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Retraction

Retracted: New Concepts in the Vague Graph Structure with an Application in Transportation

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] X. Shi and S. Kosari, "New Concepts in the Vague Graph Structure with an Application in Transportation," *Journal of Function Spaces*, vol. 2022, Article ID 1504397, 11 pages, 2022.

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Research Article

New Concepts in the Vague Graph Structure with an Application in Transportation

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Fuzzy graph (FG) models take on the presence being ubiquitous in environmental and fabricated structures by humans, specifically the vibrant processes in physical, biological, and social systems. Owing to the unpredictable and indiscriminate data which are intrinsic in real life and problems being often ambiguous, it is very challenging for an expert to exemplify these problems through applying an FG. Vague graph structure (VGS), belonging to the FG family, has good capabilities when facing with problems that cannot be expressed by FGs. VGS can handle the vagueness connected with the incompatible and determinate information of any real-world problem, where FGs may not succeed to bear satisfactory results. The previous definitions' restrictions in FGs have led us to propose new definitions in the VGS. Domination is one of the key issues that has many applications in computer science and social networks. Today, many researchers are trying to prove its application in medical sciences and psychology. Therefore, in this paper, different concepts related to domination in VGSs such as Ni-dominating set, vague full dominating set, minimal N_i-DS, and strong capacity Ni-dominating set are defined using some examples. Finally, an application of domination in medical sciences has been presented.

1. Introduction

The FG concept serves as one of the most dominant and extensively employed tools for multiple real-word problem representations, modeling, and analysis. To specify the objects and the relations between them, the graph vertices or nodes and edges or arcs are applied, respectively. Graphs have long been used to describe objects and the relationships between them. Many of the issues and phenomena around us are associated with complexities and ambiguities that make it difficult to express certainty. These difficulties were alleviated by the introduction of fuzzy sets by Zadeh [1]. This concept established well-grounded allocation membership degree to elements of a set. Rosenfeld [2] proposed the idea of the FG in 1975. The existence of a single degree for a true membership could not resolve the ambiguity on uncertain issues, so the need for a degree of membership was felt. Afterward, to overcome the existing ambiguities, Gau and Buehrer [3] introduced false-membership degrees and defined a vague set as the sum of degrees not greater than 1. Kauffman [4] represented FGs based on Zadeh's fuzzy relation [5, 6]. Samanta and Pal [7] defined fuzzy competition graphs. Vague graph notion was introduced by Ramakrishna in [8]. Borzooei et al. [9–11] investigated new concepts of VGs. Ramakrishnan and Dinesh [12, 13] studied on generalized FGSs. Harinath and Lavanya [14] represented FGSs based on wheel and star graphs. Akram et al. [15–18] defined intuitionistic fuzzy graph structures and m-polar fuzzy graph structures. VGS, belonging to the FG family, has good capabilities when facing with problems that cannot be expressed by fuzzy graphs. A VGS is referred to as a generalized structure of an FG that conveys more exactness, adaptability, and compatibility to a system when coordinated with systems running on FGs. Also, a VGS is able to concentrate on determining the uncertainly coupled with the inconsistent and indeterminate information of any real-world problem, where FGs may not lead to adequate results. Kosari et al.

[19] defined the vague graph structure and studied new concepts of irregularity on it.

Domination is one of the most important topics that has many applications in social groups and fuzzy social networks. Today, many researchers are trying to prove its application in medical sciences and psychology. For the first time, the concept of DS in graph theory was introduced by Ore [20]. The domination in the FG was defined by Somasundaram [21, 22]. Nagoor Gani et al. [23, 24] investigated several concepts of domination in FGs. Enriquez et al. [25] presented domination in the fuzzy directed graph. Parvathi and Thamizhendhi [26] described the DS in an intuitionistic fuzzy graph. Shi and Kosari [27] examined domination in product vague graphs. Talebi et al. [28] studied new concepts of domination in fuzzy graph structures. Sahoo et al. [29] gave covering and paired domination in the intuitionistic fuzzy graph. In this paper, we introduce several concepts related to domination in VGSs such as Ni-dominating set, vague full dominating set, minimal Ni-DS, and strong capacity Ni-dominating set, with some examples. Finally, an application of domination in transferring patients has been presented.

2. Preliminaries

An FG is of the form $G = (\mu, \nu)$ which is a pair of mappings $\mu\colon V \longrightarrow [0,1]$ and $\nu\colon V \times V \longrightarrow [0,1]$ as is defined as $\nu(a,b) \leq \mu(a) \wedge \mu(b)$, $\forall a, b \in V$, and ν is a symmetric fuzzy relation on μ, and $\wedge$ denotes minimum.

A VS M is a pair (t_M, f_M) on set V where t_M and f_M are taken as real-valued functions which can be defined on $V \longrightarrow [0,1]$ so that $t_M(a) + f_M(a) \leq 1$, for all $a \in V$.

Definition 1 (see [8]). A pair $G = (M, N)$ is called to be a VG on a CG G^*, where $M = (t_M, f_M)$ is a VS on V and $N = (t_N, f_N)$ is a VS on $E \subseteq V \times V$ so that $t_N(ab) \leq \min(t_M(a), t_M(b))$ and $f_N(ab) \geq \max(f_M(a), f_M(b))$, $\forall ab \in E$.

Definition 2 (see [12]). $\zeta = (\mu, \nu_1, \nu_2, \ldots, \nu_n)$ is named a fuzzy graph structure (FGS) of graph structure $\zeta^* = (V, E_1, E_2, \ldots, E_n)$ whenever $\mu, \nu_1, \nu_2, \ldots, \nu_n$ are FSs of $V, E_1, E_2, \ldots, E_n$, respectively, so that $\nu_i(ab) \leq \mu(a) \wedge \mu(b)$, $\forall a, b \in V$ and $i = 1, 2, \ldots, n$. Note that if $ab \in Supp(\nu_i)$, then ab is called a ν_i-edge of ζ, for $i = 1, 2, \ldots, n$.

Definition 3 (see [19]). $\xi = (M, N_1, N_2, \ldots, N_n)$ is called a VGS of a GS $\zeta^* = (V, E_1, E_2, \ldots, E_n)$ if $M = (t_M, f_M)$ is a VS on V and for every $i = 1, 2, \ldots, n$; $N_i = (t_{N_i}, f_{N_i})$ is a VS on E_i so that

$$t_{N_i}(ab) \leq t_M(a) \wedge t_M(b),$$
$$f_{N_i}(ab) \geq f_M(a) \vee f_M(b), \tag{1}$$

$\forall ab \in E_i \subseteq V \times V$. Note that $t_{N_i}(ab) = 0 = f_{N_i}(ab)$, $\forall ab \in V \times V - E_i$, and $0 \leq t_{N_i}(ab) \leq 1$ and $0 \leq f_{N_i}(ab) \leq 1$, $\forall ab \in E_i$, where V and $E_i\,(i = 1, 2, \ldots, n)$ are called the underlying vertex set and underlying i-edge set of ξ, respectively.

Example 1 (see [19]). Consider the VGS $\xi = (M, N_1, N_2)$ as shown in Figure 1, where $M = \{(a, 0.2, 0.3), (b, 0.2, 0.4), (c, 0.1, 0.2), (d, 0.2, 0.2)\}$, $N_1 = \{(ab, 0.2, 0.4), (bc, 0.1, 0.4)\}$, and $N_2 = \{(c\ d, 0.1, 0.2), (a\ d, 0.1, 0.3)\}$. Clearly, ξ is a VGS.

Definition 4 (see [19]). A VGS $\xi = (M, N_1, N_2, \ldots, N_n)$ is called SVGS whenever $t_{N_i}(ab) = t_M(a) \wedge t_M(b)$ and $f_{N_i}(ab) = f_M(a) \vee f_M(b)$, $\forall ab \in N_i$, $i = 1, 2, \ldots, n$.

Definition 5 (see [19]). A VGS $\xi = (M, N_1, N_2, \ldots, N_n)$ is called the CVGS if

(i) ξ is a SVGS
(ii) $Supp(N_i) \neq \emptyset$, $\forall i = 1, 2, \ldots, n$
(iii) For every pair of nodes $ab \in V$, ab is an N_i-edge for some i

Definition 6 (see [19]). Suppose that $\xi = (M, N_1, N_2, \ldots, N_n)$ is a VGS. Then,

$$\begin{aligned} p &= |M| = \left(\sum_{a \in V} t_M(a), \sum_{a \in V} f_M(a) \right), \\ q_i &= |N_i| = \left(\sum_{ab \in E_i} t_M(ab), \sum_{ab \in E_i} f_M(ab) \right), \\ q &= \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} |N_i|. \end{aligned} \tag{2}$$

Definition 7 (see [19]). A VGS $\xi = (M, N_1, N_2, \ldots, N_n)$ is named CBVGS if

(i) V can be partitioned into two subsets V_1 and V_2 so that each N_i-edge joins one node of V_1 to one node of V_2, for some i
(ii) ξ is a SVGS
(iii) $Supp(N_i) \neq \emptyset$, $\forall i, 1 \leq i \leq n$

Definition 8. Let $\xi = (M, N_1, N_2, \ldots, N_n)$ be a VGS; then, the VC of $T \subseteq V$ is defined as

$$|T| = \left| \sum_{a \in T} \frac{1 + t_M(a) - f_M(a)}{2} \right|. \tag{3}$$

Some of the basic notations are listed in Table 1.

3. Domination in Vague Graph Structures

Definition 9. Suppose that $\xi = (M, N_1, N_2, \ldots, N_n)$ is a VGS of $\zeta^* = (V, E_1, E_2, \ldots, E_n)$. We say that aN_i-dominates b, and conversely, ab is an N_i-edge in ξ. $K \subseteq V$ is named the N_i-DS if for each $b \in V - K$, $\exists$ some $a \in K$ so that aN_i-dominates b. The minimum cardinality of N_i-DSs is called N_i-DN and is described by $\gamma^{(i)}(\xi)$ or $\gamma^{(i)}$.

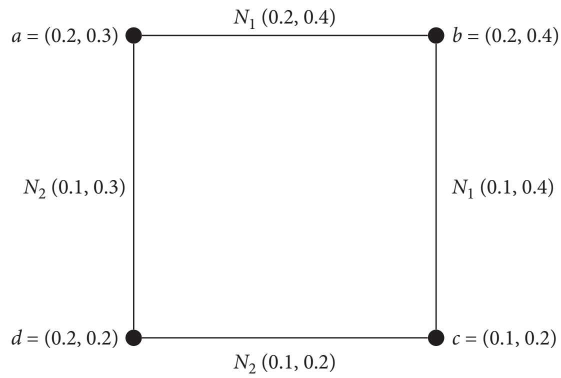

Figure 1: Vague graph structure ξ.

Table 1: Some basic notations.

Notation	Meaning
FG	Fuzzy graph
VG	Vague graph
VC	Vertex cardinality
DN	Domination number
MI-C	Minimal cardinality
FE	Full edge
CGS	Complete graph structure
FGS	Fuzzy graph structure
CN	Closed neighborhood
Ni-IV	Ni-isolated vertex
NiMI-DS	Ni minimal-dominating set
CVGS	Complete vague graph structure
VFDS	Vague full dominating set
CBVGS	Complete bipartite vague graph structure
VS	Vague set
VGS	Vague graph structure
DS	Dominating set
CG	Crisp graph
FD	Fully dominated
GS	Graph structure
SC	Strong capacity
FN	Full neighborhood
ND	Neighborhood degree
FIV	Full isolated vertex
Ni-ND	Ni-neighborhood degree
SVGS	Strong vague graph structure
WC	Weak capacity

Definition 10. $K \subseteq V$ is called the vague $N_{i_1,i_2,\ldots,i_t}$-dominating set ($N_{i_1,i_2,\ldots,i_t}$-DS) in ξ if for every $b \in V - K$, there exists $a \in K$ so that a dominated b for one of the edges $\{N_{i_1}, N_{i_2}, \ldots, N_{i_t}\}$. The $N_{i_1,i_2,\ldots,i_t}$-dominating number $\gamma^{(i_1,i_2,\ldots,i_t)}(\xi)$ or $\gamma^{(i_1,i_2,\ldots,i_t)}$ is the MI-C of a $N_{i_1,i_2,\ldots,i_t}$-DS. It is obvious that $\gamma^{(i_1,i_2,\ldots,i_t)} \leq P$.

Definition 11. $T \subseteq V$ is called the VFDS if, for each $b \in V - T$, there is some $a \in T$ so that a dominates b for one of the edges $N_1, N_2, \ldots, N_n$. The full dominating number on VGS $\xi = (M, N_1, N_2, \ldots, N_n)$ is the MI-C of VDSs in ξ and is described by $\gamma^f(\xi)$ or γ^f. A VFDS is a $N_{1,2,\ldots,k}$-DS.

Example 2. Consider the VGS $\xi = (M, N_1, N_2, N_3)$ as shown in Figure 2, where

$$M = \{(a, 0.1, 0.2), (b, 0.3, 0.5), (c, 0.2, 0.4), (d, 0.1, 0.3), (e, 0.3, 0.4), (f, 0.2, 0.3)\},$$
$$N_1 = \{(af, 0.1, 0.3), (ac, 0.1, 0.5), (be, 0.2, 0.5), (c\ d, 0.1, 0.4)\},$$

$$N_2 = \{(ab, 0.1, 0.6), (df, 0.1, 0.3), (ce, 0.2, 0.6)\},$$
$$N_3 = \{(bf, 0.2, 0.7), (de, 0.1, 0.4)\}. \tag{4}$$

The N_1-DSs of ξ are as follows:

$$\begin{aligned} K_1 &= \{a, b, d\}, \\ K_2 &= \{b, c, f\}, \\ K_3 &= \{b, d, f\}, \\ K_4 &= \{a, d, e\}, \\ K_5 &= \{c, e, f\}, \\ K_6 &= \{d, e, f\}. \end{aligned} \tag{5}$$

With a simple calculation, $\gamma^{(1)} = 1.25$. The N_2-DSs of ξ are as follows:

$$\begin{aligned} K_1 &= \{a, c, f\}, \\ K_2 &= \{a, c, d\}, \\ K_3 &= \{a, e, f\}, \\ K_4 &= \{b, e, f\}, \\ K_5 &= \{a, c, d\}, \\ K_6 &= \{b, c, f\}, \\ K_7 &= \{a, d, e\}, \\ K_8 &= \{b, d, e\}, \end{aligned} \tag{6}$$

in which $\gamma^{(2)} = 1.2$. Accordingly, the N_3-DSs are as follows:

$$\begin{aligned} K_1 &= \{a, b, c, e\}, \\ K_2 &= \{a, c, e, f\}, \\ K_3 &= \{a, b, c, d\}, \\ K_4 &= \{a, c, d, f\}, \end{aligned} \tag{7}$$

where $\gamma^{(3)} = 1.65$. The $N_{1,2}$-DSs of ξ are as follows:

$$\begin{aligned} K_1 &= \{a, d, e\}, \\ K_2 &= \{b, c, f\}, \\ K_3 &= \{b, d\}, \\ K_4 &= \{e, f\}, \end{aligned} \tag{8}$$

where $\gamma^{(1,2)} = 0.8$. Similarly, $\gamma^{(1,3)} = 0.8$, and $\gamma^{(2,3)} = 0.85$. The VFDSs of ξ are as follows:

$$\begin{aligned} T_1 &= \{b, c\}, \\ T_2 &= \{b, d\}, \\ T_3 &= \{a, d\}, \\ T_4 &= \{e, f\}, \\ T_5 &= \{a, e\}, \\ T_6 &= \{c, f\}, \end{aligned} \tag{9}$$

where $\gamma^f = 0.8$.

Theorem 1. *A VGS $\xi = (M, N_1, \ldots, N_n)$ is a CVGS if and only if every node is a VFDS.*

Proof. Let $\xi = (M, N_1, \ldots, N_n)$ be a CVGS. Consider an arbitrary node $a \in V$. Since a is incident with each node of $b \in V - \{a\}$ by some N_i-edge, a is FD by b. Therefore, $T = \{a\}$ is a VFDS, $\forall a \in V$.

Conversely, assume that $T = \{a\}$ is a VFDS, $\forall a \in V$. So, a FD b, $\forall b \in V - T$. In other words, there exists a FE from a to b, $\forall b \in V - T$. Hence, $t_N(ab) = t_M(a) \wedge t_M(b)$ and $f_N(ab) = f_M(a) \vee f_M(b)$, for all $ab \in E_i$, $i = 1, 2, \ldots, n$ and $a, b \in V$. □

Remark 1. If $\xi = (M, N_1, N_2, \ldots, N_n)$ is a CBVGS with $V = V_1 \cup V_2$, then

(i) $\gamma^{(1)} = \gamma^{(2)} = \cdots = \gamma^{(n)}$

(ii) $\gamma^{(f)} = \min\{|V_1|, |V_2|\}$

Example 3. Consider a CBVGS $\xi = (M, N_1, N_2, N_3)$ as shown in Figure 3, where

$$\begin{aligned} M &= \{(a, 0.2, 0.3), (b, 0.3, 0.4), (c, 0.1, 0.2), \\ &\quad \cdot (d, 0.2, 0.4), (e, 0.3, 0.5)\}, \\ N_1 &= \{(ac, 0.1, 0.3), (bc, 0.1, 0.4)\}, \\ N_2 &= \{(a\ d, 0.2, 0.4), (b\ d, 0.2, 0.4)\}, \\ N_3 &= \{(ae, 0.2, 0.5), (be, 0.3, 0.5)\}. \end{aligned} \tag{10}$$

The DSs and DN for Figure 3 are as follows:

$$\begin{aligned} &|\{c, d, e\}| = 1.25, |\{a, b, d, e\}| = 1.7, \gamma^{(1)} = 1.25, \\ &|\{c, d, e\}| = 1.25, |\{a, b, c, e\}| = 1.75, \gamma^{(2)} = 1.25, \\ &|\{e, c, d\}| = 1.25, |\{a, b, c, d\}| = 1.75, \gamma^{(3)} = 1.25. \end{aligned} \tag{11}$$

So, $\gamma^{(1)} = \gamma^{(2)} = \gamma^{(3)} = 1.25$. Furthermore, $|\{a, b\}| = 0.9$, $|\{c, d, e\}| = 1.25$, and $\gamma^{(f)} = 0.9$. It is clear that $\gamma^{(f)} = \min\{|V_1|, |V_2|\}$ so that $V_1 = \{a, b\}$ and $V_2 = \{c, d, e\}$.

Definition 12. Let $\xi = (M, N_1, N_2, \ldots, N_n)$ be a VGS. The N_i-neighborhood of a is described by

$$N^{(i)}(a) = \{b \in V | ab \text{ is an } N_i - \text{edge}\}. \tag{12}$$

Likewise, $N_{i_1, i_2, \ldots, i_t}$-neighborhood and FN of a are described as

$$\begin{aligned} N^{(i_1, i_2, \ldots, i_t)}(a) &= \left\{b \in V | ab \text{ is an } N_{i_1, i_2, \ldots, i_t} - \text{edge}\right\}, \\ N^f(a) &= \{b \in V | ab \text{ is a full } \mu - \text{edge}\}. \end{aligned} \tag{13}$$

The N_i-CN of a is described as

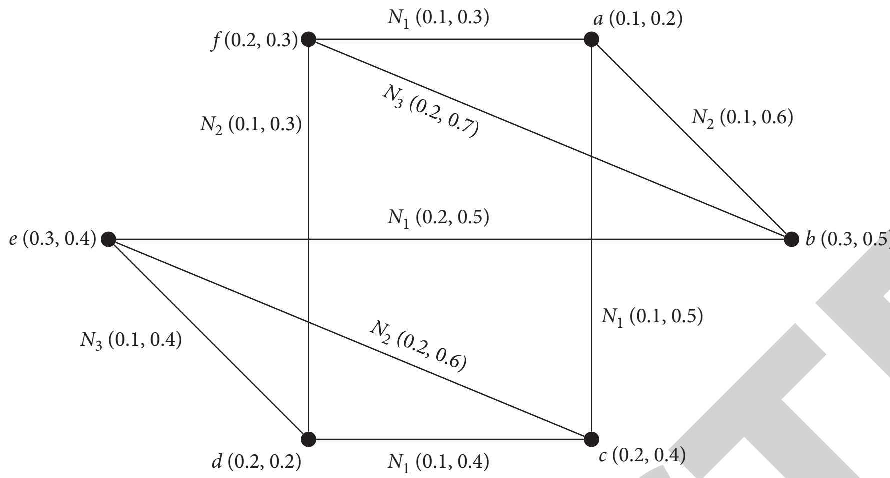

Figure 2: VGS $\xi = (M, N_1, N_2, N_3)$.

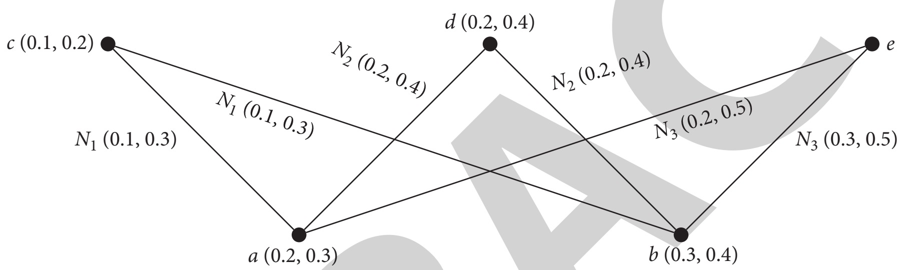

Figure 3: A complete bipartite VGS ξ.

$$N^{(i)}[a] = N^{(i)}(a) \cup \{a\}. \tag{14}$$

In the same way, $N^{(i_1,i_2,\ldots,i_t)}[x] = N^{(i_1,i_2,\ldots,i_t)}(a) \cup \{a\}$ and $N^f[a] = N^f(a) \cup \{a\}$. For nonempty set $B \subseteq V$, we have

$$\begin{aligned} N^{(i)}(B) &= \left\{N^{(i)}(a) \mid a \in B\right\}, \\ N^{(i)}[B] &= N^{(i)}(B) \cup B, \\ N^f(B) &= \left\{N^f(a) \mid a \in B\right\}, \\ N^f[B] &= N^f(B) \cup B. \end{aligned} \tag{15}$$

Definition 13. In a VGS $\xi = (M, N_1, N_2, \ldots, N_n)$, the N_i-ND of node a is described as

$$d\left(N^{(i)}(a)\right) = \left(\sum_{b \in N^{(i)}(a)} M_A(b), \sum_{b \in N^{(i)}(a)} N_A(b)\right). \tag{16}$$

Similarly,

$$d\left(N^{(i_1,i_2,\ldots,i_t)}(a)\right) = \left(\sum_{b \in N^{(i_1,i_2,\ldots,i_t)}(a)} M_A(b), \sum_{b \in N^{(i_1,i_2,\ldots,i_t)}(a)} N_A(b)\right), \tag{17}$$

$$d\left(N^f(a)\right) = \left(\sum_{b \in N^f(a)} M_A(b), \sum_{b \in N^f(a)} N_A(b)\right). \tag{18}$$

The minimum and maximum N_i-ND of ξ are denoted by $\delta_N^{(i)}$ and $\Delta_N^{(i)}$, respectively. In the same way, for the FND, they are shown by δ_N^f and Δ_N^f.

Example 4. Consider the VGS $\Gamma = (M, N_1, N_2, N_3)$ in Figure 4. We have

$$\begin{aligned} N^{(1)}(a) &= \{b, f\}, \\ N^{(1)}(c) &= \varnothing, \\ N^{(1)}(b) &= \{a, d, f\}, \\ d\left(N^{(1)}(a)\right) &= (0.3, 0.5), \\ d\left(N^{(1)}(b)\right) &= (0.6, 1.1), \\ d\left(N^{(1)}(c)\right) &= 0. \end{aligned} \tag{19}$$

Obviously, $\delta_N^{(1)} = 0$, $\Delta_N^{(1)} = (0.6, 1.1)$, $N^{(2,3)}(a) = \{f, c\}$, $N^{(2,3)}(b) = \{d\}$, $N^{(2,3)}(d) = \{b, e\}$, $N^{(2,3)}(c) = \{a\}$, $N^{(2,3)}(e) = \{d\}$, $N^{(2,3)}(f) = \{a\}$, $d(N^{(2,3)}(a)) = (0.5, 0.7)$, $d(N^{(2,3)}(b)) = (0.2, 0.5)$, $d(N^{(2,3)}(c)) = (0.2, 0.3)$, $d(N^{(2,3)}(d)) = (0.2, 0.6)$, $d(N^{(2,3)}(e)) = (0.2, 0.5)$, and $d(N^{(2,3)}(f)) = (0.2, 0.3)$. Hence, $\delta_N^{(2,3)} = (0.2, 0.3)$ and $\Delta_N^{(2,3)} = (0.5, 0.7)$.

For full neighborhood, we have $N^f(a) = \{f\}$, $N^f(b) = \{d\}$, $N^f(c) = \varnothing$, $N^f(d) = \{b\}$, $N^f(e) = \varnothing$, $N^f(f) = \{a\}$, $\delta_N^f = 0$, and $\Delta_N^f = (0.2, 0.5)$.

Definition 14. For VGS $\xi = (M, N_1, N_2, \ldots, N_n)$, $a \in V$ is named an N_i-IV whenever $N^{(i)}(a) = \varnothing$, $\forall i = 1, \ldots, n$. In

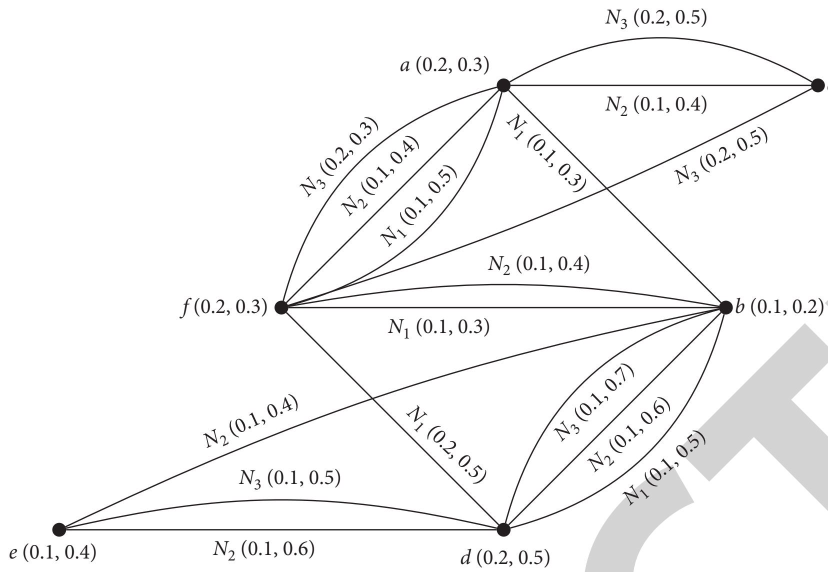

Figure 4: VGS $\xi = (M, N_1, N_2, N_3)$.

fact, for every node $b \in V$, where $b \neq a$, ab is not an N_i-edge. Also, a is FIV if $N^{f}(a) = \varnothing$. For the set $B \subseteq V$, $a \in B$ is called an N_i-IV in B if $N^{(i)}(a) \subseteq V - B$.

Example 5. For VGS $\xi = (M, N_1, N_2, N_3)$ shown in Figure 2, nodes a and c are N_3-IV. $N^{(3)}(a) = N^{(3)}(c) = \varnothing$. Note that a is N_1-IV in $B = \{a, b, d\}$ too because its N_1-neighborhood is not in B.

$$N^{(1)}(a) = \{f, c\} \subset \{c, e, f\} = V - B. \quad (20)$$

Remark 2. $\gamma^{(f)} = \gamma^{(i)} = p$ iff every node in ξ is N_i-IV, $\forall i = 1, 2, \ldots, n$. In addition, if $\forall i = 1, 2, \ldots, n$, a is an N_i-IV, then a is a FIV.

Theorem 2. *If T is a MI-VFDS of a VGS $\xi = (M, N_1, N_2, \ldots, N_n)$ without FIVs, then $V - T$ is a VFDS of ξ.*

Proof. Let $\xi = (M, N_1, N_2, \ldots, N_n)$ be a VGS without FIVs and T be a MI-VFDS of ξ. Consider an optional node $a \in T$. Since ξ has no FIVs, $\exists$ a node $b \in N^{f}(a)$ so that ab is an N_i-edge for some $i = 1, 2, \ldots, n$. Hence, $b \in V - T$ and is FD by a. Again, $b \in V - T$ is incident with some element of T. So, each node of $V - T$ is FD by some node of T. Therefore, $V - T$ is a VFDS of ξ. □

Theorem 3. *An N_i-DS K in VGS $\xi = (M, N_1, N_2, \ldots, N_n)$ is an N_i-MI-DS if and only if for each $a \in K$, one of the two following conditions holds:*

(i) a is an N_i-IV of K

(ii) There is a node $b \in V - K$ so that $N^{(i)}(b) \cap D = \{a\}$

Proof. Suppose that K is the N_i MI-DS and $a \in D$. Since K is MI-DS, $K' = K - \{a\}$ is not a N_i-DS. Hence, $\exists$ some $b \in V - K' = V - (K - \{a\}) = (V - K) \cup \{a\}$ so that b is not N_i-D by each element of $K - \{a\}$. If $b = a$, then a is an N_i-IV of K. If $b \in V - K$, then b is N_i-D by $a \in K$ since b is not N_i-D by $K - \{a\}$, but it is N_i-D by K. Hence, $N^{(i)}(b) \cap K = \{a\}$.

Conversely, assume that K is an N_i-DS, and for each node $a \in K$, one of the two conditions holds. We show that K is an N_i MI-DS. Assume K is not an N_i MI-DS. Hence, $\exists$ a node $a \in K$ so that $K - \{a\}$ is an N_i-DS. Therefore, a is N_i-D by at least one node in $K - \{a\}$. So, condition (i) does not hold. Also, if $K - \{a\}$ is an N_i-DS, then each node in $V - K$ is N_i-D by at least one node in $K - \{a\}$. Thus, condition (ii) does not hold, and this contradicts our assumption. □

Proposition 1. *If $\xi = (M, N_1, N_2, \ldots, N_n)$ is a VGS without N_i-IV, then*

$$\gamma^{(i)} \leq \frac{p}{2}. \quad (21)$$

Proof. Suppose that K is an N_i MI-DS of ξ. Since $V - K$ is an N_i-DS of ξ, we have $|K| \leq |V - K|$. So, $2|K| \leq |V|$ and $|K| \leq |V|/2$. Therefore, $\gamma^{(i)} \leq p/2$. □

Theorem 4. *If $\xi = (M, N_1, N_2, \ldots, N_n)$ is a VGS without N_i-IV, then*

$$\gamma^{(i)} \leq p - \Delta_N^{(i)} \leq p - \delta_N^{(i)}. \quad (22)$$

Proof. Let $\xi = (M, N_1, N_2, \ldots, N_n)$ be a VGS and $a \in V$ be a node with the maximum N_i-ND of ξ. Clearly, $V - N^{(i)}(a)$ is an N_i-DS of ξ. If K is a minimum N_i-DS of ξ, then $|K| \leq |V - N^{(i)}(a)|$, $\gamma^{(i)} \leq p - \Delta_N^{(i)}$. Since $\Delta_N^{(i)} \geq \delta_N^{(i)}$, $\gamma^{(i)} \leq p - \Delta_N^{(i)} \leq p - \delta_N^{(i)}$. □

Corollary 1. *In a VGS $\xi = (M, N_1, N_2, \ldots, N_n)$ without N_i-IV,*

$$\gamma^f \le p - \Delta_N^f \le p - \delta_N^f. \tag{23}$$

Definition 15. Suppose $\xi = (M, N_1, N_2, \ldots, N_n)$ is a VGS and $a \in V$. The N_i-degree of a is shown as $d^{(i)}(a)$ in that we have

$$d^{(i)}(a) = \left(\sum_{ab \in N_i} t_N(ab), \sum_{ab \in N_i} f_N(ab) \right). \tag{24}$$

The minimum N_i-degree and the maximum N_i-degree of nodes in ξ are defined as

$$\begin{aligned} \delta^{(i)}(\xi) &= \min\{d^{(i)}(a) | a \in V\}, \\ \Delta^{(i)}(\xi) &= \max\{d^{(i)}(a) | a \in V\}. \end{aligned} \tag{25}$$

The $N_{i_1, i_2, \ldots, i_t}$-degree and full degree of a are defined by

$$\begin{aligned} d^{(i_1, i_2, \ldots, i_t)}(a) &= \left(\sum_{\substack{m=1 \\ ab \in N_{i_m}}}^{t} t_{N_{i_m}}(ab), \sum_{\substack{m=1 \\ ab \in N_{i_m}}}^{t} f_{N_{i_m}}(ab) \right), \\ d^f(a) &= \left(\sum_{\substack{i=1 \\ ab \in N_i}}^{n} t_{N_i}(ab), \sum_{\substack{i=1 \\ ab \in N_i}}^{n} f_{N_i}(ab) \right), \end{aligned} \tag{26}$$

respectively. Also, $\delta^f(\xi) = \min\{d^f(a) | a \in V\}$ and $\Delta^f(\xi) = \max\{d^f(a) | a \in V\}$. For nonempty set $B \subseteq V$, the degree is described as follows:

$$\begin{aligned} d^{(i)}(B) &= \sum_{a \in B} d^{(i)}(a), \\ d^f(B) &= \sum_{a \in B} d^f(a), \\ d^{(i_1, i_2, \ldots, i_t)}(B) &= \sum_{a \in B} d^{(i_1, i_2, \ldots, i_t)}(a). \end{aligned} \tag{27}$$

Example 6. Consider the VGS $\xi = (M, N_1, N_2, N_3)$ as shown in Figure 5, where

$$\begin{aligned} M &= \{(a, 0.1, 0.2), (b, 0.2, 0.4), (c, 0.3, 0.5), \\ &\quad \cdot (d, 0.2, 0.3), (e, 0.2, 0.5)\}, \\ N_1 &= \{(ac, 0.1, 0.6), (be, 0.1, 0.5), (de\ , 0.1, 0.5)\}, \\ N_2 &= \{(ab, 0.1, 0.4), (ce, 0.1, 0.6)\}, \\ N_3 &= \{(ae, 0.1, 0.7), (bc, 0.2, 0.5), (c\ d, 0.2, 0.6)\}. \end{aligned} \tag{28}$$

The N_3-degree of $B = \{a, d, e\}$ is as follows: $d^{(3)}(a) = (0.1, 0.5)$, $d^{(3)}(d) = (0.2, 0.6)$, and $d^{(3)}(e) = (0.1, 0.5)$. Hence, $d^{(3)}(B) = (0.4, 1.6)$. Also, $d^{(3)}(\xi) = (0.1, 0.5)$ and $\Delta^{(3)}(\xi) = (0.4, 1.1)$. The $N_{1,2}$-degree of B is as follows:

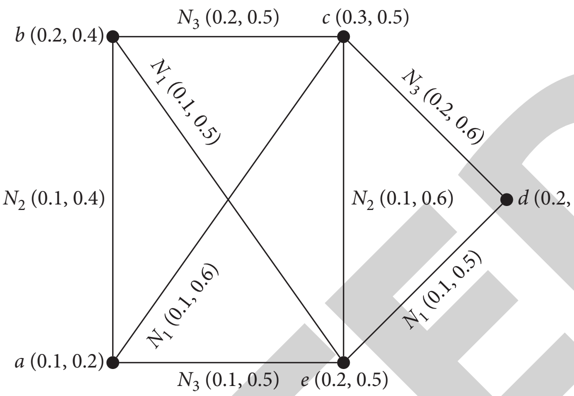

Figure 5: VGS $\xi = (M, N_1, N_2, N_3)$.

$$\begin{aligned} N^{(1,2)}(a) &= (0.2, 1), \\ N^{(1,2)}(d) &= (0.1, 0.5), \\ N^{(1,2)}(e) &= (0.3, 1.6). \end{aligned} \tag{29}$$

Hence, $N^{(1,2)}(B) = (0.6, 3.1)$. The full degree of B is as follows: $d^f(a) = (0.3, 1.5)$, $d^f(d) = (0.3, 1.1)$, and $d^f(e) = (0.4, 2.1)$. So, $d^f(B) = (1, 4.7)$.

Proposition 2. *If $\xi = (M, N_1, N_2, \ldots, N_n)$ is a VGS, then*

(i) $\gamma^{(i)} \le p - \Delta^{(i)}(\xi)$

(ii) $\gamma^f \le p - d^f(T)$, T is a γ^f-set

Proof. Assume that $\Delta^{(i)}(\xi)$ is the maximum N_i-degree of nodes in ξ and K is a $\gamma^{(i)}$-set. Then, $V - K$ is an N_i-DS. So, $|K| \le |V - K|$ and $\gamma^{(i)} \le p - |K| \le p - \Delta^{(i)}(\xi)$. Hence, (i) holds. Suppose T is a set of nodes with MI-C such that every N_i-edge, $1 \le i \le k$, is incident with some nodes in T. Then, $d^f(T) \le \Delta^{(i)}(\xi)$, and we have $\gamma^f \le \gamma^{(i)} \le p - \Delta^{(i)}(\xi) \le p - d^f(T)$. So, $\gamma^f \le p - d^f(T)$. □

Definition 16. The capacity of a node a in the VGS $\xi = (M, N_1, N_2, \ldots, N_n)$ is the sum membership of different N_i-edges in a, and it is shown by $C(a)$. $\delta_C(a)$ and $\Delta_C(a)$ are MI-C and MA-C of a for different N_i-edges in a. Clearly, $\delta_C(a) \le C(a) \le \Delta_C(a) \le n$ and $C(a) \le d^f(a)$. Likewise, $\delta_C(\xi) = \min\{\delta_C(a) | a \in V\}$, and $\Delta_C(\xi) = \max\{\Delta_C(a) | a \in V\}$. For $B \subseteq V$, the capacity of B is described by

$$C(B) = \sum_{a \in B} C(a). \tag{30}$$

Example 7. Consider the VGS $\xi = (M, N_1, N_2, N_3)$ as in Figure 5. We have

$$
\begin{aligned}
C(a) &= (0.3, 1.5),\\
C(b) &= (0.4, 1.4),\\
C(d) &= (0.3, 1.1),\\
\delta_C(c) &= (0.4, 1.7),\\
\Delta_C(c) &= (0.4, 1.8),\\
\delta_C(e) &= (0.3, 1.6) = \Delta_C(e).
\end{aligned} \tag{31}
$$

Definition 17. Let K be an N_i-DS in the VGS $\xi = (M, N_1, N_2, \ldots, N_n)$. Then, K is named the N_i-SCDS if for each node $b \in V - K$, $\exists$ a node $a \in D$ neighbor to b so that $C(a) \geq C(b)$.

The MI-C of N_i-SCDSs is named SC N_i-DN $\gamma_{SC}^{(i)}$, and the MI-C of N_i-WCDSs is called WC N_i-DN $\gamma_{WC}^{(i)}$. Similarly, the MI-C of SC and WC of VFDSs are denoted by γ_{SC}^{f} and γ_{WC}^{f}, respectively.

Example 8. For VGS $\xi = (M, N_1, N_2, N_3)$ as shown in Figure 2, $K_1 = \{a, b, c, e\}$ is a SC N_3-DS among other N_3-DSs, and $K_4 = \{d, f, a, c\}$ is a WC N_3-DS.

Theorem 5. *If $\xi = (M, N_1, N_2, \ldots, N_n)$ is a VGS, then*

(i) $\gamma_{SC}^{f} \leq p - \Delta_C(\xi)$

(ii) $\gamma_{WC}^{(i)} \leq p - \delta_C(\xi)$

Proof. (i) Let a be a node with capacity $\Delta_C(\xi)$ and K be a set of nodes neighbor to a. In this case, each node $b \in K$ is a neighbor to $a \in V - K$, and $C(b) \leq C(a)$. Hence, $V - K$ is a SC-VFDS. Therefore,

$$\gamma_{SC}^{f} \leq |V - K| \leq p - \Delta_C(\xi). \tag{32}$$

(ii) Let a be a node of capacity $\delta_C(\xi)$ and K' be the set of nodes neighbor to a by N_i-edge. Then, each node $b \in K'$ is a neighbor to $a \in V - K'$, and $C(b) \geq C(a)$. So, $V - K'$ is an N_i-WCDS. Hence, $\gamma_{WC}^{(i)} \leq |V - K'| \leq p - \delta_C(\xi)$. □

4. Application of Dominating Sets in Transferring Patients

Nowadays, many hospitals in cities have to transfer patients to neighboring cities that have the necessary facilities owing to the lack of facilities and equipment for treatment, but there are many factors that can be important in deciding which hospital to choose. One of them is the suitability of the roads in terms of smoothness and having the necessary traffic signs for ambulance drivers because the unsuitability of the roads can be dangerous and stressful for both the patient and the vehicle. Another factor that can be very useful is the amount of congestion and traffic between roads because if this congestion is less, then the patient will be transported to the hospital faster and easier and will receive the required treatment. As we see today, many patients lose their lives because of not arriving in hospitals on time and not receiving immediate care. Therefore, the amount of traffic and congestion can be very vital factors in this case. Additionally, another factor that can be effective in this decision is the patients' admission rate in the desired hospital because the more patients a hospital accepts, the easier it is for the patients to be treated more effortlessly and quickly. Note that this issue plays an important role in a patient's treatment process. Therefore, in this paper, we intend to express the use of the vague full dominating set in transferring a patient to the most effective hospital in the shortest possible time. For this purpose, we consider four hospitals in Sari, Iran. Suppose a patient lives in place A (Ghaemshahr) and has to go to one of the medical centers in Sari for treatment called Ibn Sina (B), Amir Mazandarani (C), Imam Khomeini (D), and Fatemeh Zahra (E). Hospitals are shown in the graph with the symbols B, C, D, and E.

In this vague graph structure, the vertices represent the hospitals, and the edges indicate the level of smoothness and quality of roads, road traffic, and the patients' admission rate in the desired hospitals, respectively. The weight of vertices and edges is shown in Tables 2 and 3. The location of the hospitals is shown in Figure6.

The vertex $C(0.5, 0.2)$ shows that Amir Mazandarani hospital has 50% of medical devices and equipment to treat a patient, but unfortunately, it does not have 20% of the necessary facilities for treatment. The edge $N_1(AC)$ shows that the ACroute meets only 50% of the global transport standards in terms of quality and construction and has 30% breakdowns. The edge $N_3(A\ D)$ indicates that 30% of patients can be admitted to Imam Khomeini hospital, but the hospital is not able to accept another 50% of patients. The edge $N_2(AB)$ shows this route has 40% of city traffic for most hours of the day, and 60% of it is free of traffic. The vague full dominating sets for Figure 7 are as follows:

$$
\begin{aligned}
T_1 &= \{A, B\},\\
T_2 &= \{A, C\},\\
T_3 &= \{A, D\},\\
T_4 &= \{B, D\},\\
T_5 &= \{B, E\},\\
T_6 &= \{A, E\},\\
T_7 &= \{C, E\},\\
T_8 &= \{C, D\}.
\end{aligned} \tag{33}
$$

After calculating the cardinality of $T_1, \ldots, T_n$, we obtain

$$
\begin{aligned}
|T_1| &= 1,\\
|T_2| &= 1.25,\\
|T_3| &= 1.15,\\
|T_4| &= 0.95,\\
|T_5| &= 0.9,\\
|T_6| &= 1.1,\\
|T_7| &= 1.15,\\
|T_8| &= 1.2.
\end{aligned} \tag{34}
$$

Clearly, T_2 has the largest cardinal wards among other vague full dominating sets, so it turns out that it can be the best choice because firstly, the path from A to C has the

Table 2: Weight of vertices.

	A	B	C	D	E
M_A	0.5	0.2	0.5	0.3	0.3
N_A	0.3	0.4	0.2	0.2	0.3

Table 3: Weight of edges.

ξ	$N_1(AB)$	$N_2(AB)$	$N_1(BC)$	$N_2(BC)$
(t_N, f_N)	$(0.4, 0.6)$	$(0.4, 0.2)$	$(0.1, 0.5)$	$(0.2, 0.4)$
ξ	$N_1(AC)$	$N_2(AC)$	$N_3(AC)$	$N_2(AE)$
(t_N, f_N)	$(0.5, 0.3)$	$(0.2, 0.6)$	$(0.4, 0.3)$	$(0.3, 0.4)$
ξ	$N_1(A\ D)$	$N_3(A\ D)$	$N_1(DE)$	$N_2(DE)$
(t_N, f_N)	$(0.2, 0.5)$	$(0.3, 0.5)$	$(0.2, 0.6)$	$(0.3, 0.4)$
ξ	$N_3(DE)$			
(t_N, f_N)	$(0.2, 0.6)$			

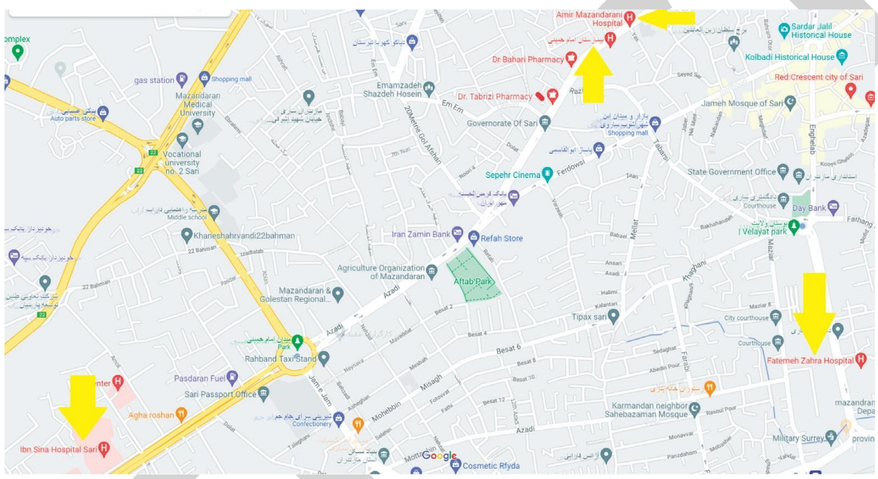

Figure 6: Location of hospitals in Sari city.

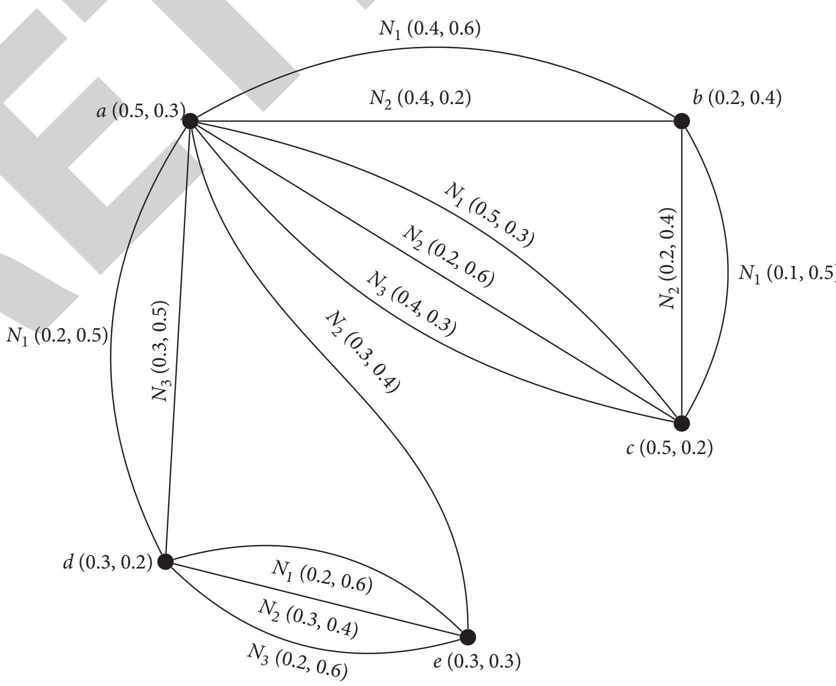

Figure 7: VGS ξ.

smoothest and best communication path, and secondly, only 20% of this path has traffic, and congestion is caused by cars, and thirdly, hospital C has 40% admission for patients, which is more than other hospitals. Note that hospital C has 50% of the necessary medical equipment, which is significant compared with other hospitals. Therefore, it is concluded that the government should provide the necessary facilities for hospitals so that patients do not have to go to hospitals far away for treatment. Also, in terms of transportation, the roads must be of good quality so that ambulances can transport patients for treatment as soon as possible.

5. Conclusion

Fuzzy graph has various applications in modern science and technology, especially in the fields of neural networks, computer science, operation research, and decision-making. Vague graph structures have more precision, flexibility, and compatibility, as compared to the fuzzy graphs. Today, VGSs play an important role in social networks and allow users to find the most effective person in a group or organization. One of the most important features of VGSs that has many applications in real problems is the concept of domination. Domination has many applications in psychology, medical science, social groups, and computer networks. Therefore, in this research, we defined different concepts related to domination in VGSs such as N_i-dominating set, vague full dominating set, and minimal N_i-dominating set, with several examples. Finally, an application of domination in medical sciences has been presented. In our future work, we will introduce ψ-complement, self-complement, strong self-complement, and totally strong self-complement in VGSs and investigate some of their properties.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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