Hindawi
Journal of Function Spaces
Volume 2022, Article ID 2419196, 6 pages
https://doi.org/10.1155/2022/2419196

Research Article

The Sufficient and Necessary Conditions for the Poisson Distribution Series to Be in Some Subclasses of Analytic Functions

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Received 5 March 2022; Revised 23 March 2022; Accepted 6 April 2022; Published 5 May 2022

Academic Editor: John R. Akeroyd

In this paper, we introduce new subclasses of analytic functions in the open unit disc. Furthermore, the necessary and sufficient conditions for the Poisson distribution series to be in these new subclasses are found.

1. Introduction

Let $\mathscr{A}$ be the class of all analytic functions f in the open unit disc $U=\{z\in\mathbb{C}:|z|<1\}$ and normalized by $f(0)=0$ and $f'(0)=1$. A function $f\in\mathscr{A}$ has the Taylor series expansion of the form

$$f(z)=z+\sum_{k=2}^{\infty}a_k z^k. \tag{1}$$

We denote by $\mathscr{S}$ the subclass of $\mathscr{A}$ consisting of normalized functions of the form (1) which are univalent in U. Further, we denote by $\mathscr{T}$ the subclass of $\mathscr{S}$ consisting of functions with negative coefficients of the form

$$f(z)=z-\sum_{k=2}^{\infty}a_k z^k,\ a_k\geq 0. \tag{2}$$

If $f,g\in\mathscr{A}$ such that f is given by (1) and g is given by $g(z)=z+\sum_{k=2}^{\infty}b_k z^k$, then, the Hadamard product $(f*g)(z)$ is defined by

$$(f*g)(z)=z+\sum_{k=2}^{\infty}a_k b_k z^k. \tag{3}$$

In 1837, the French mathematician Siméon Denis Poisson created the Poisson distribution which is a popular distribution expresses the probability of a given number of events occurring in a fixed interval of time or space. In [1], Porwal introduced a power series such that its coefficients are probabilities of the Poisson distribution

$$N(m,z)=z+\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}e^{-m}z^k,\ m>0,\ z\in U. \tag{4}$$

In addition, he introduced the series

$$R(m,z)=2z-N(m,z)=z-\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}e^{-m}z^k,\ m>0,\ z\in U. \tag{5}$$

In [2], Porwal and Kumar introduced a new linear operator defined by

$$N(m,z)*f(z)=z+\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}e^{-m}a_k z^k,\ m>0,\ z\in U. \tag{6}$$

In [3, 4], El-Ashwah and Kota presented the functions

$H_v(m,z)$ and $H_v^*(m,z)$ as below:

$$H_v(m,z) = (1-v)N(m,z) + vz(N(m,z))' = z + \sum_{k=2}^{\infty}(1+v(k-1))\frac{m^{k-1}}{(k-1)!}e^{-m}z^k, \quad (7)$$

and

$$H_v^*(m,z) = 2z - H_v(m,z) = z - \sum_{k=2}^{\infty}(1+v(k-1))\frac{m^{k-1}}{(k-1)!}e^{-m}z^k, \quad (8)$$

where $m>0$, $0\le v\le 1$, and $z\in U$. Suppose the functions ϕ, ψ, and ζ are given by

$$\phi(z) = z + \sum_{k=2}^{\infty}\gamma_k z^k, \gamma_k \ge 0, \psi(z) = z + \sum_{k=2}^{\infty}\mu_k z^k, \mu_k \ge 0, \quad (9)$$

$$\zeta(z) = z + \sum_{k=2}^{\infty}\rho_k z^k, \rho_k \ge 0, \quad (10)$$

for every $z\in U$.

Definition 1. Let $0\le\lambda,\beta<1$. A function $f\in\mathcal{A}$ is said to be in the class $\mathcal{A}(\phi,\psi,\zeta,\lambda,\beta)$ if the following condition is satisfied

$$\mathfrak{R}\left\{\frac{(1-\lambda)(f*\phi)(z)+\lambda(f*\psi)(z)}{(f*\zeta)(z)}\right\} > \beta. \quad (11)$$

Further, we define the class $\mathcal{T}(\phi,\psi,\zeta,\lambda,\beta)$ by

$$\mathcal{T}(\phi,\psi,\zeta,\lambda,\beta) = \mathcal{A}(\phi,\psi,\zeta,\lambda,\beta)\cap\mathcal{T}. \quad (12)$$

Indeed, we have

(1)

$$\mathcal{A}\left(\frac{z}{(1-z)^2},\frac{z+z^2}{(1-z)^3},z,\lambda,\beta\right) \equiv \mathcal{F}(\lambda,\beta), = \left\{f\in\mathcal{A}:\mathfrak{R}\left\{f'(z)+\lambda z f''(z)\right\} > \beta, 0\le\lambda,\beta<1\right\}, \quad (13)$$

where the class $\mathcal{F}(\lambda,\beta)$ was studied by Chichra in [5]. We define the class $\mathcal{F}(\lambda,\beta)\cap\mathcal{T} = \mathcal{R}^1(\lambda,\beta)$ which was introduced and studied by Orhan [6].

(2)

$$\mathcal{A}\left(\frac{z}{(1-z)^2},\frac{z+z^2}{(1-z)^3},\frac{z}{1-z},\lambda,\beta\right) \equiv H(\lambda,\beta), = \left\{f\in\mathcal{A}:\mathfrak{R}\left\{\frac{zf'(z)+\lambda z^2 f''(z)}{f(z)}\right\} > \beta, 0\le\lambda,\beta<1\right\}, \quad (14)$$

where the class $H(\lambda,\beta)$ was studied by Obradovic and Joshi in [7], and the class $H(\lambda,0)$ was intoduced and studied by Ramesha et al. in [8]. We define the class $H(\lambda,\beta)\cap\mathcal{T} = \bar{H}(\lambda,\beta)$ which was studied by Lashin [9].

(3)

$$\mathcal{A}\left(\frac{z}{1-z},\frac{z}{(1-z)^2},z,\lambda,\beta\right) \equiv \mathcal{J}(\lambda,\beta), = \left\{f\in\mathcal{A}:\mathfrak{R}\left\{\frac{(1-\lambda)f(z)}{z}+\lambda f'(z)\right\} > \beta, 0\le\lambda,\beta<1\right\}, \quad (15)$$

where the class $\mathcal{J}(\lambda,\beta)$ was studied by Ding et al. [10]. We define the class $\mathcal{J}(\lambda,\beta)\cap\mathcal{T} = \mathcal{J}^*(\lambda,\beta)$ which was introduced by Hassan [11].

(4)

$$\mathcal{A}\left(\frac{z}{1-z},\frac{z+z^2}{(1-z)^3},\frac{z}{(1-z)^2},\lambda,\beta\right) \equiv \mathcal{X}(\lambda,\beta), = \left\{f\in\mathcal{A}:\mathfrak{R}\left\{(1-\lambda)\frac{f(z)}{zf'(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\right\} > \beta, 0\le\lambda,\beta<1\right\}, \quad (16)$$

where the classes $\mathcal{X}(\lambda,\beta)$ and $\mathcal{X}(\lambda,\beta)\cap\mathcal{T} \ll \mathcal{X}^*(\lambda,\beta)$ were introduced by Lashin et al. in [12].

In this paper, we find the necessary and sufficient conditions for the Poisson distribution series to be in the classes $\mathcal{J}^*(\lambda,\beta)$, $\bar{H}(\lambda,\beta)$, and $\mathcal{X}^*(\lambda,\beta)$.

2. Coefficient Inequalities and Related Properties

We first derive the sufficient and necessary conditions for the function f to be in the aforementioned classes.

Theorem 2. *The sufficient condition for f to be in the class $\mathcal{A}(\phi,\psi,\zeta,\lambda,\beta)$ is*

$$\sum_{k=2}^{\infty}(|\gamma_k-\rho_k+\lambda(\mu_k-\gamma_k)| + (1-\beta)\rho_k)|a_k| \le 1-\beta. \quad (17)$$

Proof. We need to show that

$$\mathfrak{R}\left\{\frac{(1-\lambda)(f*\phi)(z)+\lambda(f*\psi)(z)}{(f*\zeta)(z)}\right\} > \beta. \quad (18)$$

Then, we have

$$\left|\frac{(1-\lambda)(f*\phi)(z)+\lambda(f*\psi)(z)}{(f*\zeta)(z)} - 1\right| = \left|\frac{\sum_{k=2}^{\infty}(\gamma_k-\rho_k+\lambda(\mu_k-\gamma_k))a_k z^{k-1}}{1+\sum_{k=2}^{\infty}\rho_k a_k z^{k-1}}\right|, \le \frac{\sum_{k=2}^{\infty}|\gamma_k-\rho_k+\lambda(\mu_k-\gamma_k)||a_k|}{1-\sum_{k=2}^{\infty}\rho_k|a_k|} \le 1-\beta, \quad (19)$$

if condition (17) holds. This implies that $f \in \mathscr{A}(\phi, \psi, \zeta, \lambda, \beta)$ which completes the proof. □

Theorem 3. *Let $F(k) = \gamma_k - \rho_k + \lambda(\mu_k - \gamma_k)$ be an increasing function of k and $\lambda \geq \max\{0, (\rho_2 - \gamma_2)/(\mu_2 - \gamma_2)\}$. Then, the necessary and sufficient condition for the function f to be in the class $\mathscr{T}(\phi, \psi, \zeta, \lambda, \beta)$ is*

$$\sum_{k=2}^{\infty}\{\gamma_k - \rho_k + \lambda(\mu_k - \gamma_k) + (1-\beta)\rho_k\}a_k \leq 1-\beta. \quad (20)$$

Proof. In view of Theorem 2, it suffices to show the necessary condition only. Assume that $f \in \mathscr{T}(\phi, \psi, \xi, \lambda, \beta)$. Then

$$\Re\left\{\frac{(1-\lambda)(f*\phi)(z) + \lambda(f*\psi)(z)}{(f*\zeta)(z)}\right\} > \beta, \quad (21)$$

which is equivalent to

$$\Re\left\{\frac{1 - \sum_{k=2}^{\infty}(\gamma_k + \lambda(\mu_k - \gamma_k))a_k z^{k-1}}{1 - \sum_{k=2}^{\infty}\rho_k a_k z^{k-1}}\right\} > \beta. \quad (22)$$

Choosing z on the real axis, then

$$\left\{\frac{1 - \sum_{k=2}^{\infty}(\gamma_k + \lambda(\mu_k - \gamma_k))a_k z^{k-1}}{1 - \sum_{k=2}^{\infty}\rho_k a_k z^{k-1}}\right\}, \quad (23)$$

is also real. Let $z \longrightarrow 1^-$ through real values, we get

$$1 - \sum_{k=2}^{\infty}(\gamma_k + \lambda(\mu_k - \gamma_k))a_k \geq \beta\left(1 - \sum_{k=2}^{\infty}\rho_k a_k\right), \quad (24)$$

which is equivalent to (20), and this completes the proof. □

Putting $\phi = z/(1-z)^2$, $\psi = (z+z^2)/(1-z)^3$, and $\zeta = z/(1-z)$ in Theorems 3, we get the following corollary due to Lashin [9].

Corollary 4 (see [9]). *A function $f \in \mathscr{T}$ is in the class $\bar{H}(\lambda, \beta)$ if and only if*

$$\sum_{k=2}^{\infty}\{(k-1)(\lambda k + 1) + (1-\beta)\}a_k \leq 1-\beta, 0 \leq \lambda, \beta < 1. \quad (25)$$

Putting $\phi = z/(1-z)$, $\psi = z/(1-z)^2$, and $\zeta = z$ in Theorem 3, we get the following corollary due to Hassan [11].

Corollary 5 (see [11]). *A function $f \in \mathscr{T}$ is in the class $\mathscr{J}^*(\lambda, \beta)$ if and only if*

$$\sum_{k=2}^{\infty}[\lambda(k-1) + 1]a_k \leq 1-\beta, 0 \leq \lambda, \beta < 1. \quad (26)$$

Putting $\phi = z/(1-z)$, $\psi = (z+z^2)/(1-z)^3$, and $\zeta = z/(1-z)^2$ in Theorem 3, we get the following corollary.

Corollary 6 (see [12]). *Let $1/3 \leq \lambda < 1$. A function $f \in \mathscr{T}$ is in the class $\mathscr{X}^*(\lambda, \beta)$ if and only if*

$$\sum_{k=2}^{\infty}\{(k-1)[\lambda(k+1) - 1] + (1-\beta)k\}a_k \leq 1-\beta, 0 \leq \beta < 1. \quad (27)$$

Making use of the techniques and methodology given by Porwal [1] (see also [13–18]), we get the following theorems.

Theorem 7. *The sufficient condition for $H_v(m, z)$ to be in the class $H(\lambda, \beta)$ is*

$$\lambda v m^3 + (4v\lambda + v + \lambda)m^2 + [(1+2\lambda)(1+v) + v(1-\beta)]m \leq (1-\beta)e^{-m}. \quad (28)$$

Also, condition (28) is necessary and sufficient for $H_v^*(m, z)$ to be in the class $\bar{H}(\lambda, \beta)$.

Proof. According to Theorem 2, we need to show that

$$\sum_{k=2}^{\infty}[(k-1)(\lambda k + 1) + (1-\beta)][1 + v(k-1)]\frac{m^{k-1}}{(k-1)!}e^{-m} \leq 1-\beta. \quad (29)$$

Thus,

$$\begin{aligned}
&\sum_{k=2}^{\infty}[(k-1)(\lambda k + 1) + (1-\beta)][1 + v(k-1)]\frac{m^{k-1}}{(k-1)!}e^{-m} \\
&= e^{-m}\sum_{k=2}^{\infty}\{(k-1)[\lambda(k-2) + (1+2\lambda)] + (1-\beta)\}\frac{m^{k-1}}{(k-1)!} \\
&\quad + e^{-m}\sum_{k=2}^{\infty}\{v\lambda(k-1)(k-2)(k-3) + 2v\lambda(k-1)(k-2)\}\frac{m^{k-1}}{(k-1)!} \\
&\quad + e^{-m}\sum_{k=2}^{\infty}\{v(1+2\lambda)(k-1)(k-2) + v(1+2\lambda)(k-1) \\
&\quad + v(1-\beta)(k-1)\}\frac{m^{k-1}}{(k-1)!}, = v\lambda e^{-m}\sum_{k=4}^{\infty}\frac{m^{k-1}}{(k-4)!} \\
&\quad + (4v\lambda + v + \lambda)e^{-m}\sum_{k=3}^{\infty}\frac{m^{k-1}}{(k-3)!} \\
&\quad + [(1+2\lambda)(1+v) + v(1-\beta)]e^{-m}\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-2)!} \\
&\quad + (1-\beta)e^{-m}\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}, = \lambda v m^3 + (4v\lambda + v + \lambda)m^2 \\
&\quad + [(1+2\lambda)(1+v) + v(1-\beta)]m + (1-\beta)(1-e^{-m}) \leq (1-\beta),
\end{aligned} \quad (30)$$

if condition (28) holds. Then, from Theorem 3, it follows that the condition (28) is necessary and sufficient for $H_v^*(m, z) \in \bar{H}(\lambda, \beta)$. Hence, the proof is completed. □

In Theorem 7, if we put $v = 0$, then, we get the following corollary which was obtained by Murugusundaramoorthy et al. [19].

Corollary 8. *The sufficient condition for $N(m, z)$ to be in the class $H(\lambda, \beta)$ is*

$$\lambda m^2 + (1 + 2\lambda)m \leq (1 - \beta)e^{-m}. \tag{31}$$

Also, condition (31) is necessary and sufficient for $R(m, z)$ to be in the class $\bar{H}(\lambda, \beta)$.

Theorem 9. *The sufficient condition for $H_v(m, z)$ to be in the class $\mathcal{J}(\lambda, \beta)$ is*

$$\lambda v m^2 + (\lambda + v(1 + \lambda))m + 1 - e^{-m} \leq 1 - \beta. \tag{32}$$

Also, condition (32) is necessary and sufficient for $H_v^*(m, z)$ to be in the class $\mathcal{J}^*(\lambda, \beta)$.

Proof. According to Theorem 2, we need to show that

$$\sum_{k=2}^{\infty} [1 + \lambda(k - 1)][1 + v(k - 1)] \frac{m^{k-1}}{(k - 1)!} e^{-m} \leq 1 - \beta. \tag{33}$$

Thus,

$$\begin{aligned}
&\sum_{k=2}^{\infty} (1 + \lambda(k - 1))(1 + v(k - 1)) \frac{m^{k-1}}{(k - 1)!} e^{-m} \\
&\quad = e^{-m} \sum_{k=2}^{\infty} \{\lambda v(k - 1)(k - 2) + (\lambda v + \lambda + v)(k - 1) + 1\} \frac{m^{k-1}}{(k - 1)!}, \\
&\quad = e^{-m} \left\{\lambda v \sum_{k=3}^{\infty} \frac{m^{k-1}}{(k - 3)!} + [\lambda + v(1 + \lambda)] \sum_{k=2}^{\infty} \frac{m^{k-1}}{(k - 2)!} + \sum_{k=2}^{\infty} \frac{m^{k-1}}{(k - 1)!}\right\}, \\
&\quad = \lambda v m^2 + (\lambda + v(1 + \lambda))m + 1 - e^{-m} \leq 1 - \beta,
\end{aligned} \tag{34}$$

if condition (32) holds. Then, from Theorem 3, it follows that condition (32) is necessary and sufficient for $H_v^*(m, z) \in \mathcal{J}^*(\lambda, \beta)$. Hence, the proof is completed. □

In Theorem 9, if we put $v = 0$, then, we get the following corollary which was obtained by Frasin [20].

Corollary 10. *The sufficient condition for $N(m, z)$ to be in the class $\mathcal{J}(\lambda, \beta)$ is*

$$\lambda m + 1 - e^{-m} \leq 1 - \beta. \tag{35}$$

Also, condition (35) is necessary and sufficient for $R(m, z)$ to be in the class $\mathcal{J}^*(\lambda, \beta)$.

Theorem 11. *The sufficient condition for $N(m, z)$ to be in the class $\mathcal{X}(\lambda, \beta)$ is*

$$\lambda m^2 + (3\lambda - \beta)m \leq (1 - \beta)e^{-m}, \tag{36}$$

where $1/3 \leq \lambda < 1$. Also, condition (36) is necessary and sufficient for $R(m, z)$ to be in the class $\mathcal{X}^*(\lambda, \beta)$.

Proof. According to Theorem 2, we need to show that

$$\sum_{k=2}^{\infty} \{(k - 1)[\lambda(k + 1) - 1] + (1 - \beta)k\} \frac{m^{k-1}}{(k - 1)!} e^{-m} \leq 1 - \beta. \tag{37}$$

Thus,

$$\begin{aligned}
&\sum_{k=2}^{\infty} \{(k - 1)[\lambda(k + 1) - 1] + (1 - \beta)k\} \frac{m^{k-1}}{(k - 1)!} e^{-m} \\
&\quad = e^{-m} \sum_{k=2}^{\infty} \{\lambda(k - 1)(k - 2) + (3\lambda - \beta)(k - 1) + (1 - \beta)\} \frac{m^{k-1}}{(k - 1)!}, \\
&\quad = e^{-m} \left\{\lambda \sum_{k=3}^{\infty} \frac{m^{k-1}}{(k - 3)!} + (3\lambda - \beta) \sum_{k=2}^{\infty} \frac{m^{k-1}}{(k - 2)!} + (1 - \beta) \sum_{k=2}^{\infty} \frac{m^{k-1}}{(k - 1)!}\right\}, \\
&\quad = \lambda m^2 + (3\lambda - \beta)m + (1 - \beta)(1 - e^{-m}) \leq (1 - \beta),
\end{aligned} \tag{38}$$

if condition (36) holds. Then, from Theorem 3, it follows that the condition (36) is necessary and sufficient for $R(m, z) \in \mathcal{X}^*(\lambda, \beta)$. Hence, the proof is completed. □

Theorem 12. *The sufficient condition for $H_v(m, z)$ to be in the class $\mathcal{X}(\lambda, \beta)$ is*

$$\begin{aligned}
&\lambda v m^3 + [\lambda + v(5\lambda - \beta)]m^2 + [(3\lambda - \beta)(v + 1) + v(1 - \beta)]m \\
&\quad \leq (1 - \beta)e^{-m},
\end{aligned} \tag{39}$$

where $1/3 \leq \lambda < 1$. Also, condition (39) is necessary and sufficient for $H_v^*(m, z)$ to be in the class $\mathcal{X}^*(\lambda, \beta)$.

Proof. According to Theorem 2, we need to show that

$$\sum_{k=2}^{\infty}[(k-1)(\lambda(k+1)-1)+(1-\beta)k](1+v(k-1))\frac{m^{k-1}}{(k-1)!}e^{-m}\leq 1-\beta. \tag{40}$$

Thus,

$$\begin{aligned}\sum_{k=2}^{\infty}[(k-1)(\lambda(k+1)-1)+(1-\beta)k](1+v(k-1))\frac{m^{k-1}}{(k-1)!}e^{-m} &= e^{-m}\sum_{k=2}^{\infty}\{\lambda v(k-1)(k-2)(k-3)\\ &+[\lambda+v(5\lambda-\beta)](k-1)(k-2)+[(3\lambda-\beta)(v+1)+v(1-\beta)](k-1)+(1-\beta)\}\frac{m^{k-1}}{(k-1)!},\\ &= e^{-m}\left\{\lambda v\sum_{k=4}^{\infty}\frac{m^{k-1}}{(k-4)!}+[\lambda+v(5\lambda-\beta)]\sum_{k=3}^{\infty}\frac{m^{k-1}}{(k-3)!}+[(3\lambda-\beta)(v+1)+v(1-\beta)]\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-2)!}+(1-\beta)\sum_{k=2}^{\infty}\frac{m^{k-1}}{(k-1)!}\right\},\\ &= \lambda v m^3+[\lambda+v(5\lambda-\beta)]m^2+[(3\lambda-\beta)(v+1)+v(1-\beta)]m+(1-\beta)(1-e^{-m})\leq 1-\beta,\end{aligned} \tag{41}$$

if condition (39) holds. Then, from Theorem 3, it follows that the condition (39) is necessary and sufficient for $H_v^*(m,z)\in\mathcal{X}^*(\lambda,\beta)$. Hence, the proof is completed. □

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors would like to thank the Deanship of Scientific Research (DSR) at King Abdulaziz University (KAU), Jeddah, Saudi Arabia, for funding this project under grant no. (KEP-PhD:49-130-1443). Also, they would like to express their thanks to the referees for their helpful comments and suggestions, which improved the presentation of the paper.

References

[1] S. Porwal, "An application of a Poisson distribution series on certain analytic functions," *Complex Analysis and Operator Theory*, vol. 18, pp. 1–3, 2014.

[2] S. Porwal and M. Kumar, "A unified study on starlike and convex functions associated with Poisson distribution series," *Afrika Matematika*, vol. 27, no. 5-6, pp. 1021–1027, 2016.

[3] R. M. El-Ashwah and W. Y. Kota, "Some application of Poisson distribution series on subclasses of univalent functions," *Fractional Calculus and Applied Analysis*, vol. 9, no. 1, pp. 167–179, 2018.

[4] R. M. El-Ashwah and W. Y. Kota, "Some condition on a Poisson distribution series to be in subclasses of univalent functions," *Acta Universitatis Apulensis*, vol. 51, pp. 89–103, 2017.

[5] P. Chichra, "New subclasses of the class of close-to-convex functions," *Proceedings of the American Mathematical Society*, vol. 62, no. 1, pp. 37–43, 1977.

[6] H. Orhan, "On a subclass of analytic functions with negative coefficients," *Applied Mathematics and Computation*, vol. 142, no. 2-3, pp. 243–253, 2003.

[7] M. Obradović and S. B. Joshi, "On certain classes of strongly starlike functions," *Taiwanese Journal of Mathematics*, vol. 2, no. 3, pp. 297–302, 1998.

[8] C. Ramesha, S. Kumar, and K. S. Padmanabhan, "A sufficient condition for starlikeness," *Chinese Journal of Mathematics*, vol. 23, 171 pages, 1995.

[9] A. Y. Lashin, "On a certain subclass of starlike functions with negative coefficients," *Journal of Inequalities in Pure and Applied Mathematics*, vol. 10, no. 2, pp. 1–8, 2009.

[10] S. S. Ding, Y. Ling, and G. J. Bao, "Some properties of a class of analytic functions," *Journal of Mathematical Analysis and Applications*, vol. 195, no. 1, pp. 71–81, 1995.

[11] B. F. Hassan, *An Application of Certain Integral and Differential Operators, [M.S. thesis]*, Mansoura University, 2016.

[12] A. Y. Lashin, A. O. Badghaish, and A. Z. Bajamal, "Certain subclasses of univalent functions involving Pascal distribution series," *Boletn de la Sociedad Matemática Mexicana*, vol. 28, no. 1, pp. 1–11, 2022.

[13] Ş. Altınkaya and S. Yalçın, "Poisson distribution series for analytic univalent functions," *Complex Analysis and Operator Theory*, vol. 12, no. 5, pp. 1315–1319, 2018.

[14] B. A. Frasin, "On certain subclasses of analytic functions associated with Poisson distribution series," *Acta Univ. Sapientiae Math.*, vol. 11, no. 1, pp. 78–86, 2019.

[15] N. Magesh, S. Porwal, and C. Abirami, "Starlike and convex properties for Poisson distribution series," *Universitatis Babeş-Bolyai*, vol. 63, no. 1, pp. 71–78, 2018.

[16] G. Murugusundaramoorthy, K. Vijaya, K. Uma, and T. Janani, "Inclusion results on subclass of analytic functions associated

with Poisson distribution series," *International Journal of Pure and Applied Mathematics*, vol. 109, no. 7, pp. 83–90, 2016.

[17] G. Murugusundaramoorthy, "Subclasses of starlike and convex functions involving Poisson distribution series," *Afrika Matematika*, vol. 28, no. 7-8, pp. 1357–1366, 2017.

[18] S. Porwal, S. Altınkaya, and S. Yalçın, "The Poisson distribution series of general subclasses of univalent functions," *Acta Universitatis Apulensis*, vol. 58, pp. 45–52, 2019.

[19] G. Murugusundaramoorthy, K. Vijaya, and S. Porwal, "Some inclusion results of certain subclass of analytic functions associated with Poisson distribution series," *Hacettepe Journal of Mathematics and Statistics*, vol. 45, no. 4, pp. 1101–1107, 2016.

[20] B. A. Frasin, "Poisson distribution series on a general class of analytic functions," *Acta Comment. Univ. Tartu. Math.*, vol. 24, no. 2, pp. 241–251, 2020.