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Research Article

Some New Aspects in the Intuitionistic Fuzzy and Neutrosophic Fixed Point Theory

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In this manuscript, we use the concepts of continuous t-norms and continuous t-conorms to introduce some definitions, in which intuitionistic fuzzy rectangular metric spaces, intuitionistic fuzzy rectangular metric-like spaces, intuitionistic fuzzy rectangular b-metric spaces, intuitionistic fuzzy rectangular b-metric-like spaces, neutrosophic rectangular metric spaces, neutrosophic rectangular metric-like spaces, neutrosophic rectangular b-metric spaces, and neutrosophic rectangular b-metric-like spaces are included. Continuous t-norms and continuous t-conorms are used to generalize the probability distribution of triangular inequalities in metric space axioms. Nontrivial examples, some fixed point results, and an application to the integral equation are imparted in this manuscript.

1. Introduction

Fuzzy set (FS) presented by Zadeh [1] is a useful tool for those situations in which the data are imprecise and the idea of degree of membership is involved in FS theory. Intuitionistic fuzzy sets (IFSs) introduced by Atanassov [2] are the generalization of the FS, in which degrees of membership and nonmembership are involved. Smarandache [3] presented the idea of neutrosophic sets (NSs) that are the generalization of the IFS, in which degrees of membership, nonmembership, and uncertainty are involved.

By combining the concepts of FS and metric spaces, fuzzy metric spaces (FMSs) were presented by Kramosil and Michalek [4]. Kaleva and Seikkala [5] coined FMS in which they defined a distance between two points to be a nonnegative fuzzy number, and Garbiec [6] presented the fuzzy interpretation of the Banach contraction principle in the FMS. Park [7] presented the intuitionistic fuzzy metric space (IFMS), in which he used George and Veeramani's [8] approach of applying continuous t-norm (CTN) and continuous t-conorm (CTCN) to the FMS. Kirişci and Şimşek [9] presented the notion of neutrosophic metric space (NMS), in which they used the idea of NS and probabilistic metric spaces. FMS deals with membership functions, and IFMS deals with membership and nonmembership functions. NMS is a generalization of the IFMS that deals with membership, nonmembership, and inconsistent functions. Altun et al. [10] and Aslantas et al. [11] proved some interesting results for cyclic p-contractions and KW-type nonlinear contractions. Al-Omeri et al. [12, 13] proved several neutrosophic fixed point results and generalized theorems in the sense of neutrosophic cone metric spaces.

Javed et al. [14] presented the idea of fuzzy b-metric-like spaces (FBMLSs) and proved several fixed point results. Mehmood et al. [15] presented the concept of fuzzy rectangular b-metric spaces (FRBMSs) and proved the Banach contraction principle in the sense of FRBMS. For some necessary definitions and related fixed point results, see [16–19].

In this manuscript, we generalized the concepts used in [14, 15]. The main objectives of this manuscript are as follows:

(i) To present different notions in the intuitionistic fuzzy and neutrosophic fixed point theory

(ii) To prove certain fixed point theorems

(iii) To enhance the existing literature of the FMS and fuzzy fixed point theory

This study is organized with some basic notions of FRBMS, FBMLS, IFMS, and NMS. The notions of intuitionistic fuzzy rectangular metric spaces (IFRMSs), intuitionistic fuzzy rectangular metric-like spaces (IFRMLSs), intuitionistic fuzzy rectangular b-metric spaces (IFRBMSs), intuitionistic fuzzy rectangular b-metric-like spaces (IFRBMLSs), neutrosophic rectangular metric spaces (NRMSs), neutrosophic rectangular metric-like spaces (NRMLSs), neutrosophic rectangular b-metric spaces (NRBMSs), and neutrosophic rectangular b-metric-like spaces (NRBMLSs) are discussed in detail, and several fixed point results, nontrivial examples, and an application to the integral equation are imparted. At the end, conclusion is given for the examined results.

2. Preliminaries

In this section, some basic definitions are imparted that are helpful to understand the main section.

Definition 1 (see [7]). A binary operation $*: [0,1]\times[0,1]\longrightarrow[0,1]$ is called a CTN if it meets the following assertions:

C1. $\zeta * b = b * \zeta$, $(\forall)\,\zeta, b \in [0,1]$

C2. $*$ is continuous

C3. $\zeta * 1 = \zeta$, $(\forall)\,\zeta \in [0,1]$

C4. $(\zeta * b) * c = \zeta * (b * c)$, $(\forall)\,\zeta, b, c \in [0,1]$

C5. If $\zeta \le c$ and $b \le \sigma$, with $\zeta, b, c, \sigma \in [0,1]$, then $\zeta * b \le c * \sigma$

Definition 2 (see [7]). A binary operation $\bigcirc: [0,1]\times[0,1]\longrightarrow[0,1]$ is called a CTCN if it meets the following assertions:

T1. $\zeta \bigcirc b = b \bigcirc \zeta$, for all $\zeta, b \in [0,1]$

T2. $\bigcirc$ is continuous

T3. $\zeta \bigcirc 0 = 0$, for all $\zeta \in [0,1]$

T4. $(\zeta \bigcirc b) \bigcirc c = \zeta \bigcirc (b \bigcirc c)$, for all $\zeta, b, c \in [0,1]$

T5. If $\zeta \le c$ and $b \le \sigma$, with $\zeta, b, c, \sigma \in [0,1]$, then $\zeta \bigcirc b \le c \bigcirc \sigma$

Definition 3 (see [19]). Let a set $E \ne \varnothing$ and $\vartheta \in E$. A NS G in E is categorized by a truth membership function $\mathfrak{B}_G(\vartheta)$, an indeterminacy membership function $\mathfrak{D}_G(\vartheta)$, and a falsity membership function $\mathcal{Q}_G(\vartheta)$. The functions $\mathfrak{B}_G(\vartheta)$, $\mathfrak{D}_G(\vartheta)$, and $\mathcal{Q}_G(\vartheta)$ are real standard or nonstandard subsets of $]0^-, 1^+[$; that is, $\mathfrak{B}_G(\vartheta): E \longrightarrow 0^-, 1^+[$, $\mathfrak{D}_G(\vartheta): E \longrightarrow 0^-, 1^+[$ and $\mathcal{Q}_G(\vartheta): E \longrightarrow 0^-, 1^+[$. So,

$$0^- \le \sup \mathfrak{B}_G(\vartheta) + \sup \mathfrak{D}_G(\vartheta) + \sup \mathcal{Q}_G(\vartheta) \le 3^+. \tag{1}$$

Definition 4 (see [14]). Let E be a nonempty set. A triplet $(E, F_b, *)$ is called a FBMLS if $*$ is a CTN and F_b is a FS on $E \times E \times (0,\infty)$ and fulfills the following assertions for all $\vartheta, \delta, g \in E$ and $\tau, z > 0$:

A1. $F_b(\vartheta, \delta, \tau) > 0$

A2. $F_b(\vartheta, \delta, \tau) = 1$; then, $\vartheta = \delta$

A3. $F_b(\vartheta, \delta, \tau) = F_b(\delta, \vartheta, \tau)$

A4. $F_b(\vartheta, g, b(\tau + z)) \ge F_b(\vartheta, \delta, \tau) * F_b(\delta, g, z)$

A5. $F_b(\vartheta, \delta, .): (0,\infty) \longrightarrow [0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty} F_b(\vartheta, \delta, \tau) = 1$

Definition 5 (see [15]). Let E be a nonempty set. A triplet $(E, R_b, *)$ is called a FRMS if $*$ is a CTN and R_b is a FS on $E \times E \times [0,\infty)$ and fulfills the following assertions for all $\vartheta, \delta, g \in E$ and $\tau, z, w > 0$:

(1) $R_b(\vartheta, \delta, 0) = 0$

(2) $R_b(\vartheta, \delta, \tau) = 1$ if and only if $\vartheta = \delta$

(3) $R_b(\vartheta, \delta, \tau) = R_b(\delta, \vartheta, \tau)$

(4) $R_b(\vartheta, g, \tau + z + w) \ge R_b(\vartheta, \delta, \tau) * R_b(\delta, u, z) + R_b(u, g, w)$ for all distinct $\delta, u \in E\backslash\{\vartheta, g\}$

(5) $R_b(\vartheta, \delta, .): (0,\infty) \longrightarrow [0,1]$ is left continuous, and $\lim_{\tau\longrightarrow\infty} R_b(\vartheta, \delta, \tau) = 1$

Definition 6 (see [7]). Take $E \ne \varnothing$. Let $*$ be a CTN, $\bigcirc$ be a CTCN, and F, V be FSs on $E \times E \times (0,\infty)$. If $(E, F, V, *, \bigcirc)$ verifies the following assertions for all $\vartheta, \delta \in E$ and $z, \tau > 0$,

F1. $F(\vartheta, \delta, \tau) + V(\vartheta, \delta, \tau) \le 1$

F2. $F(\vartheta, \delta, \tau) > 0$

F3. $F(\vartheta, \delta, \tau) = 1 \Longleftrightarrow \vartheta = \delta$

F4. $F(\vartheta, \delta, \tau) = F(\delta, \vartheta, \tau)$

F5. $F(\vartheta, g, \tau + z) \ge F(\vartheta, \delta, \tau) * F(\delta, g, z)$

F6. $F(\vartheta, \delta, z): (0,\infty) \longrightarrow [0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty} F(\vartheta, \delta, \tau) = 1$ for all $\tau > 0$

F7. $V(\vartheta, \delta, \tau) > 0$

F8. $V(\vartheta, \delta, \tau) = 0 \Longleftrightarrow z\vartheta = \delta$

F9. $V(\vartheta, \delta, \tau) = V(\delta, \vartheta, \tau)$

F10. $V(\vartheta, g, \tau + z) \le V(\vartheta, \delta, \tau) \bigcirc V(\delta, g, z)$

F11. $V(\vartheta, \delta, z): (0,\infty) \longrightarrow [0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty} V(\vartheta, \delta, \tau) = 0$ for all $\tau > 0$

Then, $(E, F, V, *, \bigcirc)$ is an IFMS

Definition 7 (see [8]). Let $E \ne \varnothing$, $*$ be a CTN, and $\bigcirc$ be a CTCN. L, W, and Q are NSs on $E \times E \times (0,\infty)$ which are said to be a neutrosophic metric on E if for all $\vartheta, \delta, g \in E$, the following circumstances fulfill:

S1. $L(\vartheta, \delta, \tau) + W(\vartheta, \delta, \tau) + Q(\vartheta, \delta, \tau) \le 3$ for all $\tau \in \mathbb{R}^+$

S2. $L(\vartheta, \delta, \tau) > 0$ for all $\tau > 0$

S3. $L(\vartheta, \delta, \tau) = 1$ for all $\tau > 0$ if and only if $\vartheta = \delta$

S4. $L(\vartheta, \delta, \tau) = L(\delta, \vartheta, \tau)$ for all $\tau > 0$

S5. $L(\vartheta, g, \tau + z) \ge L(\vartheta, \delta, \tau) * L(\delta, g, z)$ for all $\tau, z > 0$

S6. $L(\vartheta, \delta, z)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} L(\vartheta, \delta, \tau) = 1$ for all $\tau > 0$
S7. $W(\vartheta, \delta, \tau) < 1$ for all $\tau > 0$
S8. $W(\vartheta, \delta, \tau) = 0$ for all $\tau > 0$ if and only if $\vartheta = \delta$
S9. $W(\vartheta, \delta, \tau) = W(\delta, \vartheta, \tau)$ for all $\tau > 0$
S10. $W(\vartheta, g, \tau + z) \leq W(\vartheta, \delta, \tau) \bigcirc W(\delta, g, z)$ for all $\tau, z > 0$
S11. $W(\vartheta, \delta, z)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} W(\vartheta, \delta, \tau) = 0$ for all $\tau > 0$
S12. $Q(\vartheta, \delta, \tau) < 1$ for all $\tau > 0$
S13. $Q(\vartheta, \delta, \tau) = 0$ for all $\tau > 0$ if and only if $\vartheta = \delta$
S14. $Q(\vartheta, \delta, \tau) = Q(\delta, \vartheta, \tau)$ for all $\tau > 0$
S15. $Q(\vartheta, g, \tau + z) \leq Q(\vartheta, \delta, \tau) \bigcirc Q(\delta, g, z)$ for all $\tau, z > 0$
S16. $Q(\vartheta, \delta, z)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} Q(\vartheta, \delta, \tau) = 0$ for all $\tau > 0$

Then, $(E, L, W, Q, *, \bigcirc)$ is called a NMS

3. Main Results

In this section, we present some new notions as generalizations of intuitionistic fuzzy and neutrosophic metric spaces; also, some fixed point results are proved.

Definition 8. Let E be a nonempty set. A five-tuple $(E, \mathfrak{B}_i, \mathfrak{D}_i, *, \bigcirc)$ is called an IFRMS if $*$ is a CTN, $\bigcirc$ is a CTCN, and $\mathfrak{B}_i$ and $\mathfrak{D}_i$ are two FSs on $E \times E \times [0, \infty)$ which fulfill the following assertions for all $\vartheta, \delta, g \in E$ and $\tau, z, w > 0$:

R1. $\mathfrak{B}_i(\vartheta, \delta, \tau) + \mathfrak{D}_i(\vartheta, \delta, \tau) \leq 1$
R2. $\mathfrak{B}_i(\vartheta, \delta, 0) = 0$
R3. $\mathfrak{B}_i(\vartheta, \delta, \tau) = 1$ if and only if $\vartheta = \delta$
R4. $\mathfrak{B}_i(\vartheta, \delta, \tau) = \mathfrak{B}_i(\delta, \vartheta, \tau)$
R5. $\mathfrak{B}_i(\vartheta, g, \tau + z + w) \geq \mathfrak{B}_i(\vartheta, \delta, \tau) * \mathfrak{B}_i(\delta, u, z) + \mathfrak{B}_i(u, g, w)$ for all distinct $\delta, u \in E \backslash \{\vartheta, g\}$
R6. $\mathfrak{B}_i(\vartheta, \delta, .)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} \mathfrak{B}_i(\vartheta, \delta, \tau) = 1$
R7. $\mathfrak{D}_i(\vartheta, \delta, 0) = 1$
R8. $\mathfrak{D}_i(\vartheta, \delta, \tau) = 0$ if and only if $\vartheta = \delta$
R9. $\mathfrak{D}_i(\vartheta, \delta, \tau) = \mathfrak{D}_i(\delta, \vartheta, \tau)$
R10. $\mathfrak{D}_i(\vartheta, g, \tau + z + w) \leq \mathfrak{D}_i(\vartheta, \delta, \tau) \bigcirc \mathfrak{D}_i(\delta, u, z) + \mathfrak{D}_i(u, g, w)$ for all distinct $\delta, u \in E \backslash \{\vartheta, g\}$
R11. $\mathfrak{D}_i(\vartheta, \delta, .)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} \mathfrak{D}_i(\vartheta, \delta, \tau) = 0$

Example 1. Let (E, d) be a rectangular metric space, define $\mathfrak{B}_i, \mathfrak{D}_i$: $E \times E \times [0, \infty) \longrightarrow [0, 1]$ by

$$\begin{aligned} \mathfrak{B}_i(\vartheta, \delta, \tau) &= \frac{\tau}{\tau + d(\vartheta, \delta)}, \\ \mathfrak{D}_i(\vartheta, \delta, \tau) &= 1 - \frac{\tau}{\tau + d(\vartheta, \delta)} \text{ for all } \vartheta, \delta \in E \text{ and } \tau > 0, \end{aligned} \tag{2}$$

and let $*$ be a CTN and $\bigcirc$ be a CTCN on E. Then, it is easy to see that $(E, \mathfrak{B}_i, \mathfrak{D}_i, *, \bigcirc)$ is an IRFMS.

Definition 9. Let E be a nonempty set. A five-tuple $(E, \mathfrak{B}_b, \mathfrak{D}_b, *, \bigcirc)$ is called an IFRBMS if there is $b \geq 1$, $*$ is a CTN, $\bigcirc$ is a CTCN, and $\mathfrak{B}_b$ and $\mathfrak{D}$ are two FSs on $E \times E \times [0, \infty)$ verifying the following assertions for all $\vartheta, \delta, g \in E$ and $\tau, z, w > 0$:

I. $\mathfrak{B}_b(\vartheta, \delta, \tau) + \mathfrak{D}_b(\vartheta, \delta, \tau) \leq 1$
II. $\mathfrak{B}_b(\vartheta, \delta, 0) = 0$
III. $\mathfrak{B}_b(\vartheta, \delta, \tau) = 1$ if and only if $\vartheta = \delta$
IV. $\mathfrak{B}_b(\vartheta, \delta, \tau) = \mathfrak{B}_b(\delta, \vartheta, \tau)$
V. $\mathfrak{B}_b(\vartheta, g, b(\tau + z + w)) \geq \mathfrak{B}_b(\vartheta, \delta, \tau) * \mathfrak{B}_b(\delta, u, z) + \mathfrak{B}_b(u, g, w)$ for all distinct $\delta, u \in E \backslash \{\vartheta, g\}$
VI. $\mathfrak{B}_b(\vartheta, \delta, .)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} \mathfrak{B}_b(\vartheta, \delta, \tau) = 1$
VII. $\mathfrak{D}_b(\vartheta, \delta, 0) = 1$
VIII. $\mathfrak{D}_b(\vartheta, \delta, \tau) = 0$ if and only if $\vartheta = \delta$
IX. $\mathfrak{D}_b(\vartheta, \delta, \tau) = \mathfrak{D}_b(\delta, \vartheta, \tau)$
X. $\mathfrak{D}_b(\vartheta, g, b(\tau + z + w)) \leq \mathfrak{D}_b(\vartheta, \delta, \tau) \bigcirc \mathfrak{D}_b(\delta, u, z) + \mathfrak{D}_b(u, g, w)$ for all distinct $\delta, u \in E \backslash \{\vartheta, g\}$
XI. $\mathfrak{D}_b(\vartheta, \delta, .)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} \mathfrak{D}_b(\vartheta, \delta, \tau) = 0$

Example 2. Let (E, d) be a rectangular b-metric space, and define $\mathfrak{B}_b, \mathfrak{D}_b$: $E \times E \times [0, \infty) \longrightarrow [0, 1]$ by

$$\begin{aligned} \mathfrak{B}_b(\vartheta, \delta, \tau) &= \frac{\tau}{\tau + d(\vartheta, \delta)}, \\ \mathfrak{D}_b(\vartheta, \delta, \tau) &= \frac{d(\vartheta, \delta)}{\tau + d(\vartheta, \delta)} \text{ for all } \vartheta, \delta \in E \text{ and } \tau > 0, \end{aligned} \tag{3}$$

with CTN $\zeta * b = \min\{\zeta, b\}$ and CTCN $\zeta \bigcirc b = \max\{\zeta, b\}$. Then, it is easy to see that $(E, \mathfrak{B}_b, \mathfrak{D}_b, * \bigcirc)$ is an IFRBMS.

Example 3. Let (E, d) be a rectangular b-metric space, and define $\mathfrak{B}_b, \mathfrak{D}_b$: $E \times E \times [0, \infty) \longrightarrow [0, 1]$ by

$$\begin{aligned} \mathfrak{B}_b(\vartheta, \delta, \tau) &= e^{-d(\vartheta, \delta)/\tau}, \\ \mathfrak{D}_b(\vartheta, \delta, \tau) &= 1 - e^{-d(\vartheta, \delta)/\tau} \text{ for all } \vartheta, \delta \in E \text{ and } \tau > 0, \end{aligned} \tag{4}$$

with CTN $\zeta * b = \min\{\zeta, b\}$ and CTCN $\zeta \bigcirc b = \max\{\zeta, b\}$. Then, it is easy to see that $(E, \mathfrak{B}_b, \mathfrak{D}_b, * \bigcirc)$ is an IFRBMS.

Remark 1. The above Examples 2 and 3 are also an IFRBMS with CTN $\zeta * b = \zeta$�b and CTCN $\zeta \bigcirc b = \max\{\zeta, b\}$.

Remark 2. Every IFRMS is an IFRBMS, but the converse may not be true.

Definition 10. Let E be a nonempty set. A five-tuple $(E, \mathfrak{B}_l, \mathfrak{D}_l, *, \bigcirc)$ is called an IFRBMLS if there is $b \geq 1$, $*$ is a CTN, $\bigcirc$ is a CTCN, and $\mathfrak{B}_l$ and $\mathfrak{D}_l$ are two FSs on $E \times E \times [0, \infty)$ fulfilling the following assertions for all $\vartheta, \delta, g \in E$ and $\tau, z, w > 0$:

A. $\mathfrak{B}_l(\vartheta, \delta, \tau) + \mathfrak{D}_l(\vartheta, \delta, \tau) \leq 1$
B. $\mathfrak{B}_l(\vartheta, \delta, 0) = 0$
C. $\mathfrak{B}_l(\vartheta, \delta, \tau) = 1$ implies $\vartheta = \delta$
D. $\mathfrak{B}_l(\vartheta, \delta, \tau) = \mathfrak{B}_l(\delta, \vartheta, \tau)$
E. $\mathfrak{B}_l(\vartheta, g, b(\tau + z + w)) \geq \mathfrak{B}_l(\vartheta, \delta, \tau) * \mathfrak{B}_l(\delta, u, z) + \mathfrak{B}_l(u, g, w)$ for all distinct $\delta, u \in E \backslash \{\vartheta, g\}$
F. $\mathfrak{B}_l(\vartheta, \delta, .)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} \mathfrak{B}_l(\vartheta, \delta, \tau) = 1$
G. $\mathfrak{D}_l(\vartheta, \delta, 0) = 1$
H. $\mathfrak{D}_l(\vartheta, \delta, \tau) = 0$ implies if $\vartheta = \delta$
I. $\mathfrak{D}_l(\vartheta, \delta, \tau) = \mathfrak{D}_l(\delta, \vartheta, \tau)$
J. $\mathfrak{D}_l(\vartheta, g, b(\tau + z + w)) \leq \mathfrak{D}_l(\vartheta, \delta, \tau) \bigcirc \mathfrak{D}_l(\delta, u, z) + \mathfrak{D}_l(u, g, w)$ for all distinct $\delta, u \in E \backslash \{\vartheta, g\}$
K. $\mathfrak{D}_l(\vartheta, \delta, .)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} \mathfrak{D}_l(\vartheta, \delta, \tau) = 0$

Definition 11. In the above Definition 10, if we take $b = 1$, then it becomes an IFRMLS.

Example 4. Define $\mathfrak{B}_l, \mathfrak{D}_l$: $E \times E \times [0, \infty) \longrightarrow [0, 1]$ by

$$\begin{aligned} \mathfrak{B}_l(\vartheta, \delta, \tau) &= \frac{\tau}{\tau + \max\{\vartheta, \delta\}^p}, \\ \mathfrak{D}_l(\vartheta, \delta, \tau) &= \frac{\max\{\vartheta, \delta\}^p}{\tau + \max\{\vartheta, \delta\}^p} \text{ for all } \vartheta, \delta \in E \text{ and } \tau > 0, \end{aligned} \tag{5}$$

with CTN $\zeta * b = \min\{\zeta, b\}$ and CTCN $\zeta \bigcirc b = \max\{\zeta, b\}$. Then, it is easy to see that $(E, \mathfrak{B}_l, \mathfrak{D}_l, * \bigcirc)$ is an IFRBMLS, and if we take $p = 1$, then it becomes an IFRMLS.

Example 5. Define $\mathfrak{B}_l, \mathfrak{D}_l$: $E \times E \times [0, \infty) \longrightarrow [0, 1]$ by

$$\begin{aligned} \mathfrak{B}_l(\vartheta, \delta, \tau) &= e^{-\max\{\vartheta, \delta\}^p/\tau}, \\ \mathfrak{D}_l(\vartheta, \delta, \tau) &= 1 - e^{-\max\{\vartheta, \delta\}^p/\tau} \text{ for all } \vartheta, \delta \in E, \ p \geq 1, \text{ and } \tau > 0, \end{aligned} \tag{6}$$

with CTN $\zeta * b = \min\{\zeta, b\}$ and CTCN $\zeta \bigcirc b = \max\{\zeta, b\}$. Then, it is easy to see that $(E, \mathfrak{B}_l, \mathfrak{D}_l, * \bigcirc)$ is an IFRBMLS.

Remark 3. The above Examples 4 and 5 are also an IFRBMLS with CTN $\zeta * b = \zeta �b$ and CTCN $\zeta \bigcirc b = \max\{\zeta, b\}$.

Remark 4. In an IFRBMLS, the self-distance may not be equal to 1 and 0.

For this, consider the above Example 5; then, we have

$$\begin{aligned} \mathfrak{B}_l(\vartheta, \vartheta, \tau) &= e^{-\max\{\vartheta, \vartheta\}^p/\tau} = e^{-\vartheta^p/\tau} \neq 1, \\ \mathfrak{D}_l(\vartheta, \vartheta, \tau) &= 1 - e^{-\max\{\vartheta, \vartheta\}^p/\tau} = 1 - e^{-\vartheta^p/\tau} \neq 0. \end{aligned} \tag{7}$$

Remark 5. Every IFRBMS is an IFRBMLS, but the converse may not be true.

Remark 6. IFRBMLS may not be continuous.

Example 6. Let $E = [0, \infty)$, $\mathfrak{B}_l(\vartheta, \delta, \tau) = e^{-d(\vartheta, \delta)/\tau}$, and $\mathfrak{D}_l(\vartheta, \delta, \tau) = 1 - e^{-d(\vartheta, \delta)/\tau}$ for all $\vartheta, \delta \in E$, $\tau > 0$, and

$$d(\vartheta, \delta) = \begin{cases} 0, & \text{if } \vartheta = \delta, \\ 2(\vartheta + \delta)^2, & \text{if } \vartheta, \delta \in [0, 1], \\ \dfrac{1}{2}(\vartheta + \delta)^2, & \text{otherwise.} \end{cases} \tag{8}$$

If we define CTN by $\zeta * b = \zeta �b$ and CTCN by $\zeta \bigcirc b = \max\{\zeta, b\}$, then $(E, \mathfrak{B}_l, \mathfrak{D}_l, *, \bigcirc)$ is an IFRBMLS. Now, to illustrate continuity, we have

$$\begin{aligned} \lim_{n \longrightarrow \infty} \mathfrak{B}_l\left(0, 1 - \frac{1}{n}, \tau\right) &= \lim_{n \longrightarrow \infty} e^{-2(1-(1/n))^2/\tau} \\ &= e^{-2/\tau} = \mathfrak{B}_l(0, 1, \tau), \\ \lim_{n \longrightarrow \infty} \mathfrak{D}_l\left(0, 1 - \frac{1}{n}, \tau\right) &= 1 - \lim_{n \longrightarrow \infty} e^{-2(1-(1/n))^2/\tau} \\ &= 1 - e^{-2/\tau} = \mathfrak{D}_l(0, 1, \tau). \end{aligned} \tag{9}$$

However,

$$\begin{aligned} \lim_{n \longrightarrow \infty} \mathfrak{B}_l\left(1, 1 - \frac{1}{n}, \tau\right) &= \lim_{n \longrightarrow \infty} e^{-2(2-(1/n))^2/\tau} \\ &= e^{-8/\tau} \neq 1 = \mathfrak{B}_l(1, 1, \tau), \\ \lim_{n \longrightarrow \infty} \mathfrak{D}_l\left(1, 1 - \frac{1}{n}, \tau\right) &= 1 - \lim_{n \longrightarrow \infty} e^{-2(2-(1/n))^2/\tau} \\ &= 1 - e^{-8/\tau} \neq 0 = \mathfrak{D}_l(1, 1, \tau). \end{aligned} \tag{10}$$

Hence, $(E, \mathfrak{B}_l, \mathfrak{D}_l, \mathcal{Q}, *, \bigcirc)$ is not continuous.

Definition 12. Let E be a nonempty set. A six-tuple $(E, \mathfrak{B}_e, \mathfrak{D}_e, \mathcal{Q}_e, *, \bigcirc)$ is called a NRMS if $*$ is a CTN, $\bigcirc$ is a CTCN, and $\mathfrak{B}_e$, $\mathfrak{D}_e$, and $\mathcal{Q}_e$ are three NSs on $E \times E \times [0, \infty)$ fulfilling the following assertions for all $\vartheta, \delta, g \in E$ and $\tau, z, w > 0$:

(i) $\mathfrak{B}_e(\vartheta, \delta, \tau) + \mathfrak{D}_e(\vartheta, \delta, \tau) + \mathcal{Q}(\vartheta, \delta, \tau) \leq 3$
(ii) $\mathfrak{B}_e(\vartheta, \delta, 0) = 0$
(iii) $\mathfrak{B}_e(\vartheta, \delta, \tau) = 1$ if and only if $\vartheta = \delta$
(iv) $\mathfrak{B}_e(\vartheta, \delta, \tau) = \mathfrak{B}_e(\delta, \vartheta, \tau)$
(v) $\mathfrak{B}_e(\vartheta, g, \tau + z + w) \geq \mathfrak{B}_e(\vartheta, \delta, \tau) * \mathfrak{B}_e(\delta, u, z) + \mathfrak{B}_e(u, g, w)$ for all distinct $\delta, u \in E \backslash \{\vartheta, g\}$
(vi) $\mathfrak{B}_e(\vartheta, \delta, .)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} \mathfrak{B}_e(\vartheta, \delta, \tau) = 1$
(vii) $\mathfrak{D}_e(\vartheta, \delta, 0) = 1$
(viii) $\mathfrak{D}_e(\vartheta, \delta, \tau) = 0$ if and only if $\vartheta = \delta$
(ix) $\mathfrak{D}_e(\vartheta, \delta, \tau) = \mathfrak{D}_e(\delta, \vartheta, \tau)$
(x) $\mathfrak{D}_e(\vartheta, g, \tau + z + w) \leq \mathfrak{D}_e(\vartheta, \delta, \tau) \bigcirc \mathfrak{D}_e(\delta, u, z) + \mathfrak{D}_e(u, g, w)$ for all distinct $\delta, u \in E \backslash \{\vartheta, g\}$
(xi) $\mathfrak{D}_e(\vartheta, \delta, .)$: $(0, \infty) \longrightarrow [0, 1]$ is continuous, and $\lim_{\tau \longrightarrow \infty} \mathfrak{D}_e(\vartheta, \delta, \tau) = 0$

(xii) $\mathcal{Q}_e(\vartheta,\delta,0)=1$
(xiii) $\mathcal{Q}_e(\vartheta,\delta,\tau)=0$ if and only if $\vartheta=\delta$
(xiv) $\mathcal{Q}_e(\vartheta,\delta,\tau)=\mathcal{Q}_e(\delta,\vartheta,\tau)$
(xv) $\mathcal{Q}_e(\vartheta,g,\tau+z+w)\le\mathcal{Q}_e(\vartheta,\delta,\tau)\bigcirc\mathcal{Q}_e(\delta,u,z)+\mathcal{Q}_e(u,g,w)$ for all distinct $\delta,u\in E\backslash\{\vartheta,g\}$
(xvi) $\mathcal{Q}_e(\vartheta,\delta,.)\colon(0,\infty)\longrightarrow[0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty}\mathcal{Q}_e(\vartheta,\delta,\tau)=0$

Example 7. Let (E,d) be a rectangular metric space, and define $\mathfrak{B}_e,\mathfrak{D},\mathcal{Q}_e\colon E\times E\times[0,\infty)\longrightarrow[0,1]$ by

$$\begin{aligned}\mathfrak{B}_e(\vartheta,\delta,\tau)&=\frac{\tau}{\tau+d(\vartheta,\delta)},\\ \mathfrak{D}_e(\vartheta,\delta,\tau)&=1-\frac{\tau}{\tau+d(\vartheta,\delta)},\\ \mathcal{Q}_e(\vartheta,\delta,\tau)&=\frac{d(\vartheta,\delta)}{\tau},\end{aligned}\tag{11}$$

for all $\vartheta,\delta\in E$ and $\tau>0$, with CTN $\zeta*b=\min\{\zeta,b\}$ and CTCN $\zeta\bigcirc b=\max\{\zeta,b\}$. Then, it is easy to see that $(E,\mathfrak{B}_e,\mathfrak{D}_e,\mathcal{Q}_e,*\bigcirc)$ is a NRMS.

Definition 13. Let E be a nonempty set. A six-tuple $(E,\mathfrak{B},\mathfrak{D},\mathcal{Q},*,\bigcirc)$ is called a NRBMS if there is $b\ge1$, $*$ is a CTN, $\bigcirc$ is a CTCN, and $\mathfrak{B}$, $\mathfrak{D}$, and $\mathfrak{Q}$ are three NSs on $E\times E\times[0,\infty)$ fulfilling the following assertions for all $\vartheta,\delta,g\in E$ and $\tau,z,w>0$:

(a) $\mathfrak{B}(\vartheta,\delta,\tau)+\mathfrak{D}(\vartheta,\delta,\tau)+\mathcal{Q}(\vartheta,\delta,\tau)\le3$
(b) $\mathfrak{B}(\vartheta,\delta,0)=0$
(c) $\mathfrak{B}(\vartheta,\delta,\tau)=1$ if and only if $\vartheta=\delta$
(d) $\mathfrak{B}(\vartheta,\delta,\tau)=\mathfrak{B}(\delta,\vartheta,\tau)$
(e) $\mathfrak{B}(\vartheta,g,b(\tau+z+w))\ge\mathfrak{B}(\vartheta,\delta,\tau)*\mathfrak{B}(\delta,u,z)+\mathfrak{B}(u,g,w)$ for all distinct $\delta,u\in E\backslash\{\vartheta,g\}$
(f) $\mathfrak{B}(\vartheta,\delta,.)\colon(0,\infty)\longrightarrow[0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty}\mathfrak{B}(\vartheta,\delta,\tau)=1$
(g) $\mathfrak{D}(\vartheta,\delta,0)=1$
(h) $\mathfrak{D}(\vartheta,\delta,\tau)=0$ if and only if $\vartheta=\delta$
(i) $\mathfrak{D}(\vartheta,\delta,\tau)=\mathfrak{D}(\delta,\vartheta,\tau)$
(j) $\mathfrak{D}(\vartheta,g,b(\tau+z+w))\le\mathfrak{D}(\vartheta,\delta,\tau)\bigcirc\mathfrak{D}(\delta,u,z)+\mathfrak{D}(u,g,w)$ for all distinct $\delta,u\in E\backslash\{\vartheta,g\}$
(k) $\mathfrak{D}(\vartheta,\delta,.)\colon(0,\infty)\longrightarrow[0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty}\mathfrak{D}(\vartheta,\delta,\tau)=0$
(l) $\mathcal{Q}(\vartheta,\delta,0)=1$
(m) $\mathcal{Q}(\vartheta,\delta,\tau)=0$ if and only if $\vartheta=\delta$
(n) $\mathcal{Q}(\vartheta,\delta,\tau)=\mathcal{Q}(\delta,\vartheta,\tau)$
(o) $\mathcal{Q}(\vartheta,g,b(\tau+z+w))\le\mathcal{Q}(\vartheta,\delta,\tau)\bigcirc\mathcal{Q}(\delta,u,z)+\mathcal{Q}(u,g,w)$ for all distinct $\delta,u\in E\backslash\{\vartheta,g\}$
(p) $\mathcal{Q}(\vartheta,\delta,.)\colon(0,\infty)\longrightarrow[0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty}\mathcal{Q}(\vartheta,\delta,\tau)=0$

Example 8. Let (E,d) be a rectangular b-metric space, and define $\mathfrak{B},\mathfrak{D},\mathcal{Q}\colon E\times E\times[0,\infty)\longrightarrow[0,1]$ by

$$\begin{aligned}\mathfrak{B}(\vartheta,\delta,\tau)&=\frac{\tau}{\tau+d(\vartheta,\delta)},\\ \mathfrak{D}(\vartheta,\delta,\tau)&=1-\frac{\tau}{\tau+d(\vartheta,\delta)},\\ \mathcal{Q}(\vartheta,\delta,\tau)&=\frac{d(\vartheta,\delta)}{\tau},\end{aligned}\tag{12}$$

for all $\vartheta,\delta\in E$ and $\tau>0$, with CTN $\zeta*b=\min\{\zeta,b\}$ and CTCN $\zeta\bigcirc b=\max\{\zeta,b\}$. Then, it is easy to see that $(E,\mathfrak{B},\mathfrak{D},\mathcal{Q},*\bigcirc)$ is a NRBMS.

Remark 7. The above Example 6 is also a NRBMS with CTN $\zeta*b=\zeta�b$ and CTCN $\zeta\bigcirc b=\max\{\zeta,b\}$.

Remark 8. Every NRMS is a NRBMS, but the converse may not be true.

Definition 14. Let $(E,\mathfrak{B},\mathfrak{D},\mathcal{Q},*,\bigcirc)$ be a NRBMS, and assume $\{\vartheta_n\}$ to be a sequence in E. Then,

(i) $\{\vartheta_n\}$ is named to be a convergent sequence if there exists $\vartheta\in E$ such that

$$\begin{aligned}&\lim_{n\longrightarrow\infty}\mathfrak{B}(\vartheta_n,\vartheta,\tau)=1,\\ &\lim_{n\longrightarrow\infty}\mathfrak{D}(\vartheta_n,\vartheta,\tau)=0,\\ &\lim_{n\longrightarrow\infty}\mathcal{Q}(\vartheta_n,\vartheta,\tau)=0\text{ for all }\tau>0.\end{aligned}\tag{13}$$

(ii) $\{\vartheta_n\}$ is named to be a Cauchy sequence if

$$\begin{aligned}&\lim_{n\longrightarrow\infty}\mathfrak{B}(\vartheta_n,\vartheta_{n+q},\tau)=1,\\ &\lim_{n\longrightarrow\infty}\mathfrak{D}(\vartheta_n,\vartheta_{n+q},\tau)=0,\\ &\lim_{n\longrightarrow\infty}\mathcal{Q}(\vartheta_n,\vartheta_{n+q},\tau)=0.\end{aligned}\tag{14}$$

(iii) If every Cauchy sequence is convergent in E, then $(E,\mathfrak{B},\mathfrak{D},\mathcal{Q},*,\bigcirc)$ is said to be a complete NRBMS.

Definition 15. Let E be a nonempty set. A six-tuple $(E,\mathfrak{B}_h,\mathfrak{D}_h,\mathcal{Q}_h,*,\bigcirc)$ is called a NRBMLS if there is $b\ge1$, $*$ is a CTN, $\bigcirc$ is a CTCN, and $\mathfrak{B}_h$, $\mathfrak{D}_h$, and $\mathcal{Q}_h$ are three NSs on $E\times E\times[0,\infty)$ fulfilling the following assertions for all $\vartheta,\delta,g\in E$ and $\tau,z,w>0$:

L1. $\mathfrak{B}_h(\vartheta,\delta,\tau)+\mathfrak{D}_h(\vartheta,\delta,\tau)+\mathcal{Q}_h(\vartheta,\delta,\tau)\le3$
L2. $\mathfrak{B}_h(\vartheta,\delta,0)=0$
L3. $\mathfrak{B}_h(\vartheta,\delta,\tau)=1$ implies if $\vartheta=\delta$
L4. $\mathfrak{B}_h(\vartheta,\delta,\tau)=\mathfrak{B}_h(\delta,\vartheta,\tau)$
L5. $\mathfrak{B}_h(\vartheta,g,b(\tau+z+w))\ge\mathfrak{B}_h(\vartheta,\delta,\tau)*\mathfrak{B}_h(\delta,u,z)+\mathfrak{B}_h(u,g,w)$ for all distinct $\delta,u\in E\backslash\{\vartheta,g\}$
L6. $\mathfrak{B}_h(\vartheta,\delta,.)\colon(0,\infty)\longrightarrow[0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty}\mathfrak{B}_h(\vartheta,\delta,\tau)=1$
L7. $\mathfrak{D}_h(\vartheta,\delta,0)=1$
L8. $\mathfrak{D}_h(\vartheta,\delta,\tau)=0$ implies $\vartheta=\delta$

L9. $\mathfrak{D}_h(\vartheta,\delta,\tau)=\mathfrak{D}_h(\delta,\vartheta,\tau)$

L10. $\mathfrak{D}_h(\vartheta,g,b(\tau+z+w))\le\mathfrak{D}_h(\vartheta,\delta,\tau)\bigcirc\mathfrak{D}_h(\delta,u,z)+\mathfrak{D}_h(u,g,w)$ for all distinct $\delta,u\in E\backslash\{\vartheta,g\}$

L11. $\mathfrak{D}_h(\vartheta,\delta,.)$: $(0,\infty)\longrightarrow[0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty}\mathfrak{D}_h(\vartheta,\delta,\tau)=0$

L12. $\mathcal{Q}_h(\vartheta,\delta,0)=1$

L13. $\mathcal{Q}_h(\vartheta,\delta,\tau)=0$ implies $\vartheta=\delta$

L14. $\mathcal{Q}_h(\vartheta,\delta,\tau)=\mathcal{Q}_h(\delta,\vartheta,\tau)$

L15. $\mathcal{Q}_h(\vartheta,g,b(\tau+z+w))\le\mathcal{Q}_h(\vartheta,\delta,\tau)\bigcirc\mathcal{Q}_h(\delta,u,z)+\mathcal{Q}_h(u,g,w)$ for all distinct $\delta,u\in E\backslash\{\vartheta,g\}$

L16. $\mathcal{Q}_h(\vartheta,\delta,.)$: $(0,\infty)\longrightarrow[0,1]$ is continuous, and $\lim_{\tau\longrightarrow\infty}\mathcal{Q}_h(\vartheta,\delta,\tau)=0$

Definition 16. In the above definition, if we take $b=1$, then it becomes a NRMLS.

Example 9. Define $\mathfrak{B}_h,\mathfrak{D}_h,\mathcal{Q}_h$: $E\times E\times[0,\infty)\longrightarrow[0,1]$ by

$$\begin{aligned}\mathfrak{B}_h(\vartheta,\delta,\tau)&=\frac{\tau}{\tau+(\vartheta+\delta)^p},\\ \mathfrak{D}_h(\vartheta,\delta,\tau)&=1-\frac{\tau}{\tau+(\vartheta+\delta)^p},\\ \mathcal{Q}_h(\vartheta,\delta,\tau)&=\frac{(\vartheta+\delta)^p}{\tau},\end{aligned}\tag{15}$$

for all $\vartheta,\delta\in E$, $p\ge1$, and $\tau>0$, with CTN $\zeta*b=\min\{\zeta,b\}$ and CTCN $\zeta\bigcirc b=\max\{\zeta,b\}$. Then, it is easy to see that $(E,\mathfrak{B}_h,\mathfrak{D}_h,\mathcal{Q}_h,*\bigcirc)$ is a NRBMLS, and if we take $p=1$, then it becomes a NRMLS.

Remark 9. The above Example 9 is also a NRBMLS with CTN $\zeta*b=\zeta$�b and CTCN $\zeta\bigcirc b=\max\{\zeta,b\}$.

Remark 10. Every NRBMS is a NRBMLS, but the converse may not be true.

Remark 11. From Remark 4 and Example 9, it is clear that, in the NRBMLS, the self-distances $\mathfrak{B}_h(\vartheta,\vartheta,\tau)\ne1$, $\mathfrak{D}_h(\vartheta,\vartheta,\tau)\ne0$, and $\mathcal{Q}_h(\vartheta,\vartheta,\tau)\ne0$.

Remark 12. It is clear from Example 6 that the NRBMLS may not be continuous.

Definition 17. Let $(E,\mathfrak{B}_h,\mathfrak{D}_h,\mathcal{Q}_h,*,\bigcirc)$ be a NRBMLS, and assume $\{\vartheta_n\}$ to be a sequence in E. Then,

(i) $\{\vartheta_n\}$ is named to be a convergent sequence if there exists $\vartheta\in E$ such that

$$\begin{aligned}&\lim_{n\longrightarrow\infty}\mathfrak{B}_h(\vartheta_n,\vartheta,\tau)=\mathfrak{B}_h(\vartheta,\vartheta,\tau),\\ &\lim_{n\longrightarrow\infty}\mathfrak{D}_h(\vartheta_n,\vartheta,\tau)=\mathfrak{D}_h(\vartheta,\vartheta,\tau),\\ &\lim_{n\longrightarrow\infty}\mathcal{Q}_h(\vartheta_n,\vartheta,\tau)=\mathcal{Q}_h(\vartheta,\vartheta,\tau)\text{ for all }\tau>0.\end{aligned}\tag{16}$$

(ii) $\{\vartheta_n\}$ is named to be a Cauchy sequence if

$$\begin{aligned}&\lim_{n\longrightarrow\infty}\mathfrak{B}_h(\vartheta_n,\vartheta_{n+q},\tau),\\ &\lim_{n\longrightarrow\infty}\mathfrak{D}_h(\vartheta_n,\vartheta_{n+q},\tau),\\ &\lim_{n\longrightarrow\infty}\mathcal{Q}_h(\vartheta_n,\vartheta_{n+q},\tau),\end{aligned}\tag{17}$$

exist and are finite for all $\tau>0$.

(iii) If every Cauchy sequence is convergent in E, then $(E,\mathfrak{B}_h,\mathfrak{D}_h,\mathcal{Q}_h,*,\bigcirc)$ is said to be a complete NRBMLS such that

$$\begin{aligned}&\lim_{n\longrightarrow\infty}\mathfrak{B}_h(\vartheta_n,\vartheta,\tau)=\mathfrak{B}_h(\vartheta,\vartheta,\tau)=\lim_{n\longrightarrow\infty}\mathfrak{B}_h(\vartheta_n,\vartheta_{n+q},\tau),\\ &\lim_{n\longrightarrow\infty}\mathfrak{D}_h(\vartheta_n,\vartheta,\tau)=\mathfrak{D}_h(\vartheta,\vartheta,\tau)=\lim_{n\longrightarrow\infty}\mathfrak{D}_h(\vartheta_n,\vartheta_{n+q},\tau),\\ &\lim_{n\longrightarrow\infty}\mathcal{Q}_h(\vartheta_n,\vartheta,\tau)=\mathcal{Q}_h(\vartheta,\vartheta,\tau)=\lim_{n\longrightarrow\infty}\mathcal{Q}_h(\vartheta_n,\vartheta_{n+q},\tau),\end{aligned}\tag{18}$$

for all $\tau>0$ and $q\ge1$.

Definition 18. Let $(E,\mathfrak{B}_h,\mathfrak{D}_h,\mathcal{Q}_h,*,\bigcirc)$ be a NRBMLS. For, $\vartheta\in E$, $\theta\in(0,1)$, and $\tau>0$, we define the open ball as

$$\begin{aligned}B(\vartheta,x,\tau)=\{&\delta\in E\colon\mathfrak{B}_h(\vartheta,\delta,\tau)>1-x,\ \mathfrak{D}_h(\vartheta,\delta,\tau)<x,\\ &\mathcal{Q}_h(\vartheta,\delta,\tau)<x\}\\ &(\text{center }\vartheta,\text{ radius }x\text{ with respect to }\tau).\end{aligned}\tag{19}$$

Lemma 1. *Let $(E,\mathfrak{B},\mathfrak{D},\mathcal{Q},*,\bigcirc)$ be a NRBMS and*

$$\begin{aligned}\mathfrak{B}(\vartheta,\delta,k\tau)&\ge\mathfrak{B}(\vartheta,\delta,\tau),\\ \mathfrak{D}(\vartheta,\delta,k\tau)&\le\mathfrak{D}(\vartheta,\delta,\tau),\\ \mathcal{Q}(\vartheta,\delta,k\tau)&\le\mathcal{Q}(\vartheta,\delta,\tau),\end{aligned}\tag{20}$$

for all $\vartheta,\delta\in E$, $0<k<1$, and $\tau>0$; then, $\vartheta=\delta$.

Proof. It is immediate from (f), (k), and (p). □

Theorem 1 (Banach contraction theorem in neutrosophic rectangular b-metric spaces). *Let $(E,\mathfrak{B},\mathfrak{D},\mathcal{Q},*,\bigcirc)$ be a NRBMS with $b\ge1$ such that*

$$\begin{aligned}&\lim_{\tau\longrightarrow\infty}\mathfrak{B}(\vartheta,\delta,\tau)=1,\\ &\lim_{\tau\longrightarrow\infty}\mathfrak{D}(\vartheta,\delta,\tau)=0,\\ &\lim_{\tau\longrightarrow\infty}\mathcal{Q}(\vartheta,\delta,\tau)=0\text{ for all }\vartheta,\delta\in E.\end{aligned}\tag{21}$$

Let Ψ: $E\longrightarrow E$ be a mapping satisfying

$$\begin{aligned}\mathfrak{B}(\Psi\vartheta,\Psi\delta,k\tau)&\ge\mathfrak{B}(\vartheta,\delta,\tau),\\ \mathfrak{D}(\Psi\vartheta,\Psi\delta,k\tau)&\le\mathfrak{D}(\vartheta,\delta,\tau),\\ \mathcal{Q}(\Psi\vartheta,\Psi\delta,k\tau)&\le\mathcal{Q}(\vartheta,\delta,\tau),\end{aligned}\tag{22}$$

for all $\vartheta,\delta\in E$ and $k\in[0,1/b)$. Then, Ψ has a unique fixed point.

Proof. Fix an arbitrary point $\zeta_0 \in E$, and for $n = 0, 1, 2, \ldots$, start an iterative process $\zeta_{n+1} = \Psi\zeta_n$. Successively applying inequality (22), we get for all $n, \tau > 0$,

$$\begin{aligned} \mathfrak{B}(\zeta_n, \zeta_{n+1}, \tau) &\geq \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{k^n}\right), \\ \mathfrak{D}(\zeta_n, \zeta_{n+1}, \tau) &\leq \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{k^n}\right), \\ \mathcal{Q}(\zeta_n, \zeta_{n+1}, \tau) &\leq \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{k^n}\right). \end{aligned} \tag{23}$$

Since $(E, \mathfrak{B}, \mathfrak{D}, \mathcal{Q}, *, \bigcirc)$ is a NRBMS for the sequence $\{\zeta_n\}$, write $\tau = (\tau/3) + (\tau/3) + (\tau/3)$ and use the rectangular inequalities given in (e), (j), and (o) on $\mathfrak{B}(\zeta_n, \zeta_{n+p}, \tau)$, $\mathfrak{D}(\zeta_n, \zeta_{n+p}, \tau)$, and $\mathfrak{Q}(\zeta_n, \zeta_{n+p}, \tau)$ in the following cases. □

Case 1. If p is odd, then $p = 2m + 1$ where $m \in \{1, 2, 3, \ldots\}$, and we have

$$\begin{aligned} \mathfrak{B}(\zeta_n, \zeta_{n+2m+1}, \tau) \geq{}& \mathfrak{B}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+2}, \zeta_{n+2m+1}, \frac{\tau}{3b}\right) \\ \geq{}& \mathfrak{B}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\ & * \mathfrak{B}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) * \mathfrak{B}\left(\zeta_{n+4}, \zeta_{n+2m+1}, \frac{\tau}{(3b)^2}\right) \\ \geq{}& \mathfrak{B}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\ & * \mathfrak{B}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) * \mathfrak{B}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3}\right) * \cdots * \mathfrak{B}\left(\zeta_{n+2m}, \zeta_{n+2m+1}, \frac{\tau}{(3b)^m}\right), \\ \mathfrak{D}(\zeta_n, \zeta_{n+2m+1}, \tau) \leq{}& \mathfrak{D}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+2}, \zeta_{n+2m+1}, \frac{\tau}{3b}\right) \\ \leq{}& \mathfrak{D}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\ & \bigcirc \mathfrak{D}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+4}, \zeta_{n+2m+1}, \frac{\tau}{(3b)^2}\right) \\ \leq{}& \mathfrak{D}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\ & \bigcirc \mathfrak{D}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3}\right) \bigcirc \cdots \bigcirc \mathfrak{D}\left(\zeta_{n+2m}, \zeta_{n+2m+1}, \frac{\tau}{(3b)^m}\right), \\ \mathcal{Q}(\zeta_n, \zeta_{n+2m+1}, \tau) \leq{}& \mathcal{Q}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+2}, \zeta_{n+2m+1}, \frac{\tau}{3b}\right) \\ \leq{}& \mathcal{Q}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\ & \bigcirc \mathcal{Q}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+4}, \zeta_{n+2m+1}, \frac{\tau}{(3b)^2}\right) \\ \leq{}& \mathcal{Q}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\ & \bigcirc \mathcal{Q}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3}\right) \bigcirc \cdots * \bigcirc\left(\zeta_{n+2m}, \zeta_{n+2m+1}, \frac{\tau}{(3b)^m}\right). \end{aligned} \tag{24}$$

Using (23) in the above inequalities, we deduce

$$
\begin{aligned}
\mathfrak{B}(\zeta_n, \zeta_{n+2m+1}, \tau) &\geq \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^{n+1}}\right) * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+2}}\right) \\
&\quad * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+3}}\right) * \mathfrak{B}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3 k^{n+4}}\right) * \cdots * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^m k^{n+m}}\right) \\
&\geq \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)k^n}\right) * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^n}\right) \\
&\quad * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^{n+1}}\right) * \mathfrak{B}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3bk)^3 k^{n+1}}\right) * \cdots * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^m k^{n+m}}\right), \\
\mathfrak{D}(\zeta_n, \zeta_{n+2m+1}, \tau) &\leq \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^{n+1}}\right) \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+2}}\right) \\
&\quad \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+3}}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3 k^{n+4}}\right) \bigcirc \cdots \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^m k^{n+m}}\right) \\
&\leq \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)k^n}\right) \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^n}\right) \\
&\quad \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^{n+1}}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3bk)^3 k^{n+1}}\right) \bigcirc \cdots \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^m k^{n+m}}\right), \\
\mathcal{Q}(\zeta_n, \zeta_{n+2m+1}, \tau) &\leq \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^{n+1}}\right) \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+2}}\right) \\
&\quad \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+3}}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3 k^{n+4}}\right) \bigcirc \cdots \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^m k^{n+m}}\right) \\
&\leq \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)k^n}\right) \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^n}\right) \\
&\quad \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^{n+1}}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3bk)^3 k^{n+1}}\right) \bigcirc \cdots \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^m k^{n+m}}\right).
\end{aligned}
\tag{25}
$$

Case 2. If p is even, then $p = 2m; m \in \{1, 2, 3, \ldots\}$; then, we have

$$
\begin{aligned}
\mathfrak{B}(\zeta_n, \zeta_{n+2m}, \tau) &\geq \mathfrak{B}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+2}, \zeta_{n+2m}, \frac{\tau}{3b}\right) \\
&\geq \mathfrak{B}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\
&\quad * \mathfrak{B}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) * \mathfrak{B}\left(\zeta_{n+4}, \zeta_{n+2m}, \frac{\tau}{(3b)^2}\right) \\
&\geq \mathfrak{B}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\
&\quad * \mathfrak{B}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) * \mathfrak{B}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3}\right) * \cdots * \mathfrak{B}\left(\zeta_{n+2m-4}, \zeta_{n+2m-3}, \frac{\tau}{(3b)^{m-1}}\right) \\
&\quad * \mathfrak{B}\left(\zeta_{n+2m-3}, \zeta_{n+2m-2}, \frac{\tau}{(3b)^{m-1}}\right) * \mathfrak{B}\left(\zeta_{n+2m-2}, \zeta_{n+2m}, \frac{\tau}{(3b)^{m-1}}\right),
\end{aligned}
$$

$$\begin{aligned}
\mathfrak{D}(\zeta_n, \zeta_{n+2m}, \tau) \leq{}& \mathfrak{D}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+2}, \zeta_{n+2m}, \frac{\tau}{3b}\right) \\
\leq{}& \mathfrak{D}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\
&\bigcirc \mathfrak{D}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+4}, \zeta_{n+2m}, \frac{\tau}{(3b)^2}\right) \\
\leq{}& \mathfrak{D}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\
&\bigcirc \mathfrak{D}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3}\right) \bigcirc \cdots \bigcirc \mathfrak{D}\left(\zeta_{n+2m-4}, \zeta_{n+2m-3}, \frac{\tau}{(3b)^{m-1}}\right) \\
&\bigcirc \mathfrak{D}\left(\zeta_{n+2m-3}, \zeta_{n+2m-2}, \frac{\tau}{(3b)^{m-1}}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+2m-2}, \zeta_{n+2m}, \frac{\tau}{(3b)^{m-1}}\right), \\
\mathcal{Q}(\zeta_n, \zeta_{n+2m}, \tau) \leq{}& \mathcal{Q}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+2}, \zeta_{n+2m}, \frac{\tau}{3b}\right) \\
\leq{}& \mathcal{Q}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\
&\bigcirc \mathcal{Q}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+4}, \zeta_{n+2m}, \frac{\tau}{(3b)^2}\right) \\
\leq{}& \mathcal{Q}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+1}, \zeta_{n+2}, \frac{\tau}{3b}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+2}, \zeta_{n+3}, \frac{\tau}{(3b)^2}\right) \\
&\bigcirc \mathcal{Q}\left(\zeta_{n+3}, \zeta_{n+4}, \frac{\tau}{(3b)^2}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3}\right) \bigcirc \cdots \bigcirc \left(\zeta_{n+2m-4}, \zeta_{n+2m-3}, \frac{\tau}{(3b)^{m-1}}\right) \\
&\bigcirc \mathcal{Q}\left(\zeta_{n+2m-3}, \zeta_{n+2m-2}, \frac{\tau}{(3b)^{m-1}}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+2m-2}, \zeta_{n+2m}, \frac{\tau}{(3b)^{m-1}}\right).
\end{aligned} \tag{26}$$

Using (23) in the above inequalities, we deduce

$$\begin{aligned}
\mathfrak{B}(\zeta_n, \zeta_{n+2m}, \tau) \geq{}& \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^{n+1}}\right) * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+2}}\right) \\
&* \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+3}}\right) * \mathfrak{B}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3 k^{n+4}}\right) * \cdots * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^{m-1} k^{n+2m-2}}\right) \\
\geq{}& \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)k^n}\right) * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^n}\right) \\
&* \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^{n+1}}\right) * \mathfrak{B}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3bk)^3 k^{n+1}}\right) * \cdots * \mathfrak{B}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^{m-1} k^{n+m-1}}\right),
\end{aligned}$$

$$\begin{aligned}
\mathfrak{D}(\zeta_n, \zeta_{n+2m}, \tau) \leq{} & \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^{n+1}}\right) \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+2}}\right) \\
& \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+3}}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3 k^{n+4}}\right) \bigcirc \cdots \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^{m-1} k^{n+2m-2}}\right) \\
\leq{} & \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)k^n}\right) \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^n}\right) \\
& \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^{n+1}}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3bk)^3 k^{n+1}}\right) \bigcirc \cdots \bigcirc \mathfrak{D}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^{m-1} k^{n+m-1}}\right), \\
\mathcal{Q}(\zeta_n, \zeta_{n+2m}, \tau) \leq{} & \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^{n+1}}\right) \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+2}}\right) \\
& \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^2 k^{n+3}}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3b)^3 k^{n+4}}\right) \bigcirc \cdots \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3b)^{m-1} k^{n+2m-2}}\right) \\
\leq{} & \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{3bk^n}\right) \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)k^n}\right) \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^n}\right) \\
& \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^2 k^{n+1}}\right) \bigcirc \mathcal{Q}\left(\zeta_{n+4}, \zeta_{n+5}, \frac{\tau}{(3bk)^3 k^{n+1}}\right) \bigcirc \cdots \bigcirc \mathcal{Q}\left(\zeta_0, \zeta_1, \frac{\tau}{(3bk)^{m-1} k^{n+m-1}}\right).
\end{aligned} \tag{27}$$

Therefore, from Cases 1 and 2 and together with (21), it follows that, for all $p \in \{1, 2, 3, \ldots\}$, we have

$$\begin{aligned}
&\lim_{n \longrightarrow \infty} \mathfrak{B}(\zeta_n, \zeta_{n+p}, \tau) = 1 * 1 * \cdots * 1 = 1, \\
&\lim_{n \longrightarrow \infty} \mathfrak{D}(\zeta_n, \zeta_{n+p}, \tau) = 0 \bigcirc 0 \bigcirc \cdots \bigcirc 0 = 0, \\
&\lim_{n \longrightarrow \infty} \mathfrak{D}(\zeta_n, \zeta_{n+p}, \tau) = 0 \bigcirc 0 \bigcirc \cdots \bigcirc 0 = 0.
\end{aligned} \tag{28}$$

Hence, $\{\zeta_n\}$ is a Cauchy sequence. Since $(E, \mathfrak{B}, \mathfrak{D}, \mathcal{Q}, *, \bigcirc)$ is a complete NRBMS, there exists $u \in E$ such that $\lim_{n \longrightarrow \infty} \zeta_n = u$.

Now, we examine that u is a fixed point of Ψ.

$$\begin{aligned}
\mathfrak{B}(u, \Psi u, \tau) \geq{} & \mathfrak{B}\left(u, \zeta_n, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n+1}, \Psi u, \frac{\tau}{3b}\right) \\
\geq{} & \mathfrak{B}\left(u, \zeta_n, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\Psi\zeta_{n-1}, \Psi\zeta_n, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\Psi\zeta_n, \Psi u, \frac{\tau}{3b}\right) \\
\geq{} & \mathfrak{B}\left(u, \zeta_n, \frac{\tau}{3b}\right) * \mathfrak{B}\left(\zeta_{n-1}, \zeta_n, \frac{\tau}{3bk}\right) * \mathfrak{B}\left(\zeta_n, u, \frac{\tau}{3bk}\right) \\
\longrightarrow{} & 1 * 1 * 1 = 1 \text{ as } n \longrightarrow \infty, \\
\mathfrak{D}(u, \Psi u, \tau) \leq{} & \mathfrak{D}\left(u, \zeta_n, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_n, \zeta_{n+1}, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n+1}, \Psi u, \frac{\tau}{3b}\right) \\
\leq{} & \mathfrak{D}\left(u, \zeta_n, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\Psi\zeta_{n-1}, \Psi\zeta_n, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\Psi\zeta_n, \Psi u, \frac{\tau}{3b}\right) \\
\leq{} & \mathfrak{D}\left(u, \zeta_n, \frac{\tau}{3b}\right) \bigcirc \mathfrak{D}\left(\zeta_{n-1}, \zeta_n, \frac{\tau}{3bk}\right) \bigcirc \mathfrak{D}\left(\zeta_n, u, \frac{\tau}{3bk}\right) \\
\longrightarrow{} & 0 \bigcirc 0 \bigcirc 0 = 0 \text{ as } n \longrightarrow \infty,
\end{aligned}$$

$$\begin{aligned}\mathcal{Q}(u,\Psi u,\tau) &\leq \mathcal{Q}\left(u,\zeta_n,\frac{\tau}{3b}\right)\bigcirc\mathcal{Q}\left(\zeta_n,\zeta_{n+1},\frac{\tau}{3b}\right)\bigcirc\mathcal{Q}\left(\zeta_{n+1},\Psi u,\frac{\tau}{3b}\right)\\ &\leq \mathcal{Q}\left(u,\zeta_n,\frac{\tau}{3b}\right)\bigcirc\mathcal{Q}\left(\Psi\zeta_{n-1},\Psi\zeta_n,\frac{\tau}{3b}\right)\bigcirc\mathcal{Q}\left(\Psi\zeta_n,\Psi u,\frac{\tau}{3b}\right)\\ &\leq \mathcal{Q}\left(u,\zeta_n,\frac{\tau}{3b}\right)\bigcirc\mathcal{Q}\left(\zeta_{n-1},\zeta_n,\frac{\tau}{3bk}\right)\bigcirc\mathcal{Q}\left(\zeta_n,u,\frac{\tau}{3bk}\right)\\ &\longrightarrow 0\bigcirc 0\bigcirc 0 = 0 \text{ as } n\longrightarrow\infty,\end{aligned} \tag{29}$$

which shows that u is a fixed point of Ψ.

Now, we show the uniqueness.

Assume v is another fixed point of Ψ for some $v \in E$; then,

$$\begin{aligned}\mathfrak{B}(v,u,\tau) &= \mathfrak{B}(\Psi v,\Psi u,\tau)\geq\mathfrak{B}\left(v,u,\frac{\tau}{k}\right) = \mathfrak{B}\left(\Psi v,\Psi u,\frac{\tau}{k}\right)\\ &\geq\mathfrak{B}\left(v,u,\frac{\tau}{k^2}\right)\geq\cdots\geq\mathfrak{B}\left(v,u,\frac{\tau}{k^n}\right)\longrightarrow 1 \text{ as } n\longrightarrow\infty,\\ \mathfrak{D}(v,u,\tau) &= \mathfrak{D}(\Psi v,\Psi u,\tau)\leq\mathfrak{D}\left(v,u,\frac{\tau}{k}\right) = \mathfrak{D}\left(\Psi v,\Psi u,\frac{\tau}{k}\right)\\ &\leq\mathfrak{D}\left(v,u,\frac{\tau}{k^2}\right)\leq\cdots\leq\mathfrak{D}\left(v,u,\frac{\tau}{k^n}\right)\longrightarrow 0 \text{ as } n\longrightarrow\infty,\\ \mathcal{Q}(v,u,\tau) &= \mathcal{Q}(\Psi v,\Psi u,\tau)\leq\mathcal{Q}\left(v,u,\frac{\tau}{k}\right) = \mathcal{Q}\left(\Psi v,\Psi u,\frac{\tau}{k}\right)\\ &\leq\mathcal{Q}\left(v,u,\frac{\tau}{k^2}\right)\leq\cdots\leq\mathcal{Q}\left(v,u,\frac{\tau}{k^n}\right)\longrightarrow 0 \text{ as } n\longrightarrow\infty.\end{aligned} \tag{30}$$

Thus, $u = v$. Hence, the fixed point is unique.

Remark 13. In Theorem 1, if we take $b = 1$, then it will become a Banach contraction theorem in the sense of NRMS.

Example 10. Let $E = [0,1]$, and define $\mathfrak{B},\mathfrak{D},\mathcal{Q}: E\times E\times[0,\infty)\longrightarrow[0,1]$ by

$$\begin{aligned}\mathfrak{B}(\vartheta,\delta,\tau) &= \frac{\tau}{\tau+|\vartheta-\delta|^2},\\ \mathfrak{D}(\vartheta,\delta,\tau) &= 1-\frac{\tau}{\tau+|\vartheta-\delta|^2},\\ \mathcal{Q}(\vartheta,\delta,\tau) &= \frac{|\vartheta-\delta|^2}{\tau},\end{aligned} \tag{31}$$

for all $\vartheta,\delta\in E$ and $\tau>0$, with CTN $\zeta * b = \zeta.b$ and CTCN $\zeta\bigcirc b = \max\{\zeta, b\}$. Then, it is easy to see that $(E,\mathfrak{B},\mathfrak{D}, *\bigcirc)$ is a complete NRBMS.

Define $\Psi: E \longrightarrow E$ by $\Psi(\vartheta) = 1 - 2^{-\vartheta}/3$. Then,

$$\begin{aligned}
\mathfrak{B}(\Psi\vartheta, \Psi\delta, k\tau) &= \mathfrak{B}\left(\frac{1-2^{-\vartheta}}{3}, \frac{1-2^{-\delta}}{3}, k\tau\right) = \frac{k\tau}{k\tau + \left|\left(1 - 2^{-\vartheta}/3\right) - \left(1 - 2^{-\delta}/3\right)\right|^2} \\
&= \frac{9k\tau}{9k\tau + |2^{-\vartheta} - 2^{-\delta}|^2} \geq \frac{9k\tau}{9k\tau + |\vartheta - \delta|^2} \geq \frac{\tau}{\tau + |\vartheta - \delta|^2} = \mathfrak{B}(\vartheta, \delta, \tau), \\
\mathfrak{D}(\Psi\vartheta, \Psi\delta, k\tau) &= \mathfrak{D}\left(\frac{1-2^{-\vartheta}}{3}, \frac{1-2^{-\delta}}{3}, k\tau\right) = 1 - \frac{k\tau}{k\tau + \left|\left(1 - 2^{-\vartheta}/3\right) - \left(1 - 2^{-\delta}/3\right)\right|^2} \\
&= 1 - \frac{9k\tau}{9k\tau + |2^{-\vartheta} - 2^{-\delta}|^2} \leq 1 - \frac{9k\tau}{9k\tau + |\vartheta - \delta|^2} \leq 1 - \frac{\tau}{\tau + |\vartheta - \delta|^2} = \mathfrak{D}(\vartheta, \delta, \tau), \\
\mathcal{Q}(\Psi\vartheta, \Psi\delta, k\tau) &= \mathcal{Q}\left(\frac{1-2^{-\vartheta}}{3}, \frac{1-2^{-\delta}}{3}, k\tau\right) = \frac{\left|\left(1 - 2^{-\vartheta}/3\right) - \left(1 - 2^{-\delta}/3\right)\right|^2}{k\tau} \\
&= \frac{\left|2^{-\vartheta} - 2^{-\delta}\right|^2}{9k\tau} \leq \frac{|\vartheta - \delta|^2}{9k\tau} \leq \frac{|\vartheta - \delta|^2}{\tau} = \mathcal{Q}(\vartheta, \delta, \tau),
\end{aligned} \tag{32}$$

for all $\vartheta, \delta \in E$, where $k \in [1/2, 1)$. Thus, all the conditions of Theorem 1 are satisfied, and hence, 0 is a unique fixed point of Ψ.

Corollary 1. *Let $(E, \mathfrak{B}_b, \mathfrak{D}_b, *, \bigcirc)$ be a IFRBMS with $b \geq 1$ such that*

$$\begin{aligned}
&\lim_{\tau \longrightarrow \infty} \mathfrak{B}_b(\vartheta, \delta, \tau) = 1, \\
&\lim_{\tau \longrightarrow \infty} \mathfrak{D}_b(\vartheta, \delta, \tau) = 0, \text{ for all } \vartheta, \delta \in E.
\end{aligned} \tag{33}$$

Let $\Psi: E \longrightarrow E$ be a mapping satisfying

$$\begin{aligned}
&\mathfrak{B}_b(\Psi\vartheta, \Psi\delta, k\tau) \geq \mathfrak{B}_b(\vartheta, \delta, \tau), \\
&\mathfrak{D}_b(\Psi\vartheta, \Psi\delta, k\tau) \leq \mathfrak{D}_b(\vartheta, \delta, \tau),
\end{aligned} \tag{34}$$

for all $\vartheta, \delta \in E$ and $k \in [0, 1/b)$. Then, Ψ has a unique fixed point.

Proof. It is clear from Theorem 1. □

Theorem 2. *Let $(E, \mathfrak{B}_h, \mathfrak{D}_h, \mathcal{Q}_h, *, \bigcirc)$ be a NRBMLS with $b \geq 1$ such that*

$$\begin{aligned}
&\lim_{\tau \longrightarrow \infty} \mathfrak{B}_h(\vartheta, \delta, \tau) = 1, \\
&\lim_{\tau \longrightarrow \infty} \mathfrak{D}_h(\vartheta, \delta, \tau) = 0, \\
&\lim_{\tau \longrightarrow \infty} \mathcal{Q}_h(\vartheta, \delta, \tau) = 0 \text{ for all } \vartheta, \delta \in E.
\end{aligned} \tag{35}$$

Let $\Psi: E \longrightarrow E$ be a mapping satisfying

$$\begin{aligned}
&\mathfrak{B}_h(\Psi\vartheta, \Psi\delta, k\tau) \geq \mathfrak{B}_h(\vartheta, \delta, \tau), \\
&\mathfrak{D}_h(\Psi\vartheta, \Psi\delta, k\tau) \leq \mathfrak{D}_h(\vartheta, \delta, \tau), \\
&\mathcal{Q}_h(\Psi\vartheta, \Psi\delta, k\tau) \leq \mathcal{Q}_h(\vartheta, \delta, \tau),
\end{aligned} \tag{36}$$

for all $\vartheta, \delta \in E$ and $k \in [0, 1/b)$. Then, Ψ has a unique fixed point.

Proof. It is easy to show on the lines of Theorems 1 and 2 in [14]. □

4. Application

In this section, we present an application to the integral equation of Theorem 1. In particular, we show the existence of the solution of an integral equation of the form

$$\vartheta(j) = g(j) + \int_0^j F(j, r, \vartheta(r))\mathrm{d}r, \tag{37}$$

for all $j \in [0, l]$ where $l > 0$. Let $C([0, l], \mathbb{R})$ be the space of all continuous functions defined on $[0, l]$ with CTN $\zeta * b = \zeta.b$ and CTCN $\zeta \bigcirc b = \max\{\zeta, b\}$ for all $\zeta, b \in [0, 1]$, and define a complete NRBMS by

$$\begin{aligned}
&\mathfrak{B}(\vartheta, \delta, \tau) = \sup_{j \in [0,l]} \frac{\tau}{\tau + |\vartheta(j) - \delta(j)|^2}, \\
&\mathfrak{D}(\vartheta, \delta, \tau) = \sup_{j \in [0,l]} \frac{|\vartheta(j) - \delta(j)|^2}{\tau + |\vartheta(j) - \delta(j)|^2}, \\
&\mathcal{Q}(\vartheta, \delta, \tau) = \sup_{j \in [0,l]} \frac{|\vartheta(j) - \delta(j)|^2}{\tau} \text{ for all } \vartheta, \delta \in C([0, l], \mathbb{R}) \text{ and } \tau > 0.
\end{aligned} \tag{38}$$

Theorem 3. *Let* $\Psi: C([0,l],\mathbb{R}) \longrightarrow C([0,l],\mathbb{R})$ *be the integral operator given by*

$$\Psi(\vartheta(j)) = g(j) + \int_0^j F(j,r,\vartheta(r))\mathrm{d}r, \quad g \in C([0,l],\mathbb{R}), \tag{39}$$

where $F \in C([0,l]\times[0,l]\times\mathbb{R},\mathbb{R})$ *satisfies the following conditions:*

(i) *There exists* $f: [0,l]\times[0,l] \longrightarrow [0,+\infty]$ *such that, for all* $r, j \in [0,l]$, $f(j,r) \in L^1([0,l],\mathbb{R})$ *and for all* $\vartheta, \delta \in C([0,l],\mathbb{R})$, *we have*

$$|F(j,r,\vartheta(r)) - F(j,r,\delta(r))|^2 \le f^2(j,r)|\vartheta(r)-\delta(r)|^2. \tag{40}$$

(ii) Also,

$$\sup_{j\in[0,l]} \int_0^j f^2(j,r)\mathrm{d}r \le k < 1. \tag{41}$$

Then, the integral equation has the solution $\vartheta_* \in C([0,l],\mathbb{R})$.

Proof. For all $\vartheta, \delta \in C([0,l],\mathbb{R})$, we have

$$\begin{aligned}
\mathfrak{B}(\Psi)(\vartheta(j),\Psi(\delta(j)),k\tau) &= \sup_{j\in[0,l]} \frac{k\tau}{k\tau + |\Psi(\vartheta(j)) - \Psi(\delta(j))|^2} \\
&\ge \sup_{j\in[0,l]} \frac{k\tau}{k\tau + \int_0^j |F(j,r,\vartheta(r)) - F(j,r,\delta(r))|^2\mathrm{d}r} \\
&\ge \sup_{j\in[0,l]} \frac{k\tau}{k\tau + \int_0^j f^2(j,r)|\vartheta(r)-\delta(r)|^2\mathrm{d}r} \\
&\ge \frac{k\tau}{k\tau + |\vartheta(r)-\delta(r)|^2 \sup_{j\in[0,l]} \int_0^j f^2(j,r)\mathrm{d}r} \\
&\ge \frac{k\tau}{k\tau + |\vartheta(r)-\delta(r)|^2} \ge \frac{\tau}{\tau + |\vartheta(r)-\delta(r)|^2} = \mathfrak{B}(\vartheta,\delta,\tau), \\
\mathfrak{D}(\Psi)(\vartheta(j),\Psi(\delta(j)),k\tau) &= \sup_{j\in[0,l]} \frac{|\Psi(\vartheta(j)) - \Psi(\delta(j))|^2}{k\tau + |\Psi(\vartheta(j)) - \Psi(\delta(j))|^2} \\
&\le \sup_{j\in[0,l]} \frac{\int_0^j |F(j,r,\vartheta(r)) - F(j,r,\delta(r))|^2\mathrm{d}r}{k\tau + \int_0^j |F(j,r,\vartheta(r)) - F(j,r,\delta(r))|^2\mathrm{d}r} \\
&\le \sup_{j\in[0,l]} \frac{\int_0^j f^2(j,r)|\vartheta(r)-\delta(r)|^2\mathrm{d}r}{k\tau + \int_0^j f^2(j,r)|\vartheta(r)-\delta(r)|^2\mathrm{d}r} \\
&\le \frac{|\vartheta(r)-\delta(r)|^2 \sup_{j\in[0,l]} \int_0^j f^2(j,r)\mathrm{d}r}{k\tau + |\vartheta(r)-\delta(r)|^2 \sup_{j\in[0,l]} \int_0^j f^2(j,r)\mathrm{d}r} \\
&\le \frac{|\vartheta(r)-\delta(r)|^2}{k\tau + |\vartheta(r)-\delta(r)|^2} \le \frac{|\vartheta(r)-\delta(r)|^2}{\tau + |\vartheta(r)-\delta(r)|^2} = \mathfrak{D}(\vartheta,\delta,\tau),
\end{aligned}$$

$$
\begin{aligned}
\mathcal{Q}(\Psi)(\vartheta(j),\Psi(\delta(j)),k\tau) &= \sup_{j\in[0,l]} \frac{|\Psi(\vartheta(j))-\Psi(\delta(j))|^2}{k\tau} \\
&\le \sup_{j\in[0,l]} \frac{\int_0^j |F(j,r,\vartheta(r))-F(j,r,\delta(r))|^2 \mathrm{d}r}{k\tau} \\
&\le \sup_{j\in[0,l]} \frac{\int_0^j f^2(j,r)|\vartheta(r)-\delta(r)|^2 \mathrm{d}r}{k\tau} \\
&\le \frac{|\vartheta(r)-\delta(r)|^2 \sup_{j\in[0,l]} \int_0^j f^2(j,r)\mathrm{d}r}{k\tau} \\
&\le \frac{|\vartheta(r)-\delta(r)|^2}{k\tau} \le \frac{|\vartheta(r)-\delta(r)|^2}{\tau} = \mathcal{Q}(\vartheta,\delta,\tau).
\end{aligned} \tag{42}
$$

Hence, ϑ_* is a fixed point of Ψ, which is the solution of integral equation (37). □

5. Conclusion

The aim of this study is to present the notions of intuitionistic fuzzy rectangular metric spaces, intuitionistic fuzzy rectangular metric-like spaces, intuitionistic fuzzy rectangular b-metric spaces, intuitionistic fuzzy rectangular b-metric-like spaces, neutrosophic rectangular metric spaces, neutrosophic rectangular metric-like spaces, neutrosophic rectangular b-metric spaces, and neutrosophic rectangular b-metric-like spaces and prove the Banach contraction theorem in these spaces, and nontrivial examples and an application to the integral equation are also given to support our results. Due to a diverse range of applications of the metric fixed point theory in mathematics, science, and economics, it is researched widely. Different types of fixed point results for single- and multivalued mappings can be proven in the sense of the above-defined notions in this manuscript. Also, presented notions can be extended in different mathematical structures, i.e., intuitionistic fuzzy controlled rectangular metric spaces, intuitionistic fuzzy triple controlled rectangular metric spaces, neutrosophic extended rectangular metric spaces, etc.

Data Availability

No such data were used for this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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