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Research Article

New Results in Vague Incidence Graphs with Application

Yongsheng Rao ,[1] Saeed Kosari ,[1] and Mehdi Gheisari[2]

[1]*Institute of Computing Science and Technology, Guangzhou University, Guangzhou 510006, China*
[2]*Young Researchers and Elite Club, Parand Branch, Islamic Azad University, Iran*

Correspondence should be addressed to Saeed Kosari; saeedkosari38@gzhu.edu.cn

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Vague incidence graph (VIG), belonging to the FG family has good capabilities when facing with problems that cannot be expressed by FGs. When an element membership is not clear, neutrality is a good option that can be well-supported by a VIG. The previous definitions limitations in connectivity concept have led us to offer new definitions in VIGs. Hence, in this paper, the VIG and its matrix form are proposed. Vague incidence subgraph (VISG) is defined with several properties. Incidence pairs, paths, and connectivities between pairs in VIGs are introduced. Likewise, different types of strong and cut pair in VIGs are examined with their properties. Universities are one of the most important centers for human education and play an important role in the development of the country. But the point to be very careful is that the employees of a university must do their job in the best possible way. Therefore, we have tried to identify the most effective person in a university according to its performance by presenting an application.

1. Introduction

The FG concept serves as one of the most dominant and extensively employed tools for multiple real-word problem representations, modeling, and analysis. To specify the objects and the relations between them, the graph vertices or nodes and edges or arcs are applied, respectively. Graphs have long been used to describe objects and the relationships between them. Many of the issues and phenomena around us are associated with complexities and ambiguities that make it difficult to express certainty. These difficulties were alleviated by the introduction of fuzzy sets by Zadeh [1]. The fuzzy set focuses on the membership degree of an object in a particular set. Kaufmann [2] represented FGs based on Zadeh's fuzzy relation [3, 4]. Rosenfeld [5] described the structure of FGs obtaining analogs of several graph theoretical concepts. The notion of vague set theory, the generalization of Zadeh's fuzzy set theory, was introduced by Gau and Buehrer [6] in 1993. Akram et al. [7, 8] studied regularity in vague intersection graphs. Samanta and Pal [9, 10] defined fuzzy competition graphs and some remarks on bipolar fuzzy graphs. Ramakrishna [11] introduced the concept of VGs and studied some of their properties. Borzooei et al. [12, 13] investigated domination in VGs. The strong path between nodes in FG are formulated in [14]. Darabian and Borzooei [15] presented new results in vague graphs. Many operations with their properties in FG theory have been clearly explained in [16]. Ghorai and Pal [17] established various types of FG with their several properties. Some basic definitions of paths, circuit, strong and complete FG, and their applications are briefly discussed in [18]. Strong arcs and paths of generalized FGs and their real applications are given in [19]. Dinesh [20] first defined the notion of FIGs. Different typs of nodes and properties of FIG have been discussed in [21, 22]. The geodesic distance and different types of nodes in bipolar fuzzy graphs are introduced in [23]. Poulik and Ghorai [24–26] initiated degree of nodes and indices of bipolar fuzzy graphs with applications in real life systems. Kosari et al. [27, 28] investigated new concepts in VGs. Zeng et al. [29] introduced certain properties of single-valued neutrosophic graphs. Rashmanlou et al. [30, 31] defined product vague graphs and cubic graphs. Hussain

Table 1: Some basic notations.

Notation	Meaning
FG	Fuzzy graph
VS	Vague set
IG	Incidence graph
FSS	Fuzzy subset
VI	Vague incidence
VIPSG	Vague incidence partial subgraph
VISG	Vague incidence subgraph
VSG	Vague subgraph
CVIG	Connected vague incidence graph
SP	Strong pair
FI	Fuzzy incidence
CP	Cut pair

et al. [32] studied neutrosophic vague incidence graph. Rao et al. [33] defined domination in vague incidence graph.

VIGs have a wide range of applications in the field of psychological sciences as well as the identification of individuals based on oncological behaviors. With the help of VIGs, the most efficient person in an organization can be identified according to the important factors that can be useful for an institution. Likewise, a VIG is capable of focusing on determining the uncertainly combined with the inconsistent and indeterminate information of any real-world problem, in which FGs may not lead to adequate results. Therefore, in this paper, the VIG and its matrix form are introduced. VISG is defined with several properties. Incidence pairs, paths, and connectivities between pairs in VIGs are proposed. Also, different types of strong and cut pair in VIGs are examined with their properties. Finally, an application of VIG has given.

2. Preliminaries

Definition 1 (see [34]). Let $G=(V,E)$ be a graph. Then, $G^*=(V,E,I)$ is called an incidence graph (IG), so that $I\subseteq V\times E$. If $V=\{p,q\}$, $E=\{pq\}$ and $I=\{(p,pq)\}$, then (V,E,I) is an IG even though $(q,pq)\notin I$. The pair (p,pq) is called an incidence pair or simply a pair. If $(p,pq),(q,pq),(q,qr),(r,qr)\in I$, then pq or qr are called neighbor edges.

Definition 2 (see [20]). Let G^* be an IG and σ be a FSS of V and μ, a FSS of E. Let ψ be a FSS of I. If $\psi(v,pq)\le\sigma(v)\wedge\mu(pq)$, $\forall v\in V$ and $pq\in E$, then, ψ is called a FI of graph G^* and $G=(\sigma,\mu,\psi)$ is named a FIG of G^*.

Definition 3 (see [6]). A VS A is a pair (t_A,f_A) on set V which t_A and f_A are taken as real valued functions which can be defined on $V\longrightarrow[0,1]$ so that $t_A(p)+f_A(p)\le 1$, $\forall p\in V$.

Definition 4 (see [11]). A VG is defined to be a pair $G=(A,B)$, which $A=(t_A,f_A)$ is a VS on V and $B=(t_B,f_B)$ is a VS on $E\subseteq V\times V$ so that for each $pq\in E$, $t_B(pq)\le t_A(p)\wedge t_A(q)$, $f_B(pq)\ge f_B(p)\vee f_B(q)$.

Definition 5 (see [12]). Let G be a VG and $m,n\in V$.

(i) A path $p:m=m_0,m_1,\cdots,m_{t-1},m_t=n$ in G is a sequence of distinct nodes so that $(t_B(m_{i-1},m_i)>0, f_B(m_{i-1},m_i)>0)$, $i=1,2,\cdots,t$ and the length of the path is t

(ii) If $p:m=m_0,m_1,\cdots,m_{t-1},m_t=n$ be a path of length t between m and n, then $(t_B(mn))^k$ and $(f_B(mn))^k$ are defined as

$$(t_B(mn))^k=\sup\{t_B(m,m_1)\wedge t_B(m_1,m_2)\wedge\cdots,\wedge t_B(m_{k-1},n)\},$$
$$(f_B(mn))^k=\inf\{f_B(m,m_1)\vee f_B(m_1,m_2)\vee\cdots\vee f_B(m_{k-1},n)\}. \quad (1)$$

$((t_B(mn))^\infty,(f_B(mn))^\infty)$ is said to be the strength of connectedness between two nodes m and n in G, which $(t_B(mn))^\infty=\sup_{k\in N}\{(t_B(mn))^k\}$ and $(f_B(mn))^\infty=\inf_{k\in N}\{(f_B(mn))^k\}$.

(iii) If $t_B(mn)\ge t_B^\infty(mn)$ and $f_B(mn)\le f_B^\infty(mn)$, then the arc mn in G is called a strong arc. A path $m-n$ is strong path if all arcs on the path are strong

Definition 6 (see [12]). For a VG G, if $t_B(mn)\ge(t_B(mn))^\infty$ and $f_B(mn)\le(f_B(mn))^\infty$, then the edge ab is called a strong edge of G.

All the basic notations are shown in Table 1.

3. New Concepts of Vague Incidence Graph

Definition 7. $G=(A,B,C)$ is called a VIG of underlying crisp-IG $G^*=(V,E,I)$, if:

$$\mathscr{A}=\{\langle t_A(m),f_A(m)\rangle\mid m\in V\},$$
$$\mathscr{B}=\{\langle t_B(pq),f_B(pq)\rangle\mid pq\in E\}, \quad (2)$$
$$\mathscr{C}=\{\langle t_C(m,pq),f_C(m,pq)\rangle\mid(m,pq)\in I\},$$

so that A, B are VSs on V and $E\subseteq V\times V$, respectively. $C=(C^t,C^f)$ is a mapping from $V\times E$ to $([0,1],[0,1])$ and we have: $t_B(pq)\le t_A(p)\wedge t_A(q)$, $f_B(pq)\ge f_A(p)\vee f_A(q)$, $t_C(m,pq)\le t_A(m)\wedge t_B(pq)$, $f_C(m,ab)\ge f_A(m)\vee f_B(pq)$, $\forall m\in V, pq\in E$, and $0\le t_A(m)+f_A(m)\le 1$, $0\le t_B(pq)+f_B(pq)\le 1$, $0\le t_C(m,pq)+f_C(m,pq)\le 1$.

Example 1. Consider an incidence graph $G^*=(V,E,I)$ so that $V=\{p,q,r,s\}$, $E=\{pq,qr,qs,rs,ps\}$ and $I=\{(p,pq),(q,pq),(q,qr),(r,qr),(q,qs),(s,qs),(r,rs),(s,rs),(s,ps),(p,ps)\}$ as shown in Figure 1. Clearly, $G=(A,B,C)$ is a VIG of G^*, denoted in Figure 2, which

$$A=\left\{\frac{p}{(0.2,0.4)},\frac{q}{(0.3,0.5)},\frac{r}{(0.4,0.6)},\frac{s}{(0.5,0.5)}\right\},$$
$$B=\left\{\frac{pq}{(0.2,0.5)},\frac{qr}{(0.2,0.6)},\frac{qs}{(0.2,0.6)},\frac{rs}{(0.3,0.6)},\frac{ps}{(0.2,0.5)}\right\},$$
$$C=\left\{\frac{(p,pq)}{(0.1,0.5)},\frac{(q,pq)}{(0.2,0.5)},\frac{(q,qr)}{(0.1,0.6)},\frac{(r,qr)}{(0.2,0.6)},\frac{(q,qs)}{(0.2,0.7)},\frac{(s,qs)}{(0.1,0.6)},\frac{(r,rs)}{(0.3,0.7)},\frac{(s,rs)}{(0.2,0.6)},\frac{(s,ps)}{(0.2,0.5)},\frac{(p,ps)}{(0.1,0.5)}\right\}. \quad (3)$$

Definition 8. Let $G=(A,B,C)$ be a VIG of underlying crisp-IG G^*, and ζ has n nodes $p_1,p_2,p_3,\cdots,p_n$ and m edges $e_1,e_2,e_3,\cdots,e_m$. Then, the matrix form of the VI (C) is denoted by $[\psi_{ij}]_{n\times m}$ and is defined by

$$\left[\psi_{ij}\right]_{n\times m}=\begin{matrix} & e_1 & e_2 & e_3 & \cdots & e_m \\ p_1 & \psi_{11} & \psi_{12} & \psi_{13} & \cdots & \psi_{1m} \\ p_2 & \psi_{21} & \psi_{22} & \psi_{23} & \cdots & \psi_{2m} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ p_n & \psi_{n1} & \psi_{n2} & \psi_{n3} & \cdots & \psi_{nm}\end{matrix}, \quad (4)$$

where $\psi_{ij}=(t_C(p_i,p_ip_j),f_C(p_i,p_ip_j))$.

Example 2. Consider the VIG G of Figure 2. Here, the number of nodes in G is 4 and the number of edges is 5. So, the matrix form of C is given below as

$$\left[\psi_{ij}\right]_{4\times 5}=\begin{matrix} & qr & qs & pq & rs & ps \\ q & (0.1,0.6) & (0.2,0.7) & (0.2,0.5) & (0,0) & (0,0) \\ r & (0.2,0.6) & (0,0) & (0,0) & (0.3,0.7) & (0,0) \\ s & (0,0) & (0.1,0.6) & (0,0) & (0.2,0.6) & (0.2,0.5) \\ p & (0,0) & (0,0) & (0.1,0.5) & (0,0) & (0.1,0.5)\end{matrix}. \quad (5)$$

Definition 9. A VIG $H=(A',B',C')$ called a VIPSG of the VIG $G=(A,B,C)$ if $t_{A'}(p)\le t_A(p)$, $f_{A'}(p)\ge f_A(p)$, $t_{B'}(pq)\le t_B(pq)$, $f_{B'}(pq)\ge f_B(pq)$, $t_{C'}(p,pq)\le t_C(p,pq)$, and $f_{C'}(p,pq)\ge f_C(p,pq)$, for all $p\in V$ and $pq\in E$. Also, H is called a VISG of the VIG G, if $V'\subseteq V$, $E'\subseteq E$, $C'\subseteq C$, $t_{A'}(p)=t_A(p)$, $f_{A'}(p)=f_A(p)$, $t_{B'}(pq)=t_B(pq)$, $f_{B'}(pq)=f_B(pq)$, $t_{C'}(p,pq)=t_C(p,pq)$, and $f_{C'}(p,pq)=f_C(p,pq)$, $\forall p\in V$ and $\forall pq\in E$.
If we delete a node or an edge from a VIG, then, the effects are initiated in next propositions.

Proposition 10. *A VISG of a VIG G must be a VIPSG of G.*

Proof. Let H be a VISG of the VIG G. Then, from the definitions, we have H satisfies all the conditions to be a VIPSG of the VIG G. So, H is a VISG of the VIG G. □

Proposition 11. *If $\tilde{H}$ be a VISG of the VIG $\tilde{G}$, then the VG H is a VSG of the VG G.*

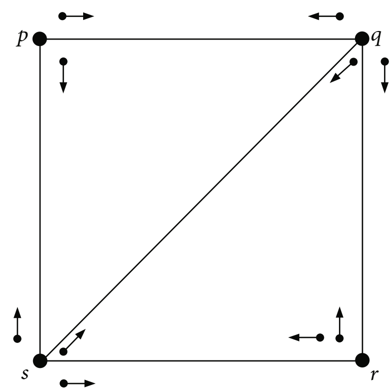

Figure 1: Incidence graph G^*.

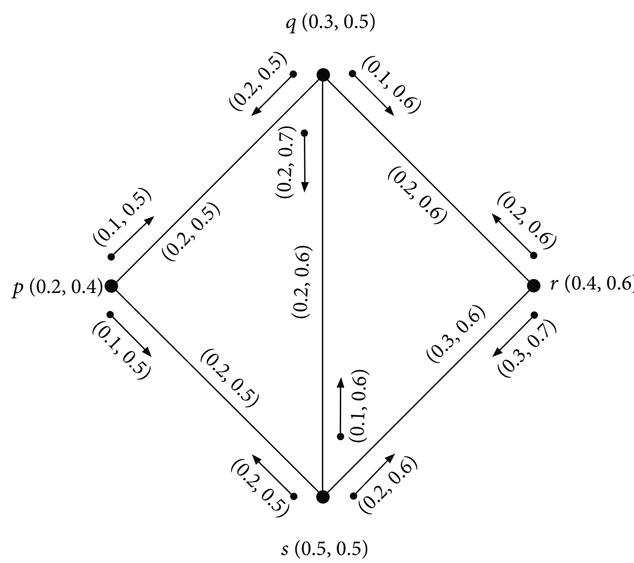

Figure 2: Vague incidence graph G.

Proof. $\tilde{H}$ is a VISG of the VIG $\tilde{G}$. Then, from the definitions, we have the corresponding VG H satisfies all the conditions to be a VSG of the VG G. Hence, H is a VSG of the VG G. □

Definition 12. A VIG $G=(A,B,C)$ is called complete-VIG if $t_C(p,pq)=\min\{t_A(p),t_B(pq)\}$ and $f_C(p,pq)=\max\{f_A(p),f_B(pq)\}$, $\forall p\in V$ and $\forall pq\in E$. A VIG G is called strong if $t_C(p,pq)=\min\{t_A(p),t_B(pq)\}$ and $f_C(p,pq)=\max\{f_A(p),f_B(pq)\}$, $\forall$ pairs (p,pq) in G.

If $G=(A,B,C)$ is a complete-VIG and the nodes p,q are neighbor to the edge pq, then $t_C(p,pq)=\min\{t_A(p),t_B(pq)\}=t_B(pq)=\min\{t_A(q),t_B(pq)\}=t_C(q,pq)$ and $f_C(p,pq)=\max\{f_A(p),f_B(pq)\}=f_B(pq)=\max\{f_A(q),f_B(pq)\}=f_C(q,pq)$.

Example 3. Consider the VIG G as Figure 3. Clearly, G is a complete-VIG and also strong-VIG.

Theorem 13. *A complete-VIG is a strong VIG.*

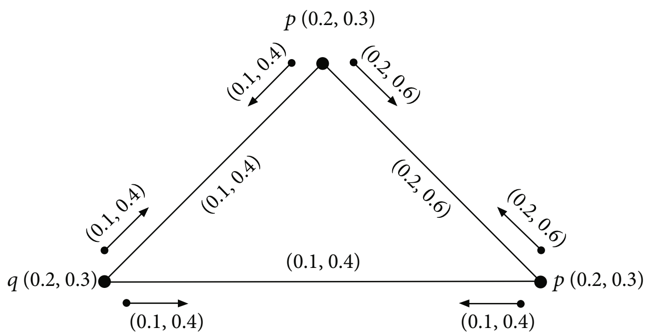

Figure 3: Complete VIG G.

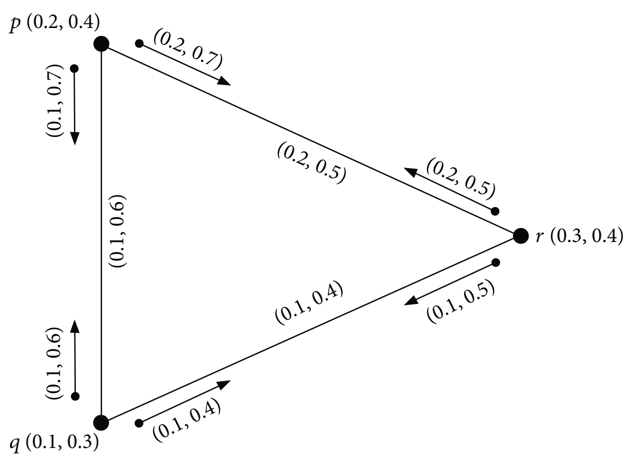

Figure 4: VIG G.

Proof. Let G be a complete-VIG and (p, pq) be a pair in G. Then, $t_C(p, pq) = \min\{t_A(p), t_B(pq)\}$ and $f_C(p, pq) = \max\{f_A(p), f_B(pq)\}$, $\forall p \in V$ and $pq \in E$. Hence, $t_C(p, pq) = \min\{t_A(p), t_B(pq)\}$ and $f_C(p, pq) = \max\{f_A(p), f_B(pq)\}$, $\forall$ pairs (p, pq) in G. □

Definition 14. Let $p = p_1, p_2, \cdots, p_{n-1} = q, p_n = r$ are the n nodes in a VIG G. Then, $p_1, (p_1, p_1p_2), p_1p_2, (p_2, p_1p_2), p_2, \cdots, q, (q, qr), qr, (r, qr)$ is called an incidence path in G. The incidence strength of this path is shown by $(C^t_{p,qr}, C^f_{p,qr})$ and is described as $C^t_{p,qr} = C^t(p_1, p_1p_2) \wedge C^t(p_2, p_1p_2) \wedge \cdots \wedge C^t(q, qr) \wedge C^t(r, qr)$ and $C^f_{p,qr} = C^f(p_1, p_1p_2) \vee C^f(p_2, p_1p_2) \vee \cdots \vee C^f(q, qr) \vee C^f(r, qr)$. The incidence strength of connectedness between p and qr in G is $\mathrm{ICONN}_G(p, qr) = (\mathrm{ICONN}^t_G(p, qr), \mathrm{ICONN}^f_G(p, qr),)$, where $\mathrm{ICONN}^t_G(p, qr) = \vee\{C^t_{p,qr}\}$ is the maximum value of true part of incidence strength of all the paths between p and qr and $\mathrm{ICONN}^f_G(p, qr) = \wedge\{C^f_{p,qr}\}$ is the minimum value of the false part of incidence strengths of all the paths between p and qr.

Example 4. Consider the CVIG G of Figure 4.

Here, $C^t(p, pq) \wedge C^t(q, pq) \wedge C^t(q, qr) = 0.1$, $C^f(p, pq) \vee C^f(q, pq) \vee C^f(q, qr) = 0.7$, $C^t(p, pr) \wedge C^t(r, pr) \wedge C^t(r, qr) = 0.1$, $C^f(p, pr) \vee C^f(r, pr) \vee C^f(r, qr) = 0.7$. So, $\mathrm{ICONN}^t_G(p, qr) = \vee\{0.1, 0.1\} = 0.1$ and $\mathrm{ICONN}^f_G(p, qr) = \wedge\{0.7, 0.7\} = 0.7$. Therefore, the incidence strength of connectedness between p and qr is $\mathrm{ICONN}_G(p, qr) = (0.1, 0.7)$.

Definition 15. Let ab be an edge of a VIG G. If $C^t(p, pq) > 0$, $C^t(q, pq) > 0$, $\psi^f(p, pq) > 0$, and $\psi^f(q, pq) > 0$, then (p, pq) and (q, pq) are called pairs. G is said to be connected if there is an incidence path between every pair.

Theorem 16. *Let $G = (A, B, C)$ be a VIG and $H = (A', B', C')$ be a VISG of G. Then, for any pair (p, pq) in G, $\mathrm{ICONN}^t_G(p, pq) \geq \mathrm{ICONN}^t_H(p, pq)$ and $\mathrm{ICONN}^f_G(p, pq) \leq \mathrm{ICONN}^f_H(p, pq)$.*

Proof. Consider H is a VISG of G. From the definition of VISG, we have $C'^t(p, pq) = C^t(p, pq)$ and $C'^f(p, pq) = C^f(p, pq)$, for all pairs (p, pq) in H. But, $\mathrm{ICONN}^t_G(p, pq)$, $\mathrm{ICONN}^t_H(p, pq)$, and $\mathrm{ICONN}^f_G(p, pq)$, $\mathrm{ICONN}^f_H(p, pq)$ lies on same incidence pair of H and $\tilde{H}$ or lies on different pairs of H and $\tilde{H}$. Now, there arises two cases.

Case 1. Suppose $\mathrm{ICONN}^t_G(p, pq)$, $\mathrm{ICONN}^t_H(p, pq)$, and $\mathrm{ICONN}^f_G(p, pq)$, $\mathrm{ICONN}^f_H(p, pq)$ lies on same pair (w, wk) of H and G. Then, from the definition of VISG we have $C'^t(w, wk) = C^t(w, wk)$ and $C'^f(w, wk) = C^f(w, wk)$. Then, $\mathrm{ICONN}^t_G(p, pq) = \mathrm{ICONN}^t_H(p, pq)$ and $\mathrm{ICONN}^f_G(p, pq) = \mathrm{ICONN}^f_H(p, pq)$.

Case 2. Suppose $\mathrm{ICONN}^t_G(p, pq)$, $\mathrm{ICONN}^t_H(p, pq)$, and $\mathrm{ICONN}^f_G(p, pq)$, $\mathrm{ICONN}^f_H(p, pq)$ lies on the pairs (w_1, w_1k_1) in G and (w_2, w_2k_2) in H. This means both the pairs (w_1, w_1k_1) and (w_2, w_2k_2) are the pairs of G. If $C^t(w_1, w_1k_1) = C^t(w_2, w_2k_2)$ and $C^f(w_1, w_1k_1) = C^f(w_2, w_2k_2)$, then $\mathrm{ICONN}^t_G(p, pq) = \mathrm{ICONN}^t_H(p, pq)$ and $\mathrm{ICONN}^f_G(p, pq) = \mathrm{ICONN}^f_H(p, pq)$. If $C^t(w_1, w_1k_1) \neq C^t(w_2, w_2k_2)$ or $C^f(w_1, w_1k_1) \neq C^f(w_2, w_2k_2)$ or both, then $\mathrm{ICONN}^t_G(p, pq) > \mathrm{ICONN}^t_H(p, pq)$ or $\mathrm{ICONN}^f_G(p, pq) < \mathrm{ICONN}^f_H(p, pq)$ or both. Hence, in any case, $\mathrm{ICONN}^t_G(p, pq) \geq \mathrm{ICONN}^t_H(p, pq)$ and $\mathrm{ICONN}^f_G(p, pq) \leq \mathrm{ICONN}^f_H(p, pq)$. □

Definition 17. A pair (p, pq) in $G = (A, B, C)$ is SP if $C^t(p, pq) \geq \mathrm{ICONN}^t_G(p, pq)$ and $C^f(p, pq) \leq \mathrm{ICONN}^f_G(p, pq)$.

Example 5. Consider the VIG G as Figure 5.

Here, $C^t(p, pq) = 0.1$, $C^f(p, pq) = 0.5$, $C^t(q, qr) = 0.1$, $C^f(q, qr) = 0.6$, $\mathrm{ICONN}^t_G(p, pq) = 0.1$, $\mathrm{ICONN}^f_G(p, pq) = 0.7$, $\mathrm{ICONN}^t_G(q, qr) = 0.1$, $\mathrm{ICONN}^f_G(q, qr) = 0.6$.

Hence, $C^t(p, pq) = \mathrm{ICONN}^t_G(p, pq) = 0.1$, $C^f(p, pq) = 0.5 < \mathrm{ICONN}^f_G(p, pq) = 0.7$ and $C^t(q, qr) = \mathrm{ICONN}^t_G(q, qr) = 0.1$, $C^f(q, qr) = \mathrm{ICONN}^f_G(b, bc) = 0.6$. Therefore, (p, pq) and (q, qr) are SPs.

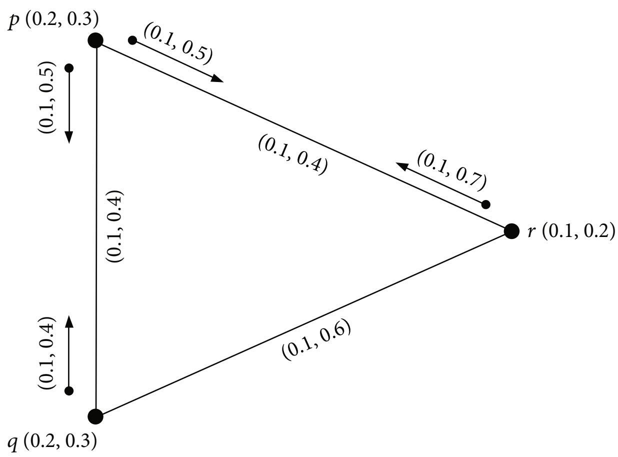

Figure 5: Vague incidence graph G with strong pairs.

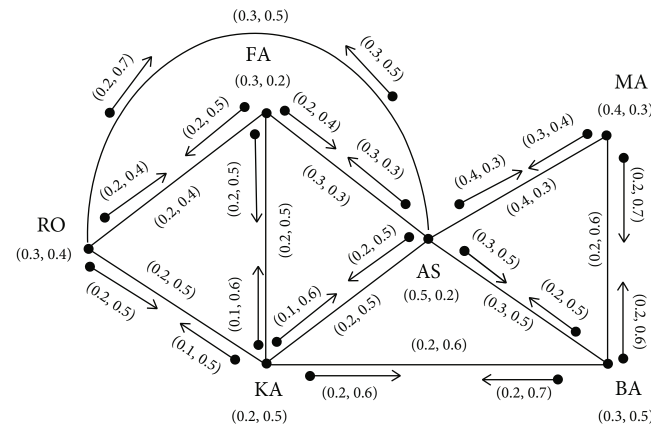

Figure 6: Vague incidence graph.

Table 2: Name of employees in an university and their services.

Name	Services
Asadi	Head of the university
Rostami	University educational director
Falah	Director of research
Bagheri	Financial affairs manager
Karami	Director of student nutrition
Mahdavi	Head of security

Table 3: The level of staff capability.

	Rostami	Falah	Asadi	Mahdavi	Bagheri	Karami
t_A	0.3	0.3	0.5	0.4	0.3	0.2
f_A	0.4	0.2	0.2	0.3	0.5	0.5

Theorem 18. *Let $G = (A, B, C)$ be a VIG and for a pair (p, pq), $(C^t(p, pq), C^f(p, pq)) = (\vee\{C^t(w, wk)\}, \wedge\{C^f(w, wk)\})$, for all pairs (w, wk) in G. Then, the pair (p, pq) is a SP.*

Proof. Let (p, pq) be a pair in the VIG G. If there is an unique pair (w, wk) in G so that $C^t(p, pq) = \vee\{C^t(w, wk)\}$ and $C^f(p, pq) = \wedge\{C^f(w, wk)\}$, for all pairs (w, wk) in G, then the true part of the strength of all the paths without (p, pq) must be less than $C^t(p, pq)$ and the false part of the strength of all the paths without (p, pq) must be greater than $C^f(p, pq)$. Then, (p, pq) is a strong pair.

Again, if there are more than one pair (p, pq) in G so that $C^t(p, pq) = \vee\{C^t(p, pq)\}$ and $C^f(p, pq) = \wedge\{C^f(w, wk)\}$, for all pairs (w, wk) in G, then $C^t(p, pq) = C^t(w, wk)$ and $C^f(p, pq) = C^f(w, wk)$, for all pairs (w, wk) in $G - \{(p, pq)\}$.

Now from the definition, we have $C^t(p, pq) \geq \mathrm{ICONN}^t_G(w, wk)$, $C^f(p, pq) \leq \mathrm{ICONN}^f_G(w, wk)$, for all pairs (w, wk) in G. Hence, (p, pq) is a SP. □

Definition 19. Let G be a VIG and H be a VISG of G so that for a pair (p, pq) in G, $H = G - \{(p, pq)\}$. If, $\mathrm{ICONN}^t_G(w, wk) > \mathrm{ICONN}^t_H(w, wk)$ and $\mathrm{ICONN}^f_G(w, wk) < \mathrm{ICONN}^f_H(w, wk)$, for some pair (w, wk) in G, then, the pair (p, pq) is called an incidence-CP of G.

Theorem 20. *Let (p, pq) be a pair in a VIG $G = (A, B, C)$ so that $C^t(p, pq) = \wedge\{t_A(p), t_B(pq)\}$ and $C^f(p, pq) = \vee\{f_A(p), f_B(pq)\}$. Then, the pair (p, pq) is a SP of G.*

Proof. Let $H = (A', B', C')$ be a VISG of a VIG $G = (A, B, C)$ so that $H = G - \{(p, pq)\}$, where (p, pq) is a pair of G. If H is disconnected, then (p, pq) should be a CP of G. Then, $\mathrm{ICONN}^t_G(p, pq) > \mathrm{ICONN}^t_H(p, pq)$ and $\mathrm{ICONN}^f_G(p, pq) < \mathrm{ICONN}^f_H(p, pq)$. Then by definition, (p, pq) is a sp. If H is connected, then $\exists$ pairs (p, pr) for some $p \neq r$, so that (p, pr) and (q, pq) lies in an incidence path from p to pq in H. Then, $\mathrm{ICONN}^t_H(p, pq) \leq \wedge\{C^t(p, pr), C^t(q, pq)\} \leq \wedge\{t_A(p), t_B(pr), t_A(q), t_B(pq)\} \leq \wedge\{t_A(p), t_B(pq)\} = C^t(p, pq)$ and $\mathrm{ICONN}^f_H(p, pq) \geq \wedge\{C^f(p, pr), C^f(q, pq)\} \geq \wedge\{f_A(p), f_B(pr), f_A(q), f_B(pq)\} \geq \wedge\{f_A(p), f_B(pq)\} = C^f(p, pq)$. Thus, $C^t(p, pq) \geq \mathrm{ICONN}^t_H(p, pq)$ and $C^f(p, pq) \geq \mathrm{ICONN}^f_H(p, pq)$. Therefore, (p, pq) is a SP of G. □

Theorem 21. *Every pair of a complete VIG is a sp.*

Proof. Let (p, pq) be a pair in a complete-VIG $G = (A, B, C)$. Then $C^t(p, pq) = \wedge\{t_A(p), t_B(pq)\}$ and $C^f(p, pq) = \vee\{f_A(p), f_B(pq)\}$. Now, by Theorem 20, it can be proved that (p, pq) is a SP of G. Since (p, pq) is arbitrary in the complete-VIG G, so all the pairs of G are SP. □

4. Application of Vague Incidence Graph to Find the Most Effective Person in a University

Science and education are always very important issues for any country that has a very high status. Because if any society has higher education, it will naturally have higher progress and prosperity. Note that the emergence of science and knowledge is equal to the creation of man. Man has always sought to understand and comprehend. Science and knowledge are very important in human life. The role of

Table 4: Adjacency matrix corresponding to Figure 6.

	Rostami	Falah	Karami	Asadi	Mahdavi	Bagheri
Rostami	(0, 0)	(0.2,0.4)	(0.2,0.5)	(0, 0)	(0, 0)	(0, 0)
Falah	(0.2,0.5)	(0, 0)	(0.2,0.5)	(0.2,0.4)	(0, 0)	(0, 0)
Karami	(0.1,0.5)	(0.1,0.6)	(0, 0)	(0.1,0.6)	(0, 0)	(0.2,0.6)
Asadi	(0.3,0.5)	(0.3,0.3)	(0.2,0.5)	(0, 0)	(0.4,0.3)	(0.3,0.5)
Mahdavi	(0, 0)	(0, 0)	(0, 0)	(0.3,0.4)	(0, 0)	(0.2,0.7)
Bagheri	(0, 0)	(0, 0)	(0.2,0.7)	(0.2,0.5)	(0.2,0.6)	(0, 0)

science in human life is to teach human beings the path to happiness, evolution, and construction. Science enables man to build the future the way he wants. Science is given as a tool at the will of man and makes nature as man wants and commands. Today, the importance of science and knowledge on humanity is not hidden, and all human schools and heavenly religions emphasize the acquisition of science and knowledge and consider the progress and advancement in the path of science as honorable. Science and knowledge are two wings that human beings can fly to infinity. The value of each human being is determined by how they are used. But one of the educational centers that play an important role in educating people are universities. Therefore, the university staff must fulfill their responsibilities in the best possible way so that there is no disruption in education. Therefore, in this section, we try to introduce the most effective staff of a university with the help of an VIG. To do this, we consider the nodes of this graph as the staff of an university and the edges as the influence of one employee on another employee. For this university, the staff is as follows: $E = \{\text{Rostami(RO)}, \text{Falah(FA)}, \text{Karami(KA)}, \text{Asadi(AS)}, \text{Mahdavi(MA)}, \text{Bagheri(BA)}\}$.

(i) Rostami has been working with Falah for 8 years and values his views on issues

(ii) Asadi has been the head of the university for a long time, and not only Falah but also Karami is very satisfied with Asadi's performance

(iii) For a university, the protection of educational tools and student files is very important. Mahdavi is a suitable person for this job

(iv) Rostami and Karami have a long history of conflict

(v) Falah has a very important role in the research work of the university

Given the above, we consider a VIG. The nodes shows each of the university staff. Each staff member has the desired ability as well as shortcomings in the performance of their duties. Therefore, we use of vague set to express the weight of the nodes. The true membership shows the efficiency of the employee, and the false membership shows the lack of management and shortcomings of each staff. But the edges shows the level of relationships and friendships between employees. If these relationships are stronger, then the student education process will be faster. Hence, the edges can be considered as a vague set so that the true membership shows a friendly relationship between both employees and the false membership shows the degree of conflict and difference between the two officials. Name of employees and level of staff capability are shown in Tables 2 and 3. The adjacency matrix corresponding to Figure 6 is shown in Table 4.

Figure 6 shows that Rostami has 30% of the power needed to do the university work as university educational director but does not have the 40% knowledge needed to be the boss. The incidence edge Karami-Bagheri shows that there is only 20% interaction and friendship between these two employees and unfortunately they have 60% disagreement and conflict. It is clear that Rostami, Kazemi and Asadi obey Fallah. Fallah's dominance rate over all three is equal to 20%. Clearly, Asadi is the most influential employee of the university because he controls all five of the university staff and also he has the highest amount of knowledge among the university staff, which is equal to 50%.

5. Conclusion

Vague incidence graph has various applications in modern science and technology, especially in the fields of neural networks, computer science, operation research, and decision making. Hence, VIG and VISG are defined and their properties are explained by several examples. Also, different incidence paths and their strengths, connectedness, and properties are introduced. Finally, the strength of connectedness between pairs in VIG and VISG are investigated. In our future work, we will study the concepts of covering, matchings, and independent dominating on VIGs and investigate their properties with some examples.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] A. Kauffman, "Introduction a la theories des sous-emsembles 503 ous," *Masson Cie*, vol. 1, pp. 607–618, 1973.

[3] L. A. Zadeh, "Similarity relations and fuzzy orderings," *Information Sciences*, vol. 3, no. 2, pp. 177–200, 1971.

[4] L. A. Zadeh, "Is there a need for fuzzy logic?," *Information Sciences*, vol. 178, no. 13, pp. 2751–2779, 2008.

[5] A. Rosenfeld, *Fuzzy Graphs, Fuzzy Sets and Their Applications*, L. A. Zadeh, K. S. Fu, and M. Shimura, Eds., Academic Press, New York, NY, USA, 1975.

[6] W.-L. Gau and D. J. Buehrer, "Vague sets," *IEEE Transactions on Systems, Man, and Cy- bernetics*, vol. 23, no. 2, pp. 610–614, 1993.

[7] M. Akram, W. A. Dudek, and M. Murtaza Yousof, "Regularity in vague intersection graphs and vague line graphs," *Abstract and Applied Analysis*, vol. 2014, 10 pages, 2014.

[8] M. Akram, N. Gani, and A. B. Saeid, "Vague hypergraphs," *Journal of Intelligent Fuzzy Systems*, vol. 26, no. 2, pp. 647–653, 2014.

[9] S. Samanta and M. Pal, "Irregular bipolar fuzzy graphs," *International Journal of Applications of Fuzzy Sets*, vol. 2, pp. 91–102, 2012.

[10] S. Samanta and M. Pal, "Fuzzy k-competition graphs and p-competition fuzzy graphs," *Fuzzy Information and Engineering*, vol. 5, no. 2, pp. 191–204, 2013.

[11] N. Ramakrishna, "Vague graphs," *International Journal of Cognitive Computing*, vol. 7, pp. 51–58, 2009.

[12] R. A. Borzooei and H. Rashmanlou, "Domination in vague graphs and its applications," *Journal of Intelligent Fuzzy Systems*, vol. 29, no. 5, pp. 1933–1940, 2015.

[13] S. Banitalebi, R. A. Borzooei, and E. Mohammaszadeh, "Irregular vague graphs," *Algebraic Structures and Their Applications*, vol. 2, no. 2, pp. 73–90, 2021.

[14] K. R. Bhutani and A. Rosenfeld, "Strong arcs in fuzzy graphs," *Information Sciences*, vol. 152, pp. 319–322, 2003.

[15] E. Darabian and R. A. Borzooei, "Results on vague graphs with applications in human trafficking," *New Mathematics and Natural Computation*, vol. 14, no. 1, pp. 37–52, 2018.

[16] J. N. Mordeson and C. S. Peng, "Operations on fuzzy graphs," *Information Sciences*, vol. 79, no. 3-4, pp. 159–170, 1994.

[17] G. Ghorai and M. Pal, "A note on "regular bipolar fuzzy graphs" Neural Computing and Applications 21(1) (2012) 197–205," *Neural Computing and Applications*, vol. 30, no. 5, pp. 1569–1572, 2018.

[18] G. Ghorai and K. Jacob, "Recent developments on the basic of fuzzy graph theory," in *Handbook of Research on Advanced Applications of Graph Theory in Modern Society*, IGI Global, 2019.

[19] S. Mandal, S. Sahoo, G. Ghorai, and M. Pal, "Application of strong arcs in m-polar fuzzy graphs," *Neural Processing Letters*, vol. 50, no. 1, pp. 771–784, 2019.

[20] T. Dinesh, "Fuzzy incidence graph - an introduction," *Advances in Fuzzy sta and systems*, vol. 21, no. 1, pp. 33–48, 2016.

[21] J. N. Mordeson, S. Mathew, and D. S. Malik, *Fuzzy Graph Theory with Application to Human Trafficking*, Springer, 2018.

[22] D. S. Malik, S. Mathew, and J. N. Mordeson, "Fuzzy incidence graphs: applications to human trafficking," *Information Sciences*, vol. 447, pp. 244–255, 2018.

[23] S. Poulik and G. Ghorai, "Detour g-interior nodes and detour g-boundary nodes in bipolar fuzzy graph with applications," *Hacettepe Journal of Mathematics and Statistics*, vol. 49, no. 3, pp. 106–119, 2019.

[24] S. Poulik and G. Ghorai, "Certain indices of graphs under bipolar fuzzy environment with applications," *Soft Computing*, vol. 24, no. 7, pp. 5119–5131, 2020.

[25] S. Poulik and G. Ghorai, "Note on "Bipolar fuzzy graphs with applications"," *Knowledge-Based Systems*, vol. 192, article 105315, 2020.

[26] S. Poulik and G. Ghorai, "Determination of journeys order based on graph's Wiener absolute index with bipolar fuzzy information," *Information Sciences*, vol. 545, pp. 608–619, 2021.

[27] S. Kosari, Y. Rao, H. Jiang, X. Liu, P. Wu, and Z. Shao, "Vague graph structure with application in medical diagnosis," *Symmetry*, vol. 12, no. 10, pp. 1582–1582, 2020.

[28] Z. Shao, S. Kosari, M. Shoaib, and H. Rashmanlou, "Certain concepts of vague graphs with applications to medical diagnosis," *Frontiers in Physics*, vol. 8, pp. 3–57, 2020.

[29] S. Zeng, M. Shoaib, S. Ali, F. Smarandache, H. Rashmanlou, and F. Mofidnakhaei, "Certain properties of single-valued neutrosophic graph with application in food and agriculture organization," *International Journal of Computational Intelligence Systems*, vol. 14, no. 1, pp. 1516–1540, 2021.

[30] H. Rashmanlou and R. A. Borzooei, "Product vague graphs and its applications," *Journal of Intelligent Fuzzy Systems*, vol. 30, no. 1, pp. 371–382, 2015.

[31] H. Rashmanlou, G. Muhiuddin, S. K. Amanathulla, F. Mofidnakhaei, and M. Pal, "A study on cubic graphs with novel application," *Journal of Intelligent Fuzzy Systems*, vol. 40, no. 1, pp. 89–101, 2021.

[32] S. Satham Hussain, R. Jahir Hussain, and M. Vignesh Babu, "Neutrosophic vague incidence graph," *Science*, vol. 12, no. 1, pp. 29–38, 2020.

[33] Y. Rao, S. Kosari, Z. Shao, R. Cai, and L. Xinyue, "A study on domination in vague incidence graph and its application in medical sciences," *Symmetry*, vol. 12, no. 11, p. 1885, 2020.

[34] R. A. Brualdi and J. L. Q. Massey, "Incidence and strong edge colorings of graphs," *Discrete Mathematics*, vol. 122, no. 1-3, pp. 51–58, 1993.