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Research Article

Analytical Fuzzy Analysis of a Fractional-Order Newell-Whitehead-Segel Model with Mittag-Leffler Kernel

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In this paper, the method for evaluating an analytical solution of fuzzy Newell-Whitehead-Segel equation with certain affecting terms of force has been given. The notions of an Atangana-Baleanu-Caputo derivative in the vague or uncertainty form are used to reach this type of result for the solution as mentioned earlier. The fuzzy Laplace transformation is implemented at the first attempt to achieve the series form result. Secondly, the iterative method is applied to investigate the suggested solution by inverse Laplace transform. Some new solutions on the Laplace transform of an arbitrary derivative under uncertainty are presented. The solution has been provided in terms of infinite series for the research, which reduces the problem to a few equations. The required results are then calculated in a series solution form that quickly leads to the analytical answer. The solution is divided into two sections, or fuzzy branches, the lower and upper branches. We proved certain test problems to demonstrate the effectiveness of the recommended approach.

1. Introduction

Fuzzy set theory is a valuable method for modeling uncertain issues. As a result, fuzzy concepts have been used to simulate a variety of natural phenomena. In particular, fuzzy partial differential equations (PDEs) have been used in engineering, pattern formation theory, control systems, population dynamics, knowledge-based systems, power engineering, image processing, trial automation, consumer electronics, robotics, management, operations research, and artificial expert/intelligence. Due to its importance in a variety of scientific areas, fuzzy set theory has a close link with fractional calculus [1–4]. Byatt and Kandel [5] proposed fuzzy DEs in 1978, while Agarwal et al. [6] were the first to address the Riemann-Liouville differentiability and fuzziness concept under the Hukuhara differentiability. FC and fuzzy set theory integrates several numerical methodologies that allow for more in-depth knowledge of complex systems while also lowering the computing cost of solving them.

Physical models of real-world phenomena usually contain considerable uncertainty for many reasons. Fuzzy sets appear to be a good tool for modeling the uncertainty that ambiguity and impreciseness bring up. We use it in disciplines including medical, environmental, economic, medical, social, economic, physical, and social science, where data contains uncertainty. To investigate these issues, Zadeh introduced fuzziness to set theory in 1965. Fractional calculus has risen in prominence over the last forty years due to its various applications in engineering and physics [7–11]. In the behavior of defined system processes, many examples exhibit fuzzy instead of stochastic uncertainty. Several writers have been interested in studying the theoretical underpinnings of fuzzy issues in recent years. Fractional fuzziness differential equations (FFDEs) are particularly valuable in modeling scientific and technical problems, like population models, civil engineering, electrohydraulics modeling, and weapon systems evaluation. In mathematics, fractional calculus, along with the help of fuzzy theory, is a key instrument for dealing with ambiguity and determining subjective or ambiguous status and providing more general solutions. This has been addressed in a variety of real-world situations, including the golden mean [12], practical systems [13], medicine [14], gravity [15], engineering phenomena, and quantum physics. For the very first time, Zadeh [16] became acquainted with fuzzy sets. Then, there was work on the concept of a fuzzy number and its use in fuzzy control [17].

The necessity to model some real-world challenges while accounting for data uncertainty led to fuzzy partial differential equations. As we will see subsequently, partial differential equations are important in many domains of engineering and science. Heat transfer is an important part of mechanical and aerospace engineering study because many equipment and systems in both aerospace and mechanical engineering disciplines are subject to heat [18, 19]. The governing differential equation for the above-mentioned physical setting may be derived using heat conduction equations. An engineer can foresee possible form changes of the plate in vibrations based on the equations mentioned earlier in simulation findings. Numerous engineering problems fall into this category by nature, and engineers should address them using numerical approaches; see for more details [20, 21].

The classic Newell-Whitehead-Segel model (NWS) is among the most widely used amplitude equations for presenting the occurrence of stationary spatial stripe patterns in a two-dimensional system, as well as the dynamic behavior near the Rayleigh-Benard convection bifurcation point of binary fluid mixtures [22]. The rolling pattern, in which the cylinders are made using fluid streamlines that may be twisted to form the hexagonal, and spiral-like pattern, in which the striped cells and honeycomb are formed by splitting the flow of the fluid, are both visible. For example, stripe patterns can be observed in the human fingerprints, visual cortex, and zebra skin. It is important to note that hexagonal patterns may be created using laser beam propagation across the nonlinear optical medium in a diffusion and chemical reaction model [23].

Nonlinear fractional partial differential equations (FPDEs) are seen in many mathematical structures. Solving these equations, on the other hand, is frequently challenging. Effective and established techniques are required to find approximate or analytical solutions to these equations. There are many standard numeric-analytic strategies, like Sumudu transform method [24], homotopy analysis method [25], variational iteration method [26], fractional complex transform method [27], and homotopy perturbation algorithm [28].

2. Basic Definitions

Definition 1. Let us consider fuzzy continuous term of $\tilde{\Theta}(\eta)$ on $[0,\wp] \subset R$ in sense of Atangana-Baleanu-Caputo (ABC) operator with respect to η as [29].

The ABC derivative of $\tilde{\Theta}(\eta) \in \mathscr{H}^1(0,\eta)$ expressed as

$$D_\eta^\gamma \tilde{\Theta}(\eta) = \frac{\mathrm{ABC}(\gamma)}{1-\gamma}\int_0^\eta \frac{d}{d\varepsilon}\tilde{\Theta}(\varepsilon) M_\gamma\left[\frac{-\gamma}{1-\gamma}(\eta-\varepsilon)^\gamma\right] d\varepsilon. \quad (1)$$

Replacing $E_\gamma[(-\gamma/(1-\gamma))(\eta-\varepsilon)^\gamma]$ by $E_1[(-\gamma/(1-\gamma))(\eta-\varepsilon)]$, we have "differential Caputo-Fabrizio derivative." Further, if $\tilde{\Theta}(\eta) \in C^F[0,\wp] \cap L^F[0,\wp]$, such that $\tilde{\Theta}(\eta) = [\underline{\Theta}_\gamma, \bar{\Theta}_\gamma(\eta)]$, $\gamma \in [0,1]$ and $\eta_0 \in (0,\wp)$, then the fractional fuzzy ABC derivative is defined as

$$\left[D_\eta^\gamma \tilde{\Theta}(\eta)\right]_\delta = \left[D_\eta^\gamma \underline{\Theta}_\gamma(\eta), D_\eta^\gamma \bar{\Theta}_\delta(\eta)\right], \quad 0 \le \delta \le 1, \quad (2)$$

such that

$$D_\eta^\gamma \underline{\Theta}_\gamma(\eta) = \frac{\mathrm{ABC}(\gamma)}{1-\gamma}\int_0^\eta \frac{d}{d\varepsilon}\underline{\Theta}(\varepsilon) E_\gamma\left[\frac{-\gamma}{1-\gamma}(\eta-\varepsilon)^\gamma\right] d\varepsilon,$$
$$D_\eta^\gamma \tilde{\Theta}_\gamma(\eta) = \frac{\mathrm{ABC}(\gamma)}{1-\gamma}\int_0^\eta \frac{d}{d\varepsilon}\tilde{\Theta}(\varepsilon) E_\gamma\left[\frac{-\gamma}{1-\gamma}(\eta-\varepsilon)^\gamma\right] d\varepsilon.$$
$$D_\eta^\gamma[\text{constant}] = 0. \quad (3)$$

Here, $\mathrm{ABC}(\gamma)$ show "function of normalization" and defined by $\kappa(0) = \kappa(1) = 1$, and E_γ is named as the "Mittag-Leffler" term.

Definition 2. Then, the ABC integral is defined as [29]

$$\mathbb{I}_\eta^\gamma \tilde{\Theta}(\eta) = \frac{(1-\gamma)\tilde{\Theta}(\eta)}{\mathrm{ABC}(\gamma)} + \frac{\gamma}{\mathrm{ABC}(\gamma)}\int_0^\eta \frac{(\eta-\varepsilon)^{\gamma-1}}{\Gamma(\gamma)}\tilde{\Theta}(\varepsilon) d\varepsilon. \quad (4)$$

Further, if $\tilde{\Theta}(\eta) \in C^F[0,\wp] \cap L^F[0,\wp]$, where $L^F[0,\wp]$ and $C^F[0,\wp]$ define the "space continues fuzzy term 1" is the space of "fuzzy integrable Lebesgue terms," respectively. Then, fuzzy fractional ABC integral is defined as

$$\left[\mathbb{I}_0^\gamma \tilde{\Theta}(\eta)\right]_\delta = \left[\mathbb{I}_0^\delta \underline{\Theta}_\delta(\eta), \mathbb{I}_0^\delta \bar{\Theta}_\delta(\eta)\right], \quad 0 \le \delta \le 1, \quad (5)$$

such that

$$\mathbb{I}_0^\delta \underline{\Theta}_\delta(\eta) = \frac{1-\gamma}{\mathrm{ABC}(\gamma)}\underline{\Theta}(\eta) + \frac{\gamma}{\mathrm{ABC}(\gamma)\Gamma(\gamma)}\int_0^\eta (\eta-\varepsilon)^{\gamma-1}\underline{\Theta}(\varepsilon) d\varepsilon,$$
$$\mathbb{I}_0^\delta \bar{\Theta}_\delta(\eta) = \frac{1-\gamma}{\mathrm{ABC}(\gamma)}\bar{\Theta}(\eta) + \frac{\gamma}{\mathrm{ABC}(\gamma)\Gamma(\gamma)}\int_0^\eta (\eta-\varepsilon)^{\gamma-1}\bar{\Theta}(\varepsilon) d\varepsilon. \quad (6)$$

Definition 3. The "Fuzzy Laplace transform" of ABC derivative of $\tilde{\Theta}(\eta)$ is given as [29]

$$\mathscr{L}\left[D_0^\gamma \tilde{\Theta}(\eta)\right] = \frac{\mathrm{ABC}(\gamma)}{[s^\gamma(1-\gamma)+\gamma]}\left[s^\gamma \mathscr{L}\left[\tilde{\Theta}(\eta) - s^{\gamma-1}\tilde{\Theta}(0)\right]\right]. \quad (7)$$

Definition 4. The function of "Mittag-Leffler" $E_\beta(\eta)$ is defined as [29]

$$E_{\beta}(\eta) = \sum_{n=0}^{\infty} \frac{\eta^n}{\Gamma(n\beta + 1)}, \quad \beta > 0. \tag{8}$$

Definition 5. A mapping $\kappa : R \longrightarrow [0, 1]$. If it holds, it is considered to be a fuzzy number [29]. $(i)\kappa$ is upper semicontinuous;

(ii) $\kappa\{\mu(\varepsilon_1) + \mu(\varepsilon_2)\} \geq \min\{\kappa(\varepsilon_1), \kappa(\varepsilon_2)\}$

(iii) $\exists \varepsilon_0 \in R$ such that $\kappa(\varepsilon_0) = 1$

(iv) $\mathrm{cl}\{r \in R, \kappa(r) > 0\}$ is compact

Definition 6. The parametric form of a fuzzy number is $(\underline{\kappa}(\delta), \bar{\kappa}(\delta))$ such that $0 \leq \delta \leq 1$ and conditions [29]

(i) $\underline{k}(\delta)$ increasing, left-continuous over $(0, 1]$ and right continues at 0

(ii) $\bar{k}(\delta)$ decreasing, left-continuous over $(0, 1]$ and right continues at 0

(iii) $k(\delta) \leq \bar{k}(\delta)$

3. Methodology

In this section, we apply Laplace transform to analyze the general solution of PDE. On both sides implementing Laplace transform, we get

$$\mathscr{L}\left[D_{\eta}^{\gamma}\left(\tilde{\Theta}(\psi, \eta)\right)\right] = \mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}(\psi, \eta)\right)\right]. \tag{9}$$

Evaluating the Laplace transformation, Equation (9) implies that

$$\frac{\mathrm{ABC}(\gamma)}{[s^{\gamma}(1-\gamma)+\gamma]}\left[s^{\gamma}\mathscr{L}\left[\tilde{\Theta}(\psi, \eta)\right] - s^{\gamma-1}\tilde{\Theta}(\psi, 0)\right] = \mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}(\psi, \eta)\right)\right]. \tag{10}$$

By applying the initial condition, we have

$$s^{\gamma}\mathscr{L}\left[\tilde{\Theta}(\psi, \eta)\right] = s^{\gamma-1}\tilde{g}(\psi, \eta) + \frac{[s^{\gamma}(1-\gamma)+\gamma]}{\mathrm{ABC}(\gamma)} \cdot \mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}(\psi, \eta)\right)\right], \tag{11}$$

or

$$\mathscr{L}\left[\tilde{\Theta}(\psi, \eta)\right] = \frac{1}{s}\tilde{g}(\psi, \eta) + \frac{[s^{\gamma}(1-\gamma)+\gamma]}{s^{\gamma}\mathrm{ABC}(\gamma)} \cdot \mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}(\psi, \eta)\right)\right]. \tag{12}$$

We can write the unknown functions as to investigate the series-type solution $\tilde{\Theta}(\psi, \eta) = \sum_{n=0}^{\infty}\tilde{\Theta}_n(\psi, \eta)$. We can write of Equation (9), and we get

$$\mathscr{L}\left[\sum_{n=0}^{\infty}\tilde{\Theta}_n(\psi, \eta)\right] = \frac{1}{s}\tilde{g}(\psi, \eta) + \frac{[s^{\gamma}(1-\gamma)+\gamma]}{s^{\gamma}\mathrm{ABC}(\gamma)} \cdot \mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\sum_{n=0}^{\infty}\tilde{\Theta}_n(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\sum_{n=0}^{\infty}\tilde{\Theta}_n(\psi, \eta)\right)\right]. \tag{13}$$

Comparisons of term by term of Equation (13), we have

$$\begin{aligned}
\mathscr{L}\left[\tilde{\Theta}_0(\psi, \eta)\right] &= \frac{1}{s}\tilde{g}(\psi, \eta), \\
\mathscr{L}\left[\tilde{\Theta}_1(\psi, \eta)\right] &= \frac{[s^{\gamma}(1-\gamma)+\gamma]}{s^{\gamma}\mathrm{ABC}(\gamma)}\mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}_0(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}_0(\psi, \eta)\right)\right], \\
\mathscr{L}\left[\tilde{\Theta}_2(\psi, \eta)\right] &= \frac{[s^{\gamma}(1-\gamma)+\gamma]}{s^{\gamma}\mathrm{ABC}(\gamma)}\mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}_1(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}_1(\psi, \eta)\right)\right], \\
&\vdots \\
\mathscr{L}\left[\tilde{\Theta}_{n+1}(\psi, \eta)\right] &= \frac{[s^{\gamma}(1-\gamma)+\gamma]}{s^{\gamma}\mathrm{ABC}(\gamma)}\mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}_n(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}_n(\psi, \eta)\right)\right], \quad n \geq 0.
\end{aligned} \tag{14}$$

Using inverse Laplace transformation in Equation (14), we get

$$\begin{aligned}
\tilde{\Theta}_0(\psi, \eta) &= \mathscr{L}^{-1}\left[\frac{1}{s}\tilde{g}(\psi, \eta)\right], \\
\tilde{\Theta}_1(\psi, \eta) &= \mathscr{L}^{-1}\left[\frac{[s^{\gamma}(1-\gamma)+\gamma]}{s^{\gamma}\mathrm{ABC}(\gamma)}\mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}_0(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}_0(\psi, \eta)\right)\right]\right], \\
\tilde{\Theta}_2(\psi, \eta) &= \mathscr{L}^{-1}\left[\frac{[s^{\gamma}(1-\gamma)+\gamma]}{s^{\gamma}\mathrm{ABC}(\gamma)}\mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}_1(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}_1(\psi, \eta)\right)\right]\right], \\
&\vdots \\
\tilde{\Theta}_{n+1}(\psi, \eta) &= \mathscr{L}^{-1}\left[\frac{[s^{\gamma}(1-\gamma)+\gamma]}{s^{\gamma}\mathrm{ABC}(\gamma)}\mathscr{L}\left[A \frac{\partial^2}{\partial \psi^2}\left(\tilde{\Theta}_n(\psi, \eta)\right) + \frac{\partial}{\partial x}\left(h(\psi)\tilde{\Theta}_n(\psi, \eta)\right)\right]\right], \quad n \geq 0.
\end{aligned} \tag{15}$$

Thus, the fuzzy solutions are obtained as

$$\begin{aligned}
\underline{\Theta}(\psi, \eta) &= \sum_{n=0}^{\infty}\underline{\Theta}_n(\psi, \eta), \\
\bar{\Theta}(\psi, \eta) &= \sum_{n=0}^{\infty}\bar{\Theta}_n(\psi, \eta).
\end{aligned} \tag{16}$$

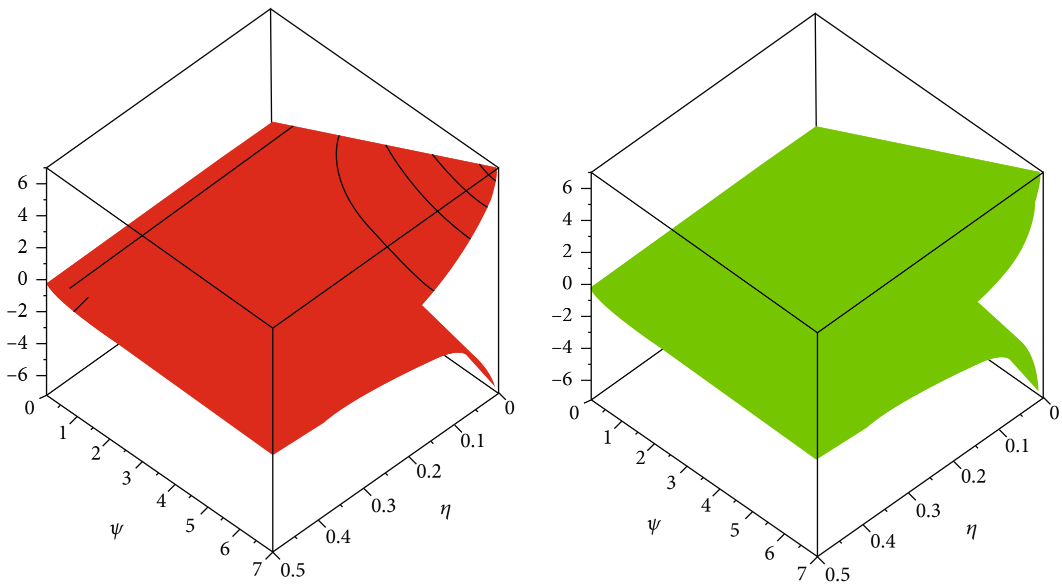

Figure 1: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 1$ and the second graph at fractional of $\gamma = 0.8$.

4. Numerical Results

4.1. Case I. Consider the fractional fuzzy Newell-Whitehead-Segel equation:

$$ {}^{\mathrm{ABC}}D_{\eta}^{\gamma}\tilde{\Theta} - \tilde{\Theta}_{\psi\psi} - 2\tilde{\Theta} + 3\tilde{\Theta}^2 = 0, \quad \eta > 0, \quad \psi \in R, \quad 0 < \gamma \le 1, \tag{17} $$

with the initial condition

$$ \begin{aligned} \tilde{\Theta}(\psi, 0) &= \lambda, \\ \tilde{\kappa} &= (\underline{\kappa}(\delta)\bar{\kappa}(\delta)) = (\delta - 1, 1 - \delta). \end{aligned} \tag{18} $$

Applying the suggested technique, we get

$$ \begin{aligned} \underline{\Theta}_0(\psi,\eta) &= \underline{\kappa}(\delta)\lambda, \\ \bar{\Theta}_0(\psi,\eta) &= \bar{\kappa}(\delta)\lambda, \\ \underline{\Theta}_1(\psi,\eta) &= \frac{\underline{\kappa}(\delta)\lambda(2-3\lambda)}{\mathrm{ABC}(\gamma)}\left[1 - \gamma + \frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right], \\ \bar{\Theta}_1(\psi,\eta) &= \frac{\bar{\kappa}(\delta)\lambda(2-3\lambda)}{\mathrm{ABC}(\gamma)}\left[1 - \gamma + \frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right], \\ \underline{\Theta}_2(\psi,\eta) &= \underline{\kappa}(\delta)\frac{2\lambda(2-3\lambda)(1-3\lambda)}{(\mathrm{ABC}(\gamma))^2}\left((1-\gamma)^2 + \frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)} + \frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right), \\ \bar{\Theta}_2(\psi,\eta) &= \bar{\kappa}(\delta)\frac{2\lambda(2-3\lambda)(1-3\lambda)}{(\mathrm{ABC}(\gamma))^2}\left((1-\gamma)^2 + \frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)} + \frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right). \end{aligned} \tag{19} $$

The higher terms can be obtained in a similar way. We used to find the series solution Equation (17); therefore, we write

$$ \tilde{\Theta}(\psi,\eta) = \tilde{\Theta}_0(\psi,\eta) + \tilde{\Theta}_1(\psi,\eta) + \tilde{\Theta}_2(\psi,\eta) + \tilde{\Theta}_3(\psi,\eta) + \tilde{\Theta}_4(\psi,\eta) + \cdots. \tag{20} $$

The upper and lower fuzzy portion type can be written as

$$ \begin{aligned} \underline{\Theta}(\psi,\eta) &= \underline{\Theta}_0(\psi,\eta) + \underline{\Theta}_1(\psi,\eta) + \underline{\Theta}_2(\psi,\eta) + \underline{\Theta}_3(\psi,\eta) + \underline{\Theta}_4(\psi,\eta) + \cdots, \\ \bar{\Theta}(\psi,\eta) &= \bar{\Theta}_0(\psi,\eta) + \bar{\Theta}_1(\psi,\eta) + \bar{\Theta}_2(\psi,\eta) + \bar{\Theta}_3(\psi,\eta) + \bar{\Theta}_4(\psi,\eta) + \cdots, \\ \underline{\Theta}(\psi,\eta) &= \underline{\kappa}(\delta)\lambda + \frac{\underline{\kappa}(\delta)\lambda(2-3\lambda)}{\mathrm{ABC}(\gamma)}\left[1 - \gamma + \frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right] \\ &\quad + \underline{\kappa}(\delta)\frac{2\lambda(2-3\lambda)(1-3\lambda)}{(\mathrm{ABC}(\gamma))^2} \\ &\quad \cdot \left[(1-\gamma)^2 + \frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)} + \frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right] + \cdots, \\ \bar{\Theta}(\psi,\eta) &= \bar{\kappa}(\delta)\lambda + \frac{\bar{\kappa}(\delta)\lambda(2-3\lambda)}{\mathrm{ABC}(\gamma)}\left[1 - \gamma + \frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right] \\ &\quad + \bar{\kappa}(\delta)\frac{2\lambda(2-3\lambda)(1-3\lambda)}{(\mathrm{ABC}(\gamma))^2} \\ &\quad \cdot \left[(1-\gamma)^2 + \frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)} + \frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right] + \cdots. \end{aligned} \tag{21} $$

The exact solution is

$$ \tilde{\Theta}(\psi,\eta) = \tilde{\kappa}\frac{(-2/3)\lambda\exp^{2\eta}}{(-2/3) + \lambda - \lambda\exp^{2\eta}}. \tag{22} $$

In Figure 1, the first graph shows the 3D fuzzy upper and lower figure of an analytical solution at $\gamma = 1$ and second the fractional graph of $\gamma = 0.8$. Figure 2 shows the 3D fuzzy figure of upper and lower of an analytical solution at $\gamma = 0.6$ and the second figure at fractional order of $\gamma = 0.4$. In Figure 3, the figure shows the 3D fuzzy figure of upper and lower of various fractional orders of γ. Figure 4 shows the 2D fuzzy figure of the upper and lower of various fractional orders of γ.

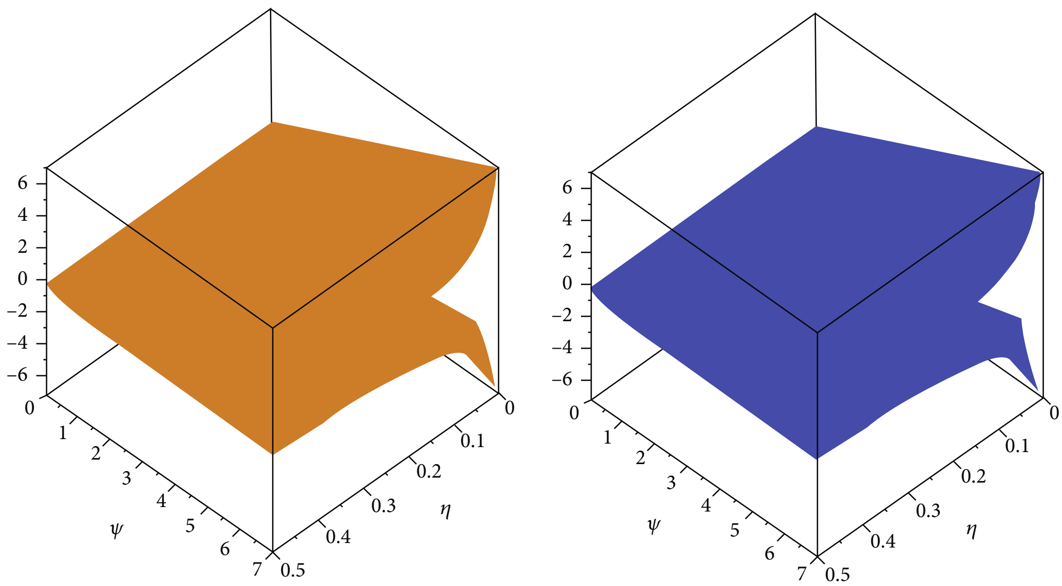

Figure 2: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 0.6$ and the second graph at fractional of $\gamma = 0.4$.

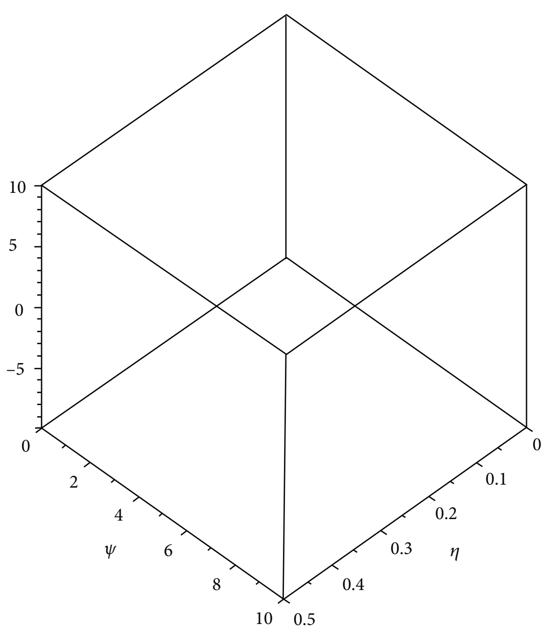

Figure 3: The first graph shows the fuzzy 3D upper and lower figure of the various fractional orders of γ.

4.2. Case II. Consider the fractional fuzzy Newell-Whitehead-Segel equation:

$$ {}^{ABC}D_\eta^\gamma \tilde{\Theta} - \tilde{\Theta}_{\psi\psi} - \tilde{\Theta}\left(1 - \tilde{\Theta}\right) = 0, \quad \eta > 0, \quad \psi \in R, \quad 0 < \gamma \le 1. \tag{23} $$

The initial condition is

$$ \tilde{\Theta}(\psi, 0) = \frac{1}{\left(1 + \exp^{\psi/\sqrt{6}}\right)^2}, \tag{24} $$
$$ \tilde{\kappa} = (\underline{\kappa}(\delta)\bar{\kappa}(\delta)) = (\delta - 1, 1 - \delta). $$

Using the proposed method, we get

$$ \begin{aligned} \underline{\Theta}_0(\psi, \eta) &= \underline{\kappa}(\delta)\frac{1}{\left(1 + \exp^{\psi/\sqrt{6}}\right)^2}, \\ \bar{\Theta}_0(\psi, \eta) &= \bar{\kappa}(\delta)\frac{1}{\left(1 + \exp^{\psi/\sqrt{6}}\right)^2}, \\ \underline{\Theta}_1(\psi, \eta) &= \frac{\underline{\kappa}(\delta)}{ABC(\gamma)}\frac{5}{3}\frac{\exp^{\psi/\sqrt{6}}}{\left(1 + \exp^{\psi/\sqrt{6}}\right)^3}\left[1 - \gamma + \frac{\gamma\eta^\gamma}{\Gamma(\gamma + 1)}\right], \\ \bar{\Theta}_1(\psi, \eta) &= \frac{\bar{\kappa}(\delta)}{ABC(\gamma)}\frac{5}{3}\frac{\exp^{\psi/\sqrt{6}}}{\left(1 + \exp^{\psi/\sqrt{6}}\right)^3}\left[1 - \gamma + \frac{\gamma\eta^\gamma}{\Gamma(\gamma + 1)}\right], \\ \underline{\Theta}_2(\psi, \eta) &= \frac{\underline{\kappa}(\delta)}{(ABC(\gamma))^2}\frac{25}{18}\left(\frac{\exp^{\psi/\sqrt{6}}\left(-1 + 2\exp^{\psi/\sqrt{6}}\right)}{\left(1 + \exp^{\psi/\sqrt{6}}\right)^4}\right) \\ &\quad \cdot \left((1 - \gamma)^2 + \frac{2\gamma(1 - \gamma)\eta^\gamma}{\Gamma(\gamma + 1)} + \frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma + 1)}\right), \\ \bar{\Theta}_2(\psi, \eta) &= \frac{\bar{\kappa}(\delta)}{(ABC(\gamma))^2}\frac{25}{18}\left(\frac{\exp^{\psi/\sqrt{6}}\left(-1 + 2\exp^{\psi/\sqrt{6}}\right)}{\left(1 + \exp^{\psi/\sqrt{6}}\right)^4}\right) \\ &\quad \cdot \left((1 - \gamma)^2 + \frac{2\gamma(1 - \gamma)\eta^\gamma}{\Gamma(\gamma + 1)} + \frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma + 1)}\right). \end{aligned} \tag{25} $$

The higher terms can be obtained in a similar way. We used to find the series solution Equation (23); therefore, we write

$$ \tilde{\Theta}(\psi, \eta) = \tilde{\Theta}_0(\psi, \eta) + \tilde{\Theta}_1(\psi, \eta) + \tilde{\Theta}_2(\psi, \eta) + \tilde{\Theta}_3(\psi, \eta) + \tilde{\Theta}_4(\psi, \eta) + \cdots. \tag{26} $$

The upper and lower fuzzy portion type can be written

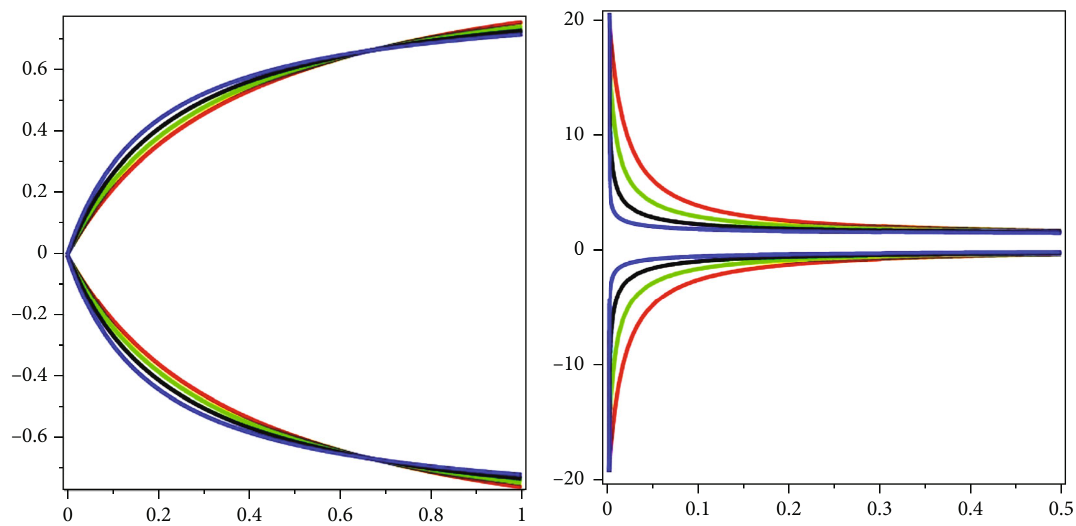

FIGURE 4: The first graph shows the fuzzy 2D upper and lower figure of the various fractional orders of γ.

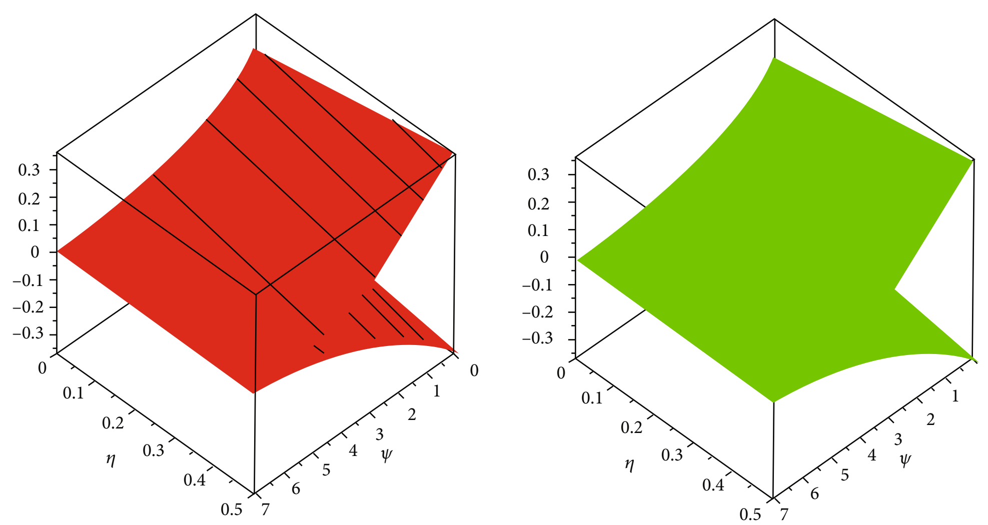

FIGURE 5: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 1$ and the second graph at fractional of $\gamma = 0.8$.

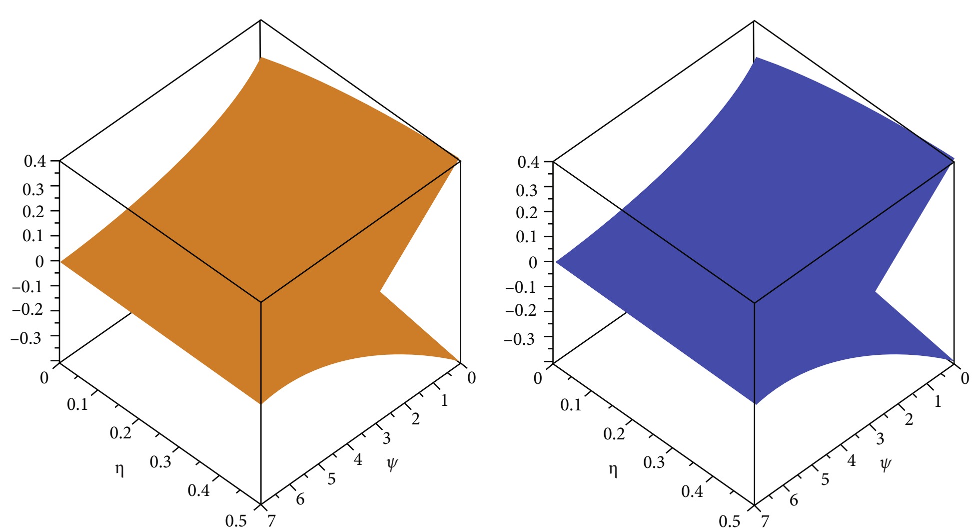

FIGURE 6: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 0.6$ and the second graph at fractional of $\gamma = 0.4$.

as

$$\underline{\Theta}(\psi,\eta)=\underline{\Theta}_0(\psi,\eta)+\underline{\Theta}_1(\psi,\eta)+\underline{\Theta}_2(\psi,\eta)+\underline{\Theta}_3(\psi,\eta)+\underline{\Theta}_4(\psi,\eta)+\cdots,$$
$$\bar{\Theta}(\psi,\eta)=\bar{\Theta}_0(\psi,\eta)+\bar{\Theta}_1(\psi,\eta)+\bar{\Theta}_2(\psi,\eta)+\bar{\Theta}_3(\psi,\eta)+\bar{\Theta}_4(\psi,\eta)+\cdots,$$
$$\underline{\Theta}(\psi,\eta)=\underline{\kappa}(\delta)\frac{1}{\left(1+\exp^{\psi/\sqrt{6}}\right)^2}+\frac{\underline{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{5}{3}\frac{\exp^{\psi/\sqrt{6}}}{\left(1+\exp^{\psi/\sqrt{6}}\right)^3}\cdot\left[1-\gamma+\frac{\gamma\eta^\gamma}{\Gamma(\gamma+1)}\right]+\frac{\underline{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{25}{18}\cdot\left(\frac{\exp^{\psi/\sqrt{6}}\left(-1+2\exp^{\psi/\sqrt{6}}\right)}{\left(1+\exp^{\psi/\sqrt{6}}\right)^4}\right)\cdot\left[(1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^\gamma}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right]+\cdots,$$
$$\bar{\Theta}(\psi,\eta)=\bar{\kappa}(\delta)\frac{1}{\left(1+\exp^{\psi/\sqrt{6}}\right)^2}+\frac{\bar{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{5}{3}\frac{\exp^{\psi/\sqrt{6}}}{\left(1+\exp^{\psi/\sqrt{6}}\right)^3}\cdot\left[1-\gamma+\frac{\gamma\eta^\gamma}{\Gamma(\gamma+1)}\right]+\frac{\bar{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{25}{18}\cdot\left(\frac{\exp^{\psi/\sqrt{6}}\left(-1+2\exp^{\psi/\sqrt{6}}\right)}{\left(1+\exp^{\psi/\sqrt{6}}\right)^4}\right)\cdot\left[(1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^\gamma}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right]+\cdots. \quad (27)$$

The exact solution is

$$\tilde{\Theta}(\psi,\eta)=\tilde{\kappa}\left(\frac{1}{1+\exp^{\psi/\sqrt{6}-(5/6)\eta}}\right)^2. \quad (28)$$

In Figure 5, the first graph shows the 3D fuzzy upper and lower figure of an analytical solution at $\gamma=1$ and the second the fractional graph of $\gamma=0.8$. Figure 6 shows the 3D fuzzy figure of upper and lower of an analytical solution at $\gamma=0.6$ and the second figure at fractional order of $\gamma=0.4$. In Figure 7, the figure shows the 3D fuzzy figure of upper and lower of various fractional orders of γ. Figure 8 shows the 2D fuzzy figure of the upper and lower of various fractional orders of γ.

4.3. Case III. Consider the fractional fuzzy Newell-Whitehead-Segel equation:

$$^{\mathrm{ABC}}D_\eta^\gamma\tilde{\Theta}-\tilde{\Theta}_{\psi\psi}-\tilde{\Theta}+\tilde{\Theta}^4=0,\quad \eta>0,\quad \psi\in R,\quad 0<\gamma\le 1. \quad (29)$$

The initial condition is

$$\tilde{\Theta}(\psi,0)=\frac{1}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{2/3}},$$
$$\tilde{\kappa}=(\underline{\kappa}(\delta)\bar{\kappa}(\delta))=(\delta-1,1-\delta). \quad (30)$$

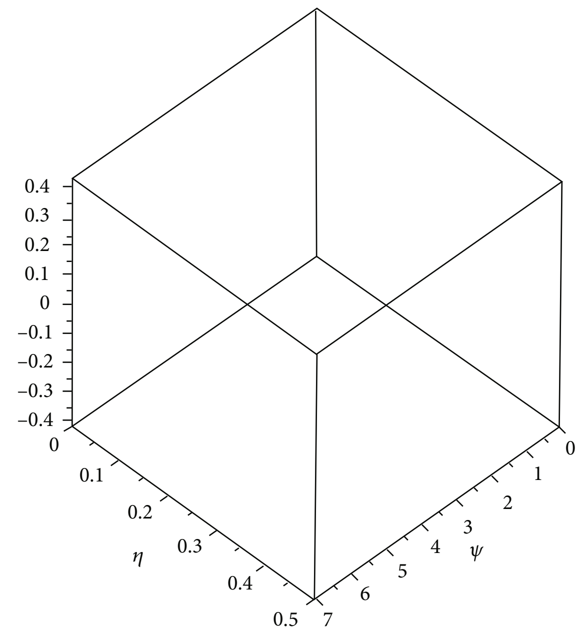

Figure 7: The first graph shows the fuzzy 3D upper and lower figure of the various fractional orders of γ.

Using the proposed method, we get

$$\underline{\Theta}_0(\psi,\eta)=\underline{\kappa}(\delta)\frac{1}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{2/3}},$$
$$\bar{\Theta}_0(\psi,\eta)=\bar{\kappa}(\delta)\frac{1}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{2/3}}$$
$$\underline{\Theta}_1(\psi,\eta)=\frac{\underline{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{7}{5}\frac{\exp^{(3/\sqrt{10})\psi}}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{5/3}}\left[1-\gamma+\frac{\gamma\eta^\gamma}{\Gamma(\gamma+1)}\right],$$
$$\bar{\Theta}_1(\psi,\eta)=\frac{\bar{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{7}{5}\frac{\exp^{(3/\sqrt{10})\psi}}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{5/3}}\left[1-\gamma+\frac{\gamma\eta^\gamma}{\Gamma(\gamma+1)}\right],$$
$$\underline{\Theta}_2(\psi,\eta)=\frac{\underline{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{25}{18}\left(\frac{\exp^{\psi/\sqrt{6}}\left(-1+2\exp^{\psi/\sqrt{6}}\right)}{\left(1+\exp^{\psi/\sqrt{6}}\right)^4}\right)\cdot\left((1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^\gamma}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right),$$
$$\bar{\Theta}_2(\psi,\eta)=\frac{\bar{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{25}{18}\left(\frac{\exp^{\psi/\sqrt{6}}\left(-1+2\exp^{\psi/\sqrt{6}}\right)}{\left(1+\exp^{\psi/\sqrt{6}}\right)^4}\right)\cdot\left((1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^\gamma}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right). \quad (31)$$

The higher terms can be obtained in a similar way. We used to find the series solution Equation (29); therefore, we write

$$\tilde{\Theta}(\psi,\eta)=\tilde{\Theta}_0(\psi,\eta)+\tilde{\Theta}_1(\psi,\eta)+\tilde{\Theta}_2(\psi,\eta)+\tilde{\Theta}_3(\psi,\eta)+\tilde{\Theta}_4(\psi,\eta)+\cdots. \quad (32)$$

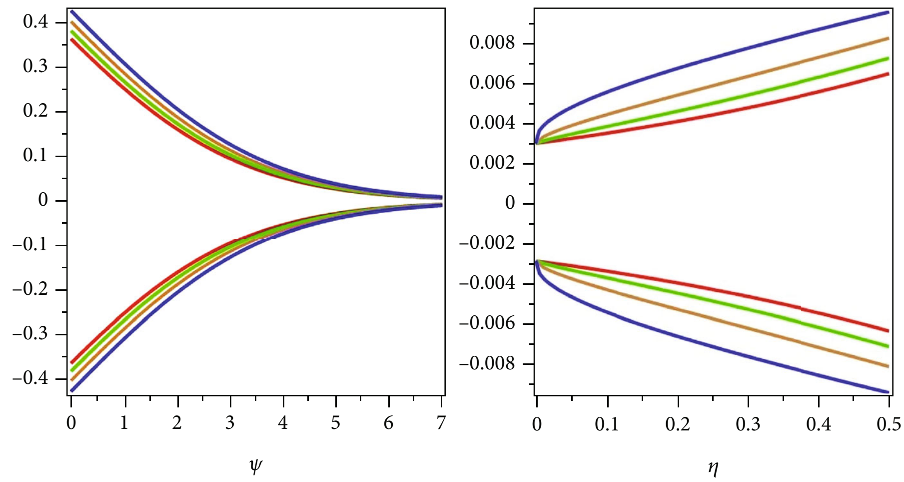

FIGURE 8: The first graph shows the fuzzy 3D upper and lower figure of the various fractional orders of γ.

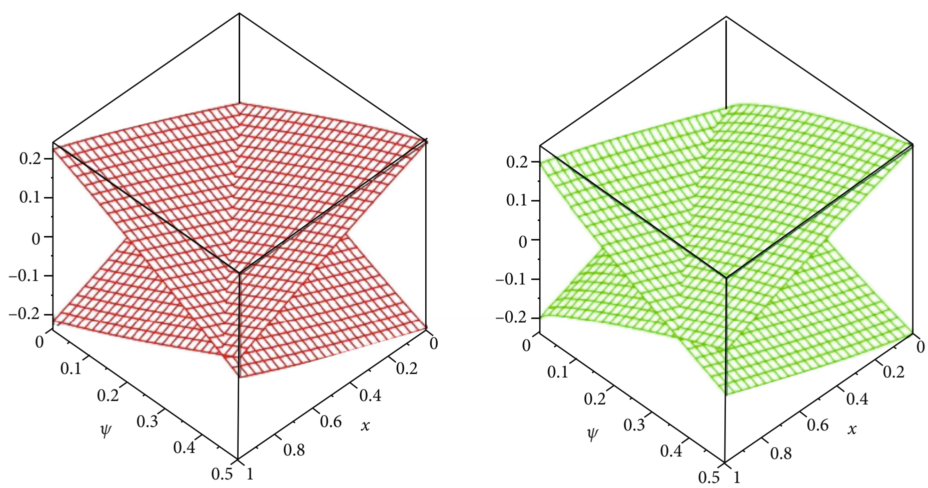

FIGURE 9: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 1$ and the second graph at fractional of $\gamma = 0.8$.

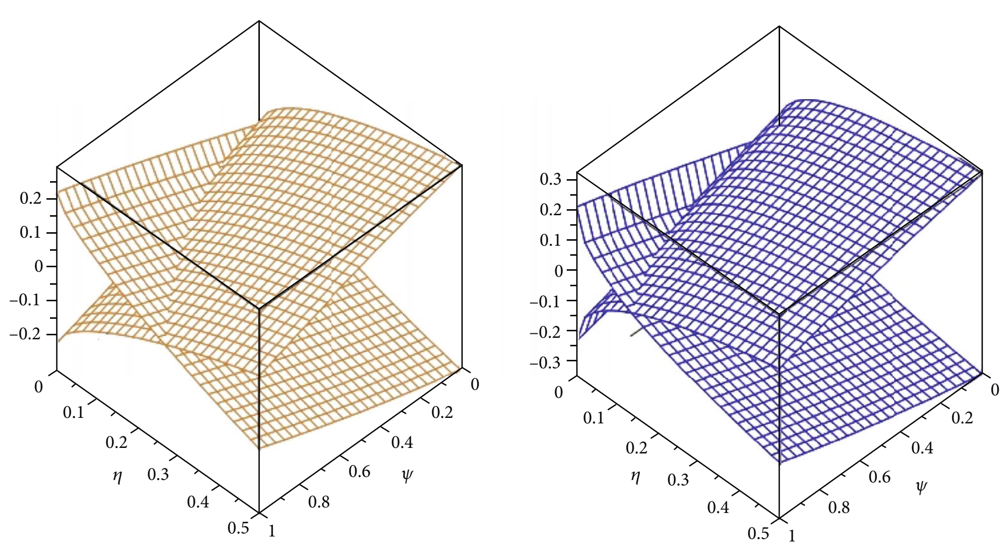

FIGURE 10: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 0.6$ and the second graph at fractional of $\gamma = 0.4$.

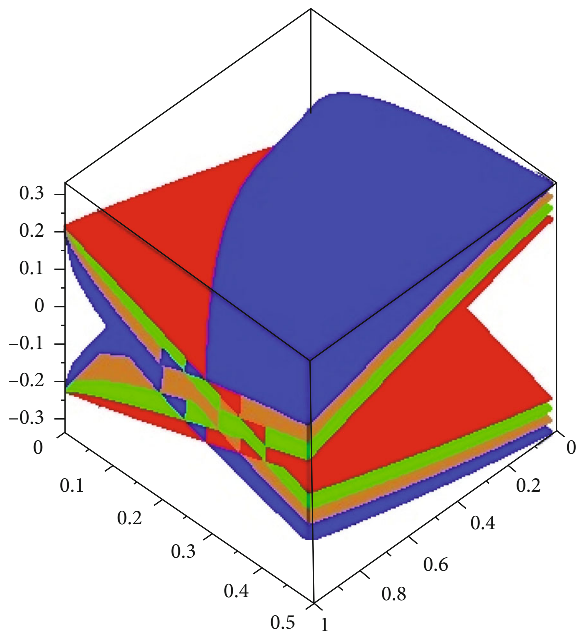

Figure 11: The first graph shows the fuzzy 3D upper and lower figure of the various fractional orders of γ.

The upper and lower fuzzy portion type can be written as

$$\begin{aligned}
\underline{\Theta}(\psi,\eta) &= \underline{\Theta}_0(\psi,\eta)+\underline{\Theta}_1(\psi,\eta)+\underline{\Theta}_2(\psi,\eta)+\underline{\Theta}_3(\psi,\eta)+\underline{\Theta}_4(\psi,\eta)+\cdots,\\
\bar{\Theta}(\psi,\eta) &= \bar{\Theta}_0(\psi,\eta)+\bar{\Theta}_1(\psi,\eta)+\bar{\Theta}_2(\psi,\eta)+\bar{\Theta}_3(\psi,\eta)+\bar{\Theta}_4(\psi,\eta)+\cdots,\\
\underline{\Theta}(\psi,\eta) &= \underline{\kappa}(\delta)\frac{1}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{2/3}}+\frac{\underline{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{7}{5}\frac{\exp^{(3/\sqrt{10})\psi}}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{5/3}}\\
&\quad\cdot\left[1-\gamma+\frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right]+\frac{\underline{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{25}{18}\\
&\quad\cdot\left(\frac{\exp^{\psi/\sqrt{6}}\left(-1+2\exp^{\psi/\sqrt{6}}\right)}{\left(1+\exp^{\psi/\sqrt{6}}\right)^4}\right)\\
&\quad\cdot\left((1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right)+\cdots,\\
\bar{\Theta}(\psi,\eta) &= \bar{\kappa}(\delta)\frac{1}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{2/3}}+\frac{\bar{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{7}{5}\frac{\exp^{(3/\sqrt{10})\psi}}{\left(1+\exp^{3\psi/\sqrt{10}}\right)^{5/3}}\\
&\quad\cdot\left[1-\gamma+\frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right]+\frac{\bar{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{25}{18}\\
&\quad\cdot\left(\frac{\exp^{\psi/\sqrt{6}}\left(-1+2\exp^{\psi/\sqrt{6}}\right)}{\left(1+\exp^{\psi/\sqrt{6}}\right)^4}\right)\\
&\quad\cdot\left((1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right)+\cdots.
\end{aligned}\tag{33}$$

The exact solution is

$$\tilde{\Theta}(\psi,\eta)=\tilde{\kappa}\left(\frac{1}{2}\tanh\left(-\frac{3}{2\sqrt{10}}\left(\psi-\frac{7}{\sqrt{10}}\eta\right)\right)\right).\tag{34}$$

In Figure 9, the first graph shows the 3D fuzzy upper and lower figure of an analytical solution at $\gamma=1$ and the second the fractional graph of $\gamma=0.8$. Figure 10 shows the 3D fuzzy figure of upper and lower of an analytical solution at $\gamma=0.6$ and the second figure at fractional order of $\gamma=0.4$. In Figure 11, the figure shows the 3D fuzzy figure of upper and lower of various fractional orders of γ.

4.4. Case IV. Consider the fractional fuzzy Newell-Whitehead-Segel equation:

$${}^{\mathrm{ABC}}D_{\eta}^{\gamma}\tilde{\Theta}-\tilde{\Theta}_{\psi\psi}-3\tilde{\Theta}+4\tilde{\Theta}^3=0,\quad \eta>0,\quad \psi\in R,\quad 0<\gamma\le 1.\tag{35}$$

The initial condition is

$$\begin{aligned}
\tilde{\Theta}(\psi,0)&=\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}}{\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}},\\
\tilde{\kappa}&=(\underline{\kappa}(\delta)\bar{\kappa}(\delta))=(\delta-1,1-\delta).
\end{aligned}\tag{36}$$

Using the proposed method, we get

$$\begin{aligned}
\underline{\Theta}_0(\psi,\eta)&=\underline{\kappa}(\delta)\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}}{\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}},\\
\bar{\Theta}_0(\psi,\eta)&=\bar{\kappa}(\delta)\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}}{\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}},\\
\underline{\Theta}_1(\psi,\eta)&=\frac{\underline{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{9}{2}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}\exp^{(\sqrt{6}/2)\psi}}{\left(\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)^2}\left[1-\gamma+\frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right],\\
\bar{\Theta}_1(\psi,\eta)&=\frac{\bar{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{9}{2}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}\exp^{(\sqrt{6}/2)\psi}}{\left(\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)^2}\left[1-\gamma+\frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right],\\
\underline{\Theta}_2(\psi,\eta)&=\frac{\underline{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{81}{4}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}\exp^{(\sqrt{6}/2)\psi}\left(-\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)}{\left(\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)^3}\\
&\quad\cdot\left((1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right),\\
\bar{\Theta}_2(\psi,\eta)&=\frac{\bar{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{81}{4}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}\exp^{(\sqrt{6}/2)\psi}\left(-\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)}{\left(\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)^3}\\
&\quad\cdot\left((1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right).
\end{aligned}\tag{37}$$

The higher terms can be obtained in a similar way. We used to find the series solution Equation (35); therefore, we write

$$\tilde{\Theta}(\psi,\eta)=\tilde{\Theta}_0(\psi,\eta)+\tilde{\Theta}_1(\psi,\eta)+\tilde{\Theta}_2(\psi,\eta)+\tilde{\Theta}_3(\psi,\eta)+\tilde{\Theta}_4(\psi,\eta)+\cdots.\tag{38}$$

The upper and lower fuzzy portion type can be written as

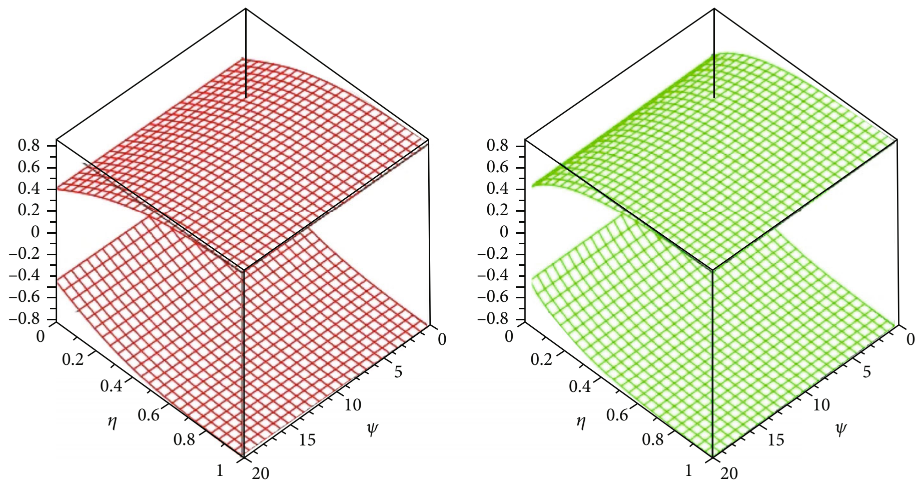

FIGURE 12: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 1$ and the second graph at fractional of $\gamma = 0.8$.

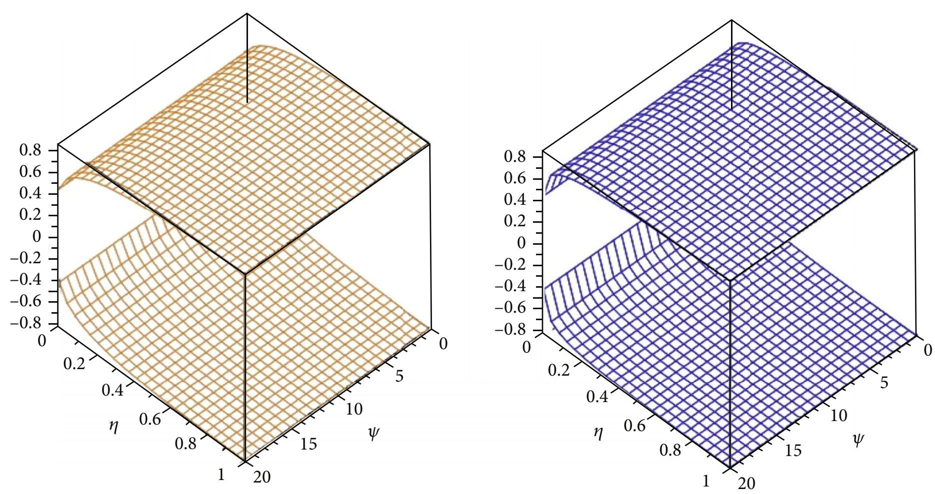

FIGURE 13: The first graph shows the fuzzy 3D upper and lower figure of analytical solution at $\gamma = 0.6$ and the second graph at fractional of $\gamma = 0.4$.

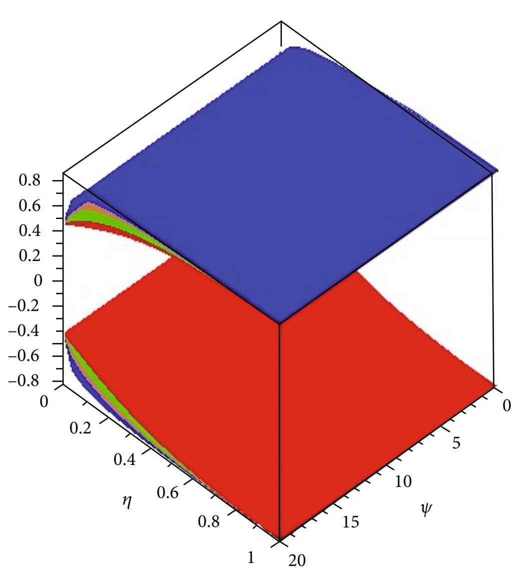

FIGURE 14: The first graph shows the fuzzy 3D upper and lower figure of the various fractional orders of γ.

$$
\begin{aligned}
\underline{\Theta}(\psi,\eta) &= \underline{\Theta}_0(\psi,\eta) + \underline{\Theta}_1(\psi,\eta) + \underline{\Theta}_2(\psi,\eta) + \underline{\Theta}_3(\psi,\eta) + \underline{\Theta}_4(\psi,\eta) + \cdots,\\
\bar{\Theta}(\psi,\eta) &= \bar{\Theta}_0(\psi,\eta) + \bar{\Theta}_1(\psi,\eta) + \bar{\Theta}_2(\psi,\eta) + \bar{\Theta}_3(\psi,\eta) + \bar{\Theta}_4(\psi,\eta) + \cdots,\\
\underline{\Theta}(\psi,\eta) &= \underline{\kappa}(\delta)\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}}{\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}} + \frac{\underline{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{9}{2}\\
&\quad \cdot \sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}\exp^{(\sqrt{6}/2)\psi}}{\left(\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)^2}\left[1-\gamma+\frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right]\\
&\quad + \frac{\underline{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{81}{4}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}\exp^{(\sqrt{6}/2)\psi}\left(-\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)}{\left(\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)^3}\\
&\quad \cdot \left((1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right)+\cdots,\\
\bar{\Theta}(\psi,\eta) &= \bar{\kappa}(\delta)\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}}{\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}} + \frac{\bar{\kappa}(\delta)}{\mathrm{ABC}(\gamma)}\frac{9}{2}\\
&\quad \cdot \sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}\exp^{(\sqrt{6}/2)\psi}}{\left(\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)^2}\left[1-\gamma+\frac{\gamma\eta^{\gamma}}{\Gamma(\gamma+1)}\right]\\
&\quad + \frac{\bar{\kappa}(\delta)}{(\mathrm{ABC}(\gamma))^2}\frac{81}{4}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\psi}\exp^{(\sqrt{6}/2)\psi}\left(-\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)}{\left(\exp^{\sqrt{6}\psi}+\exp^{(\sqrt{6}/2)\psi}\right)^3}\\
&\quad \cdot \left((1-\gamma)^2+\frac{2\gamma(1-\gamma)\eta^{\gamma}}{\Gamma(\gamma+1)}+\frac{\gamma^2\eta^{2\gamma}}{\Gamma(2\gamma+1)}\right)+\cdots.
\end{aligned}
\tag{39}
$$

The exact solution is

$$\tilde{\Theta}(\psi,\eta)=\tilde{\kappa}\sqrt{\frac{3}{4}\frac{\exp^{\sqrt{6}\psi}}{\exp^{\sqrt{6}\psi}+\exp^{((\sqrt{6}/2)\psi-(9/2)\eta)}}}. \tag{40}$$

In Figure 12, the first graph shows the 3D fuzzy upper and lower figure of an analytical solution at $\gamma=1$ and the second the fractional graph of $\gamma=0.8$. Figure 13 shows the 3D fuzzy figure of upper and lower of an analytical solution at $\gamma=0.6$ and the second figure at fractional order of $\gamma=0.4$. In Figure 14, the figure shows the 3D fuzzy figure of upper and lower of various fractional orders of γ.

5. Conclusion

In this paper, investigating the fractional Newell-Whitehead-Segel equation with uncertainty is not an easy problem to analyze, especially in advanced differentiability such as Atangana-Baleanu-Caputo on the fuzzy valued functions. Because the generated models are more complex to solve as parametric coupled systems than standard fuzzy differential equations, we must use the meaning of the uncertain Atangana-Baleanu-Caputo derivative to identify the significance of these solutions to the uncertain models. It can be seen that the solution for each level is an interval at each point, implying that our solutions are fuzzy number functions at each point in the domain.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors' Contributions

Yousuf Alkhezi and Nehad Ali Shah contributed equally to this work and are the co-first authors.

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