Hindawi
Journal of Function Spaces
Volume 2022, Article ID 2613133, 21 pages
https://doi.org/10.1155/2022/2613133

Research Article

An Efficient Method for Solving Fractional Black-Scholes Model with Index and Exponential Decay Kernels

Saima Rashid,[1] **Saad Ihsan Butt**,[2] **Zakia Hammouch**,[3,4,5] **and Ebenezer Bonyah**[6]

[1]*Department of Mathematics, Government College University, Faisalabad 38000, Pakistan*
[2]*Department of Mathematics, COMSATS University, Lahore Campus, Islamabad, Pakistan*
[3]*Division of Applied Mathematics, Thu Dau Mot University, Binh Duong Province, Vietnam*
[4]*Department of Mathematics, Ecole Normale Supéerieure de Meknés, Université Moulay Ismail, Meknes 50000, Morocco*
[5]*Department of Medical Research, China Medical University Hospital, Taichung, Taiwan*
[6]*Department of Mathematics Education, University of Education, Winneba, Kumasi Campus, Ghana*

Correspondence should be addressed to Zakia Hammouch; hammouch_zakia@tdmu.edu.vn

Received 16 April 2022; Revised 1 August 2022; Accepted 9 August 2022; Published 21 September 2022

Academic Editor: Alexander Meskhi

The Black-Scholes equation (BSe) is fascinating in the business world for predicting the performance of financial investment valuation systems. The Caputo fractional derivative (CFD) and Caputo-Fabrizio fractional derivative operators are used in this research to analyze the BSe. The Adomian decomposition method (ADM) and the new iterative transform (NIM) approach are combined alongside the Yang transform. In addition, the convergence and uniqueness results for the aforementioned framework have been calculated. The existence and uniqueness results have been established and frequently accompanied innovative aspects of the prospective system in fixed point terminologies. To provide additional insight into such concepts, a variety of illustrations and tabulations are used. Additionally, the provided techniques regulate and modify the obtained analytical results in a really productive fashion, allowing us to modify and regulate the converging domains of the series solution in a pragmatic manner.

1. Introduction

Recently, the investigation of modified derivatives and integrals has grown in prominence in recent decades, owing to its appealing implications in a wide range of disciplines, including Maxwell fluids [1, 2], circuit theory [3], and epidemics [4, 5]. As an outgrowth of conventional integer analysis, fractional calculus (FC) has been exploited to examine the implications and integrals of indefinite powers. Because integer-order derivative and integral operators are being used to simulate all real-world processes, numerous researchers have proposed multiple variations of fractional operators as a modification of the fractional formulations [6–9]. The interaction effect in FC has been utilized to represent numerous processes in thermodynamics, chemical engineering, biomechanics, and other disciplines, despite the fact that the analyzed formulae in FC are typically reluctant to analyze complicated phenomena [10–14]. In addition, fractional differential formulas have a higher granularity than integer differential operators. Illustrations comprise Katugampola, Weyl, Hadamard, Caputo, Riesz, Riemann, and Liouville, Weyl, Jumarie, Caputo and Fabrizio [15], and Grünwald and Letnikov [16]. Likewise, the Liouville-Caputo and Caputo-Fabrizio fractional filtrations are thought to be ideal.

It is imperative to address fractional-order nonlinear partial differential equations (NPDEs) that regulate the foregoing experimental results in order to successfully comprehend such occurrences. There is, however, no universal comprehensive principle that applies to NPDEs in an attempt to obtain a numerical approach. Researchers have determined successful approaches to derive meaningful numerical methods for NPDEs in recent times, including the inverse scattering transform Sin-Cos method (SCM) [17], homotopy perturbation method (HPM) [18], Adomian decomposition method (ADM) [19, 20], new iterative transform method (NITM) [21], variation iteration method

(VIM) [22], G/G' expansion method [23], Lie symmetry analysis (LSA) [24], Haar wavelet method [25], inverse scattering transform [26], simple equation method [27], Bäcklund transformation [28], and henceforth.

In 1973, Fischer Black and Myron Scholes formulated a mathematical formula for passive investment valuation. The pioneering Black-Scholes equation (BSe) is at the core of contemporary financial economics, and it is indeed tough to communicate about mainstream capitalism without mentioning the revolutionary BSe.

The objective of this paper is to leverage the Yang decomposition method (YDM) and the Yang iterative transform method (YITM) to modify the results into a BSe. The fractional interpretation of BSe is characterized in financial services by [29]:

$$\mathbf{D}_{\varrho}^{\delta}\mathscr{U} + \frac{\varpi^2}{2}\mathscr{S}^2\frac{\partial^2\mathscr{U}}{\partial\mathscr{S}^2} + \zeta\mathscr{S}\frac{\partial\mathscr{U}}{\partial\mathscr{S}} - \zeta\mathscr{U} = 0, \tag{1}$$

subject to the playoff mapping

$$\mathscr{U}(\mathscr{S},\mathscr{T}) = \max\,(\mathscr{S} - E, 0), \tag{2}$$

where $\mathscr{U}(\mathscr{S},\varrho)$ denotes the alternative means worth at $\mathscr{S}$ asset prices of the moment, ϱ, and $\mathscr{T}$ indicates the termination term. The symbol E symbolizes share value. The parameter ζ represents the uncertainty of borrowing until it matures. The continual ϖ indicates the unpredictability of a trading asset. The required assumptions are also entailed: a continuous uncertainty risk premium u, no operating charges, the capacity to transact an unrestricted quantity of inventory, and no restrictions on market manipulation. Ultimately, we provide European alternatives. It is also worth mentioning that $\mathscr{U}(0,\varrho) = 0$ and $\mathscr{U}(\mathscr{S},\mathscr{T}) \approx \mathscr{S}$ as $\mathscr{S} \mapsto \infty$. The parabolic diffusion problem can perhaps be described as the BSe in (1). Inducing the modifications that follow:

$$\begin{aligned}\mathscr{S} &= E\exp\,(\mathbf{y}_1),\\ \varrho &= \mathscr{T} - \frac{2\tau}{\varpi^2},\\ \mathscr{U} &= E\mathbf{U}(\mathbf{y}_1,\varrho),\end{aligned} \tag{3}$$

then (1) diminishes to

$$\mathbf{D}_{\varrho}^{\delta}\mathbf{U}(\mathbf{y}_1,\varrho) = \frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2} + (\zeta - 1)\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1} - \zeta\mathbf{U}(\mathbf{y}_1,\varrho), \tag{4}$$

related initial condition (ICs) are

$$\mathbf{U}(\mathbf{y}_1, 0) = \max\,(\exp\,(\mathbf{y}_1) - 1, 0), \tag{5}$$

where ζ designates the threshold when the direct connection involving wage growth and market instability coincides. Cen and Le proposed the generalized fractional BSe in [30]. The BSe is stated as follows:

$$\begin{aligned}\mathbf{D}_{\varrho}^{\delta}\mathbf{U}(\mathbf{y}_1,\varrho) = {}&-0.08(2 + \sin\mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}\\ &- 0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1} + 0.06\mathbf{U}(\mathbf{y}_1,\varrho),\end{aligned} \tag{6}$$

supplemented ICs

$$\mathbf{U}(\mathbf{y}_1, 0) = \max\,(\mathbf{y}_1 - 25\exp\,(-0.06), 0). \tag{7}$$

The fractional BSe considering a particular resource has been widely explored ([31, 32]). The fractional BSe is a version of the classical BSe that expands its restrictions. The BSe was implemented by Meng and Wang [33] to analyze fractional potential assessment. The fractional BSe was used to determine the insured guarantee valuation for treasury foreign trade in China. Their results indicate that the fractional BSe surpasses the traditional BSe when it pertains to measuring the impact of the pricing system [34]. The Black-Scholes financial theory was calculated using the HPM by Fall et al. [35]. By adopting the Ornstein-Uhlenbeck Procedures, Matadi and Zondi [36] explored the consistent values of BSe. The computational estimation of fractional BSe emerging in the banking system was demonstrated by Kumar et al. [37]. Employing a novel fractional operator, Yavuz and Özdemir [38] suggested a novel strategy for the European efficient market hypothesis.

The ADM introduced a well-known concept during George Adomian's significant surge in 1980. For example, it has been frequently applied to deal with a variety of complex PDEs like the $K(2,2)$ and $K(3,3)$ models [19], biological population model [39], Swift-Hohenberg model [40], and henceforth. The ADM is essential because it overcomes the necessity for a smaller component in the considerations, eliminating the challenges that occur with classic Adomian approaches. The main objective of this research was to leverage the ADM to analyze fractional-order BSe using a recently designed integral transformation known as the "Yang transformation" [41].

Daftardar-Gejji and Jafari [42] proposed NITM in 2006, which is frequently adopted by scholars owing to its usefulness in fractional ODES and PDEs. If a precise result emerges, the iterative method leads to it through repeated estimates. For methodological concerns, a significant fraction of projections can be considered with a satisfactory amount of precision for specific issues. For managing nonlinearity components, the NITM sometimes does not require a restrictive assumption. For instance, researchers exploited NITM to develop analytical results for the fractional Schrödinger equation in [43], and Wang and Liu used NITM to address the fractional Fornberg-Whitham model in [44]. Widatalla and Liu used NITM to develop the Laplace decomposition algorithm in [22].

Due to the aforesaid tendency, we apply the YDM and the YITM to achieve the expressive result of the fractional-order BSe. For renewability algorithmic techniques, the Yang transform efficiently integrates the ADM and NITM. The

Yang transform is a combination of a few different transforms. Both these proposed techniques produce interpretive findings in the sense of a convergent series. The Caputo-Fabrizio fractional derivative operator is used to explain quantitative categorizations of the BSe. The offered methodologies are well demonstrated in modeling and enumeration investigations. The exact-analytical findings are a valuable way to analyze the dynamics of systems that are problematic to computationally analyze, notably for fractional PDEs. Financial and monetary phenomena can be investigated using this approximate expression.

2. Preliminaries

In this part, we address several key ideas, conceptions, and terminologies related to fractional derivative operators involving index and exponential decay as a kernel, as well as the Yang transform's specific repercussions.

Definition 1 (see [9]). The Caputo fractional derivative (CFD) is described as follows:

$$ {}_0^c\mathbf{D}_\varrho^\delta \mathbf{U}(\varrho) = \begin{cases} \dfrac{1}{\Gamma(r-\delta)} \displaystyle\int_0^\varrho \dfrac{\mathbf{U}^{(r)}(\mathbf{y}_1)}{(\varrho-\mathbf{y}_1)^{\delta+1-r}} d\mathbf{y}_1, r-1<\delta<r, \\ \dfrac{d^r}{d\varrho^r}\mathbf{U}(\varrho), \delta = r. \end{cases} \tag{8} $$

Definition 2 (see [15]). The Caputo fractional derivative operator is described as follows:

$$ {}^{CF}\mathbf{D}_\varrho^\delta(\mathbf{U}(\varrho)) = \frac{(2-\delta)\mathbb{A}(\delta)}{2(1-\delta)} \int_0^\varrho \exp\left(-\frac{\delta(\varrho-\mathbf{y}_1)}{1-\delta}\right)\mathbf{U}'(\varrho)d\varrho, \tag{9} $$

where $\mathbf{U} \in \mathbf{H}^1(a_1, a_2)(\textit{Sobolev space}), a_1 < a_2, \delta \in [0, 1]$, and $\mathbb{A}(\delta)$ signifies a normalization function as $\mathbb{A}(\delta) = \mathbb{A}(0) = \mathbb{A}(1) = 1$.

Definition 3 (see [15]). The fractional integral of the Caputo-Fabrizio operator is defined as

$$ {}^{CF}\mathcal{I}_\varrho^\delta(\mathbf{U}(\varrho)) = \frac{2(1-\delta)}{(2-\delta)\mathbb{A}(\delta)}\mathbf{U}(\varrho) + \frac{2\delta}{(2-\delta)\mathbb{A}(\delta)}\int_0^\varrho \mathbf{U}(\mathbf{y}_1)d\mathbf{y}_1. \tag{10} $$

Definition 4 (see [41]). The Yang transform is described as follows:

$$ \mathbf{Y}[\mathbf{U}(\varphi)] = \mathbb{Y}(s_1) = \int_0^\infty \mathbf{U}(\varphi)\exp\left(-\frac{\varphi}{s_1}\right)d\varphi, \varphi > 0. \tag{11} $$

The Yang transform of a range of vital expressions is as follows:

$$ \begin{aligned} \mathbf{Y}[1] &= s_1, \\ \mathbf{Y}[\varphi] &= s_1^2, \\ &\vdots \\ \mathbf{Y}\left[\frac{\varphi^\delta}{\Gamma(\delta+1)}\right] &= s_1^{\delta+1}. \end{aligned} \tag{12} $$

Definition 5 (see [41]). The Yang transform of the CFD operator is mentioned as

$$ \mathbf{Y}\left\{{}_0^c\mathbf{D}_\varrho^\delta(\mathbf{U}(\varrho)), \mathfrak{s}\right\} = \varphi^{-\delta}\mathbf{Q}(\mathfrak{s}) - \sum_{\kappa=0}^{\delta-1}\varphi^{1-\delta-\kappa}(\mathfrak{s})\mathbf{U}^{(\kappa)}(0), r-1 < \delta < r, \varphi > 0. \tag{13} $$

Definition 6 (see [45]). The Yang transform of the Caputo-Fabrizio fractional derivative operator is stated as

$$ \mathbf{Y}\left\{{}_0^{CF}\mathbf{D}_\varrho^\delta(\mathbf{U}(\varphi)), s_1\right\} = \frac{\mathbf{Y}[\mathbf{U}(\varphi) - s_1\mathbf{U}(0)]}{1+\delta(s_1-1)}. \tag{14} $$

Definition 7 (see [46]). The Mittag-Leffler function for single parameter is defined as

$$ E_\delta(z) = \sum_{\kappa=0}^{\infty}\frac{z_1^\kappa}{\Gamma(\kappa\delta+1)}, \delta, z_1 \in \mathbb{C}, \Re(\delta) \geq 0. \tag{15} $$

3. Algorithmic Configuration for Nonlinear PDEs

Let us surmise the fractional version of nonlinear PDE:

$$ \mathbf{D}_\varrho^\delta\mathbf{U}(\mathbf{y}_1,\varrho) + \mathbf{L}\mathbf{U}(\mathbf{y}_1,\varrho) + \mathbf{N}\mathbf{U}(\mathbf{y}_1,\varrho) = \mathbf{Q}(\mathbf{y}_1,\varrho), \varrho > 0, 0 < \delta \leq 1 \tag{16} $$

having ICs

$$ \mathbf{U}(\mathbf{y}_1, 0) = \mathcal{G}(\mathbf{y}_1), \tag{17} $$

where $\mathbf{D}_\varrho^\delta = \partial^\delta\mathbf{U}(\mathbf{y}_1,\varrho)/\partial\varrho^\delta$ represents the Caputo-Fabrizio fractional derivative considering the order $\delta \in (0, 1]$ whilst $\mathbf{L}$ and $\mathbf{N}$ indicates the linear and nonlinear functionals, respectively. Furthermore, $\mathbf{Q}(\mathbf{y}_1,\varrho)$ indicates the source term.

3.1. Construction of Yang Decomposition Method. Incorporating the Yang transformation to (16), we obtain

$$ \mathbf{Y}\left[\mathbf{D}_\varrho^\delta\mathbf{U}(\mathbf{y}_1,\varrho) + \bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho) + \bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right] = \mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)]. \tag{18} $$

Initially, we implement the Yang transform differentiabilty criteria to CFD, and then further implement the Caputo-Fabrizio fractional derivative operator as described in the following:

$$\varphi^{-\delta}\mathscr{U}(\mathbf{y}_1,\varrho)=\sum_{p=0}^{n-1}\varphi^{1-\delta-p}\mathbf{U}^{(p)}(0)+\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]+\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)], \tag{19}$$

$$\frac{1}{1+\delta(\varphi-1)}\mathscr{U}(\mathbf{y}_1,\varrho)=\frac{\varphi}{1+\delta(\varphi-1)}\mathbf{U}(0)+\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]+\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)]. \tag{20}$$

The inverse Yang transform of (19) and (20), respectively, gives

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\mathbf{Y}^{-1}\left[\sum_{p=0}^{n-1}\varphi^{1-\delta-p}\mathbf{U}^{(p)}(0)+\varphi^{\delta}\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)]\right]-\mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]\right], \tag{21}$$

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\mathbf{Y}^{-1}[\varphi\mathbf{U}(0)+(1+\delta(\varphi-1))\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)]]-\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]\right]. \tag{22}$$

The infinite series $\mathbf{U}(\mathbf{y}_1,\varrho)$ illustrates the result of the Yang decomposition approach:

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\sum_{p=0}^{\infty}\mathbf{U}_p(\mathbf{y}_1,\varrho). \tag{23}$$

As a consequence, the nonlinear component $\bar{\mathbf{N}}(\mathbf{y}_1,\varrho)$ can be assessed employing the Adomian decomposition approach, as follows:

$$\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)=\sum_{p=0}^{\infty}\tilde{A}_p(\mathbf{U}_0,\mathbf{U}_1,\cdots),p=0,1,\cdots, \tag{24}$$

where

$$\tilde{A}_p(\mathbf{U}_0,\mathbf{U}_1,\cdots)=\frac{1}{q!}\left[\frac{d^p}{d\delta^p}\bar{\mathbf{N}}\left(\sum_{j=0}^{\infty}\delta^j\mathbf{U}_j\right)\right]_{\delta=0},q>0. \tag{25}$$

Putting (20) and (24) into (21) and (22), respectively, we attain

$$\sum_{p=0}^{\infty}\mathbf{U}_p(\mathbf{y}_1,\varrho)=\mathscr{G}(\mathbf{y}_1)+\tilde{\mathscr{G}}(\mathbf{y}_1)-\mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\sum_{p=0}^{\infty}\tilde{A}_p\right]\right], \tag{26}$$

$$\sum_{p=0}^{\infty}\mathbf{U}_p(\mathbf{y}_1,\varrho)=\mathscr{G}(\mathbf{y}_1)+\tilde{\mathscr{G}}(\mathbf{y}_1)-\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\sum_{p=0}^{\infty}\tilde{A}_p\right]\right]. \tag{27}$$

As a nutshell, the iterative approach for (26) and (27) is as follows:

$$\begin{aligned}
\mathbf{U}_0(\mathbf{y}_1,\varrho)&=\mathscr{G}(\mathbf{y}_1)+\tilde{\mathscr{G}}(\mathbf{y}_1),p=0,\\
\mathbf{U}_{q+1}(\mathbf{y}_1,\varrho)&=-\mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[\bar{\mathbf{L}}(\mathbf{U}_p(\mathbf{y}_1,\varrho))+\sum_{p=0}^{\infty}\tilde{A}_p\right]\right],q\geq 1,\\
\mathbf{U}_{q+1}(\mathbf{y}_1,\varrho)&=-\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\bar{\mathbf{L}}(\mathbf{U}_p(\mathbf{y}_1,\varrho))+\sum_{p=0}^{\infty}\tilde{A}_p\right]\right],q\geq 1.
\end{aligned} \tag{28}$$

3.2. Formation of Yang Iterative Transform Method. Implementing the Yang transform to (16) incorporates the ICs (17), ones obtain

$$\mathbf{Y}\left[\mathbf{D}_{\varrho}^{\delta}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]=\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)]. \tag{29}$$

First, we apply the differentiation rule of Yang transform for CFD, and then we consider for Caputo-Fabrizio fractional derivative operator, respectively, and we get

$$\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]=\varphi^{\delta}\sum_{p=0}^{n-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(\varphi,0)-\varphi^{\delta}\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]+\varphi^{\delta}\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)], \tag{30}$$

$$\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]=\varphi^{\delta}\sum_{p=0}^{n-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(\varphi,0)-(1+\delta(\varphi-1))\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]+(1+\delta(\varphi-1))\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)]. \tag{31}$$

Using the fact of the inverse Yang transform of (30) and (31), respectively, produces

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\mathscr{G}(\varphi)+\mathbf{Y}^{-1}\left\{\varphi^{\delta}\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)]\right\}-\mathbf{Y}^{-1}\left\{\varphi^{\delta}\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]\right\}, \tag{32}$$

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\mathscr{G}(\varphi)+\mathbf{Y}^{-1}\{(1+\delta(\varphi-1))\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1,\varrho)]\}-\mathbf{Y}^{-1}\left\{(1+\delta(\varphi-1))\mathbf{Y}\left[\bar{\mathbf{L}}\mathbf{U}(\mathbf{y}_1,\varrho)+\bar{\mathbf{N}}\mathbf{U}(\mathbf{y}_1,\varrho)\right]\right\}. \tag{33}$$

Employing the recursive approach, we determine

$$\mathbf{U}(\mathbf{y}_1, \varrho) = \sum_{p=0}^{\infty} \mathbf{U}_p(\mathbf{y}_1, \varrho). \tag{34}$$

Moreover, utilizing the linearity $\bar{\mathbf{L}}$ of the operator, thus we have

$$\bar{\mathbf{L}}\left(\sum_{p=0}^{\infty} \mathbf{U}_p(\mathbf{y}_1, \varrho)\right) = \sum_{p=0}^{\infty} \bar{\mathbf{L}}\left[\mathbf{U}_p(\mathbf{y}_1, \varrho)\right], \tag{35}$$

and the nonlinearity $\bar{\mathbf{N}}$ handled by (see [42])

$$\begin{aligned} \bar{\mathbf{N}}\left(\sum_{p=0}^{\infty} \mathbf{U}_p(\mathbf{y}_1, \varrho)\right) &= \bar{\mathbf{N}}(\mathbf{U}_0(\mathbf{y}_1, \varrho)) \\ &+ \sum_{q=1}^{\infty}\left[\bar{\mathbf{N}}\left(\sum_{\kappa=0}^{p} \mathbf{U}_p(\mathbf{y}_1, \varrho)\right) - \bar{\mathbf{N}}\left(\sum_{\kappa=0}^{p-1} \mathbf{U}_p(\mathbf{y}_1, \varrho)\right)\right] \\ &= \bar{\mathbf{N}}(\mathbf{U}_0) + \sum_{q=1}^{\infty} D_p, \end{aligned} \tag{36}$$

where $D_p = \bar{\mathbf{N}}(\sum_{\kappa=0}^{p} \mathbf{U}_p) - \bar{\mathbf{N}}(\sum_{\kappa=0}^{p-1} \mathbf{U}_p)$.

Inserting (37), (39), and (36) into (32) and (33), respectively, we observe

$$\begin{aligned} \sum_{p=0}^{\infty} \mathbf{U}_p(\mathbf{y}_1, \varrho) &= \mathscr{G}(\varphi) + \mathbf{Y}^{-1}\left\{\varphi^{\delta}\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1, \varrho)]\right\} \\ &- \mathbf{Y}^{-1}\left\{\varphi^{\delta}\mathbf{Y}\left[\bar{\mathbf{L}}\left(\sum_{\kappa=0}^{p} \mathbf{U}_p(\mathbf{y}_1, \varrho)\right) + \bar{\mathbf{N}}(\mathbf{U}_0) + \sum_{\kappa=1}^{p} D_p\right]\right\}, \end{aligned} \tag{37}$$

$$\begin{aligned} \sum_{p=0}^{\infty} \mathbf{U}_p(\mathbf{y}_1, \varrho) &= \mathscr{G}(\varphi) + \mathbf{Y}^{-1}\{(1 + \delta(\varphi - 1))\mathbf{Y}[\mathbf{Q}(\mathbf{y}_1, \varrho)]\} \\ &- \mathbf{Y}^{-1}\left\{(1 + \delta(\varphi - 1))\mathbf{Y}\left[\bar{\mathbf{L}}\left(\sum_{p=0}^{p} \mathbf{U}_p(\mathbf{y}_1, \varrho)\right) + \bar{\mathbf{N}}(\mathbf{U}_0) + \sum_{q=1}^{p} D_p\right]\right\}. \end{aligned} \tag{38}$$

Ultimately, for CFD, we develop appropriate analysis procedure:

$$\begin{aligned} \mathbf{U}_0(\mathbf{y}_1, \varrho) &= \mathscr{G}(\varphi) + \mathbf{Y}^{-1}\left\{\varphi^{\delta}\mathscr{L}[\mathbf{Q}(\mathbf{y}_1, \varrho)]\right\}, \\ \mathbf{U}_1(\mathbf{y}_1, \varrho) &= -\mathbf{Y}^{-1}\left\{\varphi^{\delta}\mathbf{Y}\left[\bar{\mathbf{L}}(\mathbf{U}_0(\mathbf{y}_1, \varrho)) + \bar{\mathbf{N}}(\mathbf{U}_0(\mathbf{y}_1, \varrho))\right]\right\}, \\ &\vdots \\ \mathbf{U}_{q+1}(\mathbf{y}_1, \varrho) &= -\mathbf{Y}^{-1}\left\{\varphi^{\delta}\mathbf{Y}\left[\bar{\mathbf{L}}\left(\mathbf{U}_p(\mathbf{y}_1, \varrho)\right) + D_p\right]\right\}. \end{aligned} \tag{39}$$

The exploratory procedure for the Caputo-Fabrizio fractional derivative operator is shown then:

$$\begin{aligned} \mathbf{U}_0(\mathbf{y}_1, \varrho) &= \mathscr{G}(\varphi) + \mathbf{Y}^{-1}\{(1 + \delta(\varphi - 1))\mathscr{L}[\mathbf{Q}(\mathbf{y}_1, \varrho)]\}, \\ \mathbf{U}_1(\mathbf{y}_1, \varrho) &= -\mathbf{Y}^{-1}\{(1 + \delta(\varphi - 1))\mathbf{Y}\left[\bar{\mathbf{L}}(\mathbf{U}_0(\mathbf{y}_1, \varrho)) + \bar{\mathbf{N}}(\mathbf{U}_0(\mathbf{y}_1, \varrho))\right]\}, \\ &\vdots \\ \mathbf{U}_{q+1}(\mathbf{y}_1, \varrho) &= -\mathbf{Y}^{-1}\{(1 + \delta(\varphi - 1))\mathbf{Y}\left[\bar{\mathbf{L}}\left(\mathbf{U}_p(\mathbf{y}_1, \varrho)\right) + D_p\right]\}. \end{aligned} \tag{40}$$

Eventually, the q-term result in series formulation is generated by (37), (39), and (40), and we have

$$\begin{aligned} \mathbf{U}(\mathbf{y}_1, \varrho) \cong{}& \mathbf{U}_0(\mathbf{y}_1, \varrho) + \mathbf{U}_1(\mathbf{y}_1, \varrho) + \mathbf{U}_2(\mathbf{y}_1, \varrho) \\ &+ \cdots + \mathbf{U}_p(\mathbf{y}_1, \varrho), q \in \mathbb{N}. \end{aligned} \tag{41}$$

4. Mathematical Formulations of BSM via Caputo-Fabrizio Fractional Derivative Operator

The coming parts will illustrate how well the adequate conditions ensure the formation of a unique solution. Our hypothesis of the existence of solutions in the scenario of YDM is developed by [47].

Theorem 8 (Uniqueness theorem). *For $0 < \varepsilon < 1$, then system (24) has a unique solution, where $\varepsilon = (K_1 + K_2 + K_3)(1 + \delta(\varrho - 1))$.*

Proof. Surmise that there is a set of continuous mappings in the Banach space $\Omega = (\mathbb{C}[\mathscr{I}], \|.\|)$ Considering $\mathscr{I} = [0, \mathscr{T}]$, present the norm $\|.\|$ To continue this, suppose a mapping

$\mathscr{V} : \Omega \mapsto \Omega$ such that

$$\begin{aligned}\mathbf{U}_{\ell+1}(\mathbf{y}_1, \varrho) = \mathbf{U}(\mathbf{y}_1, \varrho) + \mathbf{Y}^{-1}[(1+\delta(\varphi-1))\mathbf{Y}[\mathbf{L}[\mathbf{U}_\ell(\mathbf{y}_1, \varrho)] \\ + \bar{P}[\mathbf{U}_\ell(\mathbf{y}_1, \varrho)] + \mathbf{N}[\mathbf{U}_\ell(\mathbf{y}_1, \varrho)]]], \ell \geq 0,\end{aligned} \tag{42}$$

where $\mathbf{L}[\mathbf{U}(\mathbf{y}_1, \varrho)] \equiv \partial^3 \mathbf{U}(\mathbf{y}_1, \varrho)/\partial \mathbf{y}_1^2$ and $\bar{P}[\mathbf{U}(\mathbf{y}_1, \varrho)] \equiv \partial \mathbf{U}(\mathbf{y}_1, \varrho)/\partial \mathbf{y}_1$. Here, suppose that $\mathbf{L}[\mathbf{U}(\mathbf{y}_1, \varrho)]$ and $\mathbf{N}[\mathbf{U}(\mathbf{y}_1, \varrho)]$ are also Lipschitzian with $|\bar{P}\mathbf{U} - \bar{P}\widehat{\mathbf{U}}| < \mathbf{K}_1|\mathbf{U} - \widehat{\mathbf{U}}|$ and $|\mathbf{L}\mathbf{U} - \mathbf{L}\widehat{\mathbf{U}}| < \mathbf{K}_2|\mathbf{U} - \widehat{\mathbf{U}}|$, where $\mathbf{K}_1$ and $\mathbf{K}_2$ are Lipschitz constant, respectively, and $\mathbf{U}, \widehat{\mathbf{U}}$ are distinct functional values.

$$\begin{aligned}
&\|\mathscr{V}\mathbf{U} - \mathscr{V}\widehat{\mathbf{U}}\| = \max_{\varrho \in \mathscr{J}} |\mathbf{Y}^{-1}[(1+\delta(\varphi-1))\mathbf{Y}[\mathbf{L}[\mathbf{U}(\mathbf{y}_1, \varrho)] \\
&\quad + \bar{P}[\mathbf{U}(\mathbf{y}_1, \varrho)] + \mathbf{N}[\mathbf{U}(\mathbf{y}_1, \varrho)]]] \\
&\quad - \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\mathbf{L}\left[\widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right]\right.\right. \\
&\quad + \bar{P}\left[\widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right] + \mathbf{N}\left[\widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right]]]| \\
&\qquad \leq \max_{\varrho \in \mathscr{J}} |\mathbf{Y}^{-1}[(1+\delta(\varphi-1))\mathbf{Y}[\mathbf{L}[\mathbf{U}(\mathbf{y}_1, \varrho)] \\
&\quad - \mathbf{L}\left[\widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right]]] + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\bar{P}[\mathbf{U}(\mathbf{y}_1, \varrho)]\right.\right. \\
&\quad - \bar{P}\left[\widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right]]] + \mathbf{Y}^{-1}[(1+\delta(\varphi-1))\mathbf{Y}[\mathbf{N}[\mathbf{U}(\mathbf{y}_1, \varrho)] \\
&\quad - \mathbf{N}\left[\widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right]]]| \leq \max_{\varrho \in \mathscr{J}} \left[\mathbf{K}_1 \mathbf{Y}^{-1}[(1+\delta(\varphi-1))\mathbf{Y}|\mathbf{U}(\mathbf{y}_1, \varrho)\right. \\
&\quad - \widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)|] + \mathbf{K}_2 \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left|\mathbf{U}(\mathbf{y}_1, \varrho) - \widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right|\right] \\
&\quad + \mathbf{K}_3 \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left|\mathbf{U}(\mathbf{y}_1, \varrho) - \widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right|\right]] \\
&\qquad \leq \max_{\varrho \in \mathscr{J}} (\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3)\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left|\mathbf{U}(\mathbf{y}_1, \varrho) - \widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right|\right] \\
&\qquad \leq (\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3)\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left\|\mathbf{U}(\mathbf{y}_1, \varrho) - \widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right\|\right] \\
&\qquad = (\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3)\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\varphi\left\|\mathbf{U}(\mathbf{y}_1, \varrho) - \widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right\|\right] \\
&\qquad = (\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3)(1+\delta(\varrho-1))\left\|\mathbf{U}(\mathbf{y}_1, \varrho) - \widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right\|.
\end{aligned} \tag{43}$$

For $0 < \varepsilon < 1$, the functional is contraction. As a result of the Banach contraction fixed point hypothesis, (16) has a fixed value. This produces the intended outcome. □

Theorem 9 (Convergence analysis). *Equation (16) has a generic type solution and will be convergent.*

Proof. Surmise that $\widehat{S}_\ell$ be the *nth* partial sum, that is, $\widehat{S}_\ell = \sum_{m=0}^{\ell} \mathbf{U}_\ell(\mathbf{y}_1, \varrho)$. Further, we exhibit $\{\widehat{S}_\ell\}$ is a Cauchy sequence in Banach space U.

We do it by contemplating a novel kind of Adomian polynomials.

$$\begin{aligned}\bar{R}\left(\widehat{S}_\ell\right) &= \tilde{\mathbf{H}}_\ell + \sum_{p=0}^{\ell-1} \tilde{\mathbf{H}}_p, \\ \mathbf{N}\left(\widehat{S}_\ell\right) &= \tilde{\mathbf{H}}_\ell + \sum_{c=0}^{\ell-1} \tilde{\mathbf{H}}_c.\end{aligned} \tag{44}$$

Now

$$\begin{aligned}
&\left\|\widehat{S}_\ell - \widehat{S}_p\right\| = \max_{\varrho \in \mathscr{J}} \left|\widehat{S}_\ell - \widehat{S}_p\right| = \max_{\varrho \in \mathscr{J}} \left|\sum_{m=q+1}^{\ell} \widehat{\mathbf{U}}(\mathbf{y}_1, \varrho)\right|, (m = 1, 2, 3, \cdots) \\
&\leq \max_{\varrho \in \mathscr{J}} \left|\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q+1}^{\ell} \mathbf{L}[\mathbf{U}_{\ell-1}(\mathbf{y}_1, \varrho)]\right]\right]\right. \\
&\quad + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q+1}^{\ell} \bar{P}[\mathbf{U}_{\ell-1}(\mathbf{y}_1, \varrho)]\right]\right] \\
&\quad \left. + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q+1}^{\ell} \tilde{\mathbf{H}}_{\ell-1}(\mathbf{y}_1, \varrho)\right]\right]\right| \\
&= \max_{\varrho \in \mathscr{J}} \left|\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q}^{\ell-1} \mathbf{L}[\mathbf{U}_\ell(\mathbf{y}_1, \varrho)]\right]\right]\right. \\
&\quad + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q}^{\ell-1} \bar{P}[\mathbf{U}_\ell(\mathbf{y}_1, \varrho)]\right]\right] \\
&\quad \left. + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q}^{\ell-1} \tilde{\mathbf{H}}_\ell(\mathbf{y}_1, \varrho)\right]\right]\right| \\
&\leq \max_{\varrho \in \mathscr{J}} \left|\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q}^{\ell-1} \mathbf{L}\left(\widehat{S}_{\ell-1}\right) - \mathbf{L}\left(\widehat{S}_{q-1}\right)\right]\right]\right. \\
&\quad + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q}^{\ell-1} \bar{P}\left(\widehat{S}_{\ell-1}\right) - \bar{P}\left(\widehat{S}_{q-1}\right)\right]\right] \\
&\quad \left. + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{m=q}^{\ell-1} \mathbf{N}\left(\widehat{S}_{\ell-1}\right) - \mathbf{N}\left(\widehat{S}_{q-1}\right)\right]\right]\right| \\
&\leq \max_{\varrho \in \mathscr{J}} \left|\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\mathbf{L}\left(\widehat{S}_{\ell-1}\right) - \mathbf{L}\left(\widehat{S}_{q-1}\right)\right]\right]\right. \\
&\quad + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\bar{P}\left(\widehat{S}_{\ell-1}\right) - \bar{P}\left(\widehat{S}_{q-1}\right)\right]\right] \\
&\quad \left. + \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\mathbf{N}\left(\widehat{S}_{\ell-1}\right) - \mathbf{N}\left(\widehat{S}_{q-1}\right)\right]\right]\right| \\
&\leq \mathbf{K}_1 \max_{\varrho \in \mathscr{J}} \mathbf{Y}^{-1}\left|\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\left(\widehat{S}_{\ell-1}\right) - \left(\widehat{S}_{q-1}\right)\right]\right]\right| \\
&\quad + \mathbf{K}_2 \max_{\varrho \in \mathscr{J}} \left|\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\left(\widehat{S}_{\ell-1}\right) - \left(\widehat{S}_{q-1}\right)\right]\right]\right| \\
&\quad + \mathbf{K}_3 \max_{\varrho \in \mathscr{J}} \left|\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\left(\widehat{S}_{\ell-1}\right) - \left(\widehat{S}_{q-1}\right)\right]\right]\right| \\
&= (\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3)\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\varphi\left\|\widehat{S}_{\ell-1} - \widehat{S}_{q-1}\right\|\right. . \\
&= (\mathbf{K}_1 + \mathbf{K}_2 + \mathbf{K}_3)(1+\delta(\varrho-1))\left\|\widehat{S}_{\ell-1} - \widehat{S}_{q-1}\right\|.
\end{aligned} \tag{45}$$

Assume $n = q + 1$; then

$$\left\|\widehat{S}_{q+1} - \widehat{S}_p\right\| \leq \varepsilon\left\|\widehat{S}_p - \widehat{S}_{q-1}\right\| \leq \varepsilon^2\left\|\widehat{S}_{q-1} - \widehat{S}_{q-2}\right\| \leq \cdots \leq \varepsilon^p\left\|\widehat{S}_1 - \widehat{S}_0\right\|, \tag{46}$$

where $\varepsilon = (1 + \delta(\varrho - 1))$. In view of triangular variant, we have

$$\begin{aligned}
\left\|\widehat{S}_\ell - \widehat{S}_p\right\| &\leq \left\|\widehat{S}_{q+1} - \widehat{S}_p\right\| + \left\|\widehat{S}_{q+2} - \widehat{S}_{q+1}\right\| + \cdots + \left\|\widehat{S}_\ell - \widehat{S}_{\ell-1}\right\| \\
&\leq \left[\varepsilon^p + \varepsilon^{q+1} + \cdots + \varepsilon^{\ell-1}\right]\left\|\widehat{S}_1 - \widehat{S}_0\right\| \\
&\leq \varepsilon^p\left(\frac{1 - \varepsilon^{\ell-p}}{\varepsilon}\right)\|\mathbf{U}_1\|,
\end{aligned} \tag{47}$$

since $0 < \varepsilon < 1$, we have $(1 - \varepsilon^{\ell-p}) < 1$, and then

$$\|\widehat{S}_\ell - \widehat{S}_p\| \le \frac{\varepsilon^p}{1-\varepsilon} \max_{\varrho \in \mathcal{J}} \|\mathbf{U}_1\|. \quad (48)$$

Thus, $|\mathbf{U}_1| < \infty$ (since $\mathbf{U}(\mathbf{y}_1, \varrho)$ is bounded). Also, as $q \mapsto \infty$, then $\|\widehat{S}_\ell - \widehat{S}_p\| \mapsto 0$. Therefore, $\{\widehat{S}_1\}$ is a Cauchy sequence in K. Ultimately, $\sum_{n=0}^{\infty} \mathbf{U}_\ell$ is convergent, and the direct result is obtained. □

Theorem 10 (see [47]) (Error estimate). *The absolute inaccuracy of the (16) through (24) sum is determined as*

$$\max_{\varrho \in \mathcal{J}} \left| \mathbf{U}(\mathbf{y}_1, \varrho) - \sum_{\ell=1}^{p} \mathbf{U}_\ell(\mathbf{y}_1, \varrho) \right| \le \frac{\varepsilon^p}{1-\varepsilon} \max_{\varrho \in \mathcal{J}} \|\mathbf{U}_1\|. \quad (49)$$

5. Mathematical Description of BSM Time-Fractional Systems

Here, we construct the estimated analytical solution of BSM considering the CFD and Caputo-Fabrizio fractional derivative operators utilizing the Yang decomposition approach.

5.1. Yang Decomposition Method

Example 1 (see [29]). Surmise the fractional-order BSM (4) supplemented with the (5).

Case 1. To begin, we utilize the Caputo fractional derivative operator employing the Yang decomposition approach to analyze the (4). Implementing the Yang transform on (4), we get

$$\mathbf{Y}\left[\mathbf{D}_\varrho^\delta \mathbf{U}(\mathbf{y}_1, \varrho)\right] = \mathbf{Y}\left[\frac{\partial^2 \mathbf{U}(\mathbf{y}_1, \varrho)}{\partial \mathbf{y}_1^2} + (\zeta - 1)\frac{\partial \mathbf{U}(\mathbf{y}_1, \varrho)}{\partial \mathbf{y}_1} - \zeta \mathbf{U}(\mathbf{y}_1, \varrho)\right]. \quad (50)$$

Utilizing the Yang transform's differentiation criteria gives

$$\begin{aligned} \varphi^{-\delta}(\varrho)\mathbf{Y}[\mathbf{U}(\mathbf{y}_1, \varrho)] &= \phi(\varrho) \sum_{p=0}^{p-1} \varphi^{1-\delta-p} \mathbf{U}^{(p)}(0) \\ &+ \mathbf{Y}\left[\frac{\partial^2 \mathbf{U}(\mathbf{y}_1, \varrho)}{\partial \mathbf{y}_1^2} + (\zeta - 1)\frac{\partial \mathbf{U}(\mathbf{y}_1, \varrho)}{\partial \mathbf{y}_1} - \zeta \mathbf{U}(\mathbf{y}_1, \varrho)\right]. \end{aligned} \quad (51)$$

Utilizing (5), we find

$$\begin{aligned} \mathbf{Y}[\mathbf{U}(\mathbf{y}_1, \varrho)] &= \varphi \max(\exp(\mathbf{y}_1) - 1, 0) \\ &+ \varphi^\delta \mathbf{Y}\left[\frac{\partial^2 \mathbf{U}(\mathbf{y}_1, \varrho)}{\partial \mathbf{y}_1^2} + (\zeta - 1)\frac{\partial \mathbf{U}(\mathbf{y}_1, \varrho)}{\partial \mathbf{y}_1} - \zeta \mathbf{U}(\mathbf{y}_1, \varrho)\right]. \end{aligned} \quad (52)$$

Applying the inverse Yang transform produces

$$\begin{aligned} \mathbf{U}(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}[\varphi.\max(\exp(\mathbf{y}_1) - 1, 0)] \\ &+ \mathbf{Y}^{-1}\left[\varphi^\delta \mathbf{Y}\left[\frac{\partial^2 \mathbf{U}(\mathbf{y}_1, \varrho)}{\partial \mathbf{y}_1^2} + (\zeta - 1)\frac{\partial \mathbf{U}(\mathbf{y}_1, \varrho)}{\partial \mathbf{y}_1} - \zeta \mathbf{U}(\mathbf{y}_1, \varrho)\right]\right]. \end{aligned} \quad (53)$$

To determine this, apply the Yang decomposition approach as follows:

$$\begin{aligned} \mathbf{U}_0(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}[\varphi.\max(\exp(\mathbf{y}_1) - 1, 0)] \\ &= \max(\exp(\mathbf{y}_1) - 1, 0). \end{aligned} \quad (54)$$

We predict that the unidentified mapping $\mathbf{U}(\mathbf{y}_1, \varrho)$ may be expressed as an infinite series of the pattern

$$\begin{aligned} \mathbf{U}(\mathbf{y}_1, \varrho) &= \sum_{p=0}^{\infty} \mathbf{U}_p(\mathbf{y}_1, \varrho), \\ \sum_{p=0}^{\infty} \mathbf{U}_{q+1}(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}\left[\varphi^\delta \mathbf{Y}\left[\sum_{p=0}^{\infty} (\mathbf{U}(\mathbf{y}_1, \varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right. \\ &\left.\left. + (\zeta - 1)\sum_{p=0}^{\infty} (\mathbf{U}(\mathbf{y}_1, \varrho))_{\mathbf{y}_1} - \zeta \sum_{p=0}^{\infty} (\mathbf{U}(\mathbf{y}_1, \varrho))\right]\right], p = 0, 1, 2, \cdots, \end{aligned} \quad (55)$$

$$\begin{aligned} \mathbf{U}_1(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}\left[\varphi^\delta \mathbf{Y}\left[(\mathbf{U}_0(\mathbf{y}_1, \varrho))_{\mathbf{y}_1\mathbf{y}_1} + (\zeta - 1)(\mathbf{U}_0(\mathbf{y}_1, \varrho))_{\mathbf{y}_1} + \zeta \mathbf{U}_0\right]\right] \\ &= [\zeta \max(\exp(\mathbf{y}_1), 0) - \zeta \max(\exp(\mathbf{y}_1 - 1), 0)]\mathbf{Y}^{-1}\left[\varphi^{\delta+1}\right] \\ &= [\zeta \max(\exp(\mathbf{y}_1), 0) - \zeta \max(\exp(\mathbf{y}_1 - 1), 0)]\frac{\varrho^\delta}{\Gamma(\delta + 1)}, \\ \mathbf{U}_2(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}\left[\frac{1}{\varphi^\delta(\varrho)}\mathbf{Y}\left[(\mathbf{U}_1(\mathbf{y}_1, \varrho))_{\mathbf{y}_1\mathbf{y}_1} + (\zeta - 1)(\mathbf{U}_1(\mathbf{y}_1, \varrho))_{\mathbf{y}_1} + \zeta \mathbf{U}_1\right]\right] \\ &= \left[-\zeta^2 \max(\exp(\mathbf{y}_1), 0) + \zeta^2 \max(\exp(\mathbf{y}_1 - 1), 0)\right]\frac{\varrho^{2\delta}}{\Gamma(2\delta + 1)}, \\ \mathbf{U}_3(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}\left[\frac{1}{\varphi^\delta(\varrho)}\mathbf{Y}\left[(\mathbf{U}_2(\mathbf{y}_1, \varrho))_{\mathbf{y}_1\mathbf{y}_1} + (\zeta - 1)(\mathbf{U}_2(\mathbf{y}_1, \varrho))_{\mathbf{y}_1} + \zeta \mathbf{U}_2\right]\right] \\ &= \left[-\zeta^3 \max(\exp(\mathbf{y}_1), 0) + \zeta^3 \max(\exp(\mathbf{y}_1 - 1), 0)\right]\frac{\varrho^{3\delta}}{\Gamma(3\delta + 1)}. \\ &\vdots \end{aligned} \quad (56)$$

For Example 1, the series form solution is developed as follows:

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1,\varrho) &= \mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)+\cdots\\ &= \max(\exp(\mathbf{y}_1-1),0)\\ &\quad\cdot\left[1-\frac{\zeta\varrho^{\delta}}{\Gamma(\delta+1)}+\frac{\zeta^2\varrho^{2\delta}}{\Gamma(2\delta+1)}-\frac{\zeta^3\varrho^{3\delta}}{\Gamma(3\delta+1)}+\cdots\right]\\ &\quad+\max(\exp(\mathbf{y}_1),0)\\ &\quad\cdot\left[1-1+\frac{\zeta\varrho^{\delta}}{\Gamma(\delta+1)}-\frac{\zeta^2\varrho^{2\delta}}{\Gamma(2\delta+1)}+\frac{\zeta^3\varrho^{3\delta}}{\Gamma(3\delta+1)}+\cdots\right]\\ &= \max(\exp(\mathbf{y}_1-1),0)E_\delta\left(-\zeta(\varrho)^{\delta}\right)\\ &\quad+\max(\exp(\mathbf{y}_1),0)E_\delta\left(1-\zeta(\varrho)^{\delta}\right).\end{aligned} \tag{57}$$

Case 2. The Caputo-Fabrizio fractional derivative operator and the Yang decomposition approach are now used to solve equation (4). Assuming (50) and implementing the Yang transform's differentiation criteria, we obtain

$$\frac{1}{1+\delta(1-\varphi)}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]=\frac{1}{1+\delta(1-\varphi)}\sum_{p=0}^{p-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(0)+\mathbf{Y}\left[\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}+(\zeta-1)\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}-\zeta\mathbf{U}(\mathbf{y}_1,\varrho)\right]. \tag{58}$$

Utilizing (5), we obtain

$$\begin{aligned}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)] &= \varphi\max(\exp(\mathbf{y}_1)-1,0)+(1+\delta(\varphi-1))\mathbf{Y}\\ &\quad\cdot\left[\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}+(\zeta-1)\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}-\zeta\mathbf{U}(\mathbf{y}_1,\varrho)\right],\\ \mathbf{U}(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}[\varphi.\max(\exp(\mathbf{y}_1)-1,0)]+\mathbf{Y}^{-1}\\ &\quad\cdot\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}+(\zeta-1)\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}-\zeta\mathbf{U}(\mathbf{y}_1,\varrho)\right]\right].\end{aligned} \tag{59}$$

Employing the Yang decomposition approach produces

$$Q_0(\mathbf{y}_1,\varrho)=\mathbf{Y}^{-1}[\varphi\max(\exp(\mathbf{y}_1)-1,0)]=\max(\exp(\mathbf{y}_1)-1,0). \tag{60}$$

We predict that the unidentified mapping $\mathbf{U}(\mathbf{y}_1,\varrho)$ may be expressed as an infinite series of the pattern

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1,\varrho) &= \sum_{p=0}^{\infty}\mathbf{U}_p(\mathbf{y}_1,\varrho),\\ \sum_{p=0}^{\infty}\mathbf{U}_{q+1}(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\ &\quad\left.\left.+(\zeta-1)\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-\zeta\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1,\varrho))\right]\right], p=0,1,2,\cdots,\end{aligned} \tag{61}$$

$$\begin{aligned}\mathbf{U}_1(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\ &\quad\left.\left.+(\zeta-1)(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_0\right]\right]\\ &= [\zeta\max(\exp(\mathbf{y}_1),0)-\zeta\max(\exp(\mathbf{y}_1-1),0)]\\ &\quad\cdot(1+\delta(\varrho-1)),\\ \mathbf{U}_2(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\ &\quad\left.\left.+(\zeta-1)(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_1\right]\right]\\ &= -\left[\zeta^2\max(\exp(\mathbf{y}_1),0)+\zeta^2\max(\exp(\mathbf{y}_1-1),0)\right]\\ &\quad\cdot\left((1-\delta)^2+2\varrho\delta(1-\delta)+\frac{\rho^2\delta^2}{2}\right),\\ \mathbf{U}_3(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\ &\quad\left.\left.+(\zeta-1)(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_2\right]\right]\\ &= -\frac{\left[\zeta^3\max(\exp(\mathbf{y}_1),0)+\zeta^3\max(\exp(\mathbf{y}_1-1),0)\right]}{\mathbb{A}^3(\delta)}\\ &\quad\times\left((1-\delta)^3+3\varrho^2\delta^2(1-\delta)\frac{\rho^2}{2}+3\varrho\delta(1-\delta)^2+\frac{\varrho^3\delta^3}{3}\right),\vdots\end{aligned} \tag{62}$$

For Example 1, the series form solution is developed as follows:

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1,\varrho) &= \mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)\\ &\quad+\cdots,=\max(\exp(\mathbf{y}_1-1),0)\\ &\quad\cdot\left[1-\frac{\zeta\varrho^{\delta}}{\Gamma(\delta+1)}+\frac{\zeta^2\varrho^{2\delta}}{\Gamma(2\delta+1)}-\frac{\zeta^3\varrho^{3\delta}}{\Gamma(3\delta+1)}+\cdots\right]\\ &\quad+\max(\exp(\mathbf{y}_1),0)\\ &\quad\cdot\left[1-1+\frac{\zeta\varrho^{\delta}}{\Gamma(\delta+1)}-\frac{\zeta^2\varrho^{2\delta}}{\Gamma(2\delta+1)}+\frac{\zeta^3\varrho^{3\delta}}{\Gamma(3\delta+1)}+\cdots\right]\\ &= \max(\exp(\mathbf{y}_1-1),0)E_\delta\left(-\zeta(\varrho)^{\delta}\right)\\ &\quad+\max(\exp(\mathbf{y}_1),0)E_\delta\left(1-\zeta(\varrho)^{\delta}\right).\end{aligned} \tag{63}$$

Considering the Taylor series expansion and assigning

$\delta = 1$, the exact findings of Example 1 can be determined as

$$\mathbf{U}(\mathbf{y}_1, \varrho) = \max(\exp(\mathbf{y}_1 - 1), 0)\exp(-\zeta\varrho) + \max(\exp(\mathbf{y}_1), 0)[1 - \exp(-\zeta\varrho)]. \tag{64}$$

Example 2 (see [30]). Surmise the fractional-order BSM (6) supplemented with the (7).

Case 1. To begin, we utilize the Caputo fractional derivative operator, employing the Yang decomposition approach to analyze the (6). Implementing the Yang transform on (6), we get

$$\mathbf{Y}\left[\mathbf{D}_{\varrho}^{\delta}\mathbf{U}(\mathbf{y}_1, \varrho)\right] = \mathbf{Y}\left[-0.08(2 + \sin \mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1, \varrho)}{\partial\mathbf{y}_1^2} - 0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1, \varrho)}{\partial\mathbf{y}_1} + 0.06\mathbf{U}(\mathbf{y}_1, \varrho)\right]. \tag{65}$$

Utilizing the Yang transform's differentiation criteria, gives

$$\varphi^{-\delta}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1, \varrho)] = \sum_{p=0}^{n-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(0) + \mathbf{Y}\left[-0.08(2 + \sin \mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1, \rho)}{\partial\mathbf{y}_1^2} - 0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1, \varrho)}{\partial\mathbf{y}_1} + 0.06\mathbf{U}(\mathbf{y}_1, \varrho)\right]. \tag{66}$$

Utilizing (7), we find

$$\mathbf{Y}[\mathbf{U}(\mathbf{y}_1, \varrho)] = \varphi.\max(\mathbf{y}_1 - 25\exp(-0.06), 0) + \varphi^{\delta}\mathbf{Y}\left[-0.08(2 + \sin \mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1, \varrho)}{\partial\mathbf{y}_1^2} - 0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1, \varrho)}{\partial\mathbf{y}_1} + 0.06\mathbf{U}(\mathbf{y}_1, \varrho)\right]. \tag{67}$$

Applying the inverse Yang transform produces

$$\mathbf{U}(\mathbf{y}_1, \varrho) = \mathbf{Y}^{-1}[\varphi.\max(\mathbf{y}_1 - 25\exp(-0.06), 0)] + \mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[-0.08(2 + \sin \mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1, \varrho)}{\partial\mathbf{y}_1^2} - 0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1, \varrho)}{\partial\mathbf{y}_1} + 0.06\mathbf{U}(\mathbf{y}_1, \varrho)\right]\right]. \tag{68}$$

To determine this, apply the Yang decomposition approach as follows:

$$Q_0(\mathbf{y}_1, \varrho) = \mathbf{Y}^{-1}[\varphi.\max(\mathbf{y}_1 - 25\exp(-0.06), 0)] = \max(\mathbf{y}_1 - 25\exp(-0.06), 0). \tag{69}$$

We predict that the unidentified mapping $\mathbf{U}(\mathbf{y}_1, \varrho)$ may be expressed as an infinite series of the pattern

$$\mathbf{U}(\mathbf{y}_1, \varrho) = \sum_{p=0}^{\infty}\mathbf{U}_p(\mathbf{y}_1, \varrho),$$
$$\sum_{p=0}^{\infty}\mathbf{U}_{q+1}(\mathbf{y}_1, \varrho) = \mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1, \varrho))_{\mathbf{y}_1\mathbf{y}_1} + (\zeta - 1)\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1, \varrho))_{\mathbf{y}_1} - \zeta\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1, \varrho))\right]\right], p = 0, 1, 2, \cdots, \tag{70}$$

$$\begin{aligned}\mathbf{U}_1(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[(\mathbf{U}_0(\mathbf{y}_1, \varrho))_{\mathbf{y}_1\mathbf{y}_1} + (\zeta - 1)(\mathbf{U}_0(\mathbf{y}_1, \varrho))_{\mathbf{y}_1} + \zeta\mathbf{U}_0\right]\right] \\ &= [\zeta\max(\exp(\mathbf{y}_1), 0) - \zeta\max(\exp(\mathbf{y}_1 - 1), 0)]\mathbf{Y}^{-1}\left[\varphi^{\delta+1}\right] \\ &= [\zeta\max(\exp(\mathbf{y}_1), 0) - \zeta\max(\exp(\mathbf{y}_1 - 1), 0)]\frac{\varrho^{\delta}}{\Gamma(\delta + 1)},\end{aligned}$$

$$\begin{aligned}\mathbf{U}_2(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}\left[\frac{1}{\varphi^{\delta}(\varrho)}\mathbf{Y}\left[(\mathbf{U}_1(\mathbf{y}_1, \varrho))_{\mathbf{y}_1\mathbf{y}_1} + (\zeta - 1)(\mathbf{U}_1(\mathbf{y}_1, \varrho))_{\mathbf{y}_1} + \zeta\mathbf{U}_1\right]\right] \\ &= \left[-\zeta^2\max(\exp(\mathbf{y}_1), 0) + \zeta^2\max(\exp(\mathbf{y}_1 - 1), 0)\right]\frac{\varrho^{2\delta}}{\Gamma(2\delta + 1)},\end{aligned}$$

$$\begin{aligned}\mathbf{U}_3(\mathbf{y}_1, \varrho) &= \mathbf{Y}^{-1}\left[\frac{1}{\varphi^{\delta}(\varrho)}\mathbf{Y}\left[(\mathbf{U}_2(\mathbf{y}_1, \varrho))_{\mathbf{y}_1\mathbf{y}_1} + (\zeta - 1)(\mathbf{U}_2(\mathbf{y}_1, \varrho))_{\mathbf{y}_1} + \zeta\mathbf{U}_2\right]\right] \\ &= \left[-\zeta^3\max(\exp(\mathbf{y}_1), 0) + \zeta^3\max(\exp(\mathbf{y}_1 - 1), 0)\right]\frac{\varrho^{3\delta}}{\Gamma(3\delta + 1)}. \\ &\vdots\end{aligned} \tag{71}$$

For Example 2, the series form solution is developed as follows:

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1, \varrho) &= \mathbf{U}_0(\mathbf{y}_1, \varrho) + \mathbf{U}_1(\mathbf{y}_1, \varrho) + \mathbf{U}_2(\mathbf{y}_1, \varrho) + \mathbf{U}_3(\mathbf{y}_1, \varrho) \\ &\quad + \cdots = \max(\mathbf{y}_1 - 25\exp(-0.06), 0) \\ &\quad + (\mathbf{y}_1 - \max(\mathbf{y}_1 - 25\exp(-0.06)), 0) \\ &\quad \times\left[1 - 1 - \frac{0.06\varrho^{\delta}}{\Gamma(\delta + 1)} - \frac{0.0036\varrho^{2\delta}}{\Gamma(2\delta + 1)} - \frac{0.000216\varrho^{3\delta}}{\Gamma(3\delta + 1)} + \cdots\right] \\ &= \max(\mathbf{y}_1 - 25\exp(-0.06), 0) \\ &\quad + (\mathbf{y}_1 - \max(\mathbf{y}_1 - 25\exp(-0.06), 0))\left[1 - E_{\delta}\left(0.06(\varrho)^{\delta}\right)\right].\end{aligned} \tag{72}$$

Case 2. The Caputo-Fabrizio fractional derivative operator and the Yang decomposition approach are now used to solve the (6).

Assuming (65) and implementing the Yang transform's differentiation criteria, we obtain

$$\frac{1}{1+\delta(\varphi-1)}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]=\frac{1}{1+\delta(\varphi-1)}\sum_{p=0}^{n-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(0)+\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}-0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}+0.06\mathbf{U}(\mathbf{y}_1,\varrho)\right]. \tag{73}$$

Utilizing (7), we obtain

$$\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]=\varphi.\max(\mathbf{y}_1-25\exp(-0.06),0)+(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}-0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}+0.06\mathbf{U}(\mathbf{y}_1,\varrho)\right]. \tag{74}$$

Employing the inverse Yang transform gives

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\mathbf{Y}^{-1}[\varphi.\max(\mathbf{y}_1-25\exp(-0.06),0)]+\mathbf{Y}^{-1}\cdot\left[(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}-0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}+0.06\mathbf{U}(\mathbf{y}_1,\varrho)\right]\right]. \tag{75}$$

Employing the Yang decomposition approach, we obtain

$$\begin{aligned}Q_0(\mathbf{y}_1,\varrho)&=\mathbf{Y}^{-1}[\varphi.\max(\mathbf{y}_1-25\exp(-0.06),0)]\\&=\max(\mathbf{y}_1-25\exp(-0.06),0).\end{aligned} \tag{76}$$

We predict the unidentified mapping $\mathbf{U}(\mathbf{y}_1,\varrho)$ may be expressed as an infinite series of the pattern

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\sum_{p=0}^{\infty}\mathbf{U}_p(\mathbf{y}_1,\varrho),$$

$$\sum_{p=0}^{\infty}\mathbf{U}_{q+1}(\mathbf{y}_1,\varrho)=\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}-0.06\mathbf{y}_1\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+0.06\sum_{p=0}^{\infty}(\mathbf{U}(\mathbf{y}_1,\varrho))\right]\right],\ p=0,1,2,\cdots, \tag{77}$$

$$\begin{aligned}\mathbf{U}_1(\mathbf{y}_1,\varrho)&=\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}-0.06\mathbf{y}_1(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_0\right]\right]\\&=[-0.06\mathbf{y}_1+0.06\max(\mathbf{y}_1-25\exp(-0.06),0)](1-\delta(\varrho-1)),\\\mathbf{U}_2(\mathbf{y}_1,\varrho)&=\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}-0.06\mathbf{y}_1(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_1\right]\right]\\&=([-0.0036\mathbf{y}_1+0.0036\max(\mathbf{y}_1-25\exp(-0.06),0)])\left((1-\delta)^2+2\varrho\delta(1-\delta)+\frac{\varrho^2\delta^2}{2}\right),\\\mathbf{U}_3(\mathbf{y}_1,\varrho)&=\mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}-0.06\mathbf{y}_1(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_2\right]\right]\\&=-\frac{[-0.000216\mathbf{y}_1+0.00216\max(\mathbf{y}_1-25\exp(-0.06),0)]}{\mathbb{A}^3(\delta)}\\&\quad\times\left((1-\delta)^3+3\varrho^2\delta^2(1-\delta)\frac{\varrho^2}{2}+3\varrho\delta(1-\delta)^2+\frac{\varrho^3\delta^3}{3}\right),\\&\ \ \vdots\end{aligned} \tag{78}$$

For Example 2, the series form solution is developed as follows:

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1,\varrho)&=\mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)\\&+\cdots=\max(\mathbf{y}_1-25\exp(-0.06),0)\\&-(0.06\mathbf{y}_1-0.06\max(\mathbf{y}_1-25\exp(-0.06),0))(1-\delta(\varrho-1))\\&-(0.0036\mathbf{y}_1-0.0036\max(\mathbf{y}_1-25\exp(-0.06),0))\\&\cdot\left((1-\delta)^2+2\varrho\delta(1-\delta)+\frac{\varrho^2\delta^2}{2}\right)-.\cdots\end{aligned}\tag{79}$$

Considering the Taylor series expansion and assigning $\delta=1$, the exact findings of Example 2 can be determined as

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1,\varrho)&=\max(\mathbf{y}_1-25\exp(-0.06),0)\\&+(\mathbf{y}_1-\max(\mathbf{y}_1-25\exp(-0.06),0))[1-\exp(-0.06\varrho)].\end{aligned}\tag{80}$$

5.2. Yang Iterative Transform Method

Example 3 (see [29]). Surmise the fractional-order BSM (4) supplemented with the (5).

Case 1. To begin, we utilize the Caputo fractional derivative operator, employing the Yang decomposition approach to analyze the (4). Implementing the Yang transform on (4), we get

$$\begin{aligned}\varphi^{-\delta}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]&=\sum_{p=0}^{n-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(0)\\&+\mathbf{Y}\left[\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}+(\zeta-1)\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}-\zeta\mathbf{U}(\mathbf{y}_1,\varrho)\right].\end{aligned}\tag{81}$$

It follows that

$$\begin{aligned}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]&=\varphi.\max(\exp(\mathbf{y}_1)-1,0)+\varphi^{\delta}(\varrho)\mathbf{Y}\\&\cdot\left[\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}+(\zeta-1)\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}-\zeta\mathbf{U}(\mathbf{y}_1,\varrho)\right].\end{aligned}\tag{82}$$

In view of the proposed algorithm in Section 3.2, we find

$$\begin{aligned}\mathbf{U}_0(\mathbf{y}_1,\varrho)&=\mathbf{Y}^{-1}\left[\varphi^{\delta}\max(\exp(\mathbf{y}_1)-1,0)\right]=\max(\exp(\mathbf{y}_1)-1,0),\\
\mathbf{U}_1(\mathbf{y}_1,\varrho)&=\mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}+(\zeta-1)(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_0\right]\right]\\&=[\zeta\max(\exp(\mathbf{y}_1),0)-\zeta\max(\exp(\mathbf{y}_1-1),0)]\mathbf{Y}^{-1}\left[\frac{\phi(\mathfrak{s})}{\varphi^{\delta+1}(\mathfrak{s})}\right]\\&=[\zeta\max(\exp(\mathbf{y}_1),0)-\zeta\max(\exp(\mathbf{y}_1-1),0)]\frac{\varrho^{\delta}}{\Gamma(\delta+1)},\\
\mathbf{U}_2(\mathbf{y}_1,\varrho)&=\mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}+(\zeta-1)(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_1\right]\right]\\&=\left[-\zeta^2\max(\exp(\mathbf{y}_1),0)+\zeta^2\max(\exp(\mathbf{y}_1-1),0)\right]\frac{\varrho^{2\delta}}{\Gamma(2\delta+1)},\\
\mathbf{U}_3(\mathbf{y}_1,\varrho)&=\mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}+(\zeta-1)(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_2\right]\right]\\&=\left[-\zeta^3\max(\exp(\mathbf{y}_1),0)+\zeta^3\max(\exp(\mathbf{y}_1-1),0)\right]\frac{\varrho^{3\delta}}{\Gamma(3\delta+1)}.\\&\vdots\end{aligned}\tag{83}$$

The result in series representation is

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)+.\cdots\tag{84}$$

Eventually, we have

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1,\varrho)&=\mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)\\&+\cdots=\max(\exp(\mathbf{y}_1-1),0)\\&\cdot\left[1-\frac{\zeta\varrho^{\delta}}{\Gamma(\delta+1)}+\frac{\zeta^2\varrho^{2\delta}}{\Gamma(2\delta+1)}-\frac{\zeta^3\varrho^{3\delta}}{\Gamma(3\delta+1)}+\cdots\right]\\&+\max(\exp(\mathbf{y}_1),0)\left[1-1+\frac{\zeta\varrho^{\delta}}{\Gamma(\delta+1)}-\frac{\zeta^2\varrho^{2\delta}}{\Gamma(2\delta+1)}\right.\\&\left.+\frac{\zeta^3\varrho^{3\delta}}{\Gamma(3\delta+1)}+\cdots\right]=\max(\exp(\mathbf{y}_1-1),0)E_{\delta}\left(-\zeta(\varrho)^{\delta}\right)\\&+\max(\exp(\mathbf{y}_1),0)E_{\delta}\left(1-\zeta(\varrho)^{\delta}\right).\end{aligned}\tag{85}$$

Case 2. The (4) is now addressed utilizing the Caputo-Fabrizio fractional derivative operator and the Yang iterative transform method.

Assuming (4) and implementing the Yang transform's differentiation criteria, we obtain

$$\begin{aligned}\frac{1}{1+\delta(\varphi-1)}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]&=\frac{1}{1+\delta(\varphi-1)}\sum_{p=0}^{p-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(0)\\&+\mathbf{Y}\left[\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}+(\zeta-1)\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}-\zeta\mathbf{U}(\mathbf{y}_1,\varrho)\right].\end{aligned}\tag{86}$$

It follows that

$$\begin{aligned}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)]&=\varphi.\max(\exp(\mathbf{y}_1)-1,0)+(1+\delta(\varphi-1))\mathbf{Y}\\&\cdot\left[\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}+(\zeta-1)\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}-\zeta\mathbf{U}(\mathbf{y}_1,\varrho)\right].\end{aligned}\tag{87}$$

In view of the proposed algorithm in Section 3.2, we find

$$\mathbf{U}_0(\mathbf{y}_1,\varrho)=\mathbf{Y}^{-1}[\varphi.\max(\exp(\mathbf{y}_1)-1,0)]=\max(\exp(\mathbf{y}_1)-1,0),$$

$$\begin{aligned}
\mathbf{U}_1(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\
&\quad\left.\left.+(\zeta-1)(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_0\right]\right]\\
&= (\zeta \max(\exp(\mathbf{y}_1),0)-\zeta\max(\exp(\mathbf{y}_1-1),0))(1-\delta(\varrho-1)),\\
\mathbf{U}_2(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\
&\quad\left.\left.+(\zeta-1)(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_1\right]\right]\\
&= -\left(\left[\zeta^2\max(\exp(\mathbf{y}_1),0)+\zeta^2\max(\exp(\mathbf{y}_1-1),0)\right]\right)\\
&\quad\cdot\left((1-\delta)^2+2\varrho\delta(1-\delta)+\frac{\varrho^2\delta^2}{2}\right),\\
\mathbf{U}_3(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}+(\zeta-1)(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}+\zeta\mathbf{U}_2\right]\right]\\
&= -\left(\left[\zeta^3\max(\exp(\mathbf{y}_1),0)+\zeta^3\max(\exp(\mathbf{y}_1-1),0)\right]\right)\\
&\quad\times\left((1-\delta)^3+3\varrho\delta(1-\delta)^2+3\frac{\varrho^2}{2}\delta^2(1-\delta)+\frac{\varrho^3\delta^3}{3}\right).\\
&\vdots
\end{aligned} \tag{88}$$

The result in series representation is

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)+.\cdots \tag{89}$$

Eventually, we have

$$\begin{aligned}
\mathbf{U}(\mathbf{y}_1,\varrho) &= \mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)\\
&\quad+\cdots=\max(\exp(\mathbf{y}_1-1),0)\\
&\quad\cdot\left[1-\zeta(1+\delta(\varrho-1))-\zeta^2\left((1-\delta)^2+2\varrho\delta(1-\delta)+\frac{\varrho^2\delta^2}{2}\right)\right]-\cdots\Big]\\
&\quad+\max(\exp(\mathbf{y}_1),0)\left[-\zeta(1+\delta(\varrho-1))-\zeta^2\right.\\
&\quad\left.\cdot\left((1-\delta)^2+2\varrho\delta(1-\delta)+\frac{\varrho^2\delta^2}{2}\right)-\cdots\right].
\end{aligned} \tag{90}$$

Example 4 (see [30]). Surmise the fractional-order BSM (6) supplemented with the (7).

Case 1. To begin, we utilize the Caputo fractional derivative operator employing the Yang iterative transform method to analyze the (6). Implementing the Yang transform on (6), we get

$$\begin{aligned}
\varphi^{-\delta}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)] &= \sum_{p=0}^{p-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(0)+\mathbf{Y}\\
&\quad\cdot\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}\right.\\
&\quad\left.-0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}+0.06\mathbf{U}(\mathbf{y}_1,\varrho)\right].
\end{aligned} \tag{91}$$

It follows that

$$\begin{aligned}
\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)] &= \varphi.\max(\mathbf{y}_1-25\exp(-0.06),0)+\varphi^{\delta}\mathbf{Y}\\
&\quad\cdot\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}\right.\\
&\quad\left.-0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}+0.06\mathbf{U}(\mathbf{y}_1,\varrho)\right].
\end{aligned} \tag{92}$$

In view of the proposed algorithm in Section 3.2, we have

$$\begin{aligned}
\mathbf{U}_0(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}[\varphi.\max(\mathbf{y}_1-25\exp(-0.06),0)]\\
&= \max(\mathbf{y}_1-25\exp(-0.06),0),\\
\mathbf{U}_1(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\
&\quad\left.\left.-0.06\mathbf{y}_1(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_0\right]\right]\\
&= [-0.06\mathbf{y}_1+0.06\max(\mathbf{y}_1-25\exp(-0.06),0)]\mathbf{Y}^{-1}\\
&\quad\cdot\left[\varphi^{\delta+1}(\mathfrak{s})\right]=[-0.06\mathbf{y}_1+0.06\max\\
&\quad\cdot(\mathbf{y}_1-25\exp(-0.06),0)]\frac{\varrho^{\delta}}{\Gamma(\delta+1)},\\
\mathbf{U}_2(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\
&\quad\left.\left.-0.06\mathbf{y}_1(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_1\right]\right]\\
&= [-0.0036\mathbf{y}_1+0.0036\max\\
&\quad\cdot(\mathbf{y}_1-25\exp(-0.06),0)]\frac{\varrho^{2\delta}}{\Gamma(2\delta+1)},\\
\mathbf{U}_3(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[\varphi^{\delta}\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\
&\quad\left.\left.-0.06\mathbf{y}_1(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_2\right]\right]\\
&= [-0.000216\mathbf{y}_1+0.00216\max\\
&\quad\cdot(\mathbf{y}_1-25\exp(-0.06),0)]\frac{\varrho^{3\delta}}{\Gamma(3\delta+1)},\vdots
\end{aligned} \tag{93}$$

The result in series representation is

$$\mathbf{U}(\mathbf{y}_1,\varrho)=\mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)+.\cdots \tag{94}$$

Consequently, we have

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1,\varrho) &= \mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)\\ &\quad +\cdots, = \max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right)\\ &\quad +\left(\mathbf{y}_1-\max\left(\mathbf{y}_1-25\exp\left(-0.06\right)\right)\right)\\ &\quad \times\left[1-1-\frac{0.06\varrho^{\delta}}{\Gamma(\delta+1)}-\frac{0.0036\varrho^{2\delta}}{\Gamma(2\delta+1)}-\frac{0.000216\varrho^{3\delta}}{\Gamma(3\delta+1)}+\cdots\right]\\ &= \max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right)\\ &\quad +\left(\mathbf{y}_1-\max\left(\mathbf{y}_1-25\exp\left(-0.06\right)\right)\right)\left[1-E_{\delta}\left(0.06(\varrho)^{\delta}\right)\right].\end{aligned} \tag{95}$$

Case 2. The Caputo-Fabrizio fractional derivative operator and the Yang iterative transform method are now used to solve the (6).

Assuming (6) and implementing the Yang transform's differentiation criteria, we obtain

$$\begin{aligned}\frac{1}{1+\delta(\varphi-1)}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)] &= \frac{1}{1+\delta(\varphi-1)}\sum_{p=0}^{n-1}\varphi^{1-\delta-p}(\varrho)\mathbf{U}^{(p)}(0)\\ &\quad +\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}-0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}+0.06\mathbf{U}(\mathbf{y}_1,\varrho)\right].\end{aligned} \tag{96}$$

It follows that

$$\begin{aligned}\mathbf{Y}[\mathbf{U}(\mathbf{y}_1,\varrho)] &= \varphi.\max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right)+(1+\delta(\varphi-1))\mathbf{Y}\\ &\quad\cdot\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2\frac{\partial^2\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1^2}-0.06\mathbf{y}_1\frac{\partial\mathbf{U}(\mathbf{y}_1,\varrho)}{\partial\mathbf{y}_1}+0.06\mathbf{U}(\mathbf{y}_1,\varrho)\right].\end{aligned} \tag{97}$$

In view of the proposed algorithm in Section 3.2, we have

$$\begin{aligned}\mathbf{U}_0(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[\varphi.\max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right)\right]\\ &= \max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right),\end{aligned}$$

$$\begin{aligned}\mathbf{U}_1(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\ &\quad\left.\left.-0.06\mathbf{y}_1(\mathbf{U}_0(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_0\right]\right] = (-0.06\mathbf{y}_1+0.06\max\\ &\quad\cdot\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right))(1-\delta(\varrho-1)),\end{aligned}$$

$$\begin{aligned}\mathbf{U}_2(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\ &\quad\left.\left.-0.06\mathbf{y}_1(\mathbf{U}_1(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_1\right]\right]\\ &= (-0.0036\mathbf{y}_1+0.0036\max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right))\\ &\quad\cdot\left((1-\delta)^2+2\varrho\delta(1-\delta)+\frac{\varrho^2\delta^2}{2}\right),\end{aligned}$$

$$\begin{aligned}\mathbf{U}_3(\mathbf{y}_1,\varrho) &= \mathbf{Y}^{-1}\left[(1+\delta(\varphi-1))\mathbf{Y}\left[-0.08(2+\sin\mathbf{y}_1)^2\mathbf{y}_1^2(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1\mathbf{y}_1}\right.\right.\\ &\quad\left.\left.-0.06\mathbf{y}_1(\mathbf{U}_2(\mathbf{y}_1,\varrho))_{\mathbf{y}_1}-0.06\mathbf{U}_2\right]\right]\\ &= -([-0.000216\mathbf{y}_1+0.00216\max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right)])\\ &\quad\times\left((1-\delta)^3+3\varrho\delta(1-\delta)^2+3\delta^2(1-\delta)\frac{\varrho^2\delta^2}{2}+\frac{\varrho^3\delta^3}{3}\right),\vdots\end{aligned} \tag{98}$$

The result in series representation is

$$\mathbf{U}(\mathbf{y}_1,\varrho) = \mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)+.\cdots \tag{99}$$

Consequently, we have

$$\begin{aligned}\mathbf{U}(\mathbf{y}_1,\varrho) &= \mathbf{U}_0(\mathbf{y}_1,\varrho)+\mathbf{U}_1(\mathbf{y}_1,\varrho)+\mathbf{U}_2(\mathbf{y}_1,\varrho)+\mathbf{U}_3(\mathbf{y}_1,\varrho)\\ &\quad +\cdots = \max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right)\\ &\quad -\left(0.06\mathbf{y}_1-0.06\max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right)\right)(1+\delta(\varrho-1))\\ &\quad -\left(0.0036\mathbf{y}_1-0.0036\max\left(\mathbf{y}_1-25\exp\left(-0.06\right),0\right)\right)\\ &\quad\cdot\left((1-\delta)^2+2\varrho\delta(1-\delta)+\frac{\varrho^2\delta^2}{2}\right)-\cdots.\end{aligned} \tag{100}$$

5.3. Results and Explanation. Throughout this investigation, two distinct methodologies are being employed to assess the precise analytical solutions of fractional-order BSe. For various spatial and temporal parameters, the CFD and Caputo-Fabrizio fractional derivative operators in MATLAB package 21 facilitate appropriate numerical findings for the BSe option revenue frameworks utilizing multiple orders.

We built modeling tests for many Brownian deformations involving different $\mathbf{y}_1$ parameters, and the results are shown in Table 1 for Examples 1 and 3, respectively. Table 2 illustrates a computational evaluation of the HPM [35] and the Yang decomposition technique for (4) in accordance with absolute error, considering both fractional derivative operators into account.

Table 3 illustrates the results of a mathematical model for the BSe used in Examples 2 and 4. Table 4 reports the interpretation of an evaluation of the HPM [35] and predicted approaches. The synthetically produced profiles are significantly better reliable and pragmatic than the old ones, as evidenced by this analysis.

For Example 1, Figure 1 displays the evolution of the Yang decomposition technique's data from $\mathbf{U}(\mathbf{y}_1,\varrho)$. Figures 1(a) and 1(b) exhibit the performance of precise and approximation BSe option pricing findings using the CFD operator, whilst Figures 2(a) and 2(b) presents the profile for different Brownian motion $\delta = 0.9$ and $\delta = 0.8$, respectively. Figures 3(a) and 3(b) indicate the absolute errors conducted and fractional-order fluctuation of $\mathbf{U}(\mathbf{y}_1,\varrho)$. At $\delta =$ 0.7,0.8,0.9,1.0. The multiple fractional orders act similarly.

Trying to continue in the analogous trend, Figures 4(a) and 4(b) visually depict the precise-approximate repercussions $\mathbf{U}(\mathbf{y}_1,\varrho)$ for (6) using the Yang decomposition

Table 1: The actual, YDM_{CFD} and YDM_{CF} results of Example 1 for multiple fractional-orders with changing terms of $\mathbf{y}_1$ and ϱ.

$\mathbf{y}_1$	ϱ	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1(YDM_{CFD})$	$\delta = 1(YDM_{CF})$	Exact
	0.6	0.4705192388	0.4756690138	0.4741190479	0.4705192388	0.4704162259	0.4705168133
	0.7	0.6142259772	0.5911676480	0.5697105760	0.5502060671	0.5502060671	0.5501940469
0.1	0.8	0.4844025326	0.4904274856	0.4919950943	0.4905118580	0.4902676792	0.4905042402
	0.9	0.6956692898	0.6956692898	0.6576549272	0.6380690943	0.6380690943	0.6378819374
	1.0	0.7281682904	0.7121411619	0.6954314260	0.6777823827	0.6777823827	0.6773315511
	0.6	0.5786689713	0.5809834455	0.5790903128	0.5746934962	0.5745676759	0.5746905335
	0.7	0.5855031178	0.5903021597	0.5902216376	0.5870000028	0.5868002049	0.5869945315
0.3	0.8	0.8043692685	0.7782694482	0.7525069160	0.7275064817	0.7275064817	0.7274331662
	0.9	0.4889649311	0.4971196426	0.5004383409	0.5002728118	0.4999251430	0.5002606476
	1.0	0.4931268549	0.5034252607	0.5085881420	0.5098806767	0.5094037649	0.5098621947
	0.6	0.7067878775	0.7096147828	0.7073025053	0.7019322213	0.7017785440	0.7019286027
	0.7	0.7151351230	0.7209966860	0.7209966860	0.7208983361	0.7169634225	0.7169567397
0.5	0.8	0.7226436619	0.7316318354	0.7339704329	0.7317577044	0.7313934324	0.7317463400
	0.9	0.7294499607	0.7416153606	0.7465662767	0.7463193362	0.7458006755	0.7463011895
	1.0	0.7356588213	0.7510222374	0.7587243512	0.7606525865	0.7599411177	0.7606250146
	0.6	0.8632726629	0.8667254528	0.8639012308	0.8573419511	0.8571542493	0.8573375314
	0.7	0.8734680117	0.8806273409	0.8805072160	0.8757011017	0.8754030381	0.8756929394
0.7	0.8	0.8826389617	0.8936171417	0.8964735111	0.8937708784	0.8933259556	0.8937569979
	0.9	0.8909521937	0.9058110468	0.9118581095	0.9115564957	0.9109230019	0.9115343312
	1.0	0.8985357133	0.9173006321	0.9267080152	0.9290631671	0.9281941771	0.9290294906
	0.6	1.054403612	1.058620859	1.055171346	1.047159824	1.046930564	1.047154426
	0.7	1.066856239	1.075600663	1.075453942	1.069583741	1.069219685	1.069573771
0.9	0.8	1.078057662	1.091466442	1.094955219	1.091654216	1.091110786	1.091637262
	0.9	1.088211467	1.106360111	1.113746010	1.113377618	1.112603867	1.113350546
	1.0	1.097473998	1.120393522	1.131883726	1.134760315	1.133698928	1.134719182

Table 2: For estimated outcomes of $\mathbf{U}(\mathbf{y}_1, \varrho)$ at $\delta = 1$ considering multiple choices of $\mathbf{y}_1$ and ϱ, examine HPM [35], YDM_{CFD}, and YDM_{CF} of Example 1.

$\mathbf{y}_1$	ϱ	$\lVert\text{Exact} - \text{HPM}\rVert$	$\lVert\text{Exact} - \text{YDM}_{\text{CFD}}\rVert$	$\lVert\text{Exact} - \text{YDM}_{\text{CF}}\rVert$
	0.6	7.900000×10^{-3}	2.425509×10^{-6}	1.005999×10^{-4}
	0.7	1.099099×10^{-3}	4.479656×10^{-6}	1.591007×10^{-4}
0.1	0.8	4.244800×10^{-2}	7.617867×10^{-6}	2.365698×10^{-4}
	0.9	9.107607×10^{-2}	1.216420×10^{-6}	3.355046×10^{-4}
	1.0	5.478550×10^{-2}	1.848200×10^{-6}	4.584298×10^{-4}
	0.6	9.980000×10^{-4}	2.962700×10^{-6}	1.228576×10^{-4}
	0.7	5.400763×10^{-3}	5.471300×10^{-6}	1.943266×10^{-4}
0.3	0.8	7.800998×10^{-2}	9.304400×10^{-6}	2.889363×10^{-4}
	0.9	3.900567×10^{-3}	1.485740×10^{-6}	4.097862×10^{-4}
	1.0	5.100562×10^{-2}	2.257400×10^{-6}	5.599274×10^{-4}
	0.6	3.009801×10^{-3}	3.618600×10^{-6}	1.500587×10^{-4}
	0.7	6.400789×10^{-3}	6.682800×10^{-6}	2.373511×10^{-4}
0.5	0.8	9.660009×10^{-3}	1.136440×10^{-5}	3.529076×10^{-4}
	0.9	2.934890×10^{-2}	1.814670×10^{-5}	5.005140×10^{-4}
	1.0	6.000989×10^{-3}	2.757190×10^{-5}	6.838969×10^{-4}
	0.6	2.560000×10^{-3}	4.419700×10^{-6}	1.832821×10^{-4}
	0.7	2.200000×10^{-2}	8.162300×10^{-6}	2.899013×10^{-4}
0.7	0.8	1.056900×10^{-3}	1.388050×10^{-5}	4.310423×10^{-4}
	0.9	34.008890×10^{-3}	2.216450×10^{-5}	6.113293×10^{-4}
	1.0	8.000956×10^{-3}	3.367650×10^{-5}	8.353135×10^{-4}
	0.6	7.789435×10^{-2}	5.398000×10^{-6}	2.238620×10^{-6}
	0.7	3.000897×10^{-2}	9.970000×10^{-6}	3.540860×10^{-4}
0.9	0.8	2.788609×10^{-2}	1.695400×10^{-5}	5.264760×10^{-4}
	0.9	6.560000×10^{-3}	2.707200×10^{-5}	7.466790×10^{-4}
	1.0	2.000043×10^{-2}	4.113300×10^{-5}	1.020254×10^{-3}

Table 3: The actual, YDM_{CFD} and YDM_{CF} results of Example 2 for multiple fractional-orders with changing terms of $\mathbf{y}_1$ and ϱ.

$\mathbf{y}_1$	ϱ	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1(JDM_{CFD})$	$\delta = 1(JDM_{ABC})$	Exact
	0.6	0.0026121375	0.0020293922	0.0015639815	0.0011964072	0.0011964072	0.001196407
	0.7	0.4793694085	0.4832985317	0.4832326058	0.4805949543	0.4804313735	0.4805904747
0.1	0.8	0.0055797208	0.0048449971	0.0041732529	0.0035677944	0.0035677944	0.003567793
	0.9	0.0067961194	0.0060754949	0.0053880675	0.0047428608	0.0047428608	0.0047428580
	1.0	0.0079145429	0.0072365565	0.0065645469	0.0059109000	0.0059109000	0.0059108933
	0.6	0.0052242750	0.0040587845	0.0031279630	0.0023928144	0.0023928144	0.0023928143
	0.7	0.0084408706	0.0070345154	0.0058151760	0.0047713152	0.0047713152	0.0047713148
0.3	0.8	0.011159441	0.0096899942	0.0083465059	0.0071355888	0.0071355888	0.0071355870
	0.9	0.013592238	0.012150989	0.010776135	0.0094857216	0.0094857216	0.0094857160
	1.0	0.015829085	0.014473113	0.013129093	0.011821800	0.011821800	0.0118217866
	0.6	0.0078364125	0.0060881767	0.0046919445	0.0035892216	0.0035892216	0.0035892215
	0.7	0.012661305	0.010551773	0.0087227641	0.0071569728	0.0071569728	0.0071569722
0.5	0.8	0.016739162	0.014534991	0.012519758	0.010703383	0.010703383	0.0107033805
	0.9	0.020388358	0.018226484	0.016164202	0.014228582	0.014228582	0.0142285741
	1.0	0.023743628	0.021709669	0.019693640	0.017732700	0.017732700	0.0177326799
	0.6	0.010448550	0.0081175690	0.0062559261	0.0047856288	0.0047856288	0.0047856287
	0.7	0.016881741	0.014069030	0.011630352	0.0095426304	0.0095426304	0.0095426296
0.7	0.8	0.022318883	0.019379988	0.016693011	0.016693011	0.016693011	0.0142711740
	0.9	0.027184477	0.024301979	0.021552270	0.018971443	0.018971443	0.0189714321
	1.0	0.031658171	0.028946226	0.026258187	0.023643600	0.023643600	0.0236435732
	0.6	0.013060687	0.010146961	0.0078199076	0.0059820360	0.0059820360	0.005982035
	0.7	0.021102176	0.017586288	0.014537940	0.011928288	0.011928288	0.011928287
0.9	0.8	0.027898604	0.024224985	0.020866264	0.017838972	0.017838972	0.017838967
	0.9	0.033980597	0.030377474	0.026940337	0.023714304	0.023714304	0.023714290
	1.0	0.039572714	0.036182782	0.032822734	0.029554500	0.029554500	0.029554466

Table 4: For estimated outcomes of $\mathbf{U}(\mathbf{y}_1, \varrho)$ at $\delta = 1$ considering multiple choices of $\mathbf{y}_1$ and ϱ, examine HPM [35], YDM_{CFD},and YDM_{CF} of Example 2.

$\mathbf{y}_1$	ϱ	$\|Exact – HPM\|$	$\|Exact – YDM_{CFD}\|$	$\|Exact – YDM_{CF}\|$
	0.6	3.00450×10^{-10}	2.00000×10^{-11}	2.00000×10^{-11}
	0.7	2.00000×10^{-9}	4.4796×10^{-6}	1.80000×10^{-10}
0.1	0.8	9.50000×10^{-9}	8.80000×10^{-10}	8.80000×10^{-10}
	0.9	3.89000×10^{-8}	2.76000×10^{-9}	2.76000×10^{-9}
	1.0	7.70000×10^{-8}	6.70000×10^{-9}	6.70000×10^{-9}
	0.6	5.780000×10^{-10}	4.00000×10^{-11}	4.00000×10^{-11}
	0.7	4.80000×10^{-9}	3.60000×10^{-10}	3.60000×10^{-10}
0.3	0.8	2.98000×10^{-8}	1.76000×10^{-9}	1.76000×10^{-9}
	0.9	6.45000×10^{-8}	5.52000×10^{-9}	5.52000×10^{-9}
	1.0	2.47000×10^{-7}	1.34000×10^{-8}	1.34000×10^{-8}
	0.6	7.96600×10^{-10}	6.00000×10^{-11}	6.00000×10^{-11}
	0.7	6.785000×10^{-9}	5.40000×10^{-10}	5.40000×10^{-10}
0.5	0.8	3.98000×10^{-8}	2.64000×10^{-9}	2.64000×10^{-9}
	0.9	9.31000×10^{-8}	8.28000×10^{-9}	8.28000×10^{-9}
	1.0	3.003000×10^{-7}	2.01000×10^{-8}	2.01000×10^{-8}
	0.6	9.890000×10^{-10}	8.00000×10^{-11}	8.00000×10^{-11}
	0.7	9.80000×10^{-9}	7.20000×10^{-10}	7.20000×10^{-10}
0.7	0.8	4.94000×10^{-8}	3.52000×10^{-9}	3.52000×10^{-9}
	0.9	2.89000×10^{-7}	1.10400×10^{-8}	1.10400×10^{-8}
	1.0	3.60089×10^{-7}	2.68000×10^{-8}	2.68000×10^{-8}
	0.6	2.9900×10^{-9}	1.00000×10^{-10}	1.00000×10^{-10}
	0.7	11.00011×10^{-9}	9.00000×10^{-10}	9.00000×10^{-10}
0.9	0.8	6.40000×10^{-8}	4.40000×10^{-9}	4.40000×10^{-9}
	0.9	2.87000×10^{-7}	1.38000×10^{-8}	1.38000×10^{-8}
	1.0	4.89000×10^{-7}	3.35000×10^{-8}	3.35000×10^{-8}

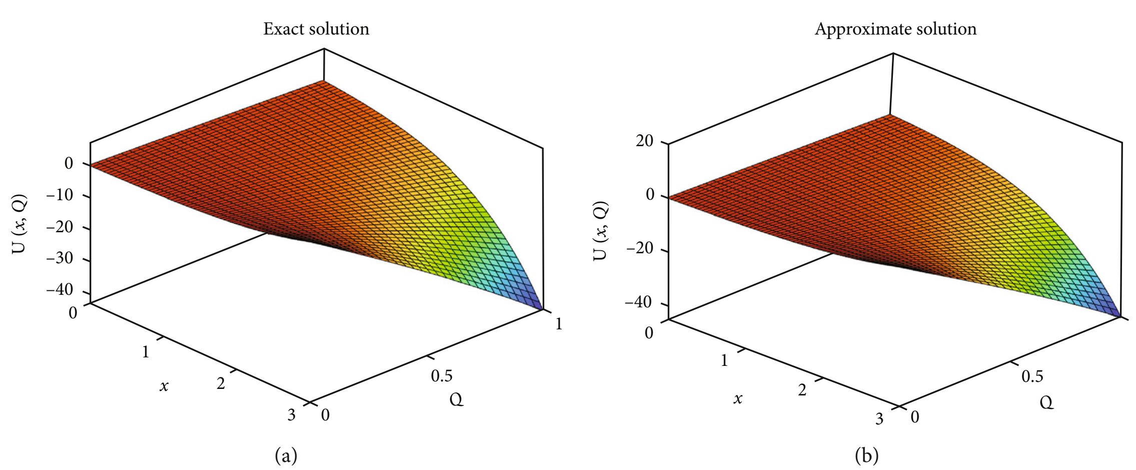


Figure 1: Three-dimensional illustration via CFD of Example 1 when $\delta = 1$. (a) Exact solution. (b) Approximate solution.

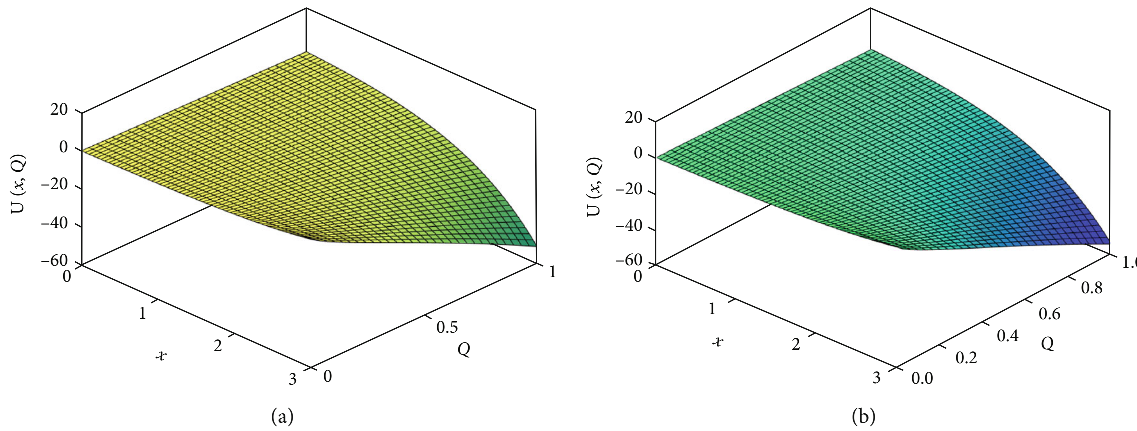


FIGURE 2: Three-dimensional illustration of the approximate solution via CFD of Example 1 when (a) $\delta = 0.9$ and $\delta = 0.8$.

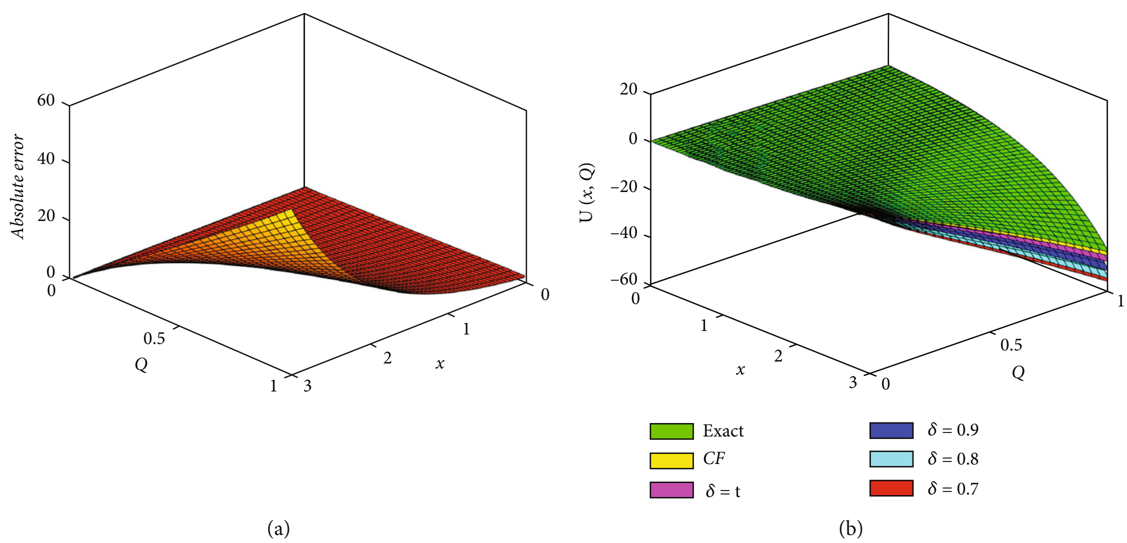


FIGURE 3: Three-dimensional illustration via the CFD of Example 1. (a) Absolute error. (b) Multiple fractional-order.

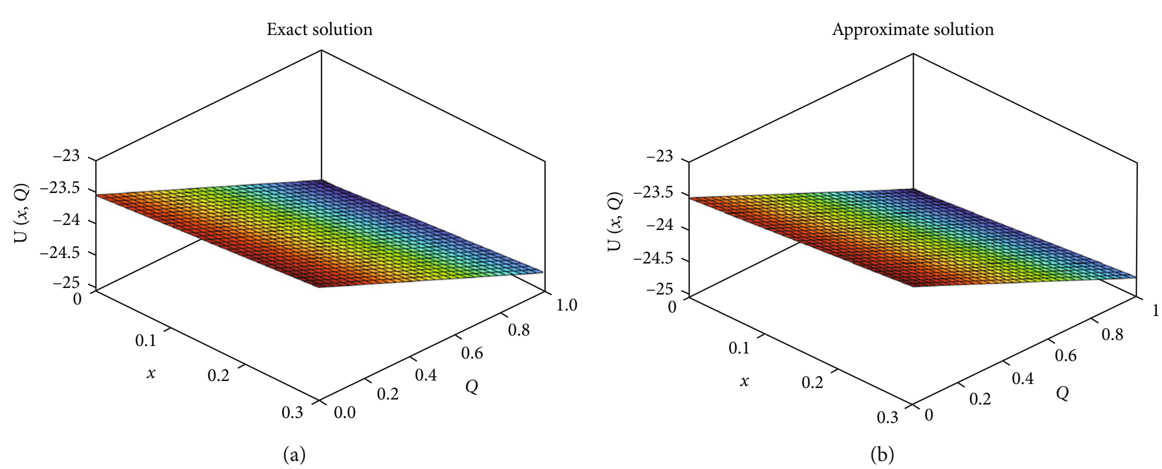
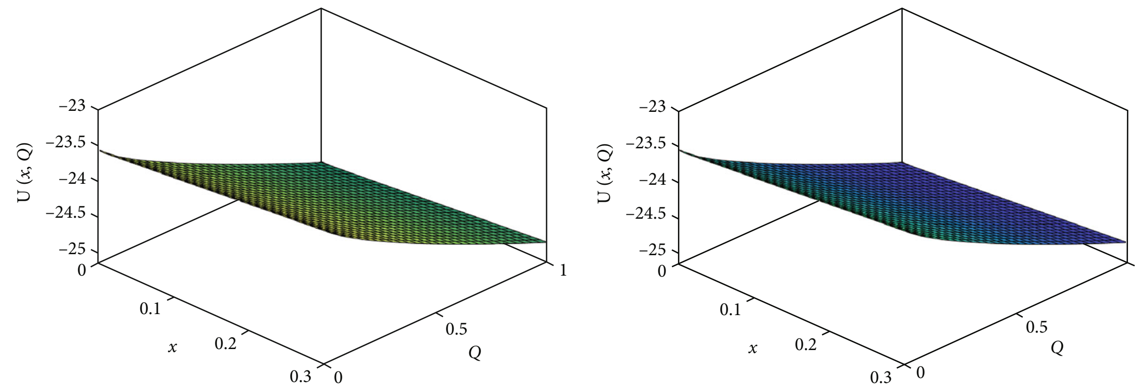
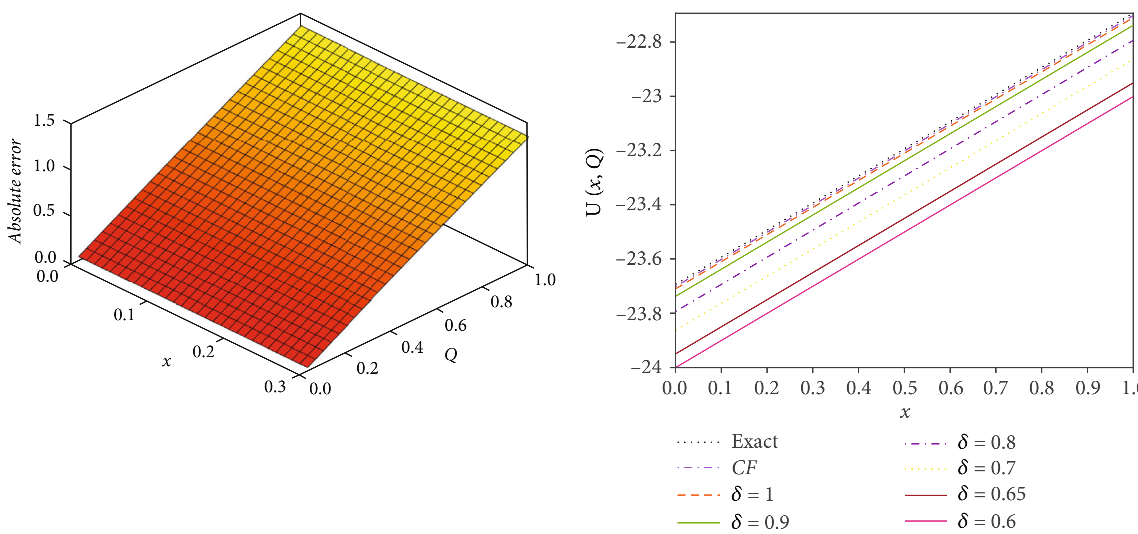


FIGURE 4: Three-dimensional illustration via CFD of Example 1 when $\delta = 1$. (a) Exact solution. (b) Approximate solution.



(a) (b)

Figure 5: Three-dimensional illustration of the approximate solution via CFD of Example 1 when (a) $\delta = 0.9$ and $\delta = 0.8$.



(a) (b)

Figure 6: Three-dimensional illustration via the CFD of Example 1. (a) Absolute error. (b) Multiple fractional-order.

approach on the contents of the choices, whilst Figures 5(a) and 5(b) present the profile for Brownian motion $\delta = 0.9$ and $\delta = 0.8$, respectively. The absolute error and sensitivity of gathered information for (6) involving different numerical and Brownian movements of $\delta = 0.7, 0.8, 0.9$ and 1 are illustrated in Figures 6(a) and 6(a). Furthermore, Figure 6(b) refers to the dynamic of the two-dimensional alternatives of the analysis values $\mathbf{U}(\mathbf{y}_1, \varrho)$ for (6). Finally, we deduce that as the amount of the time-dependent component improves, the hierarchy of the feature images tends to rise as well. It is important to remember that the fractional order has a simulatory effect on the diffusion mechanism.

6. Conclusion

The Adomian decomposition approach and the new iterative transform procedure have been leveraged to analyze the Yang transform. To interact effectively with the BSe, the Caputo and Caputo-Fabrizio fractional derivative operators have been constructed. Considering the supposition of fractional order, numerous new outcomes have been presented. To clarify the crucial aspects of the fractional frameworks under evaluation, diverse visualizations were attempted to explicate these results. The suggested scheme identifies the findings without any underlying limitations, deconvolution, or quantization. Our transformation has been described in terms of refinement and inventiveness. When comparing our results to those discovered in existing academic publications, it becomes clear that our approaches in the European Choice Valuation framework are exceptional. The schemes' effective and comprehensive execution is investigated and confirmed in an attempt to display that it may be applicable to other nonlinear evolutionary models that emerge in business and accountancy.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All authors read and approved the final manuscript.

References

[1] Y.-M. Chu, S. Bashir, M. Ramzan, and M. Y. Malik, "Model-based comparative study of magnetohydrodynamics unsteady hybrid nanofluid flow between two infinite parallel plates with particle shape effects," *Mathematical Methods in the Applied Sciences*, 2022.

[2] K. Karthikeyan, P. Karthikeyan, H. M. Baskonus, K. Venkatachalam, and Y.-M. Chu, "Almost sectorial operators on Ψ-Hilfer derivative fractional impulsive integro-differential equations," *Mathematical Methods in the Applied Sciences*, vol. 45, no. 13, pp. 8045–8059, 2022.

[3] S. A. Iqbal, M. G. Hafez, Y.-M. Chu, and C. Park, "Dynamical analysis of nonautonomous *RLC* circuit with the absence and presence of Atangana-Baleanu fractional derivative," *Journal of Applied Analysis and Computation*, vol. 12, no. 2, pp. 770–789, 2022.

[4] T.-H. Zhao, O. Castillo, H. Jahanshahi et al., "A fuzzy-based strategy to suppress the novel coronavirus (2019-NCOV) massive outbreak," *Applied and Computational Mathematics*, vol. 20, no. 1, pp. 160–176, 2021.

[5] S. Rashid, F. Jarad, and A. A. G. Ahmad, "A novel fractal-fractional order model for the understanding of an oscillatory and complex behavior of human liver with non-singular kernel," *Results in Physics*, vol. 35, no. 2, article 105292, 2022.

[6] M. Alqhtani, K. M. Owolabi, and K. M. Saad, "Spatiotemporal (target) patterns in sub-diffusive predator-prey system with the Caputo operator," *Chaos, Solitons & Fractals*, vol. 160, article 112267, 2022.

[7] M. Alqhtani and K. M. Saad, "Numerical solutions of space-fractional diffusion equations via the exponential decay kernel," *AIMS Mathematics*, vol. 7, no. 4, pp. 6535–6549, 2022.

[8] S. Aljhani, M. S. Md Noorani, K. M. Saad, and A. K. Alomari, "Numerical solutions of certain new models of the time-fractional Gray-Scott," *Journal of Function Spaces*, vol. 2021, Article ID 2544688, 12 pages, 2021.

[9] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, CA, USA, 1999.

[10] S. Rashid, S. Sultana, Y. Karaca, A. Khalid, and Y.-M. Chu, "Some further extensions considering discrete proportional fractional operators," *Fractals*, vol. 30, no. 1, article 2240026, 2022.

[11] S. Narges Hajiseyedazizi, M. E. Samei, J. Alzabut, and Y.-M. Chu, "On multi-step methods for singular fractional q-integro-differential equations," *Open Mathematics*, vol. 19, no. 1, pp. 1378–1405, 2021.

[12] S. Rashid, E. I. Abouelmagd, A. Khalid, F. B. Farooq, and Y.-M. Chu, "Some recent developments on dynamical $\hbar$-discrete fractional type inequalities in the frame of nonsingular and nonlocal kernels," *Fractals*, vol. 30, no. 2, article 2240110, 2022.

[13] F.-Z. Wang, M. N. Khan, I. Ahmad, H. Ahmad, H. Abu-Zinadah, and Y.-M. Chu, "Numerical solution of traveling waves in chemical kinetics: time-fractional fishers equations," *Fractals*, vol. 30, no. 2, article 2240051, 2022.

[14] S. Rashid, E. I. Abouelmagd, S. Sultana, and Y.-M. Chu, "New developments in weighted n-fold type inequalities via discrete generalized h-proportional fractional operators," *Fractals*, vol. 30, no. 2, article 2240056, 2022.

[15] M. Caputo and M. Fabrizio, "A new definition of fractional derivative without singular kernel," *Progress in Fractional Differentiation & Applications*, vol. 73, pp. 1–13, 2015.

[16] R. Scherer, S. L. Kalla, Y. Tang, and J. Huanget, "The Grunwald-Letnikov method for fractional differential equations," *Computers & Mathematcs with Applications*, vol. 62, no. 3, pp. 902–917, 2011.

[17] S. Bibi and S. T. Mohyud-Din, "Traveling wave solutions of KdVs using sine-cosine method," *Journal of the Association of Arab Universities for Basic and Applied Sciences*, vol. 15, no. 1, pp. 90–93, 2014.

[18] A. H. Zedan and E. El-Adrous, "The application of the homotopy perturbation method and the homotopy analysis method to the generalized Zakharov equations," *Abstract and Applied Analysis*, vol. 2012, Article ID 561252, 19 pages, 2012.

[19] S. Rashid, R. Ashraf, and F. Jarad, "Strong interaction of Jafari decomposition method with nonlinear fractional-order partial differential equations arising in plasma via the singular and nonsingular kernels," *AIMS Mathematics*, vol. 7, no. 5, 2022.

[20] S. Rashid, Z. Hammouch, H. Aydi, A. G. Ahmad, and A. M. Alsharif, "Novel computations of the time-fractional Fisher's model via generalized fractional integral operators by means of the Elzaki transform," *Fractal and Fractional*, vol. 5, no. 3, p. 94, 2021.

[21] S. Widatalla and M. Z. Liu, "New iterative method based on Laplace decomposition algorithm," *Journal of Applied Mathematics*, vol. 2013, Article ID 286529, 7 pages, 2013.

[22] A. -M. Wazwaz, "The variational iteration method: a reliable analytic tool for solving linear and nonlinear wave equations," *Computational and Applied Mathematics*, vol. 54, no. 7-8, pp. 926–932, 2007.

[23] H. O. Roshid, M. A. Akbar, N. Alam, F. Hoque, and N. Rahman, "New extended G/G'-expansion method to solve nonlinear evolution equation: the (3+1)-dimensional potential-YTSF equation," *Springer Plus*, vol. 3, no. 122, 2014http://www.springerplus.com/content/3/1/122.

[24] E. Yasar, Y. Yildirim, and C. M. Khalique, "Lie symmetry analysis, conservation laws and exact solutions of the seventh-order time fractional Sawada-Kotera-Ito equation," *Results in Physics*, vol. 6, pp. 322–328, 2016.

[25] H. A. Zedan and E. Alaidarous, "Haar Wavelet method for the system of integral equations," *Abstract and Applied Analysis*, vol. 2014, Article ID 418909, 9 pages, 2014.

[26] M. J. Ablowitz and P. A. Clarkson, *Solitons, Nonlinear Evolution Equations and Inverse Scattering*, Cambridge University Press, 2010.

[27] N. A. Kudryashov, "Simplest equation method to look for exact solutions of nonlinear differential equations," *Chaos, Solitons & Fractals*, vol. 24, no. 5, pp. 1217–1231, 2005.

[28] C. H. Gu, *Soliton theory and its application*, Zhejiang Science and Technology Press, 1990.

[29] N. Özdemir and M. Yavuz, "Numerical solution of fractional Black-Scholes equation by using the multivariate Padé approximation," *Acta Physica Polonica A*, vol. 132, no. 3-II, pp. 1050–1053, 2017.

[30] Z. Cen and A. Le, "A robust and accurate finite difference method for a generalized Black–Scholes equation," *Journal of Computational and Applied Mathematics*, vol. 235, no. 13, pp. 3728–3733, 2011.

[31] P. Wilmott, J. Dewynne, and S. Howison, *Option Pricing: Mathematical Models and Computation*, Oxford Financial Press, Oxford, 1993.

[32] C. Vazquez, "An upwind numerical approach for an American and European option pricing model," *Applied Mathematics and Computation*, vol. 97, no. 2-3, pp. 273–286, 1998.

[33] L. Meng and M. Wang, "Comparison of Black-Scholes formula with fractional Black-Scholes formula in the foreign exchange option market with changing volatility," *Asia-Pacific Financial Markets*, vol. 17, no. 2, pp. 99–111, 2010.

[34] Q. Fu, K. Wang, and X. Liu, "The foreign exchange option pricing based on the fractional Black-Scholes model and valuation," *Price Monthly*, vol. 369, pp. 68–70, 2008.

[35] A. N. Fall, S. N. Ndiaye, and N. Sene, "Black–Scholes option pricing equations described by the Caputo generalized fractional derivative," *Chaos, Solitons & Fractals*, vol. 125, pp. 108–118, 2019.

[36] M. B. Matadi and P. L. Zondi, "Invariant solutions of Black-Scholes equation with Ornstein-Uhlenbeck process," *Symmetry*, vol. 13, no. 5, p. 847, 2021.

[37] S. Kumar, D. Kumar, and J. Singh, "Numerical computation of fractional Black-Scholes equation arising in financial market," *Egyptian Journal of Basic and Applied Sciences*, vol. 1, no. 3-4, pp. 177–183, 2014.

[38] M. Yavuz and N. Özdemir, "A different approach to the European option pricing model with new fractional operator," *Mathematical Modelling of Natural Phenomena*, vol. 13, no. 1, p. 12, 2018.

[39] S. Rashid, R. Ashraf, and E. Bonyah, "On analytical solution of time-fractional biological population model by means of generalized integral transform with their uniqueness and convergence analysis," *Journal of Function Spaces*, vol. 2022, Article ID 7021288, 29 pages, 2022.

[40] S. Rashid, R. Ashraf, and F. S. Bayones, "A novel treatment of fuzzy fractional Swift–Hohenberg equation for a hybrid transform within the fractional derivative operator," *Fractal and Fractional*, vol. 5, no. 4, p. 209, 2021.

[41] X. -J. Yang, "A new integral transform method for solving steady heat-transfer problem," *Thermal Science*, vol. 20, supplement 3, pp. 639–642, 2016.

[42] V. Daftardar-Gejji and H. Jafari, "An iterative method for solving nonlinear functional equations," *Journal of Mathematical Analysis and Applications*, vol. 316, no. 2, pp. 753–763, 2006.

[43] S. Rashid, R. Ashraf, and M. Tahir, "On novel analytical solution of time-fractional Schrödinger equation within a hybrid transform," *Mathematical Sciences*, pp. 1–19, 2022.

[44] K. Wang and S. Liu, "Application of new iterative transform method and modified fractional homotopy analysis transform method for fractional Fornberg-Whitham equation," *Journal of Nonlinear Sciences and Applications*, vol. 9, no. 5, pp. 2419–2433, 2016.

[45] S. Ahmad, A. Ullah, A. Akgül, and M. De la Sen, "A novel homotopy perturbation method with applications to nonlinear fractional order KdV and Burger equation with exponential-decay kernel," *Journal of Function Spaces*, vol. 2021, Article ID 8770488, 11 pages, 2021.

[46] M. G. Mittag-Leffler, "Sur la nouvelle fonction Ea(x)," *Comptes Rendus. Académie des Sciences*, vol. 2, p. 1003, 1903.

[47] I. El-Kalla, "Convergence of the Adomian method applied to a class of nonlinear integral equations," *Applied Mathematics Letters*, vol. 21, no. 4, pp. 372–376, 2008.