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Research Article

Analysis of Fractional-Order Regularized Long-Wave Models via a Novel Transform

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A new integral transform method for regularized long-wave (RLW) models having fractional-order is presented in this study. Although analytical approaches are challenging to apply to such models, semianalytical or numerical techniques have received much attention in the literature. We propose a new technique combining integral transformation, the Elzaki transform (ET), and apply it to regularized long-wave equations in this study. The RLW equations describe ion-acoustic waves in plasma and shallow water waves in seas. The results obtained are extremely important and necessary for describing various physical phenomena. This work considers an up-to-date approach and fractional operators in this context to obtain satisfactory approximate solutions to the proposed problems. We first define the Elzaki transforms of the Caputo fractional derivative (CFD) and Atangana-Baleanu fractional derivative (ABFD) and implement them for solving RLW equations. We can readily obtain numerical results that provide us with improved approximations after only a few iterations. The derived solutions were found to be in close contact with the exact solutions. Furthermore, the suggested procedure has attained the best level of accuracy. In fact, when compared to other analytical techniques for solving nonlinear fractional partial differential equations, the present method might be considered one of the finest.

1. Introduction

Fractional calculus (FC) is a model discipline of mathematics that focuses entirely on fractional-order derivatives and integration. Fractional derivatives and fractional integrations are noninteger-order derivatives and integration that can model various phenomena in engineering and science. FC began in 1695 when L'Hospital asked Leibniz, "What would be the physical meaning of fractional derivative?" This question inspired many great scientists in the eighteenth and nineteenth centuries to focus on fractional calculus, which has a wide range of applications in applied science and technology. Many researchers have demonstrated that fractional generalizations of integer-order models efficiently represent natural phenomena [1–5]. The classical derivatives are local. In contrast, the Caputo fractional derivative is nonlocal, i.e., we can study changes in the neighborhood of a point using classical derivatives. Still, we may analyze changes in an interval using Caputo fractional derivatives. Because of this quality, the Caputo fractional derivative can be used to model a wider range of physical phenomena, including solid mechanics [6, 7], diffusion procedures [8], continuum and statistical mechanics [9], electromagnetism [10], viscoelastic materials [11], fluid mechanics [12], propagation of spherical flames [13], viscoelastic materials [14], and so on [15–17].

Fractional differential equations have been studied for decades due to their widespread application in science and engineering. Fractional partial differential equations are used to describe various phenomena in acoustics, electromagnetics, material science, viscoelasticity, electrochemistry, and plasma physics. Fractional differential equations have numerical solutions that are of great interest. For fractional differential equations, no method provides an accurate solution. Only series solution methods or linearization can generate approximate solutions [18–21]. Nonlinear phenomena can be found in various engineering and science domains, including chemical kinetics, nonlinear spectroscopy, solid state physics, fluid physics, computational biology, quantum mechanics, and thermodynamics. Different higher-order nonlinear partial differential equations (PDEs) define the idea of nonlinearity. Nonlinear models for all physical systems describe basic phenomena. The literature has presented integrative transform approaches for solving fractional differential equations. Elzaki transform (ET) is an integral transform [22] in this context. Several scholars [23–30] have looked at some essential solution approaches for real-world issues, as well as numerical simulations obtained using the novel integral transformation.

In this article, three alternative fractional homogeneous RLW equations are studied; the RLW equations, according to some scientists, are the best equations than the classical Korteweg-de Vries (KdV) equation [31]. We use the Elzaki transform combined with the CFD and ABC operator [32] to solve three special RLW problems. The approximate solutions are then obtained, and the numerical simulations of the solutions are analyzed [33, 34] and provided the nonlinear RLW equations.

$$D^{\delta}_{\mathfrak{J}}\varphi(\psi,\mathfrak{J}) - \varphi_{\psi\psi\mathfrak{J}}(\psi,\mathfrak{J}) + \varphi_{\psi}(\psi,\mathfrak{J}) + \varphi(\psi,\mathfrak{J})\varphi_{\psi}(\psi,\mathfrak{J}) = 0, \quad (1)$$

having initial source

$$\varphi(\psi, 0) = 3\alpha \sec h^2(\delta\zeta), \quad (2)$$

$$\mathrm{D}^{\delta}_{\mathfrak{J}}\varphi(\psi,\mathfrak{J}) - 2\varphi_{\psi\psi\mathfrak{J}}(\psi,\mathfrak{J}) + \varphi_{\psi}(\psi,\mathfrak{J}) = 0, \quad (3)$$

having initial source

$$\varphi(\psi, 0) = \mathrm{e}^{-\psi}, \quad (4)$$

$$\mathrm{D}^{\delta}_{\mathfrak{J}}\varphi(\psi,\mathfrak{J}) + \varphi_{\psi\psi\psi\psi}(\psi,\mathfrak{J}) = 0, \quad (5)$$

having initial source

$$\varphi(\psi, 0) = \sin\psi. \quad (6)$$

Equation (1) is known as a general regularized long-wave equation (GRLWE) having fractional-order, whereas Equations (3) and (5) represents fractional regularized long-wave equations (RLWEs).

Magnetohydrodynamic waves in plasma, ion-acoustic waves in plasma, stress waves in compressed gas bubble mixes, rotating tube flow, and longitudinal dispersive waves in elastic rods are just some of the applications of the RLW equations. The RLW equations are suitable models for many significant physical structures in applied physics and engineering. They also work on a variety of liquid flow phenomena where diffusion is essential, such as in viscous or shock situations. It can be used to simulate any nonlinear wave diffusion problem, including dissipation. Depending on the problem modeling [35], this dissipation could result in heat conduction, viscosity, thermal radiation, chemical reaction, mass diffusion, or other sources. Many necessary ocean research and engineering phenomena, such as minor frequency shallow-water waves and long-waves, are defined by fractional RLW equations. Several experts in ocean shallow liquid waves are interested in nonlinear waves described using the RLW equations having fractional-order. The fractional RLW equations were used to represent nonlinear waves in the ocean. Indeed, the tsunami's massive surface waves are defined by fractional RLW equations. Huge internal waves in the ocean's interior caused by temperature differences that can destroy marine ships could be defined as fractional RLW equations in the existing, exceedingly complex framework.

The article is given as follows: In Section 2, some basic definitions are essential for the formulation of the problem. The method is described in Section 3, using a novel integral transformation. Section 4 presents the main results, numerical simulations, and graphical representations. Finally, Section 5 presents all of the research study's significant findings.

2. Preliminaries

The fundamental concepts with and without a singular kernel of fractional derivatives, fractional integrals, and their Elzaki transform are presented in this section.

Definition 1. The Caputo fractional derivative (CFD) is defined as [1]

$${}^{C}D^{\delta}_{\mathfrak{J}}(\mu(\mathfrak{J})) = \begin{cases} \dfrac{1}{\Gamma(m-\delta)}\displaystyle\int_0^{\mathfrak{J}} \dfrac{\mu^{m}(\eta)}{(\mathfrak{J}-\eta)^{\delta+1-m}}\,d\eta, & m-1<\delta<m, \\ \dfrac{d^m}{d\mathfrak{J}^m}\mu(\mathfrak{J}), & \delta = m. \end{cases} \quad (7)$$

Definition 2. The Atangana-Baleanu derivative having fractional-order in the Caputo manner (ABC) is defined as [36]

$${}^{ABC}\mathrm{D}^{\delta}_{\mathfrak{J}}(\mu(\mathfrak{J})) = \frac{\mathrm{N}(\delta)}{1-\delta}\int_{\mathrm{m}}^{\mathfrak{J}} \mu'(\eta)\mathrm{E}_{\delta}\left[-\frac{\delta(\mathfrak{J}-\eta)^{\delta}}{1-\delta}\right]d\eta, \quad (8)$$

where $\mu \in H^1(\alpha,\beta), \beta > \alpha, \delta \in [0,1]$. A normalization function equal to 1 when $\delta = 0$ and $\delta = 1$ is represented by $N(\delta)$ in Equation (8).

Definition 3. The ABC operator fractional integral is given by [36].

$$I_{\mathfrak{J}}^{\delta}(\mu(\mathfrak{J}))=\frac{1-\delta}{N(\delta)}\mu(\mathfrak{J})+\frac{\delta}{\Gamma(\delta)N(\delta)}\int_{m}^{\mathfrak{J}}\mu(\eta)(\mathfrak{J}-\eta)^{\delta-1}d\eta. \tag{9}$$

Definition 4. In set A, the exponential function Elzaki transform is defined as [37, 38]

$$A=\left\{\mu(\mathfrak{J})\colon \exists G, p_1, p_2>0, |\mu(\mathfrak{J})|<Ge^{|\mathfrak{J}|/p_j}, \text{if} \quad \mathfrak{J}\in(-1)^j\times[0,\infty)\right\}. \tag{10}$$

G is a finite number for a specific function in the set, but p_1, p_2 can be finite or infinite.

Definition 5. The Elzaki transformation of a function $\mu(\mathfrak{J})$ is given by [38].

$$\mathscr{E}\{\mu(\mathfrak{J})\}(\varpi)=\tilde{U}(\varpi)=\varpi\int_0^{\infty}e^{-\mathfrak{J}/\varpi}\mu(\mathfrak{J})d\mathfrak{J}, \tag{11}$$

where $\mathfrak{J}\geq 0$, $\mathrm{p}_1\leq\varpi\leq\mathrm{p}_2$.

Theorem 6 (Elzaki transformation convolution theorem, [39]). *The following equality holds:*

$$\mathscr{E}\{\mu*\nu\}=\frac{1}{\varpi}\mathscr{E}(\mu)\mathscr{E}(\nu), \tag{12}$$

where Elzaki transform is represented by $\mathscr{E}\{.\}$.

Definition 7. The Elzaki transform of the CFD operator $D_{\mathfrak{J}}^{\delta}(\mu(\mathfrak{J}))$ is given by [40].

$$\mathscr{E}\left\{{}^{C}D_{\mathfrak{J}}^{\delta}(\mu(\mathfrak{J}))\right\}(\varpi)=\varpi^{-\delta}\tilde{U}(\varpi)-\sum_{k=0}^{m-1}\varpi^{2-\delta+k}\mu^{k}(0), \tag{13}$$

where $m-1<\delta<m$.

Theorem 8. *The ABC fractional derivative $D_{\mathfrak{J}}^{\delta}(\mu(\mathfrak{J}))$ Elzaki transform is defined as [32].*

$$\mathscr{E}\left\{{}^{ABC}D_{\mathfrak{J}}^{\delta}(\mu(\mathfrak{J}))\right\}(\varpi)=\frac{N(\delta)\varpi}{\delta\varpi^{\delta}+1-\delta}\left(\frac{\tilde{U}(\varpi)}{\varpi}-\varpi\mu(0)\right), \tag{14}$$

where $\mathscr{E}\{\mu(\mathfrak{J})\}\varpi=\tilde{U}(\varpi)$.

Proof. From Definition 2, we have

$$\mathscr{E}\left\{{}^{ABC}D_{\mathfrak{J}}^{\delta}(\mu(\mathfrak{J}))\right\}(\varpi)=\mathscr{E}\left\{\frac{N(\delta)}{1-\delta}\int_0^{\mathfrak{J}}\mu'(\eta)E_{\delta}\left[-\frac{\delta(\mathfrak{J}-\eta)^{\delta}}{1-\delta}\right]d\eta\right\}(\varpi). \tag{15}$$

Then, taking into account the Elzaki transform's definition and convolution, we get

$$\begin{aligned}\mathscr{E}\left\{{}^{ABC}D_{\mathfrak{J}}^{\delta}(\mu(\mathfrak{J}))\right\}(\varpi)&=\mathscr{E}\left\{\frac{N(\delta)}{1-\delta}\int_0^{\mathfrak{J}}\mu'(\eta)E_{\delta}\left[-\frac{\delta(\mathfrak{J}-\eta)^{\delta}}{1-\delta}\right]d\eta\right\}\\&=\frac{N(\delta)}{1-\delta}\frac{1}{\varpi}\mathscr{E}\left\{\mu'(\eta)\right\}\mathscr{E}\left\{E_{\delta}\left[-\frac{\delta\mathfrak{J}^{\delta}}{1-\delta}\right]d\eta\right\}\\&=\frac{N(\delta)}{1-\delta}\left[\frac{\tilde{U}(\varpi)}{\varpi}-\varpi\mu(0)\right]\\&\quad\cdot\left[\int_0^{\infty}e^{-1/\varpi}E_{\delta}\left[-\frac{\delta\mathfrak{J}^{\delta}}{1-\delta}\right]d\mathfrak{J}\right]\\&=\frac{N(\delta)\varpi}{\delta\varpi^{\delta}+1-\delta}\left[\frac{\tilde{U}(\varpi)}{\varpi}-\varpi\mu(0)\right].\end{aligned} \tag{16}$$

□

3. Description of the Technique via a New Integral Transform

The essential technique that was employed in this research will be presented in this section of the study. We use the following fractional nonlinear PDE general form to study this methodology:

$$\begin{gathered}D_{\mathfrak{J}}^{\delta}\varphi(\psi,\mathfrak{J})+L(\varphi(\psi,\mathfrak{J}))+N(\varphi(\psi,\mathfrak{J}))=\theta(\psi,\mathfrak{J}),\\(\psi,\mathfrak{J})\in[0,1]\times[0,T],\quad \kappa-1<\delta<\kappa,\end{gathered} \tag{17}$$

having initial source

$$\frac{\partial^{z}\varphi}{\partial\mathfrak{J}^{z}}(\psi,0)=\mu_{z}(\psi),\quad \mathrm{z}=0,1,\cdots,\kappa-1, \tag{18}$$

and the boundary sources

$$\begin{aligned}\varphi(0,\mathfrak{J})&=\gamma_0(\mathfrak{J}),\\\varphi(1,\mathfrak{J})&=\gamma_1(\mathfrak{J}),\\\mathfrak{J}&\geq 0,\end{aligned} \tag{19}$$

where known functions are μ_z,θ,γ_0, and γ_1. In Equation (17), $D_{\mathfrak{J}}^{\delta}\varphi(\psi,\mathfrak{J})$ represents the Caputo or ABC fractional derivatives, $L(.)$ and $N(.)$ denote the linear and nonlinear terms. The recursive steps for handling the specified problems are described (1)-(2), (3)-(4), and (5)-(6). We investigate $E\{\varphi(\psi,\mathfrak{J})\}(\varpi)=\tilde{\zeta}(\psi,\varpi)$ for Equation (17) by taking the Elzaki transform with the aid of CFD in Equation (13) and ABC in Equation (14). The modified functions for the Caputo fractional derivative can then be obtained.

$$\tilde{\zeta}(\psi,\varpi)=\varpi^{\delta}\left(\widetilde{\theta}(\varpi,\mathfrak{I})-\mathscr{E}[L(\varphi(\psi,\mathfrak{I}))+N(\varphi(\psi,\mathfrak{I}))]\right)+\varpi^{2}\varphi(\psi,0). \tag{20}$$

We also get the modified functions for the ABC derivative, which are as follows.

$$\tilde{\zeta}(\psi,\varpi)=\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\left(\widetilde{\theta}(\psi,\varpi)-\mathscr{E}[L(\varphi(\psi,\mathfrak{I}))+N(\varphi(\psi,\mathfrak{I}))]\right)+\varpi^{2}\varphi(\psi,0), \tag{21}$$

where $\mathscr{E}[\theta(\psi,\mathfrak{I})]=\widetilde{\theta}(\psi,\varpi)$. We also get when we consider the Elzaki transforms of the boundary conditions

$$\begin{aligned}\mathscr{E}[\gamma_0(\mathfrak{I})]&=\tilde{\zeta}(0,\varpi),\\ \mathscr{E}[\gamma_1(\mathfrak{I})]&=\tilde{\zeta}(1,\varpi),\\ \varpi&\geq 0.\end{aligned} \tag{22}$$

The solution to Equations (17)–(19) is then obtained by using the perturbation method as

$$\tilde{\zeta}(\psi,\varpi)=\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\tilde{\zeta}_{\mathscr{E}}(\psi,\varpi),\ \mathscr{E}=0,1,2,\cdots. \tag{23}$$

The nonlinear component in Equation (17) can be calculated as

$$N[\varphi(\psi,\mathfrak{I})]=\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\Psi_{\mathscr{E}}(\psi,\mathfrak{I}), \tag{24}$$

and the parts $\Psi_{\mathscr{E}}(\psi,\mathfrak{I})$ are define in as

$$\Psi_{\mathscr{E}}(\varphi_0,\varphi_1,\cdots,\varphi_{\mathscr{E}})=\frac{1}{\mathscr{E}!}\frac{\partial^{\mathscr{E}}}{\partial\nu^{\mathscr{E}}}\left[N\left(\sum_{i=0}^{\infty}\nu^{i}\varphi_i\right)\right]_{\lambda=0},\ \mathscr{E}=0,1,2,\cdots. \tag{25}$$

By putting Equations (23) and (24) into Equation (20), we obtain the components of the Caputo operator's solution:

$$\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\tilde{\zeta}(\psi,\varpi)=-\mathscr{X}\varpi^{\delta}\left(\mathscr{E}\left[L\left(\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi,\mathfrak{I})\right)+\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\Psi_{\mathscr{E}}(\psi,\mathfrak{I})\right]\right)+\varpi^{\delta}\left(\widetilde{\theta}(\psi,\varpi)\right)+\varpi^{2}\varphi(\psi,0). \tag{26}$$

and by putting Equations (23) and (24) into Equation (21), we have the recursive relation that provides the Atangana-Baleanu operator's solution:

$$\begin{aligned}\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\tilde{\zeta}(\psi,\varpi)=&-\mathscr{X}\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\left(\mathscr{E}\left[L\left(\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi,\mathfrak{I})\right)\right.\right.\\ &\left.\left.+\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\Psi_{\mathscr{E}}(\psi,\mathfrak{I})\right]\right)\\ &+\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\left(\widetilde{\theta}(\psi,\varpi)\right)+\varpi^{2}\varphi(\psi,0).\end{aligned} \tag{27}$$

Thus, on solving Equations (26) and (27) with respect to $\mathscr{X}$, the given Caputo homotopies are identified:

$$\begin{aligned}\mathscr{X}^{0}&:\tilde{\zeta}_0(\psi,\varpi)=\varpi^{\delta}\left(\widetilde{\theta}(\psi,\varpi)\right)+\varpi^{2}\varphi(\psi,0),\\ \mathscr{X}^{1}&:\tilde{\zeta}_1(\psi,\varpi)=-\varpi^{\delta}\mathscr{E}[L(\varphi_0(\psi,\mathfrak{I}))+\Psi_0(\psi,\mathfrak{I})],\\ \mathscr{X}^{2}&:\tilde{\zeta}_2(\psi,\varpi)=-\varpi^{\delta}\mathscr{E}[L(\varphi_1(\psi,\mathfrak{I}))+\Psi_1(\psi,\mathfrak{I})],\\ &\vdots\\ \mathscr{X}^{n+1}&:\tilde{\zeta}_{n+1}(\psi,\varpi)=-\varpi^{\delta}\mathscr{E}[L(\varphi_n(\psi,\mathfrak{I}))+\Psi_n(\psi,\mathfrak{I})].\end{aligned} \tag{28}$$

Furthermore, we determine the ABC homotopies as follows:

$$\begin{aligned}\mathscr{X}^{0}&:\tilde{\zeta}_0(\psi,\varpi)=\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\widetilde{\theta}(\psi,\varpi)+\varpi^{2}\varphi(\psi,0),\\ \mathscr{X}^{1}&:\tilde{\zeta}_1(\psi,\varpi)=-\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}[L(\varphi_0(\psi,\mathfrak{I}))+\Psi_0(\psi,\mathfrak{I})],\\ \mathscr{X}^{2}&:\tilde{\zeta}_2(\psi,\varpi)=-\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}[L(\varphi_1(\psi,\mathfrak{I}))+\Psi_1(\psi,\mathfrak{I})],\\ &\vdots\\ \mathscr{X}^{n+1}&:\tilde{\zeta}_{n+1}(\psi,\varpi)=-\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}[L(\varphi_n(\psi,\mathfrak{I}))+\Psi_n(\psi,\mathfrak{I})].\end{aligned} \tag{29}$$

When $\mathscr{X}\longrightarrow 1$, we can assume that Equations (28) and (29) represent the approximate solution to Equations (26) and (27); thus, the result is determined by

$$\Delta_n(\psi,\varpi)=\sum_{\sigma=0}^{n}\tilde{\zeta}_{\sigma}(\psi,\varpi). \tag{30}$$

We get the approximate solution of Equation (17), by taking the inverse ET to Equation (30).

$$\varphi(\psi,\varpi)\cong\varphi_n(\psi,\mathfrak{I})=\mathscr{E}^{-1}\left|\{\Delta_n(\psi,\varpi)\}\right.. \tag{31}$$

4. Applications

In this part, we will examine the problems in Equations (1)–(6) by means of Elzaki transform. First, we implement the Elzaki transform technique with the aid of Caputo derivative to solve problem (1) having initial source (2). By taking the Elzaki transform, we get

$$\tilde{\zeta}(\psi, \varpi) = \varpi^{\delta}\mathscr{E}\left[\varphi_{\psi\psi\mathfrak{I}}(\psi, \mathfrak{I}) - \varphi_{\psi}(\psi, \mathfrak{I}) - \varphi(\psi, \mathfrak{I})\varphi_{\psi}(\psi, \mathfrak{I})\right] + \varpi^{2}\varphi(\psi, 0). \quad (32)$$

We use the Elzaki perturbation transform approach to solve Equation (32) and obtain

$$\begin{aligned}\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\tilde{\zeta}_{\mathscr{E}}(\psi, \varpi) = \mathscr{X}\varpi^{\delta}\mathscr{E}&\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi\psi\mathfrak{I}} - \left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi}\right] \\ &- \mathscr{X}\varpi^{\delta}\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\Psi_{\mathscr{E}}(\psi, \mathfrak{I})\right)\right] + \varpi^{2}\varphi(\psi, 0).\end{aligned} \quad (33)$$

We now have by taking the Elzaki inverse transform to Equation (33),

$$\begin{aligned}\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \varpi) = \mathscr{X}\mathscr{E}^{-1}&\left[\varpi^{\delta}\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi\psi\mathfrak{I}} - \left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi}\right]\right] \\ &- \mathscr{X}\mathscr{E}^{-1}\left[\varpi^{\delta}\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\Psi_{\mathscr{E}}(\psi, \mathfrak{I})\right)\right]\right] \\ &+ \mathscr{E}^{-1}\left[\varpi^{2}\varphi(\psi, 0)\right].\end{aligned} \quad (34)$$

The $\Psi_{\mathscr{E}}(.)$ values in Equation (34) are functions that indicate the nonlinear terms assumed in Equation (26) and are analyzed as follows:

$$\begin{aligned}\Psi_0(\varphi) &= \varphi_0(\varphi_0)_{\psi}, \\ \Psi_1(\varphi) &= \varphi_0(\varphi_1)_{\psi} + \varphi_1(\varphi_0)_{\psi}, \\ \Psi_2(\varphi) &= \varphi_0(\varphi_2)_{\psi} + \varphi_1(\varphi_1)_{\psi} + \varphi_2(\varphi_0)_{\psi}, \\ &\vdots\end{aligned} \quad (35)$$

Then, by examining the associated powers of $\mathscr{X}$, we obtain the terms of the Caputo operator solution:

$$\mathscr{X}^{0}: \tilde{\zeta}_0(\psi, \mathfrak{I}) = \mathscr{E}^{-1}\left[\varpi^{2}3\alpha \sec h^{2}(\delta\psi)\right] = 3\alpha \sec h^{2}(\delta\psi),$$

$$\begin{aligned}\mathscr{X}^{1}: \tilde{\zeta}_1(\psi, \mathfrak{I}) &= \mathscr{E}^{-1}\left[\varpi^{\delta}\mathscr{E}[\mathrm{L}(\varphi_0(\psi, \mathfrak{I}))]\right] - \mathscr{E}^{-1}\left[\varpi^{\delta}\mathscr{E}[\Psi_0(\psi, \mathfrak{I})]\right] \\ &= 3\alpha\delta\{1 + 6\alpha\delta + \cos h(2\delta\psi)\} \sec h^{4}(\delta\psi) \tan h(\delta\psi)\frac{\mathfrak{I}^{\delta}}{\Gamma(\delta + 1)},\end{aligned}$$

$$\begin{aligned}\mathscr{X}^{2}: \tilde{\zeta}_2(\psi, \mathfrak{I}) &= \mathscr{E}^{-1}\left[\varpi^{\delta}\mathscr{E}[L(\varphi_1(\psi, \mathfrak{I}))]\right] - \mathscr{E}^{-1}\left[\varpi^{\delta}\mathscr{E}[\Psi_1(\psi, \mathfrak{I})]\right] \\ &= -\frac{3}{32}\alpha\delta^{2}\{-8 - 96\alpha - 576\alpha^{2} \\ &\quad + 3(-3 - 16\alpha + 144\alpha^{2}) \cos h(2\delta\psi) \\ &\quad + 48\alpha \cos h(4\delta\psi) \\ &\quad + \cos h(6\delta\psi)\} \sec h^{8}(\delta\psi)\frac{\mathfrak{I}^{2\delta}}{\Gamma(2\delta + 1)}, \\ &\vdots\end{aligned} \quad (36)$$

As a result, the approximate solution to the problem is

$$\begin{aligned}\varphi(\psi, \mathfrak{I}) = \Bigg(&3\alpha \sec \mathrm{h}^{2}(\delta\psi) + 3\alpha\delta\{1 + 6\alpha\delta \\ &+ \cos h(2\delta\psi)\} \sec \mathrm{h}^{4}(\delta\psi) \tan h(\delta\psi)\frac{\mathfrak{I}^{\delta}}{\Gamma(\delta + 1)} \\ &- \frac{3}{32}\alpha\delta^{2}\{-8 - 96\alpha - 576\alpha^{2} + 3(-3 - 16\alpha + 144\alpha^{2}) \cosh(2\delta\psi) \\ &+ 48\alpha \cosh(4\delta\psi) + \cosh(6\delta\psi)\} \sec \mathrm{h}^{8}(\delta\psi)\frac{\mathfrak{I}^{2\delta}}{\Gamma(2\delta + 1)}, + \cdots\Bigg),\end{aligned} \quad (37)$$

providing the problem's integer-order $(\delta = 1)$ solution, $\varphi(\psi, \mathfrak{I}) = 3\alpha \sec \mathrm{h}^{2}(\delta(\psi - (1 + \alpha)\mathfrak{I}))$.

On the other hand, we use the Elzaki transform in combination with the Atangana-Baleanu operator to solve the problem. First, we use the Elzaki transform to solve the problem:

$$\begin{aligned}\tilde{\zeta}(\psi, \varpi) = \left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right)\mathscr{E}&\left[\varphi_{\psi\psi\mathfrak{I}}(\psi, \mathfrak{I}) - \varphi_{\psi}(\psi, \mathfrak{I}) - \varphi(\psi, \mathfrak{I})\varphi_{\psi}(\psi, \mathfrak{I})\right] \\ &+ \varpi^{2}\varphi(\psi, 0).\end{aligned} \quad (38)$$

To Equation (38), we use the Elzaki perturbation transform approach and get

$$\begin{aligned}\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\tilde{\zeta}_{\mathscr{E}}(\psi, \varpi) = \mathscr{X}\left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right)\mathscr{E} &\cdot \left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi\psi\mathfrak{I}} - \left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi}\right] \\ &- \mathscr{X}\left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right)\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\Psi_{\mathscr{E}}(\psi, \mathfrak{I})\right)\right] + \varpi^{2}\varphi(\psi, 0).\end{aligned} \quad (39)$$

By taking the inverse ET of Equation (39), we get

$$\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \varphi_{\mathscr{E}}(\psi, \mathfrak{J}) = \mathscr{X}\mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{J})\right)_{\psi\psi\mathfrak{J}} - \left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{J})\right)_{\psi}\right]\right] - \mathscr{X}\mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}}\Psi_{\mathscr{E}}(\psi, \mathfrak{J})\right)\right]\right] + \mathscr{E}^{-1}\left[\varpi^{2}\varphi(\psi, 0)\right]. \tag{40}$$

The $\Psi_{\mathscr{E}}(.)$ terms in Equation (40) are nonlinear polynomials that were described in Equation (25). We derive the following results by repeating the methods for nonlinear polynomials:

$$\mathscr{X}^{0} : \varphi_0(\psi, \mathfrak{J}) = \mathscr{E}^{-1}\left[\varpi^{2} 3\alpha \sec \mathrm{h}^{2}(\delta\psi)\right] = 3\alpha \sec \mathrm{h}^{2}(\delta\psi),$$

$$\begin{aligned}\mathscr{X}^{1} : \varphi_1(\psi, \mathfrak{J}) &= \mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}[\varphi_0(\psi, \mathfrak{J})]\right] - \mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}[\Psi_0(\psi, \mathfrak{J})]\right] \\ &= -\frac{3\alpha\delta\{1+6\alpha\delta+\cosh(2\delta\psi)\} \sec \mathrm{h}^{4}(\delta\psi) \tanh(\delta\psi)}{N(\delta)}\left(\frac{\delta\mathfrak{J}^{\delta}}{\Gamma(\delta+1)}+1-\delta\right),\end{aligned} \tag{41}$$

$$\begin{aligned}\mathscr{X}^{2} : \varphi_2(\psi, \mathfrak{J}) &= \mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}[\varphi_1(\psi, \mathfrak{J})]\right] - \mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta}+1-\delta}{N(\delta)}\right)\mathscr{E}[\Psi_1(\psi, \mathfrak{J})]\right] \\ &= -\frac{-3/32\alpha\delta^{2}\left\{-8-96\alpha-576\alpha^{2}+3\left(-3-16\alpha+144\alpha^{2}\right)\cosh(2\delta\psi)+48\alpha\cosh(4\delta\psi)+\cosh(6\delta\psi)\right\} \sec \mathrm{h}^{8}(\delta\psi)}{(N(\delta))^{2}} \\ &\quad \cdot \left(\frac{\left(\delta\mathfrak{J}^{\delta}\right)^{2}}{\Gamma(2\delta+1)}+\frac{2\delta(1-\delta)\mathfrak{J}^{\delta}}{\Gamma(\delta+1)}+(1-\delta)^{2}\right), \\ &\vdots\end{aligned} \tag{42}$$

As a result, based on the ABC operator, the approximate solution is as follows:

$$\begin{aligned}\varphi(\psi, \mathfrak{J}) &= \sum_{\sigma=0}^{n} \varphi_{\sigma}(\psi, \mathfrak{J}) = 3\alpha \sec \mathrm{h}^{2}(\delta\psi) - \frac{3\alpha\delta\{1+6\alpha\delta+\cosh(2\delta\psi)\} \sec \mathrm{h}^{4}(\delta\psi) \tanh(\delta\psi)}{N(\delta)}\left(\frac{\delta\mathfrak{J}^{\delta}}{\Gamma(\delta+1)}+1-\delta\right) \\ &\quad - \frac{-3/32\alpha\delta^{2}\left\{-8-96\alpha-576\alpha^{2}+3\left(-3-16\alpha+144\alpha^{2}\right)\cosh(2\delta\psi)+48\alpha\cosh(4\delta\psi)+\cosh(6\delta\psi)\right\} \sec \mathrm{h}^{8}(\delta\psi)^{2}}{(N(\delta))^{2}} \\ &\quad \cdot \left(\frac{\left(\delta\mathfrak{J}^{\delta}\right)^{2}}{\Gamma(2\delta+1)}+\frac{2\delta(1-\delta)\mathfrak{J}^{\delta}}{\Gamma(\delta+1)}+(1-\delta)^{2}\right)+\cdots,\end{aligned} \tag{43}$$

providing the problem's integer-order ($\delta = 1$) solution, $\varphi(\psi, \mathfrak{I}) = 3\alpha \sec h^2(\delta(\psi - (1+\alpha)\mathfrak{I}))$.

Secondly, we implement the Elzaki transform technique with the aid of Caputo derivative to solve problem (3) having initial source (4). By taking the Elzaki transform, we get

$$\tilde{\zeta}(\psi, \varpi) = \varpi^{\delta} \mathscr{E}\left[2\varphi_{\psi\psi\mathfrak{I}}(\psi, \mathfrak{I}) - \varphi_{\psi}(\psi, \mathfrak{I})\right] + \varpi^2 \varphi(\psi, 0). \quad (44)$$

We use the Elzaki perturbation transform approach to solve Equation (44) and obtain

$$\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \tilde{\zeta}_{\mathscr{E}}(\psi, \varpi) = \mathscr{X} \varpi^{\delta} \mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi\psi\mathfrak{I}} - \left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi}\right] + \varpi^2 \varphi(\psi, 0). \quad (45)$$

We now have by taking the Elzaki inverse transform to Equation (45),

$$\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \tilde{\zeta}_{\mathscr{E}}(\psi, \mathfrak{I}) = \mathscr{X} \mathscr{E}^{-1}\left[\varpi^{\delta} \mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)\right]\right] + \mathscr{E}^{-1}\left[\varpi^2 \varphi(\psi, 0)\right]. \quad (46)$$

Then, by examining the associated powers of $\mathscr{X}$, we obtain the terms of the Caputo operator solution:

$$\begin{aligned}
\mathscr{X}^0 &: \varphi_0(\psi, \mathfrak{I}) = \mathscr{E}^{-1}\left[\varpi^2 e^{-\psi}\right] = e^{-\psi}, \\
\mathscr{X}^1 &: \varphi_1(\psi, \mathfrak{I}) = \mathscr{E}^{-1}\left[\varpi^{\delta} \mathscr{E}[L(\varphi_0(\psi, \mathfrak{I}))]\right], \\
&= e^{-\psi} \frac{\mathfrak{I}^{\delta}}{\Gamma(\delta+1)}, \\
\mathscr{X}^2 &: \varphi_1(\psi, \mathfrak{I}) = \mathscr{E}^{-1}\left[\varpi^{\delta} \mathscr{E}[L(\varphi_1(\psi, \mathfrak{I}))]\right], \\
&= e^{-\psi} \frac{\mathfrak{I}^{2\delta}}{\Gamma(2\delta+1)}, \\
&\vdots
\end{aligned} \quad (47)$$

As a result, the approximate solution to the problem is

$$\varphi(\psi, \mathfrak{I}) = e^{-\psi} + e^{-\psi} \frac{\mathfrak{I}^{\delta}}{\Gamma(\delta+1)} + e^{-\psi} \frac{\mathfrak{I}^{2\delta}}{\Gamma(2\delta+1)} + \cdots, \quad (48)$$

providing the problem's integer-order ($\delta = 1$) solution $\varphi(\psi, \mathfrak{I}) = e^{(\mathfrak{I} - \psi)}$.

On the other hand, we use the Elzaki transform in combination with the Atangana-Baleanu operator to solve the problem. First, we use the Elzaki transform to solve the problem:

$$\tilde{\zeta}(\psi, \varpi) = \left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right)\left(\mathscr{E}\left[\varphi_{\psi\psi\mathfrak{I}}(\psi, \mathfrak{I}) - \varphi_{\psi}(\psi, \mathfrak{I})\right]\right) + \varpi^2 \varphi(\psi, 0). \quad (49)$$

To Equation (49), we use the Elzaki perturbation transform approach and get

$$\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \tilde{\zeta}_{\mathscr{E}}(\psi, \varpi) = \left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right) \cdot \left(\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi\psi\mathfrak{I}} - \left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)_{\psi}\right]\right) + \varpi^2 \varphi(\psi, 0). \quad (50)$$

By taking the inverse ET of the last equation, we get

$$\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \tilde{\zeta}_{\mathscr{E}}(\psi, \mathfrak{I}) = \mathscr{X} \mathscr{E}^{-1} \cdot \left[\left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right) \mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty} \mathscr{X}^{\mathscr{E}} \varphi_{\mathscr{E}}(\psi, \mathfrak{I})\right)\right]\right] + \mathscr{E}^{-1}\left[\varpi^2 \varphi(\psi, 0)\right]. \quad (51)$$

Thus, on comparing both sides

$$\mathscr{X}^0 : \varphi_0(\psi, \mathfrak{I}) = \mathscr{E}^{-1}\left[\varpi^2 e^{-\psi}\right] = e^{-\psi},$$

$$\begin{aligned}
\mathscr{X}^1 : \varphi_1(\psi, \mathfrak{I}) &= \mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right) \mathscr{E}\left[(\varphi_0)_{\psi\psi\mathfrak{I}} - (\varphi_0)_{\psi}\right]\right] \\
&= \frac{e^{-\psi}}{N(\delta)}\left(\frac{\mathfrak{I}^{\delta}}{\Gamma(\delta)} + 1 - \delta\right),
\end{aligned}$$

$$\begin{aligned}
\mathscr{X}^2 : \varphi_2(\psi, \mathfrak{I}) &= \mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right) \mathscr{E}\left[(\varphi_1)_{\psi\psi\mathfrak{I}} - (\varphi_1)_{\psi}\right]\right] \\
&= \frac{e^{-\psi}}{(N(\delta))^2}\left(\frac{\delta^2 \mathfrak{I}^{2\delta}}{\Gamma(2\delta+1)} + \frac{(1-\delta)\delta\mathfrak{I}^{2\delta}}{\Gamma(\delta+1)} + (1-\delta)^2\right)
\end{aligned} \quad (52)$$

As a result, based on the ABC operator, the approximate solution is as follows:

$$\varphi(\psi, \mathfrak{J}) = e^{-\psi} + \frac{e^{-\psi}}{N(\delta)}\left(\frac{\mathfrak{J}^{\delta}}{\Gamma(\delta)} + 1 - \delta\right) + \frac{e^{-\psi}}{(N(\delta))^2}\left(\frac{\delta^2\mathfrak{J}^{2\delta}}{\Gamma(2\delta+1)} + \frac{(1-\delta)\delta\mathfrak{J}^{2\delta}}{\Gamma(\delta+1)} + (1-\delta)^2\right) + \cdots, \tag{53}$$

providing the problem's integer-order $(\delta = 1)$ solution, $\varphi(\psi, \mathfrak{J}) = e^{(\mathfrak{J}-\psi)}$.

Finally, we use the Elzaki transform approach with the aid of Caputo and ABC derivative operators to solve the problem in Equations (5)–(6). To Equations (5)–(6), we first apply the Elzaki transform with the aid of Caputo derivative:

$$\tilde{\zeta}(\psi, \varpi) = \varpi^{\delta}\mathscr{E}\left[\varphi_{\psi\psi\psi\psi}(\psi, \mathfrak{J})\right] + \varpi^2\varphi(\psi, 0). \tag{54}$$

We use the Elzaki perturbation transform approach to solve Equation (54) and obtain

$$\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\tilde{\zeta}_{\mathscr{E}}(\psi, \varpi) = \mathscr{X}\varpi^{\delta}\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \varpi)\right)_{\psi\psi\psi\psi}\right] + \varpi^2\varphi(\psi, 0). \tag{55}$$

We now have by taking the Elzaki inverse transform to Equation (55)

$$\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{J}) = \mathscr{X}\mathscr{E}^{-1}\left[\varpi^{\delta}\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{J})\right)\right]\right] + \mathscr{E}^{-1}\left[\varpi^2\varphi(\psi, 0)\right]. \tag{56}$$

Then, by examining thsse associated powers of $\mathscr{X}$, we obtain the terms of the Caputo operator solution:

$$\begin{aligned}
\mathscr{X}^0 &: \varphi_0(\psi, \mathfrak{J}) = \mathscr{E}^{-1}\left[\varpi^2 \sin\psi\right] = \sin\psi,\\
\mathscr{X}^1 &: \varphi_1(\psi, \mathfrak{J}) = \mathscr{E}^{-1}\left[\varpi^{\delta}\mathscr{E}[L(\varphi_0(\psi, \mathfrak{J}))]\right],\\
&= -\sin\psi\frac{\mathfrak{J}^{\delta}}{\Gamma(\delta+1)},\\
\mathscr{X}^2 &: \varphi_1(\psi, \mathfrak{J}) = \mathscr{E}^{-1}\left[\varpi^{\delta}\mathscr{E}[L(\varphi_1(\psi, \mathfrak{J}))]\right],\\
&= \sin\psi\frac{\mathfrak{J}^{2\delta}}{\Gamma(2\delta+1)},\\
&\vdots
\end{aligned} \tag{57}$$

As a result, the approximate solution to the problem is

$$\varphi(\psi, \mathfrak{J}) = \sin\psi - \sin\psi\frac{\mathfrak{J}^{\delta}}{\Gamma(\delta+1)} + \sin\psi\frac{\mathfrak{J}^{2\delta}}{\Gamma(2\delta+1)} + \cdots, \tag{58}$$

providing the problems integer-order $(\delta = 1)$ solution $\varphi(\psi, \mathfrak{J}) = \sin\psi e^{(-\mathfrak{J})}$.

On the other hand, we use the Elzaki transform in combination with the Atangana-Baleanu operator to solve the problem. First, we use the Elzaki transform to solve the problem:

$$\tilde{\zeta}(\psi, \varpi) = \left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right)\left(\mathscr{E}\left[\varphi_{\psi\psi\psi\psi}(\psi, \mathfrak{J})\right]\right) + \varpi^2\varphi(\psi, 0). \tag{59}$$

To Equation (59), we use the Elzaki perturbation transform approach and get

$$\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\tilde{\zeta}_{\mathscr{E}}(\psi, \varpi) = \left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right) \cdot \left(\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{J})\right)_{\psi\psi\psi\psi}\right]\right) + \varpi^2\varphi(\psi, 0). \tag{60}$$

By taking the inverse ET of the last equation, we get

$$\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{J}) = \mathscr{X}\mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right)\mathscr{E}\left[\left(\sum_{\mathscr{E}=0}^{\infty}\mathscr{X}^{\mathscr{E}}\varphi_{\mathscr{E}}(\psi, \mathfrak{J})\right)\right]\right] + \mathscr{E}^{-1}\left[\varpi^2\varphi(\psi, 0)\right]. \tag{61}$$

Thus, on comparing both sides

$$\mathscr{X}^0 : \varphi_0(\psi, \mathfrak{J}) = \mathscr{E}^{-1}\left[\varpi^2 \sin\psi\right] = \sin\psi,$$

$$\mathscr{X}^1 : \varphi_1(\psi, \mathfrak{J}) = \mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta} + 1 - \delta}{N(\delta)}\right)\mathscr{E}\left[(\varphi_0)_{\psi\psi\psi\psi}\right]\right] = \frac{\sin\psi}{\mathrm{N}(\delta)}\left(\frac{\mathfrak{J}^{\delta}}{\Gamma(\delta)} + 1 - \delta\right),$$

$$\mathscr{X}^2 : \varphi_2(\psi, \mathfrak{J}) = \mathscr{E}^{-1}\left[\left(\frac{\delta\varpi^{\delta} + 1 - \delta}{\mathrm{N}(\delta)}\right)\mathscr{E}\left[(\varphi_1)_{\psi\psi\psi\psi}\right]\right] = \frac{\sin\psi}{(\mathrm{N}(\delta))^2}\left(\frac{\delta^2\mathfrak{J}^{2\delta}}{\Gamma(2\delta+1)} + \frac{(1-\delta)\delta\mathfrak{J}^{2\delta}}{\Gamma(\delta+1)} + (1-\delta)^2\right). \tag{62}$$

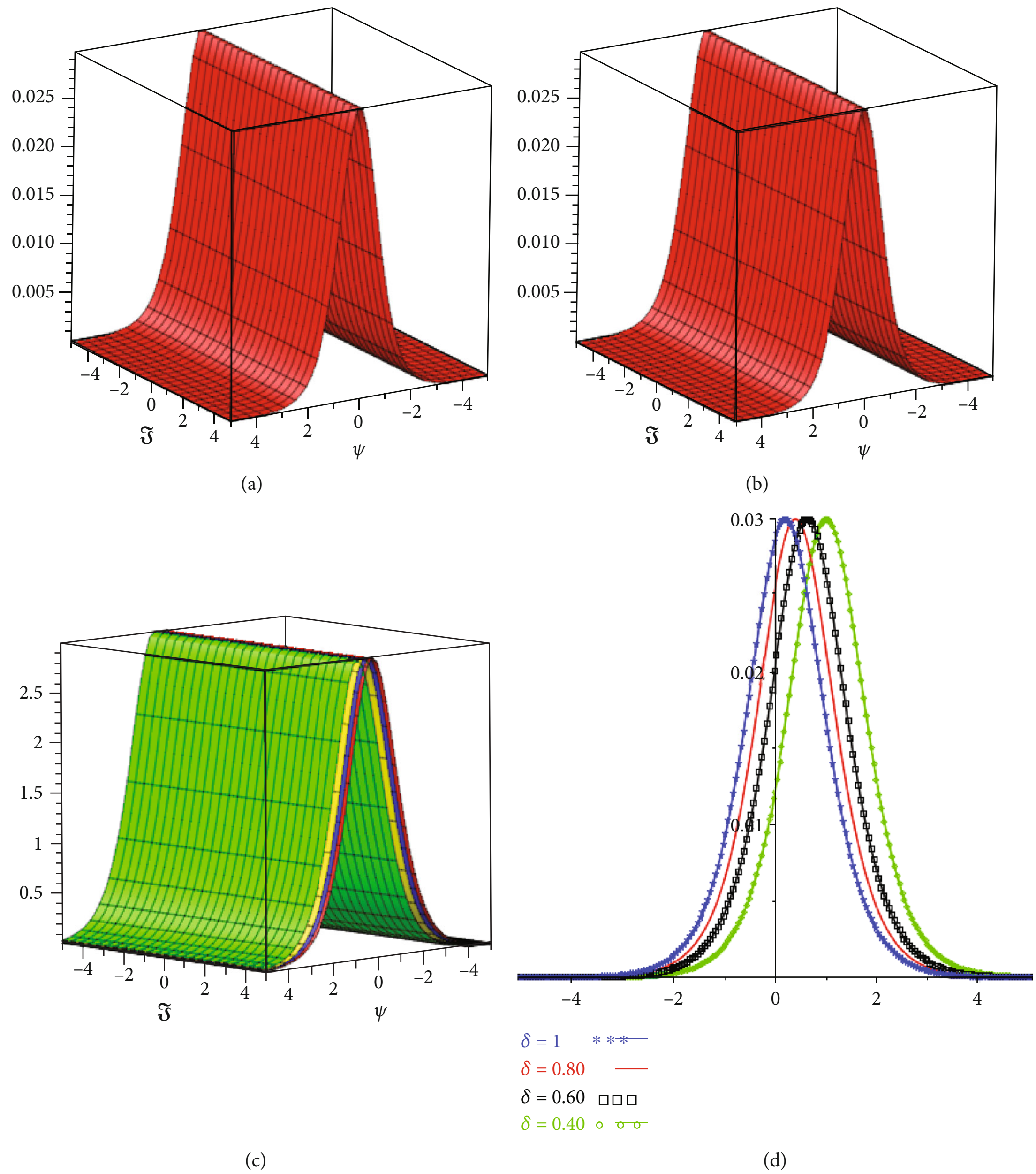

Figure 1: Example 1 solution graph (a) exact solution, (b) analytical solution at $\delta = 1$, (c) analytical solution at various fractional-orders of δ, and (d) $\mathfrak{J} = 0.5$.

As a result, based on the ABC operator, the approximate solution is as follows:

$$\varphi(\psi, \mathfrak{J}) = \sin \psi + \frac{\sin \psi}{\mathrm{N}(\delta)} \left(\frac{\mathfrak{J}^{\delta}}{\Gamma(\delta)} + 1 - \delta \right) + \frac{\sin \psi}{(\mathrm{N}(\delta))^2} \left(\frac{\delta^2 \mathfrak{J}^{2\delta}}{\Gamma(2\delta + 1)} + \frac{(1-\delta)\delta \mathfrak{J}^{2\delta}}{\Gamma(\delta + 1)} + (1-\delta)^2 \right) + \cdots, \tag{63}$$

providing the problems integer-order $(\delta = 1)$ solution, $\varphi(\psi, \mathfrak{J}) = \sin \psi \exp(-\mathfrak{J})$.

5. Results and Discussion

Figures 1(a) and 1(b) demonstrates the comparison between approximate solution and exact solution, while Figures 1(c) and 1(d) shows the 3D and 2D behavior of proposed methods results at different fractional-orders of the problem given by Equation (1). Figure 1 indicates that approximate solution obtained by the suggested techniques is more close to exact solution. We have shown the exact and approximate solutions in Figures 2(a) and 2(b), and the results to the problem given by Equation (3) with respect to various values of fractional parameter in Caputo and Atangana-Baleanu manner can be seen in Figures 2(c) and 2(d). Figures 3(a) and 3(b) represents the comparison between proposed

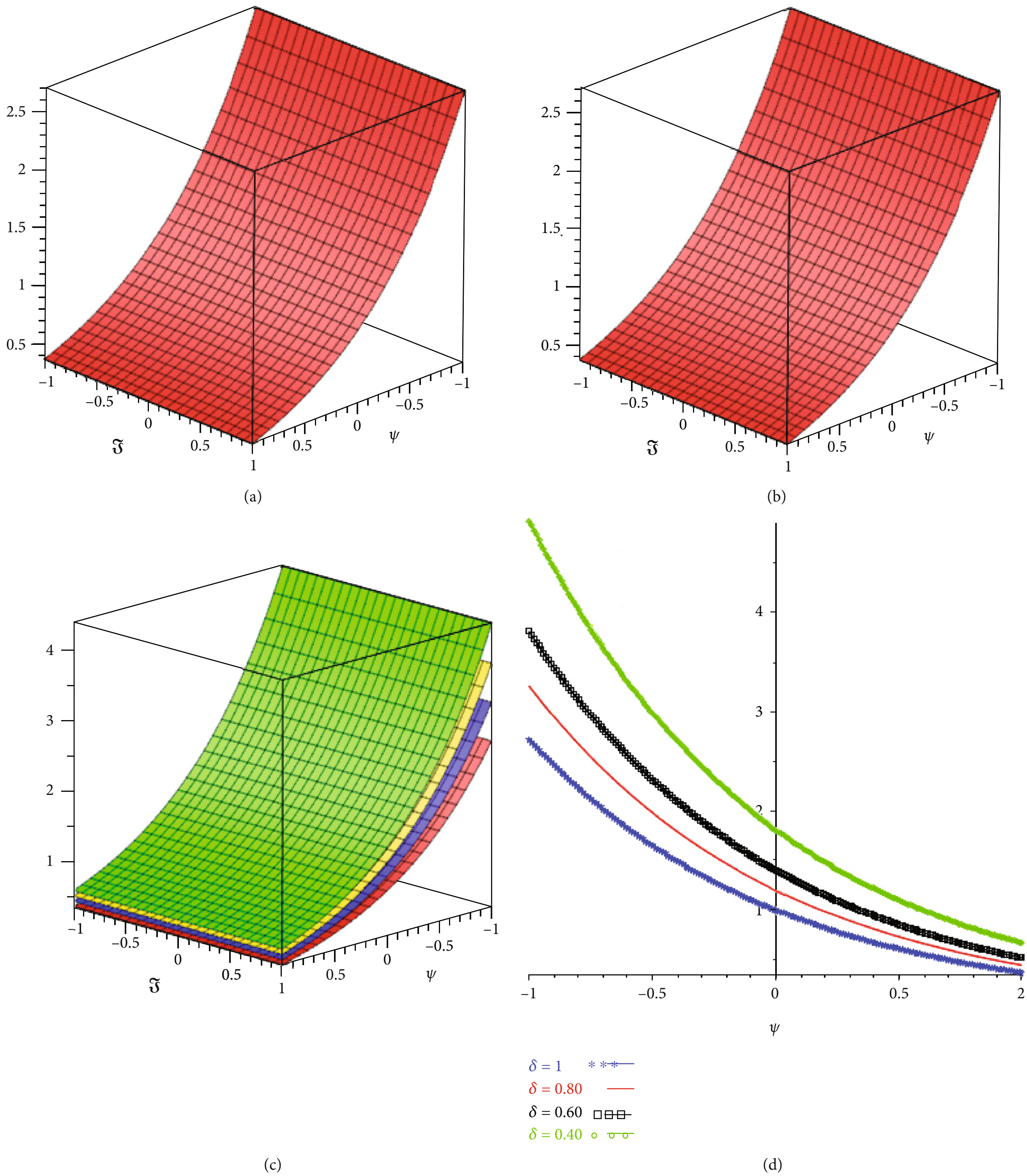

Figure 2: Example 2 solution graph (a) exact solution, (b) analytical solution at $\lambda = 1$, (c) analytical solution at various fractional-orders of δ, and (d) $\mathfrak{I} = 0.5$.

method solution and exact solution, whereas Figures 3(c) and 3(d) shows the behavior of the proposed methods as various fractional-orders of the problem given by Equation (5). On the other hand, in Tables 1–3 we presented the absolute error analysis of RLW equation obtained with the help of proposed method at various values of ψ and $\mathfrak{I}$. It is observed from tables that proposed method solution are in good contact with the exact solution and have high level of precisions between results and shows absolute error between results.

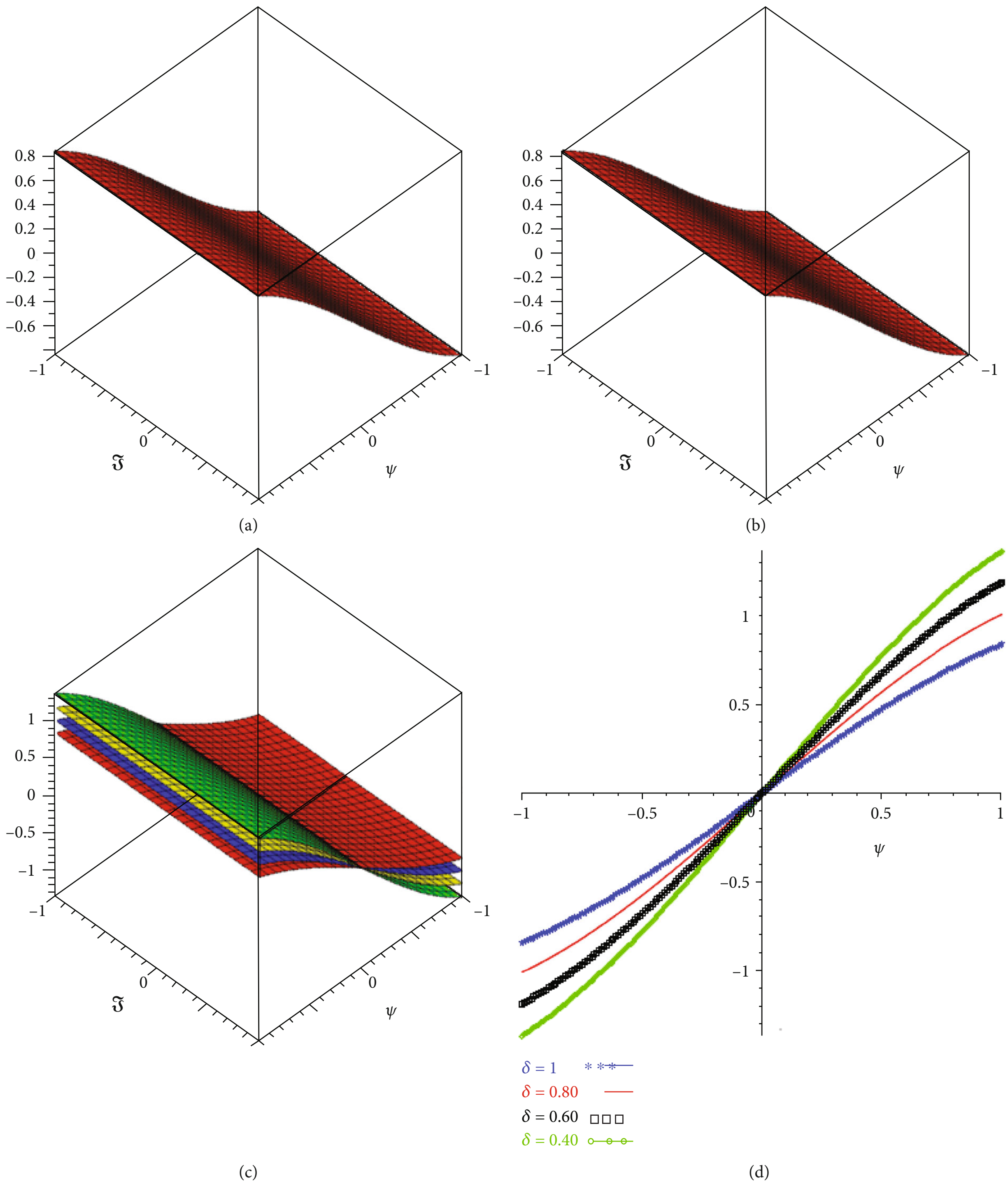

FIGURE 3: Example 3 solution graph (a) exact solution, (b) analytical solution at $\delta = 1$, (c) analytical solution at various fractional-orders of δ, and (d) $\mathfrak{J} = 0.5$.

Table 1: Comparison at different fractional-order of δ on the basis of error for example 1.

$\mathfrak{J}$	ψ	$\delta = 0.4$	$\delta = 0.6$	$\delta = 0.8$	$\delta = 1(ETM_{CFD})$	$\delta = 1(ETM_{ABC})$
	0.2	3.2544637000E-05	2.2120827000E-05	1.1245160000E-06	4.6950000000E-09	4.6950000000E-09
	0.4	5.6252371000E-05	3.8122367000E-05	1.9326610000E-06	4.5160000000E-09	4.5160000000E-09
1	0.6	6.6977543000E-05	4.5190884000E-05	2.2817165000E-06	2.8920000000E-09	2.8920000000E-09
	0.8	6.6088137000E-05	4.4352512000E-05	2.2282931000E-06	9.6700000000E-10	9.6700000000E-10
	1	5.7917905000E-05	3.8643903000E-05	1.9309485000E-06	4.3500000000E-10	4.3500000000E-10
	0.2	3.2724604000E-05	2.2243772000E-05	1.1309350000E-06	1.4503000000E-08	1.4503000000E-08
	0.4	5.6554664000E-05	3.8325285000E-05	1.9428321000E-06	1.1962000000E-08	1.1962000000E-08
2	0.6	6.7332527000E-05	4.5426381000E-05	2.2932956000E-06	6.3710000000E-09	6.3710000000E-09
	0.8	6.6435085000E-05	4.4580264000E-05	2.2393475000E-06	8.5500000000E-10	8.5500000000E-10
	1	5.8219810000E-05	3.8840174000E-05	1.9403804000E-06	2.7360000000E-09	2.7360000000E-09
	0.2	3.2889983000E-05	2.2359041000E-05	1.1372439000E-06	2.9430000000E-08	2.9430000000E-08
	0.4	5.6826239000E-05	3.8509365000E-05	1.9522428000E-06	2.2347000000E-08	2.2347000000E-08
3	0.6	6.7648218000E-05	4.5636779000E-05	2.3036960000E-06	1.0447000000E-08	1.0447000000E-08
	0.8	6.6741663000E-05	4.4781765000E-05	2.2490874000E-06	3.3000000000E-10	3.3000000000E-10
	1	5.8485407000E-05	3.9012637000E-05	1.9485796000E-06	6.9030000000E-09	6.9030000000E-09
	0.2	3.3047770000E-05	2.2471143000E-05	1.1436601000E-06	4.9484000000E-08	4.9484000000E-08
	0.4	5.7079185000E-05	3.8682371000E-05	1.9612665000E-06	3.5686000000E-08	3.5686000000E-08
4	0.6	6.7939008000E-05	4.5831283000E-05	2.3133582000E-06	1.5130000000E-08	1.5130000000E-08
	0.8	6.7022070000E-05	4.4966048000E-05	2.2579430000E-06	2.5830000000E-09	2.5830000000E-09
	1	5.8727131000E-05	3.9169160000E-05	1.9559187000E-06	1.2932000000E-08	1.2932000000E-08
	0.2	3.3201253000E-05	2.2582205000E-05	1.1502870000E-06	7.4676000000E-08	7.4676000000E-08
	0.4	5.7319182000E-05	3.8847969000E-05	1.9700805000E-06	5.1991000000E-08	5.1991000000E-08
5	0.6	6.8211657000E-05	4.6014233000E-05	2.3224915000E-06	2.0433000000E-08	2.0433000000E-08
	0.8	6.7282969000E-05	4.5137364000E-05	2.2661180000E-06	5.8960000000E-09	5.8960000000E-09
	1	5.8950823000E-05	3.9313444000E-05	1.9625736000E-06	2.0821000000E-08	2.0821000000E-08

Table 2: Comparison at different fractional-order of δ on the basis of error for example 2.

$\mathfrak{I}$	ψ	$\delta = 0.4$	$\delta = 0.6$	$\delta = 0.8$	$\delta = 1(ETM_{CFD})$	$\delta = 1(ETM_{ABC})$
	0.2	2.4586042500E-02	1.6390008900E-02	8.1946786000E-03	4.0000000000E-09	4.0000000000E-09
	0.4	2.0129349000E-02	1.3419004200E-02	6.7092353000E-03	3.4000000000E-09	3.4000000000E-09
1	0.6	1.6480517100E-02	1.0986551500E-02	5.4930573000E-03	2.7000000000E-09	2.7000000000E-09
	0.8	1.3493106100E-02	8.9950275000E-03	4.4973349000E-03	2.3000000000E-09	2.3000000000E-09
	1	1.1047220900E-02	7.3645057000E-03	3.6821064000E-03	1.9000000000E-09	1.9000000000E-09
	0.2	2.4605791800E-02	1.6402716400E-02	8.2008084000E-03	1.6300000000E-08	1.6300000000E-08
	0.4	2.0145518400E-02	1.3429408200E-02	6.7142540000E-03	1.3500000000E-08	1.3500000000E-08
2	0.6	1.6493755500E-02	1.0995069600E-02	5.4971663000E-03	1.1000000000E-08	1.1000000000E-08
	0.8	1.3503944800E-02	9.0020016000E-03	4.5006991000E-03	9.0000000000E-09	9.0000000000E-09
	1	1.1056095000E-02	7.3702156000E-03	3.6848608000E-03	7.3000000000E-09	7.3000000000E-09
	0.2	2.4623948800E-02	1.6414429900E-02	8.2064699000E-03	3.6900000000E-08	3.6900000000E-08
	0.4	2.0160384100E-02	1.3438998600E-02	6.7188893000E-03	3.0200000000E-08	3.0200000000E-08
3	0.6	1.6505926500E-02	1.1002921400E-02	5.5009613000E-03	2.4700000000E-08	2.4700000000E-08
	0.8	1.3513909600E-02	9.0084302000E-03	4.5038062000E-03	2.0200000000E-08	2.0200000000E-08
	1	1.1064253400E-02	7.3754788000E-03	3.6874046000E-03	1.6600000000E-08	1.6600000000E-08
	0.2	2.4641082400E-02	1.6425501600E-02	8.2118269000E-03	6.5500000000E-08	6.5500000000E-08
	0.4	2.0174411900E-02	1.3448063300E-02	6.7232751000E-03	5.3700000000E-08	5.3700000000E-08
4	0.6	1.6517411400E-02	1.1010343000E-02	5.5045521000E-03	4.3900000000E-08	4.3900000000E-08
	0.8	1.3523312700E-02	9.0145064000E-03	4.5067461000E-03	3.6000000000E-08	3.6000000000E-08
	1	1.1071952000E-02	7.3804536000E-03	3.6898117000E-03	2.9400000000E-08	2.9400000000E-08
	0.2	2.4657459500E-02	1.6436096000E-02	8.2169554000E-03	1.0230000000E-07	1.0230000000E-07
	0.4	2.0187820300E-02	1.3456737200E-02	6.7274740000E-03	8.3900000000E-08	8.3900000000E-08
5	0.6	1.6528389400E-02	1.1017444700E-02	5.5079899000E-03	6.8600000000E-08	6.8600000000E-08
	0.8	1.3532300600E-02	9.0203207000E-03	4.5095607000E-03	5.6200000000E-08	5.6200000000E-08
	1	1.1079310700E-02	7.3852140000E-03	3.6921161000E-03	4.6000000000E-08	4.6000000000E-08

Table 3: Comparison at different fractional-order of δ on the basis of error for example 3.

$\mathfrak{I}$	ψ	$\delta = 0.4$	$\delta = 0.6$	$\delta = 0.8$	$\delta = 1(\text{ETM}_{\text{CFD}})$	$\delta = 1(\text{ETM}_{\text{ABC}})$
	0.2	5.9659347000E-03	3.9771240000E-03	1.9884840000E-03	1.0000000000E-09	1.0000000000E-09
	0.4	1.1694026400E-02	7.7956926000E-03	3.8976932000E-03	1.9000000000E-09	1.9000000000E-09
1	0.6	1.6955914200E-02	1.1303471500E-02	5.6515139000E-03	2.8000000000E-09	2.8000000000E-09
	0.8	2.1541823200E-02	1.4360616700E-02	7.1800265000E-03	3.6000000000E-09	3.6000000000E-09
	1	2.5268927700E-02	1.6845249400E-02	8.4222940000E-03	4.2000000000E-09	4.2000000000E-09
	0.2	5.9707330000E-03	3.9802135000E-03	1.9899773000E-03	4.0000000000E-09	4.0000000000E-09
	0.4	1.1703431600E-02	7.8017484000E-03	3.9006205000E-03	7.8000000000E-09	7.8000000000E-09
2	0.6	1.6969551500E-02	1.1312252300E-02	5.6557583000E-03	1.1300000000E-08	1.1300000000E-08
	0.8	2.1559148800E-02	1.4371772300E-02	7.1854188000E-03	1.4300000000E-08	1.4300000000E-08
	1	2.5289250900E-02	1.6858335100E-02	8.4286193000E-03	1.6800000000E-08	1.6800000000E-08
	0.2	5.9751488000E-03	3.9830657000E-03	1.9913610000E-03	8.9000000000E-09	8.9000000000E-09
	0.4	1.1712087200E-02	7.8073393000E-03	3.9033328000E-03	1.7500000000E-08	1.7500000000E-08
3	0.6	1.6982101800E-02	1.1320358900E-02	5.6596910000E-03	2.5400000000E-08	2.5400000000E-08
	0.8	2.1575093500E-02	1.4382071500E-02	7.1904152000E-03	3.2300000000E-08	3.2300000000E-08
	1	2.5307954300E-02	1.6870416200E-02	8.4344802000E-03	3.7900000000E-08	3.7900000000E-08
	0.2	5.9793203000E-03	3.9857663000E-03	1.9926749000E-03	1.5900000000E-08	1.5900000000E-08
	0.4	1.1720263900E-02	7.8126327000E-03	3.9059080000E-03	3.1100000000E-08	3.1100000000E-08
4	0.6	1.6993957600E-02	1.1328034100E-02	5.6634250000E-03	4.5200000000E-08	4.5200000000E-08
	0.8	2.1590155800E-02	1.4391822500E-02	7.1951591000E-03	5.7400000000E-08	5.7400000000E-08
	1	2.5325622600E-02	1.6881854300E-02	8.4400448000E-03	6.7300000000E-08	6.7300000000E-08
	0.2	5.9833121000E-03	3.9883550000E-03	1.9939372000E-03	2.4900000000E-08	2.4900000000E-08
	0.4	1.1728088500E-02	7.8177068000E-03	3.9083824000E-03	4.8700000000E-08	4.8700000000E-08
5	0.6	1.7005302900E-02	1.1335391300E-02	5.6670127000E-03	7.0500000000E-08	7.0500000000E-08
	0.8	2.1604569600E-02	1.4401169600E-02	7.1997171000E-03	8.9600000000E-08	8.9600000000E-08
	1	2.5342530300E-02	1.6892818700E-02	8.4453916000E-03	1.0520000000E-07	1.0520000000E-07

6. Conclusion

The approximate solutions of some particular regularized long-wave equations of fractional-order are determined in this paper using a new integral transform method known as the Elzaki transformation. To begin, we consider the Elzaki transform of the fractional Atangana-Baleanu operator and used it to solve the proposed problems. The used scheme's trustworthiness and efficiency are based on its capacity to provide an appropriate convergence zone for the solution. The excellent accuracy of the findings and the simplicity of the solution approach confirm suggested method supremacy over other numerical methods. Also, we have shown how the Caputo and Atangana-Baleanu fractional operators differ when it comes to finding approximate solutions to the illustrative examples. To ensure the validity of the suggested technique, we showed the results in graphs and tables. The representations of graphs and tables demonstrate that the results obtained by suggested scheme are very accurate. In addition, the behavior of fractional-order results is discussed which confirm that the solution gets closer as the fractional-order tends toward integer-order. Finally, the approximation solution strategy employed is highly efficient and applicable to a wide range of nonlinear equations defining real systems.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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