Hindawi
Journal of Function Spaces
Volume 2024, Article ID 9818595, 1 page
https://doi.org/10.1155/2024/9818595

Retraction

Retracted: Multiplicative Topological Properties on Degree Based for Fourth Type of Hex-Derived Networks

Journal of Function Spaces

Received 23 January 2024; Accepted 23 January 2024; Published 24 January 2024

This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] H. Ali, G. Dustigeer, Y.-M. Li, M. K. Shafiq, and P. Ali, "Multiplicative Topological Properties on Degree Based for Fourth Type of Hex-Derived Networks," *Journal of Function Spaces*, vol. 2022, Article ID 2376289, 10 pages, 2022.

Hindawi
Journal of Function Spaces
Volume 2022, Article ID 2376289, 10 pages
https://doi.org/10.1155/2022/2376289

Research Article

Multiplicative Topological Properties on Degree Based for Fourth Type of Hex-Derived Networks

Haidar Ali,[1] **Ghulam Dustigeer**,[2] **Yong-Min Li**,[3,4] **Muhammad Kashif Shafiq**,[5] **and Parvez Ali**[6]

[1]*Department of Mathematics, Riphah International University, Faisalabad, Pakistan*
[2]*Department of Mathematics and Statistics, University of Agriculture, Faisalabad, Pakistan*
[3]*Department of Mathematics, Huzhou University, Huzhou 313000, China*
[4]*Institute for Advanced Study Honoring Chen Jian Gong, Hangzhou Normal University, Hangzhou 311121, China*
[5]*Department of Mathematics, University of Management and Technology, Sialkot Campus, Pakistan*
[6]*Department of Mechanical Engineering, College of Engineering, Qassim University, Unaizah, Saudi Arabia*

Correspondence should be addressed to Yong-Min Li; ymlwww@zjhu.edu.cn

Received 21 January 2022; Accepted 11 April 2022; Published 10 May 2022

Academic Editor: Hanan Alolaiyan

Chemical graph theory is a subfield of graph theory that uses a molecular graph to describe a chemical compound. When there is at least one connection between the vertices of a graph, it is said to be connected. Topology of graph has been expressed by numerical quantity which is known as topological index. Cheminformatics is a product field that combines chemistry, mathematics, and computer science. The graph plays a key role in modelling and coming up with any chemical arrangement. In this paper, we computed the multiplicative degree-based indices like Randić, Zagreb, Harmonic, augmented Zagreb, atom-bond connectivity, and geometric-arithmetic indices for newly developed fourth type of hex-derived networks and also present the graphical representations of results.

1. Introduction

Graph theory has provided chemists with a number of useful methods, such as topological indices. A molecular graph is often used to represent molecules and molecular compounds. A molecular graph is a graph theory definition of a compound's molecular formula, the vertices that correspond to the compound's atoms, and hence the edges that correspond to chemical bonds. Cheminformatics may be a trendy subject that would be a mix of chemistry, arithmetic, and data science. Topological indices, given by graph theory, square measure a vital tool. The topology index could be a quantity associated with a graph that unambiguously characterizes that graph. The chemical graph theory could be a combination of chemistry and graph theory. It is the mathematical chemistry branch that applies the graph hypothesis for modelling chemical structure. A graph will acknowledge a network, a meeting of numbers, a numeric variety, and a polynomial that speaks to the structure of that chart. The vertices and the edges of any chart moreover speak to the topological records. Cheminformatics is a modern scholarly field that brings together the fields of chemistry, mathematics, and information science. It examines the relationships between QSAR and QSPR, which are used to estimate biological activities and chemical compound properties. Wiener is the pioneer of TIs; he developed this theory in 1947, when he was working on the boiling points of Paraffins. Wiener called it a path number but afterwards; it is introduced by Wiener [1]. It is the first distance-based topological index. Topological indices are valuable in QSAR and QSPR studies, as they can alter the chemical structure into numerical values. More than 100 topological descriptors evaluated to get the connection between the atoms. There are a few strategies for evaluating atomic structures, the topological index of which is the most common since it can be derived specifically from atomic structures and measured effectively for a huge number of atoms.

1.1. Method for Drawing HDN4 Networks

Step 1. Let take a benzene network with dimension r.

Step 2. Place benzene graph in each K_3 subgraph of hexagonal network.

Step 3. Connect alternating vertices of each benzene graph to every corner of triangle.

Step 4. At the end, the derived result of the graph is called the fourth type of hex-derived network HDN4 (see Figure 1). In this way, we can also construct THDN4 (see Figure 2) and RHDN4 (see Figure 3).
On the newly developed graphs, the degree-based TIs have been calculated in this paper. First of all, Randić index computes on the fourth type of hex-derived network.

Let Y be e simple graph. The general form of the Randić index $R_\gamma(Y)$, where $\gamma \in \mathbb{R}$ is the sum of $(\kappa(\acute{L})\kappa(\acute{M}))^\gamma$ over all edges $e = \acute{c}\acute{d} \in E(Y)$, defined as follows:

$$R_\gamma(Y) = \sum_{\acute{c}\acute{d} \in E(Y)} \left(\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)\right)^\gamma \text{for } \gamma = 1, \frac{1}{2}, -1, -\frac{1}{2}. \tag{1}$$

The Zagreb index, denoted by $M_1(Y)$, introduced by Gutman and Das [2] was familiar, and mathematically, it can be written as follows:

$$M_1(Y) = \sum_{\acute{c}\acute{d} \in E(Y)} \left(\kappa\left(\acute{L}\right) + \kappa\left(\acute{M}\right)\right). \tag{2}$$

Zhong [3] calculated the harmonic index, and it can be written as follows:

$$H(Y) = \sum_{\acute{c}\acute{d} \in E(Y)} \left(\frac{2}{\kappa\left(\acute{L}\right) + \kappa\left(\acute{M}\right)}\right). \tag{3}$$

Furtula et al. [4] calculated the augmented Zagreb index, and this index is defined as follows:

$$\mathrm{AZI}(Y) = \sum_{\acute{c}\acute{d} \in E(Y)} \left(\frac{\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)}{\kappa\left(\acute{L}\right) + \kappa\left(\acute{M}\right) - 2}\right)^3. \tag{4}$$

The mathematical form of ABC-index has been computed by Estrada et al. [5] and defined as follows:

$$\mathrm{ABC}(Y) = \sum_{\acute{c}\acute{d} \in E(Y)} \sqrt{\frac{\kappa\left(\acute{L}\right) + \kappa\left(\acute{M}\right) - 2}{\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)}}. \tag{5}$$

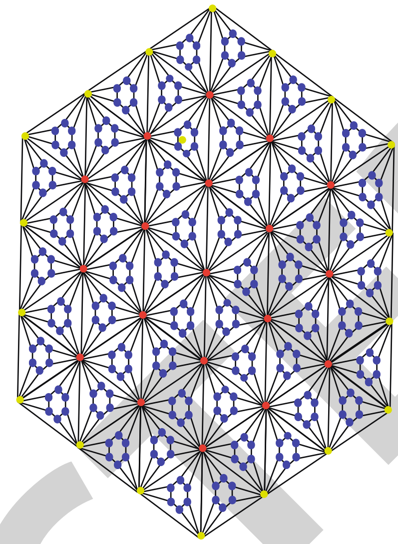

Figure 1: Fourth type of the hex-derived network (HDN4 (4)).

The mathematical form of GA-index has been developed by Vukičević and Furtula [6] and defined as follows:

$$\mathrm{GA}(Y) = \sum_{\acute{c}\acute{d} \in E(Y)} \frac{2\sqrt{\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)}}{\left(\kappa\left(\acute{L}\right) + \kappa\left(\acute{M}\right)\right)}. \tag{6}$$

2. Main Results

In this research article, we introduced the different kinds of fourth type of hex-derived networks. The degree-based topological indices have been calculated in this research on the above networks. Currently, there is exhaustive study on the topological indices and with different kinds, see [7–9].For the basic definitions about graph theory and notations, see [10, 11].

2.1. Results for Fourth Type of the Hex-Derived Network HDN4(r). For the first time, we compute the exact result on the above mentioned indices in Section 1 for newly developed graphs in this section.

Theorem 1. *Consider the fourth type of hex-derived network HDN4(r), the general form of the Randić index is equal to:*

$$R_\gamma(HDN4(r)) = \begin{cases} 11010 + r(-14610 + 5184r), & \gamma = 1, \\ 801 + r(-1262 + 535r), & \gamma = \frac{1}{2}, \\ \frac{169}{36}r^2 - \frac{43235}{5292}r + \frac{8195}{2646}, & \gamma = -1, \\ 12 - 31r + 17r^2, & \gamma = -\frac{1}{2}. \end{cases} \tag{7}$$

Proof. The $Y_1 \cong \mathrm{HDN4}(r)$ is shown in Figure 1, where

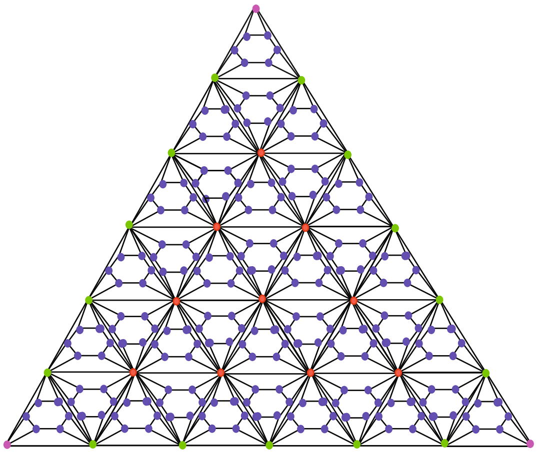

FIGURE 2: Fourth type of the triangular hex-derived network (THDN4 (7)).

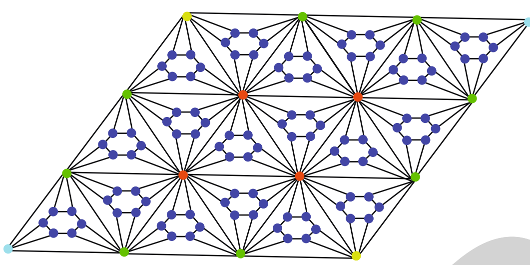

FIGURE 3: Fourth type of rectangular hex-derived network (RHDN4 (4,4)).

condition $r \geq 4$. Thus, by using equation (1), it follows that

$$R_\gamma(Y_1) = \sum_{\acute{c}\acute{d}\in E(Y_1)} \left(\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)\right)^\gamma. \quad (8)$$

□

For $\gamma = 1$, the general form of the Randić index $R_\gamma(Y_1)$ has been computed as follows:

$$R_1(Y_1) = \sum_{k=1}^{6} \sum_{\acute{c}\acute{d}\in E_k(Y_1)} \left(\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)\right). \quad (9)$$

Using Table 1, we get

$$R_1(Y_1) = 9|E_1(\mho)| + 21|E_2(Y_1)| + 54|E_3(Y_1)| + 49|E_4(Y_1)| + 126|E_5(Y_1)| + 324|E_6(Y_1)|, \Rightarrow R_1(Y_1) = 11010 + r(-14610 + 5184r)). \quad (10)$$

For$\gamma = 1/2$, we apply the formula of$R_\gamma(Y_1)$:

$$R_{1/2}(Y_1) = \sum_{k=1}^{6} \sum_{\acute{c}\acute{d}\in E_k(Y_1)} \sqrt{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)⊠}. \quad (11)$$

TABLE 1: Degree-based edge partition of HDN4(r) of the end vertices on each edge.

$\left(\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)\right)$ where $\acute{c}\acute{d}\in E(Y_1)$	Number of edges	$\left(\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)\right)$ where $\acute{c}\acute{d}\in E(Y_1)$	Number of edges
(3, 3)	$36r^2 - 72r + 36$	(7, 18)	$12m - 18$
(3, 7)	$36r - 48$	(18, 18)	$9r^2 - 33r + 30$
(3, 18)	$36r^2 - 108r + 84$		
(7, 7)	$6r - 6$		

Using Table 1, we get

$$R_1(Y_1) = 9|E_1(\mho)| + 21|E_2(Y_1)| + 54|E_3(Y_1)| + 49|E_4(Y_1)| + 126|E_5(Y_1)| + 324|E_6(Y_1)|, \Rightarrow R_1(Y_1) = 11010 + r(-14610 + 5184r)). \quad (12)$$

For$\gamma = -1$, by using formula of$R_\gamma(Y_1)$:

$$R_{-1}(Y_1) = \sum_{k=1}^{6} \sum_{\acute{c}\acute{d}\in E_k(Y_1)} \frac{1}{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)}, R_{-1}Y_1 = \frac{1}{9}|E_1(Y_1)| + \frac{1}{21}|E_2(Y_1)| + \frac{1}{54}|E_3(Y_1)| + \frac{1}{49}|E_4(Y_1)| + \frac{1}{126}|E_5(Y_1)| + \frac{1}{324}|E_6(Y_1)|, \Rightarrow R_{-1}(Y_1) = \frac{169}{36}r^2 - \frac{43235}{5292}r + \frac{8195}{2646}. \quad (13)$$

For$\gamma = -1/2$, we apply the formula of$R_\gamma(Y_1)$:

$$R_{-1/2}(Y_1) = \sum_{k=1}^{6} \sum_{\acute{c}\acute{d}\in E_k(Y_1)} \frac{1}{\sqrt{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)}}, R_{-1/2}(Y_1) = \frac{1}{3}|E_1(Y_1)| + \frac{1}{\sqrt{21}}|E_2(Y_1)| + \frac{1}{3\sqrt{6}}|E_3(Y_1)| + \frac{1}{7}|E_4(Y_1)| + \frac{1}{3\sqrt{14}}|E_5(Y_1)| + \frac{1}{18}|E_6(Y_1)|, \Rightarrow R_{-1/2}(Y_1) = 12 - 31r + 17r^2. \quad (14)$$

Theorem 2. *For fourth type of hex-derived network Y_1, the first form of the Zagreb index is equal to:*

$$M_1(Y_1) = 2\left(1023 - 1572r + 648r^2\right). \quad (15)$$

Proof. Let $Y_1 \cong$HDN4(r) be the hex-derived network. The below is the result of using the Table 1. Equation (2) can

Table 2: Edge partition of a THDN4(r) based on degrees of end vertices of each edge.

$(\kappa(\acute{L})\kappa(\acute{M}))$ where $\acute{c}\acute{d} \in E(Y_2)$	Number of edges	$(\kappa(\acute{L})\kappa(\acute{M}))$ where $\acute{c}\acute{d} \in E(Y_2)$	Number of edges
$(3,3)$	$6r^2-12r+6$	$(4,10)$	6
$(3,4)$	6	$(10,10)$	$3(r-2)$
$(3,10)$	$9(2r-4)$	$(10,18)$	$6(m-3)$
$(3,18)$	$2(3r^2-15r+18)$	$(18,18)$	$\frac{3(r^2-7r+12)}{2}$

Table 3: The edge partition of fourth type of rectangular hex-derived network.

$(\kappa(\acute{L})\kappa(\acute{M}))$ where $\acute{c}\acute{d} \in E(Y_3)$	Number of edges	$(\kappa(\acute{L})\kappa(\acute{M}))$ where $\acute{c}\acute{d} \in E(Y_3)$	Number of edges
$(3,3)$	$12r^2-24r+12$	$(7,10)$	4
$(3,4)$	4	$(7,18)$	2
$(3,7)$	8	$(10,10)$	$2(2r-5)$
$(3,10)$	$6(4r-8)$	$(10,18)$	$4(2r-5)$
$(3,18)$	$6(2r^2-8r+8)$	$(18,18)$	$3r^2-16r+21$
$(4,10)$	4		

be used to calculate the Zagreb index as follows:

$$\begin{aligned} M_1(Y_1) &= \sum_{\acute{c}\acute{d}\in E(Y_1)} \left(\kappa(\acute{L}) + \kappa(\acute{M})\right) \\ &= \sum_{k=1}^{6} \sum_{\acute{c}\acute{d}\in E_k(Y_1)} \left(\kappa(\acute{L}) + \kappa(\acute{M})\right), M_1(Y_1) \\ &= 6|E_1(Y_1)| + 10|E_2(Y_1)| + 21|E_3(Y_1)| + 14|E_4(Y_1)| \\ &\quad + 25|E_5(Y_1)| + 36|E_6(Y_1)|. \end{aligned} \tag{16}$$

By doing some calculations, we get:

$$\Rightarrow M_1(Y_1) = 2\left(1023 - 1572r + 648r^2\right). \tag{17}$$

The H-index, AZI-index, ABC-index, and GA-index have been computed for the fourth type of hex-derived network Y_1. □

Theorem 3. *Let Y_1 be the fourth form of the hex-derived network, then:*

(i) $H(Y_1) = (223/14)r^2 - (28961/1050)r + 5507/525$

(ii) $AZI(Y_1) = (4865917013307/539172272)r^2 - (35453987024262256/1230020443767)r + 88794905734397152/3690061331301$

(iii) $ABC(Y_1) = 8r^2 - 92r + 43$

(iv) $GA(Y_1) = 70r^2 - 131r + 59$

Proof. Using Table 1 and equation (3) to calculate the Harmonic index.

$$\begin{aligned} H(Y_1) &= \sum_{\acute{c}\acute{d}\in E(Y_1)} \left(\frac{2}{\kappa(\acute{L}) + \kappa(\acute{M})}\right) \\ &= \sum_{k=1}^{6} \sum_{\acute{c}\acute{d}\in E_k(Y_1)} \left(\frac{2}{\kappa(\acute{L}) + \kappa(\acute{M})}\right), H(Y_1) \\ &= \frac{1}{3}|E_1(Y_1)| + \frac{1}{5}|E_2(Y_1)| + \frac{2}{21}|E_3(Y_1)| + \frac{1}{7}|E_4(Y_1)| \\ &\quad + \frac{2}{25}|E_5(Y_1)| + \frac{1}{18}|E_6(Y_1)|. \end{aligned} \tag{18}$$

By doing some calculations, we get

$$\Rightarrow H(Y_1) = \frac{223}{14}r^2 - \frac{28961}{1050}r + \frac{5507}{525}. \tag{19}$$

By using equation (4) to calculate the augmented Zagreb index is equal to:

$$\begin{aligned} \mathrm{AZI}(Y_1) &= \sum_{\acute{c}\acute{d}\in E(Y_1)} \left(\frac{\kappa(\acute{L})\cdot\kappa(\acute{M})}{\kappa(\acute{L}) + \kappa(\acute{M}) - 2}\right)^3 \\ &= \sum_{k=1}^{6} \sum_{\acute{c}\acute{d}\in E_k(Y_1)} \left(\frac{\kappa(\acute{L})\cdot\kappa(\acute{M})}{\kappa(\acute{L}) + \kappa(\acute{M}) - 2}\right)^3, \mathrm{AZI}(Y_1) \\ &= \frac{729}{64}|E_1(Y_1)| + \frac{9261}{512}|E_2(Y_1)| + \frac{157464}{6859}|E_3(Y_1)| \\ &\quad + \frac{117649}{1728}|E_4(Y_1)| + \frac{2000376}{12167}|E_5(Y_1)| \\ &\quad + \frac{34012224}{39304}|E_6(Y_1)|. \end{aligned} \tag{20}$$

By doing some calculations, we get

$$\begin{aligned} \Rightarrow \mathrm{AZI}(Y_1) &= \frac{4865917013307}{539172272}r^2 - \frac{35453987024262256}{1230020443767}r \\ &\quad + \frac{88794905734397152}{3690061331301}. \end{aligned} \tag{21}$$

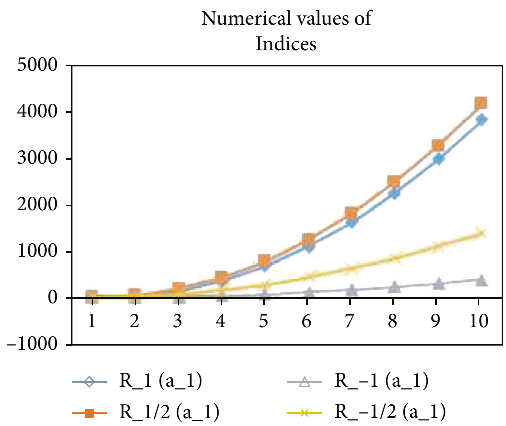

Figure 4: Comparison of indices for HDN4(4).

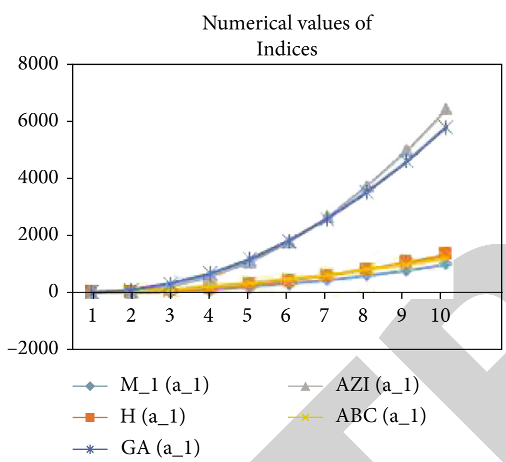

Figure 5: Comparison of indices for HDN4(4).

By using equation (5) to calculate the ABC-index:

$$\begin{aligned}\mathrm{ABC}(Y_1) &= \sum_{\acute{c}\acute{d}\in E(Y_1)} \sqrt{\frac{\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)-2}{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)}} \\ &= \sum_{k=1}^{6}\sum_{\acute{c}\acute{d}\in E_k(Y_1)} \sqrt{\frac{\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)-2}{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)}}, \mathrm{ABC}(Y_1) \\ &= \frac{2}{3}|E_1(Y_1)| + \frac{2\sqrt{42}}{21}|E_2(Y_1)| + \frac{\sqrt{114}}{18}|E_3(Y_1)| \\ &\quad + \frac{2\sqrt{3}}{7}|E_4(Y_1)| + \frac{\sqrt{322}}{42}|E_5(Y_1)| + \frac{\sqrt{34}}{18}|E_6(Y_1)|.\end{aligned} \tag{22}$$

By doing some calculations, we get

$$\Rightarrow \mathrm{ABC}(Y_1) = 48r^2 - 92r + 43. \tag{23}$$

By using equation (6) to calculate the geometric arithmetic index

$$\mathrm{GA}(Y_1) = \sum_{\acute{c}\acute{d}\in E(Y_1)} \frac{2\sqrt{\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)}}{\left(\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)\right)} = \sum_{k=1}^{6}\sum_{\acute{c}\acute{d}\in E_k(Y_1)} \frac{2\sqrt{\kappa\left(\acute{L}\right)\kappa\left(\acute{M}\right)}}{\left(\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)\right)}. \tag{24}$$

By doing some calculations, we get

$$\begin{aligned}\mathrm{GA}(Y_1) &= |E_1(Y_1)| + \frac{\sqrt{21}}{5}|E_2(Y_1)| + \frac{2\sqrt{6}}{7}|E_3(Y_1)| \\ &\quad + |E_4(Y_1)| + \frac{6\sqrt{14}}{25}|E_5(Y_1)| + |E_6(Y_1)|, \\ &\Rightarrow \mathrm{GA}(Y_1) = 70r^2 - 131r + 59.\end{aligned} \tag{25}$$

□

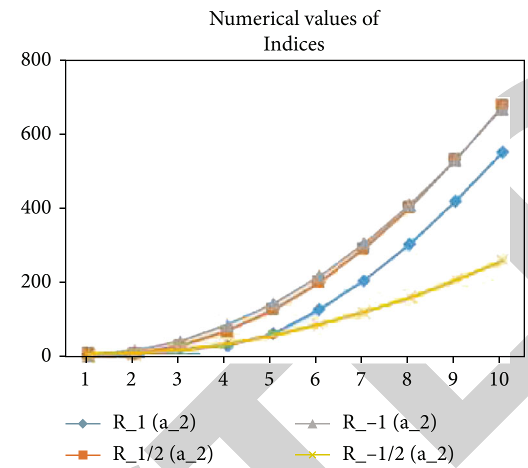

Figure 6: Comparison of indices for THDN4(7).

2.2. Results for Fourth Type of Triangular Hex-Derived Network THDN4(r). The degree-based TIs have been computed for the fourth form of the triangular hex-derived network in this portion. We calculate general form of Randić index R_γ with $\gamma = \{1, -1, 1/2, -1/2\}$, M_1-index, H-index, AZI-index, ABC-index, and GA-index in the coming theorems.

Theorem 4. *Consider the THDN4(r), then general form of the Randić index is equal to:*

$$R_\gamma(THDN4(r)) = \begin{cases} 6\left(144r^2 - 535r + 53\right), & \gamma = 1, \\ 167 - r(236 - 89r), & \gamma = \dfrac{1}{2}, \\ \dfrac{169}{216}r^2 - \dfrac{6793}{5400}r + \dfrac{611}{900}, & \gamma = -1, \\ \dfrac{299}{100}r^2 - \dfrac{46}{10}r + 2, & \gamma = -\dfrac{1}{2}. \end{cases} \tag{26}$$

Proof. Let $Y_2 \cong (\mathrm{THDN4}(r))$, using Table 2 and equation

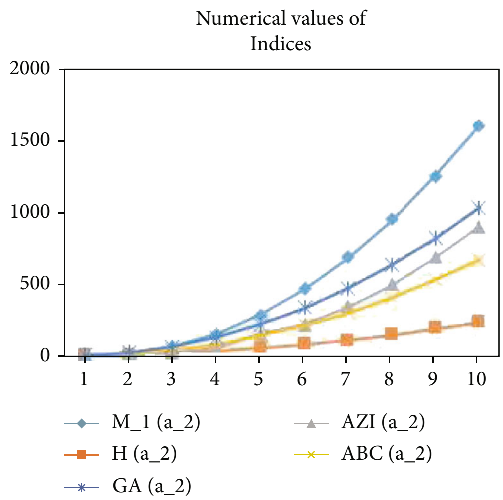

Figure 7: Comparison of indices for THDN4(7).

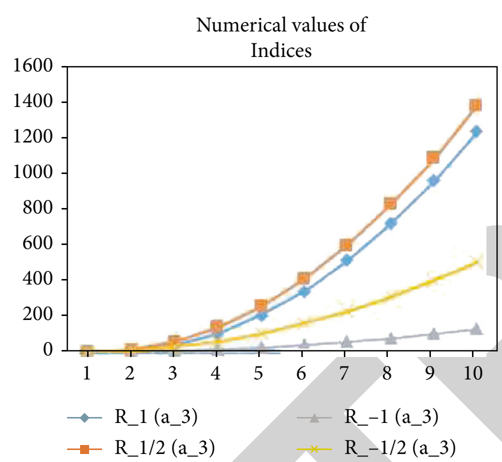

Figure 8: Comparison of indices for RHDN4(4, 4).

(1), we have

$$R_\gamma(Y_2) = \sum_{\acute{c}\acute{d} \in E(Y_2)} \left(\kappa(\acute{L})\kappa(\acute{M})\right)^\gamma. \tag{27}$$

□

For $\gamma = 1$, the general form of the Randić index $R_\gamma(Y_2)$ can be computed as follows:

$$R_1(Y_2) = \sum_{k=1}^{8} \sum_{\acute{c}\acute{d} \in E_k(Y_2)} \left(\kappa(\acute{L}) \cdot \kappa(\acute{L})\right). \tag{28}$$

Using Table 2, we get

$$\begin{aligned} R_1(Y_2) &= 9|E_1(Y_2)| + 12|E_2(Y_2)| + 30|E_3(Y_2)| + 54|E_4(Y_2)| \\ &\quad + 40|E_5(Y_2)| + 100|E_6(Y_2)| + 180|E_7(Y_2)| \\ &\quad + 324|E_8(Y_2)|, \Rightarrow R_1(Y_2) = 6\left(144r^2 - 535r + 537\right). \end{aligned} \tag{29}$$

For $\gamma = 1/2$, we apply the formula of $R_\gamma(Y_2)$:

$$R_{1/2}(Y_2) = \sum_{k=1}^{8} \sum_{\acute{c}\acute{d} \in E_k(Y_2)} \sqrt{\kappa(\acute{L}) \cdot \kappa(\acute{M})}. \tag{30}$$

Using Table 2, we get

$$\begin{aligned} R_{1/2}(Y_2) &= 3|E_1(Y_2)| + 2\sqrt{3}|E_2(Y_2)| + \sqrt{30}|E_3(Y_2)| \\ &\quad + 3\sqrt{6}|E_4(Y_2)| + 2\sqrt{10}|E_5(Y_2)| + 10|E_6(Y_2)| \\ &\quad + 6\sqrt{5}|E_7(Y_2)| + 18|E_8(Y_2)|, \Rightarrow R_{1/2}(Y_2) = 89r^2 \\ &\quad - 236r + 167. \end{aligned} \tag{31}$$

For $\gamma = -1$, by using the formula of $R_\gamma(Y_2)$:

$$\begin{aligned} R_{-1}(Y_2) &= \sum_{k=1}^{8} \sum_{\acute{c}\acute{d} \in E_k(Y_2)} \frac{1}{\kappa(\acute{L}) \cdot \kappa(\acute{M})}, R_{-1}(Y_2) = \frac{1}{9}|E_1(Y_2)| \\ &\quad + \frac{1}{12}|E_2(Y_2)| + \frac{1}{30}|E_3(Y_2)| + \frac{1}{54}|E_4(Y_2)| \\ &\quad + \frac{1}{40}|E_5(Y_2)| + \frac{1}{100}|E_6(Y_2)| + \frac{1}{180}|E_7(Y_2)| \\ &\quad + \frac{1}{324}|E_8(Y_2)|, \Rightarrow R_{-1}(Y_2) = \frac{169}{216}r^2 - \frac{6793}{5400}r + \frac{611}{900}. \end{aligned} \tag{32}$$

For $\gamma = -1/2$, by using the formula of $R_\gamma(Y_2)$:

$$\begin{aligned} R_{-1/2}(Y_2) &= \sum_{k=1}^{8} \sum_{\acute{c}\acute{d} \in E_k(Y_2)} \frac{1}{\sqrt{\kappa(\acute{L}) \cdot \kappa(\acute{M})}}, R_{-1/2}(Y_2) \\ &= \frac{1}{3}|E_1(Y_2)Y_1| + \frac{1}{\sqrt{12}}|E_2(Y_2)| + \frac{1}{\sqrt{30}}|E_3(Y_2)| \\ &\quad + \frac{1}{\sqrt{54}}|E_4(Y_2)| + \frac{1}{\sqrt{40}}|E_5(Y_2)| + \frac{1}{10}|E_6(Y_2)| \\ &\quad + \frac{1}{\sqrt{180}}|E_7(Y_2)| + \frac{1}{18}|E_8(Y_2)|, \Rightarrow R_{-1/2}(Y_2) \\ &= \frac{299}{100}r^2 - \frac{46}{10}r + 2. \end{aligned} \tag{33}$$

Theorem 5. *The first form of the Zagreb index for fourth type of triangular hex-derived network Y_2 is equivalent to:*

$$M_1(Y_2) = 6\left(36r^2 - 103r + 79\right). \tag{34}$$

Proof. Let $Y_2 \cong$ THDN3(r). Using Table 2 and equation (2),

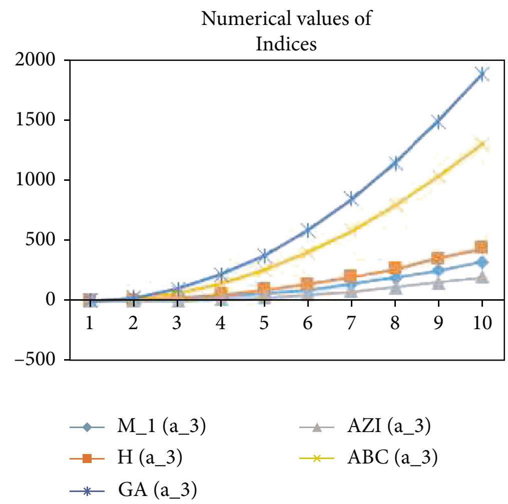

Figure 9: Comparison of indices for RHDN4(4, 4).

the first Zagreb index is equal to

$$
\begin{aligned}
M_1(Y_2) &= \sum_{\acute{c}\acute{d}\in E(Y_2)} \left(\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)\right) \\
&= \sum_{k=1}^{8}\sum_{\acute{c}\acute{d}\in E_k(Y_2)} \left(\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)\right), M_1(Y_2) \\
&= 6|E_1(Y_2)|+7|E_2(Y_2)|+13|E_3(Y_2)|+21|E_4(Y_2)| \\
&\quad +14|E_5(Y_2)|+20|E_6(Y_2)|+28|E_5(Y_2)|+36|E_8(Y_2)|.
\end{aligned} \tag{35}
$$

By doing some calculations, we get:

$$
\Rightarrow M_1(Y_2)=6\left(36r^2-103r+79\right). \tag{36}
$$

□

Theorem 6. *Let Y_2 be the THDN4, then:*

(i) $H(Y_2)=(223/84)r^2-(21527/5460)r+717/455$

(ii) $AZI(Y_2)=(1203267/800)r^2-(2816367/400)r+325590119/36000$

(iii) $ABC(Y_2)=8r^2-15r+7$

(iv) $GA(Y_2)=12r^2-19r+7$

Proof. Using Table 2 to calculate the Harmonic index and using equation (3):

$$
\begin{aligned}
H(Y_2) &= \sum_{\acute{c}\acute{d}\in E(Y_2)} \left(\frac{2}{\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)}\right) \\
&= \sum_{k=1}^{8}\sum_{\acute{c}\acute{d}\in E_k(Y_2)} \left(\frac{2}{\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)}\right), H(Y_2)=\frac{1}{3}|E_1(Y_2)| \\
&\quad +\frac{2}{7}|E_2(Y_2)|+\frac{2}{13}|E_3(Y_2)|+\frac{2}{21}|E_4(Y_2)| \\
&\quad +\frac{1}{7}|E_5(Y_2)|+\frac{1}{10}|E_6(Y_2)|+\frac{1}{14}|E_7(Y_2)|+\frac{1}{18}|E_8(Y_2)|.
\end{aligned} \tag{37}
$$

By doing some calculations, we get

$$
\Rightarrow H(Y_2)=\frac{223}{84}r^2-\frac{21527}{5460}r+\frac{717}{455}. \tag{38}
$$

Using equation (5) to calculate the augmented Zagreb index.

$$
\begin{aligned}
\mathrm{AZI}(Y_2) &= \sum_{\acute{c}\acute{d}\in E(Y_2)} \left(\frac{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)}{\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)-2}\right)^3 \\
&= \sum_{k=1}^{8}\sum_{\acute{c}\acute{d}\in E_k(Y_2)} \left(\frac{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)}{\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)-2}\right)^3, \mathrm{AZI}(Y_2) \\
&= \frac{729}{64}|E_1(Y_2)|+\frac{1728}{125}|E_2(Y_2)|+\frac{2029}{100}|E_3(Y_2)| \\
&\quad +\frac{2295}{100}|E_4(Y_2)|+\frac{1000}{27}|E_5(Y_2)|+\frac{17147}{100}|E_6(Y_2)| \\
&\quad +\frac{33182}{100}|E_7(Y_2)|+\frac{86536}{100}|E_8(Y_2)|.
\end{aligned} \tag{39}
$$

By doing some calculations, we get

$$
\Rightarrow \mathrm{AZI}(Y_2)=\frac{1203267}{800}r^2-\frac{2816367}{400}r+\frac{325590119}{36000}. \tag{40}
$$

By using equation (6) to calculate the atom bond connectivity index

$$
\begin{aligned}
\mathrm{ABC}(Y_2) &= \sum_{\acute{c}\acute{d}\in E(Y_2)} \sqrt{\frac{\kappa\left(\acute{L}\right)+\kappa\left(\acute{M}\right)-2}{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right)}} \\
&= \sum_{k=1}^{8}\sum_{\acute{c}\acute{d}\in E_k(Y_2)} \sqrt{\kappa\left(\acute{L}\right)+\kappa\left(\frac{\acute{M})-2}{\kappa\left(\acute{L}\right)\cdot\kappa\left(\acute{M}\right), \mathrm{ABC}(Y_2)=\frac{2}{3}|E_1(Y_2)|+\frac{\sqrt{15}}{6}|E_2(Y_2)|+\frac{\sqrt{330}}{30}|E_3(Y_2)|+\frac{\sqrt{114}}{18}|E_4(Y_2)|+\frac{\sqrt{30}}{10}|E_5(Y_2)|+\frac{3\sqrt{2}}{10}|E_6(Y_2)|+\frac{\sqrt{130}}{30}|E_7(Y_2)|+\frac{\sqrt{34}}{18}|E_8(Y_2)|.}\right.}
\end{aligned} \tag{41}
$$

By doing some calculations, we get

$$\Rightarrow \mathrm{ABC}(Y_2) = 8r^2 - 15r + 7. \quad (42)$$

By using equation (7) to calculate the geometric arithmetic index:

$$\mathrm{GA}(Y_2) = \sum_{\acute{c}\acute{d}\in E(Y_2)} \frac{2\sqrt{\kappa(\acute{L})\kappa(\acute{M})}}{\left(\kappa(\acute{L}) + \kappa(\acute{M})\right)} = \sum_{k=1}^{8} \sum_{\acute{c}\acute{d}\in E_k(Y_2)} \frac{2\sqrt{\kappa(\acute{L})\kappa(\acute{M})}}{\left(\kappa(\acute{L}) + \kappa(\acute{M})\right)}. \quad (43)$$

By doing some calculations, we get

$$\begin{aligned}\mathrm{GA}(Y_2) &= |E_1(Y_2)| + \frac{4\sqrt{3}}{7}|E_2(Y_2)| + \frac{2\sqrt{30}}{13}|E_3(Y_2)| \\ &+ \frac{2\sqrt{6}}{7}|E_4(Y_2)| + \frac{2\sqrt{10}}{7}|E_5(Y_2)| + |E_6(Y_2)| \\ &+ \frac{3\sqrt{5}}{7}|E_7(Y_2)| + |E_8(Y_2)|, \Rightarrow \mathrm{GA}(Y_2) \\ &= 12r^2 - 19r + 7.\end{aligned} \quad (44)$$

□

2.3. Result for Fourth Type of Rectangular Hex-Derived Network RHDN4(r, s). Some topological indices which based on the degree of the RHDN4(r, s) with condition $r = s$ have been computed in this portion. We evaluate the general form Randić index $R_\gamma(\mathrm{RHDN4}(r))$ for the $\gamma = \{1, -1, 1/2, -1/2\}$, M_1-index, H-index, AZI-index, ABC-index, and GA-index in the forward theorems of RHDN4(r, s).

Theorem 7. *For the fourth type of rectangular hex-derived network RHDN4*(r)*, the general Randić index is equal to*

$$R_\gamma(RHDN4(r)) = \begin{cases} 4\left(432r^2 - 1358r + 1093\right), & \gamma = 1, \\ 178r^2 - 432r + 267, & \gamma = \frac{1}{2}, \\ \frac{169}{108}r^2 - \frac{5509}{2025}r + \frac{5153}{3780}, & \gamma = -1, \\ 2(3r-2)(r-1), & \gamma = -\frac{1}{2}. \end{cases} \quad (45)$$

Proof. Let $Y_3 \cong \mathrm{RHDN4}(r)$ be seen in Figure 3, with condition $r = s \geq 4$. The edge partition as seen in the table is as follows:

$$R_\gamma(Y_3) = \sum_{\acute{c}\acute{d}\in E(Y_3)} \left(\kappa(\acute{L})\kappa(\acute{M})\right)^\gamma. \quad (46)$$

For $\gamma = 1$, the general form of the Randić index $R_\gamma(Y_3)$ can be calculated as follows:

$$R_1(Y_3) = \sum_{k=1}^{11} \sum_{\acute{c}\acute{d}\in E_k(Y_3)} \left(\kappa(\acute{L}) \cdot \kappa(\acute{M})\right). \quad (47)$$

□

Using Table 3, we get

$$\begin{aligned}R_1(Y_3) &= 9|E_1(Y_3)| + 12|E_2(Y_3)| + 21|E_3(Y_3)| + 30|E_4(Y_3)| \\ &+ 54|E_5(Y_3)| + 40|E_6(Y_3)| + 70|E_7(Y_3)| \\ &+ 126|E_8(Y_3)| + 100|E_9(Y_3)| + 180|E_{10}(Y_3)| \\ &+ 324|E_{11}(Y_3)|, \Rightarrow R_1(Y_3) \\ &= 4\left(432r^2 - 1358r + 1093\right).\end{aligned} \quad (48)$$

For $\gamma = 1/2$, we apply the formula of $R_\gamma(Y_3)$:

$$R_{1/2}(Y_3) = \sum_{k=1}^{11} \sum_{\acute{c}\acute{d}\in E_k(G)} \sqrt{\kappa(\acute{L}) \cdot \kappa(\acute{M})}. \quad (49)$$

Using Table 3, we get

$$\begin{aligned}R_{1/2}(Y_3) &= 3|E_1(Y_3)| + 2\sqrt{3}|E_2(Y_3)| + \sqrt{21}|E_3(Y_3)| \\ &+ \sqrt{30}|E_4(Y_3)| + 3\sqrt{6}|E_5(Y_3)| + 2\sqrt{10}|E_6(Y_3)| \\ &+ \sqrt{70}|E_7(Y_3)| + 3\sqrt{14}|E_8(Y_3)| + 10|E_9(Y_3)| \\ &+ 6\sqrt{5}|E_{10}(Y_3)| + 18|E_{11}(Y_3)|, \Rightarrow R_{1/2}(Y_3) \\ &= 178r^2 - 432r + 267.\end{aligned} \quad (50)$$

For $\gamma = -1$, by using the formula of $R_\gamma(Y_3)$:

$$\begin{aligned}R_{-1}(Y_3) &= \sum_{k=1}^{11} \sum_{\acute{c}\acute{d}\in E_k(Y_3)} \frac{1}{\kappa(\acute{L}) \cdot \kappa(\acute{M})}, R_{-1}(Y_3) \\ &= \frac{1}{9}|E_1(Y_3)| + \frac{1}{12}|E_2(Y_3)| + \frac{1}{21}|E_3(Y_3)| + \frac{1}{30}|E_4(Y_3)| \\ &+ \frac{1}{54}|E_5(Y_3)| + \frac{1}{40}|E_6(Y_3)| + \frac{1}{70}|E_7(Y_3)| \\ &+ \frac{1}{126}|E_8(Y_3)| + \frac{1}{100}|E_9(Y_3)| + \frac{1}{180}|E_{10}(Y_3)| \\ &+ \frac{1}{324}|E_{11}(Y_3)|, \Rightarrow R_{-1}(Y_3) = \frac{169}{108}r^2 - \frac{5509}{2025}r + \frac{5153}{3780}.\end{aligned} \quad (51)$$

For$\gamma = -1/2$, using formula of$R_\gamma(Y_3)$:

$$
\begin{aligned}
R_{-1/2}(Y_3) &= \sum_{k=1}^{11} \boxtimes \sum_{\acute{c}\acute{d}\in E_k(Y_3)} \boxtimes \frac{1}{\sqrt{\kappa(\acute{L})\cdot\kappa(\acute{M})}}, R_{-1/2}(Y_3) \\
&= \frac{1}{3}|E_1(Y_3)| + \frac{1}{2\sqrt{3}}|E_2(Y_3)| + \frac{1}{\sqrt{21}}|E_3(Y_3)| \\
&\quad + \frac{1}{\sqrt{30}}|E_4(Y_3)| + \frac{1}{3\sqrt{6}}|E_5(Y_3)| + \frac{1}{2\sqrt{10}}|E_6(Y_3)| \\
&\quad + \frac{1}{\sqrt{70}}|E_7(Y_3)| + \frac{1}{3\sqrt{14}}|E_8(Y_3)| + \frac{1}{10}|E_9(Y_3)| \\
&\quad + \frac{1}{6\sqrt{5}}|E_{10}(Y_3)| + \frac{1}{18}|E_{11}(Y_3)|, \Rightarrow R_{-1/2}(Y_3) \\
&= 2(3r-2)(r-1).
\end{aligned} \tag{52}
$$

Theorem 8. *The first Zagreb index for the RHDN4(r) is equivalent to*

$$
M_1(Y_3) = 2\left(216r^2 - 556r + 367\right). \tag{53}
$$

Proof. Let $Y_3 \cong \text{RHDN4}(r)$. Using Table 3 and equation (2) to calculate the Zagreb index:

$$
\begin{aligned}
M_1(Y_3) &= \sum_{\acute{c}\acute{d}\in E(Y_3)} \left(\kappa(\acute{L}) + \kappa(\acute{M})\right)\boxtimes \\
&= \sum_{k=1}^{12} \sum_{\acute{c}\acute{d}\in E_k(Y_3)} \left(\kappa(\acute{L}) + \kappa(\acute{M})\right), M_1(Y_3) \\
&= 6|E_1(Y_3)| + 7|E_2(Y_3)| + 10|E_3(Y_3)| + 13|E_4(Y_3)| \\
&\quad + 21|E_5(Y_3)| + 14|E_6(Y_3)| + 17|E_7(Y_3)| \\
&\quad + 25|E_8(Y_3)| + 20|E_9(Y_3)| + 28|E_{10}(Y_3)| + 25|E_{11}(Y_3)|.
\end{aligned} \tag{54}
$$

By doing some calculations, we get

$$
\Rightarrow M_1(Y_3) = 2\left(216r^2 - 556r + 367\right). \tag{55}
$$

□

Now, we calculate the H-index, AZI-index, ABC-index, and GA-index for RHDN4(r).

Theorem 9. *Let Y_3 be the fourth type of rectangular hex-derived network, then:*

(i) $H(Y_3) = (439/84)r^2 - (4886/585)r + 1525241/464100$

(ii) $AZI(Y_3) = (240663/80)r^2 - (2278779/200)r + 268079839/24000$

(iii) $ABC(Y_3) = 2(8r-7)(r-1)$

(iv) $GA(Y_3) = 23r^2 - 42r + 18$

Proof. Using Table 3 and equation (3) to calculate the Harmonic index:

$$
\begin{aligned}
H(Y_3) &= \sum_{\acute{c}\acute{d}\in E(Y_3)} \left(\frac{2}{\kappa(\acute{L}) + \kappa(\acute{M})}\right) \\
&= \sum_{k=1}^{11} \sum_{\acute{c}\acute{d}\in E_k(Y_3)} \left(\frac{2}{\kappa(\acute{L}) + \kappa(\acute{M})}\right), H(Y_3) = \frac{1}{3}|E_1(Y_3)| \\
&\quad + \frac{2}{7}|E_2(Y_3)| + \frac{1}{5}|E_3(Y_3)| + \frac{2}{13}|E_4(Y_3)| \\
&\quad + \frac{2}{21}|E_5(Y_3)| + \frac{1}{7}|E_6(Y_3)| + \frac{2}{17}|E_7(Y_3)| \\
&\quad + \frac{2}{25}|E_8(Y_3)| + \frac{1}{10}|E_9(Y_3)| + \frac{1}{14}|E_{10}(Y_3)| + \frac{1}{36}|E_{11}(Y_3)|.
\end{aligned} \tag{56}
$$

By doing some calculations, we get

$$
\Rightarrow H(Y_3) = \frac{439}{84}r^2 - \frac{4886}{585}r + \frac{1525241}{464100}. \tag{57}
$$

Using equation (4) to calculate the augmented Zagreb index:

$$
\begin{aligned}
AZI(Y_3) &= \sum_{\acute{c}\acute{d}\in E(Y_3)} \left(\frac{\kappa(\acute{L})\cdot\kappa(\acute{M})}{\kappa(\acute{L}) + \kappa(\acute{M}) - 2}\right)^3 \\
&= \sum_{k=1}^{11} \sum_{\acute{c}\acute{d}\in E_k(Y_3)} \left(\frac{\kappa(\acute{L})\cdot\kappa(\acute{M})}{\kappa(\acute{L}) + \kappa(\acute{M}) - 2}\right)^3, \text{AZI}(Y_3) \\
&= \frac{729}{64}|E_1(Y_3)| + \frac{1728}{125}|E_2(Y_3)| + \frac{9261}{512}|E_3(Y_3)| \\
&\quad + \frac{2029}{100}|E_4(Y_3)| + \frac{2296}{100}|E_5(Y_3)| + \frac{1000}{27}|E_6(Y_3)| \\
&\quad + \frac{2744}{27}|E_7(Y_3)| + \frac{16441}{100}|E_8(Y_3)| + \frac{17147}{100}|E_9(Y_3)| \\
&\quad + \frac{33181}{100}|E_{10}(Y_3)| + \frac{86536}{100}|E_{11}(Y_3)|.
\end{aligned} \tag{58}
$$

By doing some calculations, we get

$$
\Rightarrow \text{AZI}(Y_3) = \frac{240663}{80}r^2 - \frac{2278779}{200}r + \frac{268079839}{24000}. \tag{59}
$$

Using equation (5) to calculate the atom bond connectivity index:

$$
\begin{aligned}
\text{ABC}(Y_3) &= \sum_{\acute{c}\acute{d}\in E(Y_3)} \sqrt{\frac{\kappa(\acute{L}) + \kappa(\acute{M}) - 2}{\kappa(\acute{L})\cdot\kappa(\acute{M})}} \\
&= \sum_{k=1}^{11} \sum_{\acute{c}\acute{d}\in E_k(Y_3)} \sqrt{\frac{\kappa(\acute{L}) + \kappa(\acute{M}) - 2}{\kappa(\acute{L})\cdot\kappa(\acute{M})}}, \text{ABC}(Y_3) = \frac{2}{3}|E_1(Y_3)| \\
&\quad + \frac{\sqrt{15}}{6}|E_2(Y_3)| + \frac{2\sqrt{42}}{21}|E_3(Y_3)| + \frac{\sqrt{330}}{30}|E_4(Y_3)| \\
&\quad + \frac{\sqrt{114}}{18}|E_5(Y_3)| + \frac{\sqrt{30}}{10}|E_6(Y_3)| + \frac{\sqrt{42}}{14}|E_7(Y_3)| \\
&\quad + \frac{\sqrt{322}}{42}|E_8(Y_3)| + \frac{3\sqrt{2}}{10}|E_9(Y_3)| + \frac{\sqrt{130}}{30}|E_{10}(Y_3)| + \frac{\sqrt{34}}{18}|E_{11}(Y_3)|.
\end{aligned} \tag{60}
$$

By doing some calculations, we get

$$\Rightarrow \mathrm{ABC}(Y_3) = 2(8r-7)(r-1). \tag{61}$$

Using equation (6) to geometric arithmetic index:

$$\mathrm{GA}(Y_3) = \sum_{\acute{c}\acute{d}\in E(Y_3)} \frac{2\sqrt{\kappa(\acute{L})\kappa(\acute{M})}}{\left(\kappa(\acute{L})+\kappa(\acute{M})\right)} = \sum_{k=1}^{11}\sum_{\acute{c}\acute{d}\in E_k(Y_3)} \frac{2\sqrt{\kappa(\acute{L})\kappa(\acute{M})}}{\left(\kappa(\acute{L})+\kappa(\acute{M})\right)}. \tag{62}$$

By doing some calculations, we get

$$\begin{aligned}\mathrm{GA}(Y_3) &= |E_1(Y_3)| + \frac{4\sqrt{3}}{7}|E_2(Y_3)| + \frac{\sqrt{21}}{5}|E_3(Y_3)| \\ &\quad + \frac{2\sqrt{30}}{13}|E_4(Y_3)| + \frac{2\sqrt{6}}{7}|E_5(Y_3)| + \frac{2\sqrt{10}}{7}|E_6(Y_3)| \\ &\quad + \frac{2\sqrt{70}}{17}|E_7(Y_3)| + \frac{6\sqrt{14}}{25}|E_8(Y_3)| + |E_9(Y_3)| \\ &\quad + \frac{3\sqrt{5}}{7}|E_{10}(Y_3)| + |E_{11}(Y_3)|, \Rightarrow \mathrm{GA}(Y_3) \\ &= 23r^2 - 42r + 18.\end{aligned} \tag{63}$$

□

For comparison through graphs, the comparison of the different topological indices for the HDN4, THDN4, and RHDN4, a newly developed fourth type of hex-derived networks has been evaluated for the different values. The graphical representation shows the correctness of the results as shown in Figures 4–9.

3. Conclusion

In this paper, certain degree-based topological indices, namely, the Randić, Zagreb, Harmonic, augmented Zagreb, atom-bond connectivity, and geometric-arithmetic indices for the HDN4, THDN4, and RHDN4 networks, were studied for the first time, and analytical closed formulas for these networks were determined that will help the people working in network science to understand the underlying topologies of these networks. In future, we are interested in designing some new architectures/networks and then studying their topological indices, which will be quite helpful in understanding their underlying topologies.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] H. Wiener, "Structural determination of paraffin boiling points," *Journal of the American Chemical Society*, vol. 69, no. 1, pp. 17–20, 1947.

[2] I. Gutman and K. C. Das, "The first Zagreb index 30 years after," *MATCH Communications in Mathematical and in Computer Chemistry*, vol. 50, no. 1, pp. 83–92, 2004.

[3] L. Zhong, "The harmonic index for graphs," *Applied Mathematics Letters*, vol. 25, no. 3, pp. 561–566, 2012.

[4] B. Furtula, A. Graovac, and D. Vukićevixcx, "Augmented Zagreb index," *Journal of Mathematical Chemistry*, vol. 48, no. 2, pp. 370–380, 2010.

[5] E. Estrada, L. Torres, L. Rodríguez, and I. Gutman, "An atom-bond connectivity index: modelling the enthalpy of formation of alkanes," *Indian Journal of Chemistry*, vol. 37, pp. 849–855, 1998.

[6] D. Vukičević and B. Furtula, "Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges," *Journal of Mathematical Chemistry*, vol. 46, no. 4, pp. 1369–1376, 2009.

[7] J. B. Liu, H. Ali, M. K. Shafiq, G. Dustigeer, and P. Ali, "On topological properties of planar octahedron networks," *Polycyclic Aromatic Compounds*, pp. 1–17, 2021.

[8] R. Qi, H. Ali, U. Babar, J. B. Liu, and P. Ali, "On the sum of degree-based topological indices of rhombus-type silicate and oxide structures," *Journal of Mathematics*, vol. 2021, 16 pages, 2021.

[9] H. Ali, M. A. Binyamin, M. K. Shafiq, and W. Gao, "On the degree-based topological indices of some derived networks," *Mathematics*, vol. 7, no. 7, p. 612, 2019.

[10] M. V. Diudea, I. Gutman, and J. Lorentz, *Molecular Topology*, Nova Science Publishers, Huntington, NY, USA, 2001.

[11] I. Gutman, "Topological properties of benzenoid systems," *Topics in Current Chemistry*, vol. 162, pp. 21–28, 1992.