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Research Article

Spherical Linear Diophantine Fuzzy TOPSIS Algorithm for Green Supply Chain Management System

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Enhancing competitive pressure is one of the most significant roles of supply chain management. The competitive environment and customer perception have shifted in favour of an ecological mentality. As a result, green supplier selection (GSS) has emerged as a critical problem. The challenge of green supplier selection striving for agility, durability, ecological sensitivity, leanness, and sustainability is tackled in this paper. In terms of recycling applications, environmental applications, carbon footprint, and water consumption, the environmental parameters evaluated in GSS and traditional supplier selection differ. Because of the form of the problem, a resolution is defined, which comprises an algorithm entrenched in the spherical linear Diophantine fuzzy sets (SLDFSs) Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) technique. Before discussing the approach of the SLDF model, some background information on SLDF sets is provided. To assure the uniqueness of this robust extension, different operations on SLDFSs are described, along with some concise interpretations to help the reader comprehend these ideas. A robust TOPSIS approach has been utilized in the issue of GSS by taking into consideration the multicriteria decision-making (MCDM) technique particularly useful in several areas, like analyzing and choosing traditional and environmental conventionalities. Due to linguistic criteria and the inability to assess all criteria, the fuzzy technique must be used with the TOPSIS method to lessen the consequences of instability and ambiguity. The spherical linear Diophantine fuzzy TOPSIS approach is employed, as it simplifies the evaluation of decision-makers and criteria. The hybrid technique resulting from integrating the SLDFS and TOPSIS is extremely successful in selecting which provider is more suited among the alternatives established on the criteria set by the order of significance, and this method may also be incorporated into similar issues.

1. Introduction

One of the main archetypes in the supply chain (SC) management system is the green notion, which may be thought of as an organizational philosophy. Because of environmental legislation and consumer demands on sustainability, the notion of green supply chain management (GSCM) has gained traction as explained by Govindan et al. [1]. Mishra et al. [2] in 2019 explained that GSCM is an administrative approach that incorporates the environmental thinking structure in at-most all supply chain operations such as material selection, purchasing, product design, and production process which helps the companies to achieve more acquisitions while improving their sustainability impact by downsizing the adverse accouterments of potential consequences.

The GSCM must begin from the start of the supply chain, with raw material purchase, and continue through every stage, by incorporating disposal or recycling of the product. It is not adequate to spotlight just greenness in infiltrating SC operations for sustainability strategies and goals; firms must also address the environmental burdens of retiring operations among stakeholders or partners in order to improve supplier achievements as analyzed by Banaeian et al. [3] in 2018. As a result, suppliers play an important role in helping businesses enhance their environmental achievement as explained by Mathiyazhagan et al. [4] in 2018. As a result, businesses have focused on the green supplier selection (GSS) issue when building GSCM.

MCDM techniques may be an appropriate way of dealing with the GSS situation and comparing vendors. Though there are many published studies on GSS in both fuzzy and crisp contexts, further investigation is necessary with diverse criteria, skill, and linguistic characteristics taken into consideration.

MCDM techniques may be an appropriate way of dealing with the GSS situation and comparing vendors. Further investigations are required with various skills, measures, linguistic characteristics, and even many studies published on GSS in both crisp and fuzzy contexts.

Intuitionistic fuzzy sets (IFSs) [5, 6], Pythagorean fuzzy sets (PyFS) [7–9], and q-rung orthopair fuzzy sets (q-ROPFS) [10, 11] are basic ideas in computational intelligence with numerous applications in fuzzy set (FS) [12, 13] system modeling and decision-making under uncertainty. Nonetheless, all of these ideas are subject to stringent limitations on the membership grade (MG) and nonmembership grade (NMG). Moreover, by including the neutral-membership grade in the abovementioned grades, we get the picture fuzzy sets (PFSs) [14], spherical fuzzy sets (SFSs) [15, 16], m-polar spherical fuzzy sets (m-PSFSs) [12], and q-rung ortho picture fuzzy sets (q-RPFS) [17]. All these notions have some strict restrictions over the grade functions imposed on them. To alleviate these constraints, we suggest a novel extension of fuzzy sets known as SLDFSs [18, 19], which takes into account reference parameters as well. Spherical linear Diophantine fuzzy sets can naturally classify issues and offer acceptable solutions by picking alternative pairings of reference parameters. Some fundamental procedures for spherical linear Diophantine fuzzy sets are provided.

Some famous academics have created many nondeterministic extensions of the TOPSIS approach in recent years, including fuzzy sets [20], IFSs [21], PyFSs [22, 23], and other hybrid structures. Based on intuitionistic fuzzy numbers (IFNs), Xu et al. [24–26] proposed weighted average operators, geometric operators, and induced generalised operators. In the framework of IFNs, Jose [27] explored aggregation operators associated with the score function for MCDM. The TOPSIS technique in recent years in extensions of fuzzy sets with abstinence membership values such as PFSs [28], SFSs [29], and q-RPFSs [30] has been developed and utilized in many different fields widely in medical diagnosis problems, enterprise resource planning systems, dry bulk carrier selection, green supply selection, etc. Many researchers have studied the TOPSIS method for decision-making problems, including Adeel et al. [31], Akram and Arshad [32], Xu and Zhang [33], Selvachandran and Peng [34], Kumar and Garg [35], Biswas and Sarkar [22], Boran et al. [21], Li and Nan [36], and Eraslan and Karaaslan [37].

Although PFSs, SFSs, q-RPFSs, and m-PSFSs have extensive applications in diverse domains of real life, these ideas have limits connected to membership, abstinence, and nonmembership grades. To overcome these constraints, we provide the innovative notion of spherical linear Diophantine fuzzy set (SLDFS) with reference parameters. Because of the usage of reference parameters, the suggested SLDFS model is more efficient and adaptable than existing techniques. SLDFS classifies data in MADM situations by varying the physical sense of reference parameters. With the use of reference parameters, this set covers the spaces of existing buildings and enlarges the space for MG, AG, and NMG. The reason for the suggested paradigm is presented step by step throughout the manuscript. Now, we will go through some of the paper's main goals.

(1) In some real-world scenarios, the total of the triplets MG, abstinence grade (AG), and NMG to which an option meeting an attribute specified by a decision-maker (DM) is assigned may be greater than one (ex. $0.9 + 0.3 + 0.5 > 1$), and their sum of squares may also be greater than one (ex. $0.9^2 + 0.3^2 + 0.5^2 > 1$). In such cases, PFS and SFS fail. To address these shortcomings, the constraints on MG, AG, and NMGs are changed to $\mathfrak{t}^{\mathfrak{q}} + \mathfrak{a}^{\mathfrak{q}} + \mathfrak{m}^{\mathfrak{q}}$ in the case of q-RPFS. We are capable of dealing with MG, AG, and NMGs independently to some extent even for extremely high values of "q." In some practical issues, where MG $\mathfrak{t}$, AG $\mathfrak{a}$, and NMG $\mathfrak{f}$ are equal to 1 (i.e., $\mathfrak{t} = \mathfrak{a} = \mathfrak{f} = 1$), we get $1^{\mathfrak{q}} + 1^{\mathfrak{q}} + 1^{\mathfrak{q}} \geq 1$, which defines the q-RPFS condition. In such cases, MADM with q-RPFS may not be employed

(2) Our initial goal is to close this research gap using the unique idea of SLDFS. We are capable of dealing with the picture, spherical, and q-rung picture character of alternatives under the influence of reference factors using this approach. (For example, for $(0.8 + 0.4 + 0.6 > 1)$, we can provide reference parameters like $(0.8)(0.3) + (0.4)(0.4) + (0.6)(0.2) \leq 1$, where $\langle 0.3,0.4,0.2 \rangle$ are the reference parameters for MG, AG, and NMGs, respectively.) Because the suggested technique resembles the well-defined linear equation of three variables $ax + by + cz = d$ in number theory, SLDFS is the best name to describe the suggested model

(3) Our next and second purpose is to introduce reference parameter functions in SLDFS, which cannot cope with parameterizations like PFSs, SFSs, and q-RPFSs. The suggested approach improves on previous techniques, and the decision-maker (DM) has complete freedom in selecting grades. This structure additionally categorizes the problem by modifying the tactile feel of reference parameters

(4) The final goal is to build a solid link betwixt the suggested technique and the MCDM challenges. Using this innovative viewpoint, we construct an enhanced TOPSIS technique that employs SLDF weighted geometric aggregation (SLDFWG) operators. We show an application of the recently published TOPSIS approach based on SLDFSs to an MCDM problem involving the selection of green supply chain systems

(5) After reading this paper, the reader can understand the concept of SLDFG and its operations. Also, the reader can apply real-life problems using the proposed algorithm

(6) If the reader looks for limitations, the calculation part is slightly complicated

The following is how the manuscript is structured. Review of the literature is presented in Section 2. Use of the spherical linear Diophantine fuzzy TOPSIS approach to analyze the green supplier carrier selection process is explained in Section 3. Section 4 walks you through the application and the results step by step. In Section 5, we presented the implications and perspectives. Finally, Section 6 leads the paper to an end.

2. Preliminaries

We will review and give some fundamental definitions of the SLDFSs.

Definition 1 (see [18]). *SLDFS $\mathfrak{S}_{\mathfrak{D}}$ is an element on the nonvoid set $\mathfrak{Q}$ of the form*

$$\mathfrak{S}_{\mathfrak{D}} = \{(\zeta, \langle \mathfrak{m}_{\mathfrak{D}}(\zeta), \mathfrak{a}_{\mathfrak{D}}(\zeta), \mathfrak{n}_{\mathfrak{D}}(\zeta)\rangle, \langle \alpha_{\mathfrak{D}}(\zeta), \beta_{\mathfrak{D}}(\zeta), \eta_{\mathfrak{D}}(\zeta)\rangle): \zeta \in \mathfrak{Q}\}, \tag{1}$$

where the following holds:

(1) $\mathfrak{m}_{\mathfrak{D}}(\zeta), \mathfrak{a}_{\mathfrak{D}}(\zeta), \mathfrak{n}_{\mathfrak{D}}(\zeta), \mathfrak{r}_{\mathfrak{D}}(\zeta)$ are MG, AG, NMG, and the refusal grade (RG)

(2) $\alpha(\zeta), \beta(\zeta), \eta(\zeta), \gamma(\zeta) \in [0, 1]$ are the reference parameters of MG, AG, NMG, and RG, respectively

(3) The condition $0 \leq \alpha_{\mathfrak{D}}(\zeta)\mathfrak{m}_{\mathfrak{D}}(\zeta) + \beta_{\mathfrak{D}}(\zeta)\mathfrak{a}_{\mathfrak{D}}(\zeta) + \eta_{\mathfrak{D}}(\zeta)\mathfrak{n}_{\mathfrak{D}}(\zeta) \leq 1 \forall \zeta \in Q$ and with $0 \leq \alpha(\zeta) + \beta(\zeta) + \eta(\zeta) \leq 1$

(4) $\gamma_{\mathfrak{D}}(\zeta)\mathfrak{r}_{\mathfrak{D}}(\zeta) = 1 - (\alpha_{\mathfrak{D}}(\zeta)\mathfrak{m}_{\mathfrak{D}}(\zeta) + \beta_{\mathfrak{D}}(\zeta)\mathfrak{a}_{\mathfrak{D}}(\zeta) + \eta_{\mathfrak{D}}(\zeta)\mathfrak{n}_{\mathfrak{D}}(\zeta))$

(5) $\mathfrak{S}_{\mathfrak{D}} = (\langle \mathfrak{m}_{\mathfrak{D}}, \mathfrak{a}_{\mathfrak{D}}, \mathfrak{n}_{\mathfrak{D}}\rangle, \langle \alpha_{\mathfrak{D}}, \beta_{\mathfrak{D}}, \eta_{\mathfrak{D}}\rangle)$ is the spherical linear Diophantine fuzzy number (SLDFN), with the constraints $0 \leq \alpha + \beta + \eta \leq 1$ and $0 \leq \alpha_{\mathfrak{D}}\mathfrak{m}_{\mathfrak{D}} + \beta_{\mathfrak{D}}\mathfrak{a}_{\mathfrak{D}} + \eta_{\mathfrak{D}}\mathfrak{n}_{\mathfrak{D}} \leq 1$

Definition 2. A SLDFS on $\mathfrak{Q}$ is said to be

(i) absolute SLDFS, if $\mathfrak{S}^1_{\mathfrak{D}} = \{\zeta, (\langle 1, 0, 0\rangle, \langle 1, ,0, 0\rangle): \zeta \in \mathfrak{Q}\}$

(ii) null SLDFS, if $\mathfrak{S}^0_{\mathfrak{D}} = \{\zeta, (\langle 0, 0, 1\rangle, \langle 0, 0, 1\rangle): \zeta \in \mathfrak{Q}\}$

Definition 3. Let $\mathfrak{S}_{\mathfrak{D}} = (\langle \mathfrak{m}_{\mathfrak{D}}, \mathfrak{a}_{\mathfrak{D}}, \mathfrak{n}_{\mathfrak{D}}\rangle, \langle \alpha_{\mathfrak{D}}, \beta_{\mathfrak{D}}, \eta_{\mathfrak{D}}\rangle)$ be a SLDFN, then

(1) the score function (SF) is displayed by $S_{(\mathfrak{S}_{\mathfrak{D}})}$ and is depicted as

$$S_{(\mathfrak{S}_{\mathfrak{D}})} = \frac{1}{2}[(\mathfrak{m}_{\mathfrak{D}} - \mathfrak{a}_{\mathfrak{D}} - \mathfrak{n}_{\mathfrak{D}}) + (\alpha_{\mathfrak{D}} - \beta_{\mathfrak{D}} - \eta_{\mathfrak{D}})], \tag{2}$$

where $S : \mathfrak{S}_{\mathfrak{D}}(\mathfrak{Q}) \longrightarrow [-1, 1]$

(2) the accuracy function (AF) is displayed by $A_{(\mathfrak{S}_{\mathfrak{D}})}$ and is depicted as

$$A_{(\mathfrak{S}_{\mathfrak{D}})} = \frac{1}{2}\left[\frac{(\mathfrak{m}_{\mathfrak{D}} + \mathfrak{a}_{\mathfrak{D}} + \mathfrak{n}_{\mathfrak{D}})}{3} + (\alpha_{\mathfrak{D}} + \beta_{\mathfrak{D}} + \eta_{\mathfrak{D}})\right], \tag{3}$$

where $A : \mathfrak{S}_{\mathfrak{D}}(\mathfrak{Q}) \longrightarrow [0, 1]$ and $\mathfrak{S}_{\mathfrak{D}}(\mathfrak{Q})$ is the assortment of all SLDFNs on $\mathfrak{Q}$

Definition 4. Two LDFNs $\mathfrak{S}_{\mathfrak{D}_1}$ and $\mathfrak{S}_{\mathfrak{D}_2}$ can be comparable using SF and AF. It is defined as follows:

(i) $\mathfrak{S}_{\mathfrak{D}_1} > \mathfrak{S}_{\mathfrak{D}_2}$ if $S(\mathfrak{S}_{\mathfrak{D}_1}) > S(\mathfrak{S}_{\mathfrak{D}_2})$

(ii) $\mathfrak{S}_{\mathfrak{D}_1} < \mathfrak{S}_{\mathfrak{D}_2}$ if $S(\mathfrak{S}_{\mathfrak{D}_1}) < S(\mathfrak{S}_{\mathfrak{D}_2})$

(iii) If $S(\mathfrak{S}_{\mathfrak{D}_1}) = S(\mathfrak{S}_{\mathfrak{D}_2})$, then

(a) $\mathfrak{S}_{\mathfrak{D}_1} > \mathfrak{S}_{\mathfrak{D}_2}$ if $A(\mathfrak{S}_{\mathfrak{D}_1}) > A(\mathfrak{S}_{\mathfrak{D}_2})$

(b) $\mathfrak{S}_{\mathfrak{D}_1} < \mathfrak{S}_{\mathfrak{D}_2}$ if $A(\mathfrak{S}_{\mathfrak{D}_1}) < A(\mathfrak{S}_{\mathfrak{D}_2})$

(c) $\mathfrak{S}_{\mathfrak{D}_1} = \mathfrak{S}_{\mathfrak{D}_2}$ if $A(\mathfrak{S}_{\mathfrak{D}_1}) = A(\mathfrak{S}_{\mathfrak{D}_2})$

Definition 5. Let $\mathfrak{S}_{\mathfrak{D}_i} = (\langle \mathfrak{m}_{\mathfrak{D}_i}, \mathfrak{a}_{\mathfrak{D}_i}, \mathfrak{n}_{\mathfrak{D}_i}\rangle, \langle \alpha_{D_i}, \beta_{D_i}, \eta_{D_i}\rangle)$ for $\mathfrak{i} \in \Delta$ is a convene of SLDFNs on $\mathfrak{Q}$ and $\mathfrak{X} > 0$ then

(i) $\mathfrak{S}^{\mathfrak{c}}_{\mathfrak{D}_1} = (\langle \mathfrak{n}_{\mathfrak{D}_1}, 1 - \mathfrak{a}_{\mathfrak{D}_1}, \mathfrak{m}_{\mathfrak{D}_1}\rangle, \langle \eta_{D_1}, \beta_{D_1}, \alpha_{D_1}\rangle)$

(ii) $\mathfrak{S}_{\mathfrak{D}_1} = \mathfrak{S}_{\mathfrak{D}_2} \Leftrightarrow \mathfrak{m}_{\mathfrak{D}_1} = \mathfrak{m}_{\mathfrak{D}_2}, \mathfrak{a}_{\mathfrak{D}_1} = \mathfrak{a}_{\mathfrak{D}_2}, \mathfrak{n}_{\mathfrak{D}_1} = \mathfrak{n}_{\mathfrak{D}_2}, \alpha_{D_1} = \alpha_{D_2}, \beta_{D_1} = \beta_{D_2}, \eta_{D_1} = \eta_{D_2}$

(iii) $\mathfrak{S}_{\mathfrak{D}_1} \subseteq \mathfrak{S}_{\mathfrak{D}_2} \Leftrightarrow \mathfrak{m}_{\mathfrak{D}_1} \leq \mathfrak{m}_{\mathfrak{D}_2}, \mathfrak{a}_{\mathfrak{D}_1} \geq \mathfrak{a}_{\mathfrak{D}_2}, \mathfrak{n}_{\mathfrak{D}_1} \geq \mathfrak{n}_{\mathfrak{D}_2}, \alpha_{D_1} \leq \alpha_{D_2}, \beta_{D_1} \geq \beta_{D_2}, \eta_{D_1} \geq \eta_{D_2}$

(iv) $\mathfrak{S}_{\mathfrak{D}_1} \oplus \mathfrak{S}_{\mathfrak{D}_2} = (\langle \mathfrak{m}_{\mathfrak{D}_1} + \mathfrak{m}_{\mathfrak{D}_2} - \mathfrak{m}_{\mathfrak{D}_1}\mathfrak{m}_{\mathfrak{D}_2}, \mathfrak{a}_{\mathfrak{D}_1}\mathfrak{a}_{\mathfrak{D}_2}, \mathfrak{n}_{\mathfrak{D}_1}\mathfrak{n}_{\mathfrak{D}_2}\rangle, \langle \alpha_{D_1} + \alpha_{D_2} - \alpha_{D_1}\alpha_{D_2}, \beta_{D_1}\beta_{D_2}, \eta_{D_1}\eta_{D_2}\rangle)$

(v) $\mathfrak{S}_{\mathfrak{D}_1} \otimes \mathfrak{S}_{\mathfrak{D}_2} = (\langle \mathfrak{m}_{\mathfrak{D}_1}\mathfrak{m}_{\mathfrak{D}_2}, \mathfrak{a}_{\mathfrak{D}_1} + \mathfrak{a}_{\mathfrak{D}_2} - \mathfrak{a}_{\mathfrak{D}_1}\mathfrak{a}_{\mathfrak{D}_2}, \mathfrak{n}_{\mathfrak{D}_1} + \mathfrak{n}_{\mathfrak{D}_2} - \mathfrak{n}_{\mathfrak{D}_1}\mathfrak{n}_{\mathfrak{D}_2}\rangle, \langle \alpha_{D_1}\alpha_{D_2}, \beta_{D_1} + \beta_{D_2} - \beta_{D_1}\beta_{D_2}, \eta_{D_1} + \eta_{D_2} - \eta_{D_1}\eta_{D_2}\rangle)$

(vi) $\mathfrak{S}_{\mathfrak{D}_1} \cup \mathfrak{S}_{\mathfrak{D}_2} = (\langle \mathfrak{m}_{\mathfrak{D}_1} \vee \mathfrak{m}_{\mathfrak{D}_2}, \mathfrak{a}_{\mathfrak{D}_1} \wedge \mathfrak{a}_{\mathfrak{D}_2}, \mathfrak{n}_{\mathfrak{D}_1} \wedge \mathfrak{n}_{\mathfrak{D}_2}\rangle, \langle \alpha_{D_1} \vee \alpha_{D_2}, \beta_{D_1} \wedge \beta_{D_2}, \eta_{D_1} \wedge \eta_{D_2}\rangle)$

(vii) $\mathfrak{S}_{\mathfrak{D}_1} \cap \mathfrak{S}_{\mathfrak{D}_2} = (\langle \mathfrak{m}_{\mathfrak{D}_1} \wedge \mathfrak{m}_{\mathfrak{D}_2}, \mathfrak{a}_{\mathfrak{D}_1} \vee \mathfrak{a}_{\mathfrak{D}_2}, \mathfrak{n}_{\mathfrak{D}_1} \vee \mathfrak{n}_{\mathfrak{D}_2}\rangle, \langle \alpha_{D_1} \wedge \alpha_{D_2}, \beta_{D_1} \vee \beta_{D_2}, \eta_{D_1} \vee \eta_{D_2}\rangle)$

(viii) $\mathfrak{X}\mathfrak{S}_{\mathfrak{D}_1} = (\langle (1-(1-\mathfrak{m}_{\mathfrak{D}_1})^{\mathfrak{X}}), \mathfrak{a}_{\mathfrak{D}_1}^{\mathfrak{X}}, \mathfrak{n}_{\mathfrak{D}_1}^{\mathfrak{X}}\rangle, \langle (1-(1-\alpha_{\mathfrak{D}_1})^{\mathfrak{X}}), \beta_{\mathfrak{D}_1}^{\mathfrak{X}}, \eta_{\mathfrak{D}_1}^{\mathfrak{X}}\rangle)$

(ix) $\mathfrak{S}_{\mathfrak{D}_1}^{\mathfrak{X}} = (\langle \mathfrak{m}_{\mathfrak{D}_1}^{\mathfrak{X}}, (1-(1-\mathfrak{a}_{\mathfrak{D}_1})^{\mathfrak{X}}, (1-(1-\mathfrak{n}_{\mathfrak{D}_1})^{\mathfrak{X}})\rangle, \langle \alpha_{\mathfrak{D}_1}^{\mathfrak{X}}, (1-(1-\beta_{\mathfrak{D}_1})^{\mathfrak{X}}, (1-(1-\mathfrak{n}_{\mathfrak{D}_1})^{\mathfrak{X}})\rangle)$

Example 1. In medicine, the combination of medications is for better therapy. Medicines are chemicals or substances that are used to treat, stop, or prevent disease, alleviate symptoms, or aid in the identification of disorders. Medical advancements have enabled doctors to heal numerous ailments and save lives. A combination drug, often known as a fixed-dose combination (FDC), is a medication that contains two or more active components in a single dosage form. Aspirin/paracetamol/caffeine, for example, is a combination medication used to relieve pain, particularly tension headaches and migraines. Let $G = \{\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3, \mathfrak{g}_4\}$ represent the combination of two life-saving medications. Two or more medications can be mixed in the formulation of medicine to increase its impact. If the reference or control parameter is taken into account, α = excellent impact against infection produced during surgeries; β = not high impact against infection produced during surgeries (uneffected or neutral); γ = no high impact against infection produced during surgeries.

The SLDF information can be taken as Table 1 for these reference or control parameters.

All cited mathematicians claim that these operations produce SLDFNs, and their constructed aggregation operators produce results for SLDFNs. Now, we check these operations for arbitrary SLDFNs. Let $\mathfrak{S}_{\mathfrak{D}_1} = (\langle 0.92, 0.56, 0.45\rangle, \langle 0.64, 0.26, 0.09\rangle)$ and $\mathfrak{S}_{\mathfrak{D}_2} = (\langle 0.85, 0.65, 0.58\rangle, \langle 0.43, 0.32, 0.21\rangle)$ be two SLDFNs, then

(i) $\mathfrak{S}_{\mathfrak{D}_1}^{c} = (\langle 0.45, 0.44, 0.92\rangle, \langle 0.09, 0.74, 0.64\rangle)$

(ii) $\mathfrak{S}_{\mathfrak{D}_2} \subseteq \mathfrak{S}_{\mathfrak{D}_1}$ by Definition 5 (iii)

(iii) $\mathfrak{S}_{\mathfrak{D}_1} \oplus \mathfrak{S}_{\mathfrak{D}_2} = (\langle 0.988, 0.364, 0.261\rangle, \langle 0.7948, 0.0832, 0.0189\rangle)$

(iv) $\mathfrak{S}_{\mathfrak{D}_1} \otimes \mathfrak{S}_{\mathfrak{D}_2} = (\langle 0.782, 0.846, 0.769\rangle, \langle 0.2752, 0.4968, 0.2811\rangle)$

Table 1: SLDFSs.

$\mathfrak{S}_{\mathfrak{D}}$	$(\langle \mathfrak{m}_{\mathfrak{D}}, \mathfrak{a}_{\mathfrak{D}}, \mathfrak{n}_{\mathfrak{D}}\rangle, \langle \alpha_{\mathfrak{D}}, \beta_{\mathfrak{D}}, \eta_{\mathfrak{D}}\rangle)$
$\mathfrak{g}_1$	$(\langle 0.92, 0.24, 0.41\rangle, \langle 0.54, 0.12, 0.11\rangle)$
$\mathfrak{g}_2$	$(\langle 0.51, 0.46, 0.81\rangle, \langle 0.38, 0.32, 0.11\rangle)$
$\mathfrak{g}_3$	$(\langle 0.73, 0.45, 0.53\rangle, \langle 0.34, 0.23, 0.21\rangle)$
$\mathfrak{g}_4$	$(\langle 0.72, 0.26, 0.74\rangle, \langle 0.29, 0.34, 0.21\rangle)$

(v) $\mathfrak{S}_{\mathfrak{D}_1} \cup \mathfrak{S}_{\mathfrak{D}_2} = (\langle 0.92, 0.56, 0.45\rangle, \langle 0.64, 0.26, 0.09\rangle) = \mathfrak{S}_{\mathfrak{D}_1}$

(vi) $\mathfrak{S}_{\mathfrak{D}_1} \cap \mathfrak{S}_{\mathfrak{D}_2} = (\langle 0.85, 0.65, 0.58\rangle, \langle 0.43, 0.32, 0.21\rangle) = \mathfrak{S}_{\mathfrak{D}_2}$

If $\mathfrak{X} = 0.3$, then we have the following:

(vii) $\mathfrak{X}\mathfrak{S}_{\mathfrak{D}_1} = (\langle 0.5313, 0.8403, 0.7870\rangle, \langle 0.2640, 0.6676, 0.4856\rangle)$

(viii) $\mathfrak{S}_{\mathfrak{D}_1}^{\mathfrak{X}} = (\langle 0.9753, 0.2183, 0.1642\rangle, \langle 0.8747, 0.0864, 0.0279\rangle)$

3. Methodology: SLDF-TOPSIS Algorithms

The selection of environmentally friendly vendors is seen as an MCDM problem. As an example, in order to solve the issue of TOPSIS, an MCDM approach is proposed. This section examines and summarizes the SLDFSs and the TOPSIS approach.

Algorithm 1 (SLDF-TOPSIS). There are several criteria in decision-making challenges. Certain of them are critical and required light of the topic at hand, while others are less so. So, one must choose the appropriate and significant criteria, which is done with the support of a professional viewpoint or according to the problem's requirements.

Step 1. Pinpoint the problem: $\mathfrak{E} = \{\mathfrak{e}_{\mathfrak{k}}\}$, the group of decision-makers/experts, the assemblage of alternatives/attributes is $\mathfrak{A} = \{\mathfrak{a}_{\mathfrak{i}}\}$ and $\mathfrak{C} = \{\mathfrak{c}_{\mathfrak{j}}\}$ is the family of parameters/criteria, where $\mathfrak{i}, \mathfrak{j}, \mathfrak{k} \in N$ and $\mathfrak{i} = \{1, 2, 3, \cdots, \mathfrak{p}\}$, $\mathfrak{j} = \{1, 2, 3, \cdots, \mathfrak{q}\}$, and $\mathfrak{k} = \{1, 2, 3, \cdots, \mathfrak{r}\}$.

The decision matrix is represented as

$$\mathscr{D} = [a_{\mathfrak{i}\mathfrak{j}}]_{\mathfrak{p}\times\mathfrak{q}} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1\mathfrak{q}} \\ a_{21} & a_{22} & \cdots & a_{2\mathfrak{q}} \\ \vdots & \vdots & \ddots & \vdots \\ a_{\mathfrak{p}1} & a_{\mathfrak{p}2} & \cdots & a_{\mathfrak{p}\mathfrak{q}} \end{pmatrix}. \quad (4)$$

Step 2. The rating of each alternative based on the associated criterion is created using spherical linear Diophantine fuzzy information for MADM and is presented in the decision matrix as

$$\mathfrak{D}_P = [a_{ij}]_{\mathfrak{p}\times\mathfrak{q}} = \left[\left(\langle \mathfrak{m}_{ij}, \mathfrak{a}_{ij}, \mathfrak{n}_{ij}\rangle, \langle \alpha_{ij}, \beta_{ij}, \eta_{ij}\rangle\right)\right]_{\mathfrak{p}\times\mathfrak{q}} = \begin{pmatrix} \langle \mathfrak{m}_{11}, \mathfrak{a}_{11}, \mathfrak{n}_{11}\rangle, \langle \alpha 11, \beta 11, \eta 11\rangle & \langle \mathfrak{m}_{12}, \mathfrak{a}_{12}, \mathfrak{n}_{12}\rangle, \langle \alpha_{12}, \beta_{12}, \eta_{12}\rangle & \cdots & \langle \mathfrak{m}_{1\mathfrak{q}}, \mathfrak{a}_{1\mathfrak{q}}, \mathfrak{n}_{1\mathfrak{q}}\rangle, \langle \alpha_{1\mathfrak{q}}, \beta_{1\mathfrak{q}}, \eta_{1\mathfrak{q}}\rangle \\ \langle \mathfrak{m}_{21}, \mathfrak{a}_{21}, \mathfrak{n}_{21}\rangle, \langle \alpha_{21}, \beta_{21}, \eta_{21}\rangle & \langle \mathfrak{m}_{22}, \mathfrak{a}_{22}, \mathfrak{n}_{22}\rangle, \langle \alpha_{22}, \beta_{22}, \eta_{22}\rangle & \cdots & \langle \mathfrak{m}_{2\mathfrak{q}}, \mathfrak{a}_{2\mathfrak{q}}, \mathfrak{n}_{2\mathfrak{q}}\rangle, \langle \alpha_{2\mathfrak{q}}, \beta_{2\mathfrak{q}}, \eta_{2\mathfrak{q}}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \mathfrak{m}_{\mathfrak{p}1}, \mathfrak{a}_{\mathfrak{p}1}, \mathfrak{n}_{\mathfrak{p}1}\rangle, \langle \alpha_{\mathfrak{p}1}, \beta_{\mathfrak{p}1}, \eta_{\mathfrak{p}1}\rangle & \langle \mathfrak{m}_{\mathfrak{p}2}, \mathfrak{a}_{\mathfrak{p}2}, \mathfrak{n}_{\mathfrak{p}2}\rangle, \langle \alpha_{\mathfrak{p}2}, \beta_{\mathfrak{p}2}, \eta_{\mathfrak{p}2}\rangle & \cdots & \langle \mathfrak{m}_{\mathfrak{p}\mathfrak{q}}, \mathfrak{a}_{\mathfrak{p}\mathfrak{q}}, \mathfrak{n}_{\mathfrak{p}\mathfrak{q}}\rangle, \langle \alpha_{\mathfrak{p}\mathfrak{q}}, \beta_{\mathfrak{p}\mathfrak{q}}, \eta_{\mathfrak{p}\mathfrak{q}}\rangle \end{pmatrix}. \tag{5}$$

In matrix $\mathfrak{D}_P$, the entries $\mathfrak{m}_{ij}, \mathfrak{a}_{ij}, \mathfrak{n}_{ij}, \alpha_{ij}, \beta_{ij}, \eta_{ij}$ represent membership, abstinence, nonmembership grades, and their reference parameters, respectively, where $\mathfrak{i} = \{1, 2, 3, \cdots, \mathfrak{p}\}$, $\mathfrak{j} = \{1, 2, 3, \cdots, \mathfrak{q}\}$. These grades satisfy the following properties under spherical linear Diophantine fuzzy environment:

(1) $0 \le \mathfrak{m}_{ij}, \mathfrak{a}_{ij}, \mathfrak{n}_{ij}, \alpha_{ij}, \beta_{ij}, \eta_{ij} \le 1$

(2) $0 \le \alpha_{ij} + \beta_{ij} + \eta_{ij} \le 1$

(3) $0 \le \alpha_{ij}\mathfrak{m}_{ij} + \beta_{ij}\mathfrak{a}_{ij} + \eta_{ij}\mathfrak{n}_{ij} \le 1$, where $\mathfrak{i} = 1, 2, 3, \cdots, \mathfrak{p}$, and $\mathfrak{j} = 1, 2, 3, \cdots, \mathfrak{q}$

If w_{ij} denotes the weight allocated by $\mathfrak{E}_\mathfrak{k}$ to C_j keeping into consideration the linguistic variables (LVs) (Table 2), construct weighted parameter matrix

$$\mathscr{P} = [w_{ij}]_{\mathfrak{p}\times\mathfrak{q}} = \begin{pmatrix} w_{11} & w_{12} & \cdots & w_{1\mathfrak{q}} \\ w_{21} & w_{22} & \cdots & w_{2\mathfrak{q}} \\ \vdots & \vdots & \ddots & \vdots \\ w_{\mathfrak{p}1} & w_{\mathfrak{p}2} & \cdots & w_{\mathfrak{p}\mathfrak{q}} \end{pmatrix}. \tag{6}$$

Step 3. Assume we have a group of "$\mathfrak{r}$" decision-makers, each with their own decision weights. As a result, each expert's choice and importance differ from one another. Assume that the SLD-LV takes into account the significance of each expert. Let $\mathfrak{e}_\mathfrak{k} = (\langle \mathfrak{m}_\mathfrak{k}, \mathfrak{a}_\mathfrak{k}, \mathfrak{n}_\mathfrak{k}\rangle, \langle \alpha_\mathfrak{k}, \beta_\mathfrak{k}, \eta_\mathfrak{k}\rangle)$, $\mathfrak{k} = 1, 2, 3, \cdots, \mathfrak{r}$ be SLDFNs for the assessment of "$\mathfrak{r}$" decision-makers, and the weights of $\mathfrak{k}$th decision-makers are given as follows:

$$\omega_\mathfrak{k} = \frac{1 - \sqrt{\left[(1 - \mathfrak{m}_\mathfrak{k}(\zeta))^2 + \mathfrak{a}_\mathfrak{k}(\zeta))^2 + \mathfrak{n}_\mathfrak{k}(\zeta))^2 + (1 - \alpha_\mathfrak{k}(\zeta))^2 + (\beta_\mathfrak{k}(\zeta))^2 + (\eta_\mathfrak{k}(\zeta))^2\right]/3}}{\sum_{\mathfrak{k}=1}^{\mathfrak{r}}\left[1 - \sqrt{\left[(1 - \mathfrak{m}_\mathfrak{k}(\zeta))^2 + \mathfrak{a}_\mathfrak{k}(\zeta))^2 + \mathfrak{n}_\mathfrak{k}(\zeta))^2 + (1 - \alpha_\mathfrak{k}(\zeta))^2 + (\beta_\mathfrak{k}(\zeta))^2 + \eta_\mathfrak{k}(\zeta))^2\right]/3}\right]}, \tag{7}$$

where $\mathfrak{k} = 1, 2, 3, \cdots, \mathfrak{r}$ and $\sum_{\mathfrak{k}=1}^{\mathfrak{r}}\omega_\mathfrak{k} = 1$.

Step 4. Construct SLDF decision matrix $\mathfrak{Z}_\mathfrak{i} = [\mathfrak{z}_{j\mathfrak{k}}^{\mathfrak{i}}]_{\mathfrak{p}\times\mathfrak{q}}$, where $\mathfrak{z}_{j\mathfrak{k}}^{\mathfrak{i}}$ is a SLDFS object, for $\mathfrak{i}$ expert. The aggregating matrix is $\mathfrak{A} = \mathfrak{Z}_1 + \mathfrak{Z}_2 + \cdots + \mathfrak{Z}_\mathfrak{n}/\mathfrak{n} = [\dot{\mathfrak{z}}_{j\mathfrak{k}}]_{\mathfrak{p}\times\mathfrak{q}}$, where

$$\begin{aligned} \dot{\mathfrak{z}}_{ij} &= \mathrm{SLDFWG}\left(\mathfrak{z}_{ij}{}^1, \mathfrak{z}_{ij}{}^2, \cdots, \mathfrak{z}_{ij}^{\mathfrak{k}}\right) \\ &= \omega_1\mathfrak{z}_{ij}{}^1 \otimes \omega_2\mathfrak{z}_{ij}{}^2 \otimes \cdots \otimes \omega_\mathfrak{k}\mathfrak{z}_{ij}^{\mathfrak{k}} \\ &= \left(\left\langle \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(\mathfrak{m}_{ij}{}^{\mathfrak{k}}\right)^{\omega_\mathfrak{k}}, 1 - \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(1 - \mathfrak{m}_{ij}{}^{\mathfrak{k}}\right)^{\omega_\mathfrak{k}}, 1 - \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(1 - \mathfrak{n}_{ij}{}^{\mathfrak{k}}\right)^{\omega_\mathfrak{k}}\right\rangle,\right. \\ &\quad \left.\cdot \left\langle \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(\alpha_{ij}{}^{\mathfrak{k}}\right)^{\omega_\mathfrak{k}}, 1 - \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(1 - \beta_{ij}{}^{\mathfrak{k}}\right)^{\omega_\mathfrak{k}}, 1 - \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(1 - \eta_{ij}{}^{\mathfrak{k}}\right)^{\omega_\mathfrak{k}}\right\rangle\right). \end{aligned} \tag{8}$$

Hence, we get aggregated decision matrix for $\mathfrak{i} = 1, 2, 3, \cdots, \mathfrak{p}$, $\mathfrak{j} = 1, 2, 3, \cdots, \mathfrak{q}$ and $\mathfrak{k} = 1, 2, 3, \cdots, \mathfrak{r}$.

Step 5. Acquire the weighted SLDF decision matrix

$$\mathfrak{I} = [\check{\mathfrak{z}}_{j\mathfrak{k}}]_{\mathfrak{l}\times\mathfrak{q}} = \begin{pmatrix} \check{\mathfrak{z}}_{11} & \check{\mathfrak{z}}_{12} & \cdots & \check{\mathfrak{z}}_{1\mathfrak{q}} \\ \check{\mathfrak{z}}_{21} & \check{\mathfrak{z}}_{22} & \cdots & \check{\mathfrak{z}}_{2\mathfrak{q}} \\ \vdots & \vdots & \ddots & \vdots \\ \check{\mathfrak{z}}_{\mathfrak{p}1} & \check{\mathfrak{z}}_{\mathfrak{p}2} & \cdots & \check{\mathfrak{z}}_{\mathfrak{p}\mathfrak{q}} \end{pmatrix}, \tag{9}$$

where $\check{\mathfrak{z}}_\mathfrak{k} = \xi_\mathfrak{k} \times \dot{\mathfrak{z}}_{j\mathfrak{k}}$

$$\begin{aligned} \check{\mathfrak{z}}_\mathfrak{k} &= \mathrm{SLDFWG}\left(\mathfrak{z}_{ij}{}^1, \mathfrak{z}_{ij}{}^2, \cdots, \mathfrak{z}_{ij}^{\mathfrak{k}}\right) = \xi_1\mathfrak{z}_{ij}{}^1 \otimes \xi_2\mathfrak{z}_{ij}{}^2 \otimes \cdots \otimes \xi_\mathfrak{k}\mathfrak{z}_{ij}^{\mathfrak{k}} \\ &= \left(\left\langle \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(\mathfrak{m}_{ij}{}^{\mathfrak{k}}\right)^{\xi_\mathfrak{k}}, 1 - \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(1 - \mathfrak{m}_{ij}{}^{\mathfrak{k}}\right)^{\xi_\mathfrak{k}}, 1 - \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(1 - \mathfrak{n}_{ij}{}^{\mathfrak{k}}\right)^{\xi_\mathfrak{k}}\right\rangle,\right. \\ &\quad \left.\cdot \left\langle \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(\alpha_{ij}{}^{\mathfrak{k}}\right)^{\xi_\mathfrak{k}}, 1 - \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(1 - \beta_{ij}{}^{\mathfrak{k}}\right)^{\xi_\mathfrak{k}}, 1 - \Pi_{\mathfrak{k}=1}^{\mathfrak{r}}\left(1 - \eta_{ij}{}^{\mathfrak{k}}\right)^{\xi_\mathfrak{k}}\right\rangle\right). \end{aligned} \tag{10}$$

Table 2: LTs for significant criteria weights.

LVs	SLDFN
Certainly low importance (CLI)	$(\langle 0.00,0.00,0.94\rangle, \langle 0.05,0.02,0.85\rangle)$
Very low importance (VLI)	$(\langle 0.10,0.20,0.90\rangle, \langle 0.12,0.14,0.73\rangle)$
Low importance (LI)	$(\langle 0.35,0.31,0.80\rangle, \langle 0.11,0.23,0.65\rangle)$
Below average importance (BAI)	$(\langle 0.35,0.45,0.75\rangle, \langle 0.17,0.25,0.57\rangle)$
Average importance (AI)	$(\langle 0.43,0.56,0.48\rangle, \langle 0.19,0.28,0.51\rangle)$
Above average importance (AAI)	$(\langle 0.55,0.65,0.35\rangle, \langle 0.31,0.33,0.36\rangle)$
High importance (HI)	$(\langle 0.65,0.80,0.20\rangle, \langle 0.20,0.35,0.35\rangle)$
Very high importance (VHI)	$(\langle 0.80,0.90,0.10\rangle, \langle 0.40,0.37,0.21\rangle)$
Certainly high importance (CHI)	$(\langle 0.90,1.00,0.03\rangle, \langle 0.52,0.39,0.05\rangle)$
Exactly equal (EE)	$(\langle 0.96,0.95,0.19\rangle, \langle 0.79,0.10,0.02\rangle)$

Step 6. The weights of the criterion and the aggregated decision matrix are utilized to create the aggregated weighted SLDF decision matrix in this phase. This matrix may be produced and assessed as follows:

$$\check{\mathfrak{z}}_{\mathfrak{k}} \otimes \dot{\mathfrak{z}}_{\mathfrak{i}\mathfrak{j}} = {}^{\check{\mathfrak{z}}_{\mathfrak{k}}}\dot{\mathfrak{z}}_{\mathfrak{i}\mathfrak{j}} = \mathfrak{Z}_{\mathfrak{i}} = \left[\dot{\mathfrak{z}}^{\mathfrak{i}}_{\mathfrak{j}\mathfrak{k}}\right]_{\mathfrak{p}\times\mathfrak{q}}. \tag{11}$$

Here, each entry is SLDFN and $\mathfrak{i} = 1, 2, \cdots, \mathfrak{p}$, $\mathfrak{j} = 1, 2, \cdots, \mathfrak{q}$.

Step 7. Locate SLDF-valued positive ideal solution (SLDFV-PIS) and SLDF-valued negative ideal solution (SLDFV-NIS), confined in a form

$$\begin{aligned}\mathfrak{s}^+_{\mathfrak{j}} &= \left\{\ddot{\rho}^+_1, \ddot{\rho}^+_2, \cdots, \ddot{\rho}^+_{\mathfrak{q}}\right\} \\ &= \left\{\left\langle \vee^{\mathfrak{i}}\mathfrak{m}_{\mathfrak{i}\mathfrak{j}}, \wedge^{\mathfrak{i}}\mathfrak{a}_{\mathfrak{i}\mathfrak{j}}, \wedge^{\mathfrak{i}}\mathfrak{n}_{\mathfrak{i}\mathfrak{j}}\right\rangle, \left\langle \vee^{\mathfrak{i}}\alpha_{\mathfrak{i}\mathfrak{j}}, \wedge^{\mathfrak{i}}\beta_{\mathfrak{i}\mathfrak{j}}, \wedge^{\mathfrak{i}}\gamma_{\mathfrak{i}\mathfrak{j}}\right\rangle\right\},\end{aligned} \tag{12}$$

$$\begin{aligned}\mathfrak{s}^-_{\mathfrak{j}} &= \left\{\ddot{\rho}^-_1, \ddot{\rho}^-_2, \cdots, \ddot{\rho}^-_{\mathfrak{q}}\right\} \\ &= \left\{\left\langle \wedge^{\mathfrak{i}}\mathfrak{m}_{\mathfrak{i}\mathfrak{j}}, \vee^{\mathfrak{i}}\mathfrak{a}_{\mathfrak{i}\mathfrak{j}}, \vee^{\mathfrak{i}}\mathfrak{n}_{\mathfrak{i}\mathfrak{j}}\right\rangle, \left\langle \wedge^{\mathfrak{i}}\alpha_{\mathfrak{i}\mathfrak{j}}, \vee^{\mathfrak{i}}\beta_{\mathfrak{i}\mathfrak{j}}, \vee^{\mathfrak{i}}\gamma_{\mathfrak{i}\mathfrak{j}}\right\rangle\right\}.\end{aligned} \tag{13}$$

Step 8. The normalized Euclidean distance (NED) of each alternative and its SLDFNIS can be defined as

$$\begin{aligned}\mathfrak{D}^{\mathfrak{N}+}_{\mathfrak{E}} = \frac{1}{6\mathfrak{n}}\sum_{\mathfrak{j}=1}^{\mathfrak{q}}\Big\{&\left({}^{\mathfrak{i}}\mathfrak{m}_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\mathfrak{m}_{\mathfrak{j}}{}^{+}\right)^2 + \left({}^{\mathfrak{i}}\mathfrak{a}_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\mathfrak{a}_{\mathfrak{j}}{}^{+}\right)^2 \\ &+ \left({}^{\mathfrak{i}}\mathfrak{n}_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\mathfrak{n}_{\mathfrak{j}}{}^{+}\right)^2 + \left({}^{\mathfrak{i}}\alpha_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\alpha_{\mathfrak{j}}{}^{+}\right)^2 + \left({}^{\mathfrak{i}}\beta_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\beta_{\mathfrak{j}}{}^{+}\right)^2 \\ &+ \left({}^{\mathfrak{i}}\gamma_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\gamma_{\mathfrak{j}}{}^{+}\right)^2\Big\}.\end{aligned} \tag{14}$$

The normalized Euclidean distance (NED) of each alternative and its SLDFPIS can be defined as

$$\begin{aligned}\mathfrak{D}^{\mathfrak{N}-}_{\mathfrak{E}} = \frac{1}{6\mathfrak{n}}\sum_{\mathfrak{j}=1}^{\mathfrak{q}}\Big\{&\left({}^{\mathfrak{i}}\mathfrak{m}_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\mathfrak{m}_{\mathfrak{j}}{}^{-}\right)^2 + \left({}^{\mathfrak{i}}\mathfrak{a}_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\mathfrak{a}_{\mathfrak{j}}{}^{-}\right)^2 \\ &+ \left({}^{\mathfrak{i}}\mathfrak{n}_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\mathfrak{n}_{\mathfrak{j}}{}^{-}\right)^2 + \left({}^{\mathfrak{i}}\alpha_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\alpha_{\mathfrak{j}}{}^{-}\right)^2 + \left({}^{\mathfrak{i}}\beta_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\beta_{\mathfrak{j}}{}^{-}\right)^2 \\ &+ \left({}^{\mathfrak{i}}\gamma_{\mathfrak{i}\mathfrak{j}} - {}^{\mathfrak{i}}\gamma_{\mathfrak{j}}{}^{-}\right)^2\Big\}.\end{aligned} \tag{15}$$

Step 9. Compute the SLDF relative closeness with the formula

$$\mathfrak{C}^+_{\mathfrak{j}} = \frac{\mathfrak{D}^{\mathfrak{N}-}_{\mathfrak{E}}}{\mathfrak{D}^{\mathfrak{N}+}_{\mathfrak{E}} + \mathfrak{D}^{\mathfrak{N}-}_{\mathfrak{E}}}. \tag{16}$$

Step 10. Finally, the most advantageous ranking procedure of the choices is chosen. The ideal scenario is the one with the greatest updated coefficient value.

The proposed SLDF-TOPSIS is expressed as a flow diagram in Figure 1.

4. Numerical Example

4.1. Case Study. Engenderment systems have advanced to incipient heights in tandem with advances in information technology. As a result of ecumenical competition, businesses have transmuted their core capabilities, ameliorated their subsisting environment, and developed incipient business models for themselves and their stakeholders. In this component, the recommended GSS technique is implemented by the management of an agricultural implement company (represented as ABC) in Turkey. The managers' goal is to assess supplier performance by investigating the priority ranking of the GSS criteria from the industrial expert platform. The company that primarily manufactures lawnmowers expands its product line without jeopardizing its reputation for excellence and perpetuated innovation, and it wishes to implement its environmental management system throughout the supply chain, which includes cooperation activities with all suppliers in accordance with industrial practices. GSS has been apperceived as a required decision-making activity for the ABC firm in light of this circumstance.

4.2. Problem Description. ABC Corporation wants to select the finest green supplier in light of industrial fundamentals. A committee of 4 experts was formed to evaluate vendors. Face-to-face interviews were used to obtain data. Four decision-makers/experts from various areas of the firm have been requested to offer input on the suggested method, symbolised by $\mathfrak{E} = \{\mathfrak{e}_1, \mathfrak{e}_2, \mathfrak{e}_3, \mathfrak{e}_4\}$. It has been concluded that the needed part can be obtained from one of five suppliers defined by $\mathfrak{A} = \{\mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{a}_3, \mathfrak{a}_4, \mathfrak{a}_5\}$. To pick green suppliers and construct a decision-making tool in the evaluation, the required criteria must first be appropriately

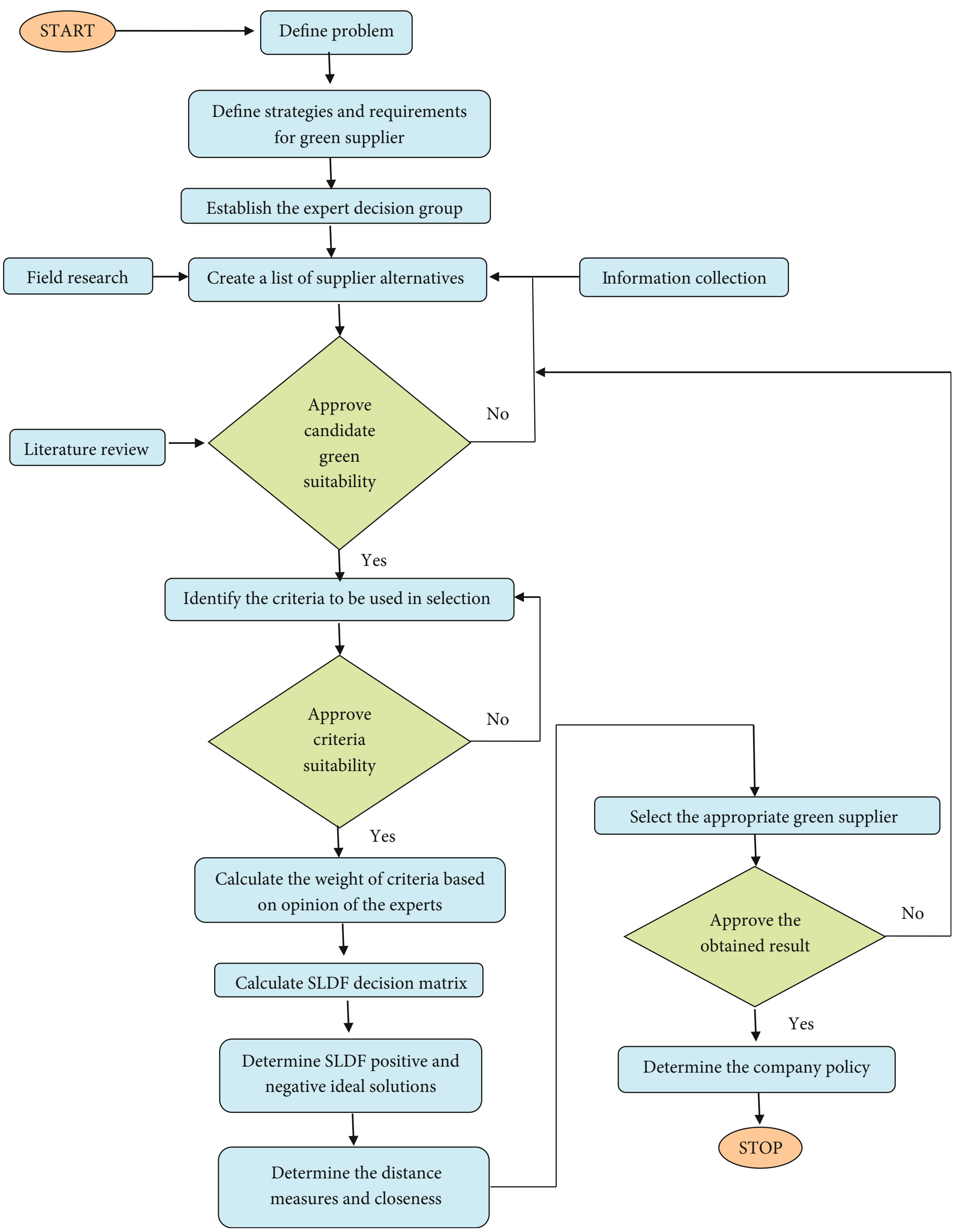

Figure 1: Flow diagram of the proposed algorithm.

identified. The six most significant and prevalent environmental factors are delivery performance, pollution control, production, quality service, environmental representation, and technology capability. The choice of criteria $\mathfrak{C} = \{\mathfrak{c}_1, \mathfrak{c}_2, \mathfrak{c}_3, \mathfrak{c}_4, \mathfrak{c}_5, \mathfrak{c}_6\}$, where the criteria are $\mathfrak{c}_1 =$ delivery performance, $\mathfrak{c}_2 =$ pollution control, $\mathfrak{c}_3 =$ production, $\mathfrak{c}_4 =$ quality service, $\mathfrak{c}_5 =$ environmental representation, and $\mathfrak{c}_6 =$ technology capability.

4.3. Application of SLDF-TOPSIS to GSS. We apply Algorithm 1 (SLDF-TOPSIS) in this example as follows.

Step 1. Let $\mathfrak{E} = \{\mathfrak{e}_\mathfrak{k} : \mathfrak{k} = 1, 2, 3, 4\}$ be the team comprising experts, $\mathfrak{A} = \{\mathfrak{a}_\mathfrak{i} : \mathfrak{i} = 1, 2, 3, \cdots, 5\}$ the set of alternatives covered by the study, and $\mathfrak{C} = \{\mathfrak{c}_\mathfrak{j} : \mathfrak{j} = 1, 2, \cdots, 6\}$ the family of criteria. The importance of each decision-maker is

Table 3: Expert's importance in terms of SLDFNs.

Expert	Linguistic variables	SLDF numbers
e_1	Very good (VG)	$(\langle 0.96,0.15,0.12\rangle, \langle 0.65,0.10,0.21\rangle)$
e_2	Good (G)	$(\langle 0.90,0.20,0.30\rangle, \langle 0.62,0.13,0.22\rangle)$
e_3	Medium good (MG)	$(\langle 0.85,0.33,0.41\rangle, \langle 0.57,0.17,0.25\rangle)$
e_4	Fair (F)	$(\langle 0.72,0.45,0.53\rangle, \langle 0.47,0.19,0.31\rangle)$

Table 4: Judgments/assessments given by four experts in terms of LTs.

Alternatives	Experts	Criteria c_1	c_2	c_3	c_4	c_5	c_6
a_1	e_1	CHI	AAI	HI	BAI	LI	EE
	e_2	AI	VHI	EE	CHI	AAI	HI
	e_3	EE	AI	AAI	LI	HI	VHI
	e_4	CHI	EE	VHI	AI	HI	AI
a_2	e_1	LI	HI	CHI	VHI	EE	AI
	e_2	AAI	HI	CHI	VHI	HI	EE
	e_3	LI	VHI	BAI	HI	AI	AAI
	e_4	AAI	VHI	EE	CHI	VHI	EE
a_3	e_1	CHI	AAI	EE	AI	LI	VHI
	e_2	EE	BAI	CHI	HI	AAI	VHI
	e_3	LI	AI	AAI	HI	EE	HI
	e_4	LI	EE	VHI	HI	AAI	CHI
a_4	e_1	EE	AAI	CHI	AI	VHI	AAI
	e_2	AI	EE	HI	HI	LI	VHI
	e_3	HI	EE	BAI	CHI	AAI	EE
	e_4	CHI	LI	VHI	AAI	EE	AI
a_5	e_1	LI	AAI	VHI	EE	HI	CHI
	e_2	HI	AAI	AI	VHI	EE	HI
	e_3	EE	LI	HI	CHI	AAI	VHI
	e_4	CHI	AAI	HI	VHI	EE	AI
Weights	e_1	G	F	MG	G	VG	VG
	e_2	G	VG	VG	F	G	G
	e_3	VG	F	G	G	VG	F
	e_4	G	G	VG	F	G	VG

expressed by LT, and each LT is an SLDFN shown in Table 2, where we have different types of level of importance as low importance (LI), average importance (AI), and high importance (VI).

Step 2. The decision power of experts is considered "linguistic terms (LTs)." Each expert has different opinions about LTs. First, we define some LTs for rating criteria and experts as shown in Table 3. For example, for the LT "very good/very important (VG/VI)," we have numeric value $\langle 0,96,0.15,0.12\rangle, \langle 0.65,0.10,0.21\rangle$ as a SLDFN. If an expert assign this LT to a criteria for a decision-making problem, then it means that criteria have 90% satisfaction value, 15% indeterminacy value, and 12% of dissatisfaction

Table 5: Aggregated SLDF decision table.

Alternatives	Criteria	SLDFNs
a_1	c_1	$(\langle 0.7451,1.0000,0.2175\rangle, \langle 0.4345,0.3000,0.2030\rangle)$
	c_2	$(\langle 0.6365,0.8160,0.2979\rangle, \langle 0.3509,0.2934,0.3121\rangle)$
	c_3	$(\langle 0.7226,0.8627,0.2192\rangle, \langle 0.3674,0.2877,0.2485\rangle)$
	c_4	$(\langle 0.4716,1.0000,0.6062\rangle, \langle 0.2132,0.2924,0.4778\rangle)$
	c_5	$(\langle 0.5132,0.6584,0.5066\rangle, \langle 0.1879,0.3095,0.4649\rangle)$
	c_6	$(\langle 0.7143,0.8721,0.2360\rangle, \langle 0.3560,0.2736,0.2662\rangle)$
a_2	c_1	$(\langle 0.4303,0.4939,0.6573\rangle, \langle 0.1766,0.2774,0.5388\rangle)$
	c_2	$(\langle 0.7087,0.8501,0.1598\rangle, \langle 0.2669,0.3584,0.2950\rangle)$
	c_3	$(\langle 0.7288,1.0000,0.3180\rangle, \langle 0.4308,0.3130,0.2075\rangle)$
	c_4	$(\langle 0.7782,1.0000,0.1127\rangle, \langle 0.3562,0.3690,0.2200\rangle)$
	c_5	$(\langle 0.6901,0.8612,0.2589\rangle, \langle 0.3416,0.2682,0.2854\rangle)$
	c_6	$(\langle 0.6577,0.8456,0.3289\rangle, \langle 0.4089,0.2161,0.2837\rangle)$
a_3	c_1	$(\langle 0.6184,1.0000,0.5217\rangle, \langle 0.3058,0.2516,0.3676\rangle)$
	c_2	$(\langle 0.5065,0.7053,0.5073\rangle, \langle 0.2770,0.2585,0.4186\rangle)$
	c_3	$(\langle 0.8002,1.0000,0.1761\rangle, \langle 0.4993,0.2930,0.1548\rangle)$
	c_4	$(\langle 0.5725,0.7451,0.2992\rangle, \langle 0.1969,0.3292,0.4040\rangle)$
	c_5	$(\langle 0.5459,0.7275,0.5234\rangle, \langle 0.2811,0.2503,0.4121\rangle)$
	c_6	$(\langle 0.7782,1.0000,0.1127\rangle, \langle 0.3562,0.3690,0.2200\rangle)$
a_4	c_1	$(\langle 0.6932,1.0000,0.2619\rangle, \langle 0.3574,0.2694,0.2696\rangle)$
	c_2	$(\langle 0.6743,0.8540,0.4118\rangle, \langle 0.4153,0.2008,0.2860\rangle)$
	c_3	$(\langle 0.6445,1.0000,0.3409\rangle, \langle 0.2926,0.3444,0.3136\rangle)$
	c_4	$(\langle 0.5998,1.0000,0.2936\rangle, \langle 0.2670,0.3356,0.3501\rangle)$
	c_5	$(\langle 0.6023,0.7977,0.4603\rangle, \langle 0.2981,0.2794,0.3759\rangle)$
	c_6	$(\langle 0.6655,0.8369,0.2805\rangle, \langle 0.3797,0.2846,0.2852\rangle)$
a_5	c_1	$(\langle 0.6248,1.0000,0.4574\rangle, \langle 0.2734,0.2690,0.3660\rangle)$
	c_2	$(\langle 0.4944,0.5892,0.5077\rangle, \langle 0.2428,0.3076,0.4449\rangle)$
	c_3	$(\langle 0.6181,0.7990,0.2636\rangle, \langle 0.2440,0.3377,0.3617\rangle)$
	c_4	$(\langle 0.8699,1.0000,0.1131\rangle, \langle 0.5246,0.3023,0.1184\rangle)$
	c_5	$(\langle 0.7468,0.8788,0.2339\rangle, \langle 0.4154,0.2404,0.2187\rangle)$
	c_6	$(\langle 0.7003,1.0000,0.1925\rangle, \langle 0.3130,0.3554,0.2732\rangle)$

Table 6: Aggregated SLDF criteria weight table.

Alternatives	Criteria	SLDFNs
w	c_1	$(\langle 0.9138,0.1885,0.2939\rangle, \langle 0.6270,0.1230,0.2176\rangle)$
	c_2	$(\langle 0.8116,0.3363,0.4259\rangle, \langle 0.5404,0.1552,0.2677\rangle)$
	c_3	$(\langle 0.9108,0.2211,0.2745\rangle, \langle 0.6174,0.1291,0.2248\rangle)$
	c_4	$(\langle 0.8019,0.3534,0.4907\rangle, \langle 0.5356,0.1672,0.2693\rangle)$
	c_5	$(\langle 0.9321,0.1732,0.2424\rangle, \langle 0.6361,0.1138,0.2146\rangle)$
	c_6	$(\langle 0.8812,0.2457,0.2990\rangle, \langle 0.5944,0.1303,0.2375\rangle)$

Table 7: Aggregated weighted SLDF decision table.

Alternatives	Criteria	SLDFNs
$\mathfrak{a}_1$	$\mathfrak{c}_1$	$(\langle 0.6809,1.0000,0.4475\rangle, \langle 0.2724,0.3861,0.3764\rangle)$
	$\mathfrak{c}_2$	$(\langle 0.5166,0.8779,0.5969\rangle, \langle 0.1896,0.4031,0.4963\rangle)$
	$\mathfrak{c}_3$	$(\langle 0.6581,0.8931,0.4335\rangle, \langle 0.2268,0.3797,0.4174\rangle)$
	$\mathfrak{c}_4$	$(\langle 0.3782,1.0000,0.7994\rangle, \langle 0.1142,0.4107,0.6184\rangle)$
	$\mathfrak{c}_5$	$(\langle 0.4784,0.7176,0.6262\rangle, \langle 0.1195,0.3881,0.5797\rangle)$
	$\mathfrak{c}_6$	$(\langle 0.6294,0.9035,0.4644\rangle, \langle 0.2116,0.3682,0.4405\rangle)$
$\mathfrak{a}_2$	$\mathfrak{c}_1$	$(\langle 0.3932,0.5893,0.7580\rangle, \langle 0.1107,0.3663,0.6392\rangle)$
	$\mathfrak{c}_2$	$(\langle 0.5752,0.9005,0.5176\rangle, \langle 0.1442,0.4580,0.4837\rangle)$
	$\mathfrak{c}_3$	$(\langle 0.6638,1.0000,0.5052\rangle, \langle 0.2660,0.4017,0.3857\rangle)$
	$\mathfrak{c}_4$	$(\langle 0.6240,1.0000,0.5481\rangle, \langle 0.1908,0.4745,0.4301\rangle)$
	$\mathfrak{c}_5$	$(\langle 0.6432,0.8852,0.4385\rangle, \langle 0.2173,0.3515,0.4388\rangle)$
	$\mathfrak{c}_6$	$(\langle 0.5796,0.8835,0.5296\rangle, \langle 0.2431,0.3182,0.4538\rangle)$
$\mathfrak{a}_3$	$\mathfrak{c}_1$	$(\langle 0.5651,1.0000,0.6623\rangle, \langle 0.1917,0.3437,0.5052\rangle)$
	$\mathfrak{c}_2$	$(\langle 0.4111,0.8044,0.7171\rangle, \langle 0.1497,0.3736,0.5742\rangle)$
	$\mathfrak{c}_3$	$(\langle 0.7288,1.0000,0.4023\rangle, \langle 0.3083,0.3843,0.3448\rangle)$
	$\mathfrak{c}_4$	$(\langle 0.4591,0.8352,0.6431\rangle, \langle 0.1055,0.4414,0.5645\rangle)$
	$\mathfrak{c}_5$	$(\langle 0.5088,0.7747,0.6389\rangle, \langle 0.1788,0.3356,0.5383\rangle)$
	$\mathfrak{c}_6$	$(\langle 0.6857,1.0000,0.3780\rangle, \langle 0.2117,0.4512,0.4053\rangle)$
$\mathfrak{a}_4$	$\mathfrak{c}_1$	$(\langle 0.6334,1.0000,0.4788\rangle, \langle 0.2241,0.3593,0.4285\rangle)$
	$\mathfrak{c}_2$	$(\langle 0.5473,0.9031,0.6623\rangle, \langle 0.2244,0.3248,0.4771\rangle)$
	$\mathfrak{c}_3$	$(\langle 0.5870,1.0000,0.5218\rangle, \langle 0.1807,0.4290,0.4679\rangle)$
	$\mathfrak{c}_4$	$(\langle 0.4810,1.0000,0.6402\rangle, \langle 0.1430,0.4467,0.5251\rangle)$
	$\mathfrak{c}_5$	$(\langle 0.5614,0.8327,0.5911\rangle, \langle 0.1896,0.3614,0.5098\rangle)$
	$\mathfrak{c}_6$	$(\langle 0.5864,0.8770,0.4956\rangle, \langle 0.2257,0.3778,0.4550\rangle)$
$\mathfrak{a}_5$	$\mathfrak{c}_1$	$(\langle 0.5709,1.0000,0.6169\rangle, \langle 0.1714,0.3589,0.5040\rangle)$
	$\mathfrak{c}_2$	$(\langle 0.4013,0.7274,0.7174\rangle, \langle 0.1312,0.4151,0.5935\rangle)$
	$\mathfrak{c}_3$	$(\langle 0.5630,0.8434,0.4657\rangle, \langle 0.1506,0.4232,0.5052\rangle)$
	$\mathfrak{c}_4$	$(\langle 0.6976,1.0000,0.5483\rangle, \langle 0.2810,0.4190,0.3558\rangle)$
	$\mathfrak{c}_5$	$(\langle 0.6961,0.8998,0.4196\rangle, \langle 0.2642,0.3268,0.3864\rangle)$
	$\mathfrak{c}_6$	$(\langle 0.6171,1.0000,0.4339\rangle, \langle 0.1860,0.4394,0.4458\rangle)$

value. For the assigned LT, the reference parameters have 65% satisfaction value and 10% indeterminacy value and 21% dissatisfaction value corresponding to the three grades, and these reference parameters control the behavior of membership, abstinence, and nonmembership grades.

Step 3. Consider we have four decision-makers or experts in the selection committee, and all experts have different opinions. The weight of each decision maker is calculated by utilizing equation (7) as follows: $\omega_1 = [1 - \sqrt{0.2151/3}/4 - \sqrt{0.2151/3} - \sqrt{0.3497/3} - \sqrt{0.5758/3} - \sqrt{0.9749/3}] = 0.3073$, and similarly, $\omega_2 = 0.2764$, $\omega_3 = 0.2358$, $\omega_4 = 0.1804$. Hence, the weight vector of four experts is $\omega = (0.3073, 0.2764, 0.2358, 0.1804)^T$ with the constraint $\sum_{\mathfrak{k}=1}^{4} \omega_{\mathfrak{k}} = 1$.

Table 8: Relative SLDF: PIS and NIS.

Alternatives	Criteria	SLDFNs
$\mathfrak{S}^+_{\mathfrak{j}}$	$\mathfrak{c}_1$	$(\langle 0.6809,0.5893,0.4475\rangle, \langle 0.2724,0.3437,0.3764\rangle)$
	$\mathfrak{c}_2$	$(\langle 0.5752,0.7274,0.5176\rangle, \langle 0.2244,0.3248,0.4771\rangle)$
	$\mathfrak{c}_3$	$(\langle 0.7288,0.8434,0.4023\rangle, \langle 0.3083,0.3797,0.3448\rangle)$
	$\mathfrak{c}_4$	$(\langle 0.6976,0.8352,0.5481\rangle, \langle 0.2810,0.4107,0.3558\rangle)$
	$\mathfrak{c}_5$	$(\langle 0.6961,0.7176,0.4196\rangle, \langle 0.2642,0.3268,0.3864\rangle)$
	$\mathfrak{c}_6$	$(\langle 0.6857,0.8770,0.3780\rangle, \langle 0.2431,0.3182,0.4053\rangle)$
$\mathfrak{S}^-_{\mathfrak{j}}$	$\mathfrak{c}_1$	$(\langle 0.3932,1.0000,0.7580\rangle, \langle 0.1107,0.3861,0.6392\rangle)$
	$\mathfrak{c}_2$	$(\langle 0.4013,0.9031,0.7174\rangle, \langle 0.1312,0.4580,0.5935\rangle)$
	$\mathfrak{c}_3$	$(\langle 0.5630,1.0000,0.5218\rangle, \langle 0.1506,0.4290,0.5052\rangle)$
	$\mathfrak{c}_4$	$(\langle 0.3782,1.0000,0.7994\rangle, \langle 0.1055,0.4745,0.6184\rangle)$
	$\mathfrak{c}_5$	$(\langle 0.4784,0.8998,0.6389\rangle, \langle 0.1195,0.3881,0.5797\rangle)$
	$\mathfrak{c}_6$	$(\langle 0.5796,1.0000,0.5296\rangle, \langle 0.1860,0.4512,0.4550\rangle)$

Table 9: Distance measure of each alternative.

Alternatives	$\mathfrak{D}^{\mathfrak{N}+}_{\mathfrak{C}}$	$\mathfrak{D}^{\mathfrak{N}-}_{\mathfrak{C}}$
$\mathfrak{a}_1$	0.0191	0.0117
$\mathfrak{a}_2$	0.0138	0.0161
$\mathfrak{a}_3$	0.0183	0.0091
$\mathfrak{a}_4$	0.0158	0.0097
$\mathfrak{a}_5$	0.0145	0.0170

Table 10: SLDF closeness coefficient of each alternative.

Alternatives	$\mathfrak{C}^+_{\mathfrak{j}}$	Rank
$\mathfrak{a}_1$	0.3796	4
$\mathfrak{a}_2$	0.5391	1
$\mathfrak{a}_3$	0.3330	5
$\mathfrak{a}_4$	0.3804	3
$\mathfrak{a}_5$	0.5385	2

Step 4. Table 2 contains the LT as well as the SLDFNs. These LTs are used to rate the alternatives based on the supplied criteria. The assessment values of each alternative $\mathfrak{A}_{\mathfrak{i}}$: $\mathfrak{i} = \{1, 2, 3, \cdots, \mathfrak{p} = 5\}$ according to each criteria $\mathfrak{C}_{\mathfrak{j}}$: $\mathfrak{j} = \{1, 2, 3, \cdots, \mathfrak{q} = 6\}$ are provided by four experts given in Table 4, and it is based on SLDFNs. For alternative $\mathfrak{A}_{\mathfrak{i}}$, $\mathfrak{i} = 1, 2, 3, 4, 5$, and criteria $\mathfrak{C}_{\mathfrak{j}}$, $\mathfrak{j} = 1, 2, 3, 4, 5, 6$, we have the following LTs according to these four decision-makers. For the alternative $\mathfrak{A}_{\mathfrak{i}}$ and criteria $\mathfrak{C}_{\mathfrak{j}}$, we have four LTs for four decision-makers: HI, VHI, CHI, and EE, respectively. We have six SLDFNs corresponding to six criteria. We now apply equation (8) to do further input data computations. We apply equation (8) to Table 4 to obtain the aggregated SLDF decision table shown in Table 5. For the alternative $\mathfrak{a}_1$ and for criteria $\mathfrak{c}_1$, we will use the LTs "MG, G, MG, and F"

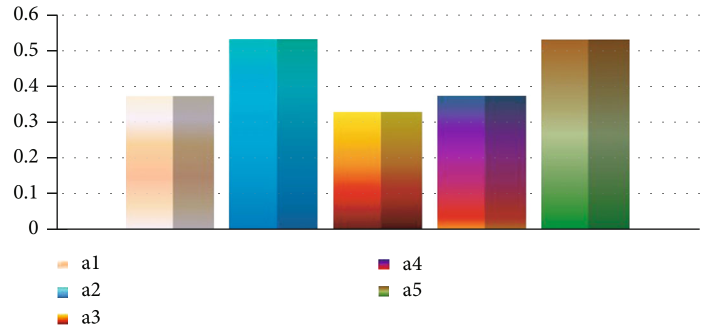

Figure 2: Priority order of the green supplier.

from Table 3. We use the weight vector $\omega = (0.3073, 0.2764, 0.2358, 0.1804)^T$ calculated in Step 3 and calculate the aggregated value by using equation (8) given in Table 4.

Step 5. We compute the aggregated weights for the six criteria listed in Table 6 using equation (10). Because equation (10) is the same aggregated formula as equation (8), we use it to aggregate weights for six criteria, and all of the computations are the same as in Step 4.

Step 6. We now assess the aggregated SLDF decision table using aggregated weights and alternative ratings. Using equation (11), we get Table 7. Table 7 is obtained by multiplying Table 5 with Table 6. This multiplication is done according to the definition of multiplication of two SLDFNs. The first SLDFN for option $\mathfrak{a}_1$ and criteria $\mathfrak{c}_1$ is $(\langle 0.7451, 1.0000, 0.2175\rangle, \langle 0.4345, 0.3000, 0.2030\rangle)$ shown in Table 5, while the second SLDFN for criteria $\mathfrak{c}_1$ is $(\langle 0.9138, 0.1885, 0.2939\rangle, \langle 0.6270, 0.1230, 0.2176\rangle)$ shown in Table 6. We acquire an SLDFN as an aggregated decision SLDFN $(\langle 0.6809, 1.0000, 0.4475\rangle, \langle 0.2724, 0.3861, 0.3764\rangle)$ for option $\mathfrak{a}_1$ and criteria $\mathfrak{c}_1$ by multiplying both SLDFNs. The same approach may be used to compute all other values.

Step 7. Now, using equations (12) and (13) in Table 7, we determine Table 8 SLDF relative PIS and NIS for the calculated aggregated weight. The mathematical methods for selecting MG, AG, and NMG from the aggregated decision Table 7 are briefly described in equations (12) and (13).

Step 8. Let us calculate the distance measure by using the proposed algorithm from relative PIS and relative NIS provided in Tables 7 and 8. For positive ideal solution, we find the normalized Euclidean distance between six criteria of the alternative $\mathfrak{a}_1$ given in Table 7 and $\mathfrak{S}_j^+$ given in Table 8. The value $\mathfrak{D}_{\mathfrak{E}}^{\mathfrak{N}+}$ for alternative $\mathfrak{a}_1$ is calculated as follows: $\mathfrak{D}_{\mathfrak{E}}^{\mathfrak{N}+} = 1/6 \times 6[(0.6809 - 0.6809)^2 + (1.0000 - 0.5893)^2 + (0.4475 - 0.4475)^2 + (0.2724 - 0.2724)^2 + (0.3861 - 0.3437)^2 + (0.3764 - 0.3764)^2 + \cdots + (0.6294 - 0.6857)^2 + (0.9035 - 0.8770)^2 + (0.4644 - 0.3780)^2 + (0.2116 - 0.2431)^2 + (0.3682 - 0.3182)^2 + (0.4405 - 0.4053)^2] = 0.0191$. On the same pattern, we can calculate all other values for PIS and NIS corresponding to all the alternatives. The distance measure of each alternative is given in Table 9.

Step 9. Calculating the SLDF relative closeness as $\mathfrak{C}_j^+ = \mathfrak{D}_{\mathfrak{E}}^{\mathfrak{N}-}/\mathfrak{D}_{\mathfrak{E}}^{\mathfrak{N}+} + \mathfrak{D}_{\mathfrak{E}}^{\mathfrak{N}-} = 0.0117/0.0191 + 0.0117 = 0.3796$ and its ranking is presented in Table 10.

Step 10. The priority structure of the green supplier as seen in Table 10 (see Figure 2) is $a_2 > a_5 > a_4 > a_1 > a_3$; thus, a_2 is the best green supplier.

5. Comparison, Limitations, and Advantages

(i) In this manuscript, we present the perception of SLDFS and therefore a TOPSIS-based algorithm. The model's most notable feature is that it envelopes the assessment spaces of FSs, IFSs, q-ROFSs, PFSs, SFSs, and q-RPFSs. It also has the reference parameters, which is an additional point, and the comparison analysis is given in Table 11

(ii) Table 12 shows the ranking results of five options using known methodologies and a novel notion utilizing the TOPSIS method. Comparison in Table 12 illustrates that the optimal selection produced by the suggested method is more like the current approaches, which is expressive in and of itself and validates the new method's dependability and validity. Now comes the question of why we need to describe a novel algorithm based on this unique structure. There are several grounds, as described above, why the suggested methodology is superior to other previous techniques

(iii) As a limitation, SLDFS does not provide information about the roughness of data and cannot handle multivalued parameterizations

(iv) This framework can also be used to define and solve MCDM problems in a broader context. The developed TOPSIS technique based on SLDFSs outperforms several current methods. There is a little disparity betwixt the suggested approach's ranking results and those found in the literature. In reality, depending on their ordering tactics, various

Table 11: Comparison between some existing sets and SLDFS.

Set	Advantages	Limitations
FS [12, 13]	It can handle imprecise information	It cannot handle nonmembership values
IFS [5, 6, 25]	It contains the membership and nonmembership values	It will not work for the case $\mathfrak{t} + \mathfrak{f} > 1$
PyFS [7–9, 23]	It can be used when the sum of membership and nonmembership grades exceeds 1	It will not work for the case $\mathfrak{t}^2 + \mathfrak{f}^2 > 1$
q-ROFS [10, 11]	It is more efficient than PyFs. We can handle the sum of q-power of membership and nonmembership values less than 1	It cannot be used for $\mathfrak{t} = \mathfrak{f} = 1$ or if "q" is small with $\mathfrak{t}^q + \mathfrak{f}^q > 1$
PFS [14]	It is efficient than IFS. This theory is used when we express the sum of membership, neutral, and nonmembership less than 1	It will not work for the case $\mathfrak{t} + \alpha + \mathfrak{f} > 1$
SFS [15]	It is better than PFS and PyFS. It works for the case $\mathfrak{t}^2 + \alpha^2 + \mathfrak{f}^2 < 1$	It will not work for the case $\mathfrak{t}^2 + \alpha^2 + \mathfrak{f}^2 > 1$
q-RPFS [17]	It is better than PFS. It could handle when the sum of q-power of membership, neutral, and nonmembership is less than 1	It cannot be used for $\mathfrak{t} = \alpha = \mathfrak{f} = 1$ or if the value of "q" is smaller with $\mathfrak{t}^q + \alpha^q + \mathfrak{f}^q > 1$
SLDF (proposed)	(1) It can handle all circumstances where FS, IFS, PyFS, q-ROFS, PFS, SFS, and q-RPFS cannot be used (2) It takes a parameterization approach and operates under the effect of reference parameters (3) Membership, abstinence, and nonmembership grades may be freely selected from [0, 1]	Calculation is complicated

Table 12: Comparison with existing approaches.

Techniques	Approach	Ranking alternatives
Wang et al. [28]	PFS	$\alpha_2 > \alpha_1 > \alpha_5 > \alpha_4 > \alpha_3$
Kahraman et al. [29]	SFS	$\alpha_2 > \alpha_4 > \alpha_2 > \alpha_1 > \alpha_3$
Akram and Shumaiza [30]	q-RPFS	$\alpha_2 > \alpha_5 > \alpha_4 > \alpha_3 > \alpha_1$
Proposed algorithm	SLDFS	$\alpha_2 > \alpha_5 > \alpha_4 > \alpha_1 > \alpha_3$

equations can allow somewhat different effects. Because of the addition of reference parameters in SLDFSs, more comprehensive information may be extracted from data input. So, when comparing to other methodologies, our SLDFS-based technique provides more precise findings

6. Conclusion

We looked into several fuzzy set extensions, such as IFSs, PFSs, and q-ROFSs. SLDFSs are a new fuzzy set extension that we started. This concept, when combined with the usage of two extra reference factors, can contribute to a more efficient and flexible structure for the modeling of fuzzy systems and DM under ambivalence circumstances. From a geometric standpoint, we compared SLDFSs to various current extensions of fuzzy sets. We introduced several fundamental operations for SLDFS and created the SLDFS-based TOPSIS approach by utilizing spherical linear Diophantine fuzzy geometric aggregation operators. Furthermore, we successfully applied the suggested strategy to a multiattribute decision-making issue involving the selection of green supply chain management systems. When compared to other current methodologies, numerical findings suggest that the SLDFS-based methodology is more realistic and versatile in real-world situations.

Data Availability

No data was used.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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