Research Article

Unsteady MHD Flow for Fractional Casson Channel Fluid in a Porous Medium: An Application of the Caputo-Fabrizio Time-Fractional Derivative

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Theoretically, this work describes the exact solutions of fractional Casson fluid through a channel under the effect of MHD and porous medium. The unsteady fluid motion of the bottom plate, which is confined by parallel but perpendicular sidewalls, supports the flow. By introducing the dimensionless parameters and variables, the momentum equation, as well as the initial and boundary conditions, has been transformed to a dimensionless form. A mix of Laplace and Fourier transformations is used to get the exact solution for the momentum equation. The constitutive equations for Caputo-Fabrizio's time-fractional derivative are also incorporated for recovering the exact solutions of the flow problem under consideration. After recovering the exact solutions for flow characteristics, three different cases at the surface of the bottom plate are discussed, by addressing the limiting cases under the influence of the side walls. Moreover, these solutions are captured graphically, and the effects of the Reynolds number Re , fractional parameter α , effective permeability K_{eff} , and dimensionless parameter for Casson fluid β on the fluid's motion are observed.

1. Introduction

Fractional calculus plays a critical part in the solving of complicated engineering issues. Because of its significance, a fractional model solution for flow issues is preferred by many scientists and researchers. A fraction model correctly depicts the motion of a flow issue when compared to ordinary differential equations (ODEs). It recovers an ODE's solution, which explains minute flow system fluctuations. Even for Newtonian fluids, the older scientists' operators including the Caputo operator employed a solitary kernel that resulted in complicated series solutions. In 2015,

Caputo and Fabrizio [1] suggested a new fractional operator that may be employed in simple ways to solve this problem. Following that, a lot of scholars have applied this notion to numerous sorts of flows using varied geometries. Alshabanat et al. [2] proposed a new fractional derivative utilizing a nonsingular form kernel of exponential and trigonometric functions. Singh et al. [3] investigated a fractional epidemiological model for viral determination in the computer utilizing fractional derivatives and numerically solved the modeled issue using the iterative technique. Shah and Khan [4] have discussed heat transmission for an oscillating second-grade fluid upon a vertical surface by employing

the Caputo-Fabrizio derivative. In this work, the authors have determined the exact solutions for flow and thermal characteristics by applying Laplace transform. The authors of this investigation have also carried out a comparative study for the time derivative of fractional and integral order both for Newtonian and second-grade fluids and have highlighted that fractional parameter enhanced the flow characteristics due to augmented velocities of fractional fluid.

Researchers have been studying mathematical models for non-Newtonian fluids because of the growing trend in technological and industrial applications. Because of their relevance at the industrial level, research into these fluids is desirable. Non-Newtonian fluids, for example, lubricant production for a variety of vehicles, spinning of metal, metal extrusion, removing nonmetallic inclusions from molten metal, food, shoe manufacturing (the shoe must be filled with a non-Newtonian fluid to protect the feet from damage), and industries that deal with medicine and coolant, have a wide range of uses. Casson fluid is known as Non-Newtonian fluid because of its rheological features. Casson's model, introduced in 1959, was shown to be extremely flexible and to best reflect the curves of silicon suspension, among other things [5]. Hussain et al. [6] used the shooting method to find the numerical solution of a Casson fluid by changing viscosity flows near a shrinking/extending sheet by slip effects in MHD stagnation point flow. Casson nanofluid flows hydromagnetically through a porous stretched cylinder under Newtonian heat and mass conditions which were discussed by Naqvi et al. [7]. Rao et al. [8] found the exact solution of Casson fluid near a plate which is infinite, exponentially accelerated, and vertical with the effect of MHD and porosity. An overview of numerical approaches for heat and mass transfer in Casson fluids is presented by Verma and Mondal [9]. Sheikh et al. [10] obtained exact solutions of free convection MHD flow of Casson fluid in a channel by using Laplace transform. A fractional model with the Mittag-Leffler memory for generalized Casson MHD fluid by Newtonian heating was discussed by Tassaddiq et al. [11]. Goud et al. [12] calculated the exact solution of natural convection MHD flow of Casson fluid near a perpendicular plate through a porous medium by finite element method. The behavior of a non-Newtonian micropolar-Casson fluid pulsatile flow in a restricted channel influenced by Lorentz force according to Darcy's law is investigated by Ali et al. [13].

The flow in the channels is also more important. Because of its relevance, numerous academics have been drawn to explore channel flows during the previous few decades. Using the Dufour effect, Jha and Ajibade [14] examined heat and mass transfer for free convective fluid flow along a vertical channel. Free convection of transient flow on a flat surface was studied by Ingham [15]. The authors of this study focused on a vertically oriented flat plate. Free convective flow with MHD effect on a flat plate was examined by Raptis and Singh [16]. The authors employed an accelerated vertical plate in this investigation. Fluid flow near an exponential plate has been explored by Singh and Kumar [17]. MHD fluid flow across a flat plate was studied by Khan et al. [18]. The authors of this study looked at the effect of sidewalls on fluid flow. Haq et al. [19] explored the flow of

MHD fluid on a porous sheet. Fetecau [20] has solved the fluid flow via a pipe analytically. The author used the Steklov expansion theorem to find the exact answer in this study. Furthermore, the same flow has been explored under the impact of side walls in this work by addressing the limiting instances. Using the Fourier transform, Fetecău and Zierep [21] examined a set of accurate solutions for the second-grade fluid modeled issue. An abrupt jerk was applied to the fluid in this experiment to get fluid motion.

This work investigates a time-dependent fractional Casson fluid through a channel with porosity and MHD effect; the flow is caused by the bottom plate's unstable motion, which is confined by sidewalls that are parallel to one another but normal to the bottom plate. For the aim of creating a dimensionless form of governing equations, a group of nondimensional variables has been used in the momentum equation and applied boundary conditions. The exact solutions for the momentum equation were then obtained using integral transforms [22, 23] such as Laplace, finite Fourier, and Fourier transforms, as well as the Caputo-Fabrizio fractional derivative. These ideas were then addressed for various bottom plate instances. We utilized Mathcad 15 to explain the graphical results for the modeled issue after recovering the exact answers for various scenarios. The impact of different parameters involved in the solution of the flow problem has been discussed upon flow characteristics.

2. Physical Description of Problem

Assume that a fractional unsteady Casson fluid flows on an infinite plate. The plate is limited by two sidewalls that are parallel to each other but normal to the bottom plate and are separated by a distance of d . The flow is caused by the bottom plate because when $t = 0$, both the plate and the fluid are at rest, and the bottom plate begins to move at an unsteady velocity $U_0 f(t)$, which satisfies the condition $f(t) = 0$ at $t = 0$ and piecewise continuous and exponential order at infinity. Because the motion is unidirectional, the equation describes the velocity of the flow system $V = u(y, z, t)i$ such that $v = w = 0$.

Assumed flow system's governing equations [24] are as follows:

$$\rho \frac{\partial u}{\partial t} = \left(1 + \frac{1}{\xi}\right) \left(\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) - \left(\sigma B_0^2 + \frac{\mu \varphi}{k}\right) u, \quad y > 0, \quad 0 \leq z \leq d, \quad (1)$$

where

$$\begin{aligned} \tau_{xy} &= \mu \frac{\partial u}{\partial y}, \\ \tau_{xz} &= \mu \frac{\partial u}{\partial z}. \end{aligned} \quad (2)$$

The initial and boundary conditions for the problem are as follows:

$$\begin{aligned}
\tau_{xy}(y, z, 0) &= \tau_{xz}(y, z, 0) = 0, \\
u(y, z, 0) &= 0, \\
u(0, z, t) &= U_0 f(t), \\
u(y, 0, t) &= u(y, d, t) = 0,
\end{aligned} \tag{3}$$

although

$$u(y, z, t) \longrightarrow 0, \quad \frac{\partial u(y, z, t)}{\partial y} \longrightarrow 0, \quad \text{for } y \longrightarrow \infty, \tag{4}$$

where μ represents the dynamic viscosity and ρ is the density. Also, $f(t) = 0$ at $t = 0$ and $f(t) \neq 0$ for $t > 0$.

Consider the collection of dimensionless variables to transform the governing equation and its initial boundary conditions for the postulated flow problem into nondimensional form.

$$\begin{aligned}
y^* &= \frac{y}{d_0}, \\
z^* &= \frac{z}{d_0}, \\
(\tau_{xy}^*, \tau_{xz}^*) &= \frac{(\tau_{xy}, \tau_{xz})}{\rho U_0^2}, \\
u^* &= \frac{u}{U_0}, \\
t^* &= \frac{t U_0}{d_0}, \\
L^* &= \frac{d}{d_0}.
\end{aligned} \tag{5}$$

By plugging Equation (5) into Equations (1) and (2), we get the following:

$$\frac{\partial u}{\partial t} = \frac{1}{\beta} \left(\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right) - K_{\text{eff}} u, \quad y > 0, \quad 0 \leq z \leq L, \tag{6}$$

$$\begin{aligned}
\tau_{xy} &= \frac{1}{\text{Re}} \frac{\partial u}{\partial y}, \\
\tau_{xz} &= \frac{1}{\text{Re}} \frac{\partial u}{\partial z}.
\end{aligned} \tag{7}$$

Here, $K_{\text{eff}} = M + (1/K)$ is the effective permeability; moreover, $1/K = \nu \phi d_0 / k U_0$ is the porous medium inverse permeability, and $M = \sigma B_0^2 d_0 / \rho U_0$ is the magnetic parameter.

In a dimensionless form, the initial and boundary conditions are as follows:

$$\begin{aligned}
u(y, z, t) &= 0, \quad t = 0, \\
u(y, z, t) &= f\left(\frac{d_0 t}{U_0}\right) = f(t), \quad y = 0, \\
u(y, 0, t) &= u(y, L, t) = 0, \\
u(y, z, t) &\longrightarrow 0, \\
\frac{\partial u(y, z, t)}{\partial y} &\longrightarrow 0, \quad y \longrightarrow \infty.
\end{aligned} \tag{8}$$

The length characteristics d_0 , Reynolds number $\text{Re} = U_0 d_0 / \nu$, and dimensionless Casson fluid parameters are depicted above.

In a generalized form, the fractional constitutive equations are written as follows:

$$\begin{aligned}
\tau_{xy} &= \frac{1}{\text{Re}} {}^{CF}D_t^\alpha \frac{\partial u}{\partial y}, \\
\tau_{xz} &= \frac{1}{\text{Re}} {}^{CF}D_t^\alpha \frac{\partial u}{\partial z}.
\end{aligned} \tag{9}$$

3. Problem Solution

Using the Laplace transform to solve Equations (6) and (7) and incorporating Equation (9), we have the following:

$$\begin{aligned}
s\bar{u} &= \frac{1}{\beta} \left(\frac{\partial \bar{\tau}_{xy}}{\partial y} + \frac{\partial \bar{\tau}_{xz}}{\partial z} \right) - K_{\text{eff}} \bar{u}, \\
\bar{\tau}_{xy} &= \frac{1}{\text{Re}} \frac{s}{(1-\alpha)s + \alpha} \frac{\partial \bar{u}}{\partial y}, \\
\bar{\tau}_{xz} &= \frac{1}{\text{Re}} \frac{s}{(1-\alpha)s + \alpha} \frac{\partial \bar{u}}{\partial z}.
\end{aligned} \tag{10}$$

By simplifying the above equations, we have the following:

$$\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\text{Re} \beta (s + K_{\text{eff}}) ((1-\alpha)s + \alpha)}{s} \bar{u}. \tag{11}$$

After using the Laplace transformation, the initial and boundary conditions were reduced to

$$\begin{aligned}
\bar{u}(y, z, s) &= 0, \quad s = 0, \\
\bar{u}(y, z, s) &= \bar{F}(s), \quad y = 0, \\
\bar{u}(y, z, s) &= \bar{u}(y, z, s) = 0, \quad z = 0, \quad z = L, \\
\bar{u}(y, z, s) &\longrightarrow 0, \\
\frac{\partial \bar{u}(y, z, s)}{\partial y} &\longrightarrow 0, \quad y \longrightarrow \infty.
\end{aligned} \tag{12}$$

After that, multiply Equation (11) by $\sqrt{2/\pi} \int_0^\infty \int_0^L \sin(\psi_n z) \sin(y\chi) dz dy$ and using Equation (12), in which $\zeta_n = n\pi/L$ and the resulting equation was rewritten as follows:

$$\bar{u}_{sn}(\chi, s) = \sqrt{\frac{2}{\pi}} \left(\frac{1 - (-1)^n}{\zeta_n} \right) \left[\frac{\chi s F(s)}{\operatorname{Re} \beta((1-\alpha)s + \alpha)(s + K_{\text{eff}}) + s(\chi^2 + \zeta_n^2)} \right]. \quad (13)$$

Rewrite Equation (13) in the following form:

$$\begin{aligned} \bar{u}_{sn}(\chi, s) = & \sqrt{\frac{2}{\pi}} \left(\frac{1 - (-1)^n}{\zeta_n} \right) \left[\frac{\chi F(s)}{\chi^2 + \zeta_n^2} \right] \\ & - \sqrt{\frac{2}{\pi}} \left(\frac{1 - (-1)^n}{\zeta_n} \right) \frac{\chi F(s)}{\chi^2 + \zeta_n^2} \\ & \cdot \left[\frac{((1-\alpha)s + \alpha)(s + K_{\text{eff}})}{((1-\alpha)s + \alpha)(s + K_{\text{eff}}) + (s(\chi^2 + \zeta_n^2)/\operatorname{Re} \beta)} \right]. \end{aligned} \quad (14)$$

Making use of Laplace inverse transform along with convolution theorem, we have the following equation:

$$\begin{aligned} \bar{u}_{sn}(\chi, t) = & \sqrt{\frac{2}{\pi}} \left(\frac{1 - (-1)^n}{\zeta_n} \right) \left[\frac{\chi f(t)}{\chi^2 + \zeta_n^2} \right] \\ & + \sqrt{\frac{2}{\pi}} \left(\frac{1 - (-1)^n}{\zeta_n} \right) (f''(t) + F_1 f'(t) + F_2 f(t)) \\ & \cdot \frac{\chi}{2A_{1n}(\chi)(\chi^2 + \zeta_n^2)} * (e^{-D_{1n}(\chi)} - e^{-D_{2n}(\chi)}). \end{aligned} \quad (15)$$

In Equation (15), we have the following:

$$\begin{aligned} F_1 &= \frac{(1-\alpha)K_{\text{eff}} + \alpha}{(1-\alpha)}, \\ F_2 &= \frac{\alpha K_{\text{eff}}}{(1-\alpha)}, \\ D_{1n}(\chi) &= \frac{C_{1n}(\chi)}{2} + A_{1n}(\chi), \\ D_{2n}(\chi) &= \frac{C_{1n}(\chi)}{2} - A_{1n}(\chi), \\ A_{1n}(\chi) &= \sqrt{\left(\frac{C_{1n}^2(\chi)}{4} - E_2 \right)}, \\ C_{1n}(\chi) &= \frac{\operatorname{Re} \beta(1-\alpha)K_{\text{eff}} + \operatorname{Re} \beta\alpha + \chi^2 + \zeta_n^2}{\operatorname{Re} \beta(1-\alpha)}. \end{aligned} \quad (16)$$

After using Fourier inversion methods to solve Equation (15) and simplifying the resulting equation,

$$\begin{aligned} u(y, z, t) = & \frac{4f(t)}{L} \sum_{n=1}^{\infty} \frac{e^{-\zeta_n y} \sin(\zeta_n z)}{\zeta_n} \\ & + \frac{4}{\pi L} (f''(t) + F_1 f'(t) + F_2 f(t)) \sum_{n=1}^{\infty} \frac{\sin(\zeta_n z)}{\zeta_n} \\ & * \int_0^\infty \frac{\chi (e^{-D_{1n}(\chi)} - e^{-D_{2n}(\chi)}) \sin(y\chi)}{A_{1n}(\chi)(\chi^2 + \zeta_n^2)} d\chi. \end{aligned} \quad (17)$$

In Equation (17), $m = 2n - 1$, while

$$\begin{aligned} A_{1m}(\chi) &= \sqrt{\left(\frac{C_{1m}^2(\chi)}{4} - F_2 \right)}, \\ C_{1m}(\chi) &= \frac{\operatorname{Re} \beta(1-\alpha)K_{\text{eff}} + \operatorname{Re} \beta\alpha + \chi^2 + \zeta_m^2}{\operatorname{Re} \beta(1-\alpha)}, \\ D_{1m}(\chi) &= \frac{C_{1m}(\chi)}{2} + A_{1m}(\chi), \\ D_{2m}(\chi) &= \frac{C_{1m}(\chi)}{2} - A_{1m}(\chi), \\ A_{1m}(\chi) &= \sqrt{\left(\frac{C_{1m}^2(\chi)}{4} - F_2 \right)}. \end{aligned} \quad (18)$$

Take $L = 2h$ by translating system of the coordinate axis by putting $z = z^* + h$ and $\xi_m = (2n - 1/2h)\pi$ with $\xi_m = \zeta_m$ in Equation (17); after that, we have the following:

$$\begin{aligned} u(y, z, t) = & \frac{2f(t)}{h} \sum_{n=1}^{\infty} \frac{e^{-\xi_n y} (-1)^{n+1} \cos(\xi_n z)}{\xi_n} \\ & + \frac{2}{\pi h} (f''(t) + F_1 f'(t) + F_2 f(t)) \\ & \times \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\xi_n z)}{\xi_n} \\ & * \int_0^\infty \frac{\chi (e^{-D_{1m}(\chi)} - e^{-D_{2m}(\chi)}) \sin(y\chi)}{A_{1m}(\chi)(\chi^2 + \xi_m^2)} d\chi, \end{aligned} \quad (19)$$

where

$$\begin{aligned} A_{1m}(\chi) &= \sqrt{\left(\frac{C_{1m}^2(\chi)}{4} - F_2 \right)}, \\ C_{1m}(\chi) &= \frac{\operatorname{Re} \beta(1-\alpha)K_{\text{eff}} + \operatorname{Re} \beta\alpha + \chi^2 + \xi_m^2}{\operatorname{Re} \beta(1-\alpha)}, \\ D_{1m}(\chi) &= \frac{C_{1m}(\chi)}{2} + A_{1m}(\chi), \\ D_{2m}(\chi) &= \frac{C_{1m}(\chi)}{2} - A_{1m}(\chi). \end{aligned} \quad (20)$$

4. Special Cases

4.1. $h \rightarrow \infty$ Means That Distance among the Sidewalls Will Be Maximized. When the distance between the sidewalls of a flow system is increased, the flow is no longer impacted by these sidewalls and Equation (19) is reduced to the following:

$$u(y, t) = f(t) + \frac{1}{\pi} \left(f''(t) + F_1 f'(t) + F_2 f(t) \right) * \int_0^\infty \frac{(e^{-L_1(\chi)t} - e^{-L_2(\chi)t}) \sin(y\chi)}{\chi c_1(\chi)} d\chi. \quad (21)$$

In Equation (21),

$$A_2(\chi) = \sqrt{\left(\frac{C_2^2(\chi)}{4} - F_2 \right)},$$

$$C_2(\chi) = \frac{\operatorname{Re} \beta(1 - \alpha) K_{\text{eff}} + \operatorname{Re} \beta \alpha + \chi^2}{\operatorname{Re} \beta(1 - \alpha)},$$

$$L_1(\chi) = \frac{C_2(\chi)}{2} + A_2(\chi),$$

$$L_2(\chi) = \frac{C_2(\chi)}{2} - A_2(\chi). \quad (22)$$

4.2. Stokes' First Problem When $f(t) = H(t)$. By choosing $f(t) = H(t)$ in Equation (19) (where $H(t)$ is Heaviside's unit step function), we will get the following:

$$u(y, z, t) = \frac{2}{h} H(t) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{-\xi_m y} \cos(\xi_m z)}{\xi_m} + \frac{2}{h\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\xi_m z)}{\xi_m} \times \int_0^\infty \frac{\chi [D_{2m}(\chi)(D_{1m}(\chi)F_1 - D_{1m}^2(\chi) - F_2)e^{-D_{1m}(\chi)t} - D_{1m}(\chi)(D_{2m}(\chi)F_1 - D_{2m}^2(\chi) - F_2)e^{-D_{2m}(\chi)t}] \sin(y\chi)}{D_{1m}(\chi)D_{2m}(\chi)A_{1m}(\chi)(\chi^2 + \xi_m^2)} d\chi. \quad (23)$$

If the side walls are ignored, Equation (23) becomes the following:

$$u(y, t) = H(t) + \frac{1}{\pi} \int_0^\infty \frac{\left(L_2(\chi)(L_1(\chi)F_1 - L_1^2(\chi) - F_2)e^{-L_1(\chi)t} - L_1(\chi)(L_2(\chi)F_1 - L_2^2(\chi) - F_2)e^{-L_2(\chi)t} \right)}{\chi L_1(\chi)L_2(\chi)c_1(\chi)} \cdot \sin(y\chi) d\chi. \quad (24)$$

4.3. If $f(t) = t^a$. By putting this case in Equation (19), after simplification, we will obtain the following:

$$u(y, z, t) = \frac{2}{h} t^a \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{-\xi_m y} \cos(\xi_m z)}{\xi_m}$$

$$+ \frac{2}{h\pi} \left(\frac{d^2(t^a)}{dt^2} + F_1 \frac{d(t^a)}{dt} + F_2 t^a \right) \times \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\xi_m z)}{\xi_m} * \int_0^\infty \frac{\chi (e^{-D_{1m}(\chi)t} - e^{-D_{2m}(\chi)t}) \sin(y\chi)}{c_{1m}(\chi)(\chi^2 + \xi_m^2)} d\chi. \quad (25)$$

The solution for an accelerating fractional Casson fluid is given by Equation (25). If the bottom plate is constantly accelerated, then $f(t) = t$; we get from Equation (19) that we obtain after simplification.

$$u(y, z, t) = \frac{2}{h} t \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{-\xi_m y} \cos(\xi_m z)}{\xi_m} + \frac{2}{h\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\xi_m z)}{\xi_m} \times \int_0^\infty \frac{\chi \sin(y\chi)}{c_{1m}(\chi)(\chi^2 + \xi_m^2)} \left[\frac{F_1}{D_{1m}(\chi)D_{2m}(\chi)} (D_{1m}(\chi)e^{-D_{2m}(\chi)t} - D_{2m}(\chi)e^{-D_{1m}(\chi)t}) + \frac{F_2}{D_{1m}^2(\chi)D_{2m}^2(\chi)} \right. \\ \left. \cdot \left(-2c_{1m}(\chi)(D_{1m}(\chi)D_{2m}(\chi)t - A_{3m}(\chi)) + D_{2m}^2(\chi)e^{-D_{1m}(\chi)t} - D_{1m}^2(\chi)e^{-D_{2m}(\chi)t} \right) \right] d\chi. \quad (26)$$

FIGURE 1: Flow characteristics for the impulsive type motion of fractional fluid when $z \in [0, 1]$.

Again, by neglecting the effects of sidewalls that are $h \rightarrow \infty$ on fractional fluid flow, the velocity field takes the following form.

$$u(y, t) = t + \frac{1}{\pi} \int_0^\infty \frac{\sin(y\chi)}{\chi c_1(\chi)} \left[\frac{F_1}{L_1(\chi)L_2(\chi)} (L_1(\chi)e^{-L_2(\chi)t} - L_2(\chi)e^{-L_1(\chi)t}) + \frac{F_2}{L_1^2(\chi)L_2^2(\chi)} \right. \\ \left. \left(-2c_1(\chi)(L_1(\chi)L_2(\chi)t - A_3(\chi)) + L_2^2(\chi)e^{-L_1(\chi)t} - L_1^2(\chi)e^{-L_2(\chi)t} \right) \right] d\xi. \quad (27)$$

4.4. $f(t) = H(t)e^{i\omega t}$. After the oscillation of the bottom plate, the fractional Casson fluid has oscillated with veloc-

ities $u_s(y, z, t)$ and $u_c(y, z, t)$, respectively, corresponding to sine and cosine oscillations. This case gives the following:

$$u_{cs}(y, z, t) = \frac{2}{h} \cos(\omega t) \sum_{n=1}^\infty \frac{(-1)^{n+1} e^{-\xi_m y} \cos(\xi_m z)}{\xi_m} \\ + \frac{2}{h\pi} \sum_{n=1}^\infty \frac{(-1)^{n+1} \cos(\xi_m z)}{\xi_m} \int_0^\infty \frac{\chi \sin(y\chi)}{A_{1m}(\chi)(\chi^2 + \xi_m^2)} \\ \times \left(\{ (D_{1m}(\chi)D_{2m}(\chi) - \omega^2)(\omega^2 - F_2) - F_1\omega^2 C_{1m}(\chi) \} \cos(\omega t) \right. \\ \left. + \{ C_{1m}(\chi)(\omega^2 - F_2) + F_1 D_{1m}(\chi)D_{2m}(\chi) - F_1\omega^2 \} \sin(\omega t) \right) \\ \times \frac{-2A_{1m}(\chi)d\chi}{(D_{1m}^2(\chi) + \omega^2)(D_{2m}^2(\chi) + \omega^2)},$$

FIGURE 2: Flow characteristics for constant acceleration of fractional fluid when $z \in [0, 1]$.

$$\begin{aligned}
 u_{ct}(y, z, t) &= \frac{2}{h\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\xi_m z)}{\xi_m} \int_0^{\infty} \frac{\chi \sin(y\chi)}{A_{1m}(\chi)(\chi^2 + \xi_m^2)} \\
 &\quad \times \left[\frac{D_{1m}(\chi)(F_2 - \omega^2) + F_1 \omega^2}{D_{1m}^2(\chi) + \omega^2} e^{-D_{1m}(\chi)t} \right. \\
 &\quad \left. + \frac{D_{2m}(\chi)(F_2 - \omega^2) - F_1 \omega^2}{D_{2m}^2(\chi) + \omega^2} e^{-D_{2m}(\chi)t} \right] d\chi, \\
 u_{ss}(y, z, t) &= \frac{2}{h} \sin(\omega t) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{-\xi_m y} \cos(\xi_m z)}{\xi_m} \\
 &\quad + \frac{2}{h\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\xi_m z)}{\xi_m} \int_0^{\infty} \frac{\chi \sin(y\chi)}{A_{1m}(\chi)(\chi^2 + \xi_m^2)} \\
 &\quad \times \left(\left\{ (D_{1m}(\chi)D_{2m}(\chi) - \omega^2)(\omega^2 - F_2) - F_1 \omega^2 C_{1m}(\chi) \right\} \sin(\omega t) \right. \\
 &\quad \left. + \left\{ F_1 \omega(\omega^2 - D_{1m}(\chi)D_{2m}(\chi)) - \omega C_{1m}(\chi)(\omega^2 + F_2) \right\} \cos(\omega t) \right) \\
 &\quad \times \frac{-2A_{1m}(\chi)d\chi}{(D_{1m}^2(\chi) + \omega^2)(D_{2m}^2(\chi) + \omega^2)}, \\
 u_{st}(y, z, t) &= \frac{2}{h\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cos(\xi_m z)}{\xi_m} \int_0^{\infty} \frac{\chi \sin(y\chi)}{A_{1m}(\chi)(\chi^2 + \xi_m^2)} \\
 &\quad \times \left[\frac{\omega(\omega^2 + F_1 D_{1m}(\chi) - F_2)}{D_{1m}^2(\chi) + \omega^2} e^{-D_{1m}(\chi)t} \right. \\
 &\quad \left. - \frac{\omega(\omega^2 + F_1 D_{2m}(\chi) - F_2)}{D_{2m}^2(\chi) + \omega^2} e^{-D_{2m}(\chi)t} \right] d\chi.
 \end{aligned} \tag{28}$$

FIGURE 3: Flow characteristics for cosine oscillations of fractional fluid when $z \in [0, 1]$.

Now, if we neglect the effects of sidewalls that are $h \rightarrow \infty$ on fractional fluid flow as a limiting case, then the velocity fields for such fluid flow caused by an oscillating bottom plate are as follows.

$$u_{cs}(y, z, t) = \cos(\omega t) + \frac{1}{\pi} \int_0^\infty \frac{\sin(y\chi)}{\chi A_2(\chi)} \cdot \left(\{L_1(\chi)L_2(\chi) - \omega^2\}(\omega^2 - F_2) - F_1\omega^2 C_2(\chi)\} \cos(\omega t) \right. \\ \left. + \{C_2(\chi)(\omega^2 - F_2) + F_1L_1(\chi)L_2(\chi) - F_1\omega^2\} \sin(\omega t) \right) \\ \times \frac{-2A_2(\chi)d\chi}{(L_1^2(\chi) + \omega^2)(L_2^2(\chi) + \omega^2)},$$

$$u_{ci}(y, z, t) = \frac{1}{\pi} \int_0^\infty \frac{\sin(y\chi)}{\chi A_2(\chi)} \left[\frac{L_1(\chi)(F_2 - \omega^2) + F_1\omega^2}{L_1^2(\chi) + \omega^2} e^{-L_1(\chi)t} \right. \\ \left. + \frac{L_2(\chi)(F_2 - \omega^2) - F_1\omega^2}{L_2^2(\chi) + \omega^2} e^{-L_2(\chi)t} \right] d\chi,$$

$$u_{ss}(y, z, t) = \sin(\omega t) + \frac{1}{\pi} \int_0^\infty \frac{\sin(y\chi)}{\chi A_2(\chi)} \cdot \left(\{L_1(\chi)L_2(\chi) - \omega^2\}(\omega^2 - F_2) - E_1\omega^2 C_2(\chi)\} \sin(\omega t) \right) \\ \cdot \left(\{F_1\omega(\omega^2 - L_1(\chi)L_2(\chi)) - \omega C_2(\chi)(\omega^2 + F_2)\} \cos(\omega t) \right) \\ \times \frac{-2A_2(\chi)d\chi}{(L_1^2(\chi) + \omega^2)(L_2^2(\chi) + \omega^2)},$$

$$u_{st}(y, z, t) = \frac{1}{\pi} \int_0^\infty \frac{\sin(y\chi)}{\chi A_2(\chi)} \left[\frac{\omega(\omega^2 + F_1L_1(\chi) - F_2)}{L_1^2(\chi) + \omega^2} e^{-L_1(\chi)t} \right. \\ \left. - \frac{\omega(\omega^2 + F_1L_2(\chi) - F_2)}{L_2^2(\chi) + \omega^2} e^{-L_2(\chi)t} \right] d\chi. \quad (29)$$

5. Numerical Results and Discussions

With the influence of MHD and porosity, three distinct forms of motion have been examined for fractional Casson fluid in a

FIGURE 4: Flow characteristics for sine oscillations of fractional fluid when $z \in [0, 1]$.

channel passing from a horizontal, unsteadily, and infinite moving plate that is confined by two parallel side walls separated by a distance d . For the objective of obtaining an exact solution, joint Laplace and Fourier transforms were used. For the problem under examination, many situations at the bottom plate have been studied. The same flow has been studied under the impact of side walls by addressing the limiting instances. After determining general solutions for any arbitrary flow, solution for impulsive, solution for constantly accelerated, and solution for the sinusoidal fluid motion of the bottom plate were retrieved.

Flow characteristics for Stokes' first issue are depicted in Figures 1(a)–1(d). It can be seen from these graphs that velocity decreases as the fractional parameter (α) increases because when the fractional parameter is increased, the fluid velocity decreases. An opposite impact is also observed for

the dimensionless parameter for Casson fluid (β) upon flow characteristics. It is also worth noting that velocity is a decreasing function of effective permeability (K_{eff}), in these graphs. On the other hand, Reynolds number (Re) has the opposite effect on flow properties. The velocity of the fluid rises as the Reynolds number increases.

Figures 2(a)–2(d) show velocity profiles for fractional fluid flow with a ramping type. Figure 2(a) indicates that when $K_{\text{eff}} = 0.7$, $\beta = 1$, and $\text{Re} = 2$ then with decreasing values of fractional parameter (α), the characteristics of the flow are also improved. The velocity profile rises with increasing Casson fluid values, as seen in Figure 2(b). In Figure 2(c), the velocity decreases as the effective permeability increases. Similarly, when the Reynolds number (Re) increases, the flow properties improve. As a result, the velocity profile is a decreasing function of the fractional and effective permeability parameters.

The effects of cosine oscillations on flow characteristics are depicted in Figures 3(a)–3(d). In the instance of cosine oscillations, we can see that velocity decreases as the fractional parameter (α) and effective permeability (K_{eff}) increase, and that flow profiles decrease as these two components increase which is observed from Figures 3(a) and 3(c). The flow profile is rising with greater values of the dimensionless parameter for Casson fluid parameter (β) and Reynolds number (Re) as seen in Figures 3(b) and 3(d). The effect of sine oscillations on flow characteristics is seen in Figures 4(a)–4(d). For sine oscillations on flow characteristics, a similar effect has been seen for various values of significant parameters.

6. Conclusions

The precise solutions for a time-dependent fractional Casson fluid traversing a channel under MHD and porosity effects are described in this paper. The flow is caused by the bottom plate's unstable motion, which is restricted by parallel but perpendicular sidewalls. By using an appropriate set of dimensionless variables, the momentum equation, as well as the initial and boundary conditions, has been changed to a dimensionless form. A mix of Laplace and Fourier transformations is used to get the exact solution of the momentum equation. For the problem under examination, many situations at the bottom plate have been studied. The same flow has been studied under the impact of side walls by addressing the limiting instances. After determining general solutions for any arbitrary flow, solutions for impulsive, solutions for constant accelerated, and solutions for the sinusoidal fluid motion of the bottom plate were retrieved. These precise solutions are also visually depicted in support of our effort and then theoretically presented. The following points are emphasized after a thorough evaluation of the work.

- (i) In this study, it was discovered that Reynolds number has a significant influence on flow characteristics since the velocity of the fluid increases as the Reynolds number grows in all limiting/special instances
- (ii) Whether the bottom plate is at rest or moving by constant acceleration, the velocity of the fluid decreases as the fractional parameter and effective permeability increase for long periods of time
- (iii) The velocity of cosine/sine oscillations increases with the rising Reynolds number and Casson parameter, since flow profiles increase with rising values of these two parameters, as seen in this work. The flow profile, on the other hand, decreases when the fractional parameter and effective permeability increase

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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