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Retraction

Retracted: On Topological Properties of Degree-Based Entropy of Hex-Derived Network of Type 3

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] C.-B. Zhu, H. Ali, B. Ali, P. Ali, and J.-B. Liu, "On Topological Properties of Degree-Based Entropy of Hex-Derived Network of Type 3," *Journal of Function Spaces*, vol. 2022, Article ID 2557696, 7 pages, 2022.

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Research Article

On Topological Properties of Degree-Based Entropy of Hex-Derived Network of Type 3

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Hex-derived network has an assortment of significant applications in medicine store, equipment, and network organization. Graph entropy depends upon distribution probability of vertex set and on graph itself. There are numerous issues in discrete math, software engineering, statistics, and data innovation where graph entropies are utilized to portray the reasonable constructions. In this paper, we talk about hex-derived network of type 3 denoted as HDN3(n). We likewise figure degree-based entropies, for example, Randic', ABC, and GA entropy of HDN3(n).

1. Introduction and Preliminary Results

A graph is set of points, where each pair of points tt also known as vertex is connected by an edge (also known as link or line). Here $\mathscr{V}$, $\mathscr{E}$, is the set of vertices and edges, respectively. Topological indices, for example, are a tool developed by graph theory for chemists. Chemical graphs are frequently used to model molecules and molecular compounds. A graph-theoretic illustration of structural formula of chemical compound is molecular graph, with vertices and edges correspond atoms and chemical bonds.

The structure that corresponds to a chemical system, used to characterize the components such as atoms and bonds between them, is a chemical graph. In it, the vertices represent the atoms, and the edges represent the chemical bonds. Molecular graph can represent the structural formula of chemical compound.

Cheminformatics is the combination of many fields like mathematics, information technology, and chemistry. Cheminformatics deals to predict physiochemical and biochemical activities of compounds like alkanes and benzenoid in QSAR and QSPR studies. To predict these characteristics, structure invariants are used, known as topological indices. These indices are uniquely defined for each structure.

Topological index is a function $Top : \Sigma \longrightarrow \mathbb{R}$, where $\mathbb{R}$ represents the real numbers and Σ represents simple graph which containing a characteristic that if $\mathscr{G}_1$ and $\mathscr{G}_2$ are isomorphic, then $\mathrm{Top}(\mathscr{G}_1) = \mathrm{Top}(\mathscr{G}_2)$. A whole graph may be expressed by a uniquely defined number set, a polynomial, a matrix, a relation table, or numerical value (known as a topological index).

The first topological indices are Wiener index [1] and written as

$$W(\mathscr{G}) = \sum_{(\check{u},\check{v})\subseteq\mathscr{V}(\mathscr{G})} d(\check{u},\check{v}). \tag{1}$$

Entropy has a variety of uses in field of biology, chemistry, information technology, and computer science [2]. Entropy can also use to decompose the graph into a special kind of subgraph. Due to the decomposition, a tree-like structure is obtained. Rather than computing the entropy of entire graph, we can simply figure the entropy of that inferred structure [3].

Graph entropy was first defined by Mowshowitz and Kö rner investigate the problem related to information and coding theory with the help of graph entropy. The definition of Entropy gave by Shannon in 1948 is [4].

For graph $\mathscr{G}$, $\mathscr{V}(\mathscr{G})$ is finite vertex set. Let $\mathscr{P}$ be the density of probability of vertex set. $\mathscr{V}\mathscr{P}(\mathscr{G})$ is vertex packing polytope of $\mathscr{G}$. Then, entropy of $\mathscr{G}$ with respect to P is

$$H(\mathscr{G},\mathscr{P})=\min_{a\in\mathscr{V}\mathscr{P}(\mathscr{G})}\sum_{i=1}^{n}p_i\log\left(\frac{1}{a_i}\right). \tag{2}$$

Graph entropy has been used comprehensively to depict the design of graph-based systems in mathematical science [5]. Rashevsky said that graph entropy is reliant upon order of vertices [6].

Hex-derived network of type 3 is built from hexagonal lattice. HX(2) is two-dimensional bunch of six triangles. Subsequent to adding a layer of triangles around the side of HX(2) structure HX(3), see Figure 1. Essentially adding n number of layers of triangles, we got HX(n). After supplant all K_3 subgraphs into a planar octahedron POH once, the subsequent graph will be hex-derived network of type 3. For point by point development, we allude the peruser to worry with the article [7, 8].

Randic' index is [9–11]

$$R_\alpha(\mathscr{G})=\sum_{\breve{u}\breve{v}\in\mathscr{E}(\mathscr{G})}\left(\mathfrak{d}_{\breve{u}}\times\mathfrak{d}_{\breve{v}}\right)^{\alpha}, \tag{3}$$

where $\alpha=1,-1,1/2,-1/2$.

ABC index is [12]

$$\mathrm{ABC}(\mathscr{G})=\sum_{\breve{u}\breve{v}\in\mathscr{E}(\mathscr{G})}\sqrt{\frac{\mathfrak{d}_{\breve{u}}+\mathfrak{d}_{\breve{v}}-2}{\mathfrak{d}_{\breve{u}}\times\mathfrak{d}_{\breve{v}}}}. \tag{4}$$

GA index is [13]

$$\mathrm{GA}(\mathscr{G})=\sum_{\breve{u}\breve{v}\in\mathscr{E}(\mathscr{G})}\frac{2\sqrt{\mathfrak{d}_{\breve{u}}\times\mathfrak{d}_{\breve{v}}}}{\left(\mathfrak{d}_{\breve{u}}+\mathfrak{d}_{\breve{v}}\right)}. \tag{5}$$

1.1. Entropy Based on Degree. Degree-based entropy is defined as

$$\mathrm{ENT}_d(\mathscr{G})=\log(2q)-\frac{1}{2q}\sum_{i=1}^{p}\left[\log\left(\mathfrak{d}(\breve{v}_i)\right)^{\mathfrak{d}(\breve{v}_i)}\right]. \tag{6}$$

1.2. Entropy Based on Edge Weight. It is defined as [14]

$$\mathrm{ENT}_{\mathfrak{d}}(\mathscr{G})=-\sum_{\breve{u}'\breve{v}'\in\mathscr{E}(\mathscr{G})}\frac{\mathfrak{d}\left(\breve{u}'\breve{v}'\right)}{\sum_{\breve{u}\breve{v}\in\mathscr{E}(\mathscr{G})}\mathfrak{d}(\breve{u}\breve{v})}\log\left[\frac{\mathfrak{d}\left(\breve{u}'\breve{v}'\right)}{\sum_{\breve{u}\breve{v}\in\mathscr{E}(\mathscr{G})}\mathfrak{d}(\breve{u}\breve{v})}\right]. \tag{7}$$

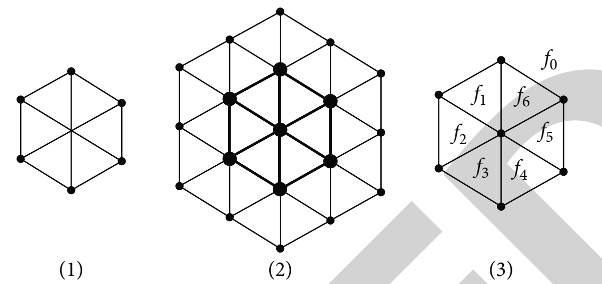

Figure 1: Hexagonal lattice.

1.2.1. Randic' Entropy. From equation (3) and equation (7), we have

$$\mathrm{ENT}_{R_\alpha}(\mathscr{G})=\log(R_\alpha)-\frac{1}{R_\alpha}\sum_{i=1}^{m}\sum_{\breve{u}\breve{v}\in\mathscr{E}_i(\mathscr{G})}\log\left[\left(\left(\mathfrak{d}(\breve{u})\times\mathfrak{d}(\breve{v})\right)^{\alpha}\right]^{\left(\left(\mathfrak{d}(\breve{u})\times\mathfrak{d}(\breve{v})\right)^{\alpha}\right.}. \tag{8}$$

1.2.2. ABC Entropy. From equation (4) and equation (7), we have

$$\mathrm{ENT}_{\mathrm{ABC}}(\mathscr{G})=\log(\mathrm{ABC})-\frac{1}{\mathrm{ABC}}\sum_{i=1}^{m}\sum_{\breve{u}\breve{v}\in\mathscr{E}_i(\mathscr{G})}\log\cdot\left[\sqrt{\frac{\mathfrak{d}(\breve{u})+\mathfrak{d}(\breve{v})-2}{\mathfrak{d}(\breve{u})\times\mathfrak{d}(\breve{v})}}\right]^{\sqrt{\mathfrak{d}(\breve{u})+\mathfrak{d}(\breve{v})-2/\mathfrak{d}(\breve{u})\times\mathfrak{d}(\breve{v})}}. \tag{9}$$

1.2.3. GA Entropy. From equation (5) and equation (7), we have

$$\mathrm{ENT}_{\mathrm{GA}}(\mathscr{G})=\log(\mathrm{GA})-\frac{1}{\mathrm{GA}}\sum_{i=1}^{m}\sum_{\breve{u}\breve{v}\in\mathscr{E}_i(\mathscr{G})}\log\cdot\left[\frac{2\sqrt{\mathfrak{d}(\breve{u})\times\mathfrak{d}(\breve{v})}}{\mathfrak{d}(\breve{u})+\mathfrak{d}(\breve{v})}\right]^{2\sqrt{\mathfrak{d}(\breve{u})\times\mathfrak{d}(\breve{v})}/\mathfrak{d}(\breve{u})+\mathfrak{d}(\breve{v})}. \tag{10}$$

2. Main Results

Xu and Fan [15] and Shao et al. [16] found the metric dimension. Imran et al. [17] found the topological indices for hex-derived network. Song et al. [18] found the entropy of HDN of types 1 and 2. Zhao et al. [19] found entropy of dominating David-derived network. In this article, we examine HDN3(n) and figure the specific outcomes for entropies dependent on edges. These entropies and their variations are right now exposed to broad examination movement, see [20–22]. For basic documentations and definitions, see [2, 23].

2.1. Result on Hex-Derived Network of Type 3. Here, we ascertain specific degree-based entropies of hex-derived network of third type. HDN3(n) is displayed in Figure 2. The edge partition of HDN3(n) is displayed in Table 1. We process Randic' entropy, ABC entropy, and GA entropy for HDN3(n).

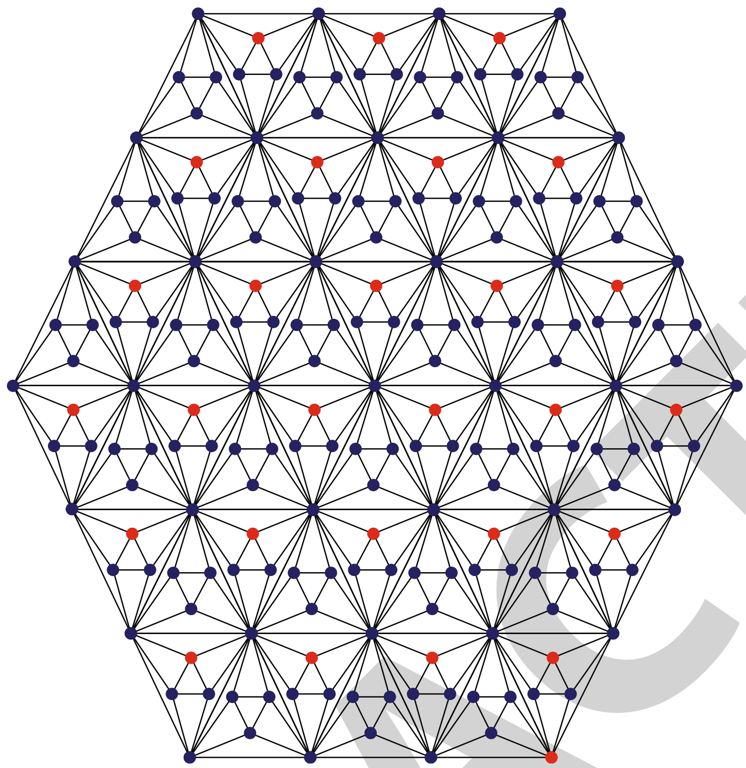

Figure 2: Hex-derived network of third type.

Table 1: Edge partition of HDN3(n).

$(d(\check{u}), d(\check{v}))$	Number of edges
(4, 4)	$18n^2 - 36n + 18$
(4, 7)	24
(4, 10)	$36n - 72$
(4, 18)	$36n^2 - 108n + 84$
(7, 10)	12
(7, 18)	6
(10, 10)	$6n - 18$
(10, 18)	$12n - 24$
(18, 18)	$9n^2 - 33n + 30$

2.2. Randic' Entropy of HDN3(n). If $\mathscr{G} \cong \mathrm{HDN3}(n)$, then from Table 1 and equation (3), we get

$$\begin{aligned} R_\alpha(\mathscr{G}) = {} & 18n^2 - 36n + 18 \times (16)^\alpha + (24) \times (28)^\alpha + (36n - 72) \\ & \times (40)^\alpha + \left(36n^2 - 108n + 84\right) \times (72)^\alpha + (12) \\ & \times (70)^\alpha + (6) \times (126)^\alpha + (6n - 18) \times (100)^\alpha \\ & + (12n - 24) \times (180)^\alpha + \left(9n^2 - 33n + 30\right) \times (324)^\alpha. \end{aligned} \tag{11}$$

For $\alpha = 1$,

$$\Rightarrow R_1(\mathscr{G}) = 5796n^2 - 14844n + 9324. \tag{12}$$

Table 2: Comparison table.

n	ENT_{R_1}	$\mathrm{ENT}_{R_{-1}}$	$\mathrm{ENT}_{R_{1/2}}$	$\mathrm{ENT}_{R_{-1/2}}$	$\mathrm{ENT}_{\mathrm{ABC}}$	$\mathrm{ENT}_{\mathrm{GA}}$
2	1.726216	1.750181	1.79585	1.799301	1.817694	1.817863
3	2.243602	2.299717	2.370664	2.378306	2.407126	2.409051
4	2.581687	2.63378	2.71567	2.722851	2.755102	2.757506
5	2.826008	2.874439	2.962203	2.968893	3.002872	3.005505
6	3.016888	3.062671	3.154083	3.160403	3.195426	3.198193
7	3.173439	3.217283	3.311178	3.31722	3.352943	3.355799
8	3.306106	3.348485	3.444179	3.450007	3.486233	3.489152
9	3.421206	3.462443	3.559499	3.56516	3.601764	3.60473
10	3.522844	3.563168	3.661292	3.666817	3.703716	3.706719

For $\alpha = -1$,

$$\Rightarrow R_{-1}(\mathscr{G}) = \frac{119}{72} n^2 - \frac{1907}{675} n + \frac{50921}{37800}. \tag{13}$$

For $\alpha = 1/2$,

$$\Rightarrow R_{1/2}(\mathscr{G}) = 539.470129n^2 - 1205.729503n + 662.146963. \tag{14}$$

For $\alpha = -1/2$,

$$\Rightarrow R_{-1/2}(\mathscr{G}) = 9.242641n^2 - 16.374728n + 7.597478. \tag{15}$$

From Table 1 and equation (8), Randic' entropy will be

$$\begin{aligned}\mathrm{ENT}_{(R_\alpha)}(\mathscr{G}) = \log\left(R_\alpha\right) - \frac{1}{R_\alpha}\Big[&\left(18n^2-36n+18\right)\times\log\left(16^\alpha\right)^{16^\alpha}\\ &+(24)\times\log\left(28^\alpha\right)^{28^\alpha}+(36n-72)\times\log\left(40^\alpha\right)^{40^\alpha}\\ &+\left(36n^2-108n+84\right)\times\log\left(72^\alpha\right)^{72^\alpha}+(12)\\ &\times\log\left(70^\alpha\right)^{70^\alpha}+(6)\times\log\left(126^\alpha\right)^{126^\alpha}+(6n-18)\\ &\times\log\left(100^\alpha\right)^{100^\alpha}+(12n-24)\times\log\left(180^\alpha\right)^{180^\alpha}\\ &+\left(9n^2-33n+30\right)\times\log\left(324^\alpha\right)^{324^\alpha}\Big].\end{aligned} \tag{16}$$

For $\alpha = 1$,

$$\begin{aligned}\mathrm{ENT}_{(R_1)}(\mathscr{G}) = \log\left(R_1\right) - \frac{1}{R_1}\Big[&\left(18n^2-36n+18\right)\times\log\left(16\right)^{16}\\ &+(24)\times\log\left(28\right)^{28}+(36n-72)\times\log\left(40\right)^{40}\\ &+\left(36n^2-108n+84\right)\times\log\left(72\right)^{72}+(12)\\ &\times\log\left(70\right)^{70}+(6)\times\log\left(126\right)^{126}+(6n-18)\\ &\times\log\left(100\right)^{100}+(12n-24)\times\log\left(180\right)^{180}\\ &+\left(9n^2-33n+30\right)\times\log\left(324\right)^{324}\Big],\end{aligned}$$

$$\begin{aligned}\Rightarrow \mathrm{ENT}_{(R_1)}(\mathscr{G}) = \log\left(R_1\right) - \frac{1}{R_1}\big(&12481.741636n^2\\ &-33600.582853n+22135.973678\big).\end{aligned} \tag{17}$$

For $\alpha = 1/2$,

$$\begin{aligned}\mathrm{ENT}_{(R_{1/2})}(\mathscr{G}) = \log\left(R_{1/2}\right) - \frac{1}{R_{1/2}}\Big[&\left(18n^2-36n+18\right)\\ &\times\log\left(\sqrt{16}\right)^{\sqrt{16}}+(24)\times\log\left(\sqrt{28}\right)^{\sqrt{28}}+(36n-72)\\ &\times\log\left(\sqrt{40}\right)^{\sqrt{40}}+\left(36n^2-108n+84\right)\times\log\left(\sqrt{72}\right)^{\sqrt{72}}\\ &+(12)\times\log\left(\sqrt{70}\right)^{\sqrt{70}}+(6)\times\log\left(\sqrt{126}\right)^{\sqrt{126}}+(6n-18)\\ &\times\log\left(\sqrt{100}\right)^{\sqrt{100}}+(12n-24)\times\log\left(\sqrt{180}\right)^{\sqrt{180}}\\ &+\left(9n^2-33n+30\right)\times\log\left(\sqrt{324}\right)^{\sqrt{324}}\Big],\end{aligned}$$

$$\begin{aligned}\Rightarrow \mathrm{ENT}_{(R_{1/2})}(\mathscr{G}) = \log\left(R_{1/2}\right) - \frac{1}{R_{1/2}}\big(&530.382264n^2\\ &-1259.440263n+730.504396\big).\end{aligned} \tag{18}$$

For $\alpha = -1$,

$$\begin{aligned}\mathrm{ENT}_{(R_{-1})}(\mathscr{G}) = \log\left(R_{-1}\right) - \frac{1}{R_{-1}}\Bigg[&\left(18n^2-36n+18\right)\times\log\left(\frac{1}{16}\right)^{1/16}+(24)\\ &\times\log\left(\frac{1}{28}\right)^{1/28}+(36n-72)\times\log\left(\frac{1}{40}\right)^{1/40}+\left(36n^2-108n+84\right)\\ &\times\log\left(\frac{1}{72}\right)^{1/72}+(12)\times\log\left(\frac{1}{70}\right)^{1/70}+(6)\times\log\left(\frac{1}{126}\right)^{1/126}\\ &+(6n-18)\times\log\left(\frac{1}{100}\right)^{1/100}+(12n-24)\times\log\left(\frac{1}{180}\right)^{1/180}\\ &+\left(9n^2-33n+30\right)\times\log\left(\frac{1}{324}\right)^{1/324}\Bigg],\end{aligned}$$

$$\Rightarrow \mathrm{ENT}_{(R_{-1})}(\mathscr{G}) = \log\left(R_{-1}\right) - \frac{1}{R_{-1}}\left(-2.353039n^2+4.038767n-1.866311\right). \tag{19}$$

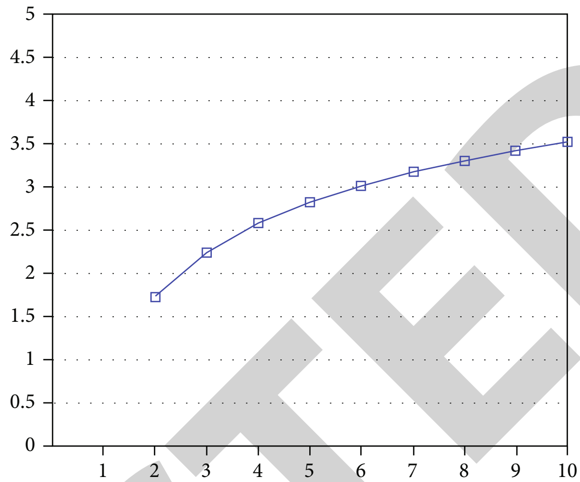

Figure 3: Randic' entropy for $\alpha = 1$.

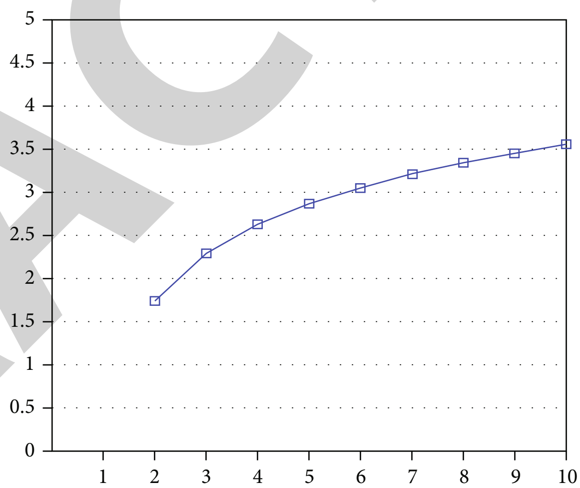

Figure 4: Randic' entropy for $\alpha = -1$.

For $\alpha = -1/2$,

$$\begin{aligned}\mathrm{ENT}_{(R_{-1/2})}(\mathscr{G}) = \log\left(R_{-1/2}\right) - \frac{1}{R_{-1/2}}\Bigg[&\left(18n^2-36n+18\right)\times\log\left(\frac{1}{\sqrt{16}}\right)^{1/\sqrt{16}}\\ &+(24)\times\log\left(\frac{1}{\sqrt{28}}\right)^{1/\sqrt{28}}+(36n-72)\times\log\left(\frac{1}{\sqrt{40}}\right)^{1/\sqrt{40}}\\ &+\left(36n^2-108n+84\right)\times\log\left(\frac{1}{\sqrt{72}}\right)^{1/\sqrt{72}}+(12)\\ &\times\log\left(\frac{1}{\sqrt{70}}\right)^{1/\sqrt{70}}+(6)\times\log\left(\frac{1}{\sqrt{126}}\right)^{1/\sqrt{126}}+(6n-18)\\ &\times\log\left(\frac{1}{\sqrt{100}}\right)^{1/\sqrt{100}}+(12n-24)\times\log\left(\frac{1}{\sqrt{180}}\right)^{1/\sqrt{180}}\\ &+\left(9n^2-33n+30\right)\times\log\left(\frac{1}{\sqrt{324}}\right)^{1/\sqrt{324}}\Bigg],\end{aligned}$$

$$\Rightarrow ENT_{(R_{-1/2})}(\mathscr{G}) = \log\left(R_{-1/2}\right) - \frac{1}{R_{-1/2}}\left(-7.276903n^2+13.371733n-6.224837\right), \tag{20}$$

where R_α for $\alpha = 1, -1, 1/2, -1/2$ is in equations (12)–(15), respectively.

2.3. ABC Entropy of HDN3(n). If $\mathcal{G} \cong \mathrm{HDN3}(n)$, then from Table 1 and equation (4), the ABC index is

$$\begin{aligned}\mathrm{ABC}(\mathcal{G}) &= \left(18n^2 - 36n + 18\right) \times \sqrt{\frac{4+4-2}{4\times4}} + (24) \times \sqrt{\frac{4+7-2}{4\times7}} \\ &+ (36n-72) \times \sqrt{\frac{4+10-2}{4\times10}} + \left(36n^2 - 108n + 84\right) \\ &\times \sqrt{\frac{4+18-2}{4\times18}} + (12) \times \sqrt{\frac{7+10-2}{7\times10}} + (6) \\ &\times \sqrt{\frac{7+18-2}{7\times18}} + (6n-18) \times \sqrt{\frac{10+10-2}{10\times10}} \\ &+ (12n-24) \times \sqrt{\frac{10+18-2}{10\times18}} + \left(9n^2 - 33n + 30\right) \\ &\times \sqrt{\frac{18+18-2}{18\times18}},\end{aligned}$$

$$\Rightarrow \mathrm{ABC}(\mathcal{G}) = 32.911846n^2 - 62.832186n + 30.543785. \tag{21}$$

From Table 1 and equation (9), ABC entropy is

$$\begin{aligned}&\mathrm{ENT}_{\mathrm{ABC}}(\mathcal{G}) \\ &= \log(ABC) - \frac{1}{\mathrm{ABC}}\left[\left(18n^2 - 36n + 18\right) \times \log\left(\sqrt{\frac{4+4-2}{4\times4}}\right)^{\sqrt{4+4-2/4\times4}}\right. \\ &+ (24) \times \log\left(\sqrt{\frac{4+7-2}{4\times7}}\right)^{\sqrt{4+7-2/4\times7}} + (36n-72) \\ &\times \log\left(\sqrt{\frac{4+10-2}{4\times10}}\right)^{\sqrt{4+10-2/4\times10}} + \left(36n^2 - 108n + 84\right) \\ &\times \log\left(\sqrt{\frac{4+18-2}{4\times18}}\right)^{\sqrt{4+18-2/4\times18}} + (12) \times \log\left(\sqrt{\frac{7+10-2}{7\times10}}\right)^{\sqrt{7+10-2/7\times10}} \\ &+ (6) \times \log\left(\sqrt{\frac{7+18-2}{7\times18}}\right)^{\sqrt{7+18-2/7\times18}} + (6n-18) \\ &\times \log\left(\sqrt{\frac{10+10-2}{10\times10}}\right)^{\sqrt{10+10-2/10\times10}} + (12n-24) \\ &\times \log\left(\sqrt{\frac{10+18-2}{10\times18}}\right)^{\sqrt{10+18-2/10\times18}} + \left(9n^2 - 33n + 30\right) \\ &\left.\times \log\left(\sqrt{\frac{18+18-2}{18\times18}}\right)^{\sqrt{18+18-2/18\times18}}\right],\end{aligned}$$

$$\Rightarrow \mathrm{ENT}_{\mathrm{ABC}}(\mathcal{G}) = \log(\mathrm{ABC}) - \frac{1}{\mathrm{ABC}}\left(-9.052434n^2 + 17.741996n - 8.591587\right), \tag{22}$$

where ABC index of HDN3(n) is in equation (21).

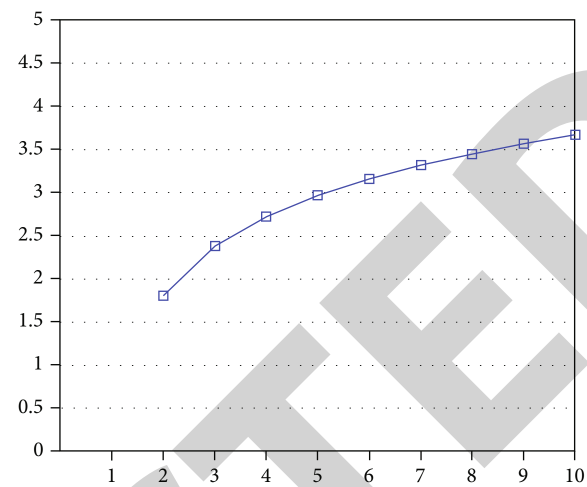

FIGURE 5: Randic' entropy for $\alpha = 1/2$.

2.4. GA Entropy of HDN3(n). If $\mathcal{G} \cong \mathrm{HDN3}(n)$, then from Table 1 and equation (5), GA index is

$$\begin{aligned}\mathrm{GA}(\mathcal{G}) &= \left(18n^2 - 36n + 18\right) \times \frac{2\sqrt{4\times4}}{4+4} + (24) \times \frac{2\sqrt{4\times7}}{4+7} \\ &+ (36n-72) \times \frac{2\sqrt{4\times10}}{4+10} + \left(36n^2 - 108n + 84\right) \\ &\times \frac{2\sqrt{4\times18}}{4+18} + (12) \times \frac{2\sqrt{7\times10}}{3+4} + (6) \times \frac{2\sqrt{7\times18}}{7+18} \\ &+ (6n-18) \times \frac{2\sqrt{10\times10}}{10+10} + (12n-24) \\ &\times \frac{2\sqrt{10\times18}}{10+18} + \left(9n^2 - 33n + 30\right) \times \frac{2\sqrt{18\times18}}{18+18},\end{aligned}$$

$$\Rightarrow GA(\mathcal{G}) = 54.770012n^2 - 102.283973n + 47.03442. \tag{23}$$

From Table 1 and equation (10), we have

$$\begin{aligned}&\mathrm{ENT}_{\mathrm{GA}}(\mathcal{G}) \\ &= \log(\mathrm{GA}) - \frac{1}{GA}\left[\left(18n^2 - 36n + 18\right) \times \log\left(\frac{2\sqrt{4\times4}}{4+4}\right)^{2\sqrt{4\times4/4+4}} + (24)\right. \\ &\times \log\left(\frac{2\sqrt{4\times7}}{4+7}\right)^{2\sqrt{4\times7/4+7}} + (36n-72) \times \log\left(\frac{2\sqrt{4\times10}}{4+10}\right)^{2\sqrt{4\times10/4+10}} \\ &+ \left(36n^2 - 108n + 84\right) \times \log\left(\frac{2\sqrt{4\times18}}{4+18}\right)^{2\sqrt{4\times18/4+18}} + (12) \\ &\times \log\left(\frac{2\sqrt{7\times10}}{3+4}\right)^{2\sqrt{7\times10/3+4}} + (6) \times \log\left(\frac{2\sqrt{7\times18}}{7+18}\right)^{2\sqrt{7\times18/7+18}} \\ &+ (6n-18) \times \log\left(\frac{2\sqrt{10\times10}}{10+10}\right)^{2\sqrt{10\times10/10+10}} + (12n-24) \\ &\times \log\left(\frac{2\sqrt{10\times18}}{10+18}\right)^{2\sqrt{10\times18/10+18}} + \left(9n^2 - 33n + 30\right) \\ &\left.\times \log\left(\frac{2\sqrt{18\times18}}{18+18}\right)^{2\sqrt{18\times18/18+18}}\right],\end{aligned}$$

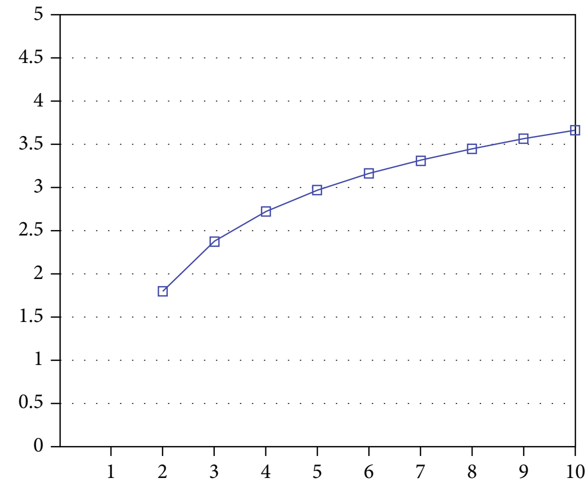

Figure 6: Randic' entropy for $\alpha = -1/2$.

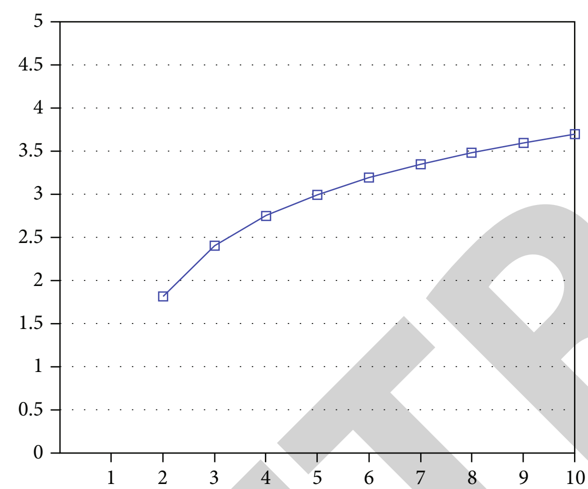

Figure 7: ABC entropy.

$$\Rightarrow \mathrm{ENT}_{\mathrm{GA}}(\mathscr{G}) = \log(\mathrm{GA}) - \frac{1}{\mathrm{GA}}\left(-3.130414n^2 + 7.745222n - 4.732694\right), \tag{24}$$

where GA index of HDN3(n) is in 16.

3. Discussion

Entropy has lot of applications in many fields. In computer science, it tells the flow of information. In the field of chemistry, it is the amount of energy that is unavailable for doing work. So, the calculation of exact numerical value of entropy is beneficial for researchers. That is the reason we register a few upsides of degree-based entropies of HDN3(n). Besides, we build the Table 2, to study the behavior of the degree-based entropies for different upsides of n. From Table 2, we can see that as n increases value of entropy for Randic' and ABC, *GA* also increases. Toward the end, we build some graphical portrayal of these entropies in Figures 3–8, which elaborate the variation in the values of indices as n increases.

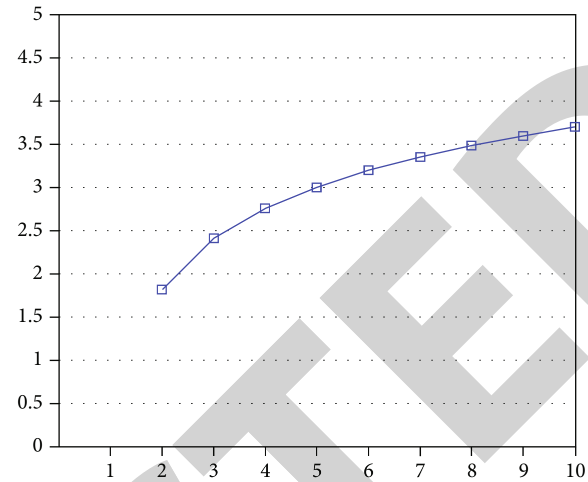

Figure 8: GA entropy.

4. Conclusion

Hex-derived network has its own roots in every field of science like biological and physical. The distribution of probability of vertex set of a graph defines its entropy which is used in physical sciences. In this paper, we talk about the entropy of various degree-based indices for HDN3(n). In Table 2, we enlist some numerical values for abovementioned indices. These numerical values are helpful in QSAR/QSPR studies of HDN. Our future work is to find the entropies of fourth type of HDN and reversed degree-based topological indices of various networks.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflict of interest.

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