Hindawi
Journal of Function Spaces
Volume 2022, Article ID 1680560, 10 pages
https://doi.org/10.1155/2022/1680560

Research Article

Diverse Exact Soliton Solutions of the Time Fractional Clannish Random Walker's Parabolic Equation via Dual Novel Techniques

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Received 12 July 2022; Revised 2 September 2022; Accepted 15 September 2022; Published 28 September 2022

Academic Editor: Yusuf Gurefe

In this article, we acquire a variety of new exact traveling wave solutions in the form of trigonometric, hyperbolic, and rational functions for the nonlinear time-fractional Clannish Random Walker's Parabolic (CRWP) equation in the sense of beta-derivative by employing the two modified methods, namely, modified $(G'/G^2)-$ expansion method and modified $F-$expansion method. The obtained solutions are verified for aforesaid equations through symbolic soft computations. To promote the essential propagated features, some investigated solutions are exhibited in the form of 2D and 3D graphics by passing on the precise values to the parameters under the constrain conditions. The obtained solutions show that the presented methods are effective, straight forward, and reliable as compared to other methods. These methods can also be used to extract the novel exact traveling wave solutions for solving any types of integer and fractional differential equations arising in mathematical physics.

1. Introduction

Investigation of the exact traveling wave solutions for fractional nonlinear partial differential equations (PDEs) plays an important role in the study of nonlinear physical phenomena. Fractional equations, both partial and ordinary ones, have been applied in modeling of many physical, engineering, chemistry, biology, etc. in recent years [1]. There are several definitions of fractional derivatives such as Riemann Liouville [2], conformable fractional derivative [3], beta derivative [4], and new truncated M-fractional derivative [5] are available in literature. Many powerful methods for obtaining exact solutions of nonlinear fractional PDEs have been presented as Hirota's bilinear method [6], sine-cosine method [7], tanh-function method [8], exponential rational function method [9], Kudryashov method [10], sine-Gordon expansion method [11], modified (G'/G)-expansion method [12], extended (G'/G)-expansion method [13], (G'/G)-expansion method [14], tanh-coth expansion method [15], Jacobi elliptic function expansion method [16], first integral method [17], sardar-subequation method [18], new subequation method [19], extended direct algebraic method [20], exp $(-\phi(\eta))$ method [21], Exp_a function method [22], $(1/G')$, $(G'/G, 1/G)$, and modified $(G'/G^2)-$ expansion methods [23, 24], Kudryashov method [25], modified expansion function method [26], new auxiliary equation method [27], extended Jacobi's elliptic expansion function method [28], extended sinh-Gordon equation expansion method [29], modified simplest equation method [30], and many more.

The time-fractional Clannish Random Walker's Parabolic (CRWP) equation [31, 32] is a model that can determine the behavior of two species A and B of random walker who execute a concurrent one- dimensional random walk characterized by an intensification of the clannishness of the members of one species A at point x at a time $t, u(x, t)$, can be expressed by the time- fractional CRWP equation as

$$D_t^{\alpha_1} u + s u_x + q u u_x + r u_{xx} = 0, \tag{1}$$

where α_1 is a parameter describing the order of the fractional time derivative and $0 < \alpha_1 \leq 1$..

The major concern of this existing study is to utilize the novel meanings of fractional-order derivative, named beta fractional derivative [4], for time-fractional CRWP equation, and to find the novel comprehensive exact traveling wave solutions in the form of hyperbolic, trigonometric, and rational functions by employ two modified methods, modified $(G'/G^2)-$ expansion method [33] and modified $F-$ expansion method [34]. Beta-derivative has some interesting consequences in diverse areas including fluid mechanics, optical physics, chaos theory, biological models, disease analysis, and circuit analysis. To the best of our knowledge, the obtained solutions are more general and in different form which have never been reported in previously published studies [31, 32]. Our results also enrich the variety of the dynamics of higher-dimensional nonlinear wave field. It is hoped that these results will provide some valuable information in the higher-dimensional nonlinear field.

By using the modified $(G'/G^2)-$ expansion method [33], traveling wave solutions have been found for the nonlinear Schrödinger equation along third-order dispersion. Different types of traveling wave solutions of the Fokas-Lenells equations have been determined in [35] by this method. Alyahdaly found the general exact traveling wave solutions to the nonlinear evolution equations in [36]. Gepreel and Nofal [37] obtained the analytical solutions for nonlinear evolution equations in mathematical physics. Siddique and Mehdi found the exact traveling wave solutions for two prolific conformable M-fractional differential equations in [23]. Exact solutions for nonlinear integral member of Kadomstev-Petviashvili hierarchy differential equations have been determined by Gepreel [38].

A modified F–expansion method is proposed by taking full advantages of F expansion method and Riccati equation in seeking exact solutions of nonlinear PDEs. Darvishi and Najafi [39] used a modified F-expansion method to handle the foam Drainage equation. Aasaraai [40] used this method to construct new solutions of the nonlinear $(1+2)$-dimensional Maccari's system. Aasaraai and Mehrlatifan [41] applied this method to coupled system of equation. Ali et al. [42] derived dispersive analytical soliton solutions of some nonlinear wave's dynamical model with the help of modified F–expansion method. Darvishi et al. [43] found traveling wave solutions for the $(3+1)$-dimensional breaking soliton equation.

This article organized it as follows: in Section 2, we present beta-derivative and its properties. The descriptions of strategies are given in Section 3. In Section 4, we present a mathematical analysis of the models and its solutions via proposed methods. In Section 5, some graphical representations for some analytical solutions are presented. Some conclusions are drawn in the last section.

2. Beta-Derivative and Its Properties

Definition: suppose a function $h(x)$ that is defined $\forall$ non-negative x. Therefore, the beta-derivative of the function $h(x)$ is given as [4]:

$$D^{\beta}(h(x)) = \lim_{\varepsilon \longrightarrow 0} \frac{h\left(x + \varepsilon(x + (1/\Gamma(\beta)))^{1-\beta}\right) - h(x)}{\varepsilon}, 0 < \beta \leq 1. \tag{2}$$

Properties: assuming that a and b are real numbers, $g(x)$ and $h(x)$ are two functions $\beta-$differentiable and $\beta \in (0, 1]$, then, the following relations can be satisfied

$$i. D^{\beta}(a g(x) + b h(x)) = a D^{\beta}(g(x)) + b D^{\beta}(h(x)), \forall a, b \in R. \tag{3}$$

$$ii. D^{\beta}(g(x)h(x)) = h(x) D^{\beta}(g(x)) + g(x) D^{\beta}(h(x)). \tag{4}$$

$$iii. D^{\beta}\left(\frac{g(x)}{h(x)}\right) = \frac{h(x) D^{\beta}(g(x)) + g(x) D^{\beta}(h(x))}{(h(x))^2}. \tag{5}$$

$$iv. D^{\beta}(g(x)) = \frac{dg(x)}{dx}\left(x + \frac{1}{\Gamma(\beta)}\right)^{1-\beta}. \tag{6}$$

3. Description of Strategies

3.1. The Modified (G'/G^2)-Expansion Method. Let us consider the nonlinear PDE is in the form

$$Q(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt}, \cdots) = 0, \tag{7}$$

where $u = u(x, t)$ is an unknown function, and Q is a polynomial depending on $u(x, t)$ and its various partial derivatives.

Step 1. By wave transformation

$$\eta = x - \nu t, \mathrm{u}(x, t) = U(\eta). \tag{8}$$

Here, ν is the speed of traveling wave.

The wave variable permits us to reduce Eq. (8) into a nonlinear ordinary differential equation (ODE) for $U = U(\eta)$:

$$R\left(U, U', U'', U''', \cdots\right) = 0, \tag{9}$$

where R is a polynomial of $U(\eta)$ and its total derivative with respect to η.

Step 2. Extend the solutions of Eq. (9) in the following form

$$U(\eta)=\sum_{i=0}^{m}\alpha_i\left(\frac{G'}{G^2}\right)^i, \tag{10}$$

where $\alpha_i(i=0,1,2,3,\cdots,m)$ are constants and find to be later. It is important that $\alpha_i\neq 0$. Integer m can be determined by considering the homogenous balance between the governing nonlinear terms and the highest order derivatives in Eq. (9).

The function $G=G(\eta)$ satisfies the following Riccati equation,

$$\left(\frac{G'}{G^2}\right)'=\lambda_1\left(\frac{G'}{G^2}\right)^2+\lambda_0, \tag{11}$$

where λ_0 and λ_1 are constants. We gain the below solutions to Eq. (11) due to different conditions of λ_0:

When $\lambda_0\lambda_1<0$,

$$\left(\frac{G'}{G^2}\right)=-\frac{\sqrt{|\lambda_0\lambda_1|}}{\lambda_1}+\frac{\sqrt{|\lambda_0\lambda_1|}}{2}\left[\frac{C_1\sinh\left(\sqrt{\lambda_0\lambda_1}\eta\right)+C_2\cosh\left(\sqrt{\lambda_0\lambda_1}\eta\right)}{C_1\cosh\left(\sqrt{\lambda_0\lambda_1}\eta\right)+C_2\sinh\left(\sqrt{\lambda_0\lambda_1}\eta\right)}\right]. \tag{12}$$

When $\lambda_0\lambda_1>0$,

$$\left(\frac{G'}{G^2}\right)=\sqrt{\frac{\lambda_0}{\lambda_1}}\left[\frac{C_1\cos\left(\sqrt{\lambda_0\lambda_1}\eta\right)+C_2\sin\left(\sqrt{\lambda_0\lambda_1}\eta\right)}{C_1\sin\left(\sqrt{\lambda_0\lambda_1}\eta\right)-C_2\sin\left(\sqrt{\lambda_0\lambda_1}\eta\right)}\right]. \tag{13}$$

When $\lambda_0=0$ and $\lambda_1\neq 0$,

$$\left(\frac{G'}{G^2}\right)=-\frac{C_1}{\lambda_1(C_1\eta+C_2)}. \tag{14}$$

where C_1 and C_2 are arbitrary constant.

Step 3. By substituting Eq. (10) into Eq. (9) along with Eq. (11) and tracing all coefficients of each$\left(G'/G^2\right)^i$ to zero, then solving that algebraic equations generated in the term a_i, $\lambda_0,\lambda_1,C_1,C_2$ and other parameters.

Step 4. By substituting Eq. (10) of which α_i,ν and other parameters that are found in step 3 into Eq. (8), we get the solutions of Eq. (7).

3.2. The Modified $F-$Expansion Method. Here, we will describe the basic steps of $F-$expansion method [34].

$$U(\eta)=a_0+\sum_{i=1}^{m}a_iF^i(\eta)+\sum_{i=1}^{m}b_iF^{-i}(\eta), \tag{15}$$

where a_0,a_i, and b_i are constants to be determined. $F(\eta)$ satisfies the Riccati equation:

$$F'(\eta)=A+BF(\eta)+CF^2(\eta), \tag{16}$$

where A,B, and C are constants to be determined. The prime denotes $d/d\eta$. Integer m can be determined by considering the homogenous balance between the governing nonlinear terms and the highest order derivatives of $U(\eta)$ in Eq. (9). Given different values of A,B, and C, the different Riccati function solution $F(\eta)$ can be obtained from Eq. (16) (see Table 1).

Step 1. Consider Eqs. (7), (8), and (9).

Step 2. Extend the solution of Eq. (9) in the following form

Step 3. Substituting Eq. (15) along with Eq. (16) into Eq. (9) and collect coefficients of $F^i(\eta)$ to zero yields a system of algebraic equations for a_i and b_i.

Step 4. Solve the system of algebraic equations, probably with the aid of Mathematica. a_i and b_i can be expressed by A, B, and C (or the coefficients of Eq. (9)). Substituting these results into (16), we can obtain the general form of traveling wave solutions to Eq. (9).

Step 5. Selecting A, B, C, and $F(\eta)$ from Table 1 and substituting them along with a_i and b_i into Eq. (15), a series of soliton-like solutions, trigonmetric function solutions, and rational solutions to Eq. (7) can be obtained.

The modified F-expansion method is more effective in obtaining the soliton-like solution, trigonometric function solutions, exponential solutions, and rational solutions of the nonlinear partial deferential equations. This method will yield more rich types solutions of the nonlinear partial deferential equations. It shows that the modified F-expansion method is more powerful in constructing exact solutions of nonlinear PDEs.

4. Application

Time-fractional Clannish Random Walker's Parabolic equation:

Let us assume the transformations:

$$u(x,t)=U(\eta),\eta=x-\frac{c}{\beta}\left(t+\frac{1}{\Gamma(\beta)}\right)^\beta, \tag{17}$$

where c is constant. By using Eq. (17) into Eq. (1), we get the following ordinary differential equation.

$$2(s-c)U+qU^2+2rU'=0. \tag{18}$$

In the following subsections, the proposed methods are applied to extract the required solutions:

4.1. Solutions with the Modified $\left(G'/G^2\right)-$Expansion Method. By applying the homogenous balance technique

TABLE 1: Relations between A, B, C and corresponding $F(\eta)$ in Eq. (16) [34].

A	B	C	$F(\eta)$
0	1	-1	$1/2 + 1/2 \tanh(\eta/2)$
0	-1	1	$1/2 - 1/2 \coth(\eta/2)$
1/2	0	-1/2	$\coth(\eta) \pm \operatorname{csch}(\eta), \tanh(\eta) \pm i \operatorname{sech}(\eta)$
1	0	-1	$\tanh(\eta), \coth(\eta)$
1/2	0	½	$\sec(\eta) + \tan(\eta), \csc(\eta) - \cot(\eta)$
-1/2	0	-1/2	$\sec(\eta) - \tan(\eta), \csc(\eta) + \cot(\eta)$
1(-1)	0	1(-1)	$\tan(\eta), \cot(\eta)$
0	0	$\neq 0$	$-(1/C\eta + m)$($m$is an arbitrary constant)
Arbitrary constant	0	0	$A\eta$
Arbitrary constant	$\neq 0$	0	$(\exp(B) - A)/B$

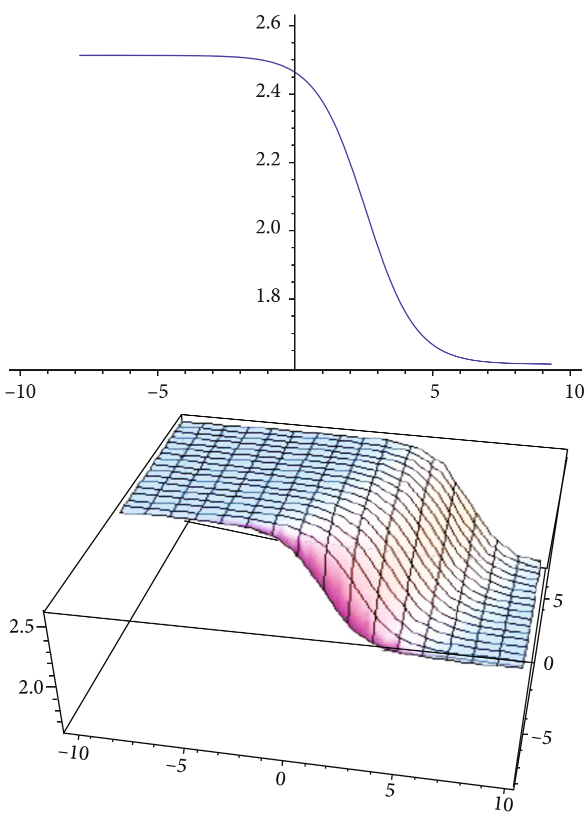

FIGURE 1: 2D and 3D graphics for hyperbolic traveling wave solution (21) at $\{\beta = 0.5, r = 1, q = 1, \lambda_0 = 0.5, \lambda_1 = 0.5,\ C_1 = 1, C_2 = 0, c = 1\}$.

between the terms U' and U^2 into Eq. (18), we get $m = 1$. For $m = 1$, Eq. (10) reduces into

$$U(\eta) = a_0 + a_1 \left(\frac{G'}{G^2}\right), \tag{19}$$

where a_0 and a_1 are unknown parameters. By using Eq. (19) along with Eq. (11) into Eq. (18) and summing up all coefficients of same order of (G'/G^2), we get the set of algebraic equations involving a_0, a_1 and other parameters. Solving the obtained set of algebraic equations with Mathematica, we reach the following results:

$$a_0 = \pm \frac{2ir\sqrt{\lambda_0}\sqrt{\lambda_1}}{q}, a_1 = \frac{-2r\lambda_1}{q}, c = s \pm 2ir\sqrt{\lambda_0}\sqrt{\lambda_1}. \tag{20}$$

Now we use the Eqs. (19) and (12)–(14) into Eq. (19) and set the below cases.

if$\lambda_0\lambda_1 < 0$, then

$$u_1(x,t) = \pm \frac{2ir\sqrt{\lambda_0\lambda_1}}{q} + \frac{2r\sqrt{|\lambda_0\lambda_1|}}{q}\left(1 - \frac{\lambda_1}{2}\left[\frac{C_1 \sinh\left(\sqrt{\lambda_0\lambda_1}\eta\right) + C_2 \cosh\left(\sqrt{\lambda_0\lambda_1}\eta\right)}{C_1 \cosh\left(\sqrt{\lambda_0\lambda_1}\eta\right) + C_2 \sinh\left(\sqrt{\lambda_0\lambda_1}\eta\right)}\right]\right). \tag{21}$$

If $\lambda_0\lambda_1 > 0$,then

$$u_2(x,t) = \pm \frac{2ir\sqrt{\lambda_0\lambda_1}}{q} - \frac{2r\lambda_1}{q}\left(\sqrt{\frac{\lambda_0}{\lambda_1}}\left[\frac{C_1 \cos\left(\sqrt{\lambda_0\lambda_1}\eta\right) + C_2 \sin\left(\sqrt{\lambda_0\lambda_1}\eta\right)}{C_1 \sin\left(\sqrt{\lambda_0\lambda_1}\eta\right) - C_2 \cos\left(\sqrt{\lambda_0\lambda_1}\eta\right)}\right]\right), \tag{22}$$

If $\lambda_0 = 0, \lambda_1 \neq 0$,then

$$u_3(x,t) = \frac{2r}{q}\left(\frac{C_1}{C_1\eta + C_2}\right). \tag{23}$$

4.2. Solutions with the Modified F – Expansion Method. By applying the homogenous balance technique between the terms U' and U^2 into Eq. (18), we get $m = 1$. For $m = 1$, Eq. (15) reduces into:

$$U(\eta) = a_0 + a_1 F + \frac{b_1}{F}, \tag{24}$$

where a_0 and a_1 are unknown parameters. By using Eq. (24) along with Eq. (16) into Eq. (18) and summing up all the coefficients of same order of F, we get the set of algebraic equations involving a_0, a_1 and other parameters. Solving the obtained set of algebraic equations with Mathematica, we reach the following results.

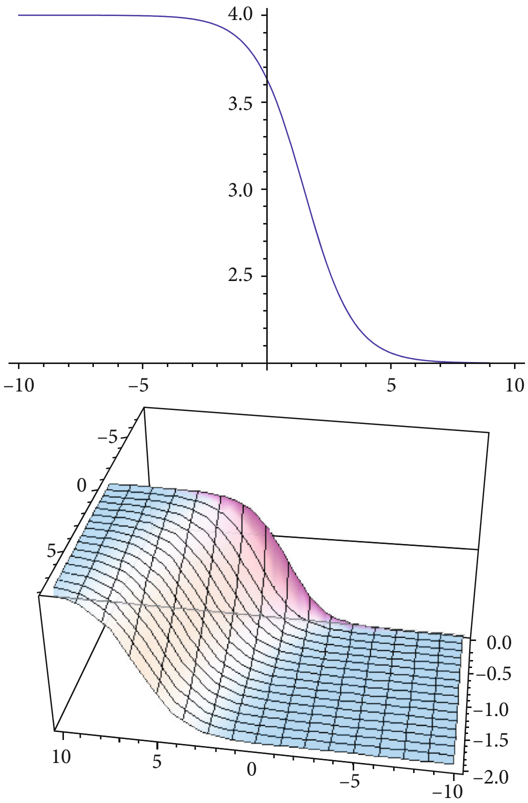

Figure 2: 2D and 3D graphics for hyperbolic traveling wave solution (26) at $\{\beta = 0.5, r = 1, q = 1, c = 1\}$.

Put Eq. (24) into Eq. (18) along with the solution of Eq. (16), we get

For $A = 0$, $B = 1$, and $C = -1$.

$$a_0 = -\frac{2r}{q}, a_1 = \frac{2r}{q}, b_1 = 0, c = s - r. \tag{25}$$

Put Eq. (25) into Eq. (24) along with the solution of Eq. (16), we get

$$u_1(x,t) = -\frac{r}{q}\left(1 - \tanh\left(\frac{\eta}{2}\right)\right). \tag{26}$$

For $A = 0$, $B = -1$, and $C = 1$.

$$a_0 = \frac{2r}{q}, a_1 = -\frac{2r}{q}, b_1 = 0, c = s + r, \tag{27}$$

$$u_2(x,t) = \frac{r}{q}\left(1 + \coth\left(\frac{\eta}{2}\right)\right). \tag{28}$$

For $A = 1/2$, $B = 0$, $C = -1/2$.

Family-I

$$a_0 = -\frac{r}{q}, a_1 = \frac{r}{q}, b_1 = 0, c = s - r, \tag{29}$$

$$u_3(x,t) = -\frac{r}{q}\left(1 - \left(\coth(\eta) + \operatorname{csch}(\eta)\right)\right). \tag{30}$$

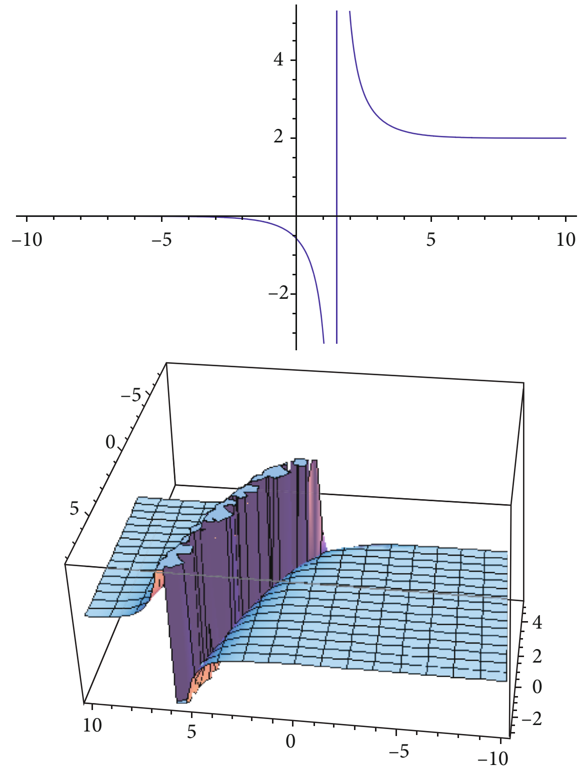

Figure 3: 2D and 3D graphics for hyperbolic traveling wave solution (28) at $\{\beta = 0.5, r = 1, q = 1, c = 1\}$.

Family-II

$$a_0 = -\frac{r}{q}, a_1 = 0, b_1 = \frac{r}{q}, c = s - r, \tag{31}$$

$$u_4(x,t) = -\frac{r}{q}\left(1 - \frac{1}{(\coth(\eta) + \operatorname{csch}(\eta))}\right). \tag{32}$$

Family-III

$$a_0 = \frac{2r}{q}, a_1 = \frac{r}{q}, b_1 = \frac{r}{q}, c = 2r + s, \tag{33}$$

$$u_5(x,t) = \frac{2r}{q}\left(2 + \left((\coth(\eta) + \operatorname{csch}(\eta)) + \frac{1}{(\coth(\eta) + \operatorname{csch}(\eta))}\right)\right). \tag{34}$$

For $A = 1$, $B = 0$, $C = -1$.

Family-I

$$a_0 = -\frac{2r}{q}, a_1 = \frac{2r}{q}, b_1 = \frac{r}{q}, c = s - 2r, \tag{35}$$

$$u_6(x,t) = -\frac{2r}{q}\left(1 - \tanh(\eta)\right). \tag{36}$$

Family-II

$$a_0 = \frac{2r}{q}, a_1 = 0, b_1 = \frac{2r}{q}, c = 2r + s, \tag{37}$$

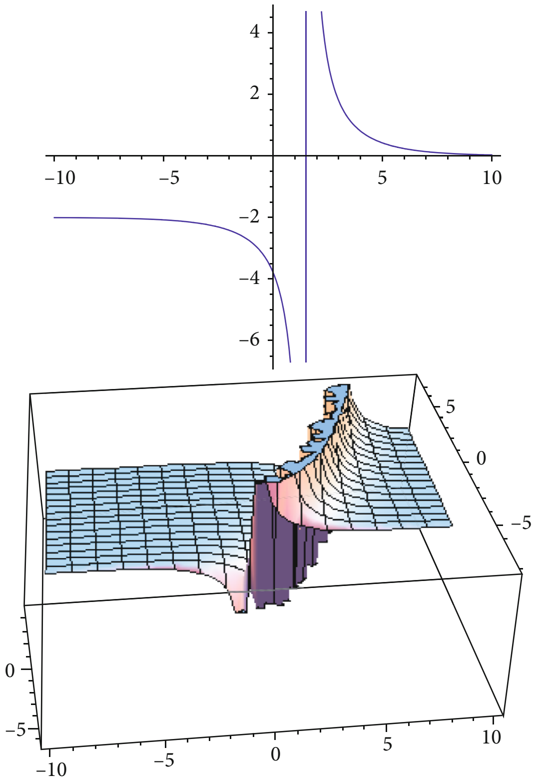

Figure 4: 2D and 3D graphics for hyperbolic traveling wave solution (30) at $\{\beta = 0.5, r = 1, q = 1, c = 1\}$.

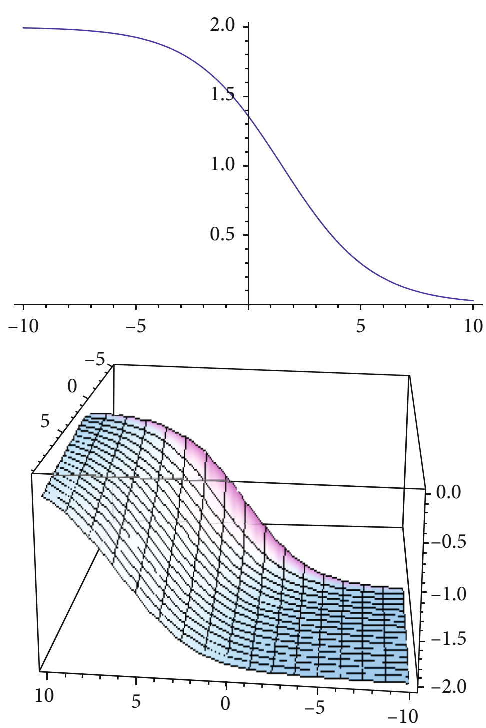

Figure 5: 2D and 3D graphics for hyperbolic traveling wave solution (32) at $\{\beta = 0.5, r = 1, q = 1, c = 1\}$.

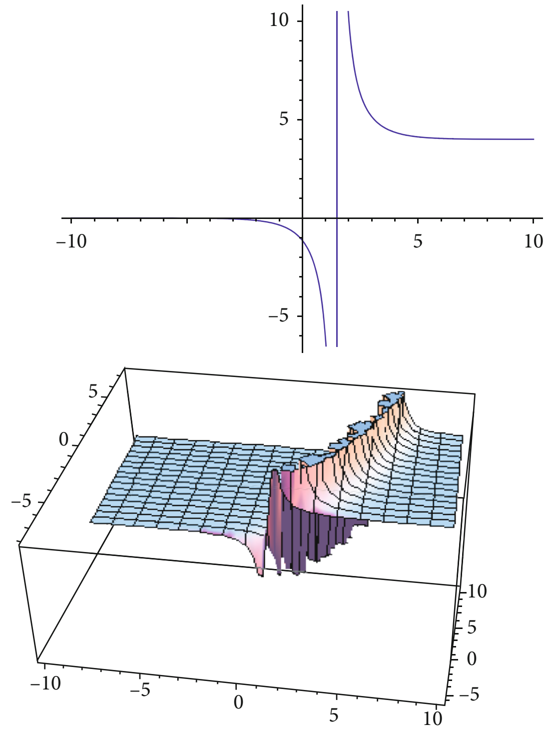

Figure 6: 2D and 3D graphics for hyperbolic traveling wave solution (34) at $\{\beta = 0.5, r = 1, q = 1, c = 1\}$.

$$u_7(x, t) = \frac{2r}{q}\left(1 + \frac{1}{\tanh(\eta)}\right). \tag{38}$$

Family-III

$$a_0 = \frac{4r}{q}, a_1 = \frac{2r}{q}, b_1 = \frac{2r}{q}, c = 4r + s, \tag{39}$$

$$u_8(x, t) = \frac{2r}{q}\left(2 + \left(\tanh(\eta) + \frac{1}{\tanh(\eta)}\right)\right). \tag{40}$$

For $A = C = 1/2$, $B = 0$.

Family-I

$$a_0 = -\frac{ir}{q}, a_1 = -\frac{r}{q}, b_1 = 0, c = s - ir, \tag{41}$$

$$u_9(x, t) = -\frac{r(\tan(\eta) + \sec(\eta))}{q} - \frac{ir}{q}, \tag{42}$$

Family-II

$$a_0 = \frac{ir}{q}, a_1 = 0, b_1 = \frac{ir}{q}, c = s + ir, \tag{43}$$

$$u_{10}(x, t) = \frac{ir}{q(\tan(\eta) + \sec(\eta))} + \frac{ir}{q}. \tag{44}$$

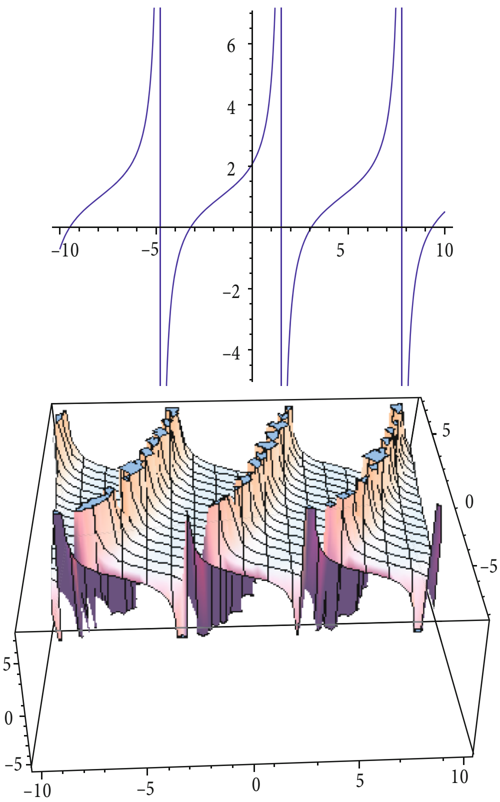

Figure 7: 2D and 3D graphics for trigonometric traveling wave solution (22) at $\{\beta = 0.5, r = 1, q = 1, \lambda_0 = 0.5, \lambda_1 = 0.5, C_1 = 0, C_2 = 1, c = 1\}$.

Family-III

$$a_0 = -\frac{2ir}{q}, a_1 = -\frac{r}{q}, b_1 = \frac{r}{q}, c = s - 2ir, \tag{45}$$

$$u_{11}(x,t) = -\frac{r((\tan(\eta) + \sec(\eta)) - (1/(\tan(\eta) + \sec(\eta))))}{q} - \frac{2ir}{q}. \tag{46}$$

For $A = C = -1/2$, $B = 0$.
Family-I

$$a_0 = \frac{ir}{q}, a_1 = \frac{r}{q}, b_1 = 0, c = s + ir, \tag{47}$$

$$u_{12}(x,t) = -\frac{r(\sec(\eta) - \tan(\eta))}{q} + \frac{ir}{q}. \tag{48}$$

Family-II

$$a_0 = \frac{ir}{q}, a_1 = 0, b_1 = -\frac{r}{q}, c = s + ir, \tag{49}$$

$$u_{13}(x,t) = -\frac{r}{q(\sec(\eta) - \tan(\eta))} + \frac{ir}{q}. \tag{50}$$

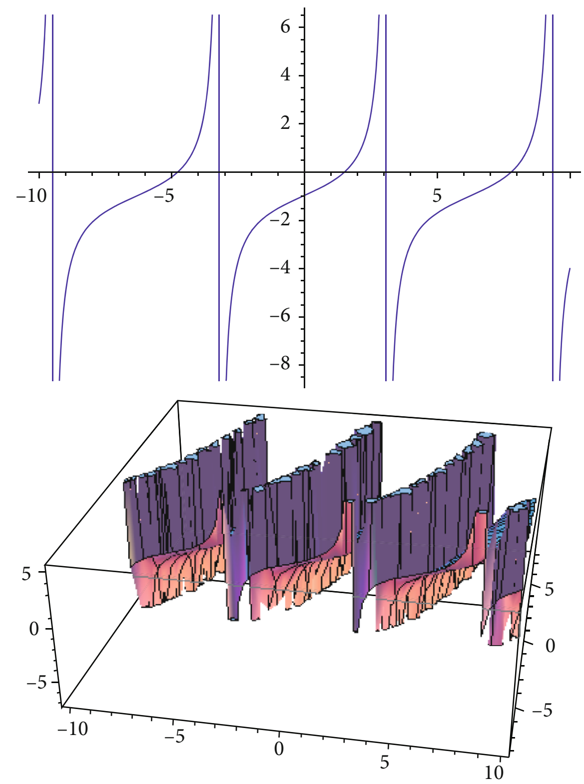

Figure 8: 2D and 3D graphics for trigonometric traveling wave solution (42) at $\{\beta = 0.5, r = 1, q = 1, c = 1\}$.

Family-III

$$a_0 = \frac{2ir}{q}, a_1 = \frac{r}{q}, b_1 = -\frac{r}{q}, c = s + 2ir, \tag{51}$$

$$u_{14}(x,t) = -\frac{r((\sec(\eta) - \tan(\eta)) + (1/(\sec(\eta) - \tan(\eta))))}{q} + \frac{2ir}{q}. \tag{52}$$

For $A = C = -1$, $B = 0$.
Family-I

$$a_0 = -\frac{2ir}{q}, a_1 = \frac{2r}{q}, b_1 = 0, c = s - 2ir, \tag{53}$$

$$u_{15}(x,t) = \frac{2r\cot(\eta)}{q} - \frac{2ir}{q}. \tag{54}$$

Family-II

$$a_0 = \frac{2ir}{q}, a_1 = 0, b_1 = -\frac{2r}{q}, c = s + 2ir, \tag{55}$$

$$u_{16}(x,t) = -\frac{2r}{q\cot(\eta)} + \frac{2ir}{q}. \tag{56}$$

Family-III

$$a_0 = \frac{4ir}{q}, a_1 = \frac{2r}{q}, b_1 = -\frac{2r}{q}, c = s + 4ir, \tag{57}$$

$$u_{17}(x,t)=\frac{2r(\cot(\eta)-(1/\cot(\eta)))}{q}+\frac{4ir}{q}. \quad (58)$$

For $A=0$, $B=0$.

$$a_0=0, a_1=-\frac{2Cr}{q}, b_1=0, c=s, \quad (59)$$

$$u_{18}(x,t)=\frac{2Cr}{q(C\eta+\varepsilon)}. \quad (60)$$

For $B=0$, $C=0$.

$$a_0=0, a_1=0, b_1=\frac{2Ar}{q}, c=s, \quad (61)$$

$$u_{19}(x,t)=\frac{2Ar}{q(A\eta)}. \quad (62)$$

For $C=0$.

$$a_0=\frac{2Br}{q}, a_1=0, b_1=\frac{2Ar}{q}, c=Br+s, \quad (63)$$

$$u_{20}(x,t)=\frac{2Ar}{q(\exp(B\eta)-A)/B}+\frac{2Br}{q}. \quad (64)$$

5. Results and Discussion

In this section, we discuss the graphical interpretation of obtained results. Two powerful analytical methods, namely, modified $(G'/G^2)-$expansion method and modified $F-$expansion method, are used to extract the trigonometric, hyperbolic, and rational wave solutions of the governing model. The physical significance of these solutions is shown by assigning particular values of free parameters. The solutions to Eqs. (22), (42), (44), (46), (48), (50), (52), (54), (56), and (58) present as trigonometric function solutions; the solutions of (21), (26), (28), (30), (32), (34), (36), (38), and (40) present as hyperbolic function solutions; and the solutions of (23), (60), (62), and (64) present as rational function solutions. We explain the dynamic performance of the hyperbolic function answers of Eqs. (21), (26), (28), (30), (32), and (34) which are illustrated in Figures 1–6. In particular, Figures 1–6 demonstrate the 3D shape and 2D graph for different values of the fractional parameter β for the trigonometric function answers of Eqs. (21), (26), (28), (30), (32), and (34). Finally, we explain the dynamic performance of the trigonometric function answers of Eqs. (22) and (34) in Figures 7 and 8, which depict the 3D shape and 2D graph for different values of the fractional parameter β for the trigonometric function answers of Eqs. (22) and (34). The implemented mathematical simulations acknowledge that the answers are of periodic wave shapes and of rational, hyperbolic, and trigonometric categorizations. Furthermore, through observing the construction of the acquired solutions, it could be understood that the parameter β of fractional derivatives has important role in the formulation of all the solutions.

6. Conclusions

In this work, we applied the modified (G'/G^2)-expansion method and modified $F-$expansion method in a satisfactory way to find the novel exact traveling wave solutions of the time-fractional CRWP equation in the sense of beta-derivative. Various obtained solutions are in the form of hyperbolic, trigonometric, and rational forms. To describe the physical phenomena of the time-fractional CRWP model, some solutions are plotted in the form of 2D and 3D by assigning the specific value to the parameters under the constrain conditions. All algebraic computations and graphical representations in this work have been provided for the obtained solutions at various parameters values with the help of Mathematica. It is essential to note that these new solutions of the time-fractional CRWP equation have not been exposed in literature by employed our two analytical modified mathematical methods. Lastly, the studied methods can be potentially applied to solve various nonlinear PDEs that are apparent in many important nonlinear scientific phenomena in physics and engineering.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The researchers would like to thank the Deanship of Scientific Research, Qassim University for funding the publication of this project.

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