

Retraction

Retracted: A Novel Multiple-Criteria Decision-Making Approach Based on Picture Fuzzy Sets

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] H. Karamti, M. S. Sindhu, M. Ahsan, I. Siddique, I. Mekawy, and H. A. El-Wahed Khalifa, "A Novel Multiple-Criteria Decision-Making Approach Based on Picture Fuzzy Sets," *Journal of Function Spaces*, vol. 2022, Article ID 2537513, 8 pages, 2022.

Research Article

A Novel Multiple-Criteria Decision-Making Approach Based on Picture Fuzzy Sets

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Experts are using picture fuzzy sets (PFSs) in their probes to resolve the uncertain and vague information during the process of decision making because PFSs describe human attitudes naturally. Divergence measure (DM) plays a dominant role in discriminating between two distributions of probability and extracting consequences from that discrimination. In the present work, a novel picture fuzzy divergence measure (PF-DM) is developed between two PFSs. Some of the suggested measure's important qualities are also discussed with particular situations to validate it. Based on the suggested PF-DM, a multiple-criteria decision-making (MCDM) model is established to grab the fuzzy information. The suggested measure's performance is compared to that of various existing measures in the literature. An MCDM model has been proven for the usefulness of the suggested technique in dealing with real-life scenarios in the context of dengue sickness and pattern identification. Validation of the suggested MCDM model has been further investigated using validity testing. To improve the generated model, a thorough comparison with several current methodologies has been carried out while taking the time complexity (TC) factor into account.

1. Introduction

Handling vague/uncertain information in real-life situations has trouble. Therefore, various techniques, such as the theory of fuzzy sets (FSs), have been examined to address the ambiguity and uncertainty found in the real world. Zadeh [1] introduced the concept of FSs in 1965 to tackle the imprecise, unclear data and applied them in decision-making complications by considering diverse perspectives. Although the FS theory has been applied to a variety of sectors, some circumstances in actual life appear to be beyond its capabilities. To capture such occurrences, Atanassov [2] suggested intuitionistic fuzzy sets (IFSs). IFSs are a

generalization of FSs made up of two degrees: membership degree (MD) and non-membership degree (NMD), with $0 \leq MD + NMD \leq 1$. Innumerable investigators contributed their work in this extension, for example, based on the Hamming distance, Szmidt and Kacprzyk [3] developed a similarity measure (SM) between IFSs. Xu and Xia [4] categorized SM and the geometric distance of IFSs for group decision-making problems. Several scholars have begun research on Atanassov's intuitionistic fuzzy theory due to its elasticity and effective applications. Furthermore, stimulating solicitations in specific areas such as risk analysis, image processing, and medical diagnosis have been established in [5, 6].

This cosmos is full of uncertainty, inconsistency, and ambiguity. In reality, the great majority of the concepts we discuss include ambiguity instead of exact information. Addressing fear or uncertainties is a major difficulty in many professions, including sociology, natural science, engineering, medicine, and economics. Recently, a lot of authors have expressed a significant interest in depicting ambiguity. Traditional hypotheses, such as FS [1], vague sets [7], and rough sets [8], are well known and play a significant role in demonstrating uncertainty. Molodtsov [9] defined soft sets as an entirely new scientific instrument for coping with uncertainty.

Regardless, IFSs have been used in many fields of life. However, there are a variety of situations in which IFSs cannot be used. For example, casting a vote and dealing with issues such as yes, neutral, rejection, and refusal, which restricts the use of IFSs. Cuong [10] developed a generalization of FS called picture fuzzy (PF) sets (PFSs) to address such issues. PFSs have considered three factors from their inception: neutral membership degree (NMD), acceptance membership degree (AMD), and refusal membership degree (RMD). The requirement $0 \leq \text{AMD} + \text{NMD} + \text{RMD} \leq 1$ was met by these three features. From its beginning to the current day, PFSs have been verified for treating unclear circumstances in real-life issues, for example, decision making, pattern recognition, clustering, etc.

Based on PFSs, numerous researchers have worked in different fields, for instance, Thong [11] put forward an algorithm for fuzzy clustering that depicts the benefits of PFSs, Wei [12] developed the PF cross-entropy model for MCDM problems, Ashraf et al. [13] explained several procedures to MCDM issues under PF framework, and Sindhu et al. [14] implemented PFSs to select the best alternative. Recently, Zhao et al. [15] presented a dynamic distance measure of PFS depending on a PF point operator to resolve the real-life problem. Duong and Thao established a dissimilarity measure on PFSs and applied it MCDM problem to choose the optimal option. Verma and Rohtagi [16] presented the similarity measures based on PFSs to diagnose diseases and pattern recognition. Gocer [17] created a group decision-making technique based on interval-valued PFSs and then used it to determine a sustainable supply chain strategy. Sustainable supply chain strategy is a complex MCDM issue involving multiple parameters that may be contradictory at the same time. Luo et al. [18] proposed the SM based on relationship matrix and showed that it satisfied all the conditions of SM and then implemented the established SM to resolve the MCDM problem.

DM is useful for discriminating between two probability distributions and extracting conclusions from that discrimination. With the use of HFSs [19] and IFSs [20], many academics have worked and developed to answer decision-making difficulties. DM has also been utilized in MCDM problems by several experts [21, 22]. The PFS has been regarded as an extension of the FS and IFSs. As a result of the findings of the study on FS measures, the IFSs for the PFSs are likewise natural and necessary. This is the motivation behind our research into the DM of PFSs, both in theory and in practice. The major aims and contributions of this study are as follows:

- (i) A novel PF divergence measure (PF-DM) is proposed along with its properties.
- (ii) PF-DM is developed to measure the degree of fuzziness.
- (iii) Two examples, one related to medical diagnosis and the other to pattern recognition, are used to reveal the strength and reliability of the newly developed PF-DM.

Since the inception of the current MCDM theory in the early 1960s, MCDM has been developing to aid decision making to meet various applications [23]. Its importance in numerous application fields has grown significantly, particularly when new techniques are developed and existing ones are improved. MCDM approaches can provide powerful tools for allocating resources inside difficult operations, as well as in complicated exercises such as picking alternatives, weighing criteria, and establishing priorities. MCDM strategies are techniques and ways that attempt to find an optimal solution when there are several competing criteria. The selection of the most appealing DM goals from a wide number of available options while taking into account a variety of physical and abstract features or attributes [24] is at the heart of MCDM techniques. MCDM is a discipline that supports decision makers in making the best decision possible from a set of options based on several factors [25]. Several MCDM approaches have been created in the current decade [26] and used in many fields, such as supplier selection [27, 28] and project management development [29]. Sindhu et al. [30, 31] proposed a TOPSIS-based assessment approach for PFSs to address MCDM problems.

The remainder of the paper is organized as follows. Some basic and related knowledge is penned in Section 2. Novel PF-DM and its properties are discussed comprehensively in Section 3. An MCDM model based on PF-DM is established to resolve the uncertain information. Section 5 comprises two MCDM problems related to medical diagnosis and pattern recognition. In Section 6, the validity of the suggested model is investigated. A comprehensive comparison based on TC is analyzed in Section 6.3. Finally, Section 7 points out some conclusions.

2. Basic Concepts

Some basic ideas connected to the present work like FSs, IFSs, and PFSs and their operational laws are briefly penned in this section.

Definition 1 (see [1]). Let Y be a discourse set such that $Y = \{y_1, y_2, \dots, y_n\}$; then, a FS \mathcal{F} over Y can be illustrated as

$$\mathcal{F} = \{(y, \mu_{\mathcal{F}}(y)) \mid y \in Y\}, \quad (1)$$

where $\mu_{\mathcal{F}}(y): X \rightarrow [0, 1]$ is a MD so that $y \in Y$ to \mathcal{F} .

Definition 2 (see [2]). Suppose that Y is a fixed set, and an IFS \mathcal{F} on Y is illustrated as

$$\mathcal{F} = \{ \langle y, \alpha_{\mathcal{F}}(y), \beta_{\mathcal{F}}(y) \rangle \mid y \in Y \}, \quad (2)$$

where $\alpha_{\mathcal{F}}(y)$ and $\beta_{\mathcal{F}}(y) \in [0, 1]$ are called the MD and NMD of $y \in Y$ to \mathcal{F} , respectively, with the condition: $0 \leq \alpha_{\mathcal{F}}(y) + \beta_{\mathcal{F}}(y) \leq 1$, for all $y \in Y$. For all $y \in Y$, $\eta_I(y)$ is known as hesitancy degree of $y \in \mathcal{F}$ where $\eta_I(y) = 1 - \alpha_{\mathcal{F}}(y) - \beta_{\mathcal{F}}(y)$.

Definition 3 (see [10]). A PFS on $Y = \{y_1, y_2, \dots, y_n\}$ is penned as follows:

$$P = \{ \langle y, \alpha_P(y), \eta_P(y), \beta_P(y) \rangle \mid y \in Y \}, \quad (3)$$

where $\alpha_P(y)$, $\beta_P(y)$, and $\eta_P(y) \in [0, 1]$ are called the MD, neutral degree, and NMD of $y \in Y$ to the set P ; subsequently, $\alpha_P(y)$, $\eta_P(y)$, and $\beta_P(y)$ fulfil the condition: $0 \leq \alpha_P(y) + \eta_P(y) + \beta_P(y) \leq 1$, for all $y \in Y$. Also, $\eta_P(y) = 1 - \alpha_P(y) - \beta_P(y)$; then, $\eta_P(y)$ is supposed to represent a degree of refusal membership of $y \in Y$ in P . For simplicity, PFS over a fixed set Y is read as PFS (Y).

Definition 4 (see [10]). Let $P_1 = (\alpha_{P_1}(y), \eta_{P_1}(y), \beta_{P_1}(y))$, $P_2 = (\alpha_{P_2}(y), \eta_{P_2}(y), \beta_{P_2}(y))$, and $P = (\alpha_P(y), \eta_P(y), \beta_P(y))$ be three PFSs on Y ; then, some arithmetic operations are described as follows:

- (1) $P_1 \subseteq P_2$ iff $\alpha_{P_1}(y) \leq \alpha_{P_2}(y)$ and $\eta_{P_1}(y) \leq \eta_{P_2}(y)$ for all $y \in Y$.
- (2) $P_1 = P_2$ iff $P_1 \subseteq P_2$ and $P_2 \subseteq P_1$.
- (3) $P_1 \sqcup P_2 = \{ y, \max(\alpha_{P_1}(y), \alpha_{P_2}(y)), \min(\eta_{P_1}(y), \eta_{P_2}(y)), \min(\beta_{P_1}(y), \beta_{P_2}(y)) \mid y \in Y \}$.
- (4) $P_1 \cap P_2 = \{ y, \max(\alpha_{P_1}(y), \alpha_{P_2}(y)), \min(\eta_{P_1}(y), \eta_{P_2}(y)), \min(\beta_{P_1}(y), \beta_{P_2}(y)) \mid y \in Y \}$.
- (5) $P^C = \{ \langle y, \beta_P(y), \eta_P(y), \alpha_P(y) \rangle \mid y \in Y \}$.

3. Novel Divergence Measure for PFSs

In the present work, a novel PF-DM is proposed.

- (i) $D_m(\mathfrak{P}_1, \mathfrak{P}_2) \geq 0$.
- (ii) $D_m(\mathfrak{P}_1, \mathfrak{P}_2) = 0 \Leftrightarrow \mathfrak{P}_1 = \mathfrak{P}_2$.

Definition 5. Let $\mathfrak{P}_1, \mathfrak{P}_2 \in$ PFSs. The function D : PFS \times PFS R is called a PF-divergence measure (PF-DM) denoted by $D_m(\mathfrak{P}_1, \mathfrak{P}_2)$ if it satisfies

Definition 6. Let $\mathfrak{P}_1, \mathfrak{P}_2 \in$ PFSs. Then, a PF-DM is defined by

$$D_m(\mathfrak{P}_1, \mathfrak{P}_2) = \frac{1}{n} \sum_{i=1}^n \left[\frac{1}{\sqrt{6}-1} \left(\sqrt[6]{\frac{\mathfrak{P}_1^6(y_i) + \mathfrak{P}_2^6(y_i)}{6}} - \frac{\mathfrak{P}_1(y_i) + \mathfrak{P}_2(y_i)}{6} \right) \right], \quad (4)$$

where $\mathfrak{P}(y_i) = (\alpha_P(y_i), \eta_P(y_i), \beta_P(y_i))$ and $\mathfrak{P}^k(y_i) = (\alpha_P^k(y_i), \eta_P^k(y_i), \beta_P^k(y_i))$, $k = 6$.

Theorem 1. Let $\mathfrak{P}_1, \mathfrak{P}_2 \in$ PFSs. Then, the measure $D_M(\mathfrak{P}_1, \mathfrak{P}_2)$ is valid PF-divergence measure.

Proof

- (i) The following inequality holds for any two real numbers $a, b \in [0, 1]$:

$$\sqrt[6]{a^6 + b^6} \geq a + b/6.$$

Since $0 \leq \mathfrak{P}_1(y_i), \mathfrak{P}_2(y_i) \leq 1$.

$$\Rightarrow \sqrt[6]{\mathfrak{P}_1^6(y_i) + \mathfrak{P}_2^6(y_i)}/6 \geq \mathfrak{P}_1(y_i) + \mathfrak{P}_2(y_i)/6.$$

$$\Rightarrow \sqrt[6]{\mathfrak{P}_1^6(y_i) + \mathfrak{P}_2^6(y_i)}/6 - \mathfrak{P}_1(y_i) + \mathfrak{P}_2(y_i)/6 \geq 0.$$

$$\Rightarrow D_m(\mathfrak{P}_1, \mathfrak{P}_2) \geq 0.$$

- (ii) Suppose that $\mathfrak{P}_1 = \mathfrak{P}_2$; therefore, $\alpha_1(y_i) = \alpha_2(y_i), \eta_1(y_i) = \eta_2(y_i), \beta_1(y_i) = \beta_2(y_i), \forall i = 1, 2, \dots, n$. Then, (4) becomes

$$D_m(\mathfrak{P}_1, \mathfrak{P}_2) = \frac{1}{n} \sum_{i=1}^n \left[\frac{1}{\sqrt{6}-1} \left(\sqrt[6]{\frac{\mathfrak{P}_1^6(y_i) + \mathfrak{P}_2^6(y_i)}{6}} - \frac{\mathfrak{P}_1(y_i) + \mathfrak{P}_2(y_i)}{6} \right) \right] = 0. \quad (5)$$

Conversely, assume $D_m(\mathfrak{P}_1, \mathfrak{P}_2) = 0$.

$$D_m(\mathfrak{P}_1, \mathfrak{P}_2) = \frac{1}{n} \sum_{i=1}^n \left[\frac{1}{\sqrt{6}-1} \left(\sqrt[6]{\frac{\mathfrak{P}_1^6(y_i) + \mathfrak{P}_2^6(y_i)}{6}} - \frac{\mathfrak{P}_1(y_i) + \mathfrak{P}_2(y_i)}{6} \right) \right] = 0. \quad (6)$$

This is possible if and only if $\alpha_1(y_i) = \alpha_2(y_i), \eta_1(y_i) = \eta_2(y_i), \beta_1(y_i) = \beta_2(y_i), \forall i = 1, 2, \dots, n$.

This proves that $\mathfrak{P}_1 = \mathfrak{P}_2$.

Since the measure $D_m(\mathfrak{P}_1, \mathfrak{P}_2)$ holds the postulates of Definition 5, $D_m(\mathfrak{P}_1, \mathfrak{P}_2)$ is an appropriate divergence measure for PFSs. \square

Theorem 2. Let $\mathfrak{P}_1, \mathfrak{P}_2, \mathfrak{P}_3 \in \text{PFSs}(Y)$; then, the measure $D_m(\mathfrak{P}_1, \mathfrak{P}_2)$ satisfies the following axioms:

- (i) $D_m(\mathfrak{P}_1, \mathfrak{P}_2) = D_m(\mathfrak{P}_2, \mathfrak{P}_1)$.
- (ii) $\forall \mathfrak{P}_1 \subseteq \mathfrak{P}_2 \subseteq \mathfrak{P}_3$, we have
 - (a) $D_m(\mathfrak{P}_1, \mathfrak{P}_2) \leq D_m(\mathfrak{P}_1, \mathfrak{P}_3)$.
 - (b) $D_m(\mathfrak{P}_2, \mathfrak{P}_3) \leq D_m(\mathfrak{P}_1, \mathfrak{P}_3)$.

Proof

- (i) It is easy to understand.
- (ii) (a) Let $\mathfrak{P}_1 \subseteq \mathfrak{P}_2 \subseteq \mathfrak{P}_3$; then, for any $0 \leq i \leq n$,

$$\begin{aligned} \alpha_1(y_i) &\leq \alpha_2(y_i) \leq \alpha_3(y_i), \\ \eta_1(y_i) &\leq \eta_2(y_i) \leq \eta_3(y_i), \\ \text{and } \beta_1(y_i) &\leq \beta_2(y_i) \leq \beta_3(y_i), \text{ and we obtain} \end{aligned}$$

$$\begin{aligned} D_m(\mathfrak{P}_1, \mathfrak{P}_2) &= 1/n \sum_{i=1}^n [1/\sqrt{6} - 1 (\sqrt[6]{\mathfrak{P}_1^6(y_i) + \mathfrak{P}_2^6(y_i)/6} - \\ &\quad \mathfrak{P}_2(y_i) + \mathfrak{P}_2(y_i)/6)] \\ &\leq 1/n \sum_{i=1}^n [1/\sqrt{6} - 1 (\sqrt[6]{\mathfrak{P}_1^6(y_i) + \mathfrak{P}_3^6(y_i)/6} - \\ &\quad \mathfrak{P}_1(y_i) + \mathfrak{P}_3(y_i)/6)] = D_m(\mathfrak{P}_1, \mathfrak{P}_3). \end{aligned}$$

Hence, $D_m(\mathfrak{P}_1, \mathfrak{P}_2) \leq D_m(\mathfrak{P}_1, \mathfrak{P}_3)$. Similarly, we can verify axiom (b). \square

Theorem 3. Let $\mathfrak{P}_1, \mathfrak{P}_2 \in \text{PFSs}$. Then, for $D_m(\mathfrak{P}_1, \mathfrak{P}_2)$, we get

- (1) $D_m(\mathfrak{P}_1^c, \mathfrak{P}_2^c) = D_m(\mathfrak{P}_1, \mathfrak{P}_2)$,
- (2) $D_m(\mathfrak{P}_1^c, \mathfrak{P}_2) = D_m(\mathfrak{P}_1, \mathfrak{P}_2^c)$,
- (3) $\forall \mathfrak{P}_1 \subseteq \mathfrak{P}_2$ or $\mathfrak{P}_2 \subseteq \mathfrak{P}_1$, we get
 - (i) $D_m(\mathfrak{P}_1 \cap \mathfrak{P}_2, \mathfrak{P}_2) = D_m(\mathfrak{P}_1, \mathfrak{P}_1 \cup \mathfrak{P}_2) \leq D_m(\mathfrak{P}_1, \mathfrak{P}_2)$,
 - (ii) $D_m(\mathfrak{P}_1 \cap \mathfrak{P}_2, \mathfrak{P}_1 \cup \mathfrak{P}_2) = D_m(\mathfrak{P}_1, \mathfrak{P}_2)$.

Proof

- (1) Since $\mathfrak{P}_1 = (\alpha_1(y_i), \eta_1(y_i), \beta_1(y_i))$, $\mathfrak{P}_2 = (\alpha_2(y_i), \eta_2(y_i), \beta_2(y_i))$, $\mathfrak{P}_1^c = (\beta_1(y_i), \eta_1(y_i), \alpha_1(y_i))$, $\mathfrak{P}_2^c = (\beta_2(y_i), \eta_2(y_i), \alpha_2(y_i))$; then,

$$D_m(\mathfrak{P}_1^c, \mathfrak{P}_2^c) = 1/n \sum_{i=1}^n [1/\sqrt{6} - 1 (\sqrt[6]{\beta_1^6(y_i) + \eta_1^6(y_i) + \alpha_1^6(y_i) + \beta_2^6(y_i) + \eta_2^6(y_i) + \alpha_2^6(y_i)/6} - \beta_1(y_i) + \eta_1(y_i) + \alpha_1(y_i) + \beta_2(y_i) + \eta_2(y_i) + \alpha_2(y_i)/6)],$$

$$= 1/n \sum_{i=1}^n [1/\sqrt{6} - 1 (\sqrt[6]{\alpha_1^6(y_i) + \eta_1^6(y_i) + \beta_1^6(y_i) + \alpha_2^6(y_i) + \eta_2^6(y_i) + \beta_2^6(y_i)/6} - \alpha_1(y_i) + \eta_1(y_i) + \beta_1(y_i) + \alpha_2(y_i) + \eta_2(y_i) + \beta_2(y_i)/6)]$$

$$\Rightarrow D_m(\mathfrak{P}_1^c, \mathfrak{P}_2^c) = D_m(\mathfrak{P}_1, \mathfrak{P}_2).$$

$$(2) D_m(\mathfrak{P}_1^c, \mathfrak{P}_2) = 1/n \sum_{i=1}^n [1/\sqrt{6} - 1 (\sqrt[6]{\beta_1^6(y_i) + \eta_1^6(y_i) + \alpha_1^6(y_i) + \alpha_2^6(y_i) + \eta_2^6(y_i) + \beta_2^6(y_i)/6} - \beta_1(y_i) + \eta_1(y_i) + \alpha_1(y_i) + \alpha_2(y_i) + \eta_2(y_i) + \beta_2(y_i)/6)],$$

$$= 1/n \sum_{i=1}^n [1/\sqrt{6} - 1 (\sqrt[6]{\alpha_1^6(y_i) + \eta_1^6(y_i) + \beta_1^6(y_i) + \beta_2^6(y_i) + \eta_2^6(y_i) + \alpha_2^6(y_i)/6} - \alpha_1(y_i) + \eta_1(y_i) + \beta_1(y_i) + \beta_2(y_i) + \eta_2(y_i) + \alpha_2(y_i)/6)],$$

$$(ii) \Rightarrow D_m(\mathfrak{P}_1^c, \mathfrak{P}_2) = D_m(\mathfrak{P}_1, \mathfrak{P}_2^c).$$

$$(3) (i) \text{ Take } \mathfrak{P}_1 \subseteq \mathfrak{P}_2; \text{ then, } D_m(\mathfrak{P}_1 \cap \mathfrak{P}_2, \mathfrak{P}_2) = D_m(\mathfrak{P}_1, \mathfrak{P}_2), \text{ also, } D_m(\mathfrak{P}_1, \mathfrak{P}_1 \cup \mathfrak{P}_2) = D_m(\mathfrak{P}_1, \mathfrak{P}_2).$$

Now if $\mathfrak{P}_2 \subseteq \mathfrak{P}_1$, then $D_m(\mathfrak{P}_1 \cap \mathfrak{P}_2, \mathfrak{P}_2) = D_m(\mathfrak{P}_2, \mathfrak{P}_2) = 0$; also, $D_m(\mathfrak{P}_1, \mathfrak{P}_1 \cup \mathfrak{P}_2) = D_m(\mathfrak{P}_1, \mathfrak{P}_1) = 0$.

That is, if $\mathfrak{P}_1 \subseteq \mathfrak{P}_2$ or $\mathfrak{P}_2 \subseteq \mathfrak{P}_1$, we get $D_m(\mathfrak{P}_1 \cap \mathfrak{P}_2, \mathfrak{P}_2) = D_m(\mathfrak{P}_1, \mathfrak{P}_1 \cup \mathfrak{P}_2) \leq D_m(\mathfrak{P}_1, \mathfrak{P}_2)$.

(ii) Its proof is simple. \square

4. MCDM Process Based on PF-DM

In this section, a decision-making process based on PF-DM has been provided, followed by practical examples to demonstrate the concept. Suppose a collection of n alternatives represented by $A = \{A_1, A_2, \dots, A_n\}$ that has been investigated by the decision maker by considering the collection of m different criteria $Q = \{Q_1, Q_2, \dots, Q_m\}$ having weight vectors is $W = (w_1, w_2, \dots, w_m)^T$ so that $\sum_{j=1}^m w_j = 1$. Consider the decision maker assessing the rating of an alternative A_i , ($1 \leq i \leq n$) on the criteria G_j , ($1 \leq j \leq m$) in the form of PFSs. To arrive at the best solution, MCDM takes the following actions.

Step 1. Gather data in the form of PF decision matrix.

$$P_c = [p_{ij}]_{n \times m}.$$

$$P_c = \begin{pmatrix} \alpha_{P_{11}}, \eta_{P_{11}}, \beta_{P_{11}} & (\alpha_{P_{12}}, \eta_{P_{12}}, \beta_{P_{12}}) \cdot (\alpha_{P_{1m}}, \eta_{P_{1m}}, \beta_{P_{1m}}) \\ \alpha_{P_{21}}, \eta_{P_{21}}, \beta_{P_{21}} & (\alpha_{P_{22}}, \eta_{P_{22}}, \beta_{P_{22}}) \cdot (\alpha_{P_{2m}}, \eta_{P_{2m}}, \beta_{P_{2m}}) \\ \vdots & \vdots \\ \alpha_{P_{n1}}, \eta_{P_{n1}}, \beta_{P_{n1}} & (\alpha_{P_{n2}}, \eta_{P_{n2}}, \beta_{P_{n2}}) \cdot (\alpha_{P_{nm}}, \eta_{P_{nm}}, \beta_{P_{nm}}) \end{pmatrix}.$$

Step 2. Evaluate the ideal solution $N = \min[p_{ij}]_{n \times m}$.

Step 3. Based on (4), compute the PF-DM.

Step 4. Arrange the values of PF-DM from lower order to top order.

Step 5. Based on Step 4, choose the best alternative.

5. Solution of Problems Based on PF-DM

The authors used the proposed MCDM model to recognize the pattern and breakout of dengue disease in this section.

Step 1. Picture fuzzy (PF) decision matrix $P_c = [p_{ij}]_{5 \times 5}$ is denoted in Table 1.

Step 2. The ideal solution $N = \{(y_1, 0.1000, 0.1000, 0.1000), (y_2, 0.0.1000, 0.3000), (y_3, 0.3000, 0.1000, 0.1000), (y_4, 0.2000, 0.1000, 0.1000), (y_5, 0.1000, 0.1000, 0.2000)\}$.

TABLE 1: PF decision matrix.

Symptoms	
B_1	$\{(y_1, 0.1, 0.4, 0.3), (y_2, 0.4, 0.1, 0.3), (y_3, 0.3, 0.2, 0.4), (y_4, 0.2, 0.3, 0.5), (y_5, 0.5, 0.1, 0.3)\}$
B_2	$\{(y_1, 0.3, 0.3, 0.3), (y_2, 0.0, 0.2, 0.7), (y_3, 0.6, 0.1, 0.1), (y_4, 0.4, 0.4, 0.2), (y_5, 0.1, 0.5, 0.3)\}$
B_3	$\{(y_1, 0.5, 0.1, 0.3), (y_2, 0.3, 0.4, 0.3), (y_3, 0.6, 0.1, 0.3), (y_4, 0.4, 0.4, 0.1), (y_5, 0.1, 0.5, 0.2)\}$
B_4	$\{(y_1, 0.6, 0.2, 0.1), (y_2, 0.3, 0.2, 0.4), (y_3, 0.4, 0.2, 0.3), (y_4, 0.6, 0.1, 0.1), (y_5, 0.4, 0.2, 0.2)\}$
B_5	$\{(y_1, 0.1, 0.4, 0.3), (y_2, 0.4, 0.1, 0.3), (y_3, 0.3, 0.2, 0.4), (y_4, 0.2, 0.3, 0.4), (y_5, 0.5, 0.1, 0.3)\}$

Step 3. The PF-DM values among the symptoms $B_1, B_2, B_3, B_4,$ and B_5 and the ideal solution N are

$$D_m(B_1, N) = 0.0629, D_m(B_2, N) = 0.1006,$$

$$D_m(B_3, N) = 0.0824,$$

$$D_m(B_4, N) = 0.0866, D_m(B_5, N) = 0.0558.$$

Step 4. Arrange the values of PF-DM from lower order to top order such that $B_5 < B_1 < B_3 < B_4 < B_2$.

Step 5. The most frequent symptom is B_5 which can be seen in Figure 1.

Example 1. In the context of dengue disease, it is critical to provide a productive path in crisis response in order to avoid additional misfortunes and save people’s lives. As a result of such a life-threatening situation, health professionals must respond quickly in order to effectively control the situation and prevent further deaths. There are five major symptoms, namely, aching muscles and joints (B_1), body rash that can disappear and then reappear (B_2), high fever (B_3), intense headache (B_4), and vomiting and feeling nauseous (B_5), that can be investigated to diagnose this disease. All symptoms, $B_1, B_2, B_3, B_4,$ and $B_5,$ are penned in terms of PFSs in Table 1.

An unknown pattern is classified into a specific pattern using numerous fuzzy information parameters such as distance, similarity, accuracy, and others. For pattern recognition, we employ the proposed PF-DM. A pattern recognition problem is phrased as follows in the picture fuzzy environment.

(i) Step 1. PF decision matrix $P_c = [p_{ij}]_{3 \times 5}$ is denoted in Table 2.

Step 2. An unknown pattern represented by N is:

$$\bar{N} = \{(0.1000, 0.1000, 0.1000), (0.2000, 0.3000, 0.1000), (0.2000, 0.1000, 0.0), (0.1000, 0.0, 0.2000), (0.3000, 0.1000, 0.1000)\}$$

Step 3. The PF-DM values among the alternatives $B_1, B_2,$ and B_3 and the unknown pattern \bar{N} are

$$D_m(B_1, \bar{N}) = 0.2953, D_m(B_2, \bar{N}) = 0.3303, D_m(B_3, \bar{N}) = 0.2495.$$

Step 4. Arrange the values of PF-DM from lower order to top order such that $B_3 < B_1 < B_2$.

Step 5. Hence, B_3 is an optimal pattern. Figure 2 illustrates the ranking order of the alternatives.

Example 2. Suppose $A = \{B_1, B_2, B_3\}$ are some well-known patterns distinguished by PFSs in the universal set $Y = \{Y_1, Y_2, Y_3, Y_4, Y_5\}$ which are given in the table below.

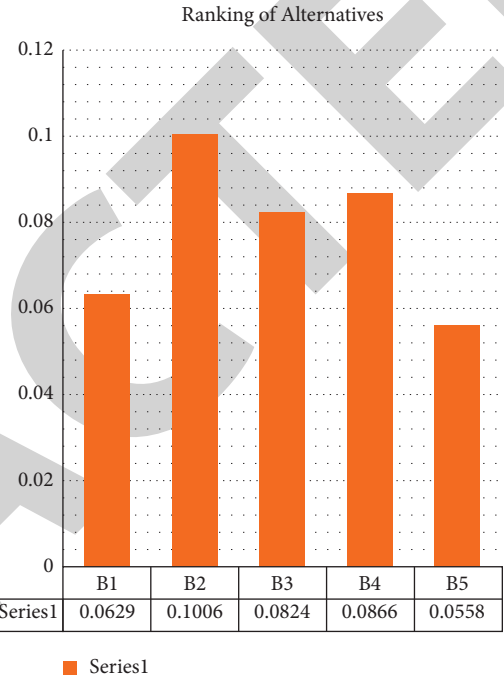


FIGURE 1: Ranking of symptoms by PF-DM.

Because determining which option is the best match for a particular choice issue is impractical in real life, Wang and Triantaphyllou [32] developed testing criteria to examine the validity of MCDM approaches, which are as follows.

6. Validity Testing of Criteria for the Proposed MCDM Model

Criterion 1. An effective MCDM approach should not change the indication of the best option when replacing a non-optimal option with a worse option without changing the relative relevance of any decision criterion.

Criterion 2. The transitive property should be followed by an effective MCDM approach.

Criterion 3. When an MCDM problem is broken down into smaller problems and the same MCDM method is used to assess the options on the smaller problems, the combined ranking of the alternatives should equal the undecomposed problem’s original rating. These testing criteria are being used to evaluate the proposed PF-DM MCDM model’s validity.

TABLE 2: PF decision matrix.

Patterns	
B_1	$\{(y_1, 0.4, 0.3, 0.1), (y_2, 0.5, 0.3, 0.2), (y_3, 0.4, 0.3, 0.0), (y_4, 0.7, 0.0, 0.2), (y_5, 0.6, 0.1, 0.1)\}$
B_2	$\{(y_1, 0.7, 0.1, 0.1), (y_2, 0.2, 0.3, 0.4), (y_3, 0.2, 0.1, 0.5), (y_4, 0.1, 0.5, 0.2), (y_5, 0.3, 0.3, 0.3)\}$
B_3	$\{(y_1, 0.1, 0.3, 0.4), (y_2, 0.4, 0.3, 0.1), (y_3, 0.3, 0.4, 0.2), (y_4, 0.2, 0.5, 0.3), (y_5, 0.5, 0.3, 0.1)\}$

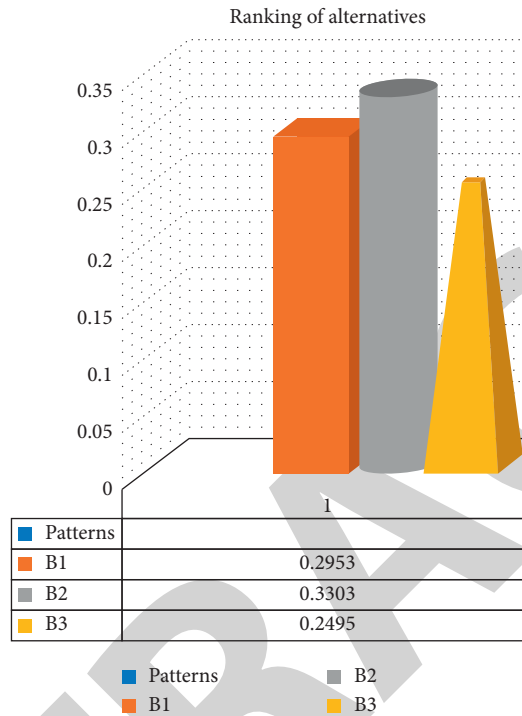


FIGURE 2: Ranking of alternatives by PF-DM.

TABLE 3: PF decision matrix.

Symptoms	
B_1	$\{(y_1, 0.3, 0.4, 0.1), (y_2, 0.3, 0.1, 0.4), (y_3, 0.4, 0.2, 0.3), (y_4, 0.5, 0.3, 0.2), (y_5, 0.3, 0.1, 0.5)\}$
B_2	$\{(y_1, 0.3, 0.3, 0.3), (y_2, 0.70, 0.2, 0.0), (y_3, 0.1, 0.1, 0.6), (y_4, 0.2, 0.4, 0.4), (y_5, 0.3, 0.5, 0.1)\}$
B_3	$\{(y_1, 0.5, 0.1, 0.3), (y_2, 0.3, 0.4, 0.3), (y_3, 0.6, 0.1, 0.3), (y_4, 0.4, 0.4, 0.1), (y_5, 0.1, 0.5, 0.2)\}$
B_4	$\{(y_1, 0.6, 0.2, 0.1), (y_2, 0.3, 0.2, 0.4), (y_3, 0.4, 0.2, 0.3), (y_4, 0.6, 0.1, 0.1), (y_5, 0.4, 0.2, 0.2)\}$
B_5	$\{(y_1, 0.1, 0.4, 0.3), (y_2, 0.4, 0.1, 0.3), (y_3, 0.3, 0.2, 0.4), (y_4, 0.2, 0.3, 0.4), (y_5, 0.5, 0.1, 0.3)\}$

6.1. *Testing Criterion 1.* Decision matrix provided in Table 3 is attained by altering the AMD and RMD of the alternative B_1 (non-favorable) and B_2 (inferior) in the original decision matrix to check the efficacy of the suggested model under test criterion 1 of Example 1.

The PF-DM values among the symptoms $B_1, B_2, B_3, B_4,$ and B_5 and the ideal solution N are $D_m(B_1, N) = 0.0660,$ $D_m(B_2, N) = 0.1037,$ $D_m(B_3, N) = 0.0855,$ $D_m(B_4, N) = 0.0896,$ and $D_m(B_5, N) = 0.0590.$ As a result, the proposed technique supported test criterion 1 provided in [32].

6.2. *Testing Criteria 2 and 3.* To validate the suggested approach, use test criterion 2 and test criterion 3. The MCDM problem presented in Example 1 has been divided into five smaller subsets such as $\{B_1, B_2, B_3, B_4\}, \{B_1, B_2, B_3, B_5\}, \{B_1, B_3, B_4, B_5\}, \{B_1, B_2, B_3, B_5\},$ and $\{B_2, B_3, B_4, B_5\}.$ We

TABLE 4: TC among the proposed and existing techniques.

Techniques	Executing time
Proposed model	0.3513 seconds
Kadian and Kumar [6]	0.4133 seconds
Thao et al. [33]	0.3910 seconds
Wang et al. [34]	0.4402 seconds
Wei et al. [35]	0.4000 seconds

may obtain the following arrangements for each subset using the suggested MCDM model: $B_1 < B_3 < B_4 < B_2, B_5 < B_1 < B_3 < B_2, B_5 < B_3 < B_4 < B_1, B_5 < B_1 < B_3 < B_1,$ and $B_5 < B_3 < B_4 < B_2,$ respectively. By aggregating all these arrangements, overall ranking $B_5 < B_1 < B_3 < B_4 < B_2$ is obtained which is identical to the original one. Thereby, the suggested MCDM model is validated under the test criteria 2 and 3 presented in [32].

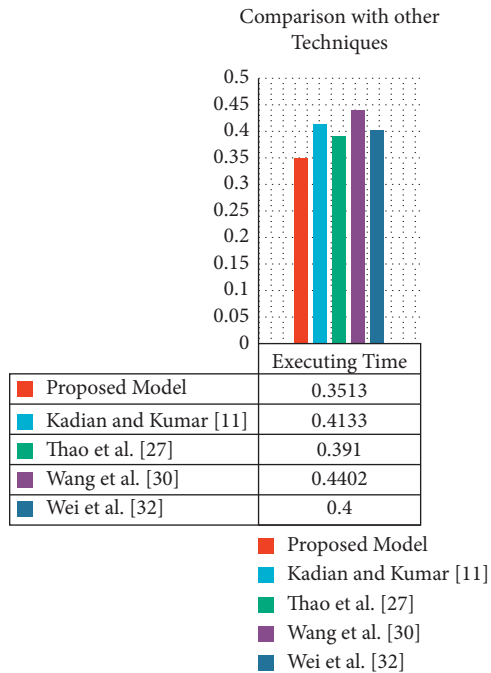


FIGURE 3: Graphical view of TC analysis.

Abbreviations	Description
DM	Divergence measure
FSs	Fuzzy sets
IFSs	Intuitionistic fuzzy sets
MD	Membership degree
MCDM	Multiple-criteria decision making
NMD	Non-membership degree
PFSs	Picture fuzzy sets
PF-DM	Picture fuzzy divergence measure
SM	Similarity measure

6.3. Comparison Based on Time Complexity. In order to strengthen the results obtained from the proposed MCDM model, TC analysis is performed in this section. TC is the time required to execute an algorithm to reach the final result. TC is measured among the proposed and the existing techniques presented by Kadian and Kumar [6], Thao et al. [33], Wang et al. [34], and Wei et al. [35]. The executing time of each technique is evaluated with the help of MATLAB which is presented in Table 4, and its graphical view is drawn in Figure 3. From Table 4, our approach takes less time as compared to others, and hence the proposed MCDM model is more effective and resolves the issues rapidly.

7. Conclusions

A PF-DM is proposed in this study, along with proof of its validity, and some of its features are studied. Existing DMs for PFSs are less feasible than the suggested DM. The suggested divergence metric surpasses other current similarities and DMs by illustrating real challenges in medical analysis and pattern recognition. It can be seen that in

pattern recognition, the suggested divergence metric can assign an unknown pattern to one of the previously recognized patterns. Diseases are diagnosed in medical analysis based on their symptoms, which are represented by visual fuzzy numbers. In addition, pattern recognition employs the proposed divergence measure. It holds in both circumstances.

As a result, the proposed DM is useful for pattern identification as well as medical diagnostics. In addition, the following are the study's long-term goals. (i) It is possible to develop a parametric generalized divergence measure. (ii) PFS can be enhanced by including a third parameter, the degree of refusal. (iii) The suggested divergence metric may be expanded and implemented in an image fuzzy soft set environment. (iv) The suggested DMs may be used to a wide range of real-world problems [36].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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