Hindawi
Journal of Function Spaces
Volume 2024, Article ID 9764097, 1 page
https://doi.org/10.1155/2024/9764097

Retraction

Retracted: Novel Concepts of *q*-Rung Orthopair Fuzzy Topology and WPM Approach for Multicriteria Decision-Making

Journal of Function Spaces

Received 30 January 2024; Accepted 30 January 2024; Published 31 January 2024

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

In addition, our investigation has also shown that one or more of the following human-subject reporting requirements has not been met in this article: ethical approval by an Institutional Review Board (IRB) committee or equivalent, patient/participant consent to participate, and/or agreement to publish patient/participant details (where relevant).

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] M. Riaz, H. M. A. Farid, S. A. Alblowi, and Y. Almalki, "Novel Concepts of *q*-Rung Orthopair Fuzzy Topology and WPM Approach for Multicriteria Decision-Making," *Journal of Function Spaces*, vol. 2022, Article ID 2094593, 16 pages, 2022.

Hindawi
Journal of Function Spaces
Volume 2022, Article ID 2094593, 16 pages
https://doi.org/10.1155/2022/2094593

Hindawi

Research Article

Novel Concepts of q-Rung Orthopair Fuzzy Topology and WPM Approach for Multicriteria Decision-Making

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Received 21 January 2022; Accepted 10 March 2022; Published 13 April 2022

Academic Editor: Ganesh Ghorai

A q-rung orthopair fuzzy set (q-ROFS) is a robust approach for fuzzy modeling, computational intelligence, and multicriteria decision-making (MCDM) problems. The aim of this paper is to study the topological structure on q-ROFSs and define the idea of q-rung orthopair fuzzy topology (q-ROF topology). The characterization of q-ROF α-continuous mappings between q-ROF topological spaces and q-ROF connectedness is investigated. Some relationships of different types of q-rung orthopair fuzzy connectedness are also investigated. Additionally, the "q-rung orthopair fuzzy weighted product model" (q-ROF WPM) is developed for MCDM of a hierarchical healthcare system. Due to limited and insufficient resources, a hierarchical healthcare system (HHS) is very effective to deal with the increasing problems of healthcare. Recognizing the stage of a disease with the symptoms, ranking the critical condition of patients, and referring patients to feasible hospitals are key features of HHS. A HHS will provide healthcare services in three levels, a primary health centers for initial stage of disease, secondary hospitals for secondary stage of disease, and tertiary hospital for the third-order stage. A numerical example is illustrated to demonstrate the efficiency of q-ROF WPM and advantages of HHS.

1. Introduction

Topological data analysis (TDA) methods are rapidly growing approaches to infer persistent key features for possibly complex data. Topology and big data have inspiration to conventional and information analysis in a variety of computational intelligence fields, learning algorithms [1], analysis techniques [2, 3], and big data [4, 5]. Topology corresponds to the link amongst spatial structures and features, and it can be utilized to explain some specific spatial functions as well as to conceptualize data sets with better reliability and stronger integrity of data. The isomorphisms of the category of topological spaces, known as homeomorphisms, contribute significantly to the theory. The concepts of topology like convergence, continuity, homeomorphisms, and simplicial complexes have robust geometrical interpretation.

The classical methods can not deal with uncertain and vague information in data analysis. To address these challenges, Zadeh [6] established the notion of fuzzy set (FS) theory and membership function. The membership of an element in a fuzzy set with a range of $[0, 1]$ is represented by a single value. Fuzzy logic is a type of multivalued logic in which the values of variables can range from 0 to 1. The idea behind fuzzy logic is that people take decisions relying on ambiguous and nonnumerical information. Fuzzy sets are mathematical interpretations of uncertainty and incorrect data. They are often referred to as fuzzy models. It is a term used to describe the notion of partial truth, which states that the truth value can range between false and true.

Chang [7] developed fuzzy topology and topological space and some fundamental topological concepts such as closed set, open set, compactness, and continuity. Lowen introduced a new concept and definition for fuzzy topological spaces [8, 9]. Fuzzy modeling provide a mathematical framework for uncertain information making judgments based on fuzzy descriptions of data. It is based on the membership function of fuzzy set, which assign a degree of membership or satisfaction. The researchers noticed that the membership grades are not sufficient to analyze data or objects. Atanassov [10] proposed a direct extension of FS, namely, "intuitionistic fuzzy set" (IFS), which depends on both membership grade (MG) and nonmembership grade (NMG).

Coker developed the idea of intuitionistic fuzzy topological space and investigated various counterpart versions of traditional topological properties such as compactness and continuity [11, 12]. Additional results on intuitionistic fuzzy topological spaces are explored in [13, 14]. Fuzzy metric, distance function, and fuzzy metric spaces are the generalization of classical metric spaces [15]. Thus, rather than specific fixed boundaries, the degree-theoretic structure of fuzziness has been incorporated in the notion of topology. Singal and Rajvanshi pioneered the notions of open map, fuzzy alpha open and closed sets, and continuous functions [16].

Ajmal and Kohli [17] developed the idea of "connectedness in fuzzy topological spaces," and Chaudhuri and Das [18] initiated the notion of "fuzzy connected sets in fuzzy topological spaces." Olgun et al. [19] and Öztürk and Yolcu [20] proposed the notion of "Pythagorean fuzzy topology and Pythagorean fuzzy topological spaces." Turkarslan et al. [21] proposed some results of "q-rung orthopair fuzzy topological spaces", and Charisma and Ajay [22] gave the idea of "Pythagorean fuzzy α-continuity." Haydar gave the notion of connectedness for Pythagorean fuzzy topological space [23].

MCDM techniques with fuzzy modeling have been increasingly studied and applied to real-world problems and computational intelligence. Numerous MCDM approaches have been developed to evaluate the robustness of fuzzy modeling in evaluating a set of available objects against a set of criteria. Numerous fields rely on information aggregation and synthesis, including machine learning, neural network, decision analysis, and pattern recognition. Aggregation, in a wide sense, is the process of combining several bits of data to obtain a result. Additionally, it was demonstrated that fundamental data handling algorithms based on crisp integers are incapable of describing working conditions in human cognitive systems. Decision-makers (DMs) are left with hazy findings and perplexing judgements as a result of these approaches. As a result, in order to deal with the world's ambiguous and fuzzy scenarios, DMs seek new philosophies that enable them to interpret ambiguous data values while maintaining their judgement demands under a variety of circumstances.

Yager and Abbasov [24] and Yager [25] introduced the notion of "Pythagorean fuzzy set" (PFS), and Yager [26] introduced the generalized membership grading concept named as "q-rung orthopair fuzzy set." The constraint of q-ROFS is that the sum of qth powers of MG and NMG must be less than or equal to one. Clearly, the higher value of q gives higher the q-rung (q-orbit), the more orthopair's fulfil the constraining requirement, and hence, a broader space is available for q-rung orthopair's [27].

Xu and Yager [28, 29] established geometric and averaging aggregation operators (AOs) for IFSs. Ashraf and Abdullah [30] proposed mathematical approach for MCDM in COVID-19 by utilizing spherical fuzzy information. Saha et al. [31] suggested a new hybrid hesitant fuzzy weighted AOs based on Archimedean and Dombi operations for MCDM. Wei and Zhang [32] utilized single-valued neutrosophic Bonferroni power AOs to select optimal strategic providers. Wei and Wei [33] developed Dombi prioritized AOs using SVNSs. Garg [34] developed AOs of intuitionistic multiplicative set. Akram et al. [35] proposed some novel Dombi AOs for m-polar fuzzy set with applications to MCDM. Sitara et al. [36] proposed q-rung picture fuzzy (q-RPF) graph structures and numerous significant results for modeling with q-RPF graph. Kanwal et al. [37] proposed the existence of fixed points in fuzzy strong b-metric spaces, and Kanwal and Azam [38] gave the notion of common fixed points of IFS maps for Meir-Keeler type contractions. Many AOs for different extensions of fuzzy sets are proposed in [39, 40]. Mahmood et al. [41] proposed the spherical fuzzy set and T spherical fuzzy set with applications to medical diagnosis. The concept of "linear Diophantine fuzzy set" (LDFS) initiated by Riaz and Hashmi [42] is strong model for fuzzy modeling and MADM. They suggested the idea LDF-topology and LDF-AOs with applications. Farid and Riaz suggested improved operational laws for q-ROF information and "q-ROF Einstein interactive geometric" [43], q-ROF hybrid AOs [44], and TOPSIS and VIKOR approaches for q-ROFSSs for MCDM [45].

Çağman et al. [46] proposed the idea of soft topology on soft sets. Shabir and Naz [47] introduced the novel concepts of soft topological spaces and characterization of soft topology. Peng and Liu [48] proposed information measures, such as similarity measures, distance measures, and entropy, for q-ROFSs with applications to medical diagnosis, pattern recognition, and clustering analysis. Feng et al. [49] proposed MADM application by using a new score function for ranking of alternatives with generalized orthopair fuzzy membership grades. Akram et al. [50] suggested a hybrid decision-making framework by using aggregation operators under a complex spherical fuzzy prioritization approach.

The main goals of this manuscript are given in the following:

(i) To define topological structure on q-rung orthopair fuzzy sets and introduce the notion of q-rung orthopair fuzzy topology

(ii) To discuss characterization of q-ROF topology and q-ROF topological spaces like closure, interior, and frontier

(iii) To investigate some significant results related to images and inverse images of q-ROFSs under q-ROF mapping

(iv) To define q-ROF α-continuity and q-ROF connectedness, some relationships between different types of q-ROF connectedness are also investigated

(v) An extension of "weighted product model" WPM to q-ROFSs is developed. A WPM is a robust approach for MCDM frameworks to analyze a set of feasible alternatives in terms of a set of choice criteria. Other key features of WPM include multiplying a number of ratios, one for each choice criterion, each decision alternative is compared to the others. Each ratio is raised to the power of the related criterion's relative weight

(vi) A hierarchical healthcare system (HHS) is suggested that will provide healthcare services in three levels, a primary health centers for initial stage of disease, secondary hospitals for secondary stage of disease, and tertiary hospital for the third-order stage

(vii) An application to HHS for COVID-19 is also presented to illustrate the feasibility and reliability of q-ROF WPM

The remainder of the paper is organized as follows. In Section 2, we review the rudiments of q-ROFSs and q-ROFNs. In Section 3, the idea of q-ROF topology is studied. In Section 4, we presented main results about q-ROFTSs. Section 5 introduces the concept of q-ROF α-continuity. Section 6 introduces certain principles related to q-ROF connectedness, and Section 7 proposes a framework of WPM under the q-ROFSs with an application related to hierarchical healthcare system for COVID-19. Section 8 summarizes the study paper's key findings.

2. Fundamental Concepts

In this section, we review some fundamental concepts and operational procedures of q-rung orthopair fuzzy sets (q-ROFSs) and q-rung orthopair fuzzy numbers (q-ROFNs). A detailed study of these concepts can be seen in [26, 27, 48, 51].

Definition 1 (see [26]). A q-ROFS $\breve{\Upsilon}$ on X is defined as

$$\breve{\Upsilon} = \left\{ \left\langle \underline{\varrho}, \mu_{\breve{\Upsilon}}\left(\underline{\varrho}\right), \nu_{\breve{\Upsilon}}\left(\underline{\varrho}\right) \right\rangle : \underline{\varrho} \in X \right\}, \tag{1}$$

here, $\mu_{\breve{\Upsilon}}, \nu_{\breve{\Upsilon}} : X \longrightarrow [0, 1]$ denotes the MF and NMF of the alternative $\underline{\varrho} \in X$ and $\forall \underline{\varrho}$; we have

$$0 \leq \mu^{q}_{\breve{\Upsilon}}\left(\underline{\varrho}\right) + \nu^{q}_{\breve{\Upsilon}}\left(\underline{\varrho}\right) \leq 1. \tag{2}$$

Furthermore, $\pi_{\breve{\Upsilon}}\left(\underline{\varrho}\right) = \sqrt[q]{1 - \mu^{q}_{\breve{\Upsilon}}\left(\underline{\varrho}\right) - \nu^{q}_{\breve{\Upsilon}}\left(\underline{\varrho}\right)}$ is called the indeterminacy degree of $\underline{\varrho}$ to $\breve{\Upsilon}$.

Liu and Wang [51] suggested some operations on q-ROFNs with the given principles.

Definition 2 (see [51]). Let $\tilde{\delta}_1^{\wedge} = \langle \mu_1, \nu_1 \rangle$ and $\tilde{\delta}_2^{\wedge} = \langle \mu_2, \nu_2 \rangle$ be q-ROFNs. $\sigma > 0$, Then,

(1) $(\tilde{\delta}_1^{\wedge})^{c} = \langle \nu_1, \mu_1 \rangle$

(2) $\tilde{\delta}_1^{\wedge} \vee \tilde{\delta}_2^{\wedge} = \langle \max\{\mu_1, \nu_1\}, \min\{\mu_2, \nu_2\} \rangle$

(3) $\tilde{\delta}_1^{\wedge} \wedge \tilde{\delta}_2^{\wedge} = \langle \min\{\mu_1, \nu_1\}, \max\{\mu_2, \nu_2\} \rangle$

(4) $\tilde{\delta}_1^{\wedge} \oplus \tilde{\delta}_2^{\wedge} = \langle \sqrt[q]{\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q}, \nu_1 \nu_2 \rangle$

(5) $\tilde{\delta}_1^{\wedge} \otimes \tilde{\delta}_2^{\wedge} = \langle \mu_1 \mu_2, \sqrt[q]{\nu_1^q + \nu_2^q - \nu_1^q \nu_2^q} \rangle$

(6) $\sigma \tilde{\delta}_1^{\wedge} = \langle \sqrt[q]{1 - (1 - \mu_1^q)^{\sigma}}, \nu_1^{\sigma} \rangle$

(7) $(\tilde{\delta}_1^{\wedge})^{\sigma} = \langle \mu_1^{\sigma}, \sqrt[q]{1 - (1 - \nu_1^q)^{\sigma}} \rangle$

Definition 3 (see [51]). Let $\tilde{\delta}^{\wedge} = \langle \mu, \nu \rangle$ be the q-ROFN; then, the "score function" (SF) $\mathcal{S}^{\top}$ of $\tilde{\delta}^{\wedge}$ is defined as

$$\mathcal{S}^{\top}\left(\tilde{\delta}^{\wedge}\right) = \mu^q - \nu^q, \mathcal{S}^{\top}\left(\tilde{\delta}^{\wedge}\right) \in [-1, 1]. \tag{3}$$

Definition 4 (see [51]). Let $\tilde{\delta}^{\wedge} = \langle \mu, \nu \rangle$ be the q-ROFN; then, the accuracy function (AF) $\breve{\mathcal{H}}$ of $\tilde{\delta}^{\wedge}$ is defined as

$$\breve{\mathcal{H}}\left(\tilde{\delta}^{\wedge}\right) = \mu^q + \nu^q, \breve{\mathcal{H}}\left(\tilde{\delta}^{\wedge}\right) \in [0, 1]. \tag{4}$$

Definition 5. Consider $\tilde{\delta}_1^{\wedge} = \langle \mu_1, \nu_1 \rangle$ and $\tilde{\delta}_2^{\wedge} = \langle \mu_2, \nu_2 \rangle$ be two q-ROFNs. Then, the subtraction and division of q-ROFNs are defined as

(1) $\tilde{\delta}_1^{\wedge} \ominus \tilde{\delta}_2^{\wedge} = (\sqrt[q]{(\mu_1^q - \mu_2^q)/(1 - \mu_2^q)}, \nu_1/\nu_2)$, if $\mu_1 \geq \mu_2, \nu_1 \leq \min\{\nu_2, \nu_2 \pi_1/\pi_2\}$

(2) $\tilde{\delta}_1^{\wedge} \oslash \tilde{\delta}_2^{\wedge} = (\mu_1/\mu_2, \sqrt[q]{(\nu_1^q - \nu_2^q)/(1 - \nu_2^q)})$ if $\nu_1 \geq \nu_2, \mu_1 \leq \min\{\mu_2, \mu_2 \pi_1/\pi_2\}$

2.1. Superiority of q-ROFNs and Comparison with Other Fuzzy Numbers. The generalized MG and NMG of q-rung orthopair fuzzy numbers (q-ROFNs) provide a robust approach for computational intelligence, fuzzy modeling, and MCDM problems. A q-ROFN is supeior than other fuzzy numbers such as (FNs), IFNs, and PFNs. Table 1 provides analysis of the merits and limitations of q-ROFNs with other fuzzy numbers.

3. q-ROF Topology

Turkarslan et al. [21] proposed the idea of q-ROF topological spaces as a generalization of Pythagorean fuzzy topological spaces [19, 20].

Definition 6 (see [21]). Let $X \neq \varnothing$ be a set and $\breve{\mathcal{E}}^{\tau}$ be a family of q-ROF subsets of X. If

Table 1: Comparative analysis of q-ROFNs.

Theories	Merits	Limitations
Fuzzy sets [6]	Assign MG in $[0, 1]$	Can not assign NMG
IFSs [10]	Assign both MG and NMG	Fails when MG + NMG > 1
PFSs [24]	Assign both MG and NMG, superior than the IFNs	Fails when $MG^2 + NMG^2 > 1$
FFSs [52]	Assign both MG and NMG, superior than IFNs and PFNs	Fails when $MG^3 + NMG^3 > 1$
q-ROFSs [26]	Assign both MG and NMG, superior than IFNs, PFNs, and FFNs, a broader space for MG and NMG	Can not deal with $MG^q + NMG^q > 1$ and MG = NMG = 1

T1 $0_X, 1_X \in \check{\mathcal{E}}^{\tau}$,

T2 for any $\mathcal{T}^{\urcorner}{}_1, \mathcal{T}^{\urcorner}{}_2 \in \check{\mathcal{E}}^{\tau}$, we have $\mathcal{T}^{\urcorner}{}_1 \cap \mathcal{T}^{\urcorner}{}_2 \in \check{\mathcal{E}}^{\tau}$,

T3 for any $\{\mathcal{T}^{\urcorner}{}_i\}_{i\in I} \subseteq \check{\mathcal{E}}^{\tau}$, we have $\cup_{i\in I}\mathcal{T}^{\urcorner}{}_i \in \check{\mathcal{E}}^{\tau}$,

then $\check{\mathcal{E}}^{\tau}$ is called a q-ROF topology on X and the pair $(X, \check{\mathcal{E}}^{\tau})$ is said to be a q-ROFTS. Each member of $\check{\mathcal{E}}^{\tau}$ is called a q-ROF open set (q-ROFOS). The complement of a q-ROF open set is called a q-ROF closed set (q-ROFCS).

Remark 7. Because any IFS or PFS may be thought of as a q-ROF set, we can conclude that any IF topological space or PF topological space is also a q-ROFTS. In contrast, it is evident that q-ROFTS does not have to be an IF or PF topological space. Even a q-ROF open set may not be an IFS or a PFS.

Example 1. Let $X = \{\breve{\alpha}_1^{\gamma}, \breve{\alpha}_2^{\gamma}, \breve{\alpha}_3^{\gamma}\}$. Consider the following family of q-ROF subsets:

$\check{\mathcal{E}}^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_1, \cdots, \mathcal{T}^{\urcorner}{}_4\}$ where

$$\begin{aligned}
\mathcal{T}^{\urcorner}{}_1 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.59, 0.79\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.69, 0.59\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.29, 0.19\right\rangle\right\},\\
\mathcal{T}^{\urcorner}{}_2 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.61, 0.77\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.73, 0.54\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.31, 0.17\right\rangle\right\},\\
\mathcal{T}^{\urcorner}{}_3 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.65, 0.73\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.75, 0.49\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.35, 0.15\right\rangle\right\},\\
\mathcal{T}^{\urcorner}{}_4 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.72, 0.68\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.81, 0.44\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.45, 0.11\right\rangle\right\}.
\end{aligned} \tag{5}$$

One can see that $(X, \check{\mathcal{E}}^{\tau})$ is a q-ROFTS.

Moreover, we see that the following collections are all q-ROFTSs:

$\check{\mathcal{E}}_1^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_1\}$, $\check{\mathcal{E}}_2^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_2\}$, $\check{\mathcal{E}}_3^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_3\}$, $\check{\mathcal{E}}_4^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_4\}$,

$\check{\mathcal{E}}_5^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_1, \mathcal{T}^{\urcorner}{}_2\}$, $\check{\mathcal{E}}_6^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_1, \mathcal{T}^{\urcorner}{}_3\}$, $\check{\mathcal{E}}_7^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_1, \mathcal{T}^{\urcorner}{}_4\}$,

$\check{\mathcal{E}}_8^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_2, \mathcal{T}^{\urcorner}{}_3\}$, $\check{\mathcal{E}}_9^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_2, \mathcal{T}^{\urcorner}{}_4\}$, $\check{\mathcal{E}}_{10}^{\tau}\{0_X, 1_X, \mathcal{T}^{\urcorner}{}_3, \mathcal{T}^{\urcorner}{}_4\}$,

$\check{\mathcal{E}}_{11}^{\tau}\{0_X, 1_X, \mathcal{T}^{\urcorner}{}_1, \mathcal{T}^{\urcorner}{}_2, \mathcal{T}^{\urcorner}{}_3\}$, $\check{\mathcal{E}}_{12}^{\tau} = \{0_X, 1_X, \mathcal{T}^{\urcorner}{}_1, \mathcal{T}^{\urcorner}{}_3, \mathcal{T}^{\urcorner}{}_4\}$.

Definition 8 (see [21]). Let X and Y be two nonempty sets, let $\mathscr{J} : X \longrightarrow Y$ be a mapping, and let $\breve{\mathscr{E}}$ and $\tilde{\mathscr{W}}$ be q-ROF subsets of X and Y, respectively. Then, the membership and nonmembership functions of image of $\breve{\mathscr{E}}$ with respect to $\mathscr{J}$ that is denoted by $\mathscr{J}[\breve{\mathscr{E}}]$ are defined by

$$\begin{aligned}
\mu_{\mathscr{J}[\breve{\mathscr{E}}]}(y) &= \begin{cases} \sup\limits_{z\in\mathscr{J}^{-1}(y)} \mu_{\breve{\mathscr{E}}}(z), & \text{if } \mathscr{J}^{-1}(y) \neq \varnothing,\\ 0, & \text{otherwise,}\end{cases}\\
\nu_{\mathscr{J}[\breve{\mathscr{E}}]}(y) &= \begin{cases} \inf\limits_{z\in\mathscr{J}^{-1}(y)} \nu_{\breve{\mathscr{E}}}(z), & \text{if } \mathscr{J}^{-1}(y) \neq \varnothing,\\ 0, & \text{otherwise,}\end{cases}
\end{aligned} \tag{6}$$

respectively. The membership and nonmembership functions of preimage of $\tilde{\mathscr{W}}$ with respect to $\mathscr{J}$ that is denoted by $\mathscr{J}^{-1}[\tilde{\mathscr{W}}]$ are defined by

$$\begin{aligned}
\mu_{\mathscr{J}^{-1}[\tilde{\mathscr{W}}]}(x) &= \mu_{\tilde{\mathscr{W}}}(\mathscr{J}(x)),\\
\nu_{\mathscr{J}^{-1}[\tilde{\mathscr{W}}]}(x) &= \nu_{\tilde{\mathscr{W}}}(\mathscr{J}(x)),
\end{aligned} \tag{7}$$

respectively.

In [21], they showed that $\mu^q_{\mathscr{J}[\breve{\mathscr{E}}]} + \nu^q_{\mathscr{J}[\breve{\mathscr{E}}]} \leq 1$ q-ROF membership condition is provided for q-ROF image and preimage.

Proposition 9 (see [21]). *Let X and Y be two nonempty sets and $\mathscr{J} : X \longrightarrow Y$ be a q-ROF function. Then, we have*

(1) $\mathscr{J}^{-1}[\tilde{\mathscr{W}}^c] = (\mathscr{J}^{-1}[\tilde{\mathscr{W}}])^c$ for any q-ROF subset $\tilde{\mathscr{W}}$ of Y

(2) $(\mathscr{J}[\breve{\mathscr{E}}])^c \subseteq \mathscr{J}[\breve{\mathscr{E}}^c]$ for any q-ROF subset $\breve{\mathscr{E}}$ of X

(3) If $\tilde{\mathscr{W}}_1 \subseteq \tilde{\mathscr{W}}_2$ then $\mathscr{J}^{-1}[\tilde{\mathscr{W}}_1] \subseteq \mathscr{J}^{-1}[\tilde{\mathscr{W}}_2]$ where $\tilde{\mathscr{W}}_1$ and $\tilde{\mathscr{W}}_2$ are q-ROF subset of Y

(4) If $\breve{\mathscr{E}}_1 \subseteq \breve{\mathscr{E}}_2$ then $\mathscr{J}[\breve{\mathscr{E}}_1] \subseteq \mathscr{J}[\breve{\mathscr{E}}_2]$ where $\breve{\mathscr{E}}_1$ and $\breve{\mathscr{E}}_2$ are q-ROF subset of X

(5) $\mathscr{J}[\mathscr{J}^{-1}[\tilde{\mathscr{W}}]] \subseteq \tilde{\mathscr{W}}$ for any q-ROF subset $\tilde{\mathscr{W}}$ of Y

(6) $\breve{\mathscr{E}} \subseteq \mathscr{J}^{-1}[\mathscr{J}[\breve{\mathscr{E}}]]$ for any q-ROF subset $\breve{\mathscr{E}}$ of X

4. Main Results

In this section, numerous results related to q-rung orthopair fuzzy topology (q-ROFT) are proposed.

Definition 10. Let $\{\mathcal{T}^{\neg}{}_i : i \in I\}$, where $\mathcal{T}^{\neg}{}_i = \{\langle \check{\aleph}, \mu_{\mathcal{T}^{\neg}{}_i}(\check{\aleph}), \nu_{\mathcal{T}^{\neg}{}_i}(\check{\aleph})\rangle : \check{\aleph} \in X\}$, be the indexed family of q-ROFSs over X. Then, the intersection and union of the family is defined as

$$\bigcap_{i\in I} \boxtimes \mathcal{T}^{\neg}{}_i = \left\{ \left\langle \check{\aleph}, \inf \left\{ \mu_{\mathcal{T}^{\neg}{}_i}(\check{\aleph}) \right\}, \sup \left\{ \nu_{\mathcal{T}^{\neg}{}_i}(\check{\aleph}) \right\} \right\rangle : \check{\aleph} \in X \right\},$$

$$\bigcup_{i\in I} \boxtimes \mathcal{T}^{\neg}{}_i = \left\{ \left\langle \check{\aleph}, \sup \left\{ \mu_{\mathcal{T}^{\neg}{}_i}(\check{\aleph}) \right\}, \inf \left\{ \nu_{\mathcal{T}^{\neg}{}_i}(\check{\aleph}) \right\} \right\rangle : \check{\aleph} \in X \right\}. \tag{8}$$

Note that $\bigcap_{i\in I} \boxtimes \mathcal{T}^{\neg}{}_i$ and $\cup_{i\in I} \mathcal{T}^{\neg}{}_i$ are q-ROFSs over X.

Theorem 11. *Let $\{\mathcal{T}^{\neg}{}_i : i \in I\}$, where $\mathcal{T}^{\neg}{}_i = \{\langle \check{\aleph}, \mu_{\mathcal{T}^{\neg}{}_i}(\check{\aleph}), \nu_{\mathcal{T}^{\neg}{}_i}(\check{\aleph})\rangle : \check{\aleph} \in X\}$, be the indexed family of q-ROFSs over X. Then,*

(1) $\overline{\bigcap_{i\in I} \boxtimes \mathcal{T}^{\neg}{}_i} = \cup_{i\in I} \overline{\mathcal{T}^{\neg}{}_i}$

(2) $\overline{\bigcup_{i\in I} \boxtimes \mathcal{T}^{\neg}{}_i} = \bigcap_{i\in I} \boxtimes \overline{\mathcal{T}^{\neg}{}_i}$

Proof. The proof is obvious by Definition 10. □

Definition 12. Let $(X, \check{Ł}^{\tau})$ be a q-ROFTS and $\mathcal{T}^{\neg} = \{\langle \check{\aleph}, \mu_{\mathcal{T}^{\neg}}(\check{\aleph}), \nu_{\mathcal{T}^{\neg}}(\check{\aleph})\rangle : \check{\aleph} \in X\}$ be a q-ROFS over X. Then, the q-ROF interior, q-ROF closure, and q-ROF frontier or boundary of $\mathcal{T}^{\neg}$ are defined as

(1) $\mathrm{Int}(\mathcal{T}^{\neg}) = \cup\{G : G \text{ is a q-ROFOS in } X \text{ and } G \subseteq \mathcal{T}^{\neg}\}$

i.e., "$\mathrm{Int}(\mathcal{T}^{\neg})$ is the q-ROF union of q-ROF open sets contained in $\mathcal{T}^{\neg}$"

(2) $\mathrm{Cl}(\mathcal{T}^{\neg}) = \cap\{K : K \text{ is a q-ROFCS in } X \text{ and } \mathcal{T}^{\neg} \subseteq K\}$

i.e., "$\mathrm{Cl}(\mathcal{T}^{\neg})$ is the q-ROF intersection of q-ROF closed supersets of $\mathcal{T}^{\gamma}$"

(3) $\mathrm{Fr}(\mathcal{T}^{\neg}) = \mathrm{Cl}(\mathcal{T}^{\neg}) \cap \mathrm{Cl}(\mathcal{T}^{\neg c})$

(4) $\mathrm{Ext}(\mathcal{T}^{\neg}) = \mathrm{Int}(\mathcal{T}^{\neg c})$

Remark 13. By Definition 10, we have the following observations:

(i) q-ROF interior, q-ROF closure, and q-ROF boundary of a q-ROFS are always q-ROFSs

(ii) $\mathrm{Int}(\mathcal{T}^{\neg})$ is the largest q-ROF open set contained $\mathcal{T}^{\neg}$

(iii) $Cl(\mathcal{T}^{\neg})$ is the smallest q-ROF closed set containing $\mathcal{T}^{\neg}$

Example 1. Let $X = \{\check{\alpha}_1^{\gamma}, \check{\alpha}_2^{\gamma}, \check{\alpha}_3^{\gamma}\}$. Consider the family of q-ROF sets given as

$$\begin{aligned}
\check{Ł}^{\tau} &= \{1_X, 0_X, \mathcal{T}^{\neg}{}_1, \mathcal{T}^{\neg}{}_2, \mathcal{T}^{\neg}{}_3, \mathcal{T}^{\neg}{}_4,\},\\
\mathcal{T}^{\neg}{}_1 &= \{\langle \check{\alpha}_1^{\gamma}, 0.59{,}0.79\rangle, \langle \check{\alpha}_2^{\gamma}, 0.69{,}0.59\rangle, \langle \check{\alpha}_3^{\gamma}, 0.29{,}0.19\rangle\},\\
\mathcal{T}^{\neg}{}_2 &= \{\langle \check{\alpha}_1^{\gamma}, 0.61{,}0.77\rangle, \langle \check{\alpha}_2^{\gamma}, 0.73{,}0.54\rangle, \langle \check{\alpha}_3^{\gamma}, 0.31{,}0.17\rangle\},\\
\mathcal{T}^{\neg}{}_3 &= \{\langle \check{\alpha}_1^{\gamma}, 0.65{,}0.73\rangle, \langle \check{\alpha}_2^{\gamma}, 0.75{,}0.49\rangle, \langle \check{\alpha}_3^{\gamma}, 0.35{,}0.15\rangle\},\\
\mathcal{T}^{\neg}{}_4 &= \{\langle \check{\alpha}_1^{\gamma}, 0.72{,}0.68\rangle, \langle \check{\alpha}_2^{\gamma}, 0.81{,}0.44\rangle, \langle \check{\alpha}_3^{\gamma}, 0.45{,}0.11\rangle\}.
\end{aligned} \tag{9}$$

It is clear that $(X, \check{Ł}^{\tau})$ is a q-ROF topological space. Now, assume that

$$\lambda = \{\langle \check{\alpha}_1^{\gamma}, 0.79{,}0.49\rangle, \langle \check{\alpha}_2^{\gamma}, 0.89{,}0.29\rangle, \langle \check{\alpha}_3^{\gamma}, 0.58{,}0.09\rangle\}, \tag{10}$$

is a q-ROF subset over X. Then,

$$\begin{aligned}
\mathrm{Int}(\lambda) &= 0_X \cup \mathcal{T}^{\neg}{}_1 \cup \mathcal{T}^{\neg}{}_2 \cup \mathcal{T}^{\neg}{}_3 \cup \mathcal{T}^{\neg}{}_4 = \mathcal{T}^{\neg}{}_4\\
&= \{\langle \check{\alpha}_1^{\gamma}, 0.72{,}0.68\rangle, \langle \check{\alpha}_2^{\gamma}, 0.81{,}0.44\rangle, \langle \check{\alpha}_3^{\gamma}, 0.45{,}0.11\rangle\}.
\end{aligned} \tag{11}$$

On the other hand, in order to find the q-ROF closure of $\mathcal{T}^{\neg}$, it necessary to determine the q-ROF closed sets over X. Then,

$$\begin{aligned}
\mathcal{T}^{\neg c}{}_1 &= \{\langle \check{\alpha}_1^{\gamma}, 0.79{,}0.59\rangle, \langle \check{\alpha}_2^{\gamma}, 0.59{,}0.69\rangle, \langle \check{\alpha}_3^{\gamma}, 0.19{,}0.29\rangle\},\\
\mathcal{T}^{\neg c}{}_2 &= \{\langle \check{\alpha}_1^{\gamma}, 0.77{,}0.61\rangle, \langle \check{\alpha}_2^{\gamma}, 0.54{,}0.73\rangle, \langle \check{\alpha}_3^{\gamma}, 0.17{,}0.31\rangle\},\\
\mathcal{T}^{\neg c}{}_3 &= \{\langle \check{\alpha}_1^{\gamma}, 0.73{,}0.65\rangle, \langle \check{\alpha}_2^{\gamma}, 0.49{,}0.75\rangle, \langle \check{\alpha}_3^{\gamma}, 0.15{,}0.35\rangle\},\\
\mathcal{T}^{\neg c}{}_4 &= \{\langle \check{\alpha}_1^{\gamma}, 0.68{,}0.72\rangle, \langle \check{\alpha}_2^{\gamma}, 0.44{,}0.81\rangle, \langle \check{\alpha}_3^{\gamma}, 0.11{,}0.45\rangle\}.
\end{aligned} \tag{12}$$

Hence,

$$\mathrm{Cl}(\lambda) = 1_X.. \tag{13}$$

Similarly to find the q-ROF boundary of $\mathcal{T}^{\neg}$,

$$\begin{aligned}
\lambda^c &= \{\langle \check{\alpha}_1^{\gamma}, 0.49{,}0.79\rangle, \langle \check{\alpha}_2^{\gamma}, 0.29{,}0.89\rangle, \langle \check{\alpha}_3^{\gamma}, 0.09{,}0.58\rangle\},\\
\mathrm{Cl}(\lambda^c) &= 1_X \cap \mathcal{T}^{\neg c}{}_1 \cap \mathcal{T}^{\neg c}{}_2 \cap \mathcal{T}^{\neg c}{}_3 \cap \mathcal{T}^{\neg c}{}_4 = \mathcal{T}^{\neg c}{}_4\\
&= \{\langle \check{\alpha}_1^{\gamma}, 0.68{,}0.72\rangle, \langle \check{\alpha}_2^{\gamma}, 0.44{,}0.81\rangle, \langle \check{\alpha}_3^{\gamma}, 0.11{,}0.45\rangle\},\\
\mathrm{Fr}(\lambda) &= \mathrm{Cl}(\lambda) \cap \mathrm{Cl}(\lambda^c) = 1_X \cap \mathcal{T}^{\neg c}{}_4\\
&= \{\langle \check{\alpha}_1^{\gamma}, 0.68{,}0.72\rangle, \langle \check{\alpha}_2^{\gamma}, 0.44{,}0.81\rangle, \langle \check{\alpha}_3^{\gamma}, 0.11{,}0.45\rangle\},\\
\mathrm{Ext}(\lambda) &= \mathrm{Int}\left(\mathcal{T}^{\neg c}\right) = 0_X.
\end{aligned} \tag{14}$$

Table 2: Comparison of some results of crisp topology and q-ROF topology.

Crisp topology	q-rung orthopair fuzzy topology
$\mathrm{Int}(\lambda)\bigcup \mathrm{Ext}(\lambda)\bigcup Fr(\lambda)=X$	$\mathrm{Int}(\lambda)\bigcup \mathrm{Ext}(\lambda)\bigcup \mathrm{Fr}(\lambda)\neq 1_X$
$\mathrm{Int}(\lambda)\bigcap \mathrm{Ext}(\lambda)=\varnothing$	$\mathrm{Int}(\lambda)\bigcap \mathrm{Ext}(\lambda)\neq 0_X$
$\mathrm{Ext}(\lambda)\bigcap \mathrm{Fr}(\lambda)=\varnothing$	$\mathrm{Ext}(\lambda)\bigcap \mathrm{Fr}(\lambda)\neq 0_X$
$\mathrm{Int}(\lambda)\bigcap \mathrm{Fr}(\lambda)=\varnothing$	$\mathrm{Int}(\lambda)\bigcap \mathrm{Fr}(\lambda)\neq 0_X$

Remark 14. Furthermore, we investigate that some results that hold in crisp topology but do not hold in q-ROFTS $(X, \check{\mathcal{E}}^{\tau})$. Table 2 shows the comparison of some results of crisp topology and q-ROF topology.

Proposition 15. *Let $(X,\check{\mathcal{E}}^{\tau})_n$ be a q-ROFTS and $\mathscr{T}^{\urcorner}, \mathscr{T}^{\urcorner}_1$, and $\mathscr{T}^{\urcorner}_2$ be q-ROFSs over X. Then, the following properties hold:*

(1) $Int(\mathscr{T}^{\urcorner}) \subseteq \mathscr{T}^{\urcorner}$

(2) $Int(Int(\mathscr{T}^{\urcorner})) = Int(\mathscr{T}^{\urcorner})$

(3) $\mathscr{T}^{\urcorner}_1 \subseteq \mathscr{T}^{\urcorner}_2 \Rightarrow Int(\mathscr{T}^{\urcorner}_1) \subseteq Int(\mathscr{T}^{\urcorner}_2)$

(4) $Int(\mathscr{T}^{\urcorner}_1 \cap \mathscr{T}^{\urcorner}_2) = Int(\mathscr{T}^{\urcorner}_1) \cap Int(\mathscr{T}^{\urcorner}_2)$

(5) $Int(1_X) = 1_X, Int(0_X) = 0_X$

(6) $Int(\mathscr{T}^{\urcorner}_1 \cup \mathscr{T}^{\urcorner}_2) \subseteq Int(\mathscr{T}^{\urcorner}_1) \cap Int(\mathscr{T}^{\urcorner}_2)$

Proof. (1), (2), (3), and (5) can be easily obtained from the definition of the q-ROF interior (4).

From $\mathrm{Int}(\mathscr{T}^{\urcorner}_1 \cap \mathscr{T}^{\urcorner}_2) \subseteq \mathrm{Int}(\mathscr{T}^{\urcorner}_1)$ and $\mathrm{Int}(\mathscr{T}^{\urcorner}_1 \cap \mathscr{T}^{\urcorner}_2) \subseteq \mathrm{Int}(\mathscr{T}^{\urcorner}_2)$, we obtain $\mathrm{Int}(\mathscr{T}^{\urcorner}_1 \cap \mathscr{T}^{\urcorner}_2) \subseteq \mathrm{Int}(\mathscr{T}^{\urcorner}_1) \cap \mathrm{Int}(\mathscr{T}^{\urcorner}_2)$. On the other hand, from the facts $\mathrm{Int}(\mathscr{T}^{\urcorner}_1) \subseteq \mathscr{T}^{\urcorner}_1$ and $\mathrm{Int}(\mathscr{T}^{\urcorner}_2) \subseteq \mathscr{T}^{\urcorner}_2 \Rightarrow \mathrm{Int}(\mathscr{T}^{\urcorner}_1) \cap \mathrm{Int}(\mathscr{T}^{\urcorner}_2) \subseteq \mathscr{T}^{\urcorner}_1 \cap \mathscr{T}^{\urcorner}_2$ and $\mathrm{Int}(\mathscr{T}^{\urcorner}_1) \cap \mathrm{Int}(\mathscr{T}^{\urcorner}_2) \in \check{\mathcal{E}}^{\tau}$, we have $\mathrm{Int}(\mathscr{T}^{\urcorner}_1) \cap \mathrm{Int}(\mathscr{T}^{\urcorner}_2) \subseteq \mathrm{Int}(\mathscr{T}^{\urcorner}_1 \cap \mathscr{T}^{\urcorner}_2)$. So, proof of the axioms (4) is obtained from these two inequalities. □

Theorem 16. *Let $\mathscr{J}$: q-ROFS$(X) \longrightarrow$ q-ROFS(X) be a mapping. The family $\check{\mathcal{E}}^{\tau} = \{\mathscr{T}^{\urcorner} \in$ q-ROFS(X): $\mathscr{J}(\mathscr{T}^{\urcorner}) = \mathscr{T}^{\urcorner}\}$ is a q-ROF topology over X, if the mapping $\mathscr{J}$ satisfies the following conditions:*

(1) $\mathscr{J}(\mathscr{T}^{\urcorner}) \subseteq \mathscr{T}^{\urcorner}$

(2) $\mathscr{J}(1_X) = 1_X$

(3) $\mathscr{J}(\mathscr{J}(\mathscr{T}^{\urcorner})) = \mathscr{J}(\mathscr{T}^{\urcorner})$

(4) $\mathscr{J}(\mathscr{T}^{\urcorner}_1 \cap \mathscr{T}^{\urcorner}_2) = \mathscr{J}(\mathscr{T}^{\urcorner}_1) \cap \mathscr{J}(\mathscr{T}^{\urcorner}_2)$

$\mathscr{J}(\mathscr{T}^{\urcorner}) = Int(\mathscr{T}^{\urcorner})$ for each q-ROF set $\mathscr{T}^{\urcorner}$ in this q-ROF topological space.

Proposition 17. *Let $(X,\check{\mathcal{E}}^{\tau})$ be a q-ROFTS and $\mathscr{T}^{\urcorner}, \mathscr{T}^{\urcorner}_1$, and $\mathscr{T}^{\urcorner}_2$ be q-ROFSs over X. Then, the following properties hold:*

(i) $\mathscr{T}^{\urcorner} \subseteq Cl(\mathscr{T}^{\urcorner})$

(ii) $Cl(Cl(\mathscr{T}^{\urcorner})) = Cl(\mathscr{T}^{\urcorner})$

(iii) $\mathscr{T}^{\urcorner}_1 \subseteq \mathscr{T}^{\urcorner}_2 \Rightarrow Cl(\mathscr{T}^{\urcorner}_1) \subseteq Cl(\mathscr{T}^{\urcorner}_2)$

(iv) $Cl(\mathscr{T}^{\urcorner}_1 \cup \mathscr{T}^{\urcorner}_2) = Cl(\mathscr{T}^{\urcorner}_1) \cup Cl(\mathscr{T}^{\urcorner}_2)$

(v) $Cl(1_X) = 1_X, Cl(0_X) = 0_X$

Proof. (1), (2), (3), and (5) can be easily obtained from the definition of the q-ROF closure (4).

From $\mathrm{Cl}(\mathscr{T}^{y}_1) \subseteq \mathrm{Cl}(\mathscr{T}^{y}_1 \cup \mathscr{T}^{y}_2)$ and $\mathrm{Cl}(\mathscr{T}^{y}_2) \subseteq \mathrm{Cl}(\mathscr{T}^{y}_1 \cup \mathscr{T}^{y}_2)$, we obtain $\mathrm{Cl}(\mathscr{T}^{y}_1) \cup \mathrm{Cl}(\mathscr{T}^{y}_2) \subseteq \mathrm{Cl}(\mathscr{T}^{y}_1 \cup \mathscr{T}^{y}_2)$. On the other hand, from the facts $\mathscr{T}^{y}_1 \subseteq \mathrm{Cl}(\mathscr{T}^{y}_1)$ and $\mathscr{T}^{y}_2 \subseteq \mathrm{Cl}(\mathscr{T}^{y}_2) \Rightarrow \mathscr{T}^{y}_1 \cup \mathscr{T}^{y}_2 \subseteq \mathrm{Cl}(\mathscr{T}^{y}_1) \cup \mathrm{Cl}(\mathscr{T}^{y}_2)$ and $\mathrm{Cl}(\mathscr{T}^{y}_1) \cup \mathrm{Cl}(\mathscr{T}^{y}_2) \in \mathscr{T}^{y}$, we have $\mathrm{Cl}(\mathscr{T}^{y}_1 \cup \mathscr{T}^{y}_2) \subseteq \mathrm{Cl}(\mathscr{T}^{y}_1) \cup \mathrm{Cl}(\mathscr{T}^{y}_2)$. Thus, proof of the axioms (4) is obtained from these two inequalities. □

Theorem 18. *Let $\mathscr{C}$: q-ROFS$(X) \longrightarrow$ q-ROFS(X) be a mapping. The family $\check{\mathcal{E}}^{\tau} = \{\mathscr{T}^{\urcorner} \in$ q-ROFS(X): $\mathscr{C}(\mathscr{T}^{\urcorner c}) = \mathscr{T}^{\urcorner c}\}$ is a q-ROF topology over X, if the mapping $\mathscr{C}$ satisfies the following conditions:*

(1) $\mathscr{T}^{\urcorner} \subseteq \mathscr{C}(\mathscr{T}^{\urcorner})$

(2) $\mathscr{C}(0_X) = 0_X$

(3) $\mathscr{C}(\mathscr{C}(\mathscr{T}^{\urcorner})) = \mathscr{C}(\mathscr{T}^{\urcorner})$

(4) $\mathscr{C}(\mathscr{T}^{\urcorner}_1 \cup \mathscr{T}^{\urcorner}_2) = \mathscr{C}(\mathscr{T}^{\urcorner}_1) \cup \mathscr{C}(\mathscr{T}^{\urcorner}_2)$

Also, $\mathscr{C}(\mathscr{T}^{\urcorner}) = Cl(\mathscr{T}^{\urcorner})$ for each q-ROF set $\mathscr{T}^{\urcorner}$ in this q-ROF topological space.

Theorem 19. *Let $(X,\check{\mathcal{E}}^{\tau})$ be a q-ROFTS and $\mathscr{T}^{\urcorner}$ be a q-ROFS over X. Then,*

(a) $Cl(\mathscr{T}^{\urcorner c}) = (Int(\mathscr{T}^{\urcorner}))^c$

(b) $Int(\mathscr{T}^{\urcorner c}) = (Cl(\mathscr{T}^{\urcorner}))^c$

Proof.

(a) Let $\mathscr{T}^{\urcorner} = \{\langle \check{\aleph}, \mu_{\mathscr{T}^{\urcorner}}(\check{\aleph}), \nu_{\mathscr{T}^{\urcorner}}(\check{\aleph})\rangle : \check{\aleph} \in X\}$ and assume that the family of q-ROFSs contained in $\mathscr{T}^{\urcorner}$ is indexed by the family $\{\mathscr{T}^{\urcorner}_i = \{\langle \check{\aleph}, \mu_{\mathscr{T}^{\urcorner}_i}(\check{\aleph}), \nu_{\mathscr{T}^{\urcorner}_i}(\check{\aleph})\rangle : \check{\aleph} \in X\}\}_{i\in I}$. Then, we see that $\mathrm{Int}(\mathscr{T}^{\urcorner}) = \{\langle \check{\aleph}, \sup\{\mu_{\mathscr{T}^{\urcorner}_i}(\check{\aleph})\}, \inf\{\nu_{\mathscr{T}^{\urcorner}_i}(\check{\aleph})\}\rangle : \check{\aleph} \in X\}$ and hence $(\mathrm{Int}(\mathscr{T}^{\urcorner}))^c = \{\langle \check{\aleph}, \inf\{\nu_{\mathscr{T}^{\urcorner}_i}(\check{\aleph})\}, \sup\{\mu_{\mathscr{T}^{\urcorner}_i}(\check{\aleph})\}\rangle : \check{\aleph} \in X\}$. Since $\mathscr{T}^{\urcorner c} = \{\langle \check{\aleph}, \nu_{\mathscr{T}^{\urcorner}}(\check{\aleph}), \mu_{\mathscr{T}^{\urcorner}}(\check{\aleph})\rangle : \check{\aleph} \in X\}$ and $\mu_{\mathscr{T}^{\urcorner}_i}(\check{\aleph}) \leq$

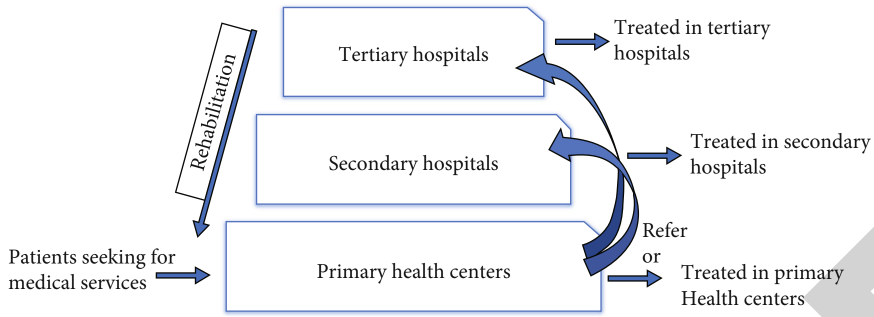

Figure 1: Procedural steps of q-ROF WPM.

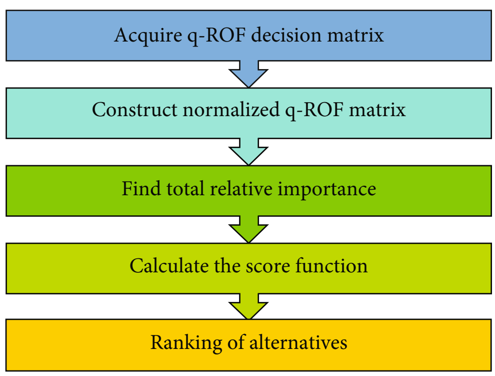

Figure 2: Procedures for patients to seek for medical services.

Table 3: Symptoms for COVID-19.

	Criterion
$\mathscr{C}_1^{\urcorner}$	Chest pain
$\mathscr{C}_2^{\urcorner}$	Breathness
$\mathscr{C}_3^{\urcorner}$	Sore throat
$\mathscr{C}_4^{\urcorner}$	Fever
$\mathscr{C}_5^{\urcorner}$	Headache
$\mathscr{C}_6^{\urcorner}$	Taste & smell
$\mathscr{C}_7^{\urcorner}$	Red or irritated eyes

$\mu_{\mathcal{T}^{\urcorner}}(\check{\aleph}), \nu_{\mathcal{T}^{\urcorner}_i}(\check{\aleph}) \geq \nu_{\mathcal{T}^{\urcorner}}(\check{\aleph})$ for each $i \in I$, we obtain that $\{\mathcal{T}^{\urcorner}_i = \{\langle \check{\aleph}, \mu_{\mathcal{T}^{\urcorner}_i}(\check{\aleph}), \nu_{\mathcal{T}^{\urcorner}_i}(\check{\aleph})\rangle : \check{\aleph} \in X\}\}_{i\in I}$ is the family of q-ROFSs containing $\mathcal{T}^{\urcorner c}$, i.e., $\mathrm{Cl}(\mathcal{T}^{\urcorner c}) = \{\langle \check{\aleph}, \inf\{\nu_{\mathcal{T}^{\urcorner}_i}(\check{\aleph})\}, \sup\{\mu_{\mathcal{T}^{\urcorner}_i}(\check{\aleph})\}\rangle : \check{\aleph} \in X\}$. Therefore, $\mathrm{Cl}(\mathcal{T}^{\urcorner c}) = (\mathrm{Int}(\mathcal{T}^{\urcorner}))^c$ immediately

(b) This analogous to (a)

□

Definition 20. A q-ROFN (or q-ROF point) $\aleph = (\mu, \nu)$ is said to be contained in q-ROFS $\check{\Upsilon} = \{\langle \varrho, \mu_{\check{\Upsilon}}(\varrho), \nu_{\check{\Upsilon}}(\varrho)\rangle : \varrho \in X\}$, written as $\aleph \in \check{\Upsilon}$, if $\mu \leq \mu_{\check{\Upsilon}}(\varrho)$ and if $\nu \geq \nu_{\check{E}}(\varrho), \forall \varrho \in X$.

Definition 21. A q-ROFN $\aleph = (\mu, \nu)$ in q-ROFS $\check{\Upsilon}$ is said to be q-ROF interior point of $\check{\Upsilon}$ if there exist q-ROF open set $\mathscr{U}$ such that $\aleph \in \mathscr{U} \subseteq \check{\Upsilon}$. Then, q-ROFS $\check{\Upsilon}$ is called q-ROF neighborhood of q-ROFN $\aleph = (\mu, \nu)$.

Theorem 22. *Consider $(X, \check{\pounds}^{\tau})$ be a q-ROFTS, then*

(1) If ϕ and φ are the neighborhood of q-ROFN $\aleph$, then $\phi \bigcap \varphi$ and $\phi \bigcup \varphi$ are also neighborhood of q-ROFN $\aleph$

(2) If ψ is neighborhood of q-ROFN $\aleph$ then each q-ROF superset $\delta \supset \psi$ is also neighborhood of q-ROFN $\aleph$

Proposition 23. *Let $(X, \check{\pounds}_1^{\tau})$ and $(Y, \check{\pounds}_2^{\tau})$ be two q-ROFTSs and $\mathcal{J} : X \longrightarrow Y$ be a q-ROF mapping. Then, the following statements are equivalent:*

(a) $\mathcal{J}$ is a q-ROF continuous mapping

(b) $\mathcal{J}[Cl(\mathcal{T}^{\urcorner})] \subseteq Cl(\mathcal{J}[\mathcal{T}^{\urcorner}])$ for each q-ROFS $\mathcal{T}^{\urcorner}$ in X

(c) $Cl(\mathcal{J}^{-1}[K]) \subseteq \mathcal{J}^{-1}[Cl(K)]$ for each q-ROFS K in Y

(d) $\mathcal{J}^{-1}[Int(K)] \subseteq Int(\mathcal{J}^{-1}[K])$ for each q-ROFS K in Y

Proof (a $\Rightarrow$ b). Let $\mathcal{J} : X \longrightarrow Y$ be a q-ROF continuous mapping and $\mathcal{T}^{\urcorner}$ be a q-ROFS over X. Then, $\mathcal{J}[\mathcal{T}^{\urcorner}] \subseteq \mathrm{Cl}(\mathcal{J}[\mathcal{T}^{\urcorner}])$ and $\mathcal{T}^{\urcorner} \subseteq \mathcal{J}^{-1}[\mathrm{Cl}(\mathcal{J}[\mathcal{T}^{\urcorner}])]$. Since $\mathrm{Cl}(\mathcal{J}[\mathcal{T}^{\urcorner}])$ is a q-ROF closed set in Y and $\mathcal{J}$ is a q-ROF continuous mapping, $\mathcal{J}^{-1}[\mathrm{Cl}(\mathcal{J}[\mathcal{T}^{\urcorner}])]$ is a q-ROF closed set in X. On the other hand, if $\mathrm{Cl}(\mathcal{T}^{\urcorner})$ is the smallest q-ROF closed set containing $\mathcal{T}^{\urcorner}$, then $\mathrm{Cl}(\mathcal{T}^{\urcorner}) \subseteq \mathcal{J}^{-1}[\mathrm{Cl}(\mathcal{J}[\mathcal{T}^{\urcorner}])]$ and so $\mathcal{J}[\mathrm{Cl}(\mathcal{T}^{\urcorner})] \subseteq \mathrm{Cl}(\mathcal{J}[\mathcal{T}^{\urcorner}])$.

(b $\Rightarrow$ c) Suppose that $\mathcal{T}^{\urcorner} = \mathcal{J}^{-1}[K]$. From (b), $\mathcal{J}[\mathrm{Cl}(\mathcal{T}^{\urcorner})] = \mathcal{J}[\mathrm{Cl}(\mathcal{J}^{-1}[K])] \subseteq \mathrm{Cl}(\mathcal{J}[\mathcal{T}^{\urcorner}]) = \mathrm{Cl}(\mathcal{J}[\mathcal{J}^{-1}[K]]) \subseteq \mathrm{Cl}(K)$. Then, $\mathrm{Cl}(\mathcal{J}^{-1}[K]) = \mathrm{Cl}(\mathcal{T}^{\urcorner}) \subseteq \mathcal{J}^{-1}[\mathcal{J}[\mathrm{Cl}(\mathcal{T}^{\urcorner})]] \subseteq \mathcal{J}^{-1}[\mathrm{Cl}(K)]$.

(c $\Rightarrow$ d) Since $\mathrm{Int}(K) = (\mathrm{Cl}(K^c))^c$, then $\mathrm{Cl}(\mathcal{J}^{-1}[K]) = \mathrm{Cl}(\mathcal{T}^{\urcorner}) \subseteq \mathcal{J}^{-1}[\mathcal{J}[\mathrm{Cl}(\mathcal{T}^{\urcorner})]] \subseteq \mathcal{J}^{-1}[\mathrm{Cl}(K)]$.

Assume that, G is a q-ROF open set in Y. Then, $\mathrm{Int}(G) = G$. From (d), $\mathcal{J}^{-1}[G] = \mathcal{J}^{-1}[\mathrm{Int}(G)] \subseteq \mathrm{Int}(\mathcal{J}^{-1}[G]) \subseteq \mathcal{J}^{-1}[G]$. Therefore, $\mathcal{J}$ is a q-ROF continuous mapping. □

Table 4: Assessment matrix acquired from DMs.

	$\mathscr{L}^{\daleth}{}_1$	$\mathscr{L}^{\daleth}{}_2$	$\mathscr{L}^{\daleth}{}_3$	$\mathscr{L}^{\daleth}{}_4$	$\mathscr{L}^{\daleth}{}_4$	$\mathscr{L}^{\daleth}{}_6$
$\mathscr{C}_1^{\urcorner}$	(0.425, 0.255)	(0.352, 0.256)	(0.359, 0.215)	(0.313, 0.243)	(0.241, 0.473)	(0.312, 0.283)
$\mathscr{C}_2^{\urcorner}$	(0.765, 0.345)	(0.236, 0.756)	(0.159, 0.715)	(0.365, 0.183)	(0.453, 0.237)	(0.478, 0.317)
$\mathscr{C}_3^{\urcorner}$	(0.142, 0.453)	(0.189, 0.421)	(0.345, 0.426)	(0.153, 0.742)	(0.135, 0.432)	(0.341, 0.532)
$\mathscr{C}_4^{\urcorner}$	(0.354, 0.321)	(0.331, 0.256)	(0.359, 0.215)	(0.145, 0.231)	(0.431, 0.135)	(0.426, 0.351)
$\mathscr{C}_5^{\urcorner}$	(0.725, 0.295)	(0.652, 0.167)	(0.942, 0.233)	(0.813, 0.064)	(0.653, 0.321)	(0.673, 0.134)
$\mathscr{C}_6^{\urcorner}$	(0.413, 0.532)	(0.152, 0.641)	(0.532, 0.542)	(0.753, 0.142)	(0.341, 0.431)	(0.241, 0.321)
$\mathscr{C}_7^{\urcorner}$	(0.341, 0.599)	(0.572, 0.531)	(0.653, 0.159)	(0.173, 0.943)	(0.378, 0.323)	(0.173, 0.493)

Table 5: $\max_j T_{ji}$ and $\min_j T_{ji}$ values.

	$\max_j T_{ji}$	$\min_j T_{ji}$
$\mathscr{C}_1^{\urcorner}$	(0.425, 0.215)	(0.241, 0.473)
$\mathscr{C}_2^{\urcorner}$	(0.765, 0.183)	(0.159, 0.756)
$\mathscr{C}_3^{\urcorner}$	(0.345, 0.421)	(0.135, 0.742)
$\mathscr{C}_4^{\urcorner}$	(0.431, 0.135)	(0.145, 0.351)
$\mathscr{C}_5^{\urcorner}$	(0.942, 0.064)	(0.652, 0.295)
$\mathscr{C}_6^{\urcorner}$	(0.753, 0.142)	(0.152, 0.641)
$\mathscr{C}_7^{\urcorner}$	(0.653, 0.159)	(0.173, 0.943)

Definition 24. Let $(X, \check{\pounds}^{\tau})$ be a q-ROFTS.

(1) A subfamily Γ of $\check{\pounds}^{\tau}$ is called a q-ROF basis (q-ROFB) for $\check{\pounds}^{\tau}$, if for each $\mathscr{T}^{\urcorner} \in \check{\pounds}^{\tau}$, $\mathscr{T}^{\urcorner} = 0_X$ or there exists $\Gamma' \subseteq \Gamma$ such that $\mathscr{T}^{\urcorner} = \cup \Gamma'$

(2) A collection Φ of some q-ROFSs over X is called a q-ROF subbase for some q-ROF topology $\check{\pounds}^{\tau}$, if the finite intersections of members of Φ form a q-ROF basis for $\check{\pounds}^{\tau}$

Theorem 25. *$(X, \check{\pounds}_1^{\tau})$ and $(Y, \check{\pounds}_2^{\tau})$ be two q-ROFTSs and $\mathscr{J} : X \longrightarrow Y$ be a q-ROF mapping. Then,*

(1) $\mathscr{J}$ is a q-ROF continuous mapping iff for each $\widetilde{\mathscr{W}} \in \Gamma$ we have $\mathscr{J}^{-1}[\widetilde{\mathscr{W}}]$ is a q-ROF open subset of X such that Γ is a q-ROF basic for $\check{\pounds}_2^{\tau}$

(2) $\mathscr{J}$ is a q-ROF continuous mapping iff for each $K \in \mathscr{T}^{\urcorner}hi$ we have $\mathscr{J}^{-1}[K]$ is a q-ROF open subset of X such that Φ is a q-ROF subbase for $\check{\pounds}_2^{\tau}$

Proof.

(i) Let $\mathscr{J}$ be a q-ROF continuous mapping. Since each $\widetilde{\mathscr{W}} \in \Gamma \subseteq \check{\pounds}_2^{\tau}$ and $\mathscr{J}$ is a q-ROF continuous mapping, then $\mathscr{J}^{-1}[\widetilde{\mathscr{W}}] \in \check{\pounds}_1^{\tau}$

Conversely, suppose that Γ is a q-ROF basic for $\check{\pounds}_2^{\tau}$ and $\mathscr{J}^{-1}[\widetilde{\mathscr{W}}] \in \check{\pounds}_1^{\tau}$ for each $\widetilde{\mathscr{W}} \in \Gamma$, then for arbitrary a q-ROF open set $\mathscr{T}^{\urcorner} \in \check{\pounds}_2^{\tau}$,

$$\mathscr{J}^{-1}\left[\mathscr{T}^{\urcorner}\right] = \mathscr{J}^{-1}\left[\cup_{\widetilde{\mathscr{W}} \in \Gamma} \widetilde{\mathscr{W}}\right] = \bigcup_{\widetilde{\mathscr{W}} \in \Gamma} \mathscr{J}^{-1}\left[\widetilde{\mathscr{W}}\right] \in \check{\pounds}_1^{\tau}. \tag{15}$$

That is, $\mathscr{J}$ is a q-ROF continuous mapping.

(ii) Let $\mathscr{J}$ be a q-ROF continuous mapping. Since each $K \in \Phi \subseteq \check{\pounds}_2^{\tau}$ and $\mathscr{J}$ is a q-ROF continuous mapping, then $\mathscr{J}^{-1}[K] \in \check{\pounds}_1^{\tau}$

Conversely, assume that Φ is a q-ROF subbase for $\check{\pounds}_2^{\tau}$ and $\mathscr{J}^{-1}[K] \in \check{\pounds}_1^{\tau}$ for each $K \in \Phi$, then for arbitrary a q-ROF open set $\mathscr{T}^{\urcorner} \in \check{\pounds}_2^{\tau}$,

$$\begin{aligned} \mathscr{J}^{-1}\left[\mathscr{T}^{\urcorner}\right] &= \mathscr{J}^{-1}\left[\cup_{i_j \in I}\left(K_{i_1} \cap K_{i_2} \cap \cdots \cap K_{i_n}\right)\right] \\ &= \bigcup_{i_j \in I} \boxtimes \left(\mathscr{J}^{-1}\left[K_{i_1}\right] \cap \mathscr{J}^{-1}\left[K_{i_2}\right] \cap \cdots \cap \mathscr{J}^{-1}\left[K_{i_n}\right]\right) \in \check{\pounds}_1^{\tau}. \end{aligned} \tag{16}$$

That is, $\mathscr{J}$ is a q-ROF continuous mapping. □

Definition 26. Let $(X, \check{\pounds}_1^{\tau})$ and $(Y, \check{\pounds}_2^{\tau})$ be two q-ROFTSs and $\mathscr{J} : X \longrightarrow Y$ be a q-ROF mapping. Then,

(1) $\mathscr{J}$ is called a q-ROF open mapping if $\mathscr{J}[\mathscr{T}^{\urcorner}]$ is a q-ROF open set over Y for every q-ROF open set $\mathscr{T}^{\urcorner}$ over X

(2) $\mathscr{J}$ is called a q-ROF closed mapping if $\mathscr{J}[K]$ is a q-ROF closed set over Y for every q-ROF closed set K over X

Example 2. Let $X = \{\check{\alpha}_1^{\gamma}, \check{\alpha}_2^{\gamma}, \check{\alpha}_3^{\gamma}\}$ and $Y = \{y_1, y_2, y_3\}$. Consider the following families of q-ROF sets $\check{\pounds}_1^{\tau} = \{0_X, 1_X, \mathscr{T}^{\urcorner}{}_1, \mathscr{T}^{\urcorner}{}_2, \mathscr{T}^{\urcorner}{}_3, \mathscr{T}^{\urcorner}{}_4\}$ and $\check{\pounds}_2^{\tau} = \{0_Y, 1_Y, S_1, S_2, S_3, S_4\}$

Table 6: Normalized decision matrix.

	$\mathscr{L}^{\daleth}{}_1$	$\mathscr{L}^{\daleth}{}_2$	$\mathscr{L}^{\daleth}{}_3$	$\mathscr{L}^{\daleth}{}_4$	$\mathscr{L}^{\daleth}{}_4$	$\mathscr{L}^{\daleth}{}_6$
$\mathscr{C}_1^{\urcorner}$	(1.000, 0.213)	(0.828, 0.216)	(0.845, 0.000)	(0.737, 0.192)	(0.567, 0.468)	(0.734, 0.256)
$\mathscr{C}_2^{\urcorner}$	(0.208, 0.750)	(0.674, 0.000)	(1.000, 0.545)	(0.436, 0.756)	(0.351, 0.755)	(0.333, 0.752)
$\mathscr{C}_3^{\urcorner}$	(0.412, 0.324)	(0.548, 0.000)	(1.000, 0.199)	(0.444, 0.728)	(0.391, 0.245)	(0.988, 0.474)
$\mathscr{C}_4^{\urcorner}$	(0.821, 0.318)	(0.768, 0.251)	(0.833, 0.206)	(0.336, 0.223)	(1.000, 0.000)	(0.988, 0.349)
$\mathscr{C}_5^{\urcorner}$	(0.769, 0.295)	(0.692, 0.166)	(1.000, 0.233)	(0.863, 0.000)	(0.693, 0.321)	(0.714, 0.132)
$\mathscr{C}_6^{\urcorner}$	(0.368, 0.557)	(1.000, 0.000)	(0.286, 0.548)	(0.201, 0.641)	(0.446, 0.611)	(0.631, 0.632)
$\mathscr{C}_7^{\urcorner}$	(0.522, 0.598)	(0.876, 0.530)	(1.000, 0.000)	(0.265, 0.943)	(0.579, 0.318)	(0.265, 0.492)

Table 7: Total relative importance and score values.

Alternatives	q-ROF WPM values	Score values
$\mathscr{L}^{\daleth}{}_1$	(0.0638991, 0.2917500)	-0.0072284
$\mathscr{L}^{\daleth}{}_2$	(0.3642640, 0.0954248)	0.01752330
$\mathscr{L}^{\daleth}{}_3$	(0.2396050, 0.1870860)	0.00207089
$\mathscr{L}^{\daleth}{}_4$	(0.0219020, 0.5043240)	-0.0646900
$\mathscr{L}^{\daleth}{}_5$	(0.0851057, 0.2975280)	-0.0077839
$\mathscr{L}^{\daleth}{}_6$	(0.1558570, 0.3159770)	-0.0093783

where

$$\begin{aligned}
\mathscr{T}^{\urcorner}{}_1 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.29, 0.45\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.59, 0.19\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.59, 0.49\right\rangle\right\},\\
\mathscr{T}^{\urcorner}{}_2 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.59, 0.49\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.79, 0.29\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.69, 0.59\right\rangle\right\},\\
\mathscr{T}^{\urcorner}{}_3 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.59, 0.40\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.79, 0.19\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.69, 0.49\right\rangle\right\},\\
\mathscr{T}^{\urcorner}{}_4 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.29, 0.42\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.59, 0.29\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.59, 0.59\right\rangle\right\},\\
S_1 &= \{\langle y_1, 0.59, 0.19\rangle, \langle y_2, 0.29, 0.45\rangle, \langle y_3, 0.59, 0.49\rangle\},\\
S_2 &= \{\langle y_1, 0.79, 0.29\rangle, \langle y_2, 0.59, 0.49\rangle, \langle y_3, 0.69, 0.59\rangle\},\\
S_3 &= \{\langle y_1, 0.79, 0.19\rangle, \langle y_2, 0.59, 0.40\rangle, \langle y_3, 0.69, 0.49\rangle\},\\
S_4 &= \{\langle y_1, 0.59, 0.29\rangle, \langle y_2, 0.29, 0.42\rangle, \langle y_3, 0.59, 0.59\rangle\}.
\end{aligned} \tag{17}$$

It is clear that $(X, \breve{\mathcal{E}}_1^{\tau})$ and $(Y, \breve{\mathcal{E}}_2^{\tau})$ are q-ROF topological spaces. If q-ROF mapping $\mathscr{J} : X \longrightarrow Y$ is defined as

$$\begin{aligned}
\mathscr{J}\left(\breve{\alpha}_1^{\gamma}\right) &= y_2,\\
\mathscr{J}\left(\breve{\alpha}_2^{\gamma}\right) &= y_1,\\
\mathscr{J}\left(\breve{\alpha}_3^{\gamma}\right) &= y_3.
\end{aligned} \tag{18}$$

Then, $\mathscr{J}$ is a q-ROF open mapping. However, $\mathscr{J}$ is not q-ROF closed mapping on q-ROF topological spaces $(X, \breve{\mathcal{E}}_1^{\tau})$.

Theorem 27. *Let $(X, \breve{\mathcal{E}}_1^{\tau})$ and $(Y, \breve{\mathcal{E}}_2^{\tau})$ be two q-ROFTSs and $\mathscr{J} : X \longrightarrow Y$ be a q-ROF mapping. Then,*

(1) *$\mathscr{J}$ is a q-ROF open mapping if $\mathscr{J}[Int(\mathscr{T}^{\urcorner})] \subseteq Int(\mathscr{J}[\mathscr{T}^{\urcorner}])$ for each q-ROF set $\mathscr{T}^{\urcorner}$ over X*

(2) *$\mathscr{J}$ is a q-ROF closed mapping if $Cl(\mathscr{J}[\mathscr{T}^{\urcorner}]) \subseteq \mathscr{J}[Cl(\mathscr{T}^{\urcorner})]$ for each q-ROF set $\mathscr{T}^{\urcorner}$ over X*

Proof.

(1) Let $\mathscr{J}$ be a q-ROF open mapping and $\mathscr{T}^{\urcorner}$ be a q-ROFS over X. Then, $\mathrm{Int}(\mathscr{T}^{\urcorner})$ is a q-ROF open set and $\mathrm{Int}(\mathscr{T}^{\urcorner}) \subseteq \mathscr{T}^{\urcorner}$. Since $\mathscr{J}$ is a q-ROF open mapping, $\mathscr{J}[\mathrm{Int}(\mathscr{T}^{\urcorner})]$ is a q-ROF open set over Y and $\mathscr{J}[\mathrm{Int}(\mathscr{T}^{\urcorner})] \subseteq \mathscr{J}[\mathscr{T}^{\urcorner}]$. Thus, $\mathscr{J}[\mathrm{Int}(\mathscr{T}^{\urcorner})] \subseteq \mathrm{Int}(\mathscr{J}[\mathscr{T}^{\urcorner}])$ is obtained

Conversely, suppose that $\mathscr{T}^{\urcorner}$ is any q-ROF open set over X. Then, $\mathscr{T}^{\urcorner} = \mathrm{Int}(\mathscr{T}^{\urcorner})$. From the condition of the theorem, we have $\mathscr{J}[\mathrm{Int}(\mathscr{T}^{\urcorner})] \subseteq \mathrm{Int}(\mathscr{J}[\mathscr{T}^{\urcorner}])$. Then, $\mathscr{J}[\mathscr{T}^{\urcorner}] = \mathscr{J}[\mathrm{Int}(\mathscr{T}^{\urcorner})] \subseteq \mathrm{Int}(\mathscr{J}[\mathscr{T}^{\urcorner}]) \subseteq \mathscr{J}[\mathscr{T}^{\urcorner}]$. This implies that $\mathscr{J}[\mathscr{T}^{\urcorner}] = \mathrm{Int}(\mathscr{J}[\mathscr{T}^{\urcorner}])$. That is, $\mathscr{J}$ is a q-ROF open mapping.

(2) Let $\mathscr{J}$ be a q-ROF closed mapping and $\mathscr{T}^{\urcorner}$ be a q-ROFS over X. Since $\mathscr{J}$ is a q-ROF closed mapping then $\mathscr{J}[\mathrm{Cl}(\mathscr{T}^{\urcorner})]$ is a q-ROF closed set over Y and $\mathscr{J}[\mathscr{T}^{\urcorner}] \subseteq \mathscr{J}[\mathrm{Cl}(\mathscr{T}^{\urcorner})]$. Thus, $\mathrm{Cl}(\mathscr{J}[\mathscr{T}^{\urcorner}]) \subseteq \mathscr{J}[\mathrm{Cl}(\mathscr{T}^{\urcorner})]$ is obtained

Conversely, assume that $\mathscr{T}^{\gamma}$ is any q-ROF closed set over X. Then, $\mathscr{T}^{\urcorner} = Cl(\mathscr{T}^{\urcorner})$. From the condition of the theorem, we have $\mathrm{Cl}(\mathscr{J}[\mathscr{T}^{\urcorner}]) \subseteq \mathscr{J}[\mathrm{Cl}(\mathscr{T}^{\urcorner})] = \mathscr{J}[\mathscr{T}^{\urcorner}] \subseteq \mathrm{Cl}(\mathscr{J}[\mathscr{T}^{\urcorner}])$. This means that $\mathrm{Cl}(\mathscr{J}[\mathscr{T}^{\urcorner}]) = \mathscr{J}[\mathscr{T}^{\urcorner}]$. That is, $\mathscr{J}$ is a q-ROF closed mapping. □

Definition 28. Let $(X, \breve{\mathcal{E}}_1^{\tau})$ and $(Y, \breve{\mathcal{E}}_2^{\tau})$ be two q-ROFTSs and $\mathscr{J} : X \longrightarrow Y$ be a q-ROF mapping. Then, $\mathscr{J}$ is a called a q-ROF homeomerphism, if

(1) $\mathscr{J}$ is a bijection

(2) $\mathscr{J}$ is a q-ROF continuous mapping

(3) $\mathscr{J}^{-1}$ is a q-ROF continuous mapping

Theorem 29. *Let $(X, \check{\mathcal{E}}_1^{\tau})$ and $(Y, \check{\mathcal{E}}_2^{\tau})$ be two q-ROFTSs and $\mathscr{J} : X \longrightarrow Y$ be a q-ROF mapping. Then, the following conditions are equivalent:*

(1) $\mathscr{J}$ is a q-ROF homeomerphism

(2) $\mathscr{J}$ is a q-ROF continuous mapping and q-ROF open mapping

(3) $\mathscr{J}$ is a q-ROF continuous mapping and q-ROF closed mapping

Proof. The proof can be easily obtained by using the previous theorems on continuity, opennes, and closedness are omitted. □

5. q-ROF α-Continuity

Definition 30. Let $(X, \check{\mathcal{E}}^{\tau})$ be a q-ROF topological space. A q-ROFS $\mathscr{T}^{\urcorner}$ over X is called a q-ROF α open set if $\mathscr{T}^{\urcorner} \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(\mathscr{T}^{\urcorner})))$. A q-ROFS whose complement is a q-ROF α open set (q-ROFαOS) is called a q-ROF α closed set (q-ROFαCS).

Proposition 31. *Let $(X, \check{\mathcal{E}}^{\tau})$ be a q-ROFTS. Then, arbitrary union of q-ROFαOS is a q-ROFαOS and arbitrary intersection of q-ROFαCSs is q-ROFαCS.*

Proof. Let $\{\mathscr{T}^{\urcorner}{}_i = \langle x, \mu_{\mathscr{T}^{\urcorner}}, \gamma_{\mathscr{T}^{\urcorner}} > | i \in I\}$ be a family of q-ROFαOSs. Then, for each $i \in I$, $\mathscr{T}^{\urcorner}{}_i \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(\mathscr{T}^{\urcorner}{}_i)))$. Thus, $\cup\mathscr{T}^{\urcorner}{}_i \subseteq \cup\mathrm{Int}((\mathrm{Cl}(\mathrm{Int}(\mathscr{T}^{\urcorner}{}_i)))) \subseteq \mathrm{Int}(\cup\mathrm{Cl}(\mathrm{Int}(\mathscr{T}^{\urcorner}{}_i)))) \subseteq \mathrm{Int}(\mathrm{Cl}(\cup\mathrm{Int}(\mathscr{T}^{\urcorner}{}_i)))) \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(\cup\mathscr{T}^{\urcorner}{}_i))))$.

Hence, $\cup\mathscr{T}^{\urcorner}{}_i$ is a q-ROFαOS set. If we take complement of this part, the consecutive will proved (ie. arbitrary intersection of q-ROFαOS is also a q-ROFαOS).

Every q-ROFOS is a q-ROFαOS, and every q-ROFCS is a q-ROFαCS but the converse is not true. □

Definition 32. The q-ROF α closure of a q-ROFS $\mathscr{T}^{\urcorner}$ in a q-ROFTS $(X\,; \check{\mathcal{E}}^{\tau})$ is represented as $Cl_{\alpha}(\mathscr{T}^{\urcorner})$ and defined by $Cl_{\alpha}(\mathscr{T}^{\urcorner}) = \bigcap\{C_i \mid C_i \text{ is a q-ROF}\alpha\text{C set and } \mathscr{T}^{\urcorner} \subseteq C_i\}$

Proposition 33. *In a q-ROFTS $(X, \check{\mathcal{E}}^{\tau})$, a q-ROFS $\mathscr{T}^{\urcorner}$ is q-ROFαC if and only if $\mathscr{T}^{\urcorner} = Cl_{\alpha}(\mathscr{T}^{\urcorner})$.*

Proof. Assume that $\mathscr{T}^{\urcorner}$ is a q-ROFαC set. Then, $\mathscr{T}^{\urcorner} \in \{C_i \mid C_i \text{ is a q-ROF}\alpha\text{C set and } \mathscr{T}^{\urcorner} \subseteq C_i\}$, so $\mathscr{T}^{\urcorner} = \cap\{C_i \mid C_i \text{ is a q-ROF}\alpha\text{C and } \mathscr{T}^{\urcorner} \subseteq C_i\} = Cl_{\alpha}(\mathscr{T}^{\urcorner})$.

Conversely, consider $\mathscr{T}^{\urcorner} = Cl_{\alpha}(\mathscr{T}^{\urcorner})$,

$$\mathscr{T}^{\urcorner} \in \left\{C_i \mid C_i \text{ is a q-ROF}\alpha\text{C set and } \mathscr{T}^{\urcorner} \subseteq C_i\right\}. \tag{19}$$

Thus, $\mathscr{T}^{\urcorner}$ is a q-ROFα-closed set. □

Proposition 34. *In a q-ROFTS $(X, \check{\mathcal{E}}^{\tau})$, the following hold for q-ROF α-closure:*

(1) $Cl_{\alpha}(\underline{0}) = \underline{0}$

(2) $Cl_{\alpha}(\mathscr{T}^{\urcorner})$ is a q-ROαC in $(X, \check{\mathcal{E}}^{\tau})$ for every q-ROFS $\mathscr{T}^{\urcorner}$ in X

(3) $Cl_{\alpha}(\mathscr{T}^{\urcorner}) \subseteq Cl_{\alpha}(R)$ whenever $\mathscr{T}^{\urcorner} \subseteq R$ for every $\mathscr{T}^{\urcorner}$ and R in X

(4) $Cl_{\alpha}(Cl_{\alpha}(\mathscr{T}^{\urcorner})) = Cl_{\alpha}(\mathscr{T}^{\urcorner})$ for every q-ROFS $\mathscr{T}^{\urcorner}$ in X

Proof.

(1) The proof is obvious

(2) By preposition, $\mathscr{T}^{\urcorner}$ is q-ROFαC iff $\mathscr{T}^{\urcorner} = Cl_{\alpha}(\mathscr{T}^{\urcorner})$ we get $Cl_{\alpha}(\mathscr{T}^{\urcorner})$ is a q-ROFαC for every $\mathscr{T}^{\urcorner}$ in X

(3) By same preposition, we get $\mathscr{T}^{\urcorner} = \mathrm{Cl}_{\alpha}(\mathscr{T}^{\urcorner})$ and $R = \mathrm{Cl}_{\alpha}(R)$. whenever $\mathscr{T}^{\urcorner} \subseteq R$, we have $\mathrm{Cl}_{\alpha}(\mathscr{T}^{\urcorner}) \subseteq \mathrm{Cl}_{\alpha}(R)$

(4) Let $\mathscr{T}^{\urcorner}$ be a q-ROFFS in X. We know that $\mathscr{T}^{\urcorner} = \mathrm{Cl}_{\alpha}(\mathscr{T}^{\urcorner})$

$\mathrm{Cl}_{\alpha}(\mathscr{T}^{\urcorner}) = \mathrm{Cl}_{\alpha}(\mathrm{Cl}_{\alpha}(\mathscr{T}^{\urcorner}))$. Thus, $\mathrm{Cl}_{\alpha}(\mathrm{Cl}_{\alpha}(\mathscr{T}^{\urcorner})) = \mathrm{Cl}_{\alpha}(\mathscr{T}^{\urcorner})$ for every $\mathscr{T}^{\urcorner}$ in X. □

Definition 35. Let $(X, \check{\mathcal{E}}_X^{\tau})$ and $(Y, \check{\mathcal{E}}_Y^{\tau})$ be q-ROFTSs. A mapping $\mathscr{J} : X \longrightarrow Y$ is named as q-ROFα-continuous (q-ROFαCN) if the inverse image of each q-ROFOS of Y is a q-ROFαO set in X.

Theorem 36. *Let $\mathscr{J} : (X, \check{\mathcal{E}}_X^{\tau}) \longrightarrow (Y, \check{\mathcal{E}}_Y^{\tau})$ be a mapping from a q-ROFTS $(X, \check{\mathcal{E}}_X^{\tau})$ to a q-ROFTS $(Y, \check{\mathcal{E}}_Y^{\tau})$. If $\mathscr{J}$ is q-ROFα-continues, then*

(1) $\mathscr{J}(Cl(Int(Cl(\mathscr{T}^{\urcorner})))) \subseteq Cl(\mathscr{J}(\mathscr{T}^{\urcorner}))$ for all q-ROFS $\mathscr{T}^{\urcorner}$ in X

(2) $Cl(Int(\mathscr{J}^{-1}(\tilde{\mathscr{W}}))) \subseteq \mathscr{J}^{-1}(Cl(\tilde{\mathscr{W}}))$ for all $\tilde{\mathscr{W}}$ in Y

6. q-ROF Connectedness

Here, we generalize the concept of IF-connected topological space, to the case of q-ROFTS.

Definition 37. Let A be a q-ROF subset in $(X, \check{\mathcal{E}}_X^{\tau})$.

(a) If there exist q-ROFOSs $\mathscr{U}^{\varsigma}$ and $\mathscr{V}^{\tau}$ in X satisfying the following properties, then $\mathscr{A}^{\zeta}$ is called q-ROF $\mathfrak{C}_i$-disconnected $(i = 1, 2, 3, 4)$:

C1 $\mathscr{A}^{\zeta} \subseteq \mathscr{U}^{\varsigma} \cup \mathscr{V}^{\tau}, \mathscr{U}^{\varsigma} \cap \mathscr{V}^{\tau} \subseteq \mathscr{A}^{\zeta^c}, \mathscr{A}^{\zeta} \cap \mathscr{U}^{\varsigma} \neq 0_x, \mathscr{A}^{\zeta} \cap \mathscr{V}^{\tau} \neq 0_x$

C2 $\mathscr{A}^{\zeta} \subseteq \mathscr{U}^{\varsigma} \cup \mathscr{V}^{\tau}, \mathscr{A}^{\zeta} \cap \mathscr{U}^{\varsigma} \cap \mathscr{V}^{\tau} \neq 0_x, \mathscr{A}^{\zeta} \cap \mathscr{U}^{\varsigma} \neq 0_x, \mathscr{A}^{\zeta} \cap \mathscr{V}^{\tau} \neq 0_x,$

C3 $\mathscr{A}^{\zeta} \subseteq \mathscr{U}^{\varsigma} \cup \mathscr{V}^{\tau}, \mathscr{U}^{\varsigma} \cap \mathscr{V}^{\tau} \subseteq \mathscr{A}^{\zeta^c}, \mathscr{U}^{\varsigma}\mathrm{U}\mathscr{A}^{\zeta^c}, \mathscr{V}^{\tau}\mathrm{U}\mathscr{A}^{\zeta^c},$

C4 $\mathscr{A}^{\zeta} \subseteq \mathscr{U}^{\varsigma} \cup \mathscr{V}^{\tau}, \mathscr{A}^{\zeta} \cap \mathscr{U}^{\varsigma} \cap \mathscr{V}^{\tau} \neq 0_x, \mathscr{U}^{\varsigma} \mathrm{U} \mathscr{A}^{\zeta^c}, \mathscr{V}^{\tau} \mathrm{U} \mathscr{A}^{\zeta^c}$.

(b) $\mathscr{A}^{\zeta}$ is said to be q-ROF $\mathfrak{C}_i$-connected $(i = 1, 2, 3, 4)$, if $\mathscr{A}^{\zeta}$ is not q-ROF $\mathfrak{C}_i$-disconnected $(i = 1, 2, 3, 4)$

It is clear that in q-ROFTSs, we have the following implications:

$$\begin{array}{ccc} \mathfrak{C}_1\text{-connectedness} & \longrightarrow & \mathfrak{C}_1\text{-connectedness} \\ \downarrow & & \downarrow \\ \mathfrak{C}_3\text{-connectedness} & \longrightarrow & \mathfrak{C}_4\text{-connectedness}. \end{array} \tag{20}$$

Example 1. Let $X = \{\breve{\alpha}_1^{\gamma}, \breve{\alpha}_2^{\gamma}, \breve{\alpha}_3^{\gamma}\}$. Consider the following family of q-ROF sets:

$$\begin{aligned} \mathscr{T}^{\neg}{}_1 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.50{,}0.20\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.50{,}0.40\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.40{,}0.40\right\rangle\right\}, \\ \mathscr{T}^{\neg}{}_2 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.40{,}0.50\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.60{,}0.30\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.20{,}0.30\right\rangle\right\}, \\ \mathscr{T}^{\neg}{}_3 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.50{,}0.20\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.60{,}0.30\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.40{,}0.30\right\rangle\right\}, \\ \mathscr{T}^{\neg}{}_4 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.40{,}0.50\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.50{,}0.40\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.20{,}0.40\right\rangle\right\}. \end{aligned} \tag{21}$$

Then, $\breve{\mathcal{E}}^{\tau} = \{1_X, 0_X, \mathscr{T}^{\neg}{}_1, \mathscr{T}^{\neg}{}_2, \mathscr{T}^{\neg}{}_3, \mathscr{T}^{\neg}{}_4\}$ is a q-ROFTS on X, and consider the q-ROFS E given below:

$$E = \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.60{,}0.20\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.50{,}0.20\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.40{,}0.30\right\rangle\right\}, \tag{22}$$

in X. Then, E is q-ROF $\mathfrak{C}_1$-connected, and E is also q-ROF$\mathfrak{C}_2$-connected, q-ROF$\mathfrak{C}_3$-connected, and q-ROF$\mathfrak{C}_4$-connected.

Example 2. Consider the q-ROFTS $(X, \breve{\mathcal{E}}_X^{\tau})$ given in Example 1 and consider the q-ROFS F given below:

$$F = \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.20{,}0.40\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.30{,}0.60\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.20{,}0.40\right\rangle\right\}. \tag{23}$$

One can check if F is q-ROF$\mathfrak{C}_1$-disconnected and hence not q-ROF$\mathfrak{C}_1$-connected.

Definition 38. Let $(X, \breve{\mathcal{E}}_X^{\tau})$ be a q-ROFTS. If there exists a q-ROFOS $\mathscr{A}^{\zeta}$ in X such that $0_x \neq \mathscr{A}^{\zeta} \neq 1_x$, then X is called q-ROF super disconnected. and X is called q-ROF fuzzy superconnected, if X is not q-ROF superdisconnected.

Example 3. Let $X = \{\breve{\alpha}_1^{\gamma}, \breve{\alpha}_2^{\gamma}, \breve{\alpha}_3^{\gamma}\}$. Consider the following family of q-ROF sets:

$$\begin{aligned} \mathscr{T}^{\neg}{}_1 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.40{,}0.50\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.60{,}0.30\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.20{,}0.30\right\rangle\right\}, \\ \mathscr{T}^{\neg}{}_2 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.50{,}0.20\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.60{,}0.30\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.40{,}0.30\right\rangle\right\}, \\ \mathscr{T}^{\neg}{}_3 &= \left\{\left\langle \breve{\alpha}_1^{\gamma}, 0.40{,}0.50\right\rangle, \left\langle \breve{\alpha}_2^{\gamma}, 0.50{,}0.40\right\rangle, \left\langle \breve{\alpha}_3^{\gamma}, 0.20{,}0.4\right\rangle\right\}. \end{aligned} \tag{24}$$

Then, $\breve{\mathcal{E}}^{\tau} = \{1_X, 0_X, \mathscr{T}^{\neg}{}_1, \mathscr{T}^{\neg}{}_2, \mathscr{T}^{\neg}{}_3,\}$ is a q-ROFTS on X , and $(X, \breve{\mathcal{E}}^{\tau})$ q-ROF superconnected.

Proposition 39. *In a q-ROFTS $(X, \breve{\mathcal{E}}_X^{\tau})$, the following conditions are equivalent:*

(i) X is q-ROF superconnected

(ii) For each q-ROFOS $\mathscr{A}^{\zeta} \neq 0_x$ in X, we have $Cl(\mathscr{A}^{\zeta}) = 0_x$

(iii) For each q-ROFCS $\mathscr{A}^{\zeta} \neq 1_x$ in X, we have $Int(\mathscr{A}^{\zeta}) = 0_x$

(iv) There exist no q-ROFOS $\mathscr{A}^{\zeta}$ and $\mathscr{S}^{\varsigma}$ in X such that $\mathscr{A}^{\zeta} \neq 0X \neq \mathscr{S}^{\varsigma}$ and $\mathscr{A}^{\zeta} \subseteq \mathscr{S}^{\varsigma c}$

(v) There exist no q-ROFOS $\mathscr{A}^{\zeta}$ and $\mathscr{S}^{\varsigma}$ in X such that $\mathscr{A}^{\zeta} \neq 0_X \neq \mathscr{S}^{\varsigma}, \mathscr{S}^{\varsigma} = [Cl(\mathscr{A}^{\zeta})]^C, \mathscr{A}^{\zeta} = [Cl(\mathscr{S}^{\varsigma})]^c$

(vi) There exist no q-ROFCS $\mathscr{A}^{\zeta}$ and $\mathscr{S}^{\varsigma}$ in X such that $\mathscr{A}^{\zeta} \neq 1_x \neq \mathscr{S}^{\varsigma}$

$$\mathscr{S}^{\varsigma} = \left[Intl\left(\mathscr{A}^{\zeta}\right)\right]^c, \mathscr{A}^{\zeta} = [Int(\mathscr{S}^{\varsigma})]^c \tag{25}$$

Proof (i=>ii): assume that there exists a q-ROFOS $\mathscr{A}^{\zeta} \neq 0_X$ such that $\mathrm{Cl}(\mathscr{A}^{\zeta}) \neq 1_x$. Since $\mathscr{S}^{\varsigma} = \mathrm{Int}(Cl(\mathscr{A}^{\zeta}))$ is a q-ROFOS in X, and this is a contradiction with the q-ROF super connectedness of X.

(ii ⇒ iii): let $\mathscr{A}^{\zeta} \neq 1_x$ be a q-ROFCS in X. If we take $\mathscr{S}^{\varsigma} = \mathscr{A}^{\zeta^c}$, then $\mathscr{S}^{\varsigma} = \mathscr{A}^{\zeta^c}$, then $\mathscr{S}^{\varsigma}$ is a q-ROFOS in X and $\mathscr{S}^{\varsigma} \neq 0x$. Hence, $\mathrm{Cl}(\mathscr{S}^{\varsigma}) = 1_x => [\mathrm{Cl}(\mathscr{S}^{\varsigma})]^C = 0$.

(iii => iv): let $\mathscr{A}^{\zeta}$ and $\mathscr{S}^{\varsigma}$ be q-ROFSs in X such that $\mathscr{A}^{\zeta} \neq 0X \neq \mathscr{S}^{\varsigma} X \neq \mathscr{S}^{\varsigma}$ and $\mathscr{A}^{\zeta} \subseteq \mathscr{S}^{\varsigma c}$. Since $\mathscr{S}^{\varsigma c}$ is a q-ROFCS in X and $\mathscr{S}^{\varsigma} \neq 0_x => \mathscr{S}^{\varsigma c} \neq 1_x$, we obtain $\mathrm{Int}(\mathscr{S}^{\varsigma c}) = 0_x$. Now, we have $0_x \neq \mathscr{A}^{\zeta} = \mathrm{Int}(\mathscr{A}^{\zeta}) \subseteq \mathrm{Int}((\mathscr{S}^{\varsigma})^C) = 0_x$, which is a contradiction.

$(iv => v)$: obvious.

$(i => v)$: let $\mathscr{A}^{\zeta}$ and $\mathscr{S}^{\varsigma}$ be q-ROFOSs in X such that $\mathscr{A}^{\zeta} \neq 0x \neq \mathscr{S}^{\varsigma}$ and $\neq \mathscr{S}^{\varsigma}$ and $\mathscr{S}^{\varsigma} = [\mathrm{Cl}(\mathscr{A}^{\zeta})]^c, \mathscr{A}^{\zeta} = [\mathrm{Cl}(\mathscr{S}^{\varsigma})]^c$. No $(\mathrm{Cl}(\mathscr{A}^{\zeta})) = \mathrm{Int}(\mathscr{S}^{\varsigma C}) = [\mathrm{Cl}(\mathscr{S}^{\varsigma})]^c = \mathscr{A}^{\zeta}$ and $\mathscr{A}^{\zeta} \neq 0X, \mathscr{A}^{\zeta} \neq 1_X$. This is a contradiction.

$(v => i)$: obvious.

$(v => vi)$: let $\mathscr{A}^{\zeta}$ and $\mathscr{S}^{\varsigma}$ q-ROFCSs in X such that $\mathscr{A}^{\zeta} \neq 1x \neq \mathscr{S}^{\varsigma}$, $\mathscr{S}^{\varsigma} = [\mathrm{Int}(\mathscr{A}^{\zeta})]^C, \mathscr{A}^{\zeta} = [\mathrm{Int}(\mathscr{S}^{\varsigma})]^C$. Taking $\mathscr{U}^{\varsigma} =$

$\mathscr{A}^{\zeta C}$ and $\mathscr{V}^\tau = (\mathscr{S}^\varsigma)^C$, $\mathscr{U}^\varsigma$ and $\mathscr{V}^\tau$ become q-ROFOSs and $\mathscr{U}^\varsigma \neq 0_x \neq \mathscr{V}^\tau$, $[Cl(\mathscr{U}^\varsigma)]^c = \mathscr{V}^\tau$ and $[Cl(\mathscr{V}^\tau)]^c = \mathscr{U}^\varsigma$. This is a contradiction.

$(vi = > v)$: this is similar to $(v = > vi)$. □

Definition 40. Let $(X, \check{\mathcal{E}}^\tau_X)$ be a q-ROFTS.

(i) X is said to be q-ROFC_5-disconnected if there exists a q-ROFOS and q-ROFCS G such that $G \neq 1_X$ and $G \neq 0_X$

(ii) X is said to be q-ROF$\mathfrak{C}_5$-connected if it is not q-ROF$\mathfrak{C}_5$-disconnected

Example 4. Let $X = \{1, 2\}$ and define the Pythagorean fuzzy subsets $\mathscr{A}^\zeta, \mathscr{S}^\varsigma, C, D$ as follows.

Let $X = \{\breve{\alpha}^\gamma_1, \breve{\alpha}^\gamma_2, \breve{\alpha}^\gamma_3\}$. Consider the following family of q-ROF sets:

$$\begin{aligned}
\mathscr{T}^\urcorner{}_1 &= \left\{\left\langle \breve{\alpha}^\gamma_1, 0.40, 0.30\right\rangle, \left\langle \breve{\alpha}^\gamma_2, 0.20, 0.70\right\rangle\right\},\\
\mathscr{T}^\urcorner{}_2 &= \left\{\left\langle \breve{\alpha}^\gamma_1, 0.30, 0.40\right\rangle, \left\langle \breve{\alpha}^\gamma_2, 0.70, 0.20\right\rangle\right\},\\
\mathscr{T}^\urcorner{}_3 &= \left\{\left\langle \breve{\alpha}^\gamma_1, 0.30, 0.40\right\rangle, \left\langle \breve{\alpha}^\gamma_2, 0.20, 0.70\right\rangle\right\},\\
\mathscr{T}^\urcorner{}_4 &= \left\{\left\langle \breve{\alpha}^\gamma_1, 0.40, 0.30\right\rangle, \left\langle \breve{\alpha}^\gamma_2, 0.70, 0.20\right\rangle\right\}.
\end{aligned} \tag{26}$$

Then, family $\check{\mathcal{E}}^\tau = \{1_X, 0_X, \mathscr{T}^\urcorner{}_1, \mathscr{T}^\urcorner{}_2, \mathscr{T}^\urcorner{}_3, \mathscr{T}^\urcorner{}_4\}$ is a q-ROFTS on X and $(X, \check{\mathcal{E}}^\tau)$ is a q-ROF$\mathfrak{C}_5$-disconnected, since $\mathscr{A}^\zeta$ is a nonzero q-ROFOS and q-ROFCS in X.

Definition 41. Let $(X, \check{\mathcal{E}}^\tau_X)$ be a q-ROFTS,

(i) X is called q-ROF disconnected, if there exist q-ROFOSs $\mathscr{A}^\zeta \neq 0_X$ and $\mathscr{S}^\varsigma \neq 0_X$ such that $\mathscr{A}^\zeta \mathscr{U}^\varsigma \mathscr{S}^\varsigma = 1_X$ and $\mathscr{A}^\zeta \cap \mathscr{S}^\varsigma = 0_X$

(ii) X is called q-ROF connected, if X is not q-ROF disconnected

Proposition 42. *q-ROF $\mathfrak{C}_5$-connectedness implies q-ROF connectedness.*

Proposition 43. *Let $(X, \check{\mathcal{E}}^\tau_1), (Y, \check{\mathcal{E}}^\tau_2)$ be two q-ROFTSs and let $f : X \longrightarrow Y$ be a q-ROF continuous surjection. If $(X, \check{\mathcal{E}}^\tau_1)$ is q-ROF connected,, then so is $(Y, \check{\mathcal{E}}^\tau_2)$.*

Proof. On the contrary, suppose that $(Y, \check{\mathcal{E}}^\tau_2)$ is q-ROF disconnected. Then, there exist q-ROFOSs $\mathscr{A}^\zeta \neq 0_Y, \mathscr{S}^\varsigma \neq 0_Y$ in Y such that $\mathscr{A}^\zeta \mathscr{U}^\varsigma \mathscr{S}^\varsigma = 1_y, \mathscr{A}^\zeta \cap \mathscr{S}^\varsigma = 0_Y$. Now, we see that $\mathscr{U}^\varsigma = f^1(\mathscr{A}^\zeta), \mathscr{V}^\tau = f^1(\mathscr{S}^\varsigma)$ are q-ROFOSs in X, since f is q-ROF continuous, From $\mathscr{A}^\zeta \neq 0_Y$, we get $\mathscr{U}^\varsigma = f^1(\mathscr{A}^\zeta) \neq 0_X$. Similarly, $\mathscr{V}^\tau \neq 0_X$. Hence, $\mathscr{A}^\zeta \mathscr{U}^\varsigma \mathscr{S}^\varsigma = 1_y = > f^1(\mathscr{A}^\zeta) f^1(\mathscr{S}^\varsigma) = f^1(1_y) = 1_X = >$ $\mathscr{U}^\varsigma_u \mathscr{V}^\tau = 1_X$; $\mathscr{A}^\zeta \cap \mathscr{S}^\varsigma = 0_Y \Rightarrow f^1(\mathscr{A}^\zeta) \cap f^1(\mathscr{S}^\varsigma) = f^1(0_Y) = 0_X \Rightarrow \mathscr{U}^\varsigma \cap \cap \mathscr{S}^\varsigma = 0_Y \Rightarrow f^1(\mathscr{A}^\zeta) \cap f^1(\mathscr{S}^\varsigma) = f^1(0_Y) = 0_X \Rightarrow \mathscr{U}^\varsigma \cap$. □

Corollary 44. *Let $(X, \check{\mathcal{E}}^\tau_1), (Y, \check{\mathcal{E}}^\tau_2)$ be two q-ROFTSs and let $f : X \longrightarrow Y$ be a q-ROF continuous surjection. If $(X, \check{\mathcal{E}}^\tau_1)$ is q-ROF $\mathfrak{C}_5$-connected, then so is $(Y, \check{\mathcal{E}}^\tau_2)$.*

Definition 45. A q-ROFTS $(X, \check{\mathcal{E}}^\tau)$ is said to be q-ROF strongly connected, if there exists nonzero q-ROFCSs $\mathscr{A}^\zeta$ and $\mathscr{S}^\varsigma$ such that $\mu_{\mathscr{A}^\zeta} + \mu_{\mathscr{S}^\varsigma} \leq 1$ and $\vartheta_{\mathscr{A}^\zeta} + \vartheta_{\mathscr{S}^\varsigma} \geq 1$.

Proposition 46. *Let $(X, \check{\mathcal{E}}^\tau_1), (Y, \check{\mathcal{E}}^\tau_2)$ be two q-ROFTSs and let $f : X \longrightarrow Y$ be a q-ROF continuous surjection. If $(X, \check{\mathcal{E}}^\tau_1)$ is q-ROF strongly connected, then so is $(Y, \check{\mathcal{E}}^\tau_2)$.*

Proof. This is analogous to proof of Proposition 43. It is clear that in q-ROFTSs, q-ROF strongly connectedness does not imply q-ROF $\mathfrak{C}_5$-connectedness, and the same is true for its converse. □

7. q-ROF Weighted Product Model

The weighted product model (WPM) is a well-known and often used MCDM framework for analyzing a set of options in terms of a set of choice criteria. By multiplying a number of ratios, one for each choice criterion, each decision alternative is compared to the others. Each ratio is raised to the power of the related criterion's relative weight.

The fundamental task in general MCDM problems is to select one or even more options from a set of available alternatives based on numerous criteria. Assume that there are n alternatives and m criterion with the criteria weight vector under the constraInt, each component of WV is +ev and the sum of WV will be unit, for a specified MCDM problem in the q-ROF domain.

Step 1. Acquire the decision matrix from DMs, DMs represent the evaluation values of the alternative $\mathscr{L}^\daleth{}_j$ in terms of the criterion $\mathscr{C}^\urcorner_i$ by $T_{ji} = (\mu_{ji}, \nu_{ji})$. DMs give the decision matrix $R = (T_{ji})_{n\times m}$ given as

$$\begin{array}{c}
\begin{array}{cccc} \mathscr{C}^\urcorner_1 & \mathscr{C}^\urcorner_2 & & \mathscr{C}^\urcorner_m \end{array}\\
\begin{bmatrix}
(\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \cdots\cdots & (\mu_{1m}, \nu_{1m})\\
(\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \cdots\cdots & (\mu_{2m}, \nu_{2m})\\
\vdots & \vdots & \ddots\ddots & \vdots\\
(\mu_{n1}, \nu_{n1}) & (\mu_{n2}, \nu_{n2}) & \cdots\cdots & (\mu_{nm}, \nu_{nm})
\end{bmatrix}
\end{array},
\begin{array}{c} \mathscr{L}^\daleth{}_1,\\ \mathscr{L}^\daleth{}_2,\\ \vdots\\ \mathscr{L}^\daleth{}_n. \end{array} \tag{27}$$

In this matrix, each element $T_{ji} = (\mu_{ji}, \nu_{ji})$ is q-ROFNs. This means that if the option $\mathscr{L}^\daleth{}_j$ fulfills the criteria $\mathscr{C}^\urcorner_i$, the grade will be the value μ_{ji}, and if the alternative $\mathscr{L}^\daleth{}_j$ fails to impress the characteristics $\mathscr{C}_i^{\urcorner}$, the grade will be the quantity ν_{ji}.

Step 2. In this step, we normalize the matrix $R = (T_{ji})_{n\times m}$ with linear approach. We classified the criteria into two types, benefit type (ξ_B) and cost type (ξ_C). The "linear normalization" for any $i \in \xi_B$ is defined as follows:

$$\bar{T}_{ji} = T_{ji} \oslash \max_j T_{ji}, \tag{28}$$

where $\max_j T_{ji}$ is defined as $\max_j T_{ji} = (\max_j \mu_{ji}, \min_j \nu_{ji})$.

And for any $i \in \xi_B$ "linear normalization" is defined as

$$\bar{T}_{ji} = \frac{\min_j T_{ji}}{T_{ji}}, \tag{29}$$

where $\min_j T_{ji}$ is defined as $\min_j T_{ji} = (\min_j \mu_{ji}, \max_j \nu_{ji})$.

By the definition of SF, we can easily see that $\min_j T_{ji}$ and $\max_j T_{ji}$ satisfied the conditions of division operation. The decision matrix $R = (T_{ji})_{n\times m}$ is transformed into the normalized matrix $\bar{R} = (\bar{T}_{ji})_{n\times m}$ and given as

$$\begin{bmatrix} \mathscr{C}_1^{\urcorner} & \mathscr{C}_2^{\urcorner} & & \mathscr{C}_m^{\urcorner} \\ (\bar{\mu}_{11}, \bar{\nu}_{11}) & (\bar{\mu}_{12}, \bar{\nu}_{12}) & \cdots\cdots & (\bar{\mu}_{1m}, \bar{\nu}_{1m}) \\ (\bar{\mu}_{21}, \bar{\nu}_{21}) & (\bar{\mu}_{22}, \bar{\nu}_{22}) & \cdots\cdots & (\bar{\mu}_{2m}, \bar{\nu}_{2m}) \\ \vdots & \vdots & \ddots\ddots & \vdots \\ (\bar{\mu}_{n1}, \bar{\nu}_{n1}) & (\bar{\mu}_{n2}, \bar{\nu}_{n2}) & \cdots\cdots & (\bar{\mu}_{nm}, \bar{\nu}_{nm}) \end{bmatrix}, \tag{30}$$

$$\begin{matrix} \mathscr{L}^{\urcorner}{}_1 \\ \mathscr{L}^{\urcorner}{}_2 \\ \vdots \\ \mathscr{L}^{\urcorner}{}_n \end{matrix}$$

Step 3. According to q-ROF-WPM, the "total relative importance" of alternative j, denoted as K_j, and defined as

$$K_j = \prod_{i=1}^{m} \boxtimes \left(\bar{T}_{ji}\right)^{w_i}. \tag{31}$$

Here, we use definition of taking power of any q-ROFN and product operation of q-ROFNs, as each K_j is also q-ROFN.

Step 4. Find the score of all evaluated K_j, which is defined in previous step.

Step 5. Rank all the alternatives as per SFs of K_j.

The pictorial view of the proposed algorithm is given in Figure 1.

7.1. Application Related to Hierarchical Healthcare System Approach for COVID-19. The COVID-19 outbreak, which began in China and has spread rapidly throughout the world, has resulted in a rise in patient and casualty numbers. The countries have suffered huge losses not only in the medical industry but also in a range of other sectors as well. As a result, governments have been tasked with the responsibility of properly implementing a variety of remedies in their own jurisdictions. However, only a few countries benefit in part from the measures implemented, while others fail to benefit at all. In this context, it is vital to choose the most critical course of action that governments should take. To evaluate the various strategies employed by various authorities, a decision problem based on the preferences of several specialists using certain contradictory and disparate parameters should be evaluated. This decision procedure is viewed as an MCDM problem in this article, which also takes into account unpredictability. q-ROFSs are used to accomplish this goal, assisting decision-makers in evaluating across a greater space and dealing more effectively with competing knowledge.

COVID-19 and other diseases are a serious threat to public health in Pakistan. Individuals seek treatment in grade III, class A hospitals because the facilities are much better than those found in other nearby hospitals. As a result, major hospitals are frequently overcrowded, with patient capacity much surpassing capacity. On the other side, small hospitals and clinics squander healthcare resources. Additional challenges for Pakistan's medical system under such circumstances include understanding how to correctly distribute scarce healthcare resources and enhancing the inlet and outlet performances of the health care system. As depicted in Figure 2, the primary health facility is responsible for the individual's initial visit.

Implementing a hierarchical medical management program is regarded as a vital and effective technique for addressing the issue of insufficient and imbalanced healthcare assets, in which medical organizations of various sizes receive patients based on the severity and urgency of their diseases. In such a system, common ailments are treated at basic centers, with patients directed to more specialized facilities if their health necessitates it. Severe diseases should be treated in more advanced facilities. At the same time, when victims' conditions improve, higher-level institutions might transfer them to lower-level facilities. As a result, determining the severity of the illness is an important action in this approach. With the increasing number of sufferers with respiratory illnesses, adopting an appropriate technique to separate patients under diverse circumstances into multiple stages of institutions is an effective way to make full use of restricted healthcare resources and cure more COVID-19 and other epidemics.

The hierarchical medical system is introduced as an effective way to relieve the burden on the number of patients in Lahore Hospital, with the purpose of categorizing the difficulty of treatment based on the ailment. The numerous medical institutes' degrees can then take on the various forms of diseases. The essential challenge in the hierarchical medical system is how to define various degrees of sickness.

Patients with different conditions can use the hierarchical healthcare system to go to different levels of hospitals, rather than all patients flocking to grade III, class A institutions. In essence, categorizing the various degrees of disease is an important step in developing the hierarchical structure.

The specific statement about the medical diagnosis problem is described as follows.

If we consider there are five patient who have +ev COVID-19 test report, namely, $\mathscr{L}^{\gimel}{}_1$, $\mathscr{L}^{\gimel}{}_2$, $\mathscr{L}^{\gimel}{}_3$, $\mathscr{L}^{\gimel}{}_4$, $\mathscr{L}^{\gimel}{}_5$, and $\mathscr{L}^{\gimel}{}_6$. Doctors (in our proposed method, these are DMs) are to be appointed for evaluating the severe condition of the patients under the symptoms (criterion) given in Table 3.

Doctors can assess the patient's condition using the diagnostic parameters for the COVID-19. Patients might be assigned to multiple levels and kinds of institutions based on the severity and urgency of COVID-19. Patients with serious diseases should be hospitalized in grade III, class A hospitals, while those with milder symptoms should be treated in grade II facilities. Other frequent ailments are treatable at local hospitals.

7.1.1. MCDM Process

(i) Step 1

Acquire the decision matrix $R = (T_{ji})_{n\times m}$ from the DMs in the format of q-ROFNs given in Table 4.

(i) Step 2

In this step, we normalize the matrix $R = (T_{ji})_{n\times m}$ with linear approach. We classified the criteria into two types, benefit type (ξ_B) and cost type (ξ_C). Here, $\mathscr{C}_2^{\urcorner}$ and $\mathscr{C}_6^{\urcorner}$ belong to the ξ_C and other are in ξ_B. First, we find $\max_j T_{ji}$ and $\min_j T_{ji}$ and $\max_j T_{ji}$ and $\min_j T_{ji}$ are given in Table 5. After this, we find normalized matrix by using the Equations (1) and (2) given in Table 6.

(i) Steps 3 and 4

According to q-ROF-WPM, the "total relative importance" of alternatives (K_j) by using Equation (3) and their score values have been calculated, given in Table 7.

(i) Step 5

Rank all the alternatives as per SFs of K_j. Final ranking will be

$$\mathscr{L}^{\gimel}{}_2 \succ \mathscr{L}^{\gimel}{}_3 \succ \mathscr{L}^{\gimel}{}_4 \succ \mathscr{L}^{\gimel}{}_6 \succ \mathscr{L}^{\gimel}{}_5 \succ \mathscr{L}^{\gimel}{}_1. \quad (32)$$

As a result, patient $\mathscr{L}^{\gimel}{}_2$ and $\mathscr{L}^{\gimel}{}_3$ status is the most critical, and they must be hospitalized in a "grade III, class A medical center". Patient $\mathscr{L}^{\gimel}{}_4$ must be served in a local hospital in the meanwhile because his condition is not life-threatening. Patients $\mathscr{L}^{\gimel}{}_6$, $\mathscr{L}^{\gimel}{}_5$, and $\mathscr{L}^{\gimel}{}_1$ may be transferred to various types of institutions based on the capacity of the ward.

8. Conclusion

This paper introduces certain properties of q-ROF topology are significant results of q-ROF topological spaces. The novel concepts of q-ROF interior, q-ROF closure, and q-ROF boundary of a q-ROFS are defined and illustrated with the help of examples. The notions of q-ROF base, q-ROF subbase, q-ROF continuous mapping, q-ROF homeomorphism, q-ROF open mapping, and q-ROF closed mapping are introduced, as well as several essential proofs. Additionally, the unique idea of "q-rung orthopair fuzzy α-continuous mapping" is introduced and explored in relation to q-ROFTSs and "q-rung orthopair fuzzy connectedness." We examine multiple relationships between different types of "q-rung orthopair fuzzy connectedness." The WPM framework is a well-known and often used MCDM technique for analyzing a set of alternatives in terms of a set of selection criteria. By multiplying a number of ratios, one for each choice criterion, each decision alternative is compared to the others. Each ratio is multiplied by the corresponding criterion's relative weight. The WPM model was extended to q-ROFSs in this study, and it was applied to a COVID-19 application important to hierarchical healthcare system. Implementing a hierarchical medical management system is viewed as a vital and effective strategy for addressing the issue of limited and uneven healthcare resources.

Data Availability

The data used to support the findings of the study are included in the article.

Conflicts of Interest

The authors declare that they have no conflict of interest.

Acknowledgments

The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University, Abha, Saudi Arabia, for funding this work through research groups program under grant number RGP.2/211/43.

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