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Research Article

Properties of Meromorphic Spiral-Like Functions Associated with Symmetric Functions

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To consolidate or adapt to many studies on meromorphic functions, we define a new subclass of meromorphic functions of complex order involving a differential operator. The defined function class combines the concept of spiral-like functions with other studies pertaining to subclasses of multivalent meromorphic functions. Inclusion relations, integral representation, geometrical interpretation, coefficient estimates and solution to the Fekete-Szegö problem of the defined classes are the highlights of this present study. Further to keep up with the present direction of research, we extend the study using quantum calculus. Applications of our main results are given as corollaries.

1. Introduction

Let $\mathscr{A}$ be the class of function of the form

$$\chi(\xi) = \xi + \sum_{n=2}^{\infty} a_n \xi^n, \tag{1}$$

which are analytic in the unit disc $\mathbb{E} = \{\xi : |\xi| < 1\}$. Also let $\mathscr{S}$ denote the class of functions $\chi \in \mathscr{A}$ which are univalent in $\mathbb{E}$. The subclasses of $\mathscr{S}$ consisting of functions which map unit disc onto a star-like and convex domain will be symbolized by $\mathscr{S}^*$ and $\mathscr{C}$, respectively. Also let $\mathscr{P}$ denote the class of functions h analytic in the unit disc, given by

$$h(\xi) = 1 + \sum_{n=1}^{\infty} R_n \xi^n, \xi \in \mathbb{E}, R_1 > 0, \tag{2}$$

and satisfies Re $(h(\xi)) > 0$, $\xi \in \mathbb{E}$. For $p \in \mathbb{N} = \{1, 2, \cdots\}$, we let $\mathscr{L}_p$ to denote the class of functions χ of the form

$$\chi(\xi) = \xi^{-p} + \sum_{n=1}^{\infty} d_{n-p} \xi^{n-p}, \tag{3}$$

which are analytic in $\mathbb{E}^* = \{\xi : \xi \in \mathbb{C} \text{ and } 0 < |\xi| < 1\}$. Shi et al. [1] defined the class $\chi(\xi) \in \mathscr{MS}_p(\sigma, \tau)$ if and only if

$$-e^{i\sigma} \frac{\xi \chi'(\xi)}{\chi(\xi)} \prec \frac{pe^{i\sigma} - (2\tau - pe^{-i\sigma})\xi}{1 - \xi} \ (\chi \in \mathscr{L}_p), \tag{4}$$

where $|\sigma| < \lambda/2$ and $\tau > p \cos \sigma$. Here, $\prec$ denotes the usual subordination of analytic function. The class $\mathscr{MS}_p(\sigma, \tau)$ is the meromorphic analogue of the class of p-valent spiral-like functions defined by Uyanik et al. in [2]. Similarly, we let $\mathscr{MC}_p(\sigma, \tau)$ to denote the class of function in $\mathscr{L}_p$ satisfying the condition

$$-e^{i\sigma} \left(1 + \frac{\xi \chi''(\xi)}{\chi'(\xi)}\right) \prec \frac{pe^{i\sigma} - (2\tau - pe^{-i\sigma})\xi}{1 - \xi}. \tag{5}$$

Extending the class of Janowski function ([3]), Aouf [4]

(Equation (4)) (also see [5]) defined the class $h(\xi) \in \mathscr{P}(X, Y, p, \tau)$ if and only if

$$h(\xi) = \frac{p + [pY + (X - Y)(p - \tau)]w(\xi)}{[1 + Yw(\xi)]}, \quad (-1 \le Y < X \le 1, 0 \le \tau < 1), \tag{6}$$

where $w(\xi)$ is the Schwartz function. Motivated by the recent study of Breaz et al. [5] and in view generalizing the superordinate function in (4), Cotîrlă and Karthikeyan in [6] defined and studied the following relation

$$\Delta_\sigma^\tau(\xi) = \frac{\left[\left(1 + Xe^{-2i\sigma}\right)pe^{i\sigma} + \tau(Y - X)\right]h(\xi) + \left[\left(1 - Xe^{-2i\sigma}\right)pe^{i\sigma} - \tau(Y - X)\right]}{[(Y + 1)h(\xi) + (1 - Y)]}, \tag{7}$$

where $-1 \le Y < X \le 1, -\pi/2 < \sigma < \pi/2, \tau > p \cos \sigma$ and $h(\xi) \in \mathscr{P}$.

It is well-known that the function $h(\xi) = 1 + \xi/1 - \xi$ maps the unit disc onto the right half plane. For an admissible choice of the parameter $X = 0.5$, $Y = -0.5$, $p = 1$, $\sigma = \pi/3$, and $\tau = 0.6$, $\Delta_\sigma^\tau(\xi)$ maps unit disc onto a domain which is convex with respect to point 0.5 if $h(\xi) = 1 + \xi/1 - \xi$ (see Figure 1). Similarly, the function $h(\xi) = \xi + \sqrt[3]{1 + \xi^3}$ which is related to the class of functions associated with leaf-like domain (see [7–9]) gets rotated and translated on the impact of $\Delta_\sigma^\tau(\xi)$ (see Figure 2) for a choice of the parameter $X = 0.5$, $Y = -0.5$, $p = 1$, $\sigma = \pi/3$, and $\tau = 0.6$.

Remark 1. The purpose to study $\Delta_\sigma^\tau(\xi)$ was mainly motivated by the study of Karthikeyan et al. [10] and Noor and Malik [11]. Here, we will list some recent studies.

(1) If we let $\sigma = 0$ in (7), then, $\Delta_\sigma^\tau(\xi)$ reduces to

$$\aleph(\xi) = \frac{[(1 + X)p + \tau(Y - X)]h(\xi) + [(1 - X)p - \tau(Y - X)]}{[(Y + 1)h(\xi) + (1 - Y)]}. \tag{8}$$

The function $\aleph(\xi)$ was defined and studied by Breaz et al. in [5].

(2) If we let $X = 1$, $Y = -1$ and $h(\xi) = (1 + \xi)/(1 - \xi)$ in (7), then, $\Delta_\sigma^\tau(\xi)$ reduces to $2\tau - pe^{-i\sigma} + (2(p \cos \sigma - \tau)/1 - \xi)$ (see the superordinate function in (4)).

It is well-known that if $\chi(\xi)$ given by (1) is in $\mathscr{S}$, then, the ℓ-symmetrical function $\left[\chi(\xi^\ell)\right]^{1/\ell}$, (ℓ is a positive integer) is also in $\mathscr{S}$. Let ℓ be a positive integer and $\varepsilon = \exp(2\pi i/\ell)$. For $\chi \in \mathscr{A}$, let

$$\chi_\ell(\xi) = \frac{1}{\ell} \sum_{\nu=0}^{\ell-1} \frac{\chi(\varepsilon^\nu \xi)}{\varepsilon^\nu}. \tag{9}$$

The function χ is said to be star-like with respect to ℓ -symmetric points if it satisfies the condition

$$\operatorname{Re} \frac{\xi \chi'(\xi)}{\chi_\ell(\xi)} > 0. \tag{10}$$

Here, we will let $\mathscr{S}_\ell^s$ to denote the class of star-like functions with respect to ℓ-symmetric points. The class $\mathscr{S}_\ell^s$ was introduced by Sakaguchi [12] in which he showed that all functions in $\mathscr{S}_\ell^s$ are univalent. Note that $\mathscr{S}_1^s = \mathscr{S}^*$.

A function $\chi \in \mathscr{L}_p$ is said to be ℓ-symmetrical if for each $\xi \in \mathbb{E}$

$$\chi(\varepsilon \xi) = \varepsilon^{-p} \chi(\xi), \tag{11}$$

For $\chi \in \mathscr{L}_p$, Equation (9) can be defined by the following equality

$$\chi_\ell(\xi) = \frac{1}{\ell} \sum_{\nu=0}^{\ell-1} \frac{\chi(\varepsilon^\nu \xi)}{\varepsilon^{-\nu p}}, (\ell = 1, 2, 3, \cdots). \tag{12}$$

Now, we extend the operator defined by Selvaraj and Karthikeyan in [13]. Using Hadamard product (or convolution), we define a operator for functions $\chi \in \mathscr{L}_p$ as follows:

$$I_\mu^m(a_1, a_2, \cdots, a_r, c_1, c_2, \cdots, c_s)\chi = \frac{1}{\xi^p} + \sum_{n=1}^{\infty} \left(\frac{\mu}{n + \mu}\right)^m \frac{(a_1)_n (a_2)_n \cdots (a_r)_n}{(c_1)_n (c_2)_n \cdots (c_s)_n} d_{n-p} \frac{\xi^{n-p}}{(n)!}, \tag{13}$$

where $(x)_n$ is the Pochhammer symbol defined by

$$(x)_n = \begin{cases} 1 & \text{if } n = 0 \\ x(x + 1)(x + 2) \cdots (x + n - 1) & \text{if } n \in N_0 = \{1, 2, , \cdots\}. \end{cases} \tag{14}$$

For convenience, we shall henceforth denote

$$I_\mu^m(a_1, a_2, \cdots, a_r, c_1, c_2, \cdots, c_s)\chi = I_\mu^m(a_1, c_1)\chi. \tag{15}$$

Note that in [13], $I_\mu^m(a_1, c_1)\chi$ was defined for $\chi \in \mathscr{L}_1$. Here, we skip the discussion on the necessity of using differential or integral operator, refer to [13–17] and reference provided therein for detailed properties of $I_\mu^m(a_1, c_1)\chi$.

Throughout this paper, we assume that $-1 \le Y < X \le 1$, $-\pi/2 < |\sigma| < \pi/2, \tau > p \cos \sigma, \lambda \ge 1, \quad \ell \in \mathbb{N}, \quad \varepsilon = \exp(2\pi i/\ell)$ and

$$\chi_\ell(m, \mu, a_1, c_1 ; \xi) = \frac{1}{\ell} \sum_{\nu=0}^{\ell-1} \varepsilon^{\nu p} \left[I_\mu^m(a_1, c_1)\chi(\varepsilon^\nu \xi)\right] = \xi^{-p} + \cdots, \tag{16}$$

$$\left(\chi \in \mathscr{L}_p ; \ell = 2, 3, \cdots\right). \tag{17}$$

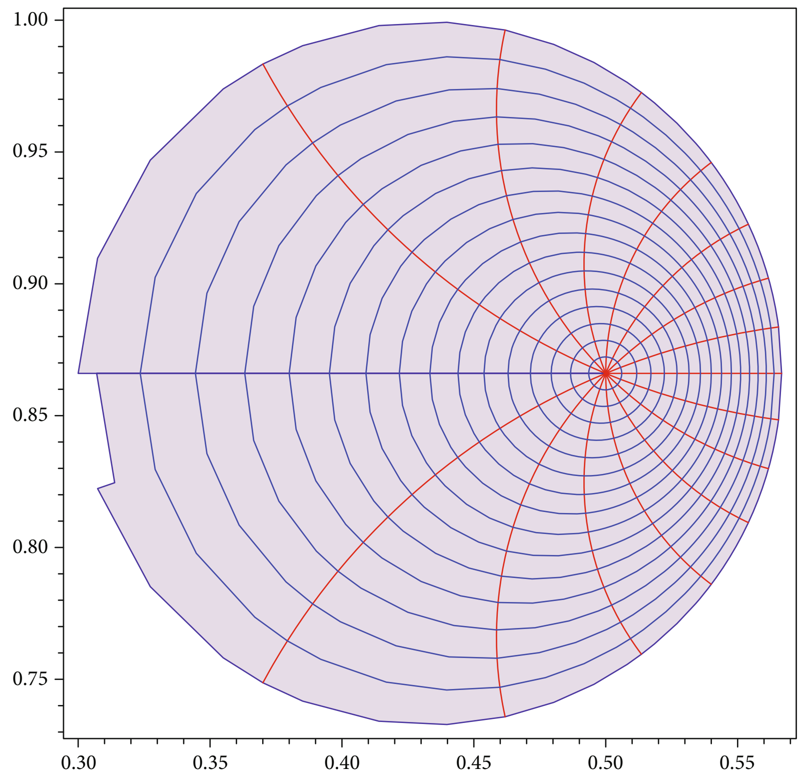

FIGURE 1: The image of the unit disc under the mapping of $\Delta_{\sigma}^{\tau}(\xi)$, if $h(\xi)=1+\xi/1-\xi$.

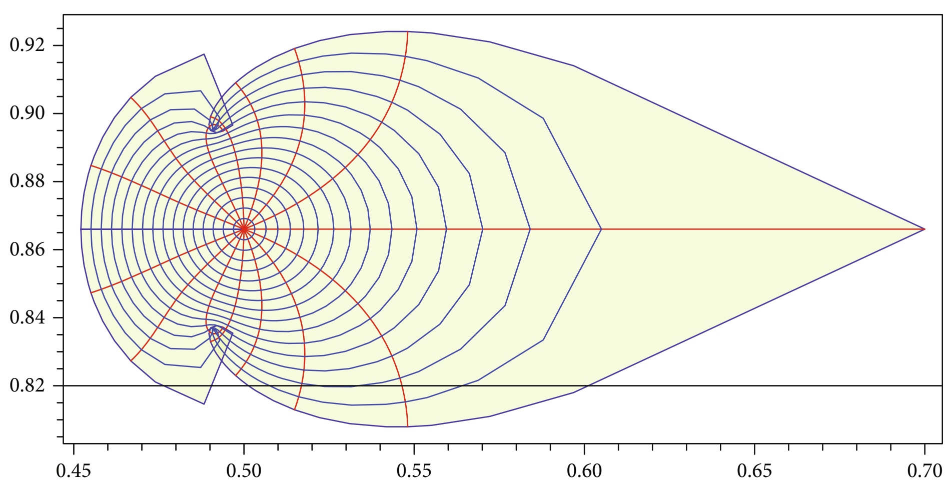

FIGURE 2: The image of the unit disc under the mapping of $\Delta_{\sigma}^{\tau}(\xi)$, if $h(\xi)=\xi+\sqrt[3]{1+\xi^3}$.

1.1. Short Introduction to Quantum Calculus. For $0<q<1$, the Jacksons q-derivative operator is defined by (see [18, 19])

$$\mathfrak{D}_q\chi(\xi) := \begin{cases} \chi'(0), & \text{if } \xi = 0, \\ \dfrac{\chi(\xi)-\chi(q\xi)}{(1-q)\xi}, & \text{if } \xi \neq 0. \end{cases} \tag{18}$$

From (18), if χ has the power series expansion (3), we can easily see that $\mathfrak{D}_q\chi(\xi)=[-p]_q\xi^{-p-1}+\sum_{n=1}^{\infty}[n-p]_q d_{n-p}\xi^{n-p-1}$, for $\xi\neq 0$, where the q *-integer number* $[n]_q$ is defined by

$$[n]_q := \frac{1-q^n}{1-q}, \tag{19}$$

and note that $\lim_{q\longrightarrow 1^-}\mathfrak{D}_q\chi(\xi)=\chi'(\xi)$. Throughout this paper, we let denote

$$\left([n]_q\right)_k := [n]_q[n+1]_q[n+2]_q\cdots[n+k-1]_q. \tag{20}$$

The q-Jackson integral is defined by (see [20])

$$I_q[\chi(\xi)] := \int_0^\xi \chi(t) d_q t = \xi(1-q) \sum_{n=0}^{\infty} q^n \chi(\xi q^n), \tag{21}$$

provided the q-series converges. Further observe that

$$\mathfrak{D}_q I_q \chi(\xi) = \chi(\xi) \text{ and } I_q \mathfrak{D}_q \chi(\xi) = \chi(\xi) - \chi(0), \tag{22}$$

where the second equality holds if χ is continuous at $\xi = 0$. For details pertaining to the significance of univalent function theory in dual with quantum calculus, refer to [21, 22] (also see [23–26]).

Meromorphic multivalent functions have been extensively studied by various authors, but motivation and references of this study are [1, 13, 27–36].

Definition 2. For $-\pi/2 < \sigma < \pi/2$, $\lambda \geq 1$, $\tau \geq p \cos \sigma$, $\mathrm{b} \in \mathbb{C} \setminus \{0\}$ and $\mathrm{I}_\mu^{\mathrm{m}}(\mathrm{a}_1, \mathrm{c}_1)\chi$ defined as in (13), a function χ belongs to the class $\mathscr{MS}_\ell^{\mathrm{m},\lambda}(a_1, c_1; b; h; X, Y)$ if it satisfies

$$e^{i\sigma}\left[p - \frac{1}{b}\left\{\frac{\xi^{(p+1)\lambda - p}\left[I_\mu^m(a_1, c_1)\chi'(\xi)\right]^\lambda}{\chi_\ell(m, \mu, a_1, c_1; \xi)} - (-p)^\lambda\right\}\right] \prec \Delta_\sigma^\tau(\xi), \tag{23}$$

where $\prec$ denotes subordination and $h(\zeta)$ is defined as in (2).

Now, we will define a class replacing ordinary derivative with a quantum derivative in $\mathscr{MS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$.

Definition 3. For $-\pi/3 < \sigma < \pi/2$, $0 \leq \eta \leq 1$, $\tau \geq p \cos \sigma$, $\mathrm{b} \in \mathbb{C} \setminus \{0\}$ and $I_\mu^m(\mathrm{a}_1, \mathrm{c}_1)\chi$ defined as in (13), a function χ belongs to the class $\mathscr{QMS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$ if

$$e^{i\sigma}\left([p]_q - \frac{1}{b}\left\{\frac{\xi^{(p+1)\lambda - p}\left[\mathfrak{D}_q I_\mu^m(a_1, c_1)\chi(\xi)\right]^\lambda}{\chi_\ell(m, \mu, a_1, c_1; \xi)} - \left(-[p]_q\right)^\lambda\right\}\right) \prec Y_q(\sigma, \tau; \xi), \tag{24}$$

where $Y_q(\sigma, \tau; \xi)$ is the $q-$analogue of $\Delta_\sigma^\tau(\xi)$, which is defined by

$$Y_q(\sigma, \tau; \xi) = \frac{\left[\left(1 + Xe^{-2i\sigma}\right)[p]_q e^{i\sigma} + \tau(Y - X)\right]h(\xi) + \left[\left(1 - Xe^{-2i\sigma}\right)[p]_q e^{i\sigma} - \tau(Y - X)\right]}{[(Y+1)h(\xi) + (1 - Y)]}. \tag{25}$$

Remark 4. We note that in the definition of $\mathscr{QMS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$, the operator $I_\mu^m(a_1, c_1)\chi$ and $\chi_\ell(m, \mu, a_1, c_1; \xi)$ are the same as used in $\mathscr{MS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$. We have not used the q-analogue operator as it would require the reader to contend with additional set of parameters.

2. Preliminaries and some Supplementary Results

Here, we will discuss the results which would help us to obtain our main results.

We note that everything in classical calculus cannot be generalized to quantum calculus, notably the chain rule needs adaptation. Hence, logarithmic differentiation needs some application of analysis. In [37], Agrawal and Sahoo obtained the following result on logarithmic differentiation. For $\chi \in \mathscr{A}$ and $0 < q < 1$, we have

$$I_q \frac{\mathfrak{D}_q \chi(\xi)}{\chi(\xi)} = \frac{q-1}{\ln q} \log \chi(\xi), \tag{26}$$

where $I_q \chi$ is the Jackson q-integral, defined as in (21). Similarly, we can see that

$$\mathfrak{D}_q\left[\{\chi(\xi)\}^{\lambda - 1/\lambda}\right] = \frac{\lambda - 1}{\lambda} \mathfrak{D}_q[\chi(\xi)]\{\chi(\xi)\}^{-1/\lambda}. \tag{27}$$

If ν is an integer, then the following identities follow directly from (16):

$$\chi_\ell(m, \mu, a_1, c_1; \varepsilon^\nu \xi) = \varepsilon^{-\nu \mathrm{p}} \chi_\ell(m, \mu, a_1, c_1; \xi). \tag{28}$$

$$\begin{aligned} \chi_\ell'(m, \mu, a_1, c_1; \varepsilon^\nu \xi) &= \varepsilon^{-\nu \mathrm{p} - \nu} \chi_\ell'(m, \mu, a_1, c_1; \xi) \\ &= \frac{1}{\ell} \sum_{\nu=0}^{\ell-1} \varepsilon^{\nu + \nu p} I_\mu^m(a_1, c_1)\chi'(\varepsilon^\nu \xi). \end{aligned} \tag{29}$$

Since q-derivative satisfies the linearity condition, (29) holds if the classical derivative is replaced with quantum derivative. That is,

$$\mathfrak{D}_q[\chi_\ell(m, \mu, a_1, c_1; \varepsilon^\nu \xi)] = \varepsilon^{-\nu \mathrm{p} - \nu} \mathfrak{D}_q[\chi_\ell(m, \mu, a_1, c_1; \xi)]. \tag{30}$$

We now state the following result which will be used to establish the coefficient inequalities.

Lemma 5 (see [38]). *Let $\vartheta(\xi) = 1 + \sum_{n=1}^\infty \vartheta_n \xi^n \in \mathscr{P}$ and also let ν be a complex number, then*

$$\left|\vartheta_2 - \nu \vartheta_1^2\right| \leq 2 \max\{1, |2\nu - 1|\}, \tag{31}$$

the result is sharp for functions given by

$$\vartheta(\xi) = \frac{1 + \xi^2}{1 - \xi^2}, \vartheta(\xi) = \frac{1 + \xi}{1 - \xi}. \tag{32}$$

The Maclaurin series for the function $\Delta_\sigma^\tau(\xi)$ (see [6]) for the function is given by

$$\Delta_\sigma^\tau(\xi) = pe^{i\sigma} + \frac{\left[X\left(pe^{-i\sigma} - \tau\right) - Y\left(pe^{i\sigma} - \tau\right)\right]R_1}{2}\xi + \cdots. \tag{33}$$

If we define the function $\vartheta(\xi)$ by

$$\vartheta(\xi) = 1 + \vartheta_1\xi + \vartheta_2\xi^2 + \cdots = \frac{1 + w(\xi)}{1 - w(\xi)} \prec \frac{1+\xi}{1-\xi}, (\xi \in \mathbb{E}). \quad (34)$$

We note that $\vartheta(0) = 1$ and $\vartheta \in \mathscr{P}$. Using (34), we have

$$\begin{aligned} w(\xi) &= \frac{\vartheta(\xi) - 1}{\vartheta(\xi) + 1} \\ &= \frac{1}{2}\left[\vartheta_1\xi + \left(\vartheta_2 - \frac{\vartheta_1^2}{2}\right)\xi^2 + \left(\vartheta_3 - \vartheta_1\vartheta_2 + \frac{\vartheta_1^3}{4}\right)\xi^3 + \cdots\right]. \end{aligned} \quad (35)$$

For some $h(\xi) = 1 + R_1\xi + R_2\xi^2 + \cdots$, we have

$$\begin{aligned} & p + b\left\{e^{-i\sigma}\Delta_\sigma^\tau[w(\xi)] - p\right\} \\ &= p + \frac{be^{-i\sigma}R_1\vartheta_1\left[X\left(pe^{-i\sigma} - \tau\right) - Y\left(pe^{i\sigma} - \tau\right)\right]}{4}\xi \\ &\quad + \frac{be^{-i\sigma}\left[X\left(pe^{-i\sigma} - \tau\right) - Y\left(pe^{i\sigma} - \tau\right)\right]R_1}{4} \\ &\quad \cdot \left[\vartheta_2 - \vartheta_1^2\left(\frac{(Y+1)R_1 + 2(1 - (R_2/R_1))}{4}\right)\right]\xi^2 + \cdots. \end{aligned} \quad (36)$$

3. Integral Representations and Closure Properties

We begin with the following.

Theorem 6. *Let $\chi \in \mathscr{MS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$, then for $\lambda = 1$, we get for $\xi \in \mathbb{E}^*$*

$$\begin{aligned} & \chi_\ell(m, \mu, a_1, c_1; \xi) \\ &= \xi^p \exp\left\{\frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\int_0^{\varepsilon^\nu\xi}\frac{1}{t}\left[b\left\{p - e^{-i\sigma}\Delta_\sigma^\tau[w(t)]\right\} - 2p\right]dt\right\}. \end{aligned} \quad (37)$$

And for $\lambda > 1$, we have for $\xi \in \mathbb{E}^*$

$$\begin{aligned} \chi_\ell(m, \mu, a_1, c_1; \xi) &= \left(\frac{\lambda - 1}{\lambda}\right) \\ &\cdot \left\{\frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\int_0^\xi\left(\frac{\left[b\left\{p - e^{-i\sigma}\Delta_\sigma^\tau[w(\varepsilon^\nu t)]\right\} + (-p)^\lambda\right]}{t^{(p+1)\lambda - p}}\right)^{1/\lambda}dt\right\}^{\lambda - 1/\lambda}, \end{aligned} \quad (38)$$

where $\chi_\ell(m, \mu, a_1, c_1; \xi)$ is defined by equality (16) and $w(\xi)$ is analytic in $\mathbb{E}$ with $w(0) = 0$ and $|w(\xi)| < 1$.

Proof. Let $\chi \in \mathscr{MS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$. In view of (23), we have

$$\frac{\xi^{(p+1)\lambda - p}\left[I_\mu^m(a_1, c_1)\chi'(\xi)\right]^\lambda}{\chi_\ell(m, \mu, a_1, c_1; \xi)} = b\left\{p - e^{-i\sigma}\Delta_\sigma^\tau[w(\xi)]\right\} + (-p)^\lambda, \quad (39)$$

where $w(\xi)$ is analytic in $\mathbb{E}$ and $w(0) = 0$, $|w(\xi)| < 1$. Substituting ξ by $\varepsilon^\nu\xi$ in the equality (39), respectively, $(\nu = 0, 1, 2, \cdots, \ell - 1, \varepsilon^\ell = 1)$, we have

$$\begin{aligned} & \frac{(\varepsilon^\nu\xi)^{(p+1)\lambda - p}\left[I_\mu^m(a_1, c_1)\chi'(\varepsilon^\nu\xi)\right]^\lambda}{\chi_\ell(m, \mu, a_1, c_1; \varepsilon^\nu\xi)} \\ &= b\left\{p - e^{-i\sigma}\Delta_\sigma^\tau[w(\varepsilon^\nu\xi)]\right\} + (-p)^\lambda. \end{aligned} \quad (40)$$

Using (28) in (40), we get

$$\begin{aligned} & \frac{\xi^{(p+1)\lambda - p}\varepsilon^{\nu(p+1)\lambda}\left[I_\mu^m(a_1, c_1)\chi'(\varepsilon^\nu\xi)\right]^\lambda}{\chi_\ell(m, \mu, a_1, c_1; \xi)} \\ &= b\left\{p - e^{-i\sigma}\Delta_\sigma^\tau[w(\varepsilon^\nu\xi)]\right\} + (-p)^\lambda. \end{aligned} \quad (41)$$

Using the equality (29) in (42), we can get

$$\frac{\varepsilon^{\nu + \nu p}I_\mu^m(a_1, c_1)\chi'(\varepsilon^\nu\xi)}{\left[\chi_\ell(m, \mu, a_1, c_1; \xi)\right]^{1/\lambda}} = \left(\frac{\left[b\left\{p - e^{-i\sigma}\Delta_\sigma^\tau[w(\varepsilon^\nu\xi)]\right\} + (-p)^\lambda\right]}{\xi^{(p+1)\lambda - p}}\right)^{1/\lambda}. \quad (42)$$

Let $\nu = 0, 1, 2, \cdots, \ell - 1$ in (42), respectively, and summing them we get

$$\frac{\chi_\ell'(m, \mu, a_1, c_1; \xi)}{\left[\chi_\ell(m, \mu, a_1, c_1; \xi)\right]^{1/\lambda}} = \frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\left(\frac{\left[b\left\{p - e^{-i\sigma}\Delta_\sigma^\tau[w(\varepsilon^\nu\xi)]\right\} + (-p)^\lambda\right]}{\xi^{(p+1)\lambda - p}}\right)^{1/\lambda}. \quad (43)$$

□

Case 1. Let $\lambda = 1$ in (43). We need to integrate from 0 to ξ to find $\chi_\ell(m, \mu, a_1, c_1; \xi)$. But from (43), we notice the presence of the first-order pole at the origin, the difficulty to integrate the above equality is avoided by integrating from ξ_0 to ξ with $\xi_0 \neq 0$, and then, let $\xi_0 \longrightarrow 0$. Therefore, on applying integration, we get

$$\begin{aligned} & \log\left(\frac{\chi_\ell(m, \mu, a_1, c_1; \xi)}{\xi^p}\right) \\ &= \frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\int_0^{\varepsilon^\nu\xi}\frac{1}{t}\left[b\left\{p - e^{-i\sigma}\Delta_\sigma^\tau[w(t)]\right\} - 2p\right]\mathrm{d}t. \end{aligned} \quad (44)$$

Hence, the proof of (37).

Case 2. If $\lambda > 1$, (43) can be rewritten as

$$\left[\{\chi_\ell(m,\mu,a_1,c_1;\xi)\}^{1-1/\lambda}\right]' = \left(1-\left(\frac{1}{\lambda}\right)\right)\frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\left(\frac{\left[b\{p-e^{-i\sigma}\Delta_\sigma^\tau[w(\varepsilon^\nu\xi)]\}+(-p)^\lambda\right]}{\xi^{(p+1)\lambda-p}}\right)^{1/\lambda}. \quad (45)$$

On integrating the above expression we obtain (38). Hence, the proof of Theorem 6.

Theorem 7. *Let $\chi \in \mathcal{QMS}_\ell^{m,\lambda}(a_1,c_1;b;h;X,Y)$, then for $\lambda = 1$, we get*

$$\chi_\ell(m,\mu,a_1,c_1;\xi) = \xi^p \exp\left\{\frac{\ln q}{(q-1)\ell}\sum_{\nu=0}^{\ell-1}\int_0^{\varepsilon^\nu\xi}\frac{1}{t}\left(b\left\{[p]_q - e^{-i\sigma}Y_q(\sigma,\tau;w(t))\right\}-2[p]_q\right)dt\right\}. \quad (46)$$

And for $\lambda > 1$, we have

$$\chi_\ell(m,\mu,a_1,c_1;\xi) = \left(\frac{\lambda-1}{\lambda}\right)\left\{\frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\int_0^{\xi}\left(\frac{\left[b\left\{[p]_q-e^{-i\sigma}\Delta_\sigma^\tau[w(\varepsilon^\nu t)]\right\}+\left([p]_q\right)^\lambda\right]}{t^{(p+1)\lambda-p}}\right)^{1/\lambda}dt\right\}^{\lambda-1/\lambda}, \quad (47)$$

where $\chi_\ell(m,\mu,a_1,c_1;\xi)$ is defined by equality (16) and $w(\xi)$ is analytic in $\mathbb{E}$ with $w(0)=0$ and $|w(\xi)|<1$.

Proof. In view of (24), (30), and (43), we have

$$\frac{\mathfrak{D}_q[\chi_\ell(m,\mu,a_1,c_1;\xi)]}{[\chi_\ell(m,\mu,a_1,c_1;\xi)]^{1/\lambda}} = \frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\left(\frac{\left(b\left\{[p]_q-e^{-i\sigma}Y_q(\sigma,\tau;w(\varepsilon^\nu\xi))\right\}+\left(-[p]_q\right)^\lambda\right)}{\xi^{(p+1)\lambda-p}}\right)^{1/\lambda}. \quad (48)$$

□

Case 1. Let $\lambda = 1$ in (48). Using the definition of logarithmic differentiation for q-derivative operator (see (26)) in (48), we get $(0<q<1)$

$$\log\left(\frac{\chi_\ell(m,\mu,a_1,c_1;\xi)}{\xi^p}\right) = \frac{\ln q}{(q-1)\ell}\sum_{\nu=0}^{\ell-1}\int_0^{\varepsilon^\nu\xi}\frac{1}{t}\left(b\left\{[p]_q-e^{-i\sigma}Y_q(\sigma,\tau;w(t))\right\}-2[p]_q\right)d_q t, \quad (49)$$

where the integral is q-Jackson integral. Hence, the proof of (37).

Case 2. If $\lambda > 1$, using chain rule (see (27)) for the q-difference operator defined in the previous section, (43) can be rewritten as

$$\mathfrak{D}_q\left[\{\chi_\ell(m,\mu,a_1,c_1;\xi)\}^{1-1/\lambda}\right] = \left(1-\left(\frac{1}{\lambda}\right)\right)\frac{1}{\ell}\sum_{\nu=0}^{\ell-1} \cdot \left(\frac{\left(b\left\{[p]_q-e^{-i\sigma}Y_q(\sigma,\tau;w(\varepsilon^\nu\xi))\right\}+\left(-[p]_q\right)^\lambda\right)}{\xi^{(p+1)\lambda-p}}\right)^{1/\lambda}. \quad (50)$$

On applying q-Jackson integral in the above expression, we obtain (47).

Corollary 8 (see [1, Theorem 1]). *Let $\chi(\xi) \in \mathcal{MS}_p(\sigma,\tau)$, then*

$$\chi(\xi) = \xi^{-p}\exp\left(2(\tau-p\cos\sigma)e^{-i\sigma}\int_0^\xi \frac{w(t)}{t[1-w(t)]}dt\right), (\xi \in \mathbb{E}^*), \quad (51)$$

where $w(\xi)$ is analytic in $\mathbb{E}$ with $w(0)=0$ and $|w(\xi)|<1$.

Proof. Letting $m=2$, $s=1$, $a_1=c_1$, $a_2=1$, $X=1$, $Y=-1$, $\ell=\lambda=b=1$, and $h(\xi)=(1+\xi)/(1-\xi)$ in Theorem 6, then (43) reduces to the form

$$-e^{i\sigma}\frac{\xi\chi'(\xi)}{\chi(\xi)} = \left(pe^{i\sigma} - \frac{2(\tau-p\cos\sigma)w(\xi)}{1-w(\xi)}\right). \quad (52)$$

Retracing the steps as in Theorem 6, we can establish the assertion of the corollary. □

Setting $m=0$, $r=2$, $s=1$, $a_1=c_1$, and $a_2=1$ in Theorem 6, we get the following

Corollary 9. *Let $\chi \in \mathcal{MS}_\ell^{0,\lambda}(2,1;b;h;X,Y)$, then, for $\lambda=1$, we get for $\xi \in \mathbb{E}^*$*

$$\chi_\ell(\xi) = -p\int_0^\xi u^{-p-1}\exp\left(\frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\int_0^{\varepsilon^\nu u}\frac{1}{t}\left[b\{p-e^{-i\sigma}\Delta_\sigma^\tau[w(t)]\}\right]dt\right)du. \quad (53)$$

And for $\lambda > 1$, we have for $\xi \in \mathbb{E}^*$

$$\chi_\ell(\xi) = -p\left(\frac{\lambda-1}{\lambda}\right)\int_0^\xi u \cdot \left\{\frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\int_0^u\left(\frac{\left[b\{p-e^{-i\sigma}\Delta_\sigma^\tau[w(\varepsilon^\nu t)]\}+(-p)^\lambda\right]}{t^{(p+1)\lambda-p}}\right)^{1/\lambda}dt\right\}^{\lambda-1/\lambda}du, \tag{54}$$

where $\chi_\ell(\xi)$ is defined by equality (12) and $w(\xi)$ is analytic in $\mathbb{E}$ with $w(0)=0$ and $|w(\xi)|<1$.

Letting $\lambda = 1X = 1$, $Y=-1$, $b=1$, and $h(\xi)=(1+\xi)/(1-\xi)$ in Corollary 9, we get the following result.

Corollary 10. (see [1]). *Let $\chi(\xi)\in\mathscr{MC}_p(\sigma,\tau)$, then for $\xi\in\mathbb{E}^*$*

$$\chi(\xi) = -p\int_0^\xi u^{-p-1}\exp\cdot\left(2(\tau-p\cos\sigma)e^{-i\sigma}\int_0^u\frac{w(t)}{t[1-w(t)]}dt\right)du, \tag{55}$$

where $w(\xi)$ is analytic in $\mathbb{E}$ with $w(0)=0$ and $|w(\xi)|<1$.

4. Fekete-Szegö Inequality of $\mathscr{MS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$ and $\mathscr{QMS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$

Very few researchers have attempted at finding solution to the Fekete-Szegö problem for class of functions with respect to ℓ-symmetric points, as it is computational tedious. Notable among those works on coefficient inequalities of classes of functions with respect to ℓ-symmetric points were done by Aouf et al. [39].

Throughout this section, we let

$$\Psi_n = \frac{1}{\ell}\sum_{\nu=0}^{\ell-1}\varepsilon^{\nu n},\ (\ell\in\mathbb{N}; n\geq 1; \varepsilon^\ell = 1),$$
$$\Theta_n = \frac{(a_1)_n(a_2)_n\cdots(a_r)_n}{n!(c_1)_n(c_2)_n\cdots(c_s)_n}\ \text{and}\ \Omega_n^m = \left(\frac{\mu}{n+\mu}\right)^m,\ (n\in\mathbb{N}). \tag{56}$$

Theorem 11. *If $\chi(\xi)\in\mathscr{MS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$, then, we have for all $\mu\in\mathbb{C}$*

$$\left|d_{2-p}-\mu d_{1-p}^2\right| \leq \frac{|b|\left|X(pe^{-i\sigma}-\tau)-Y(pe^{i\sigma}-\tau)\right|R_1}{2\left|(-p)^{\lambda-1}\{p\Psi_2+(2-p)\lambda\}\Theta_2\Omega_2^m\right|}\cdot\max\{1, |2\mathcal{Q}_1-1|\}, \tag{57}$$

where $\mathcal{Q}_1$ is given by

$$\mathcal{Q}_1 = \frac{1}{4}\left\{(Y+1)R_1 + 2\left(1-\frac{R_2}{R_1}\right) - \frac{e^{-i\sigma}b\left[X(pe^{-i\sigma}-\tau)-Y(pe^{i\sigma}-\tau)\right]\{2p^2\Psi_1^2+2p(1-p)\Psi_1+\lambda(\lambda-1)(1-p)^2\}R_1}{2(-p)^\lambda[p\Psi_1+(1-p)\lambda]^2} - \frac{\mu e^{-i\sigma}bR_1\left[X(pe^{-i\sigma}-\tau)-Y(pe^{i\sigma}-\tau)\right]\{p\Psi_2+(2-p)\lambda\}\Theta_2\Omega_2^m}{(-p)^{\lambda-1}[p\Psi_1+(1-p)\lambda]^2\Theta_1^2\Omega_1^{2m}}\right\}. \tag{58}$$

The inequality is sharp for each $\mu\in\mathbb{C}$.

Proof. As $\chi\in\mathscr{MS}_\ell^{m,\lambda}(a_1, c_1; b; h; X, Y)$, by (23), we have

$$\frac{\xi^{(p+1)\lambda-p}\left[I_\mu^m(a_1,c_1)\chi'(\xi)\right]^\lambda}{\chi_\ell(m,\mu,a_1,c_1;\xi)} - (-p)^\lambda = -b\left[e^{-i\sigma}\Delta_\sigma^\tau[w(\xi)]-p\right]. \tag{59}$$

Thus, let $\vartheta\in\mathscr{P}$ be of the form $\vartheta(\xi)=1+\sum_{\ell=1}^\infty\vartheta_n\xi^n$ and defined by

$$\vartheta(\xi) = \frac{1+w(\xi)}{1-w(\xi)},\ \xi\in\Omega. \tag{60}$$

On computation, we have

$$w(\xi) = \frac{1}{2}\vartheta_1\xi + \frac{1}{2}\left(\vartheta_2-\frac{1}{2}\vartheta_1^2\right)\xi^2 + \frac{1}{2}\left(\vartheta_3-\vartheta_1\vartheta_2+\frac{1}{4}\vartheta_1^3\right)\xi^3+\cdots,\xi\in\Omega. \tag{61}$$

The right hand side of (58)

$$-b\{e^{-i\sigma}\Delta_\sigma^\tau[w(\xi)]-p\} = -\frac{be^{-i\sigma}R_1\vartheta_1\left[X(pe^{-i\sigma}-\tau)-Y(pe^{i\sigma}-\tau)\right]}{4}\xi - \frac{be^{-i\sigma}\left[X(pe^{-i\sigma}-\tau)-Y(pe^{i\sigma}-\tau)\right]R_1}{4}\cdot\left[\vartheta_2-\vartheta_1^2\left(\frac{(Y+1)R_1+2(1-(R_2/R_1))}{4}\right)\right]\xi^2+\cdots. \tag{62}$$

From the left hand side of (58) is given by

$$
\begin{aligned}
&\frac{\xi^{(p+1)\lambda-p}\left[I_{\mu}^{m}(a_1,c_1)\chi'(\xi)\right]^{\lambda}}{\chi_{\ell}(m,\mu,a_1,c_1;\xi)}-(-p)^{\lambda}\\
&=(-p)^{\lambda-1}[p\Psi_1+(1-p)\lambda]\Theta_1\Omega_1^m d_{1-p}\xi+\frac{(-p)^{\lambda-1}}{2p}\\
&\quad\cdot\left[2p\{p\Psi_2+(2-p)\lambda\}\Theta_2\Omega_2^m d_{2-p}-\{2p^2\Psi_1^2+2p(1-p)\Psi_1\right.\\
&\quad\left.+\lambda(\lambda-1)(1-p)^2\}d_{1-p}^2\Theta_1^2\Omega_1^{2m}\right]\xi^2+\cdots.
\end{aligned}
\tag{63}
$$

From (61) and (62), we obtain

$$
\begin{aligned}
d_{1-p}&=-\frac{e^{-i\sigma}bR_1\vartheta_1\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]}{4(-p)^{\lambda-1}[p\Psi_1+(1-p)\lambda]\Theta_1\Omega_1^m},\\
d_{2-p}&=-\frac{e^{-i\sigma}b\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]R_1}{4(-p)^{\lambda-1}\{p\Psi_2+(2-p)\lambda\}\Theta_2\Omega_2^m}\left[\vartheta_2-\frac{1}{4}\left\{(Y+1)R_1+2\left(1-\frac{R_2}{R_1}\right)\right.\right.\\
&\quad\left.\left.-\frac{e^{-i\sigma}b\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]\{2p^2\Psi_1^2+2p(1-p)\Psi_1+\lambda(\lambda-1)(1-p)^2\}R_1}{2(-p)^{\lambda}[p\Psi_1+(1-p)\lambda]^2}\right\}\vartheta_1^2\right].
\end{aligned}
\tag{64}
$$

Now we consider

$$
\begin{aligned}
\left|d_{2-p}-\mu d_{1-p}^2\right|&=\left|-\frac{e^{-i\sigma}b\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]R_1}{4(-p)^{\lambda-1}\{p\Psi_2+(2-p)\lambda\}\Theta_2\Omega_2^m}\left[\vartheta_2-\frac{1}{4}\left\{(Y+1)R_1+2\left(1-\frac{R_2}{R_1}\right)\right.\right.\right.\\
&\quad\left.\left.-\frac{e^{-i\sigma}b\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]\{2p^2\Psi_1^2+2p(1-p)\Psi_1+\lambda(\lambda-1)(1-p)^2\}R_1}{2(-p)^{\lambda}[p\Psi_1+(1-p)\lambda]^2}\right\}\vartheta_1^2\right]\\
&\quad\left.-\frac{\mu e^{-2i\sigma}b^2R_1^2\vartheta_1^2\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]^2}{16(-p)^{2\lambda-2}[p\Psi_1+(1-p)\lambda]^2\Theta_1^2\Omega_1^{2m}}\right|=\left|-\frac{e^{-i\sigma}b\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]R_1}{4(-p)^{\lambda-1}\{p\Psi_2+(2-p)\lambda\}\Theta_2\Omega_2^m}\right.\\
&\quad\cdot\left[\vartheta_2-\frac{\vartheta_1^2}{4}\left\{(Y+1)R_1+2\left(1-\frac{R_2}{R_1}\right)-\frac{e^{-i\sigma}b\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]\{2p^2\Psi_1^2+2p(1-p)\Psi_1+\lambda(\lambda-1)(1-p)^2\}R_1}{2(-p)^{\lambda}[p\Psi_1+(1-p)\lambda]^2}\right.\right.\\
&\quad\left.\left.\left.-\frac{\mu e^{-i\sigma}bR_1\left[X\left(pe^{-i\sigma}-\tau\right)-Y\left(pe^{i\sigma}-\tau\right)\right]\{p\Psi_2+(2-p)\lambda\}\Theta_2\Omega_2^m}{(-p)^{\lambda-1}[p\Psi_1+(1-p)\lambda]^2\Theta_1^2\Omega_1^{2m}}\right\}\right]\right|.
\end{aligned}
\tag{65}
$$

On applying Lemma 5, we get the assertion. □

To demonstrate the applications of our results, here, we provide the most simple special case of our result. Note that the following result was obtained [[40], Theorem 6] for functions in $\chi\in\mathscr{A}$.

Corollary 12. *If $\chi(\xi)\in\mathscr{L}_1$ satisfies*

$$
-\frac{\xi\chi'(\xi)}{\chi(\xi)}\prec h(\xi),
\tag{66}
$$

and $h(\xi)=1+R_1\xi+R_2\xi^2+\cdots$, with $R_1,R_2\in\mathbb{R}$, $R_1>0$, then for all $\mu\in\mathbb{C}$ we have

$$
\left|d_1-\mu d_0^2\right|\leq\frac{R_1}{2}\max\left\{1;\left|\frac{R_2}{R_1}-R_1+2\mu R_1\right|\right\}.
\tag{67}
$$

The inequality is sharp for the function χ_* given by

$$
\chi_*(\xi)=\begin{cases}\xi\exp\displaystyle\int_0^{\xi}-\frac{h(t)+1}{t}dt, & \text{if }\left|\dfrac{R_2}{R_1}-R_1+2\mu R_1\right|\geq 1,\\[2ex] \xi\exp\displaystyle\int_0^{\xi}-\frac{h(t^2)+1}{t}dt, & \text{if }\left|\dfrac{R_2}{R_1}-R_1+2\mu R_1\right|\leq 1.\end{cases}
\tag{68}
$$

Proof. In Theorem 11, taking $r=2, s=1,\ a_1=b_1,\ a_2=1,\ X=1,\ Y=-1,\ m=\sigma=\tau=0,$ and $\ell=\lambda=p=1,$ we get the inequality

$$
\left|d_1-\mu d_0^2\right|\leq\begin{cases}\dfrac{R_1}{2}, & \text{if }\left|\dfrac{R_2}{R_1}-R_1+2\mu R_1\right|\leq 1,\\[2ex] \dfrac{R_1}{2}\left|\dfrac{R_2}{R_1}-R_1+2\mu R_1\right|, & \text{if }\left|\dfrac{R_2}{R_1}-R_1+2\mu R_1\right|\geq 1.\end{cases}
\tag{69}
$$

□

Analogous to Theorem 11, we can prove the following.

Theorem 13. *If* $\chi(\xi) \in \mathcal{QMS}_{\ell}^{m,\lambda}(a_1, c_1\,; b\,; h\,; X, Y)$*, then, we have for all* $\mu \in \mathbb{C}$

$$\left|d_{2-p} - \mu d_{1-p}^2\right| \le \frac{|b|\left|X\left([p]_q e^{-i\sigma} - \tau\right) - Y\left([p]_q e^{i\sigma} - \tau\right)\right| R_1}{2\left|\left([-p]_q\right)^{\lambda-1}\left\{[2-p]_q\lambda - [-p]_q\Psi_2\right\}\Theta_2\Omega_2^m\right|} \cdot \max\left\{1, |2\mathcal{Q}_2 - 1|\right\}, \tag{70}$$

where $\mathcal{Q}_2$ *is given by*

$$\begin{aligned}\mathcal{Q}_2 = \frac{1}{4}\Bigg\{&(Y+1)R_1 + 2\left(1 - \frac{R_2}{R_1}\right) \\ &- \frac{e^{-i\sigma}b\left[X\left([p]_q e^{-i\sigma} - \tau\right) - Y\left([p]_q e^{i\sigma} - \tau\right)\right]\left\{2[-p]_q^2\Psi_1^2 - 2[-p]_q[1-p]_q\Psi_1 + \lambda(\lambda-1)[1-p]_q^2\right\}R_1}{2(-p)^{\lambda}\left\{[1-p]_q\lambda - [-p]_q\Psi_1\right\}^2} \\ &- \frac{\mu e^{-i\sigma}bR_1\left[X\left([p]_q e^{-i\sigma} - \tau\right) - Y\left([p]_q e^{i\sigma} - \tau\right)\right]\left\{[2-p]_q\lambda - [-p]_q\Psi_2\right\}\Theta_2\Omega_2^m}{(-p)^{\lambda-1}\left\{[1-p]_q\lambda - [-p]_q\Psi_1\right\}^2\Theta_1^2\Omega_1^{2m}}\Bigg\}.\end{aligned} \tag{71}$$

The inequality is sharp.

5. Conclusions

The defined function class $\mathcal{MS}_{\ell}^{m,\lambda}(a_1, c_1\,; b\,; h\,; X, Y)$ though familiar with so called pseudo-star-like functions required lots of adaptation since it involves functions with a removable singularity of order p at the origin. Integral representation and Fekete-Szegö inequalities have been established. Further, we extend the class $\mathcal{MS}_{\ell}^{m,\lambda}(a_1, c_1\,; b\,; h\,; X, Y)$ by replacing the classical derivative with q-derivative. Since all the results involving classical derivative does not get translated to the results involving q-derivative, we used some modified conditions to obtain our main results. We note that these adaptation are essential for future research.

Data Availability

Not applicable.

Conflicts of Interest

Authors declare that they have no conflict of interest.

Authors' Contributions

All authors contributed equally to this work. All the authors have read and agreed to the published version of the manuscript.

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