, N, O, *, \bigcirc)$ is a complete NMS and T is a contraction w.r.t. $\varnothing$, then T has a unique fixed point in X.*

Proof. Assume $Þ_0 \in X$ is a point and suppose $\{Þ_n\}$ is a sequence in X such that $Þ_n = T(Þ_{n-1})$ for all $n \in \mathbb{N}$. Now, we suppose that (without the loss of generality), $Þ_n \neq Þ_{n+1}$ for all $n \in \mathbb{N}$; also, if there exists n_0 such that $Þ_n = Þ_{n_0+1}$, then $Þ_n = Þ_{n_0+1} = T(Þ_{n_0})$. This deduces that $T(Þ_{n_0}) = Þ_{n_0}$.

Currently, by contradiction, we will examine that $\lim_{n\longrightarrow+\infty} M(Þ_n, Þ_{n+1}, \Theta) = 1$, $\lim_{n\longrightarrow+\infty} N(Þ_n, Þ_{n+1}, \Theta) = 0$, and $O(Þ_n, Þ_{n+1}, \Theta) = 0$ for all $\Theta > 0$. Suppose that there exist some Θ_0 such that $\lim_{n\longrightarrow+\infty} M(Þ_n, Þ_{n+1}, \Theta) < 1$, $\lim_{n\longrightarrow+\infty} N(Þ_n, Þ_{n+1}, \Theta) > 0$, and $\lim_{n\longrightarrow+\infty} O(Þ_n, Þ_{n+1}, \Theta) > 0$ for all $n \in \mathbb{N}$. Then, we have

$$\begin{aligned}&0 \leq \varnothing(M(T(Þ), T(\varpi), \Theta), M(Þ, \varpi, \Theta)) < \frac{1}{M(Þ_{n-1}, Þ_n, \Theta_0)} - \frac{1}{M(Þ_n, Þ_{n+1}, \Theta_0)},\\&0 \geq \varnothing(N(T(Þ), T(\varpi), \Theta), N(Þ, \varpi, \Theta)) < \frac{1}{N(Þ_{n-1}, Þ_n, \Theta_0)} - \frac{1}{N(Þ_n, Þ_{n+1}, \Theta_0)},\end{aligned} \tag{17}$$

and

$$0 \geq \varnothing(O(T(Þ), T(\varpi), \Theta), O(Þ, \varpi, \Theta)) < \frac{1}{O(Þ_{n-1}, Þ_n, \Theta_0)} - \frac{1}{O(Þ_n, Þ_{n+1}, \Theta_0)} \text{ for all } n \in \mathbb{N}. \tag{18}$$

Now, since $M(Þ_{n-1}, Þ_n, \Theta_0)$ is nondecreasing sequence and $N(Þ_{n-1}, Þ_n, \Theta_0)$ and $O(Þ_{n-1}, Þ_n, \Theta_0)$ are nonincreasing sequences of $\mathbb{R}^+$, then there exist $l \leq 1$, $\sigma \geq 0$ and $h \geq 0$ such that

$$\begin{aligned}\lim_{n\longrightarrow+\infty} M(Þ_{n-1}, Þ_n, \Theta_0) &= l,\\\lim_{n\longrightarrow+\infty} N(Þ_{n-1}, Þ_n, \Theta_0) &= \sigma,\\\lim_{n\longrightarrow+\infty} O(Þ_{n-1}, Þ_n, \Theta_0) &= h.\end{aligned} \tag{19}$$

Now, by contradiction, we will prove that $l = 1$, $\sigma = 0$, and $h = 0$. Using (iii), let $l < 1$, $\Theta_n = M(Þ_n, Þ_{n+1}, \Theta_0)$, and $\partial_n = M(Þ_{n-1}, Þ_n, \Theta_0)$. So, it concluded that

$$0 \leq \varnothing(M(Þ_n, Þ_{n+1}, \Theta_0), M(Þ_{n-1}, Þ_n, \Theta_0)) < 0. \tag{20}$$

This is a contradiction, hence $l = 1$, i.e., $\lim_{n\longrightarrow+\infty} M(Þ_n, Þ_{n+1}, \Theta_0) = 1$.

Let $\sigma > 0$ and $\Theta_n = N(Þ_n, Þ_{n+1}, \Theta_0)$, and $\partial_n = N(Þ_{n-1}, Þ_n, \Theta_0)$. So, it concluded that

$$0 \geq \varnothing(N(Þ_n, Þ_{n+1}, \Theta_0), N(Þ_{n-1}, Þ_n, \Theta_0)) > 0. \tag{21}$$

This is a contradiction, hence $\sigma = 0$, i.e., $\lim_{n\longrightarrow+\infty} N(Þ_n, Þ_{n+1}, \Theta_0) = 0$.

Let $u > 0$ and $\Theta_n = O(Þ_n, Þ_{n+1}, \Theta_0)$ and $\partial_n = O(Þ_{n-1}, Þ_n, \Theta_0)$. So, it concluded that

$$0 \geq \varnothing(O(Þ_n, Þ_{n+1}, \Theta_0), O(Þ_{n-1}, Þ_n, \Theta_0)) > 0. \tag{22}$$

This is a contradiction, hence $h = 0$, i.e., $\lim_{n\longrightarrow+\infty} O(Þ_n, Þ_{n+1}, \Theta_0) = 0$.

Now, we show that $\{Þ_n\}$ is a Cauchy sequence in X. Let us assume that $\{Þ_n\}$ is not a Cauchy sequence in X, then there exist $\varepsilon \in (0,1)$ and two subsequences $\{Þ_{m_k}\}$ and $\{Þ_{n_k}\}$ of $\{Þ_n\}$ such that $n_k > m_k \geq k$, then

$$M(Þ_{m_k}, Þ_{n_k}, \Theta_0) \leq 1-\varepsilon, N(Þ_{m_k}, Þ_{n_k}, \Theta_0) \geq \varepsilon \text{ and } O(Þ_{m_k}, Þ_{n_k}, \Theta_0) \geq \varepsilon, \tag{23}$$

and

$$M(Þ_{m_k}, Þ_{n_k-1}, \Theta_0) \geq 1-\varepsilon, N(Þ_{m_k}, Þ_{n_k-1}, \Theta_0) \leq \varepsilon \text{ and } O(Þ_{m_k}, Þ_{n_k-1}, \Theta_0) \leq \varepsilon. \tag{24}$$

Using triangular inequalities (23) and (24), we have

$$1-\varepsilon \geq M(Þ_{m_k}, Þ_{n_k}, \Theta_0) \geq M(Þ_{m_k}, Þ_{n_k-1}, \Theta_0) * M(Þ_{n_k-1}, Þ_{n_k}, \Theta_0) \geq (1-\varepsilon) * M(Þ_{n_k-1}, Þ_{n_k}, \Theta_0). \tag{25}$$

Therefore, $k \longrightarrow +\infty$ $1-\varepsilon \geq \lim_{k\longrightarrow+\infty} M(Þ_{m_k}, Þ_{n_k}, \Theta_0) \geq 1-\varepsilon$, also $k \longrightarrow +\infty$,

$$\begin{aligned} &\varepsilon \leq N(Þ_{m_k}, Þ_{n_k}, \Theta_0) \leq N(Þ_{m_k}, Þ_{n_k-1}, \Theta_0) \bigcirc N(Þ_{n_k-1}, Þ_{n_k}, \Theta_0) \leq \varepsilon \bigcirc N(Þ_{n_k-1}, Þ_{n_k}, \Theta_0) \\ &\varepsilon \leq \lim_{k\longrightarrow+\infty} N(Þ_{m_k}, Þ_{n_k}, \Theta_0) \leq \varepsilon, \end{aligned} \tag{26}$$

and

$$\begin{aligned} &\varepsilon \leq O(Þ_{m_k}, Þ_{n_k}, \Theta_0) \leq O(Þ_{m_k}, Þ_{n_k-1}, \Theta_0) \bigcirc O(Þ_{n_k-1}, Þ_{n_k}, \Theta_0) \leq \varepsilon \bigcirc O(Þ_{n_k-1}, Þ_{n_k}, \Theta_0) \\ &\varepsilon \leq \lim_{k\longrightarrow+\infty} O(Þ_{m_k}, Þ_{n_k}, \Theta_0) \leq \varepsilon. \end{aligned} \tag{27}$$

Hence,

$$\begin{aligned} &\lim_{k\longrightarrow+\infty} M(Þ_{m_k}, Þ_{n_k}, \Theta_0) = 1-\varepsilon, \\ &\lim_{k\longrightarrow+\infty} N(Þ_{m_k}, Þ_{n_k}, \Theta_0) = \varepsilon, \\ &\lim_{k\longrightarrow+\infty} O(Þ_{m_k}, Þ_{n_k}, \Theta_0) = \varepsilon. \end{aligned} \tag{28}$$

Then, we obtain

$$\begin{aligned} &1-\varepsilon \geq M(Þ_{m_k}, Þ_{n_k}, \Theta_0) \geq M(Þ_{m_k}, Þ_{m_k-1}, \Theta_0) * M(Þ_{m_k-1}, Þ_{n_k-1}, \Theta_0) * M(Þ_{n_k-1}, Þ_{n_k}, \Theta_0), \\ &M(Þ_{m_k-1}, Þ_{m_k-1}, \Theta_0) \geq M(Þ_{m_k-1}, Þ_{m_k}, \Theta_0) * M(Þ_{m_k}, Þ_{n_k}, \Theta_0) * M(Þ_{n_k}, Þ_{n_k-1}, \Theta_0). \end{aligned} \tag{29}$$

As $k \longrightarrow +\infty$, we get

$$\begin{aligned} &\lim_{k\longrightarrow+\infty} M(Þ_{m_k-1}, Þ_{n_k-1}, \Theta_0) = 1-\varepsilon, \\ &\varnothing(1-\varepsilon, 1-\varepsilon) < 0, \end{aligned} \tag{30}$$

and

$$\begin{aligned} &\varepsilon \leq N(Þ_{m_k}, Þ_{n_k}, \Theta_0) \leq N(Þ_{m_k}, Þ_{m_k-1}, \Theta_0) \bigcirc N(Þ_{m_k-1}, Þ_{n_k-1}, \Theta_0) \bigcirc N(Þ_{n_k-1}, Þ_{n_k}, \Theta_0), \\ &N(Þ_{m_k-1}, Þ_{m_k-1}, \Theta_0) \leq N(Þ_{m_k-1}, Þ_{m_k}, \Theta_0) \bigcirc N(Þ_{m_k}, Þ_{n_k}, \Theta_0) \bigcirc N(Þ_{n_k}, Þ_{n_k-1}, \Theta_0). \end{aligned} \tag{31}$$

As $k \longrightarrow +\infty$, we get

$$\lim_{k\longrightarrow+\infty} N\left(Þ_{m_k-1}, Þ_{n_k-1}, \Theta_0\right) = \varepsilon,$$
$$\lim_{k\longrightarrow+\infty} \sup \varnothing\left(N\left(Þ_{m_k}, Þ_{n_k}, \Theta_0\right), N\left(Þ_{m_k-1}, Þ_{n_k-1}, \Theta_0\right)\right) > 0,$$
$$\varnothing(\varepsilon, \varepsilon) > 0, \tag{32}$$

also

$$\varepsilon \leq O\left(Þ_{m_k}, Þ_{n_k}, \Theta_0\right) \leq O\left(Þ_{m_k}, Þ_{m_k-1}, \Theta_0\right) \bigcirc O\left(Þ_{m_k-1}, Þ_{n_k-1}, \Theta_0\right) \bigcirc O\left(Þ_{n_k-1}, Þ_{n_k}, \Theta_0\right),$$
$$O\left(Þ_{m_k-1}, Þ_{m_k-1}, \Theta_0\right) \leq O\left(Þ_{m_k-1}, Þ_{m_k}, \Theta_0\right) \bigcirc O\left(Þ_{m_k}, Þ_{n_k}, \Theta_0\right) \bigcirc O\left(Þ_{n_k}, Þ_{n_k-1}, \Theta_0\right). \tag{33}$$

As $k \longrightarrow +\infty$, we get

$$\lim_{k\longrightarrow+\infty} O\left(Þ_{m_k-1}, Þ_{n_k-1}, \Theta_0\right) = \varepsilon,$$
$$\lim_{k\longrightarrow+\infty} \sup \varnothing\left(O\left(Þ_{m_k}, Þ_{n_k}, \Theta_0\right), O\left(Þ_{m_k-1}, Þ_{n_k-1}, \Theta_0\right)\right) > 0,$$
$$\varnothing(\varepsilon, \varepsilon) > 0. \tag{34}$$

These inequalities are obviously untrue and $\{Þ_n\}$ is a Cauchy sequence in X. $(X, M, N, O, *, \bigcirc)$ is complete, so $\{Þ_n\}$ converges to some point $u \in X$, i.e.,

$$\lim_{n\longrightarrow+\infty} M(Þ_n, u, \Theta) = 1, \lim_{n\longrightarrow+\infty} N(Þ_n, u, \Theta) = 0 \text{ and } \lim_{n\longrightarrow+\infty} O(Þ_n, u, \Theta) = 0 \text{ for all } \Theta > 0. \tag{35}$$

Now, we examine that u is a fixed point of T. Let $T(u) \neq u$, then

$$M(u, T(u), \Theta) < 1, N(u, T(u), \Theta) > 0 \, an \, d \lim_{n\longrightarrow+\infty} O(Þ_n, u, \Theta) = 0,$$
$$0 \leq \lim_{n\longrightarrow+\infty} \sup \varnothing(M(T(Þ_n), T(u), \Theta), M(Þ_n, u, \Theta)),$$
$$0 \leq \lim_{n\longrightarrow+\infty} \sup\left[\frac{1}{M(Þ_n, u, \Theta)} - \frac{1}{M(T(Þ_n), T(u), \Theta)}\right],$$
$$0 \leq \lim_{n\longrightarrow+\infty} \sup\left[\frac{1}{M(Þ_n, u, \Theta)} - \frac{1}{M(T(Þ_n), T(u), \Theta)}\right],$$
$$0 \leq 1 - \frac{1}{M(u, T(u), \Theta)},$$
$$1 \leq M(u, T(u), \Theta), \tag{36}$$

and

$$
\begin{aligned}
&0 \geq \lim_{n \longrightarrow +\infty} \sup \varnothing\left(N\left(T\left(Þ_n\right), T(u), \Theta\right), N\left(Þ_n, u, \Theta\right)\right),\\
&0 \geq \lim_{n \longrightarrow +\infty} \sup \left[\frac{1}{N\left(Þ_n, u, \Theta\right)}-\frac{1}{N\left(T\left(Þ_n\right), T(u), \Theta\right)}\right],\\
&0 \geq \lim_{n \longrightarrow +\infty} \sup \left[\frac{1}{N\left(Þ_n, u, \Theta\right)}-\frac{1}{N\left(T\left(Þ_{n+1}\right), T(u), \Theta\right)}\right],\\
&0 \geq \frac{1}{0}-\frac{1}{N(u, T(u), \Theta)},\\
&0 \geq N(u, T(u), \Theta),
\end{aligned}
\tag{37}
$$

also

$$
\begin{aligned}
&0 \geq \lim_{n \longrightarrow +\infty} \sup \varnothing\left(O\left(T\left(Þ_n\right), T(u), \Theta\right), O\left(Þ_n, u, \Theta\right)\right),\\
&0 \geq \lim_{n \longrightarrow +\infty} \sup \left[\frac{1}{O\left(Þ_n, u, \Theta\right)}-\frac{1}{O\left(T\left(Þ_n\right), T(u), \Theta\right)}\right],\\
&0 \geq \lim_{n \longrightarrow +\infty} \sup \left[\frac{1}{O\left(Þ_n, u, \Theta\right)}-\frac{1}{O\left(T\left(Þ_{n+1}\right), T(u), \Theta\right)}\right],\\
&0 \geq \frac{1}{0}-\frac{1}{O(u, T(u), \Theta)},\\
&0 \geq O(u, T(u), \Theta).
\end{aligned}
\tag{38}
$$

From (36)–(38), this is a contradiction, and therefore, we examine

$$
M(u, T(u), \Theta)=1,\ N(u, T(u), \Theta)=0 \text{ and } O(u, T(u), \Theta)=0. \tag{39}
$$

Thus, $T(u)=u$.

Using the idea of NSs and NMS with CTN and CTCN, we generalize the theorems in [6] in the context of NMS using the following convention:

$$
\begin{aligned}
&[M(Þ, \varpi, \Theta)]^n=\underbrace{M(Þ, \varpi, \Theta) * M(Þ, \varpi, \Theta) * \ldots * M(Þ, \varpi, \Theta)}_{n},\\
&[N(Þ, \varpi, \Theta)]^n=\underbrace{N(Þ, \varpi, \Theta) \bigcirc N(Þ, \varpi, \Theta) \bigcirc \ldots \bigcirc N(Þ, \varpi, \Theta)}_{n},\\
&[O(Þ, \varpi, \Theta)]^n=\underbrace{O(Þ, \varpi, \Theta) \bigcirc O(Þ, \varpi, \Theta) \bigcirc \ldots \bigcirc O(Þ, \varpi, \Theta)}_{n} \text{ for all } n \in \mathbb{N}.
\end{aligned}
\tag{40}
$$

□

Theorem 2. *Let $(X, M, N, O, *, \bigcirc)$ be a complete NMS with CTN '$*$' and CTCN '$\bigcirc$' of H-type. Let $T: X \times X \longrightarrow X$ and $G: X \longrightarrow X$ be two mappings and there exist $\varnothing \in \Phi$ such that*

$$
\begin{aligned}
&M(T(Þ, \varpi), T(u, v), \varnothing(\Theta)) \geq M(G(Þ), G(u), \Theta) * M(G(\varpi), G(v), \Theta),\\
&N(T(Þ, \varpi), T(u, v), \varnothing(\Theta)) \leq N(G(Þ), G(u), \Theta) \bigcirc N(G(\varpi), G(v), \Theta),
\end{aligned}
\tag{41}
$$

and

$$
O(T(Þ, \varpi), T(u, v), \varnothing(\Theta)) \leq O(G(Þ), G(u), \Theta) \bigcirc O(G(\varpi), G(v), \Theta) \text{ for all } Þ, \varpi, u, v \in X, \Theta>0. \tag{42}
$$

If $T \subseteq G$, G is continuous, T and G are compatible, then T and G have a unique fixed point.

Theorem 3. *Let $(X, M, N, O, *, \bigcirc)$ be a complete NMS with CTN '$*$' and CTCN '$\bigcirc$' of H-type. Let*

$T: X \times X \longrightarrow X$ and $G: X \longrightarrow X$ be two weakly compatible mappings and their exist $\varnothing \in \Phi$ if $T(X \times X) \subseteq G(X)$ and $T(X \times X)$ or $G(X)$ is complete, then T and G have a unique common fixed point in X.

Proof. We assume two points $Þ_0, \varpi_0 \in X$. Since, $T(X \times X) \subseteq G(X)$, we have $Þ_1, \varpi_1 \in X$ such that $G(Þ_1) = T(Þ_0, \varpi_0)$ and $G(\varpi_1) = T(\varpi_0, Þ_0)$ and two sequences $\{Þ_n\}$ and $\{\varpi_n\}$ in X can be constructed

$$G(Þ_{n+1}) = T(Þ_n, \varpi_n) \text{ and } G(\varpi_{n+1}) = T(\varpi_n, Þ_n) \text{ for all } n \geq 0. \tag{43}$$

We shall prove that $\{G(Þ_n)\}$ and $\{G(\varpi_n)\}$ are Cauchy sequences. Therefore, for any $\lambda > 0$ there exist $\mu > 0$ and following conditions are hold for all $k \in \mathbb{N}$

$$\left.\begin{array}{l} \underbrace{(1-\mu) * (1-\mu) * \ldots * (1-\mu)}_{k} \geq 1-\lambda \\ \underbrace{\mu \bigcirc \mu \ldots \bigcirc \mu \leq \lambda}_{k} \\ \underbrace{\mu \bigcirc \mu \ldots \bigcirc \mu \leq \lambda}_{k} \end{array}\right\}. \tag{44}$$

Since, $M(Þ, \varpi, \bullet)$, $N(Þ, \varpi, \bullet)$ and $O(Þ, \varpi, \bullet)$ are continuous and $\lim_{n \longrightarrow +\infty} M(Þ, \varpi, \Theta) = 1$, $\lim_{n \longrightarrow +\infty} N(Þ, \varpi, \Theta) = 0$ and $\lim_{n \longrightarrow +\infty} O(Þ, \varpi, \Theta) = 0$ *for all* $Þ, \varpi \in X$ *their exist* $\Theta_0 > 0$, such that

$$\left.\begin{array}{l} M(G(Þ_0), G(Þ_1), \Theta_0) \geq 1-\lambda, \; M(G(\varpi_0), G(\varpi_1), \Theta_0) \geq 1-\lambda \\ N(G(Þ_0), G(Þ_1), \Theta_0) \leq \mu, \; N(G(\varpi_0), G(\varpi_1), \Theta_0) \leq \mu \\ O(G(Þ_0), G(Þ_1), \Theta_0) \leq \mu, \; O(G(\varpi_0), G(\varpi_1), \Theta_0) \leq \mu \end{array}\right\}. \tag{45}$$

Since, $\varnothing \in \Phi$, therefore by (iii), we have $\sum_{n=1}^{+\infty} \varnothing^n(\Theta_0) < +\infty$, then for any $\Theta > 0$, *their exist* $n_0 \in \mathbb{N}$, such that

$$\Theta > \sum_{n=1}^{+\infty} \varnothing^n(\Theta_0). \tag{46}$$

From Theorem 2, we have

$$\begin{aligned} M(G(Þ_1), G(Þ_2), \varnothing(\Theta_0)) &= M(T(Þ_0, \varpi_0), T(Þ_1, \varpi_1), \varnothing(\Theta_0)), \\ &\geq M(G(Þ_0), G(Þ_1), \Theta_0) * M(G(\varpi_0), G(\varpi_1), \Theta_0), \\ M(G(\varpi_1), G(\varpi_2), \varnothing(\Theta_0)) &= M(T(\varpi_0, Þ_0), T(\varpi_1, Þ_1), \varnothing(\Theta_0)), \\ &\geq M(G(\varpi_0), G(\varpi_1), \Theta_0) * M(G(Þ_0), G(Þ_1), \Theta_0), \end{aligned} \tag{47}$$

and

$$\begin{aligned} N(G(Þ_1), G(Þ_2), \varnothing(\Theta_0)) &= N(T(Þ_0, \varpi_0), N(Þ_1, \varpi_1), \varnothing(\Theta_0)), \\ &\leq N(G(Þ_0), G(Þ_1), \Theta_0) \bigcirc N(G(\varpi_0), G(\varpi_1), \Theta_0), \\ N(G(\varpi_1), G(\varpi_2), \varnothing(\Theta_0)) &= N(T(\varpi_0, Þ_0), T(\varpi_1, Þ_1), \varnothing(\Theta_0)), \\ &\leq N(G(\varpi_0), G(\varpi_1), \Theta_0) \bigcirc N(G(Þ_0), G(Þ_1), \Theta_0), \end{aligned} \tag{48}$$

also

$$\begin{aligned} O(G(Þ_1), G(Þ_2), \varnothing(\Theta_0)) &= O(T(Þ_0, \varpi_0), O(Þ_1, \varpi_1), \varnothing(\Theta_0)), \\ &\leq O(G(Þ_0), G(Þ_1), \Theta_0) \bigcirc O(G(\varpi_0), G(\varpi_1), \Theta_0), \\ O(G(\varpi_1), G(\varpi_2), \varnothing(\Theta_0)) &= O(T(\varpi_0, Þ_0), T(\varpi_1, Þ_1), \varnothing(\Theta_0)), \\ &\leq O(G(\varpi_0), G(\varpi_1), \Theta_0) \bigcirc O(G(Þ_0), G(Þ_1), \Theta_0). \end{aligned} \tag{49}$$

Similarly,

$$\begin{aligned}
M\left(G(\text{Þ}_2), G(\text{Þ}_3), \varnothing^2(\Theta_0)\right) &= M\left(T(\text{Þ}_1, \varpi_1), T(\text{Þ}_2, \varpi_2), \varnothing^2(\Theta_0)\right),\\
&\geq M(G(\text{Þ}_1), G(\text{Þ}_2), \Theta_0) * M(G(\varpi_1), G(\varpi_2), \Theta_0),\\
&\geq [M(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^2 * [M(G(\varpi_0), G(\varpi_1), \Theta_0)]^2,\\
M\left(G(\varpi_2), G(\varpi_3), \varnothing^2(\Theta_0)\right) &= M\left(T(\varpi_1, \text{Þ}_1), T(\varpi_2, \text{Þ}_2), \varnothing^2(\Theta_0)\right),\\
&\geq M(G(\varpi_1), G(\varpi_2), \Theta_0) * M(G(\text{Þ}_1), G(\text{Þ}_2), \Theta_0),\\
&\geq [M(G(\varpi_0), G(\varpi_1), \Theta_0)]^2 * [M(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^2,
\end{aligned} \tag{50}$$

and

$$\begin{aligned}
N\left(G(\text{Þ}_2), G(\text{Þ}_3), \varnothing^2(\Theta_0)\right) &= N\left(T(\text{Þ}_1, \varpi_1), T(\text{Þ}_2, \varpi_2), \varnothing^2(\Theta_0)\right),\\
&\leq N(G(\text{Þ}_1), G(\text{Þ}_2), \Theta_0) \circ N(G(\varpi_1), G(\varpi_2), \Theta_0),\\
&\leq [N(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^2 \circ [N(G(\varpi_0), G(\varpi_1), \Theta_0)]^2,\\
N\left(G(\varpi_2), G(\varpi_3), \varnothing^2(\Theta_0)\right) &= N\left(T(\varpi_1, \text{Þ}_1), T(\varpi_2, \text{Þ}_2), \varnothing^2(\Theta_0)\right),\\
&\leq N(G(\varpi_1), G(\varpi_2), \Theta_0) \circ N(G(\text{Þ}_1), G(\text{Þ}_2), \Theta_0),\\
&\leq [N(G(\varpi_0), G(\varpi_1), \Theta_0)]^2 \circ [N(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^2,
\end{aligned} \tag{51}$$

also

$$\begin{aligned}
O\left(G(\text{Þ}_2), G(\text{Þ}_3), \varnothing^2(\Theta_0)\right) &= O\left(T(\text{Þ}_1, \varpi_1), T(\text{Þ}_2, \varpi_2), \varnothing^2(\Theta_0)\right),\\
&\leq O(G(\text{Þ}_1), G(\text{Þ}_2), \Theta_0) \circ O(G(\varpi_1), G(\varpi_2), \Theta_0),\\
&\leq [O(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^2 \circ [O(G(\varpi_0), G(\varpi_1), \Theta_0)]^2,\\
O\left(G(\varpi_2), G(\varpi_3), \varnothing^2(\Theta_0)\right) &= O\left(T(\varpi_1, \text{Þ}_1), T(\varpi_2, \text{Þ}_2), \varnothing^2(\Theta_0)\right),\\
&\leq O(G(\varpi_1), G(\varpi_2), \Theta_0) \circ O(G(\text{Þ}_1), G(\text{Þ}_2), \Theta_0),\\
&\leq [O(G(\varpi_0), G(\varpi_1), \Theta_0)]^2 \circ [O(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^2.
\end{aligned} \tag{52}$$

By induction, we deduce

$$\begin{aligned}
M(G(\text{Þ}_n), G(\text{Þ}_{n+1}), \varnothing^n(\Theta_0)) &\geq [M(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^{2^{n-1}} * [M(G(\varpi_0), G(\varpi_1), \Theta_0)]^{2^{n-1}},\\
M(G(\varpi_n), G(\varpi_{n+1}), \varnothing^n(\Theta_0)) &\geq [M(G(\varpi_0), G(\varpi_1), \Theta_0)]^{2^{n-1}} * [M(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^{2^{n-1}},\\
M(G(\varpi_n), G(\varpi_{n+1}), \varnothing^n(\Theta_0)) &\geq [M(G(\varpi_0), G(\varpi_1), \Theta_0)]^{2^{n-1}} * [M(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^{2^{n-1}},\\
N(G(\varpi_n), G(\varpi_{n+1}), \varnothing^n(\Theta_0)) &\leq [N(G(\varpi_0), G(\varpi_1), \Theta_0)]^{2^{n-1}} \circ [N(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^{2^{n-1}}
\end{aligned} \tag{53}$$

and

$$O(G(Þ_n),G(Þ_{n+1}),\varnothing^n(\Theta_0)) \le [O(G(Þ_0),G(Þ_1),\Theta_0)]^{2^{n-1}} \circ [O(G(\varpi_0),G(\varpi_1),\Theta_0)]^{2^{n-1}},$$
$$O(G(\varpi_n),G(\varpi_{n+1}),\varnothing^n(\Theta_0)) \le [O(G(\varpi_0),G(\varpi_1),\Theta_0)]^{2^{n-1}} \circ [O(G(Þ_0),G(Þ_1),\Theta_0)]^{2^{n-1}}. \quad (54)$$

From (45) and (46) for $m > n \ge n_0$,

$$\begin{aligned} M(G(Þ_n),G(Þ_m),\Theta) &\ge M\left(G(Þ_n),G(Þ_m),\sum_{k=n_0}^{+\infty}\varnothing^k(\Theta_0)\right) \ge M\left(G(Þ_n),G(Þ_m),\sum_{k=n}^{m-1}\varnothing^k(\Theta_0)\right),\\ &\ge M(G(Þ_n),G(Þ_{n+1}),\varnothing^n(\Theta_0)) * M(G(Þ_{n+1}),G(Þ_{n+2}),\varnothing^n(\Theta_0)) * \dots * M\left(G(Þ_{m-1}),G(Þ_m),\varnothing^{m-1}(\Theta_0)\right),\\ &\ge [M(G(\varpi_0),G(\varpi_1),\Theta_0)]^{2^{n-1}} * [M(G(Þ_0),G(Þ_1),\Theta_0)]^{2^{n-1}} * [M(G(\varpi_0),G(\varpi_1),\Theta_0)]^{2^n}\\ &\quad * [M(G(Þ_0),G(Þ_1),\Theta_0)]^{2^n} * \dots * M\left(G(Þ_{m-1}),G(Þ_m),\varnothing^{m-1}(\Theta_0)\right),\\ &= [M(G(\varpi_0),G(\varpi_1),\Theta_0)]^{2^{m-1}-2^{n-1}} * [M(G(Þ_0),G(Þ_1),\Theta_0)]^{2^{m-1}-2^{n-1}},\\ &\ge \underbrace{(1-\mu) * (1-\mu) * \dots * (1-\mu)}_{2^m-2^n} \ge 1-\lambda. \end{aligned} \quad (55)$$

and

$$\begin{aligned} N(G(Þ_n),G(Þ_m),\Theta) &\le N\left(G(Þ_n),G(Þ_m),\sum_{k=n_0}^{+\infty}\varnothing^k(\Theta_0)\right) \le N\left(G(Þ_n),G(Þ_m),\sum_{k=n}^{m-1}\varnothing^k(\Theta_0)\right),\\ &\le N(G(Þ_n),G(Þ_{n+1}),\varnothing^n(\Theta_0)) \circ N(G(Þ_{n+1}),G(Þ_{n+2}),\varnothing^n(\Theta_0)) \circ \dots \circ N\left(G(Þ_{m-1}),G(Þ_m),\varnothing^{m-1}(\Theta_0)\right),\\ &\le [N(G(\varpi_0),G(\varpi_1),\Theta_0)]^{2^{n-1}} \circ [N(G(Þ_0),G(Þ_1),\Theta_0)]^{2^{n-1}} \circ [N(G(\varpi_0),G(\varpi_1),\Theta_0)]^{2^n},\\ &\quad \circ [N(G(Þ_0),G(Þ_1),\Theta_0)]^{2^n} \circ \dots \circ N\left(G(Þ_{m-1}),G(Þ_m),\varnothing^{m-1}(\Theta_0)\right),\\ &= [N(G(\varpi_0),G(\varpi_1),\Theta_0)]^{2^{m-1}-2^{n-1}} \circ [N(G(Þ_0),G(Þ_1),\Theta_0)]^{2^{m-1}-2^{n-1}},\\ &\le \underbrace{\mu \circ \mu \circ \dots \circ \mu}_{2^m-2^n} \le \lambda, \end{aligned} \quad (56)$$

also

$$\begin{aligned}
O(G(\text{Þ}_n), G(\text{Þ}_m), \Theta) &\leq O\left(G(\text{Þ}_n), G(\text{Þ}_m), \sum_{k=n_0}^{+\infty} \varnothing^k(\Theta_0) \right) \leq O\left(G(\text{Þ}_n), G(\text{Þ}_m), \sum_{k=n}^{m-1} \varnothing^k(\Theta_0) \right),\\
&\leq O(G(\text{Þ}_n), G(\text{Þ}_{n+1}), \varnothing^n(\Theta_0)) \bigcirc O(G(\text{Þ}_{n+1}), G(\text{Þ}_{n+2}), \varnothing^n(\Theta_0)) \bigcirc \ldots \bigcirc O\big(G(\text{Þ}_{m-1}), G(\text{Þ}_m), \varnothing^{m-1}(\Theta_0)\big),\\
&\leq [O(G(\varpi_0), G(\varpi_1), \Theta_0)]^{2^{n-1}} \bigcirc [O(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^{2^{n-1}} \bigcirc [O(G(\varpi_0), G(\varpi_1), \Theta_0)]^{2^n},\\
&\quad \bigcirc [O(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^{2^n} \bigcirc \ldots \bigcirc O\big(G(\text{Þ}_{m-1}), G(\text{Þ}_m), \varnothing^{m-1}(\Theta_0)\big),\\
&= [O(G(\varpi_0), G(\varpi_1), \Theta_0)]^{2^{m-1}-2^{n-1}} \bigcirc [O(G(\text{Þ}_0), G(\text{Þ}_1), \Theta_0)]^{2^{m-1}-2^{n-1}},\\
&\leq \underbrace{\mu \bigcirc \mu \bigcirc \ldots \bigcirc \mu}_{2^m - 2^n} \leq \lambda.
\end{aligned} \tag{57}$$

From (45) and (46), we have

$$M(G(\text{Þ}_n), G(\text{Þ}_m), \Theta) > 1 - \lambda,\ N(G(\text{Þ}_n), G(\text{Þ}_m), \Theta) < \lambda \text{ and } O(G(\text{Þ}_n), G(\text{Þ}_m), \Theta) < \lambda, \tag{58}$$

for all $m, n \in \mathbb{N}$ with $m > n > n_0$ and $\Theta > 0$ and it proved that $\{G(\text{Þ}_n)\}$ is a Cauchy sequence. Similarly, $\{G(\varpi_n)\}$ is also a Cauchy sequence. Now, we prove that G and T have a coupled coincidence point. Without loss of generality, $G(X)$ is complete, and hence, there exists $\bar{a}, \bar{e} \in X\,\text{Þ}$, $\varpi \in G(X)$, then

$$\left.\begin{aligned}
\lim_{n \longrightarrow +\infty} G(\text{Þ}_n) &= \lim_{n \longrightarrow +\infty} T(\text{Þ}_n, \varpi_n) = G(\bar{a}) = \text{Þ}\\
\lim_{n \longrightarrow +\infty} G(\varpi_n) &= \lim_{n \longrightarrow +\infty} T(\varpi_n, \text{Þ}_n) = G(\bar{e}) = \varpi
\end{aligned}\right\}. \tag{59}$$

Using Theorem 2,

$$\begin{aligned}
M(T(\text{Þ}_n, \varpi_n), T(\bar{a}, \bar{e}), \varnothing(\Theta)) &\geq M(G(\text{Þ}_n), G(\bar{a}), \Theta) * M(G(\varpi_n), G(\bar{e}), \Theta),\\
N(T(\text{Þ}_n, \varpi_n), T(\bar{a}, \bar{e}), \varnothing(\Theta)) &\leq N(G(\text{Þ}_n), G(\bar{a}), \Theta) \bigcirc N(G(\varpi_n), G(\bar{e}), \Theta),
\end{aligned} \tag{60}$$

and

$$O(T(\text{Þ}_n, \varpi_n), T(\bar{a}, \bar{e}), \varnothing(\Theta)) \leq O(G(\text{Þ}_n), G(\bar{a}), \Theta) \bigcirc O(G(\varpi_n), G(\bar{e}), \Theta). \tag{61}$$

Since M, N, and O are continuous, therefore, as $n \longrightarrow +\infty$, we get

$$M(G(\bar{a}), T(\bar{a}, \bar{e}), \varnothing(\Theta)) = 1,\ N(G(\bar{a}), T(\bar{a}, \bar{e}), \varnothing(\Theta)) = 0 \text{ and } O(G(\bar{a}), T(\bar{a}, \bar{e}), \varnothing(\Theta)) = 0. \tag{62}$$

This implies that $T(\bar{a}, \bar{e}) = G(\bar{a}) = \text{Þ}$ and similarly $T(\bar{e}, \bar{a}) = G(\bar{e}) = \varpi$.

Since T and G are weekly compatible, therefore $G(T(\bar{a}, \bar{e})) = T(G(\bar{a}), G(\bar{e}))$ and $G(T(\bar{e}, \bar{a})) = T(G(\bar{e}), G(\bar{a}))$, then $G(\text{Þ}) = T(\text{Þ}, \varpi)$ and $G(\varpi) = T(\varpi, \text{Þ})$, so $G(\text{Þ}) = \varpi$ and $G(\varpi) = \text{Þ}$, then for any $\lambda > 0$, *there exist* $\mu > 0$ such that

$$\underbrace{(1-\mu) * (1-\mu) * \ldots * (1-\mu)}_{k} \geq 1 - \lambda, \tag{63}$$

and

$$\underbrace{\mu \bigcirc \mu \bigcirc \ldots \bigcirc \mu}_{k} \leq \lambda \text{ for all } k \in \mathbb{N}. \tag{64}$$

Since $M(Þ, ϖ, \bullet)$, $N(Þ, ϖ, \bullet)$, and $O(Þ, ϖ, \bullet)$ are continuous and

$$\lim_{\Theta \longrightarrow +\infty} M(Þ, ϖ, \Theta) = 1, \lim_{\Theta \longrightarrow +\infty} N(Þ, ϖ, \Theta) = 0 \text{ and } \lim_{\Theta \longrightarrow +\infty} O(Þ, ϖ, \Theta) = 0 \text{ for all } Þ, ϖ \in X, \tag{65}$$

there exist $\Theta_0 > 0$ such that

$$\begin{aligned} &M(G(Þ), ϖ, \Theta_0) \geq 1 - \mu, M(G(ϖ), Þ, \Theta_0) \geq 1 - \mu, \\ &N(G(Þ), ϖ, \Theta_0) \leq \mu, N(G(ϖ), Þ, \Theta_0) \leq \mu, \end{aligned} \tag{66}$$

and

$$O(G(Þ), ϖ, \Theta_0) \leq \mu, O(G(ϖ), Þ, \Theta_0) \leq \mu. \tag{67}$$

Also, since $\varnothing \in \Phi$, therefore, we have $\sum_{n=1}^{+\infty} \varnothing^n(\Theta_0) < +\infty$. Thus, for any $\Theta > 0$, *there exist* $n_0 \in \mathbb{N}$ such that $\Theta > \sum_{k=n_0}^{+\infty} \varnothing^k(\Theta_0)$,

$$\begin{aligned} M(G(Þ), G(ϖ_{n+1}), \varnothing(\Theta_0)) &= M(T(Þ, ϖ), T(ϖ_n, Þ_n), \varnothing(\Theta_0)), \\ &\geq M(G(Þ), G(ϖ_n), \Theta_0) * M(G(ϖ), G(Þ_n), \Theta_0), \\ N(G(Þ), G(ϖ_{n+1}), \varnothing(\Theta_0)) &= N(T(Þ, ϖ), T(ϖ_n, Þ_n), \varnothing(\Theta_0)), \\ &\leq N(G(Þ), G(ϖ_n), \Theta_0) \circ N(G(ϖ), G(Þ_n), \Theta_0), \end{aligned} \tag{68}$$

and

$$\begin{aligned} O(G(Þ), G(ϖ_{n+1}), \varnothing(\Theta_0)) &= O(T(Þ, ϖ), T(ϖ_n, Þ_n), \varnothing(\Theta_0)), \\ &\leq O(G(Þ), G(ϖ_n), \Theta_0) \circ O(G(ϖ), G(Þ_n), \Theta_0). \end{aligned} \tag{69}$$

As $n \longrightarrow +\infty$, we get

$$\left.\begin{aligned} &M(G(Þ), ϖ, \varnothing(\Theta_0)) \geq M(G(Þ), ϖ, \Theta_0) * M(G(ϖ), Þ, \Theta_0) \\ &N(G(Þ), ϖ, \varnothing(\Theta_0)) \leq N(G(Þ), ϖ, \Theta_0) \circ N(G(ϖ), Þ, \Theta_0) \\ &O(G(Þ), ϖ, \varnothing(\Theta_0)) \leq O(G(Þ), ϖ, \Theta_0) \circ O(G(ϖ), Þ, \Theta_0) \end{aligned}\right\}. \tag{70}$$

Similarly,

$$\left.\begin{aligned} &M(G(ϖ), Þ, \varnothing(\Theta_0)) \geq M(G(ϖ), Þ, \Theta_0) * M(G(Þ), ϖ, \Theta_0) \\ &N(G(ϖ), Þ, \varnothing(\Theta_0)) \leq N(G(Þ), ϖ, \Theta_0) \circ N(G(ϖ), Þ, \Theta_0) \\ &O(G(ϖ), Þ, \varnothing(\Theta_0)) \leq O(G(Þ), ϖ, \Theta_0) \circ O(G(ϖ), Þ, \Theta_0) \end{aligned}\right\}. \tag{71}$$

From (70) and (71), we obtain

$$\begin{aligned} M(G(Þ), ϖ, \varnothing(\Theta_0)) * M(G(ϖ), Þ, \varnothing(\Theta_0)) &\geq [M(G(Þ), ϖ, \Theta_0)]^2 * [M(G(ϖ), Þ, \Theta_0)]^2, \\ N(G(Þ), ϖ, \varnothing(\Theta_0)) \circ N(G(ϖ), Þ, \varnothing(\Theta_0)) &\leq [N(G(Þ), ϖ, \Theta_0)]^2 \circ [N(G(ϖ), Þ, \Theta_0)]^2, \end{aligned} \tag{72}$$

and

$$O(G(Þ), ϖ, \varnothing(\Theta_0)) \circ O(G(ϖ), Þ, \varnothing(\Theta_0)) \leq [O(G(Þ), ϖ, \Theta_0)]^2 \circ [O(G(ϖ), Þ, \Theta_0)]^2. \tag{73}$$

From these inequalities, we obtain

$$
\begin{aligned}
M\left(G(\text{Þ}), \varpi, \varnothing^{n}(\Theta_0)\right) * M\left(G(\varpi), \text{Þ}, \varnothing^{n}(\Theta_0)\right) &\geq \left[M\left(G(\text{Þ}), \varpi, \varnothing^{n-1}(\Theta_0)\right)\right]^{2} * \left[M\left(G(\varpi), \text{Þ}, \varnothing^{n-1}(\Theta_0)\right)\right]^{2},\\
&\geq \left[M\left(G(\text{Þ}), \varpi, \Theta_0\right)\right]^{2^{n}} * \left[M\left(G(\varpi), \text{Þ}, \Theta_0\right)\right]^{2^{n}},\\
N\left(G(\text{Þ}), \varpi, \varnothing^{n}(\Theta_0)\right) \bigcirc N\left(G(\varpi), \text{Þ}, \varnothing^{n}(\Theta_0)\right) &\leq \left[N\left(G(\text{Þ}), \varpi, \varnothing^{n-1}(\Theta_0)\right)\right]^{2} \bigcirc \left[N\left(G(\varpi), \text{Þ}, \varnothing^{n-1}(\Theta_0)\right)\right]^{2},\\
&\leq \left[N\left(G(\text{Þ}), \varpi, \Theta_0\right)\right]^{2^{n}} \bigcirc \left[N\left(G(\varpi), \text{Þ}, \Theta_0\right)\right]^{2^{n}},\\
O\left(G(\text{Þ}), \varpi, \varnothing^{n}(\Theta_0)\right) \bigcirc O\left(G(\varpi), \text{Þ}, \varnothing^{n}(\Theta_0)\right) &\leq \left[O\left(G(\text{Þ}), \varpi, \varnothing^{n-1}(\Theta_0)\right)\right]^{2} \bigcirc \left[O\left(G(\varpi), \text{Þ}, \varnothing^{n-1}(\Theta_0)\right)\right]^{2},\\
&\leq \left[O\left(G(\text{Þ}), \varpi, \Theta_0\right)\right]^{2^{n}} \bigcirc \left[O\left(G(\varpi), \text{Þ}, \Theta_0\right)\right]^{2^{n}}, \text{ for all } n \in \mathbb{N}.
\end{aligned}
\tag{74}
$$

Since $\Theta > \sum_{k=n_0}^{+\infty} \varnothing^{k}(\Theta_0)$, then

$$
\begin{aligned}
M\left(G(\text{Þ}), \varpi, \Theta_0\right) * M\left(G(\varpi), \text{Þ}, \Theta_0\right) &\geq M\left(G(\text{Þ}), \varpi, \sum_{k=n_0}^{+\infty} \varnothing^{k}(\Theta_0)\right) * M\left(G(\varpi), \text{Þ}, \sum_{k=n_0}^{+\infty} \varnothing^{k}(\Theta_0)\right),\\
&\geq M\left(G(\text{Þ}), \varpi, \varnothing^{n_0}(\Theta_0)\right) * M\left(G(\varpi), \text{Þ}, \varnothing^{n_0}(\Theta_0)\right),\\
&\geq \left[M\left(G(\text{Þ}), \varpi, \Theta_0\right)\right]^{2^{n_0}} * \left[M\left(G(\varpi), \text{Þ}, \Theta_0\right)\right]^{2^{n_0}},\\
&\geq \underbrace{(1-\mu) * (1-\mu) * \ldots * (1-\mu)}_{2^{2n_0}} \geq 1-\lambda,\\
N\left(G(\text{Þ}), \varpi, \Theta_0\right) \bigcirc N\left(G(\varpi), \text{Þ}, \Theta_0\right) &\leq N\left(G(\text{Þ}), \varpi, \sum_{k=n_0}^{+\infty} \varnothing^{k}(\Theta_0)\right) \bigcirc N\left(G(\varpi), \text{Þ}, \sum_{k=n_0}^{+\infty} \varnothing^{k}(\Theta_0)\right),\\
&\leq N\left(G(\text{Þ}), \varpi, \varnothing^{n_0}(\Theta_0)\right) \bigcirc N\left(G(\varpi), \text{Þ}, \varnothing^{n_0}(\Theta_0)\right),\\
&\leq \left[N\left(G(\text{Þ}), \varpi, \Theta_0\right)\right]^{2^{n_0}} \bigcirc \left[N\left(G(\varpi), \text{Þ}, \Theta_0\right)\right]^{2^{n_0}},\\
&\leq \underbrace{\mu \bigcirc \mu \bigcirc \ldots \bigcirc \mu}_{2^{2n_0}} \leq \lambda,
\end{aligned}
\tag{75}
$$

and

$$
\begin{aligned}
O\left(G(\text{Þ}), \varpi, \Theta_0\right) \bigcirc O\left(G(\varpi), \text{Þ}, \Theta_0\right) &\leq O\left(G(\text{Þ}), \varpi, \sum_{k=n_0}^{+\infty} \varnothing^{k}(\Theta_0)\right) \bigcirc O\left(G(\varpi), \text{Þ}, \sum_{k=n_0}^{+\infty} \varnothing^{k}(\Theta_0)\right),\\
&\leq O\left(G(\text{Þ}), \varpi, \varnothing^{n_0}(\Theta_0)\right) \bigcirc O\left(G(\varpi), \text{Þ}, \varnothing^{n_0}(\Theta_0)\right),\\
&\leq \left[O\left(G(\text{Þ}), \varpi, \Theta_0\right)\right]^{2^{n_0}} \bigcirc \left[O\left(G(\varpi), \text{Þ}, \Theta_0\right)\right]^{2^{n_0}},\\
&\leq \underbrace{\mu \bigcirc \mu \bigcirc \ldots \bigcirc \mu}_{2^{2n_0}} \leq \lambda.
\end{aligned}
\tag{76}
$$

For any $\lambda > 0$, we obtain

$$\left.\begin{array}{l} M(G(Þ), ϖ, \Theta) * M(G(ϖ), Þ, \Theta) \geq 1 - \lambda \\ N(G(Þ), ϖ, \Theta) \bigcirc N(G(ϖ), Þ, \Theta) \leq \lambda \\ O(G(Þ), ϖ, \Theta) \bigcirc O(G(ϖ), Þ, \Theta) \leq \lambda \end{array}\right\}. \tag{77}$$

For all $\Theta > 0$, hence, we conclude that $G(Þ) = ϖ$ and $G(ϖ) = Þ$.

We prove that $Þ = ϖ$. From Theorem 2, we have

$$\begin{aligned} M(G(Þ_{n+1}), G(ϖ_{n+1}), \varnothing(\Theta_0)) &= M(T(Þ_n, ϖ_n), T(ϖ_n, Þ_n), \varnothing(\Theta_0)), \\ &\geq M(G(Þ_n), G(ϖ_n), \Theta_0) * M(G(ϖ_n), GÞ, \Theta_0), \\ N(G(Þ_{n+1}), G(ϖ_{n+1}), \varnothing(\Theta_0)) &= N(T(Þ_n, ϖ_n), T(ϖ_n, Þ_n), \varnothing(\Theta_0)), \\ &\leq N(G(Þ_n), G(ϖ_n), \Theta_0) \bigcirc N(G(ϖ_n), GÞ, \Theta_0), \end{aligned} \tag{78}$$

and

$$\begin{aligned} O(G(Þ_{n+1}), G(ϖ_{n+1}), \varnothing(\Theta_0)) &= O(T(Þ_n, ϖ_n), T(ϖ_n, Þ_n), \varnothing(\Theta_0)), \\ &\leq O(G(Þ_n), G(ϖ_n), \Theta_0) \bigcirc O(G(ϖ_n), GÞ, \Theta_0). \end{aligned} \tag{79}$$

As $n \longrightarrow +\infty$, we obtain

$$\begin{aligned} M(Þ, ϖ, \varnothing(\Theta_0)) &\geq M(Þ, ϖ, \Theta_0) * M(ϖ, Þ, \Theta_0), \\ N(Þ, ϖ, \varnothing(\Theta_0)) &\leq N(Þ, ϖ, \Theta_0) \bigcirc N(ϖ, Þ, \Theta_0), \\ O(Þ, ϖ, \varnothing(\Theta_0)) &\leq O(Þ, ϖ, \Theta_0) \bigcirc O(ϖ, Þ, \Theta_0). \end{aligned} \tag{80}$$

Thus, we obtain

$$\begin{aligned} M(Þ, ϖ, \Theta) &\geq M\left(Þ, ϖ, \sum_{k=n_0}^{+\infty} \varnothing^k(\Theta_0)\right) \geq M(Þ, ϖ, \varnothing^{n_0}(\Theta_0)), \\ &\geq [M(Þ, ϖ, \Theta_0)]^{2^{n_0-1}} * [M(ϖ, Þ, \Theta_0)]^{2^{n_0-1}}, \\ &\geq \underbrace{(1-\mu) * (1-\mu) * \ldots * (1-\mu)}_{2^{2n_0-2}} \geq 1 - \lambda, \\ N(Þ, ϖ, \Theta) &\leq N\left(Þ, ϖ, \sum_{k=n_0}^{+\infty} \varnothing^k(\Theta_0)\right) \leq N(Þ, ϖ, \varnothing^{n_0}(\Theta_0)), \\ &\leq [N(Þ, ϖ, \Theta_0)]^{2^{n_0-1}} \bigcirc [N(ϖ, Þ, \Theta_0)]^{2^{n_0-1}}, \\ &\leq \underbrace{\mu \bigcirc \mu \bigcirc \ldots \bigcirc \mu}_{2^{2n_0-2}} \leq \lambda, \end{aligned} \tag{81}$$

and

$$\begin{aligned} O(Þ, ϖ, \Theta) &\leq O\left(Þ, ϖ, \sum_{k=n_0}^{+\infty} \varnothing^k(\Theta_0)\right) \leq O(Þ, ϖ, \varnothing^{n_0}(\Theta_0)), \\ &\leq [O(Þ, ϖ, \Theta_0)]^{2^{n_0-1}} \bigcirc [O(ϖ, Þ, \Theta_0)]^{2^{n_0-1}}, \\ &\leq \underbrace{\mu \bigcirc \mu \bigcirc \ldots \bigcirc \mu}_{2^{2n_0-2}} \leq \lambda. \end{aligned} \tag{82}$$

Hence, $Þ = ϖ$, it is clear that T and G have a common fixed point. □

Example 1. Suppose $(X, M, N, O, *, \bigcirc)$ is a complete NMS with $\bar{a} * \bar{e} = \bar{a}\bar{e}$ and $\bar{a} \bigcirc \bar{e} = \min\{1, \bar{a} + \bar{e}\}$. Let $X = [0, 10]$ with the metric $d(Þ, ϖ) = |Þ - ϖ|$ for all $Þ, ϖ \in X$, and

$$\begin{aligned} M(Þ, ϖ, \Theta) &= \frac{\Theta}{\Theta + d(Þ, ϖ)}, \\ N(Þ, ϖ, \Theta) &= \frac{d(Þ, ϖ)}{\Theta + d(Þ, ϖ)}, \\ O(Þ, ϖ, \Theta) &= \frac{d(Þ, ϖ)}{\Theta}. \end{aligned} \tag{83}$$

The map $T, S: X \longrightarrow X$ is defined by $T(Þ) = 3 + Þ/4$ and $S(Þ) = Þ$. Let $Þ_n = (1 - 1/n)$.

$$\begin{gathered}
\lim_{n\longrightarrow+\infty} M\left(T\left(S\left(\text{Þ}_n\right)\right), S\left(T\left(\text{Þ}_n\right)\right), \Theta\right) = \lim_{n\longrightarrow+\infty} M\left(T\left(\left(\text{Þ}_n\right), S\left(\frac{3+\text{Þ}_n}{4}\right), \Theta\right)\right), \\
\lim_{n\longrightarrow+\infty} M\left(\frac{3+\text{Þ}_n}{4}, \frac{3+\text{Þ}_n}{4}, \Theta\right) = 1, \\
\lim_{n\longrightarrow+\infty} N\left(T\left(S\left(\text{Þ}_n\right)\right), S\left(T\left(\text{Þ}_n\right)\right), \Theta\right) = \lim_{n\longrightarrow+\infty} N\left(T\left(\left(\text{Þ}_n\right), S\left(\frac{3+\text{Þ}_n}{4}\right), \Theta\right)\right) \\
\lim_{n\longrightarrow+\infty} N\left(\frac{3+\text{Þ}_n}{4}, \frac{3+\text{Þ}_n}{4}, \Theta\right) = 0,
\end{gathered} \tag{84}$$

and

$$\begin{gathered}
\lim_{n\longrightarrow+\infty} O\left(T\left(S\left(\text{Þ}_n\right)\right), S\left(T\left(\text{Þ}_n\right)\right), \Theta\right) = \lim_{n\longrightarrow+\infty} O\left(T\left(\left(\text{Þ}_n\right), S\left(\frac{3+\text{Þ}_n}{4}\right), \Theta\right)\right), \\
\lim_{n\longrightarrow+\infty} O\left(\frac{3+\text{Þ}_n}{4}, \frac{3+\text{Þ}_n}{4}, \Theta\right) = 0, \\
\lim_{n\longrightarrow+\infty} T\left(\text{Þ}_n\right) = \lim_{n\longrightarrow+\infty} \left(\frac{3+\text{Þ}_n}{4}\right) \\
\lim_{n\longrightarrow+\infty} S\left(\text{Þ}_n\right) = \lim_{n\longrightarrow+\infty} \text{Þ}_n
\end{gathered} \tag{85}$$

Therefore, T and S are weakly compatible mappings. We define a map α: $[0,1] \longrightarrow [0,1]$ by $\alpha(\partial) = 2\ \partial/\partial + 1$ for each $\partial \in [0,1]$ and $\alpha \in \Phi$.

$$\begin{gathered}
M\left(T(\text{Þ}), T(\varpi), \Theta\right) \geq \alpha\left(M\left(S(\text{Þ}), S(\varpi), \Theta\right)\right), \\
M\left(\frac{3+\text{Þ}}{4}, \frac{3+\varpi}{4}, \Theta\right) \geq \alpha\left(M\left(\text{Þ}, \varpi, \Theta\right)\right), \\
\frac{\Theta}{\Theta + d\left(3+\text{Þ}/4, 3+\varpi/4\right)} \geq \frac{2\Theta/\Theta + d(\text{Þ}, \varpi)}{\Theta/\Theta + d(\text{Þ}, \varpi) + 1}, \\
\frac{\Theta}{\Theta + |3+\text{Þ}/4 - 3+\varpi/4|} \geq \frac{2\Theta/\Theta + |\text{Þ}-\varpi|}{\Theta/\Theta + |\text{Þ}-\varpi| + 1}, \\
\frac{\Theta}{\Theta + |\text{Þ}-\varpi|/4} \geq \frac{\Theta}{\Theta + |\text{Þ}-\varpi|/2}, \\
\Theta + \frac{|\text{Þ}-\varpi|}{2} \geq \Theta + \frac{|\text{Þ}-\varpi|}{4}, \\
\Rightarrow 4 > 2.
\end{gathered} \tag{86}$$

A map η: $[0,1] \longrightarrow [0,1]$ by $\eta(r) = r/2 - r$ for each $r \in [0,1]$ and $\eta \in \Phi$

$$\begin{gathered}
N\left(T(\text{Þ}), T(\varpi), \Theta\right) \leq \eta\left(N\left(S(\text{Þ}), S(\varpi), \Theta\right)\right), \\
N\left(\frac{3+\text{Þ}}{4}, \frac{3+\varpi}{4}, \Theta\right) \leq \eta\left(N\left(\text{Þ}, \varpi, \Theta\right)\right), \\
\frac{d\left(3+\text{Þ}/4, 3+\varpi/4\right)}{\Theta + d\left(3+\text{Þ}/4, 3+\varpi/4\right)} \leq \frac{d(\text{Þ}, \varpi)/\Theta + d(\text{Þ}, \varpi)}{2 - d(\text{Þ}, \varpi)/\Theta + d(\text{Þ}, \varpi)}, \\
2\Theta + |\text{Þ}-\varpi| \leq 4\Theta + |\text{Þ}-\varpi|, \\
\Rightarrow 2 < 4.
\end{gathered} \tag{87}$$

A map γ: $[0,1] \longrightarrow [0,1]$ by $\gamma(w) = \begin{cases} 1 & if\ w = 1, \\ w/2 & otherwise \end{cases}$, for each $w \in [0,1]$ and $\gamma \in \Phi$,

$$\begin{gathered}
O\left(T(\text{Þ}), T(\varpi), \Theta\right) \leq \gamma\left(O\left(S(\text{Þ}), S(\varpi), \Theta\right)\right), \\
N\left(\frac{3+\text{Þ}}{4}, \frac{3+\varpi}{4}, \Theta\right) \leq \gamma\left(O\left(\text{Þ}, \varpi, \Theta\right)\right), \\
\frac{d\left(3+\varpi/4, 3+\varpi/4\right)}{\Theta} \leq \frac{d(\text{Þ}, \varpi)}{2\Theta}, \\
\frac{|\text{Þ}-\varpi|}{4\Theta} \leq \frac{|\text{Þ}-\varpi|}{2\Theta}, \\
\Rightarrow 2 < 4.
\end{gathered} \tag{88}$$

As a result, all of the assertions of Theorem 3 are met and a unique fixed point is 1. Hence, T and S have the unique fixed point in X.

Corollary 1. *Let $(X, M, N, O, \bar{e}, *, \bigcirc)$ be complete neutrosophic b-metric space (NBMS), i.e., $\bar{e} \geq 1$, multiplying in the right sides of triangle inequalities of NMS definition, such that*

$$\lim_{\Theta \longrightarrow ++\infty} M(\text{Þ}, \varpi, \Theta) = 1, \quad \lim_{\Theta \longrightarrow +\infty} N(\text{Þ}, \varpi, \Theta) = 0, \quad \lim_{\Theta \longrightarrow +\infty} O(\text{Þ}, \varpi, \Theta) = 0 \text{ for all } \text{Þ}, \varpi \in X. \tag{89}$$

Let $T: X \longrightarrow X$ be a mapping satisfying

$$M(T\text{Þ}, T\varpi, k\Theta) \geq M(\text{Þ}, \varpi, \Theta), \; N(T\text{Þ}, T\varpi, k\Theta) \leq N(\text{Þ}, \varpi, \Theta), \; O(T\text{Þ}, T\varpi, k\Theta) \leq O(\text{Þ}, \varpi, \Theta), \tag{90}$$

for all $\text{Þ}, \varpi \in X$, $k \in (0, 1/e)$. Then, T has a unique fixed point.

3. Solution of Nonlinear Fractional Differential Equations: A Fixed Point Technique

The main goal of this section is to apply Corollary 1 to examine the existence and uniqueness of solution to a nonlinear fractional differential equation (NFDE),

$$D_{0+}^{\sigma} \varpi(\Theta) = G(\Theta, \varpi(\Theta)), \quad 0 < \Theta < 1, \tag{91}$$

with the boundary condition

$$\varpi(0) + \varpi'(0) = 0, \; \varpi(1) + \varpi'(1) = 0, \tag{92}$$

where $1 < \sigma \leq 2$ is a number, D_{0+}^{σ} is the Caputo fractional derivative, and $G: [0, 1] \times [0, +\infty) \longrightarrow [0, +\infty)$ is a continuous function. Let $X = C([0, 1], \mathbb{R})$ denote the space of all continuous functions on $[0, 1]$ with CTN '$*$' $c * d = c.d$ and CTCN $c \bigcirc d = \max\{c, d\}$ for all $c, d \in [0, 1]$ and specify the complete NBMS on X as follows:

$$M(\varpi, w, \Theta) = \frac{\alpha\Theta}{\alpha\Theta + \gamma \operatorname{Sup}_{\Theta \in [0,1]} |\varpi(\Theta) - w(\Theta)|^6},$$
$$N(\varpi, w, \Theta) = \frac{\operatorname{Sup}_{\Theta \in [0,1]} |\varpi(\Theta) - w(\Theta)|^6}{\alpha\Theta + \operatorname{Sup}_{\Theta \in [0,1]} |\varpi(\Theta) - w(\Theta)|^6}, \tag{93}$$

and

$$N(\varpi, w, \Theta) = \frac{\operatorname{Sup}_{\Theta \in [0,1]} |\varpi(\Theta) - w(\Theta)|^6}{\alpha\Theta}. \tag{94}$$

For all $\Theta > 0$ and $\varpi, w \in X$. For $\varpi \in X$, we have the following integral equation:

$$\begin{aligned} \varpi(\Theta) = {} & \frac{1}{\Gamma(\sigma)} \int_0^1 (1 - \partial)^{\sigma - 1} (1 - \Theta) G(\partial, \varpi(\partial)) d\partial \\ & + \frac{1}{\Gamma(\sigma - 1)} \int_0^1 (1 - \partial)^{\sigma - 2} (1 - \Theta) G(\partial, \varpi(\partial)) d\partial \\ & + \frac{1}{\Gamma(\sigma)} \int_0^{\Theta} (\Theta - \partial)^{\sigma - 1} G(\partial, \varpi(\partial)) d\partial. \end{aligned} \tag{95}$$

Theorem 4. *The integral operator $T: X \longrightarrow X$ is given by*

$$\begin{aligned} T\varpi(\Theta) = {} & \frac{1}{\Gamma(\sigma)} \int_0^1 (1 - \partial)^{\sigma - 1} (1 - \Theta) G(\partial, \varpi(\partial)) d\partial \\ & + \frac{1}{\Gamma(\sigma - 1)} \int_0^1 (1 - \partial)^{\sigma - 2} (1 - \Theta) G(\partial, \varpi(\partial)) d\partial \\ & + \frac{1}{\Gamma(\sigma)} \int_0^{\Theta} (\Theta - \partial)^{\sigma - 1} G(\partial, \varpi(\partial)) d\partial, \end{aligned} \tag{96}$$

where $G: [0, 1] \times [0, +\infty) \longrightarrow [0, +\infty)$ fulfils the following criteria:

$$|G(\partial, \varpi(\partial)) - G(\partial, w(\partial))| \leq \frac{1}{4} |\varpi(\partial) - w(\partial)|, \text{ for all } \varpi, w \in X,$$
$$\sup_{\Theta \in (0,1)} \frac{1}{4096} \left[\frac{1 - \Theta}{\Gamma(\sigma + 1)} + \frac{1 - \Theta}{\Gamma(\sigma)} + \frac{\Theta^{\sigma}}{\Gamma(\sigma + 1)} \right]^6 \leq \eta < 1. \tag{97}$$

Then, NFDE has a unique X.

Proof. We have

$$
\begin{aligned}
|T\varpi(\Theta)-Tw(\Theta)|^6 &= \left|\frac{1-\Theta}{\Gamma(\sigma)}\int_0^1 (1-\partial)^{\sigma-1}[G(\partial,\varpi(\partial))-G(\partial,w(\partial))]d\partial+\frac{1-\Theta}{\Gamma(\sigma-1)}\int_0^1 (1-\partial)^{\sigma-2}[G(\partial,\varpi(\partial))\right.\\
&\quad \left.-G(\partial,w(\partial))]d\partial+\frac{1}{\Gamma(\sigma)}\int_0^\Theta (\Theta-\partial)^{\sigma-1}[G(\partial,\varpi(\partial))-G(\partial,w(\partial))]d\partial\right|^6,\\
&\le\left(\frac{1-\Theta}{\Gamma(\sigma)}\int_0^1 (1-\partial)^{\sigma-1}|G(\partial,\varpi(\partial))-G(\partial,w(\partial))|d\,\partial+\frac{1-\Theta}{\Gamma(\sigma-1)}\int_0^1 (1-\partial)^{\sigma-2}|G(\partial,\varpi(\partial))\right.\\
&\quad \left.-G(\partial,w(\partial))|d\,\partial+\frac{1}{\Gamma(\sigma)}\int_0^\Theta (\Theta-\partial)^{\sigma-1}|G(\partial,\varpi(\partial))-G(\partial,w(\partial))|\right)^6,\\
&\le\left(\frac{1-\Theta}{\Gamma(\sigma)}\int_0^1 (1-\partial)^{\sigma-1}\frac{|\varpi(\partial)-w(\partial)|}{4}d\partial+\frac{1-\Theta}{\Gamma(\sigma-1)}\int_0^1 (1-\partial)^{\sigma-2}\frac{|\varpi(\partial)-w(\partial)|}{4}d\partial\right.\\
&\quad \left.+\frac{1}{\Gamma(\sigma)}\int_0^\Theta (\Theta-\partial)^{\sigma-1}\frac{|\varpi(\partial)-w(\partial)|}{4}d\partial\right)^6,\\
&\le\frac{1}{4^6}\sup|\varpi(\Theta)-w(\Theta)|^6{}_{\Theta\in[0,1]}\\
&\quad\left(\frac{1-\Theta}{\Gamma(\sigma)}\int_0^1 (1-\partial)^{\sigma-1}d\partial+\frac{1-\Theta}{\Gamma(\sigma-1)}\int_0^1 (1-\partial)^{\sigma-2}d\partial+\frac{1}{\Gamma(\sigma)}\int_0^\Theta (\Theta-\partial)^{\sigma-1}d\partial\right)^6,\\
&\le\frac{1}{4^6}\sup_{\Theta\in[0,1]}|\varpi(\Theta)-w(\Theta)|^6\left[\frac{1-\Theta}{\Gamma(\sigma+1)}+\frac{1-\Theta}{\Gamma(\sigma)}+\frac{\Theta^\sigma}{\Gamma(\sigma+1)}\right]^6,\\
&=\eta\sup_{\Theta\in[0,1]}|\varpi(\Theta)-w(\Theta)|^6,
\end{aligned}
\tag{98}
$$

where

$$\eta = \sup_{\Theta\in(0,1)}\frac{1}{4096}\left[\frac{1-\Theta}{\Gamma(\sigma+1)}+\frac{1-\Theta}{\Gamma(\sigma)}+\frac{\Theta^{\sigma}}{\Gamma(\sigma+1)}\right]^{6}. \quad (99)$$

Therefore, the above inequality

$$\begin{aligned}
&\sup_{\Theta\in[0,1]}|T\varpi(\Theta)-Tw(\Theta)|^{6}\leq \eta \sup_{\Theta\in[0,1]}|\varpi(\Theta)-\ w(\Theta|^{6},\\
&\Rightarrow \alpha\Theta+\frac{\gamma}{\eta}\sup_{\Theta\in[0,1]}|\,T\varpi(\Theta)-Tw(\Theta)|^{6}\leq \alpha\Theta+\gamma\sup_{\Theta\in[0,1]}|\varpi(\Theta)-w(\Theta)|^{6},\\
&\Rightarrow \frac{\alpha(\eta\Theta)}{\alpha(\eta\Theta)+\ \gamma\sup_{\Theta\in[0,1]}|T\varpi(\Theta)-Tw(\Theta)|^{6}}\geq\frac{\alpha\Theta}{\alpha\Theta+\gamma\sup_{\Theta\in[0,1]}|\varpi(\Theta)-w(\Theta)|^{6}},\\
&\Rightarrow M(T\varpi,Tw,\eta\Theta)\geq M(\varpi,w,\Theta).
\end{aligned} \quad (100)$$

Similarly, we can deduce

$$\begin{aligned}
&\frac{\gamma\sup_{\Theta\in[0,1]}|T\varpi(\Theta)-Tw(\Theta)|^{6}}{\alpha(\eta\Theta)+\ \gamma\sup_{\Theta\in[0,1]}|T\varpi(\Theta)-Tw(\Theta)|^{6}}\leq\frac{\gamma\sup_{\Theta\in[0,1]}|\varpi(\Theta)-w(\Theta)|^{6}}{\alpha\Theta+\gamma\sup_{\Theta\in[0,1]}|\varpi(\Theta)-w(\Theta)|^{6}},\\
&\Rightarrow N(T\varpi,Tw,\eta\Theta)\leq N(\varpi,w,\Theta),
\end{aligned} \quad (101)$$

and

$$\begin{aligned}
&\frac{\gamma\sup_{\Theta\in[0,1]}|T\varpi(\Theta)-Tw(\Theta)|^{6}}{\alpha(\eta\Theta)}\leq\frac{\gamma\sup_{\Theta\in[0,1]}|\varpi(\Theta)-w(\Theta)|^{6}}{\alpha\Theta},\\
&\Rightarrow O(T\varpi,Tw,\eta\Theta)\leq O(\varpi,w,\Theta).
\end{aligned} \quad (102)$$

for some $\alpha,\gamma>0$. As a result, we can conclude that Corollary 1 assumptions are met. Hence, T has a unique fixed point and NFDE has a unique solution. □

4. Conclusion

In the presented study, various fixed point results for contraction and weakly compatible mappings are proved. As known that fixed point theory has a wide range of applications in economics, engineering, and computer science, we proved nonlinear fractional differential equations application via neutrosophic metric space and unique solution exists. This work can be extended in several structures, such as neutrosophic b-metric spaces and orthogonal neutrosophic metric spaces. Our techniques may help many researchers working in the field of plasma physics. We only remember plasma physics as an example due to its richness by several differential equations that are used to describe many nonlinear phenomena that can propagate in different plasma models [16–18].

Data Availability

On request, the data used to support the findings of this study can be obtained from the corresponding author.

Conflicts of Interest

There are no conflicts of interest declared by the authors.

Authors' Contributions

This article was written in collaboration by all of the contributors. The final manuscript was read and approved by all authors.

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