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Research Article

SR-Fuzzy Sets and Their Weighted Aggregated Operators in Application to Decision-Making

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An intuitionistic fuzzy set is one of the efficient generalizations of a fuzzy set for dealing with vagueness/uncertainties in information. Under this environment, in this manuscript, we familiarize a new type of extensions of fuzzy sets called square-root fuzzy sets (briefly, SR-Fuzzy sets) and contrast SR-Fuzzy sets with intuitionistic fuzzy sets and Pythagorean fuzzy sets. We discover the essential set of operations for the SR-Fuzzy sets along with their several properties. In addition, we define a score function for the ranking of SR-Fuzzy sets. To study multiattribute decision-making problems, we introduce four new weighted aggregated operators, namely, SR-Fuzzy weighted average (SR-FWA) operator, SR-Fuzzy weighted geometric (SR-FWG) operator, SR-Fuzzy weighted power average (SR-FWPA) operator, and SR-Fuzzy weighted power geometric (SR-FWPG) operator over SR-Fuzzy sets. We apply these operators to select the top-rank university and show how we can choose the best option by comparing the aggregate outputs through score values.

1. Introduction

The idea of fuzzy sets was proposed by Zadeh [1] to handle the imprecise information. The notion of rough set theory was originally introduced by Pawlak [2], and it was applied to many different domains (see [3–5]). The concept of soft sets was first defined by Molodtsov [6] as a general mathematical tool for dealing with uncertain objects. The merging between fuzzy sets and some uncertainty approaches such as rough sets and soft sets have been discussed in [7–9].

In several real-life situations, the degree of nonmembership is not obtained from the degree of membership. In these cases, the notion of intuitionistic fuzzy sets defined by Atanassov [10] worked very well. It is one of the interesting generalizations of fuzzy sets with best applicability. In various fields, the applications of intuitionistic fuzzy sets appear, including optimization problems, medical diagnosis, and decision-making [11–15]. However, there are numerous situations where the decision-maker may supply the degrees of membership and nonmembership of a specific attribute in such a way that their sum is greater than one. Therefore, Yager [16] put forward the concept of Pythagorean fuzzy sets which is a generalization of intuitionistic fuzzy sets, and it is a more powerful tool to solve uncertain problems. Ibrahim et al. [17] defined a new generalization of Pythagorean fuzzy sets called (3, 2)-Fuzzy sets. The main advantage of (3, 2)-Fuzzy sets is that they can characterize more vague cases than Pythagorean fuzzy sets, which can be exploited in many decision-making problems.

The idea of intuitionistic fuzzy weighted averaging operators was proposed by Xu [18]. Some geometric weighted, geometric ordered weighted, and geometric hybrid operators under the environment of intuitionistic fuzzy sets were introduced by Xu and Yager [19]. In Refs. [20–25], many researchers worked in the area of intuitionistic fuzzy sets and established various aggregation operators which are applied to group decision-making. After the advent of the Pythagorean fuzzy sets, the operators of Pythagorean fuzzy aggregation have also become an important and interesting field for research. Yager and Abbasov [26, 27], in 2013, introduced the concepts of weighted geometric, weighted averaging, ordered weighted geometric, and ordered weighted averaging operators in the frame of Pythagorean fuzzy environment. The essential properties of Pythagorean fuzzy aggregation operators were investigated by the authors of [28]. Shahzadi et al. [29] established some aggregation operators under Pythagorean fuzzy data for assessing the distinct preferences of the choice among the decision-making process. By using Pythagorean fuzzy values, Rahman et al. presented many aggregation operators like weighted geometric [30], hybrid geometric [31], weighted averaging [32], and ordered weighted geometric operators [33] and also discussed their practical applications.

The aims of writing this paper are (1) to present a novel extension of intuitionistic fuzzy set called SR-Fuzzy sets which is not obtained from q-rung orthopair fuzzy sets, (2) to introduce novel types of weighted aggregation operators and discuss their main properties, and (3) to investigate a MCDM methods depending on these operators.

In this paper, we define the concept of SR-Fuzzy sets and compare it with the other types of fuzzy sets in Section 2. Then, we introduce the set of operations for the SR-Fuzzy sets and explore their main features in Section 3. Also, the concepts of weighted aggregated operators for SR-Fuzzy sets are investigated. Thereafter, we describe MADM problems under these operators in Section 4. Finally, we outline the main achievements of the paper and propose some upcoming works in Section 5.

Before we present our main concepts and results, we recall the definitions of the intuitionistic fuzzy set (IFS) and Pythagorean fuzzy set (PFS).

Definition 1. Let S be a universal set such that $Y_\Theta : S \longrightarrow [0,1]$ and $\Psi_\Theta : S \longrightarrow [0,1]$ are mapping. Then, the intuitionistic fuzzy set (IFS) [10] (resp., Pythagorean fuzzy set (PFS) [16]) is defined by the following:

$$\Theta = \{\langle p, Y_\Theta(p), \Psi_\Theta(p)\rangle : p \in S\}, \quad (1)$$

including the condition $0 \le Y_\Theta(p) + \Psi_\Theta(p) \le 1$ (resp., $0 \le (Y_\Theta(p))^2 + (\Psi_\Theta(p))^2 \le 1$), where $Y_\Theta(p)$ is the degree of membership and $\Psi_\Theta(p)$ is the degree of nonmembership of every $p \in S$ to Θ.

2. SR-Fuzzy Sets

In this section, we initiate the notion of SR-Fuzzy sets and study its features in detail. For computations, we use only six decimal places in the whole paper.

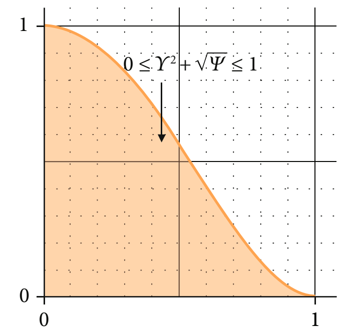

Figure 1: Grades space of SR-FSs.

Definition 2. Let S be a universal set such that $Y_\Theta : S \longrightarrow [0,1]$ and $\Psi_\Theta : S \longrightarrow [0,1]$ are mapping. Then, the SR-Fuzzy set (briefly, SR-FS) Θ is defined as following:

$$\Theta = \{\langle p, Y_\Theta(p), \Psi_\Theta(p)\rangle : p \in S\}, \quad (2)$$

where $Y_\Theta(p)$ is the degree of membership and $\Psi_\Theta(p)$ is the degree of nonmembership of $p \in S$ to Θ, such that

$$0 \le (Y_\Theta(p))^2 + \sqrt{\Psi_\Theta(p)} \le 1. \quad (3)$$

Then, there is a degree of indeterminacy of $p \in S$ to Θ defined by

$$\pi_\Theta(p) = 1 - \left[(Y_\Theta(p))^2 + \sqrt{\Psi_\Theta(p)}\right]. \quad (4)$$

It is obvious that $(Y_\Theta(p))^2 + \sqrt{\Psi_\Theta(p)} + \pi_\Theta(p) = 1$. Otherwise, $\pi_\Theta(p) = 0$ whenever $(Y_\Theta(p))^2 + \sqrt{\Psi_\Theta(p)} = 1$.

In the interest of simplicity, we shall mention the symbol $\Theta = (Y_\Theta, \Psi_\Theta)$ for the SR-FS $\Theta = \{\langle p, Y_\Theta(p), \Psi_\Theta(p)\rangle : p \in S\}$. The space of SR-Fuzzy membership grades is displayed in Figure 1.

Example 1. Assume that $Y_\Theta(p) = 0.3$ and $\Psi_\Theta(p) = 0.8$ for $S = \{p\}$. Then, $\Theta = (0.3, 0.8)$ is not an intuitionistic fuzzy set because $0.3 + 0.8 = 1.1 > 1$. In contrast, $\Theta = (0.3, 0.8)$ is an SR-FS because $(0.3)^2 + \sqrt{0.8} \approx 0.984427 \le 1$.

Note that $\pi_\Theta(p) \approx 0.015573$, and hence, $(Y_\Theta(p))^2 + \sqrt{\Psi_\Theta(p)} + \pi_\Theta(p) = 1$.

Remark 3. From Figure 2, we get that

(1) The space of Pythagorean membership grades is larger than the space of SR-Fuzzy membership grades

(2) The SR-Fuzzy and intuitionistic fuzzy sets intersect at the point $\Theta = (Y_\Theta = (-1 + \sqrt{5})/2, \Psi_\Theta = (3 - \sqrt{5})/2)$

(3) For $Y_\Theta \in (0, (-1 + \sqrt{5})/2)$ and $\Psi_\Theta \in ((3 - \sqrt{5})/2, 1)$, the space of SR-Fuzzy membership grades starts to be larger than the space of intuitionistic membership grades

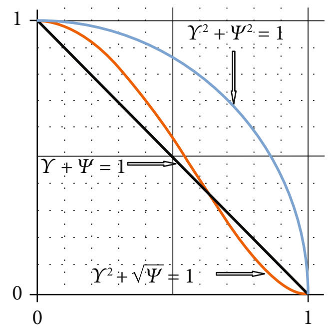

Figure 2: Comparison of grades space of IFSs, PFSs, and SR-FSs.

(4) for $\Upsilon_\Theta \in ((-1+\sqrt{5})/2, 1)$ and $\Psi_\Theta \in (0, (3-\sqrt{5})/2)$, the space of SR-Fuzzy membership grades starts to be smaller than the space of intuitionistic membership grades

Definition 4. Let $\Theta_1 = (\Upsilon_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (\Upsilon_{\Theta_2}, \Psi_{\Theta_2})$ be two SR-FSs; then

(1) $\Theta_1 = \Theta_2$ if and only if $\Upsilon_{\Theta_1} = \Upsilon_{\Theta_2}$ and $\Psi_{\Theta_1} = \Psi_{\Theta_2}$

(2) $\Theta_1 \geq \Theta_2$ if and only if $\Upsilon_{\Theta_1} \geq \Upsilon_{\Theta_2}$ and $\Psi_{\Theta_1} \leq \Psi_{\Theta_2}$

Example 2.

(1) If $\Theta_1 = (0.2,0.9)$ and $\Theta_2 = (0.2,0.9)$ for $S = \{p\}$, then $\Theta_1 = \Theta_2$

(2) If $\Theta_1 = (0.2,0.9)$ and $\Theta_2 = (0.1,0.91)$ for $S = \{p\}$, then $\Theta_2 \leq \Theta_1$

Definition 5. Let $\Theta_1 = (\Upsilon_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (\Upsilon_{\Theta_2}, \Psi_{\Theta_2})$ be two SR-Fuzzy sets (SR-FSs). Then

(1) $\Theta_1 \cap \Theta_2 = (\min\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\})$

(2) $\Theta_1 \cup \Theta_2 = (\max\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \min\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\})$

(3) $\Theta_1^c = (\sqrt[4]{\Psi_{\Theta_1}}, (\Upsilon_{\Theta_1})^4)$

Note that $(\sqrt[4]{\Psi_{\Theta_1}})^2 + \sqrt[2]{(\Upsilon_{\Theta_1})^4} = \sqrt[2]{\Psi_{\Theta_1}} + (\Upsilon_{\Theta_1})^2 \leq 1$, so Θ_1^c is an SR-Fuzzy set. It is obvious that $(\Theta^c)^c = (\sqrt[4]{\Psi_\Theta}, (\Upsilon_\Theta)^4)^c = (\Upsilon_\Theta, \Psi_\Theta)$.

Example 3. Assume that $\Theta_1 = (\Upsilon_{\Theta_1} = 0.59, \Psi_{\Theta_1} = 0.42)$ and $\Theta_2 = (\Upsilon_{\Theta_2} = 0.56, \Psi_{\Theta_2} = 0.45)$ are both SR-FSs for $S = \{p\}$. Then

(1) $\Theta_1 \cap \Theta_2 = (\min\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}) = (\min\{0.59,0.56\}, \max\{0.42,0.45\}) = (0.56,0.45)$

(2) $\Theta_1 \cup \Theta_2 = (\max\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \min\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}) = (\max\{0.59,0.56\}, \min\{0.42,0.45\}) = (0.59,0.42)$

(3) $\Theta_1^c \approx (0.805030,0.121174)$

Theorem 6. *Let $\Theta_1 = (\Upsilon_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (\Upsilon_{\Theta_2}, \Psi_{\Theta_2})$ be two SR-FSs; then the following properties hold:*

(1) $\Theta_1 \cap \Theta_2 = \Theta_2 \cap \Theta_1$

(2) $\Theta_1 \cup \Theta_2 = \Theta_2 \cup \Theta_1$

Proof. From Definition 5, we can obtain the following:

(1) $\Theta_1 \cap \Theta_2 = (\min\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}) = (\min\{\Upsilon_{\Theta_2}, \Upsilon_{\Theta_1}\}, \max\{\Psi_{\Theta_2}, \Psi_{\Theta_1}\}) = \Theta_2 \cap \Theta_1$

(2) The proof is similar to (1)

□

Theorem 7. *Let $\Theta_1 = (\Upsilon_{\Theta_1}, \Psi_{\Theta_1})$, $\Theta_2 = (\Upsilon_{\Theta_2}, \Psi_{\Theta_2})$ and $\Theta_3 = (\Upsilon_{\Theta_3}, \Psi_{\Theta_3})$ be three SR-FSs; then*

(1) $\Theta_1 \cap (\Theta_2 \cap \Theta_3) = (\Theta_1 \cap \Theta_2) \cap \Theta_3$

(2) $\Theta_1 \cup (\Theta_2 \cup \Theta_3) = (\Theta_1 \cup \Theta_2) \cup \Theta_3$

Proof. For the three SR-FSs Θ_1, Θ_2, and Θ_3, according to Definition 5, we obtain the following:

(1) $\Theta_1 \cap (\Theta_2 \cap \Theta_3) = (\Upsilon_{\Theta_1}, \Psi_{\Theta_1}) \cap (\min\{\Upsilon_{\Theta_2}, \Upsilon_{\Theta_3}\}, \max\{\Psi_{\Theta_2}, \Psi_{\Theta_3}\}) = (\min\{\Upsilon_{\Theta_1}, \min\{\Upsilon_{\Theta_2}, \Upsilon_{\Theta_3}\}\}, \max\{\Psi_{\Theta_1}, \max\{\Psi_{\Theta_2}, \Psi_{\Theta_3}\}\}) = (\min\{\min\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \Upsilon_{\Theta_3}\}, \max\{\max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}, \Psi_{\Theta_3}\}) = (\min\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}) \cap (\Upsilon_{\Theta_3}, \Psi_{\Theta_3}) = (\Theta_1 \cap \Theta_2) \cap \Theta_3$

(2) The proof is similar to (1)

□

Theorem 8. *Let $\Theta_1 = (\Upsilon_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (\Upsilon_{\Theta_2}, \Psi_{\Theta_2})$ be two SR-FSs. Then*

(1) $(\Theta_1 \cap \Theta_2) \cup \Theta_2 = \Theta_2$

(2) $(\Theta_1 \cup \Theta_2) \cap \Theta_2 = \Theta_2$

Proof. From Definition 5, we obtain the following:

(1) $(\Theta_1 \cap \Theta_2) \cup \Theta_2 = (\min\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}) \cup (\Upsilon_{\Theta_2}, \Psi_{\Theta_2}) = (\max\{\min\{\Upsilon_{\Theta_1}, \Upsilon_{\Theta_2}\}, \Upsilon_{\Theta_2}\}, \min\{\max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}, \Psi_{\Theta_2}\}) = (\Upsilon_{\Theta_2}, \Psi_{\Theta_2}) = \Theta_2$

(2) The proof is similar to (1)

□

Theorem 9. *Let $\Theta_1 = (Y_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (Y_{\Theta_2}, \Psi_{\Theta_2})$ be two SR-FSs; then*

(1) $(\Theta_1 \cap \Theta_2)^c = \Theta_1^c \cup \Theta_2^c$

(2) $(\Theta_1 \cup \Theta_2)^c = \Theta_1^c \cap \Theta_2^c$

Proof. For the two SR-FSs Θ_1 and Θ_2, according to Definition 5, we obtain the following:

(1) $(\Theta_1 \cap \Theta_2)^c = (\min\{Y_{\Theta_1}, Y_{\Theta_2}\}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\})^c = (\max\{\sqrt[4]{\Psi_{\Theta_1}}, \sqrt[4]{\Psi_{\Theta_2}}\}, \min\{(Y_{\Theta_1})^4, (Y_{\Theta_2})^4\}) = (\sqrt[4]{\Psi_{\Theta_1}}, (Y_{\Theta_1})^4) \cup (\sqrt[4]{\Psi_{\Theta_2}}, (Y_{\Theta_2})^4) = \Theta_1^c \cup \Theta_2^c$

(2) The proof is similar to (1)

□

3. Aggregation of SR-Fuzzy Sets and Its Properties

In this section, we introduce some new operations on SR-Fuzzy sets. Besides, we study the SR-Fuzzy aggregation operators and some attracted properties are indicated in detail.

3.1. Some Operations On SR-Fuzzy Sets

Definition 10. Let $\Theta = (Y_\Theta, \Psi_\Theta)$, $\Theta_1 = (Y_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (Y_{\Theta_2}, \Psi_{\Theta_2})$ be three SR-FSs and ρ be a positive real value ($\rho > 0$). Then their operations are defined as follows:

(1) $\Theta_1 \oplus \Theta_2 = (\sqrt{Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2}, \Psi_{\Theta_1}\Psi_{\Theta_2})$

(2) $\Theta_1 \otimes \Theta_2 = (Y_{\Theta_1} Y_{\Theta_2}, (\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_2}} - \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}})^2)$

(3) $\rho\Theta = (\sqrt{1 - (1 - Y_\Theta^2)^\rho}, \Psi_\Theta^\rho)$

(4) $\Theta^\rho = (Y_\Theta^\rho, (1 - (1 - \sqrt{\Psi_\Theta})^\rho)^2)$

Example 4. Suppose that $\Theta_1 = (Y_{\Theta_1} = 0.43, \Psi_{\Theta_1} = 0.64)$ and $\Theta_2 = (Y_{\Theta_2} = 0.26, \Psi_{\Theta_2} = 0.81)$ are both SR-FSs for $S = \{p\}$. Then

(1) $\Theta_1 \oplus \Theta_2 = (\sqrt{Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2}, \Psi_{\Theta_1}\Psi_{\Theta_2}) = (\sqrt{0.43^2 + 0.26^2 - (0.43)^2(0.26)^2}, (0.64)(0.81)) \approx (0.489899, 0.5184)$

(2) $\Theta_1 \otimes \Theta_2 = (Y_{\Theta_1} Y_{\Theta_2}, (\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_2}} - \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}})^2) = ((0.43)(0.26), (\sqrt{0.64} + \sqrt{0.81} - \sqrt{0.64}\sqrt{0.81})^2) = (0.1118, 0.9604)$

(3) $\rho\Theta_1 = (\sqrt{1 - (1 - Y_{\Theta_1}^2)^\rho}, \Psi_{\Theta_1}^\rho) = (\sqrt{1 - (1 - 0.43^2)^3}, 0.64^3) \approx (0.677095, 0.262144)$, for $\rho = 3$

(4) $\Theta_1^\rho = (Y_{\Theta_1}^\rho, (1 - (1 - \sqrt{\Psi_{\Theta_1}})^\rho)^2) = (0.43^3, (1 - (1 - \sqrt{0.64})^3)^2) = (0.079507, 0.984064)$, for $\rho = 3$

Theorem 11. *If $\Theta_1 = (Y_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (Y_{\Theta_2}, \Psi_{\Theta_2})$ are two SR-FSs, then $\Theta_1 \oplus \Theta_2$ and $\Theta_1 \otimes \Theta_2$ are also SR-FSs.*

Proof. For SR-FSs $\Theta_1 = (Y_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (Y_{\Theta_2}, \Psi_{\Theta_2})$, the following relations are evident:

$$0 \le Y_{\Theta_1}^2 \le 1, 0 \le \sqrt{\Psi_{\Theta_1}} \le 1, 0 \le (Y_{\Theta_1})^2 + \sqrt{\Psi_{\Theta_1}} \le 1,$$
$$0 \le Y_{\Theta_2}^2 \le 1, 0 \le \sqrt{\Psi_{\Theta_2}} \le 1, 0 \le (Y_{\Theta_2})^2 + \sqrt{\Psi_{\Theta_2}} \le 1. \quad (5)$$

Then, we have

$$Y_{\Theta_1}^2 \ge Y_{\Theta_1}^2 Y_{\Theta_2}^2, Y_{\Theta_2}^2 \ge Y_{\Theta_1}^2 Y_{\Theta_2}^2, 1 \ge Y_{\Theta_1}^2 Y_{\Theta_2}^2 \ge 0,$$
$$\sqrt{\Psi_{\Theta_1}} \ge \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}}, \sqrt{\Psi_{\Theta_2}} \ge \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}}, 1 \ge \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}} \ge 0, \quad (6)$$

which indicates that

$$Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2 \ge 0,$$
$$\text{which means that} \sqrt{Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2} \ge 0,$$

$$\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_2}} - \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}} \ge 0,$$
$$\text{which means that} \left(\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_2}} - \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}}\right)^2 \ge 0. \quad (7)$$

Since $Y_{\Theta_2}^2 \le 1$ and $0 \le 1 - Y_{\Theta_1}^2$, then $Y_{\Theta_2}^2(1 - Y_{\Theta_1}^2) \le (1 - Y_{\Theta_1}^2)$, and we get $Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2 \le 1$, and hence, $\sqrt{Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2} \le 1$.

Similarly, we get

$$\left(\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_2}} - \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}}\right)^2 \le 1. \quad (8)$$

It is obvious that

$$0 \le \sqrt{\Psi_{\Theta_1}} \le 1 - Y_{\Theta_1}^2 \text{ and } 0 \le \sqrt{\Psi_{\Theta_2}} \le 1 - Y_{\Theta_2}^2. \quad (9)$$

Then we get

$$\left(\sqrt{Y_{\Theta_1}^2+Y_{\Theta_2}^2-Y_{\Theta_1}^2Y_{\Theta_2}^2}\right)^2+\sqrt{\Psi_{\Theta_1}\Psi_{\Theta_2}}\le Y_{\Theta_1}^2+Y_{\Theta_2}^2-Y_{\Theta_1}^2Y_{\Theta_2}^2+\left(1-Y_{\Theta_1}^2\right)\left(1-Y_{\Theta_2}^2\right)=1. \quad (10)$$

Therefore

$$0\le\sqrt{Y_{\Theta_1}^2+Y_{\Theta_2}^2-Y_{\Theta_1}^2Y_{\Theta_2}^2}\le 1, 0\le\Psi_{\Theta_1}\Psi_{\Theta_2}\le 1, \quad (11)$$

and

$$0\le\left(\sqrt{Y_{\Theta_1}^2+Y_{\Theta_2}^2-Y_{\Theta_1}^2Y_{\Theta_2}^2}\right)^2+\sqrt{\Psi_{\Theta_1}\Psi_{\Theta_2}}\le 1. \quad (12)$$

Similarly, we have

$$0\le Y_{\Theta_1}Y_{\Theta_2}\le 1, 0\le\left(\sqrt{\Psi_{\Theta_1}}+\sqrt{\Psi_{\Theta_2}}-\sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}}\right)^2\le 1, \quad (13)$$

and

$$0\le\left(Y_{\Theta_1}Y_{\Theta_2}\right)^2+\sqrt{\left(\sqrt{\Psi_{\Theta_1}}+\sqrt{\Psi_{\Theta_2}}-\sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}}\right)^2}\le 1. \quad (14)$$

These indicate that both of $\Theta_1\oplus\Theta_2$ and $\Theta_1\otimes\Theta_2$ are SR-FSs. □

Theorem 12. *Let $\Theta=(Y_\Theta,\Psi_\Theta)$ be an SR-FS and ρ be a positive real value. Then, $\rho\Theta$ and Θ^ρ are also SR-FSs.*

Proof. Since $0\le Y_\Theta^2\le 1$, $0\le\sqrt{\Psi_\Theta}\le 1$, and $0\le(Y_\Theta)^2+\sqrt{\Psi_\Theta}\le 1$, then

$$0\le\sqrt{\Psi_\Theta}\le 1-Y_\Theta^2\Rightarrow 0\le\left(1-Y_\Theta^2\right)^\rho\Rightarrow 1-\left(1-Y_\Theta^2\right)^\rho\le 1\Rightarrow 0\le\sqrt{1-\left(1-Y_\Theta^2\right)^\rho}\le\sqrt{1}=1. \quad (15)$$

It is obvious that $0\le\Psi_\Theta^\rho\le 1$; then we get

$$0\le\left(\sqrt{1-\left(1-Y_\Theta^2\right)^\rho}\right)^2+\sqrt{\Psi_\Theta^\rho}\le 1-\left(1-Y_\Theta^2\right)^\rho+\left(1-Y_\Theta^2\right)^\rho=1. \quad (16)$$

Similarly, we also get

$$0\le\left(Y_\Theta^\rho\right)^2+\sqrt{\left(1-\left(1-\sqrt{\Psi_\Theta}\right)^\rho\right)^2}\le 1. \quad (17)$$

Therefore, $\rho\Theta$ and Θ^ρ are SR-FSs. □

Theorem 13. *Let $\Theta_1=(Y_{\Theta_1},\Psi_{\Theta_1})$ and $\Theta_2=(Y_{\Theta_2},\Psi_{\Theta_2})$ be two SR-FSs. Then the following properties hold:*

(1) $\Theta_1\oplus\Theta_2=\Theta_2\oplus\Theta_1$

(2) $\Theta_1\otimes\Theta_2=\Theta_2\otimes\Theta_1$

Proof. From Definition 10, we obtain the following:

(1) $\Theta_1\oplus\Theta_2=(\sqrt{Y_{\Theta_1}^2+Y_{\Theta_2}^2-Y_{\Theta_1}^2Y_{\Theta_2}^2},\Psi_{\Theta_1}\Psi_{\Theta_2})(\sqrt{Y_{\Theta_2}^2+Y_{\Theta_1}^2-Y_{\Theta_2}^2Y_{\Theta_1}^2},\Psi_{\Theta_2}\Psi_{\Theta_1})=\Theta_2\oplus\Theta_1$

(2) $\Theta_1\otimes\Theta_2=(Y_{\Theta_1}Y_{\Theta_2},(\sqrt{\Psi_{\Theta_1}}+\sqrt{\Psi_{\Theta_2}}-\sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}})^2)=(Y_{\Theta_2}Y_{\Theta_1},(\sqrt{\Psi_{\Theta_2}}+\sqrt{\Psi_{\Theta_1}}-\sqrt{\Psi_{\Theta_2}}\sqrt{\Psi_{\Theta_1}})^2)=\Theta_2\otimes\Theta_1$

□

Theorem 14. *Let $\Theta=(Y_\Theta,\Psi_\Theta)$, $\Theta_1=(Y_{\Theta_1},\Psi_{\Theta_1})$ and $\Theta_2=(Y_{\Theta_2},\Psi_{\Theta_2})$ be three SR-FSs. Then*

(1) $\rho(\Theta_1\oplus\Theta_2)=\rho\Theta_1\oplus\rho\Theta_2$, for $\rho>0$

(2) $(\rho_1+\rho_2)\Theta=\rho_1\Theta\oplus\rho_2\Theta$, for $\rho_1,\rho_2>0$

(3) $(\Theta_1\otimes\Theta_2)^\rho=\Theta_1^\rho\otimes\Theta_2^\rho$, for $\rho>0$

(4) $\Theta^{\rho_1}\otimes\Theta^{\rho_2}=\Theta^{(\rho_1+\rho_2)}$, for $\rho_1,\rho_2>0$

Proof. For the three SR-FSs Θ,Θ_1, and Θ_2, and $\rho,\rho_1,\rho_2>0$, according to Definition 10, we obtain the following:

(1) $\rho(\Theta_1\oplus\Theta_2)=\rho(\sqrt{Y_{\Theta_1}^2+Y_{\Theta_2}^2-Y_{\Theta_1}^2Y_{\Theta_2}^2},\Psi_{\Theta_1}\Psi_{\Theta_2})=(\sqrt{1-}(1-Y_{\Theta_1}^2-Y_{\Theta_2}^2+Y_{\Theta_1}^2Y_{\Theta_2}^2)^\rho,(\Psi_{\Theta_1}\Psi_{\Theta_2})^\rho)=(\sqrt{1-(1-Y_{\Theta_1}^2)^\rho(1-Y_{\Theta_2}^2)^\rho},\Psi_{\Theta_1}^\rho\Psi_{\Theta_2}^\rho)$.

And $\rho\Theta_1\oplus\rho\Theta_2=(\sqrt{1-(1-Y_{\Theta_1}^2)^\rho},\Psi_{\Theta_1}^\rho)\oplus(\sqrt{1-(1-Y_{\Theta_2}^2)^\rho},\Psi_{\Theta_2}^\rho)=(\sqrt{1-(1-Y_{\Theta_1}^2)^\rho+1-(1-Y_{\Theta_2}^2)^\rho-(1-(1-Y_{\Theta_1}^2)^\rho)(1-(1-Y_{\Theta_2}^2)^\rho)},\Psi_{\Theta_1}^\rho\Psi_{\Theta_2}^\rho)=(\sqrt{1-(1-Y_{\Theta_1}^2)^\rho(1-Y_{\Theta_2}^2)^\rho},\Psi_{\Theta_1}^\rho\Psi_{\Theta_2}^\rho)=\rho(\Theta_1\oplus\Theta_2)$

(2) $(\rho_1+\rho_2)\Theta=(\rho_1+\rho_2)(Y_\Theta,\Psi_\Theta)=(\sqrt{1-(1-Y_\Theta^2)^{\rho_1+\rho_2}},\Psi_\Theta^{\rho_1+\rho_2})=(\sqrt{1-(1-Y_\Theta^2)^{\rho_1}(1-Y_\Theta^2)^{\rho_2}},\Psi_\Theta^{\rho_1+\rho_2})=(\sqrt{1-(1-Y_\Theta^2)^{\rho_1}+1-(1-Y_\Theta^2)^{\rho_2}-(1-(1-Y_\Theta^2)^{\rho_1})(1-(1-Y_\Theta^2)^{\rho_2})},\Psi_\Theta^{\rho_1}\Psi_\Theta^{\rho_2})=(\sqrt{1-(1-Y_\Theta^2)^{\rho_1}},\Psi_\Theta^{\rho_1})\oplus(\sqrt{1-(1-Y_\Theta^2)^{\rho_2}},\Psi_\Theta^{\rho_2})=\rho_1\Theta\oplus\rho_2\Theta$

(3) $(\Theta_1\otimes\Theta_2)^\rho=(Y_{\Theta_1}Y_{\Theta_2},(\sqrt{\Psi_{\Theta_1}}+\sqrt{\Psi_{\Theta_2}}-\sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}})^2)^\rho=((Y_{\Theta_1}Y_{\Theta_2})^\rho,(1-(1-\sqrt{\Psi_{\Theta_1}}-\sqrt{\Psi_{\Theta_2}}+\sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}})^\rho)^2)=(Y_{\Theta_1}^\rho Y_{\Theta_2}^\rho,(1-(1-\sqrt{\Psi_{\Theta_1}})^\rho(1-\sqrt{\Psi_{\Theta_2}})^\rho)^2)=(Y_{\Theta_1}^\rho,(1-(1-\sqrt{\Psi_{\Theta_1}})^\rho)^2)\otimes(Y_{\Theta_2}^\rho,(1-(1-\sqrt{\Psi_{\Theta_2}})^\rho)^2)=\Theta_1^\rho\otimes\Theta_2^\rho$

(4) $\Theta^{\rho_1} \otimes \Theta^{\rho_2} = (Y_{\Theta}^{\rho_1}, (1-(1-\sqrt{\Psi_{\Theta}})^{\rho_1})^2) \otimes (Y_{\Theta}^{\rho_2}, (1-(1-\sqrt{\Psi_{\Theta}})^{\rho_2})^2) = (Y_{\Theta}^{\rho_1+\rho_2}, 1-(1-\sqrt{\Psi_{\Theta}})^{\rho_1} + 1 - (1-\sqrt{\Psi_{\Theta}})^{\rho_2} - (1-(1-\sqrt{\Psi_{\Theta}})^{\rho_1})(1-(1-\sqrt{\Psi_{\Theta}})^{\rho_2})) = (Y_{\Theta}^{\rho_1+\rho_2}, (1-(1-\sqrt{\Psi_{\Theta}})^{\rho_1+\rho_2})^2) = \Theta^{(\rho_1+\rho_2)}$

□

Theorem 15. *Let $\Theta_1 = (Y_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (Y_{\Theta_2}, \Psi_{\Theta_2})$ be two SR-FSs and $\rho > 0$. Then*

(1) $\rho(\Theta_1 \cup \Theta_2) = \rho\Theta_1 \cup \rho\Theta_2$

(2) $(\Theta_1 \cup \Theta_2)^{\rho} = \Theta_1^{\rho} \cup \Theta_2^{\rho}$

Proof. For the two SR-FSs Θ_1 and Θ_2, and $\rho > 0$, according to Definitions 5 and 10, we obtain the following:

(1) $\rho(\Theta_1 \cup \Theta_2) = \rho(\max\{Y_{\Theta_1}, Y_{\Theta_2}\}, \min\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\})$
$= (\sqrt{1-(1-\max\{Y_{\Theta_1}^2, Y_{\Theta_2}^2\})^{\rho}}, \min\{\Psi_{\Theta_1}^{\rho}, \Psi_{\Theta_2}^{\rho}\}).$

And $\rho\Theta_1 \cup \rho\Theta_2 = (\sqrt{1-(1-Y_{\Theta_1}^2)^{\rho}}, \Psi_{\Theta_1}^{\rho}) \cup (\sqrt{1-(1-Y_{\Theta_2}^2)^{\rho}}, \Psi_{\Theta_2}^{\rho}) = (\max\{\sqrt{1-(1-Y_{\Theta_1}^2)^{\rho}}, \sqrt{1-(1-Y_{\Theta_2}^2)^{\rho}}\}, \min\{\Psi_{\Theta_1}^{\rho}, \Psi_{\Theta_2}^{\rho}\}) = (\sqrt{1-(1-\max\{Y_{\Theta_1}^2, Y_{\Theta_2}^2\})^{\rho}}, \min\{\Psi_{\Theta_1}^{\rho}, \Psi_{\Theta_2}^{\rho}\}) = \rho(\Theta_1 \cup \Theta_2)$

(2) The proof is similar to (1)

□

Theorem 16. *Let $\Theta = (Y_{\Theta}, \Psi_{\Theta})$, $\Theta_1 = (Y_{\Theta_1}, \Psi_{\Theta_1})$ and $\Theta_2 = (Y_{\Theta_2}, \Psi_{\Theta_2})$ be three SR-FSs, and $\rho > 0$. Then*

(1) $(\Theta_1 \oplus \Theta_2)^c = \Theta_1^c \otimes \Theta_2^c$

(2) $(\Theta_1 \otimes \Theta_2)^c = \Theta_1^c \oplus \Theta_2^c$

(3) $(\Theta^c)^{\rho} = (\rho\Theta)^c$

(4) $\rho(\Theta)^c = (\Theta^{\rho})^c$

Proof. For the three SR-FSs Θ, Θ_1 and Θ_2, and $\rho > 0$, according to Definitions 5(3) and 10, we obtain the following:

(1) $(\Theta_1 \oplus \Theta_2)^c = (\sqrt{Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2}, \Psi_{\Theta_1}\Psi_{\Theta_2})^c$
$= (\sqrt[4]{\Psi_{\Theta_1}\Psi_{\Theta_2}}, (\sqrt{Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2})^4) = (\sqrt[4]{\Psi_{\Theta_1}} \sqrt[4]{\Psi_{\Theta_2}}, (Y_{\Theta_1}^2 + Y_{\Theta_2}^2 - Y_{\Theta_1}^2 Y_{\Theta_2}^2)^2) = (\sqrt[4]{\Psi_{\Theta_1}}, (Y_{\Theta_1})^4) \otimes (\sqrt[4]{\Psi_{\Theta_2}}, (Y_{\Theta_2})^4) = \Theta_1^c \otimes \Theta_2^c$

(2) $(\Theta_1 \otimes \Theta_2)^c = (Y_{\Theta_1} Y_{\Theta_2}, (\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_2}} - \sqrt{\Psi_{\Theta_1}} \sqrt{\Psi_{\Theta_2}})^2)^c = (\sqrt[4]{(\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_2}} - \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}})^2}, (Y_{\Theta_1} Y_{\Theta_2})^4) = (\sqrt{(\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_2}} - \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_2}})}, (Y_{\Theta_1})^4 (Y_{\Theta_2})^4) = (\sqrt[4]{\Psi_{\Theta_1}}, (Y_{\Theta_1})^4) \oplus (\sqrt[4]{\Psi_{\Theta_2}}, (Y_{\Theta_2})^4) = \Theta_1^c \oplus \Theta_2^c$

(3) $(\Theta^c)^{\rho} = (\sqrt[4]{\Psi_{\Theta}}, (Y_{\Theta})^4)^{\rho} = ((\sqrt[4]{\Psi_{\Theta}})^{\rho}, (1-(1-Y_{\Theta}^2)^{\rho})^2) = (\sqrt{1-(1-Y_{\Theta}^2)^{\rho}}, \Psi_{\Theta}^{\rho})^c = (\rho\Theta)^c$

(4) $\rho(\Theta)^c = \rho(\sqrt[4]{\Psi_{\Theta}}, (Y_{\Theta})^4) = (\sqrt{1-(1-\sqrt{\Psi_{\Theta}})^{\rho}}, ((Y_{\Theta})^4)^{\rho}) = (Y_{\Theta}^{\rho}, (1-(1-\sqrt{\Psi_{\Theta}})^{\rho})^2)^c = (\Theta^{\rho})^c$

□

Theorem 17. *Let $\Theta_1 = (Y_{\Theta_1}, \Psi_{\Theta_1})$, $\Theta_2 = (Y_{\Theta_2}, \Psi_{\Theta_2})$, and $\Theta_3 = (Y_{\Theta_3}, \Psi_{\Theta_3})$ be three SR-FSs. Then*

(1) $(\Theta_1 \cap \Theta_2) \oplus \Theta_3 = (\Theta_1 \oplus \Theta_3) \cap (\Theta_2 \oplus \Theta_3)$

(2) $(\Theta_1 \cup \Theta_2) \oplus \Theta_3 = (\Theta_1 \oplus \Theta_3) \cup (\Theta_2 \oplus \Theta_3)$

(3) $(\Theta_1 \cap \Theta_2) \otimes \Theta_3 = (\Theta_1 \otimes \Theta_3) \cap (\Theta_2 \otimes \Theta_3)$

(4) $(\Theta_1 \cup \Theta_2) \otimes \Theta_3 = (\Theta_1 \otimes \Theta_3) \cup (\Theta_2 \otimes \Theta_3)$

Proof. By Definitions 5 and 10, we obtain the following:

(1) $(\Theta_1 \cap \Theta_2) \oplus \Theta_3 = (\min\{Y_{\Theta_1}, Y_{\Theta_2}\}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}) \oplus (Y_{\Theta_3}, \Psi_{\Theta_3}) = (\sqrt{\min\{Y_{\Theta_1}^2, Y_{\Theta_2}^2\} + Y_{\Theta_3}^2 - Y_{\Theta_3}^2 \min\{Y_{\Theta_1}^2, Y_{\Theta_2}^2\}}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}\Psi_{\Theta_3}) = (\sqrt{(1-Y_{\Theta_3}^2)\min\{Y_{\Theta_1}^2, Y_{\Theta_2}^2\} + Y_{\Theta_3}^2}, \max\{\Psi_{\Theta_1}\Psi_{\Theta_3}, \Psi_{\Theta_2}\Psi_{\Theta_3}\}).$

And $(\Theta_1 \oplus \Theta_3) \cap (\Theta_2 \oplus \Theta_3) = (\sqrt{Y_{\Theta_1}^2 + Y_{\Theta_3}^2 - Y_{\Theta_1}^2 Y_{\Theta_3}^2}, \Psi_{\Theta_1}\Psi_{\Theta_3}) \cap (\sqrt{Y_{\Theta_2}^2 + Y_{\Theta_3}^2 - Y_{\Theta_2}^2 Y_{\Theta_3}^2}, \Psi_{\Theta_2}\Psi_{\Theta_3}) = (\min\{\sqrt{Y_{\Theta_1}^2 + Y_{\Theta_3}^2 - Y_{\Theta_1}^2 Y_{\Theta_3}^2}, \sqrt{Y_{\Theta_2}^2 + Y_{\Theta_3}^2 - Y_{\Theta_2}^2 Y_{\Theta_3}^2}\}, \max\{\Psi_{\Theta_1}\Psi_{\Theta_3}, \Psi_{\Theta_2}\Psi_{\Theta_3}\}) = (\min\{\sqrt{(1-Y_{\Theta_3}^2)Y_{\Theta_1}^2 + Y_{\Theta_3}^2}, \sqrt{(1-Y_{\Theta_3}^2)Y_{\Theta_2}^2 + Y_{\Theta_3}^2}\}, \max\{\Psi_{\Theta_1}\Psi_{\Theta_3}, \Psi_{\Theta_2}\Psi_{\Theta_3}\}) = (\sqrt{(1-Y_{\Theta_3}^2)\min\{Y_{\Theta_1}^2, Y_{\Theta_2}^2\} + Y_{\Theta_3}^2}, \max\{\Psi_{\Theta_1}\Psi_{\Theta_3}, \Psi_{\Theta_2}\Psi_{\Theta_3}\}).$
Thus, $(\Theta_1 \cap \Theta_2) \oplus \Theta_3 = (\Theta_1 \oplus \Theta_3) \cap (\Theta_2 \oplus \Theta_3)$

(2) The proof is similar to (1)

(3) $(\Theta_1 \cap \Theta_2) \otimes \Theta_3 = (\min\{Y_{\Theta_1}, Y_{\Theta_2}\}, \max\{\Psi_{\Theta_1}, \Psi_{\Theta_2}\}) \otimes \Theta_3 = (\min\{Y_{\Theta_1}, Y_{\Theta_2}\}Y_{\Theta_3}, (\max\{\sqrt{\Psi_{\Theta_1}}, \sqrt{\Psi_{\Theta_2}}\} + \sqrt{\Psi_{\Theta_3}} - \sqrt{\Psi_{\Theta_3}} \max\{\sqrt{\Psi_{\Theta_1}}, \sqrt{\Psi_{\Theta_2}}\})^2) = (\min\{Y_{\Theta_1}Y_{\Theta_3}, Y_{\Theta_2}Y_{\Theta_3}\}, ((1-\sqrt{\Psi_{\Theta_3}}) \max\{\sqrt{\Psi_{\Theta_1}}, \sqrt{\Psi_{\Theta_2}}\} + \sqrt{\Psi_{\Theta_3}})^2).$

And $(\Theta_1 \otimes \Theta_3) \cap (\Theta_2 \otimes \Theta_3) = (Y_{\Theta_1}Y_{\Theta_3}, (\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_3}} - \sqrt{\Psi_{\Theta_1}}\sqrt{\Psi_{\Theta_3}})^2) \cap (Y_{\Theta_2}Y_{\Theta_3}, (\sqrt{\Psi_{\Theta_2}} + \sqrt{\Psi_{\Theta_3}} - \sqrt{\Psi_{\Theta_2}}\sqrt{\Psi_{\Theta_3}})^2) = (Y_{\Theta_1}Y_{\Theta_3}, ((1-\sqrt{\Psi_{\Theta_3}})\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_3}})^2) \cap (Y_{\Theta_2}Y_{\Theta_3}, ((1-\sqrt{\Psi_{\Theta_3}})\sqrt{\Psi_{\Theta_2}} + \sqrt{\Psi_{\Theta_3}})^2) = (\min\{Y_{\Theta_1}Y_{\Theta_3},$

$Y_{\Theta_2} Y_{\Theta_3}\}$, $\max\ \{((1-\sqrt{\Psi_{\Theta_3}})\sqrt{\Psi_{\Theta_1}} + \sqrt{\Psi_{\Theta_3}})^2$, $((1-\sqrt{\Psi_{\Theta_3}})\sqrt{\Psi_{\Theta_2}} + \sqrt{\Psi_{\Theta_3}})^2\}) = (\min\{Y_{\Theta_1} Y_{\Theta_3}, Y_{\Theta_2} Y_{\Theta_3}\}$ $, ((1-\sqrt{\Psi_{\Theta_3}})\max\{\sqrt{\Psi_{\Theta_1}}, \sqrt{\Psi_{\Theta_2}}\} + \sqrt{\Psi_{\Theta_3}})^2)$.

Thus, $(\Theta_1 \cap \Theta_2) \otimes \Theta_3 = (\Theta_1 \otimes \Theta_3) \cap (\Theta_2 \otimes \Theta_3)$

(4) The proof is similar to (3)

□

Theorem 18. *Let $\Theta_1 = (Y_{\Theta_1}, \Psi_{\Theta_1})$, $\Theta_2 = (Y_{\Theta_2}, \Psi_{\Theta_2})$, and $\Theta_3 = (Y_{\Theta_3}, \Psi_{\Theta_3})$ be three SR-FSs. Then*

(1) $\Theta_1 \oplus \Theta_2 \oplus \Theta_3 = \Theta_1 \oplus \Theta_3 \oplus \Theta_2$

(2) $\Theta_1 \otimes \Theta_2 \otimes \Theta_3 = \Theta_1 \otimes \Theta_3 \otimes \Theta_2$

Proof.

(1) $\Theta_1 \oplus \Theta_2 \oplus \Theta_3 = (Y_{\Theta_1}, \Psi_{\Theta_1}) \oplus (Y_{\Theta_2}, \Psi_{\Theta_2}) \oplus (Y_{\Theta_3}, \Psi_{\Theta_3}) = (\sqrt{Y^2_{\Theta_1} + Y^2_{\Theta_2} - Y^2_{\Theta_1} Y^2_{\Theta_2}}, \Psi_{\Theta_1}\Psi_{\Theta_2}) \oplus (Y_{\Theta_3}, \Psi_{\Theta_3}) = (\sqrt{Y^2_{\Theta_1} + Y^2_{\Theta_2} - Y^2_{\Theta_1} Y^2_{\Theta_2} + Y^2_{\Theta_3} - Y^2_{\Theta_3}(Y^2_{\Theta_1} + Y^2_{\Theta_2} - Y^2_{\Theta_1} Y^2_{\Theta_2})}, \Psi_{\Theta_1}\Psi_{\Theta_2}\Psi_{\Theta_3}) = (\sqrt{Y^2_{\Theta_1} + Y^2_{\Theta_2} + Y^2_{\Theta_3} - Y^2_{\Theta_1} Y^2_{\Theta_2} - Y^2_{\Theta_1} Y^2_{\Theta_3} - Y^2_{\Theta_2} Y^2_{\Theta_3} + Y^2_{\Theta_1} Y^2_{\Theta_2} Y^2_{\Theta_3}}, \Psi_{\Theta_1}\Psi_{\Theta_2}\Psi_{\Theta_3}) = (\sqrt{Y^2_{\Theta_1} + Y^2_{\Theta_3} - Y^2_{\Theta_1} Y^2_{\Theta_3} + Y^2_{\Theta_2} - Y^2_{\Theta_2}(Y^2_{\Theta_1} + Y^2_{\Theta_3} - Y^2_{\Theta_1} Y^2_{\Theta_3})}, \Psi_{\Theta_1}\Psi_{\Theta_2}\Psi_{\Theta_3}) = (\sqrt{Y^2_{\Theta_1} + Y^2_{\Theta_3} - Y^2_{\Theta_1} Y^2_{\Theta_3}}, \Psi_{\Theta_1}\Psi_{\Theta_3}) \oplus (Y_{\Theta_2}, \Psi_{\Theta_2}) = \Theta_1 \oplus \Theta_3 \oplus \Theta_2$

(2) The proof is similar to (1)

□

In order to rank SR-FSs, we define the score function and accuracy function of the SR-FS:

Definition 19.

(1) The score function of an SR-FS $\Theta = (Y_\Theta, \Psi_\Theta)$ can be represented as $score(\Theta) = Y^2_\Theta - \sqrt{\Psi_\Theta}$

(2) The accuracy function of an SR-FS $\Theta = (Y_\Theta, \Psi_\Theta)$ can be represented as $accuracy(\Theta) = Y^2_\Theta + \sqrt{\Psi_\Theta}$

Example 5. For an SR-FS $\Theta = (0.3, 0.8)$, we find that score $(\Theta) \approx -0.804427$ and accuracy$(\Theta) \approx 0.984427$. In particular, if $\Theta = (0, 1)$, then score$(\Theta) = -1$, and if $\Theta = (1, 0)$, then score$(\Theta) = 1$.

Theorem 20. *The suggested score function of any SR-FS $\Theta = (Y_\Theta, \Psi_\Theta)$, denoted by score$(\Theta)$ lies in $[-1, 1]$.*

Proof. Since for any SR-FS Θ, we have $Y^2_\Theta + \sqrt{\Psi_\Theta} \le 1$, hence $Y^2_\Theta - \sqrt{\Psi_\Theta} \le Y^2_\Theta \le 1$ and $Y^2_\Theta - \sqrt{\Psi_\Theta} \ge -\sqrt{\Psi_\Theta} \ge -1$. Therefore, $-1 \le Y^2_\Theta - \sqrt{\Psi_\Theta} \le 1$. Hence, $score(\Theta) \in [-1, 1]$. □

Remark 21. The suggested accuracy function of any SR-FS $\Theta = (Y_\Theta, \Psi_\Theta)$, denoted by accuracy$(\Theta)$, lies in $[0, 1]$.

3.2. Aggregation of SR-Fuzzy Sets

Definition 22. Let $\Theta_i = (Y_{\Theta_i}, \Psi_{\Theta_i})(i = 1, 2, \cdots, m)$ be a value of SR-FSs and $w = (w_1, w_2, \cdots, w_m)^T$ be the weight vector of Θ_i with $w_i > 0$ and $\sum_{i=1}^m w_i = 1$. Then an SR-Fuzzy

(1) Weighted average (SR-FWA) operator is a function $SR\text{-}FWA : \Theta^m \longrightarrow \Theta$, where

$$SR-FWA(\Theta_1, \Theta_2, \cdots, \Theta_m) = \left(\sum_{i=1}^m w_i Y_{\Theta_i}, \sum_{i=1}^m w_i \Psi_{\Theta_i}\right) \tag{18}$$

(2) Weighted geometric (SR-FWG) operator is a function $SR\text{-}FWG : \Theta^m \longrightarrow \Theta$, where

$$SR-FWG(\Theta_1, \Theta_2, \cdots, \Theta_m) = \left(\prod_{i=1}^m Y^{w_i}_{\Theta_i}, \prod_{i=1}^m \Psi^{w_i}_{\Theta_i}\right) \tag{19}$$

(3) Weighted power average (SR-FWPA) operator is a function $SR\text{-}FWPA : \Theta^m \longrightarrow \Theta$, where

$$SR-FWPA(\Theta_1, \Theta_2, \cdots, \Theta_m) = \left(\left(\sum_{i=1}^m w_i Y^2_{\Theta_i}\right)^{1/2}, \left(\sum_{i=1}^m w_i \sqrt{\Psi_{\Theta_i}}\right)^2\right) \tag{20}$$

(4) Weighted power geometric (SR-FWPG) operator is a function $SR\text{-}FWPG : \Theta^m \longrightarrow \Theta$, where

$$SR-FWPG(\Theta_1, \Theta_2, \cdots, \Theta_m) = \left(\left(1 - \prod_{i=1}^m \left(1 - Y^2_{\Theta_i}\right)^{w_i}\right)^{\frac{1}{2}}, \left(1 - \prod_{i=1}^m \left(1 - \sqrt{\Psi_{\Theta_i}}\right)^{w_i}\right)^2\right) \tag{21}$$

Example 6. Suppose that $\Theta_1 = (0.53, 0.49)$, $\Theta_2 = (0.52, 0.51)$, $\Theta_3 = (0.26, 0.76)$, $\Theta_4 = (0.51, 0.53)$, $\Theta_5 = (0.50, 0.54)$, and $\Theta_6 = (0.22, 0.86)$ are six SR-Fuzzy sets, and let $w = (0.12, 0.32, 0.22, 0.13, 0.10, 0.11)^T$ be a weight vector of Θ_i $(i = 1, 2, \cdots, 6)$. Then

(1) $SR-FWA(\Theta_1,\Theta_2,\cdots,\Theta_6) = (0.53\times 0.12+0.52\times 0.32+0.26\times 0.22+0.51\times 0.13 + 0.50\times 0.10+0.22\times 0.11, 0.49\times 0.12+0.51\times 0.32+0.76 \times 0.22+0.53\times 0.13+0.54\times 0.10+0.86\times 0.11) = (0.4277, 0.6067)$

(2) $SR-FWG(\Theta_1,\Theta_2,\cdots,\Theta_6) = (0.53^{0.12}\times 0.52^{0.32}\times 0.26^{0.22}\times 0.51^{0.13}\times 0.50^{0.10}\times 0.22^{0.11}, 0.49^{0.12}\times 0.51^{0.32}\times 0.76^{0.22}\times 0.53^{0.13}\times 0.54^{0.10}\times 0.86^{0.11}) \approx (0.404460, 0.593219)$

(3) $SR-FWPA(\Theta_1,\Theta_2,\cdots,\Theta_6) = ((0.53^2\times 0.12+0.52^2\times 0.32+0.26^2\times 0.22+0.51^2\times 0.13+0.50^2\times 0.10+0.22^2\times 0.11)^{1/2}, (0.12\times\sqrt{0.49}+0.32\times\sqrt{0.51}+0.22\times\sqrt{0.76}+0.13\times\sqrt{0.53}+0.10\times\sqrt{0.54}+0.11\times\sqrt{0.86})^2) \approx (0.446369, 0.599778)$

(4) $SR-FWPG(\Theta_1,\Theta_2,\cdots,\Theta_6) = ((1-(1-0.53^2)^{0.12}\times(1-0.52^2)^{0.32}\times(1-0.26^2)^{0.22}\times(1-0.51^2)^{0.13}\times(1-0.50^2)^{0.10}\times(1-0.22^2)^{0.11})^{1/2}, (1-(1-\sqrt{0.49})^{0.12}\times(1-\sqrt{0.51})^{0.32}\times(1-\sqrt{0.76})^{0.22}\times(1-\sqrt{0.53})^{0.13}\times(1-\sqrt{0.54})^{0.10}\times(1-\sqrt{0.86})^{0.11})^2) \approx (0.452616, 0.632933)$

Remark 23. Note that the ordered values induced from the different operators introduced in Definition 22 need not be an SR-FS. To validate this matter, take the ordered values $(0.452616, 0.632933)$ which are given in (4) of the above example. By calculating, we find that $(0.452616)^2+\sqrt{0.632933}=1.0004>1$ which means that SR-FWPG$(\Theta_1,\Theta_2,\cdots,\Theta_6)$ is not an SR-FS.

Theorem 24. *Let $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})(i=1,2,\cdots,m)$ be a value of SR-FSs, $\Theta=(Y_\Theta,\Psi_\Theta)$ be SR-FS, and $w=(w_1,w_2,\cdots,w_m)^T$ be a weight vector of Θ_i with $\sum_{i=1}^m w_i=1$. Then*

(1) $SR-FWA(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR-FWA(\Theta_1\otimes\Theta,\Theta_2\otimes\Theta,\cdots,\Theta_m\otimes\Theta)$

(2) $SR-FWG(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR-FWG(\Theta_1\otimes\Theta,\Theta_2\otimes\Theta,\cdots,\Theta_m\otimes\Theta)$

(3) $SR-FWPA(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR-FWPA(\Theta_1\otimes\Theta,\Theta_2\otimes\Theta,\cdots,\Theta_m\otimes\Theta)$

(4) $SR-FWPG(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR-FWPG(\Theta_1\otimes\Theta,\Theta_2\otimes\Theta,\cdots,\Theta_m\otimes\Theta)$

Proof. We will display the proof of (1) and (4). The other affirmations are proved in a similar fashion.

(1) For any $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})(i=1,2,\cdots,m)$ and $\Theta=(Y_\Theta,\Psi_\Theta)$, we get

$$\sqrt{Y_{\Theta_i}^2+Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2}\geq\sqrt{2Y_{\Theta_i}^2Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2}=Y_{\Theta_i}Y_\Theta,$$

$$\begin{aligned}&\left(\sqrt{\Psi_{\Theta_i}}+\sqrt{\Psi_\Theta}-\sqrt{\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_\Theta}}\right)^2\\ &\geq\left(2\sqrt{\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_\Theta}}-\sqrt{\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_\Theta}}\right)^2=\Psi_{\Theta_i}\Psi_\Theta,\end{aligned}\tag{22}$$

that is

$$\sum_{i=1}^m w_i\sqrt{Y_{\Theta_i}^2+Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2}\geq\sum_{i=1}^m w_iY_{\Theta_i}Y_\Theta,\tag{23}$$

and

$$\sum_{i=1}^m w_i\left(\sqrt{\Psi_{\Theta_i}}+\sqrt{\Psi_\Theta}-\sqrt{\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_\Theta}}\right)^2\geq\sum_{i=1}^m w_i\Psi_{\Theta_i}\Psi_\Theta.\tag{24}$$

By Definition 10 (1) and (2), we have

$$\begin{aligned}&SR-FWA(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\\ &=\left(\sum_{i=1}^m w_i\sqrt{Y_{\Theta_i}^2+Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2},\sum_{i=1}^m w_i\Psi_{\Theta_i}\Psi_\Theta\right),\\ &SR-FWA(\Theta_1\otimes\Theta,\Theta_2\otimes\Theta,\cdots,\Theta_m\otimes\Theta)\\ &=\left(\sum_{i=1}^m w_iY_{\Theta_i}Y_\Theta,\sum_{i=1}^m w_i\left(\sqrt{\Psi_{\Theta_i}}+\sqrt{\Psi_\Theta}-\sqrt{\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_\Theta}}\right)^2\right).\end{aligned}\tag{25}$$

Therefore, from (25), the proof is proved

(4) For any $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})(i=1,2,\cdots,m)$ and $\Theta=(Y_\Theta,\Psi_\Theta)$, we get

$$\begin{aligned}&Y_{\Theta_i}^2+Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2\geq 2Y_{\Theta_i}^2Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2\\ &=Y_{\Theta_i}^2Y_\Theta^2\Rightarrow 1-\left(Y_{\Theta_i}^2+Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2\right)\\ &\leq 1-Y_{\Theta_i}^2Y_\Theta^2\Rightarrow\left(1-\left(Y_{\Theta_i}^2+Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2\right)\right)^{w_i}\\ &\leq\left(1-Y_{\Theta_i}^2Y_\Theta^2\right)^{w_i}\Rightarrow\prod_{i=1}^m\left(1-\left(Y_{\Theta_i}^2+Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2\right)\right)^{w_i}\\ &\leq\prod_{i=1}^m\left(1-Y_{\Theta_i}^2Y_\Theta^2\right)^{w_i}\Rightarrow 1\\ &\quad-\prod_{i=1}^m\left(1-\left(Y_{\Theta_i}^2+Y_\Theta^2-Y_{\Theta_i}^2Y_\Theta^2\right)\right)^{w_i}\\ &\geq 1-\prod_{i=1}^m\left(1-Y_{\Theta_i}^2Y_\Theta^2\right)^{w_i}.\end{aligned}\tag{26}$$

Similarly

$$\begin{aligned}&\Rightarrow 1-\prod_{i=1}^{m}\left(1-\left(\sqrt{\Psi_{\Theta_i}}+\sqrt{\Psi_{\Theta}}-\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_{\Theta}}\right)\right)^{w_i}\\&\geq 1-\prod_{i=1}^{m}\left(1-\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_{\Theta}}\right)^{w_i}.\end{aligned}\tag{27}$$

Now, by (1) and (2) of Definition 10, we have

$$\begin{aligned}&SR-FWPG(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\\&=\left(\left(1-\prod_{i=1}^{m}\left(1-\left(Y_{\Theta_i}^2+Y_{\Theta}^2-Y_{\Theta_i}^2Y_{\Theta}^2\right)\right)^{w_i}\right)^{1/2},\right.\\&\left.\left(1-\prod_{i=1}^{m}\left(1-\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_{\Theta}}\right)^{w_i}\right)^2\right),\\&SR-FWPG(\Theta_1\otimes\Theta,\Theta_2\otimes\Theta,\cdots,\Theta_m\otimes\Theta)\\&=\left(\left(1-\prod_{i=1}^{m}\left(1-Y_{\Theta_i}^2Y_{\Theta}^2\right)^{w_i}\right)^{\frac{1}{2}},\right.\\&\left.\left(1-\prod_{i=1}^{m}\left(1-\left(\sqrt{\Psi_{\Theta_i}}+\sqrt{\Psi_{\Theta}}-\sqrt{\Psi_{\Theta_i}}\sqrt{\Psi_{\Theta}}\right)\right)^{w_i}\right)^2\right).\end{aligned}\tag{28}$$

Hence, SR-$FWPG(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR$-$FWPG(\Theta_1\otimes\Theta,\Theta_2\otimes\Theta,\cdots,\Theta_m\otimes\Theta)$. □

Theorem 25. *Let $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})(i=1,2,\cdots,m)$ be a value of SR-FSs, $\Theta=(Y_{\Theta},\Psi_{\Theta})$ be SR-FS, and $w=(w_1,w_2,\cdots,w_m)^T$ be the weight vector of Θ_i with $\sum_{i=1}^{m}w_i=1$; then*

(1) SR-$FWA(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR$-$FWA(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta$

(2) SR-$FWG(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR$-$FWG(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta$

(3) SR-$FWPA(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR$-$FWPA(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta$

(4) SR-$FWPG(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\geq SR$-$FWPG(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta$

Proof. We will display the proof of (1). The other affirmations are proved in a similar fashion.

(1) For any $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})(i=1,2,\cdots,m)$ and $\Theta=(Y_{\Theta},\Psi_{\Theta})$, we get

$$\sqrt{Y_{\Theta_i}^2+Y_{\Theta}^2-Y_{\Theta_i}^2Y_{\Theta}^2}\geq\sqrt{2Y_{\Theta_i}^2Y_{\Theta}^2-Y_{\Theta_i}^2Y_{\Theta}^2}=Y_{\Theta_i}Y_{\Theta},\tag{29}$$

that is

$$\sum_{i=1}^{m}w_i\sqrt{Y_{\Theta_i}^2+Y_{\Theta}^2-Y_{\Theta_i}^2Y_{\Theta}^2}\geq\sum_{i=1}^{m}w_iY_{\Theta_i}Y_{\Theta}.\tag{30}$$

Similarly,

$$\left(\sqrt{\sum_{i=1}^{m}w_i\Psi_{\Theta_i}}+\sqrt{\Psi_{\Theta}}-\sqrt{\sum_{i=1}^{m}w_i\Psi_{\Theta_i}}\sqrt{\Psi_{\Theta}}\right)^2\geq\sum_{i=1}^{m}w_i\Psi_{\Theta_i}\Psi_{\Theta}.\tag{31}$$

By (1) and (2) of Definition 10, we have

$$\begin{aligned}&SR-FWA(\Theta_1\oplus\Theta,\Theta_2\oplus\Theta,\cdots,\Theta_m\oplus\Theta)\\&=\left(\sum_{i=1}^{m}w_i\sqrt{Y_{\Theta_i}^2+Y_{\Theta}^2-Y_{\Theta_i}^2Y_{\Theta}^2},\sum_{i=1}^{m}w_i\Psi_{\Theta_i}\Psi_{\Theta}\right),\\&SR-FWA(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta\\&=\left(\sum_{i=1}^{m}w_iY_{\Theta_i}Y_{\Theta},\left(\sqrt{\sum_{i=1}^{m}w_i\Psi_{\Theta_i}}+\sqrt{\Psi_{\Theta}}-\sqrt{\sum_{i=1}^{m}w_i\Psi_{\Theta_i}}\sqrt{\Psi_{\Theta}}\right)^2\right).\end{aligned}\tag{32}$$

Hence, the desired result is proved. □

Theorem 26. *Let $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})$ and $K_i=(Y_{K_i},\Psi_{K_i})$ $(i=1,2,\cdots,m)$ be two values of SR-FSs, and $w=(w_1,w_2,\cdots,w_m)^T$ be a weight vector of them with $\sum_{i=1}^{m}w_i=1$. Then*

(1) $SR-FWA(\Theta_1\oplus K_1,\Theta_2\oplus K_2,\cdots,\Theta_m\oplus K_m)\geq SR-FWA(\Theta_1\otimes K_1,\Theta_2\otimes K_2,\cdots,\Theta_m\otimes K_m)$

(2) $SR-FWG(\Theta_1\oplus K_1,\Theta_2\oplus K_2,\cdots,\Theta_m\oplus K_m)\geq SR-FWG(\Theta_1\otimes K_1,\Theta_2\otimes K_2,\cdots,\Theta_m\otimes K_m)$

(3) $SR-FWPA(\Theta_1\oplus K_1,\Theta_2\oplus K_2,\cdots,\Theta_m\oplus K_m)\geq SR-FWPA(\Theta_1\otimes K_1,\Theta_2\otimes K_2,\cdots,\Theta_m\otimes K_m)$

(4) $SR-FWPG(\Theta_1\oplus K_1,\Theta_2\oplus K_2,\cdots,\Theta_m\oplus K_m)\geq SR-FWPG(\Theta_1\otimes K_1,\Theta_2\otimes K_2,\cdots,\Theta_m\otimes K_m)$

Proof. We will display the proof of (1). The other affirmations are proved in a similar fashion.

(1) For any $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})$ and $K_i=(Y_{K_i},\Psi_{K_i})$ $(i=1,2,\cdots,m)$, we get

$$\sqrt{Y_{\Theta_i}^2+Y_{K_i}^2-Y_{\Theta_i}^2Y_{K_i}^2}\geq\sqrt{2Y_{\Theta_i}^2Y_{K_i}^2-Y_{\Theta_i}^2Y_{K_i}^2}=Y_{\Theta_i}Y_{K_i},\tag{33}$$

that is

$$\sum_{i=1}^{m} w_i \sqrt{Y_{\Theta_i}^2 + Y_{K_i}^2 - Y_{\Theta_i}^2 Y_{K_i}^2} \geq \sum_{i=1}^{m} w_i Y_{\Theta_i} Y_{K_i}. \quad (34)$$

Similarly

$$\sum_{i=1}^{m} w_i \left(\sqrt{\Psi_{\Theta_i}} + \sqrt{\Psi_{K_i}} - \sqrt{\Psi_{\Theta_i}} \sqrt{\Psi_{K_i}} \right)^2 \geq \sum_{i=1}^{m} w_i \Psi_{\Theta_i} \Psi_{K_i}. \quad (35)$$

By (1) and (2) of Definition 10, we have

$$SR - FWA(\Theta_1 \oplus K_1, \Theta_2 \oplus K_2, \cdots, \Theta_m \oplus K_m) = \left(\sum_{i=1}^{m} w_i \sqrt{Y_{\Theta_i}^2 + Y_{K_i}^2 - Y_{\Theta_i}^2 Y_{K_i}^2}, \sum_{i=1}^{m} w_i \Psi_{\Theta_i} \Psi_{K_i} \right),$$

$$SR - FWA(\Theta_1 \otimes K_1, \Theta_2 \otimes K_2, \cdots, \Theta_m \otimes K_m) = \left(\sum_{i=1}^{m} w_i Y_{\Theta_i} Y_{K_i}, \sum_{i=1}^{m} w_i \left(\sqrt{\Psi_{\Theta_i}} + \sqrt{\Psi_{K_i}} - \sqrt{\Psi_{\Theta_i}} \sqrt{\Psi_{K_i}} \right)^2 \right). \quad (36)$$

Thus, $SR\text{-}FWA(\Theta_1 \oplus K_1, \Theta_2 \oplus K_2, \cdots, \Theta_m \oplus K_m) \geq SR\text{-}FWA(\Theta_1 \otimes K_1, \Theta_2 \otimes K_2, \cdots, \Theta_m \otimes K_m)$. □

Theorem 27. *Let $\Theta_i = (Y_{\Theta_i}, \Psi_{\Theta_i})(i = 1, 2, \cdots, m)$ be a value of SR-FSs, and $w = (w_1, w_2, \cdots, w_m)^T$ be the weight vector of Θ_i with $\sum_{i=1}^{m} w_i = 1$ and $\rho \geq 1$; then*

(1) $SR - FWA(\rho\Theta_1, \rho\Theta_2, \cdots, \rho\Theta_m) \geq SR - FWA(\Theta_1^\rho, \Theta_2^\rho, \cdots, \Theta_m^\rho)$

(2) $SR - FWG(\rho\Theta_1, \rho\Theta_2, \cdots, \rho\Theta_m) \geq SR - FWG(\Theta_1^\rho, \Theta_2^\rho, \cdots, \Theta_m^\rho)$

(3) $SR - FWPA(\rho\Theta_1, \rho\Theta_2, \cdots, \rho\Theta_m) \geq SR - FWPA(\Theta_1^\rho, \Theta_2^\rho, \cdots, \Theta_m^\rho)$

(4) $SR - FWPG(\rho\Theta_1, \rho\Theta_2, \cdots, \rho\Theta_m) \geq SR - FWPG(\Theta_1^\rho, \Theta_2^\rho, \cdots, \Theta_m^\rho)$

Proof. We will display the proof of (1). The other affirmations are proved in a similar fashion.

(1) For any $\Theta_i = (Y_{\Theta_i}, \Psi_{\Theta_i})(i = 1, 2, \cdots, m)$, we have

$$SR - FWA(\rho\Theta_1, \rho\Theta_2, \cdots, \rho\Theta_m) = \left(\sum_{i=1}^{m} w_i \sqrt{1 - \left(1 - Y_{\Theta_i}^2\right)^\rho}, \sum_{i=1}^{m} w_i \Psi_{\Theta_i}^\rho \right),$$

$$SR - FWA(\Theta_1^\rho, \Theta_2^\rho, \cdots, \Theta_m^\rho) = \left(\sum_{i=1}^{m} w_i Y_{\Theta_i}^\rho, \sum_{i=1}^{m} w_i \left(1 - \left(1 - \sqrt{\Psi_{\Theta_i}}\right)^\rho\right)^2 \right). \quad (37)$$

Let $f(Y_{\Theta_i}) = 1 - (1 - Y_{\Theta_i}^2)^\rho - (Y_{\Theta_i}^2)^\rho$, and we have to show $f(Y_{\Theta_i}) \geq 0$. Using the Newton generalized binomial theorem, we are able to get

$$\left(1 - Y_{\Theta_i}^2\right)^\rho + \left(Y_{\Theta_i}^2\right)^\rho \leq \left(1 - Y_{\Theta_i}^2 + Y_{\Theta_i}^2\right)^\rho = 1. \quad (38)$$

Thus, $f(Y_{\Theta_i}) \geq 0$, that is

$$1 - \left(1 - Y_{\Theta_i}^2\right)^\rho - \left(Y_{\Theta_i}^2\right)^\rho \geq 0 \Rightarrow 1 - \left(1 - Y_{\Theta_i}^2\right)^\rho \geq \left(Y_{\Theta_i}^2\right)^\rho \Rightarrow \sqrt{1 - \left(1 - Y_{\Theta_i}^2\right)^\rho} \geq Y_{\Theta_i}^\rho \Rightarrow \sum_{i=1}^{m} w_i \sqrt{1 - \left(1 - Y_{\Theta_i}^2\right)^\rho} \geq \sum_{i=1}^{m} w_i Y_{\Theta_i}^\rho. \quad (39)$$

Similarly,

$$\sum_{i=1}^{m} w_i \left(1 - \left(1 - \sqrt{\Psi_{\Theta_i}}\right)^\rho\right)^2 \geq \sum_{i=1}^{m} w_i \Psi_{\Theta_i}^\rho. \quad (40)$$

Therefore $SR - FWA(\rho\Theta_1, \rho\Theta_2, \cdots, \rho\Theta_m) \geq SR - FWA(\Theta_1^\rho, \Theta_2^\rho, \cdots, \Theta_m^\rho)$. □

Theorem 28. *Let $\Theta_i = (Y_{\Theta_i}, \Psi_{\Theta_i})(i = 1, 2, \cdots, m)$ be a value of SR-FSs, $\Theta = (Y_\Theta, \Psi_\Theta)$ be SR-FS, and $w = (w_1, w_2, \cdots, w_m)^T$ be a weight vector of Θ_i with $\sum_{i=1}^{m} w_i = 1$ and $\rho \geq 1$. Then*

(1) $SR - FWA(\rho\Theta_1 \oplus \Theta, \rho\Theta_2 \oplus \Theta, \cdots, \rho\Theta_m \oplus \Theta) \geq SR - FWA(\Theta_1^\rho \otimes \Theta, \Theta_2^\rho \otimes \Theta, \cdots, \Theta_m^\rho \otimes \Theta)$

(2) $SR - FWG(\rho\Theta_1 \oplus \Theta, \rho\Theta_2 \oplus \Theta, \cdots, \rho\Theta_m \oplus \Theta) \geq SR - FWG(\Theta_1^\rho \otimes \Theta, \Theta_2^\rho \otimes \Theta, \cdots, \Theta_m^\rho \otimes \Theta)$

(3) $SR - FWPA(\rho\Theta_1 \oplus \Theta, \rho\Theta_2 \oplus \Theta, \cdots, \rho\Theta_m \oplus \Theta) \geq SR - FWPA(\Theta_1^\rho \otimes \Theta, \Theta_2^\rho \otimes \Theta, \cdots, \Theta_m^\rho \otimes \Theta)$

(4) $SR - FWPG(\rho\Theta_1 \oplus \Theta, \rho\Theta_2 \oplus \Theta, \cdots, \rho\Theta_m \oplus \Theta) \geq SR - FWPG(\Theta_1^\rho \otimes \Theta, \Theta_2^\rho \otimes \Theta, \cdots, \Theta_m^\rho \otimes \Theta)$

Proof. We will display the proof of (1). The other affirmations are proved in a similar fashion.

(1) For any $\Theta_i = (Y_{\Theta_i}, \Psi_{\Theta_i})(i = 1, 2, \cdots, m)$ and $\Theta = (Y_\Theta, \Psi_\Theta)$, we have

$$SR-FWA(\rho\Theta_1\oplus\Theta,\rho\Theta_2\oplus\Theta,\cdots,\rho\Theta_m\oplus\Theta) = \left(\sum_{i=1}^m w_i\sqrt{1-\left(1-Y_{\Theta_i}^2\right)^\rho\left(1-Y_\Theta^2\right)},\ \sum_{i=1}^m w_i\Psi_{\Theta_i}^\rho\Psi_\Theta\right),$$

$$SR-FWA\left(\Theta_1^\rho\otimes\Theta,\Theta_2^\rho\otimes\Theta,\cdots,\Theta_m^\rho\otimes\Theta\right) = \left(\sum_{i=1}^m w_iY_{\Theta_i}^\rho Y_\Theta,\ \sum_{i=1}^m w_i\left(1-\left(1-\sqrt{\Psi_{\Theta_i}}\right)^\rho\left(1-\sqrt{\Psi_\Theta}\right)\right)^2\right). \tag{41}$$

Let $f(Y_{\Theta_i})=1-(1-Y_{\Theta_i}^2)^\rho(1-Y_\Theta^2)-(Y_{\Theta_i}^2)^\rho Y_\Theta^2$, and we have to show that $f(Y_{\Theta_i})\geq 0$. At first we indicate $g(Y_{\Theta_i})=(1-Y_{\Theta_i}^2)^\rho+(Y_{\Theta_i}^2)^\rho$ and take the derivative of $g(Y_{\Theta_i})$; then

$$\begin{aligned} g'\left(Y_{\Theta_i}\right) &= -2\rho Y_{\Theta_i}\left(1-Y_{\Theta_i}^2\right)^{\rho-1}+2\rho Y_{\Theta_i}\left(Y_{\Theta_i}^2\right)^{\rho-1} \\ &= 2\rho Y_{\Theta_i}\left(\left(Y_{\Theta_i}^2\right)^{\rho-1}-\left(1-Y_{\Theta_i}^2\right)^{\rho-1}\right). \end{aligned} \tag{42}$$

Therefore, if $Y_{\Theta_i}>1/\sqrt{2}$, then $g(Y_{\Theta_i})$ is monotonic increasing, and if $Y_{\Theta_i}<1/\sqrt{2}$, then $g(Y_{\Theta_i})$ is monotonic decreasing, so $g(Y_{\Theta_i})\leq g(Y_{\Theta_i})_{\max}=\max\{g(0),g(1)\}=1$. Since $(1-Y_{\Theta_i}^2)^\rho(1-Y_\Theta^2)+(Y_{\Theta_i}^2)^\rho Y_\Theta^2\leq 1$, hence

$$\begin{aligned} f\left(Y_{\Theta_i}\right) &= 1-\left(1-Y_{\Theta_i}^2\right)^\rho\left(1-Y_\Theta^2\right)-\left(Y_{\Theta_i}^2\right)^\rho Y_\Theta^2\geq 0 \\ &\Rightarrow \sum_{i=1}^m w_i\sqrt{1-\left(1-Y_{\Theta_i}^2\right)^\rho\left(1-Y_\Theta^2\right)}\geq\sum_{i=1}^m w_iY_{\Theta_i}^\rho Y_\Theta. \end{aligned} \tag{43}$$

Similarly

$$\sum_{i=1}^m w_i\left(1-\left(1-\sqrt{\Psi_{\Theta_i}}\right)^\rho\left(1-\sqrt{\Psi_\Theta}\right)\right)^2\geq\sum_{i=1}^m w_i\Psi_{\Theta_i}^\rho\Psi_\Theta. \tag{44}$$

Hence, $SR-FWA(\rho\Theta_1\oplus\Theta,\rho\Theta_2\oplus\Theta,\cdots,\rho\Theta_m\oplus\Theta)\geq SR-FWA(\Theta_1^\rho\otimes\Theta,\Theta_2^\rho\otimes\Theta,\cdots,\Theta_m^\rho\otimes\Theta)$.

To prove the following three results, we suppose that the values obtained from the introduced operators are an SR-FS (see Remark 23). □

Theorem 29. *Let $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})(i=1,2,\cdots,m)$ be a value of SR-FSs, $\Theta=(Y_\Theta,\Psi_\Theta)$ be SR-FS, and $w=(w_1,w_2,\cdots,w_m)^T$ be the weight vector of Θ_i with $\sum_{i=1}^m w_i=1$; then*

(1) $SR\text{-}FWA(\Theta_1,\Theta_2,\cdots,\Theta_m)\oplus\Theta\geq SR\text{-}FWA(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta$

(2) $SR\text{-}FWG(\Theta_1,\Theta_2,\cdots,\Theta_m)\oplus\Theta\geq SR\text{-}FWG(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta$

(3) $SR\text{-}FWPA(\Theta_1,\Theta_2,\cdots,\Theta_m)\oplus\Theta\geq SR\text{-}FWPA(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta$

(4) $SR\text{-}FWPG(\Theta_1,\Theta_2,\cdots,\Theta_m)\oplus\Theta\geq SR\text{-}FWPG(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta$

Proof. We will display the proof of (1). The other affirmations are proved in a similar fashion.

(1) For any $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})(i=1,2,\cdots,m)$ and $\Theta=(Y_\Theta,\Psi_\Theta)$, we get

$$\begin{aligned} &\sqrt{\left(\sum_{i=1}^m w_iY_{\Theta_i}\right)^2+Y_\Theta^2-\left(\sum_{i=1}^m w_iY_{\Theta_i}\right)^2Y_\Theta^2} \\ &\geq\sqrt{2\left(\sum_{i=1}^m w_iY_{\Theta_i}\right)^2Y_\Theta^2-\left(\sum_{i=1}^m w_iY_{\Theta_i}\right)^2Y_\Theta^2}=\sum_{i=1}^m w_iY_{\Theta_i}Y_\Theta. \end{aligned} \tag{45}$$

Similarly

$$\left(\sqrt{\sum_{i=1}^m w_i\Psi_{\Theta_i}}+\sqrt{\Psi_\Theta}-\sqrt{\sum_{i=1}^m w_i\Psi_{\Theta_i}}\sqrt{\Psi_\Theta}\right)^2\geq\sum_{i=1}^m w_i\Psi_{\Theta_i}\Psi_\Theta. \tag{46}$$

By (1) and (2) of Definition 10, we obtain

$$SR-FWA(\Theta_1,\Theta_2,\cdots,\Theta_m)\oplus\Theta = \left(\sqrt{\left(\sum_{i=1}^m w_iY_{\Theta_i}\right)^2+Y_\Theta^2-\left(\sum_{i=1}^m w_iY_{\Theta_i}\right)^2Y_\Theta^2},\ \sum_{i=1}^m w_i\Psi_{\Theta_i}\Psi_\Theta\right),$$

$$SR-FWA(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes\Theta = \left(\sum_{i=1}^m w_iY_{\Theta_i}Y_\Theta,\ \left(\sqrt{\sum_{i=1}^m w_i\Psi_{\Theta_i}}+\sqrt{\Psi_\Theta}-\sqrt{\sum_{i=1}^m w_i\Psi_{\Theta_i}}\sqrt{\Psi_\Theta}\right)^2\right). \tag{47}$$

Hence, the desired result is proved. □

Theorem 30. *Let $\Theta_i=(Y_{\Theta_i},\Psi_{\Theta_i})$ and $K_i=(Y_{K_i},\Psi_{K_i})$ $(i=1,2,\cdots,m)$ be two values of SR-FSs and $w=(w_1,w_2,\cdots,w_m)^T$ be a weight vector of them with $\sum_{i=1}^m w_i=1$. Then*

(1) $SR-FWA(\Theta_1,\Theta_2,\cdots,\Theta_m)\oplus SR-FWA(K_1,K_2,\cdots,K_m)\geq SR-FWA(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes SR-FWA(K_1,K_2,\cdots,K_m)$

(2) $SR-FWG(\Theta_1,\Theta_2,\cdots,\Theta_m)\oplus SR-FWG(K_1,K_2,\cdots,K_m)\geq SR-FWG(\Theta_1,\Theta_2,\cdots,\Theta_m)\otimes SR-FWG(K_1,K_2,\cdots,K_m)$

(3) $SR-FWPA(\Theta_1, \Theta_2, \cdots, \Theta_m) \oplus SR-FWPA(K_1, K_2, \cdots, K_m) \geq SR-FWPA(\Theta_1, \Theta_2, \cdots, \Theta_m) \otimes SR-FWPA(K_1, K_2, \cdots, K_m)$

(4) $SR-FWPG(\Theta_1, \Theta_2, \cdots, \Theta_m) \oplus SR-FWPG(K_1, K_2, \cdots, K_m) \geq SR-FWPG(\Theta_1, \Theta_2, \cdots, \Theta_m) \otimes SR-FWPG(K_1, K_2, \cdots, K_m)$

Theorem 31. *Let $\Theta_i = (Y_{\Theta_i}, \Psi_{\Theta_i})(i = 1, 2, \cdots, m)$ be a value of SR-FSs and $w = (w_1, w_2, \cdots, w_m)^T$ be the weight vector of Θ_i with $\sum_{i=1}^{m} w_i = 1$ and $\rho \geq 1$; then*

(1) $\rho SR-FWA(\Theta_1, \Theta_2, \cdots, \Theta_m) \geq (SR-FWA(\Theta_1, \Theta_2, \cdots, \Theta_m))^\rho$

(2) $\rho SR-FWG(\Theta_1, \Theta_2, \cdots, \Theta_m) \geq (SR-FWG(\Theta_1, \Theta_2, \cdots, \Theta_m))^\rho$

(3) $\rho SR-FWPA(\Theta_1, \Theta_2, \cdots, \Theta_m) \geq (SR-FWPA(\Theta_1, \Theta_2, \cdots, \Theta_m))^\rho$

(4) $\rho SR-FWPG(\Theta_1, \Theta_2, \cdots, \Theta_m) \geq (SR-FWPG(\Theta_1, \Theta_2, \cdots, \Theta_m))^\rho$

4. Application of SR-FSs to Select the Top-Rank University

In this section, we apply the SR-FWA, SR-FWG, SR-FWPA, and SR-FWPG operators to select the top-rank university among different universities.

One of the following techniques to handle the multicriteria decision-making (in short, MCDM) problems is based on the different types of fuzzy weighted operators. Herein, we, first, show the steps used in the proposed methodology for MCDM:

Step 1. Represent a MCDM problem under study using the SR-Fuzzy decision matrix.

Step 2. Transmit SR-Fuzzy decision matrix into the normalized SR-Fuzzy decision matrix.

Step 3. Compute for each alternative all kinds of SR-Fuzzy weighted operators.

Step 4. Compute the scores and accuracy functions for each alternative (as we showed in Remark 23, the ordered values induced from the different operators need not be an SR-FS; however, we apply the formulas of scores and accuracy functions given in Definition 19 for those ordered values).

Step 5. Compare the given alternatives based on the score function.

Step 6. If the score functions are equal for some alternatives, then compare between them in terms of accuracy function.

Step 7. Determine the optimal ranking order of the alternatives and recognize the optimal alternative(s).

In the next example, we illustrate how the above steps are applied to select the top-rank university among different universities.

Example 7. Let $U = \{U_1, U_2, U_3\}$ be a set of alternatives (universities) and $P = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ be a set of seven attributes for the selection of universities, where

$$\begin{gathered} P_1 = \{\text{represents Academic Staffs}\}, \\ P_2 = \{\text{represents Scientific Research}\}, \\ P_3 = \{\text{represents National and International Scientific Activities}\}, \\ P_4 = \{\text{represents Student Satisfaction}\}, \\ P_5 = \{\text{represents Quality Assurance}\}, \\ P_6 = \{\text{represents Cultural and Community Activities}\} \text{ and} \\ P_7 = \{\text{represents Library}\}. \end{gathered} \tag{48}$$

Suppose that the weight vector of the attributes given by the decision-maker is $w = (0.1, 0.37, 0.14, 0.03, 0.27, 0.06, 0.03)^T$. Obviously, $\sum_{i=1}^{7} w_i = 1$. The SR-F values (Y_{p_i}, Ψ_{p_i}) of the alternatives according to different attributes are given in Table 1, where Y_{p_i} is the positive membership degree for which alternative obeys the given attribute and Ψ_{p_i} is the membership degree for which alternative does not obey the given attribute such that $0 \leq (Y_{p_i})^2 + \sqrt{\Psi_{p_i}} \leq 1$ and $Y_{p_i}, \Psi_{p_i} \in [0, 1]$.

Applying the proposed aggregation operators given in Definition 22, score and accuracy functions, we find, as demonstrated in Table 2, that the optimal ranking order of the three universities is $U_3 \succ U_1 \succ U_2$, and thus, the top alternative is U_3.

Note that the score functions for the data given in Table 2 are unequal, so they are enough to determine the optimal alternative.

The method adopted in this application is illustrated in Figure 3.

5. Conclusions

This paper contributes to the fuzzy set theory in which interest in it grew since the moment Zadeh launched it. To handle some real-life issues which are difficult to solve using fuzzy set theory, some researchers extended this theory to other fuzzy models; the most important are IFSs and PFSs.

In this paper, we have proposed a new shape of fuzzy sets called an SR-Fuzzy set and revealed its relationship with other types of the generalizations of fuzzy sets. Then, some operators on SR-Fuzzy sets have been defined, and their relationships have been presented. Furthermore, we have introduced four new weighted aggregated operators over SR-Fuzzy sets and discussed their properties in detail.

Table 1: SR-Fuzzy values.

Universities	P_1	P_2	P_3	P_4	P_5	P_6	P_7
U_1	(0.7, 0.2)	(0.4, 0.5)	(0.4, 0.7)	(0.5, 0.5)	(0.4, 0.5)	(0.7, 0.1)	(0.8, 0.1)
U_2	(0.6, 0.3)	(0.3, 0.6)	(0.3, 0.8)	(0.6, 0.4)	(0.4, 0.7)	(0.6, 0.1)	(0.7, 0.2)
U_3	(0.8, 0.1)	(0.5, 0.4)	(0.5, 0.5)	(0.6, 0.4)	(0.7, 0.2)	(0.8, 0.1)	(1, 0)

Table 2: Evaluation of scores and accuracy with SR-Fuzzy aggregation operators.

	U_1	U_2	U_3	rank order
$SR-FWA$	(0.463, 0.462)	(0.396, 0.577)	(0.62, 0.3)	
Score	-0.465337	-0.602789	-0.163323	$U_3 \succ U_1 \succ U_2$
Accuracy	0.894075	0.916421	0.932123	$U_3 \succ U_2 \succ U_1$
$SR-FWG$	(0.449660, 0.413739)	(0.379393, 0.521591)	(0.606024, 0)	
Score	-0.441031	-0.578274	0.367265	$U_3 \succ U_1 \succ U_2$
Accuracy	0.845420	0.866152	0.367265	$U_2 \succ U_1 \succ U_3$
$SR-FWPA$	(0.479479, 0.440997)	(0.414970, 0.553320)	(0.634823, 0.273865)	
Score	-0.434176	-0.571655	-0.120321	$U_3 \succ U_1 \succ U_2$
Accuracy	0.893976	0.916055	0.926321	$U_3 \succ U_2 \succ U_1$
$SR-FWPG$	(0.496476, 0.475544)	(0.425919, 0.605327)	(1, 0)	
Score	-0.443109	-0.596621	1	$U_3 \succ U_1 \succ U_2$
Accuracy	0.936085	0.959435	1	$U_3 \succ U_2 \succ U_1$

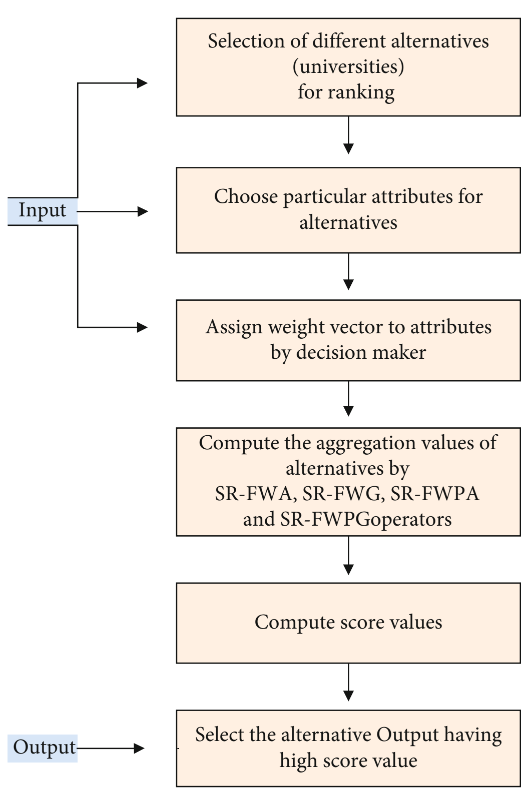

Figure 3: Flow chart to select the high-rank university.

Moreover, we have shown this procedure with one practical fully developed example.

In future works, further applications of SR-Fuzzy sets may be investigated, and also, SR-Fuzzy soft sets may be explored. Also, we will try to generate the topology from the collection of SR-Fuzzy sets and introduce the ideas of connectedness and compactness in SR-Fuzzy topology.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest.

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