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Research Article

Analytical Analysis of Fractional-Order Newell-Whitehead-Segel Equation: A Modified Homotopy Perturbation Transform Method

Naveed Iqbal,[1] **Abeer M. Albalahi**,[1] **Mohammed S. Abdo**,[2] **and Wael W. Mohammed**[1,3]

[1]*Department of Mathematics, College of Science, University of Ha'il, Ha'il 2440, Saudi Arabia*
[2]*Department of Mathematics, Hodeidah University, P.O. Box 3114, Al-Hudaydah, Yemen*
[3]*Faculty of Science, Mansoura University, Mansoura 35516, Egypt*

Correspondence should be addressed to Naveed Iqbal; naveediqbal1989@yahoo.com
and Mohammed S. Abdo; msabdo@hoduniv.net.ye

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This paper has applied a hybrid method called the homotopy perturbation transformation technique to solve fractional-order Newell-Whitehead-Segel equations. First, we used the Yang transformation to the given problem, and then, the homotopy perturbation technique was implemented to complete the procedure of the suggested method. The proposed method is simplified and requires a small calculation to achieve the solution to the targeted problem. Moreover, the derived results are in close contact with the exact results of the given models. Three examples are solved to confirm and show the feasibility of the present scenario. The findings obtained from the proposed procedure have also been in excellent alignment with other technique outcomes. It is shown that the proposed approach is effective, consistent, and straightforward to apply to various relevant problems in engineering and science.

1. Introduction

Fractional calculus (FC) is the generic generalization of integer-order calculus to arbitrary order integration and differentiation with noninteger order. FC dates back to 1695, when L'Hopital addressed Leibniz a letter regarding the probable significance of $(d^{1/2}\Phi(\vartheta))/(d\vartheta^{1/2})$, which represents the semiderivative of $\Phi(\vartheta)$ with respect to ϑ. Due to its advantageous qualities such as linearity, analyticity, and nonlocality, FC has recently become a strong tool. Furthermore, several pioneering references for various definitions of FC are accessible, laying the framework for FC. With the rapid advancement of digital computer expertise, many academics have begun to focus on the FC's theory and applications. The idea of fractional-order calculus has been used to signal processing, chaos theory, optics, noisy environments, and other disciplines [1, 2]. The numerical and analytical solutions for differential equations of any order that evolved as a result of the preceding processes are critical in explaining the characteristics of nonlinear issues encountered in everyday life [3–6].

The investigation of fractional-order integrals and derivatives is an exciting study of fractional calculus. It has increased the broad consideration of scientists in the last two decades. It has uncommon implementations in different areas of engineering and the medical field. In this specific situation, Riemann-Liouville is the pioneer who gave the ideas of fractional derivatives and integrals [7–11]. From these definitions, the scientists began to think and characterize fractional equations, which are expansions and speculations of Riemann-Liouville ideas [12–14]. For instance, Caputo [15] gave an improved formula in the area of fractional calculus. The Caputo derivative helps display wonders that assess collaborations inside the past and issues with nonlocal properties [16, 17].

In recent decades, nonlinear differential equation solutions have become increasingly important. Numerous scholars have used different methods to solve a variety of problems [18–20]. To accomplish the objective of profoundly exact solutions, numerous creators outline other procedures, for example, finite difference technique [21], Adomian decay method [22], finite element method [23], generalized differential change method [24], fractional differential change method [25], homotopy perturbation method [26, 27], iterative procedure [28, 29], and homotopy analysis strategy [30]. In the past few years, various researchers used the variational iteration method (VIM), the differential transform method, and the Adomian decomposition method (ADM) [31–33]. One of the most significant abundancy equations is the Newell-Whitehead-Segel equation [34–36], which portrays the stripe design in two-dimensional frameworks. Additionally, this equation was implemented to various assortment frameworks, e.g., nonlinear optics, Faraday instability, chemical reactions, Rayleigh-Benard convection, and organic frameworks. The Newell-Whitehead Segel equation's estimated solutions were introduced by differential transformation method, Adomian decomposition, and reduced differential transformation.

In this present work, the homotopy perturbation transformation method is used to analyze the result of fractional Newell-Whitehead-Segel equations. The solutions to the presented problems demonstrate the accuracy of the proposed technique. With the use of different fractional-order figures, the solutions of the recommended methodology are analyzed and shown. The technique presented here is useful in investigating various fractional partial differential equations.

2. Preliminaries Concepts

Definition 1. If the Caputo-Fabrizio derivative is defined as [37]

$$ {}^{CF}D_{\bar{t}}^{\beta}[\mathbb{P}(\bar{t})] = \frac{N(\beta)}{1-\beta}\int_0^{\bar{t}} \mathbb{P}'(\varrho)K(\bar{t},\varrho)d\varrho, n-1<\beta\le n, \quad (1)$$

$N(\beta)$ is the normalization function with $N(0)=N(1)=1$
.

$$ {}^{CF}D_{\bar{t}}^{\beta}[\mathbb{P}(\bar{t})] = \frac{N(\beta)}{1-\beta}\int_0^{\bar{t}} [\mathbb{P}(\bar{t})-\mathbb{P}(\varrho)]K(\bar{t},\varrho)d\varrho. \quad (2)$$

Definition 2. The fractional Caputo-Fabrizio integral is defined as [37]

$$ {}^{CF}I_{\bar{t}}^{\beta}[\mathbb{P}(\bar{t})] = \frac{1-\beta}{N(\beta)}\mathbb{P}(\bar{t}) + \frac{\beta}{N(\beta)}\int_0^{\bar{t}} \mathbb{P}(\varrho)d\varrho, \bar{t}\ge 0, \beta\in(0,1]. \quad (3)$$

Definition 3. For $N(\beta)=1$, the following solution represents the Laplace transform Caputo-Fabrizio derivative [37]:

$$ L\left[{}^{CF}D_{\bar{t}}^{\beta}[\mathbb{P}(\bar{t})]\right] = \frac{sL[\mathbb{P}(\bar{t})-\mathbb{P}(0)]}{s+\beta(1-s)}. \quad (4)$$

Definition 4. The Yang transformation of $\mathbb{P}(\bar{t})$ is given as [38]

$$ \mathbb{Y}[\mathbb{P}(\bar{t})] = \chi(s) = \int_0^{\infty} \mathbb{P}(\bar{t})e^{-\bar{t}-/s}d\bar{t}, \bar{t}>0. \quad (5)$$

Remark 5. Yang transformation of few helpful functions is defined as below.

$$ \begin{aligned} Y[1] &= s, \\ Y[\bar{t}] &= s^2, \\ Y\left[\bar{t}^i\right] &= \Gamma(i+1)s^{i+1}. \end{aligned} \quad (6)$$

Lemma 6: Laplace-Yang duality. *Let the Laplace transformation of $\mathbb{P}(\bar{t})$ be $F(s)$; then, $\chi(s)=F(1/s)$ [39].*

Proof. From Equation (5), we can gain a different manifestation of the Yang transformation by putting $\bar{t}/s=\zeta$ as

$$ L[\mathbb{P}(\bar{t})] = \chi(s) = s\int_0^{\infty} \mathbb{P}(s\zeta)e^{\zeta}d\zeta.\zeta>0. \quad (7)$$

Since $L[\mathbb{P}(\bar{t})]=F(s)$, this implies that

$$ F(s) = L[\mathbb{P}(\bar{t})] = \int_0^{\infty} \mathbb{P}(\bar{t})e^{-s\bar{t}}d\bar{t}. \quad (8)$$

Putting $\bar{t}=\zeta/s$ in (8), we have

$$ F(s) = \frac{1}{s}\int_0^{\infty} \mathbb{P}\left(\frac{\zeta}{s}\right)e^{\zeta}d\zeta. \quad (9)$$

Thus, from Equation (7), we obtain

$$ F(s) = \chi\left(\frac{1}{s}\right). \quad (10)$$

Also from Equations (5) and (8), we get

$$ F\left(\frac{1}{s}\right) = \chi(s). \quad (11)$$

The links (10) and (11) show the duality connection among the Yang and Laplace transform. □

Lemma 6. *Let $\mathbb{P}(\bar{t})$ be a continuous function; then, the Yang transform Caputo-Fabrizio derivative of $\mathbb{P}(\bar{t})$ is given as [39]*

$$ Y[\mathbb{P}(\bar{t})] = \frac{Y[\mathbb{P}(\bar{t})-s\mathbb{P}(0)]}{1+\beta(s-1)}. \quad (12)$$

Proof. The fractional Laplace transform Caputo-Fabrizio is defined as

$$ L[\mathbb{P}(\bar{t})] = \frac{L[s\mathbb{P}(\bar{t})-\mathbb{P}(0)]}{s+\beta(1-s)}. \quad (13)$$

Also, we have the link among Yang and Laplace properties, i.e., $\chi(s) = F(1/s)$. To obtain the important solution, we substitute s by $1/s$ in Equation (13), and we have

$$Y[\mathbb{P}(\bar{t})] = \frac{(1/s)Y[\mathbb{P}(\bar{t}) - \mathbb{P}(0)]}{(1/s) + \beta(1-(1/s))},$$
$$Y[\mathbb{P}(\bar{t})] = \frac{Y[\mathbb{P}(\bar{t}) - s\mathbb{P}(0)]}{1+\beta(s-1)}. \tag{14}$$

The proof is completed. □

3. General Implementation of the Given Methodology

Consider the fractional partial differential equation

$$^{CF}D_{\bar{t}}^{\beta} V(\varphi,\bar{t}) M V(\varphi,\bar{t}) + N[\varphi]V(\varphi,\bar{t}) = h(\varphi,\bar{t}), \quad \bar{t} > 0, 0 < \beta \le 1,$$
$$V(\varphi, 0) = g(\varphi), \quad \varphi \in \Re, \tag{15}$$

where $D_{\bar{t}}^{\beta} = \partial^{\beta}/\partial\bar{t}^{\beta}$ Caputo's derivative, $M[\varphi]$ and $N[\varphi]$ are the linear and nonlinear operators, respectively, and $h(\varphi,\bar{t})$ is source function.

Implementing Yang transformation to (15), we have

$$Y\left[D_{\bar{t}}^{\beta}\mathcal{V}(\varphi,\bar{t}) + M\mathcal{V}(\varphi,\bar{t}) + N[\varphi]\mathcal{V}(\varphi,\bar{t})\right] = Y[h(\varphi,\bar{t})], \bar{t} > 0, \quad 0 < \beta \le 1,$$
$$\mathcal{V}(\varphi,\bar{t}) = sg(\varphi) + (1+\beta(s-1))Y[h(\varphi,\bar{t})] - (1+\beta(s-1))Y[M\mathcal{V}(\varphi,\bar{t}) + N[\varphi]\mathcal{V}(\varphi,\bar{t})]. \tag{16}$$

Now, applying the inverse Yang transformation, we have

$$\mathcal{V}(\varphi,\bar{t}) = sg(\varphi) - Y^{-1}[(1+\beta(s-1))Y[M\mathcal{V}(\varphi,\bar{t}) + N[\varphi]\mathcal{V}(\varphi,\bar{t})]], \tag{17}$$

where

$$F(\varphi,\bar{t}) = Y^{-1}[sg(\varphi) + (1+\beta(s-1))Y\{h(\varphi,\bar{t})\}] = g(\varphi) + Y^{-1}[(1+\beta(s-1))Y\{h(\varphi,\bar{t})\}]. \tag{18}$$

Now, perturbation method having parameter P is defined as

$$\mathcal{V}(\varphi,\bar{t}) = \sum_{j=0}^{\infty} P^j \mathcal{V}_j(\varphi,\bar{t}). \tag{19}$$

$P \in [0,1]$; P is a parameter of perturbation. Then, nonlinear terms can be expressed as

$$\mathcal{V}(\varphi,\bar{t}) = \sum_{j=0}^{\infty} P^j H_j(\mathcal{V}), \tag{20}$$

where He's polynomials H_n of the form $\mathcal{V}_0, \mathcal{V}_1, \mathcal{V}_2 \cdots, \mathcal{V}_n$ can be calculated as

$$_n(\mathcal{V}_0, \mathcal{V}_1, \cdots, \mathcal{V}_n) = \frac{1}{\gamma(n+1)} D_P^j \left[N\left(\sum_{j=0}^{\infty} P^i \mathcal{V}_i\right)\right]_{P=0}, \tag{21}$$

where $D_P^j = \partial^j/\partial P^j$. Applying relations (19) and (20) in (16) and constructing the homotopy, we have

$$\sum_{j=0}^{\infty} P^j \mathcal{V}_j(\varphi,\bar{t}) = F(\varphi,\bar{t}) - P \times \left[Y^{-1}\left\{(1+\beta(s-1))Y\left\{M\sum_{j=0}^{\infty} P^j\mathcal{V}_j(\varphi,\bar{t}) + \sum_{j=0}^{\infty} P^j H_j(\mathcal{V})\right\}\right\}\right]. \tag{22}$$

Both sides comparing coefficient of P, we have

$$P^0 : \mathcal{V}_0(\varphi,\bar{t}) = F(\varphi,\bar{t}),$$
$$P^1 : \mathcal{V}_1(\varphi,\bar{t}) = Y^{-1}[(1+\beta(s-1))Y(M\mathcal{V}_0(\varphi,\bar{t}) + H_0(\mathcal{V}))],$$
$$P^2 : \mathcal{V}_2(\varphi,\bar{t}) = Y^{-1}[(1+\beta(s-1))Y(M\mathcal{V}_1(\varphi,\bar{t}) + H_1(\mathcal{V}))],$$
$$\vdots$$
$$P^j : \mathcal{V}_n(\varphi,\bar{t}) = Y^{-1}[(1+\beta(s-1))Y(M\mathcal{V}_{k-1}(\varphi,\bar{t}) + H_{k-1}(\mathcal{V}))],$$
$$k > 0, k \in N. \tag{23}$$

Easily calculating the component of $\mathcal{V}_j(\varphi,\bar{t})$, by taking $P \longrightarrow 1$, we get

$$\mathcal{V}(\varphi,\bar{t}) = \lim_{M\longrightarrow\infty} \sum_{j=1}^{M} \mathcal{V}_j(\varphi,\bar{t}). \tag{24}$$

4. Test Problems

Four cases of nonlinear diffusion equations are presented to demonstrate the suggested technique's capability and reliability.

Case 1. Consider the fractional-order Newell-Whitehead-Segel equation

$$^{CF}D_{\bar{t}}^{\beta}\mathcal{V} = \mathcal{V}_{\varphi\varphi} + 2\mathcal{V} - 3\mathcal{V}^2, 0 < \beta \le 1, \tag{25}$$

with initial conditions

$$\mathscr{V}(\varphi, 0) = \lambda. \tag{26}$$

Taking Yang transform of (25), we have

$$\begin{aligned} Y[\mathscr{V}(\varphi,\bar{t})] &= s\mathscr{V}^{(0)}(\varphi,0) + (1+\beta(s-1))Y\left(\mathscr{V}_{\varphi\varphi} + 2\mathscr{V} - 3\mathscr{V}^2\right), \\ Y[\mathscr{V}(\varphi,\bar{t})] &= s\lambda + (1+\beta(s-1))\left[Y\left(\mathscr{V}_{\varphi\varphi} + 2\mathscr{V} - 3\mathscr{V}^2\right)\right]. \end{aligned} \tag{27}$$

Taking the inverse Yang transformation, we obtain

$$\mathscr{V}(\varphi,\bar{t}) = \lambda + Y^{-1}\left[(1+\beta(s-1))\left\{Y\left(\mathscr{V}_{\varphi\varphi} + 2\mathscr{V} - 3\mathscr{V}^2\right)\right\}\right]. \tag{28}$$

Now, applying the above-mentioned homotopy perturbation method as in (8), we have

$$\sum_{j=0}^{\infty} P^j \mathscr{V}_j(\varphi,\bar{t}) = \lambda + P\left[Y^{-1}\left\{(1+\beta(s-1))Y\left(\left(\sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi,\bar{t})\right)_{\varphi\varphi} + 2\sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi,\bar{t}) - 3\left(\sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi,\bar{t})\right)^2\right)\right\}\right]. \tag{29}$$

Comparing the same power coefficient of P, we get

$$\begin{aligned} P^0 &: \mathscr{V}_0(\varphi,\bar{t}) = \lambda, \\ P^1 &: \mathscr{V}_1(\varphi,\bar{t}) = Y^{-1}\left((1+\beta(s-1))Y\left[\mathscr{V}_{0\varphi\varphi} + \mathscr{V}_0 - \mathscr{V}_0^2\right]\right) \\ &\qquad = \lambda(2-3\lambda)\{1+\beta\bar{t}-\beta\}, \\ P^2 &: \mathscr{V}_2(\varphi,\bar{t}) = Y^{-1}\left((1+\beta(s-1))Y\left[\mathscr{V}_{1\varphi\varphi} + \mathscr{V}_1 - 2\mathscr{V}_0\mathscr{V}_1\right]\right) \\ &\qquad = 2\lambda(2-3\lambda)(1-3\lambda)\left\{(1-\beta)2\beta\bar{t} + (1-\beta)^2 + \frac{\beta^2\bar{t}^2}{2}\right\}, \\ &\ \vdots \end{aligned} \tag{30}$$

We get the convergence series type solution as

$$\begin{aligned} \mathscr{V}(\varphi,\bar{t}) &= \mathscr{V}_0 + \mathscr{V}_1 + \mathscr{V}_2 + \cdots = \lambda + \lambda(2-3\lambda)\{1+\beta\bar{t}-\beta\} \\ &\quad + 2\lambda(2-3\lambda)(1-3\lambda)\left\{(1-\beta)2\beta\bar{t} + (1-\beta)^2 + \frac{\beta^2\bar{t}^2}{2}\right\} + \cdots. \end{aligned} \tag{31}$$

The exact result of Equation (25) is given as

$$\mathscr{V}(\varphi,\bar{t}) = \frac{(-2/3)\lambda\exp^{2\bar{t}}}{(-2/3) + \lambda - \lambda\exp^{2\bar{t}}}. \tag{32}$$

Case 2. Consider the fractional-order Newell-Whitehead-Segel equation

$${}^{CF}D_{\bar{t}}^{\beta}\mathscr{V} = \mathscr{V}_{\varphi\varphi} + \mathscr{V}(1-\mathscr{V}), 0 < \beta \le 1, \tag{33}$$

with initial conditions

$$\mathscr{V}(\varphi,0) = \frac{1}{\left(1+\exp^{\varphi/\sqrt{6}}\right)^2}. \tag{34}$$

Taking Yang transform of (33), we have

$$\begin{aligned} Y[\mathscr{V}(\varphi,\bar{t})] &= s\mathscr{V}^{(0)}(\varphi,0) + (1+\beta(s-1))Y\left(\mathscr{V}_{\varphi\varphi} + \mathscr{V}(1-\mathscr{V})\right), \\ Y[\mathscr{V}(\varphi,\bar{t})] &= \frac{s}{\left(1+\exp^{\varphi/\sqrt{6}}\right)^2} + (1+\beta(s-1))\left[Y\left(\mathscr{V}_{\varphi\varphi} + \mathscr{V}(1-\mathscr{V})\right)\right]. \end{aligned} \tag{35}$$

Taking the inverse Yang transformation, we obtain

$$\mathscr{V}(\varphi,\bar{t}) = \frac{1}{\left(1+\exp^{\varphi/\sqrt{6}}\right)^2} + Y^{-1}\left[(1+\beta(s-1))\left\{Y\left(\mathscr{V}_{\varphi\varphi} + \mathscr{V}(1-\mathscr{V})\right)\right\}\right]. \tag{36}$$

Now, applying the homotopy perturbation method, we get

$$\sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi,\bar{t}) = \frac{1}{\left(1+\exp^{\varphi/\sqrt{6}}\right)^2} + P\left[Y^{-1}\left\{(1+\beta(s-1))Y\left(\left(\sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi,\bar{t})\right)_{\varphi\varphi} + \sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi,\bar{t})\left(1 - \sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi,\bar{t})\right)\right)\right\}\right]. \tag{37}$$

Comparing the same power coefficient of P, we get

$$P^0 : \mathscr{V}_0(\varphi, \bar{t}) = \frac{1}{\left(1 + \exp^{\varphi/\sqrt{6}}\right)^2},$$

$$P^1 : \mathscr{V}_1(\varphi, \bar{t}) = Y^{-1}\left((1 + \beta(s-1))Y\left[\mathscr{V}_{0\varphi\varphi} + \mathscr{V}_0 - \mathscr{V}_0^2\right]\right) = \frac{5}{3}\frac{\exp^{\varphi/\sqrt{6}}}{\left(1 + \exp^{\varphi/\sqrt{6}}\right)^3}\{1 + \beta\bar{t} - \beta\},$$

$$P^2 : \mathscr{V}_2(\varphi, \bar{t}) = Y^{-1}\left((1 + \beta(s-1))Y\left[\mathscr{V}_{1\varphi\varphi} + \mathscr{V}_1 - 2\mathscr{V}_0\mathscr{V}_1\right]\right) = \frac{25}{18}\left(\frac{\exp^{\varphi/\sqrt{6}}\left(-1 + 2\exp^{\varphi/\sqrt{6}}\right)}{\left(1 + \exp^{\varphi/\sqrt{6}}\right)^4}\right) \cdot \left\{(1-\beta)2\beta\bar{t} + (1-\beta)^2 + \frac{\beta^2\bar{t}^2}{2}\right\},$$

$$\vdots \tag{38}$$

We get the convergence series type solution as

$$\mathscr{V}(\varphi, \bar{t}) = \mathscr{V}_0 + \mathscr{V}_1 + \mathscr{V}_2 + \cdots = \frac{1}{\left(1 + \exp^{\varphi/\sqrt{6}}\right)^2} + \frac{5}{3}\frac{\exp^{\varphi/\sqrt{6}}}{\left(1 + \exp^{\varphi/\sqrt{6}}\right)^3}\{1 + \beta\bar{t} - \beta\} + \frac{25}{18}\left(\frac{\exp^{\varphi/\sqrt{6}}\left(-1 + 2\exp^{\varphi/\sqrt{6}}\right)}{\left(1 + \exp^{\varphi/\sqrt{6}}\right)^4}\right) \cdot \left\{(1-\beta)2\beta\bar{t} + (1-\beta)^2 + \frac{\beta^2\bar{t}^2}{2}\right\} + \cdots. \tag{39}$$

Then, $\beta = 1$; we obtain the exact result of Equation (33):

$$\mathscr{V}(\varphi, \bar{t}) = \left(\frac{1}{1 + \exp^{(\varphi/\sqrt{6}) - ((5/6)\bar{t})}}\right)^2. \tag{40}$$

Case 3. Consider the fractional-order Newell-Whitehead-Segel equation

$$^{CF}D_{\bar{t}}^{\beta}\mathscr{V} = \mathscr{V}_{\varphi\varphi} + \mathscr{V} - \mathscr{V}^4, 0 < \beta \leq 1, \tag{41}$$

with initial conditions

$$\mathscr{V}(\varphi, 0) = \frac{1}{\left(1 + \exp^{3\varphi/\sqrt{10}}\right)^{2/3}}. \tag{42}$$

Taking Yang transform of (41), we have

$$Y[\mathscr{V}(\varphi, \bar{t})] = s\mathscr{V}^{(0)}(\varphi, 0) + (1 + \beta(s-1))Y\left(\mathscr{V}_{\varphi\varphi} + \mathscr{V} - \mathscr{V}^4\right),$$

$$Y[\mathscr{V}(\varphi, \bar{t})] = \frac{s}{\left(1 + \exp^{3\varphi/\sqrt{10}}\right)^{2/3}} + (1 + \beta(s-1))\left[Y\left(\mathscr{V}_{\varphi\varphi} + \mathscr{V} - \mathscr{V}^4\right)\right]. \tag{43}$$

Taking the inverse Yang transform, we get

$$\mathscr{V}(\varphi, \bar{t}) = \frac{1}{\left(1 + \exp^{3\varphi/\sqrt{10}}\right)^{2/3}} + Y^{-1}\left[(1 + \beta(s-1))\left\{Y\left(\mathscr{V}_{\varphi\varphi} + \mathscr{V} - \mathscr{V}^4\right)\right\}\right]. \tag{44}$$

Now, applying the HPM, we get

$$\sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi, \bar{t}) = \frac{1}{\left(1 + \exp^{3\varphi/\sqrt{10}}\right)^{2/3}} + P\left[Y^{-1}\left\{(1 + \beta(s-1))Y\left(\left(\sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi, \bar{t})\right)_{\varphi\varphi} + \sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi, \bar{t}) - \left(\sum_{j=0}^{\infty} P^j\mathscr{V}_j(\varphi, \bar{t})\right)^4\right)\right\}\right]. \tag{45}$$

Comparing the same power coefficient of P, we get

$$P^0 : \mathscr{V}_0(\varphi, \bar{t}) = \frac{1}{\left(1 + \exp^{3\varphi/\sqrt{10}}\right)^{2/3}},$$

$$P^1 : \mathscr{V}_1(\varphi, \bar{t}) = Y^{-1}\left((1 + \beta(s-1))Y\left[\mathscr{V}_{0\varphi\varphi} + \mathscr{V}_0 - \mathscr{V}_0^4\right]\right) = \frac{7}{5}\frac{\exp^{(3/\sqrt{10})\varphi}}{\left(1 + \exp^{3\varphi/\sqrt{10}}\right)^{5/3}}\{1 + \beta\bar{t} - \beta\},$$

$$P^2 : \mathscr{V}_2(\varphi, \bar{t}) = Y^{-1}\left((1 + \beta(s-1))Y\left[\mathscr{V}_{1\varphi\varphi} + \mathscr{V}_1 - 4\mathscr{V}_0^3\mathscr{V}_1\right]\right) = \frac{49}{50}\frac{\left(2\exp^{(3/\sqrt{10})\varphi} - 3\right)\exp^{(3/\sqrt{10})\varphi}}{\left(1 + \exp^{3\varphi/\sqrt{10}}\right)^{8/3}} \cdot \left\{(1-\beta)2\beta\bar{t} + (1-\beta)^2 + \frac{\beta^2\bar{t}^2}{2}\right\},$$

$$\vdots \tag{46}$$

We get the convergence series type solution as

$$\begin{aligned}\mathscr{V}(\varphi,\bar{t}) &= \mathscr{V}_0+\mathscr{V}_1+\mathscr{V}_2+\cdots\\ &= \frac{1}{\left(1+\exp^{3\varphi/\sqrt{10}}\right)^{2/3}}+\frac{7}{5}\frac{\exp^{(3/\sqrt{10})\varphi}}{\left(1+\exp^{3\varphi/\sqrt{10}}\right)^{5/3}}\{1+\beta\bar{t}-\beta\}\\ &\quad+\frac{49}{50}\frac{\left(2\exp^{(3/\sqrt{10})\varphi}-3\right)\exp^{(3/\sqrt{10})\varphi}}{\left(1+\exp^{3\varphi/\sqrt{10}}\right)^{8/3}}\\ &\quad\cdot\left\{(1-\beta)2\beta\bar{t}+(1-\beta)^2+\frac{\beta^2\bar{t}^2}{2}\right\}+\cdots.\end{aligned}\tag{47}$$

Then, $\beta=1$; we get the exact result:

$$\mathscr{V}(\varphi,\bar{t})=\left(\frac{1}{2}\tanh\left(-\frac{3}{2\sqrt{10}}\left(\varphi-\frac{7}{\sqrt{10}}\bar{t}\right)\right)\right).\tag{48}$$

Case 4. Consider the fractional-order Newell-Whitehead-Segel equation

$${}^{CF}D_{\bar{t}}^{\beta}\mathscr{V}=\mathscr{V}_{\varphi\varphi}+3\mathscr{V}-4\mathscr{V}^3,\ 0<\beta\le 1,\tag{49}$$

with initial conditions

$$\mathscr{V}(\varphi,0)=\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}}{\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}}.\tag{50}$$

Taking Yang transform of (36), we have

$$\begin{aligned}Y[\mathscr{V}(\varphi,\bar{t})] &= s\mathscr{V}^{(0)}(\varphi,0)+(1+\beta(s-1))Y\left(\mathscr{V}_{\varphi\varphi}+3\mathscr{V}-4\mathscr{V}^3\right),\\ Y[\mathscr{V}(\varphi,\bar{t})] &= s\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}}{\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}}\\ &\quad+(1+\beta(s-1))\left[Y\left(\mathscr{V}_{\varphi\varphi}+3\mathscr{V}-4\mathscr{V}^3\right)\right].\end{aligned}\tag{51}$$

Taking the inverse Yang transform, we get

$$\begin{aligned}\mathscr{V}(\varphi,\bar{t}) &= \sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}}{\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}}\\ &\quad+Y^{-1}\left[(1+\beta(s-1))\left\{Y\left(\mathscr{V}_{\varphi\varphi}+3\mathscr{V}-4\mathscr{V}^3\right)\right\}\right].\end{aligned}\tag{52}$$

Now, applying the HPM, we get

$$\sum_{j=0}^{\infty}P^j\mathscr{V}_j(\varphi,\bar{t})=\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}}{\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}}+P\left[Y^{-1}\left\{(1+\beta(s-1))Y\left(\left(\sum_{j=0}^{\infty}P^j\mathscr{V}_j(\varphi,\bar{t})\right)_{\varphi\varphi}+3\sum_{j=0}^{\infty}P^j\mathscr{V}_j(\varphi,\bar{t})-4\left(\sum_{j=0}^{\infty}P^j\mathscr{V}_j(\varphi,\bar{t})\right)^3\right)\right\}\right].\tag{53}$$

Comparing the same power coefficient of P, we get

$$\begin{aligned}P^0 &: \mathscr{V}_0(\varphi,\bar{t})=\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}}{\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}},\\ P^1 &: \mathscr{V}_1(\varphi,\bar{t})=Y^{-1}\left((1+\beta(s-1))Y\left[\mathscr{V}_{0\varphi\varphi}+\mathscr{V}_0-\mathscr{V}_0^4\right]\right)\\ &\quad=\frac{9}{2}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}\exp^{(\sqrt{6}/2)\varphi}}{\left(\exp^{\sqrt{6}\varphi}+\exp^{\sqrt{6}/2\varphi}\right)^2}\{1+\beta\bar{t}-\beta\},\\ P^2 &: \mathscr{V}_2(\varphi,\bar{t})=Y^{-1}\left((1+\beta(s-1))Y\left[\mathscr{V}_{1\varphi\varphi}+\mathscr{V}_1-4\mathscr{V}_0^3\mathscr{V}_1\right]\right)\\ &\quad=\frac{81}{4}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}\exp^{(\sqrt{6}/2)\varphi}\left(-\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}\right)}{\left(\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}\right)^3}\\ &\quad\cdot\left\{(1-\beta)2\beta\bar{t}+(1-\beta)^2+\frac{\beta^2\bar{t}^2}{2}\right\},\\ &\vdots\end{aligned}\tag{54}$$

We get the convergence series type solution as

$$\begin{aligned}\mathscr{V}(\varphi,\bar{t}) &= \mathscr{V}_0+\mathscr{V}_1+\mathscr{V}_2+\cdots\\ \mathscr{V}(\varphi,\bar{t}) &= \sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}}{\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}}\\ &\quad+\frac{9}{2}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}\exp^{(\sqrt{6}/2)\varphi}}{\left(\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}\right)^2}\{1+\beta\bar{t}-\beta\}\\ &\quad+\frac{81}{4}\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}\exp^{(\sqrt{6}/2)\varphi}\left(-\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}\right)}{\left(\exp^{\sqrt{6}\varphi}+\exp^{(\sqrt{6}/2)\varphi}\right)^3}\\ &\quad\cdot\left\{(1-\beta)2\beta\bar{t}+(1-\beta)^2+\frac{\beta^2\bar{t}^2}{2}\right\}+\cdots.\end{aligned}\tag{55}$$

Then, $\beta=1$; we get the exact result as

$$\mathscr{V}(\varphi,\bar{t})=\sqrt{\frac{3}{4}}\frac{\exp^{\sqrt{6}\varphi}}{\exp^{\sqrt{6}\varphi}+\exp^{(((\sqrt{6}/2)\varphi)-((9/2)\bar{t}))}}.\tag{56}$$

5. Graphical Discussion

Using an effective analytic approach, the article was aimed at an analytical solution to the time-fractional Newell-Whitehead-Segel equation. The relevant problems are solved

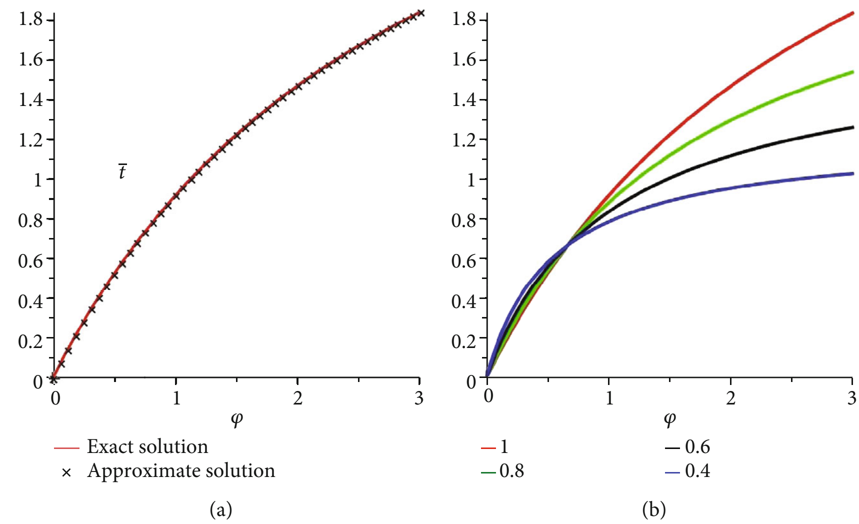

FIGURE 1: (a) Graph of approximate and exact result and (b) the second graph of various fractional order of β of Case 1.

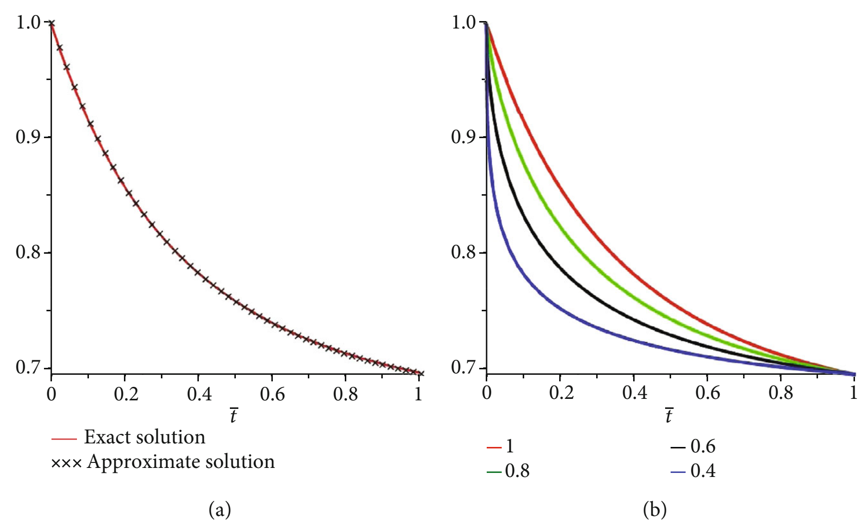

FIGURE 2: (a) The first figure shows the actual and analytical solution, and (b) the second graph is various fractional order of β of Case 1.

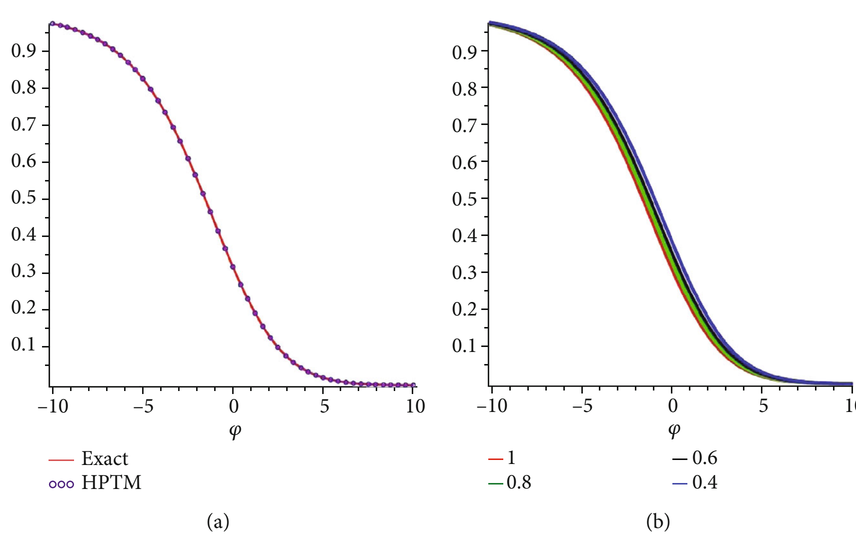

FIGURE 3: (a) The first graph of actual and analytical solution and (b) different fractional order of β of Case 2.

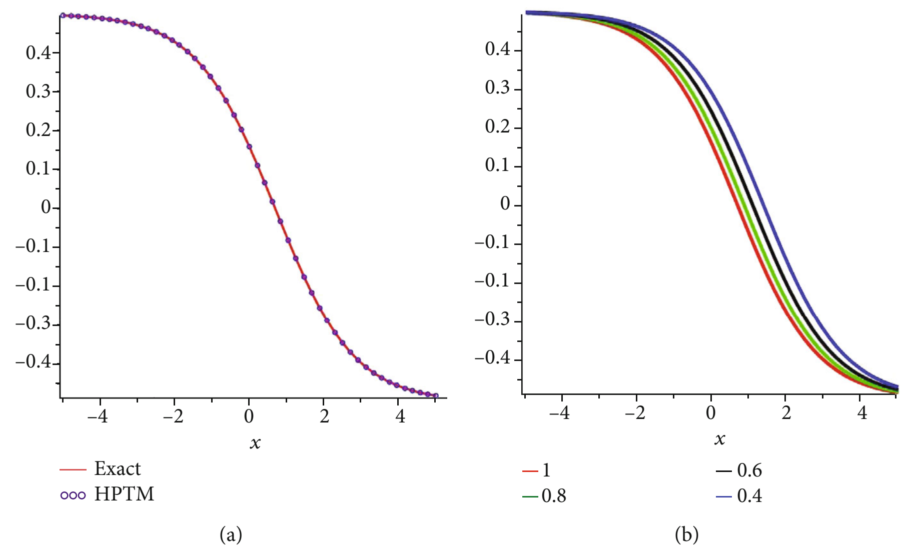

FIGURE 4: (a) The first figure of actual and analytic result and (b) and the second figure of various fractional order of β of Case 3.

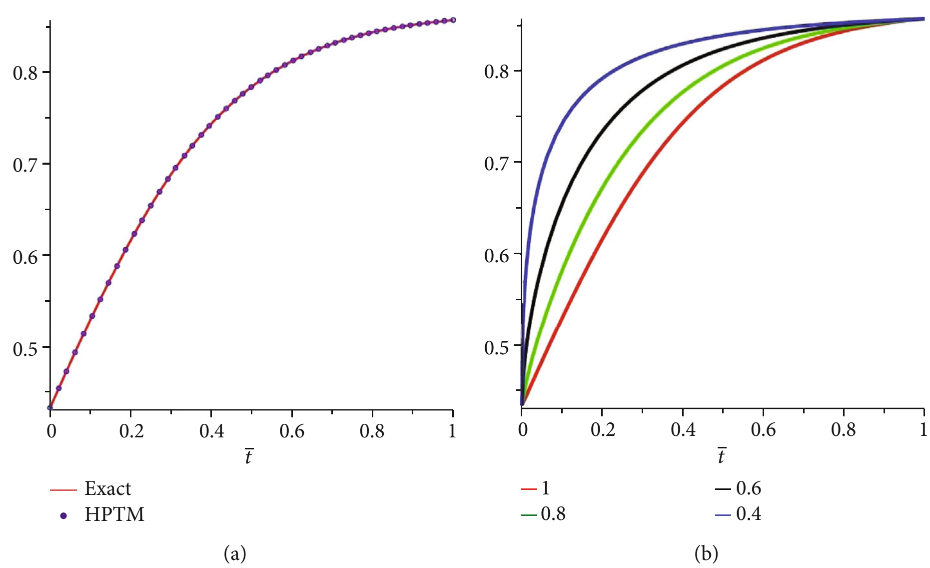

FIGURE 5: (a) The first graph of actual and analytical solution and (b) the second graph of different fractional order of β of Case 4.

using the homotopy perturbation Yang transformation methodology. The solution to certain illustrative problems is offered to test the validity of the proposed strategy. For both fractional and integer-order issues, solution graphs are displayed. Figure 1(a) depicts the precise and approximate solutions of example 1 at $\gamma = 1$, and Figure 1(b) depicts the analytical solutions of several fractional orders of $\beta = 1$, 0.8, 0.6, and 0.4. Figure 2(a) depicts the precise and approximate solutions of example 1 at $\beta = 1$, and Figure 2(b) depicts the analytical solutions of several fractional orders of $\beta = 0.4, 0.6, 0.8,$ and 1 with regard to $\bar{t}$. The precise HPETM solution is quite close to the precise result of the given problems. Figure 3 also shows the exact and HPETM solutions plots from example 2 (a) and (b) calculated at different fractional order $\beta = 0.8$, 0.6, and 0.4. Figures 4 and 5 illustrate a similar graphical examination and discussion of Cases 3 and 4. It has been demonstrated that the proposed strategies have the same accuracy. As fractional-order analysis to integer-order is examined, it is discovered that fractional-order problems are convergent to an integer-order result. A similar phenomenon of fractional-order solutions convergent to integral-order solutions is observed.

6. Conclusion

This article implements the HPTM to solve fractional-order Newell-Whitehead-Segel equations and obtain an analytical result. The HPTM has been an efficient approach to partial differential equations with Caputo operators due to the signed agreement between the approximate and actual results. A comparison was performed to demonstrate that the technique has a small computation size compared to other techniques' computational size. And its rapid convergence indicates that the procedure is accurate and dramatically improves linear and nonlinear partial differential equations.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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