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Research Article

Fermatean Cubic Fuzzy Aggregation Operators and Their Application in Multiattribute Decision-Making Problems

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The purpose of aggregation methods is to convert a list of objects of a set into a single object of the same set usually by an n-arry function, so-called aggregation operator. The key features of this work are the aggregation operators, because they are based on a novel set called Fermatean cubic fuzzy set (F-CFS). F-CFS has greater spatial scope and can deal with more ambiguous situations where other fuzzy set extensions fail to support them. For this purpose, the notion of F-CFS is defined. F-CFS is the transformation of intuitionistic cubic fuzzy set (I-CFS), Pythagorean cubic fuzzy set (P-CFS), interval-valued cubic fuzzy set, and basic orthopair fuzzy set and is grounded on the constraint that "the cube of the supremum of membership plus nonmembership degree is ≤1". We have analyzed some properties of Fermatean cubic fuzzy numbers (F-CFNs) as they are the alteration of basic properties of I-CFS and P-CFS. We also have defined the score and deviation degrees of F-CFNs. Moreover, the distance measuring function between two F-CFNs is defined which shows the space between two F-CFNs. Based on this notion, the aggregation operators namely Fermatean cubic fuzzy-weighted averaging operator (F-CFWA), Fermatean cubic fuzzy-weighted geometric operator (F-CFWG), Fermatean cubic fuzzy-ordered-weighted averaging operator (F-CFOWA), and Fermatean cubic fuzzy-ordered-weighted geometric operator (F-CFOWG) are developed. Furthermore, the notion is applied to multiattribute decision-making (MADM) problem in which we presented our objectives in the form of F-CFNs to show the effectiveness of the newly developed strategy.

1. Introduction

The multiattribute decision-making (MADM) approach is a well-known and influential procedure for selecting the best alternative for many problems in our practical life. Despite its popularity, there are two fundamental issues for MADM: (i) how decision makers suitably establish their assessment data and (ii) how the best alternative is determined. The selection may be accessible in the case of crisp data, but it becomes challenging when there are uncertainties and fuzziness in the available data. Zadeh's fuzzy set [1] has played a significant role in dealing with such data types. The proficient concept called fuzzy set theory (FST) was established in 1965. A fuzzy set F contains membership degree $\mu_F(x)$ of an object $x \in X$, such that $\mu_F(x) \in [0, 1]$. Considering the importance of FST in real-life problems, fuzzy MADM has become a hot topic for researchers. McBratney and Odeh [2] have shown the best solid example of the application of FST in real life and showed that FST provides a rich and meaningful improvement or extension of conventional logic

and explored that the Mathematics produced by FST is consistent. They exposed that the applications in soil sciences, numerical classification of soil and mapping, land evaluation, modeling and simulation of soil physical process [2], etc. are emerging roles of FST.

In a simple fuzzy set, we cannot assess complete information about an object in the universe to explain a decision-making problem more realistically. Atanassov [3] introduced an intuitionistic fuzzy set (IFS) by defining the membership grade $\mu_i(x)$ and nonmembership grade $v_i(x)$ for an element x in the set X, such that $\mu_i(x), v_i(x) \in [0, 1]$. The concept of IFS is proficient however; Yager [4] generalized the concept of IFS by introducing the Pythagorean fuzzy set (PFS). In PFS, the square sum of membership grade $\mu_i(x)$ and nonmembership grade $v_i(x)$ is no more than one. Many authors contributed to the IFS and PFS in various directions, like decision making, medical diagnoses, and information measures. Yang and Yao presented set-theoretic operations and relationships in the three systems (i.e., fuzzy sets, Atanassov's IFS, and shadowed sets) and the applications of the constructed shadowed sets for three-way decision making [5]. Recently, Verma worked on parametric information measures under the IFS and proposed four new order-α divergence measures between two IFSs [6]. Verma and Merigó [7] developed a new and flexible method for Pythagorean fuzzy decision making using some trigonometric similarity measures. Bakioglu and Atahan [8] investigated the prioritization of risks involved with self-driving vehicles by proposing new hybrid MCDM methods based on the analytic hierarchy process (AHP), the technique for order preference by similarity to an ideal solution (TOPSIS), and Vlse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) under Pythagorean fuzzy environment. Zhang and Xu [9] and Rani et al. [10] introduced the technique for order of preferences by similarity to ideal solution (TOPSIS) to tackle the MADM problems under PFS. Similarly, Sajjad et al. [11] presented a new extension of the TOPSIS method based on Pythagorean hesitant fuzzy sets with incomplete weight information. Peng and Yang [12] and Khan [13] worked on the fundamental properties of interval-valued Pythagorean fuzzy (IVPF) aggregation operators. They presented some emphasized aggregation operators based on interval-valued intuitionistic fuzzy numbers and their operation to group decision-making problems. Yager extended set measures to Pythagorean fuzzy sets [14]. Xu et al. presented the Pythagorean undefined decision-making method based on overall entropy [15], and Garg [16] generalized Pythagorean fuzzy information aggregation using Einstein operations. On the other hand, in the intuitionistic cubic fuzzy set (I-CFS) [17], both membership and nonmembership degrees are cubic sets. It is clear that the definition is more reliable and accurate for using it in multicriteria decision-making problems (MCDM), and numerous works have been done in this environment. Scholars evaluate MCDM problems through modified aggregation operators in terms of I-CFS [18]. Khan et al. [19] presented P-CFS, wherein membership and nonmembership degrees are cubic fuzzy numbers. The condition is that the supremum of the square of the membership degree plus the square of the nonmembership degree should be less than or equal to one, i.e., $0 \leq (\Lambda_{p_c}(x))^2 + (\Gamma_{p_c}(x))^2 \leq \tilde{1}, 1]$. Since P-CFNs are the modification of I-CFNs, the definition has more powerful results in MADM problems and is quite a straightforward methodology to deal with such problems. Readers can easily understand the value of the work by comparing it with the early results obtained through different aggregation operators. For example, Fahmi et al. [20, 21] defined Einstein aggregation operators for trapezoidal cubic fuzzy and cubical fuzzy sets. They also introduced geometric operators with triangular cubic linguistic hesitant fuzzy numbers [21]. Similarly, Rahman et al. in [22, 23], respectively, defined generalized intuitionistic fuzzy Einstein hybrid aggregation operators and interval-valued Pythagorean fuzzy Einstein hybrid aggregation operators and utilized them in decision-making problems.

The idea of the Pythagorean fuzzy set was further extended by Senapati and Yager [24], defining Fermatean fuzzy sets (FFS). Graphically, it can be shown that the spatial scope and area of acceptability of the Fermatean fuzzy set is greater than the intuitionistic fuzzy set and Pythagorean fuzzy set. They also presented the entire sets of operations and defined the score and accuracy functions for Fermatean fuzzy sets. Moreover, Senapati and Yager [25] aggressively worked more on the Fermatean fuzzy set and developed Fermatean fuzzy-weighted averaging/geometric operators and their application in MCDM methodologies. Deng and Wang [26] devised two novel distance-measure methods for Fermatean fuzzy sets. Gao et al., [27] worked in the area of continuities, derivatives, and differentials in the workplace of Fermatean fuzzy numbers, which is a milestone in the field of fuzzifications, while Ye et al. and Wei et al. [28, 29] worked on single variable differential calculus and Maclaurin operators under q-rung orthopair fuzzy numbers, the associated limit, continuities, and derivatives. Liu and Wang [30] developed a multiple attribute decision-making method based on Archimedean Bonferroni operators of q-rung orthopair fuzzy numbers. Subha and Sharmila [31] introduced the concept of interval-valued Fermatean fuzzy interior bi Γ-subsemihypergroup, interval-valued Fermatean fuzzy bi Γ-hypersemigroup and the relation between these ideals. Gul [32] presented three well-known multiattribute evaluation methods, namely, SAW, ARAS, and VIKOR, under Fermatean fuzzy environment and showed their applications in COVID-19 testing laboratories. Garg et al. [33] established some aggregation operators based on t-norm and t-conorm to cumulate the Fermatean fuzzy data in decision-making problems and exhibit the application in the COVID-19 testing facility. Recently, Zeb et al. [34] introduced aggregation operators in the environment of a Fermatean fuzzy soft set and presented their application in the decision-making problem of selecting the most critical COVID-19 patient. Shit and Ghorai [35] used Dombi t-norm and t-conorm operations on Fermatean fuzzy numbers. The more interesting work done by Chinnadurai et al. [36] is the introduction of Fermatean fuzzy numbers into a complex field and showing applicability in MCDM.

Discussing the novelty of our proposed work, we are compelled to say that there are still many problems where the subjected conditions over I-CFS and P-FCS may fail. For example, a panel of experts was invited to give their opinions about the feasibility of an investment plan, and they were divided into two independent groups to make a decision. One group considered the feasibility of the investment plan 0.8, while the other group considered the nonmembership degree 0.78. It was seen that $0.8 + 0.78 > 1$, $(0.8)^2 + (0.78)^2 > 1$; hence, the situation can not be described by IFS and PFS. If the above problem is taken as $(0.8)^3 + (0.78)^3 = 0.474\,55 < 1$, the situation can be handled if we use the Fermatean fuzzy set. Now without any doubt, the same situation will occur in the fuzzification of I-CFS and P-CFS. This problem motivated us to define a more reliable extension of I-CFS and P-CFS called the F-CFS. In F-CFS, the membership and nonmembership are cubic fuzzy numbers of degree 3 with the condition that the supremum of membership plus nonmembership degree is $\leq[\tilde{1}, 1]$. Henceforth, F-CFS is an alternative and more efficient tool compared to Pythagorean and intutionistic cubic fuzzy numbers in solving MADM problems. Because the space of acceptability or the spatial scope of F-CFS is greater than I-CFS and P-CFS and thus can support more ambiguous situations. The aggregation operators in this environment are more powerful and flexible in dealing with MADM problems. The rest of the paper is arranged as follows:

Section 2 is devoted to basic concepts related to F-CFS. Section 3 briefly discusses the basic operations and related properties of F-CFNs. In Section 4, we have introduced the novel aggregation operators in the environment of F-CFS. The decision-making approach and its practical application through a case study of selecting the best university based on some essential indicators for their ranking have been discussed in Section 5. The comparative analysis is provided in Section 6, and the summary is concluded in Section 7.

2. Preliminaries and Basic Concepts

In this section, we present some basic definitions and consequential properties that will help to understand the concept of F-CFS.

Definition 1 (see [1]). Let X be a fixed set; then, a fuzzy set (FS) F is an entity having the given formulations:

$$F = \{\langle x, \mu_F(x)\rangle | x \in X\}, \tag{1}$$

where μ_F is a function from X to $[0, 1]$ and $\mu_F(x)$ is called membership degree of x in X.

Definition 2 (see [3]). Let X be a fixed set; then, IFS set I is an entity having the following texture:

$$I = \{\langle x, \mu_i(x), \upsilon_i(x)\rangle | x \in X\}, \tag{2}$$

where $\mu_i(z)$ and $\upsilon_i(z)$ are functions from X to $[0, 1]$. Also the conditions $0 \leq \mu_i(z) \leq 1$ and $0 \leq \upsilon_i(z) \leq 1$ are imposed for all z in X and represent membership and nonmembership degrees of element x in X to the set I.

Definition 3 (see [37]). Let U be a fixed set; then, a cubic fuzzy set (CFS) C is a set of the form

$$C = \{\langle x, \mu_c(t), \upsilon_c(t)\rangle | t \in X\}, \tag{3}$$

where $\mu_c(t)$ is interval valued fuzzy set and $\upsilon_c(t)$ is a single fuzzy set in U.

Definition 4 (see [4]). Let R be a universal set; then, a PFS set P is an object of the form having the following structure:

$$P = \left\{\left\langle x, \mu_p(k), \upsilon_p(k)\right\rangle \middle| k \in R\right\}, \tag{4}$$

where $\mu_p(k), \upsilon_p(k)$ are functions from X to $[0, 1]$, such that $0 \leq \mu_p(k) \leq 1$, $0 \leq \upsilon_p(k) \leq 1$ for all k in R and $0 \leq \mu_i(k)^2 \leq 1$, $0 \leq \upsilon_i(k)^2 \leq 1$ for all k in R, and denote the membership and nonmembership degrees of element k in R to set P.

$$\pi_p(k) = \sqrt{1 - \mu_p{}^2 - \upsilon_p{}^2}. \tag{5}$$

Equation (5) is called the Pythagorean fuzzy index of k in R to the set P. Also $0 \leq \pi_p(k) \leq 1$ for each k in R. We denote a Pythagorean fuzzy number by $P = \langle \Lambda_p, \Gamma_p\rangle$.

Definition 5 (see [4]). Let $P = \langle \mu_p, \lambda_p\rangle$, $P_1 = \langle \mu_{p_1}, \lambda_{p_1}\rangle$, and $P_2 = \langle \mu_{p_2}, \lambda_{p_2}\rangle$ be the PFNs; then, the following are some basic operations defined for Pythagorean fuzzy numbers:

(1) $P_1 \oplus P_2 = \sqrt{\mu_{p_1}^2 + \mu_{p_2}^2 - \mu_{p_1}^2 \mu_{p_2}^2}$;

(2) $P_1 \oplus P_2 = \sqrt{\mu_{p_1}^2 + \mu_{p_2}^2 - \mu_{p_1}^2 \mu_{p_2}^2}$;

(3) $P_1 \otimes P_2 = \langle \mu_{p_1}\mu_{p_2}, \sqrt{\upsilon_{p_1}^2 + \upsilon_{p_2}^2 - \upsilon_{p_1}\upsilon_{p_2}}\rangle$;

(4) $\lambda P = \langle \sqrt{1 - (1 - \mu_p^2)^\lambda}, (\upsilon_p)^\lambda\rangle (\lambda > 0)$;

(5) $P^\lambda = \langle \mu_p^\lambda, \sqrt{1 - (1 - \upsilon_p^2)^\lambda}\rangle (\lambda > 0)$;

(6) $P^c = (\upsilon_p, \mu_p)$.

Definition 6 (see [12]). Let I be a universe of entities; the interval-valued Pythagorean fuzzy set (IVPFS) denoted by V can be defined as

$$V = \{c, a_A(c), b_A(c) | c \in I\}, \tag{6}$$

where $a_\nu(c) = [a_\nu^L(c), a_\nu^U(c)]$ and $b_\nu(c) = [b_\nu^L(c), b_\nu^U(c)] \subset [0, 1]$ are the intervals and $a_\nu^L(c) = \inf a_\nu(c)$ and $a_\nu^U(c) = \sup a_\nu(c)$ similarly $b_\nu^L(c) = \inf b_\nu(c)$ and $b_\nu^U(c) = \sup b_\nu(c)$ for each c in I. Also, $0 \leq (a_V^U(c))^2 + (b_\nu^U(c))^2 \leq 1$.

Definition 7. Let $\pi_v(c) = [\pi_v^L(c), \pi_v^U(c)]$, for all $c \in I$; then, IVPF index value of c to V is given by

$$\pi_v^U(c) = \sqrt{1 - (\mu_v^U(c))^2 - (\nu_v^U(c))^2},$$
$$\pi_v^L(c) = \sqrt{1 - (\mu_v^L(c))^2 - (\nu_v^L(c))^2}. \quad (7)$$

For above equations, the following conditions must be followed:

(1) If $a_v^L(c) = a_v^U(c)$ and $b_v^L(c) = b_v^U(c)$, then an IVPFS set reduces to PFS.

(2) If $a_v^U(c) + b_v^U(c) \leq 1$, then an IVPFS set reduces to an IVIFS.

Next, we define the score and accuracy function for (IVPFSs).

Definition 8 (see [12]). Let $V = ([a_v^L, a_v^U], [b_v^L, b_v^U])$ is an IVPFN; we can define the score function $S(v)$ in the following way:

$$S(v) = \frac{1}{2}\left[\left(a_v^L(c)^2 + \left(a_v^U(c)\right)^2 - \left(b_v^L(c)\right)^2 - \left(b_v^U(c)\right)^2\right]. \quad (8)$$

Clearly, $S(v) \in -[1, 1]$

Definition 9 (see [12]). Let $A = \langle [a_A^L, a_A^U], [b_A^L, b_A^U] \rangle$ is an IVPFN; we can define the accuracy function $H(A)$ of A as

$$H(A) = \frac{1}{2}\left[\left(\left(a_A^L(x)^2 + \left(a_A^U(x)\right)^2 + \left(b_A^L(x)\right)^2 + \left(b_A^U(x)\right)^2\right]\right., \quad (9)$$

where $H(A) \in [0, 1]$. For ranking purposes, we use the following conditions:

(1) If $S(A) < S(A_1)$, then $A < A_1$;

(2) If $S(A) = S(A_1)$, then
   (a) If $H(A) = H(A_1)$, then $A = A_1$;
   (b) If $H(A) < H(A_1)$, then $A < A_1$.

Definition 10 (see [12]). Let $A_i = \langle [a_i, b_i], [c_i, d_i] \rangle$ for $i = 1, 2, 3, \cdots, n$ be the collection of IVPFNs and $\delta > 0$; then, the following operational laws are satisfied:

$$\delta A_1 = \left\langle \left[\sqrt{1 - (1 - a_1^2)^\delta}, \sqrt{1 - (1 - b_1^2)^\delta}\right], \left[(c_1)^\delta, (d_1)^\delta\right] \right\rangle,$$
$$A_1^\delta = \left\langle \left[(c_1)^\delta, (d_1)^\delta\right], \left[\sqrt{1 - (1 - a_1^2)^\delta}, \sqrt{1 - (1 - b_1^2)^\delta}\right] \right\rangle,$$
$$A_1 \otimes A_2 = \left\langle [a_1 a_2, b_1 b_2], \left[\sqrt{(c_1)^2 + (c_2)^2 - (c_1)^2 (c_2)^2}, \sqrt{d_1^2 + d_2^2 - d_1^2 d_2^2}\right] \right\rangle,$$
$$A_1 \oplus A_2 = \left\langle \left[\sqrt{(a_1)^2 + (a_2)^2 - (a_1)^2 (a_2)^2}, \sqrt{b_1^2 + b_2^2 - b_1^2 b_2^2}\right], [c_1 c_2, d_1 d_2] \right\rangle. \quad (10)$$

Definition 11 (see [12]). Let $A_i = \langle [a_i, b_i], [c_i, d_i] \rangle$ for $i = 1, 2, 3 \cdots, n$ be the collection of IVPFNs; then, interval-valued Pythagorean fuzzy-weighted averaging IVPFWA operator is defined as

$$\mathrm{IVPFWA}_w(A_1, A_2, A_3, \cdots, A_n) = \left\langle \left[\sqrt{1 - \prod_{i=1}^{n} \left(1 - (a_i)^2\right)^{w_i}}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - (b_i)^2\right)^{w_i}}\right], \cdot \left[\prod_{i=1}^{n} (c_i)^{w_i}, \prod_{i=1}^{n} (d_i)^{w_i}\right] \right\rangle, \quad (11)$$

where $w = (w_1, w_2, w_3, \cdots, w_n)^T$ be the weight vector of for p_i for $i = 1, 2, \cdots, n$ and $w_i \in [0, 1]$ and $\sum w_i = 1$.

Definition 12 (see [12]). Let $A_i = \langle [a_i, b_i], [c_i, d_i] \rangle$ for $i = 1, 2, 3, \cdots, n$ be the collection of IVPFNs; then, interval-valued Pythagorean fuzzy-weighted geometric IVPFWG operator is defined as

$$\mathrm{IVPFWG}_w(A_1, A_2, A_3, \cdots, A_n) = \left\langle \left[\prod_{i=1}^{n} (a_i)^{w_i}, \prod_{i=1}^{n} (b_i)^{w_i}\right], \left[\sqrt{1 - \prod_{i=1}^{n} \left(1 - (c_i)^2\right)^{w_i}}, \cdot \sqrt{1 - \prod_{i=1}^{n} \left(1 - (d_i)^2\right)^{w_i}}\right] \right\rangle, \quad (12)$$

where $w = (w_1, w_2, w_3, \cdots, w_n)^T$ be the weight vector of for p_i for $i = 1, 2, \cdots, n$ and $w_i \in [0, 1]$ and also $\sum w_i = 1$.

Definition 13 (see [19]). Let X be a fixed set; then, a P-CFS can be defined as

$$P_c = \left\{x, \left\langle \left(\Lambda_{p_c}(x), \Gamma_{p_c}(x)\right) \right\rangle \middle| x \in X\right\}, \quad (13)$$

where $\Lambda_{p_c}(x) = \langle A_{p_c}(x); \lambda_{p_c}(x) \rangle$, $\Gamma_{p_c}(x) = \langle \tilde{A}_{p_c}(x); \mu_{p_c}(x) \rangle$, $A_{p_c}(x) = [a_{p_c}(x), b_{p_c}(x)]$ and $\tilde{A}_{p_c}(x) = [\tilde{a}_{p_c}(x), \tilde{b}_{p_c}(x)]$. Also,

$$0 \leq \left(\Lambda_{p_c}(x)\right)^2 + \left(\Gamma_{p_c}(x)\right)^2 \leq [\tilde{1}, 1]. \quad (14)$$

The degree of indeterminacy for P-CFS is defined as

$$\pi_{\tilde{c}} = \left\langle \sqrt{1 - \left(\sup\left(A_{p_c}(x)\right)\right)^2 - \left(\sup\left(\tilde{A}_{p_c}(x)\right)\right)^2}\,; \cdot \sqrt{1 - \left(\lambda^2_{p_c}(x) - \mu^2_{p_c}(x)\right)}\right\rangle. \tag{15}$$

Now for simplicity we call $P_c = (\langle A\,;\lambda\rangle, \langle \tilde{A}\,;\mu\rangle)$ a Pythagorean fuzzy number denoted by P_c.

Definition 14 (see [19]). Let $P_c = (\langle A\,;\lambda\rangle, \langle \tilde{A}\,;\mu\rangle)$, $P_{c_1} = (\langle A_1\,;\lambda\rangle, \langle \tilde{A}_1\,;\mu\rangle)$, and $P_{c_2} = (\langle A_2\,;\lambda\rangle, \langle \tilde{A}_2\,;\mu\rangle)$ be three P-CFNs and $\delta > 0$ where $A_1 = [a_1, b_1]$, $\tilde{A}_1 = [\tilde{a}_1, \tilde{b}_1]$, $A_2 = [a_2, b_2]$, $\tilde{A}_2 = [\tilde{a}_2, \tilde{b}_2]$, $A = [a, b]$, and $\tilde{A} = [\tilde{a}, \tilde{b}]$; then, the operation laws are also satisfied.

$$\begin{aligned}
P_{c_1} \oplus P_{c_2} &= \left(\left\langle \begin{array}{c} \left[\sqrt{(a_1)^2 + (a_2)^2 - (a_1)^2(a_2)^2}, \sqrt{b_1^2 + b_2^2 - b_1^2 b_2^2}\right]\,; \sqrt{\lambda_1^2 + \lambda_2^2 - \lambda_1^2\lambda_2^2} \\ \left\langle \left[\tilde{a}_1\tilde{a}_2, \tilde{b}_1\tilde{b}_2\right]\,; \mu_1\mu_2 \right\rangle \end{array}\right\rangle,\right);\\
P_{c1} \otimes P_{c_2} &= \left(\left\langle \begin{array}{c} [a_1a_2, b_1b_2]\,; \lambda_1\lambda_2, \\ \left[\sqrt{\tilde{a}_1 + (\tilde{a}_2)^2 - (\tilde{a}_1)^2(\tilde{a}_2)^2}, \sqrt{\tilde{b}_1^2 + \tilde{b}_2^2 - \tilde{b}_1^2\tilde{b}_2^2}\right]\,; \sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2} \end{array}\right\rangle\right);\\
\delta P_{c_1} &= \left(\begin{array}{c} \left\langle \left[1 - (1 - a_1^2)^\delta, 1 - (1 - b_1^2)^\delta\right]\,; 1 - (1 - \lambda_1^2)^\delta \right\rangle, \\ \left\langle (\tilde{a}_1\tilde{a}_{2,})^\delta, \left(\tilde{b}_1\tilde{b}_2\right)^\delta\,; (\mu_1\mu_2)^\delta \right\rangle \end{array}\right);\\
P_c^c &= \left(\left\langle \tilde{A}\,;\mu\right\rangle, \langle A\,;\lambda\rangle\right).
\end{aligned} \tag{16}$$

Theorem 15. *Let $P_c = (\langle A\,;\lambda\rangle, \langle \tilde{A}\,;\mu\rangle)$, $P_{c_1} = (\langle A_1\,;\lambda_1\rangle, \langle \tilde{A}_1\,;\mu_1\rangle)$, and $P_{c_2} = (\langle A_2\,;\lambda_2\rangle, \langle \tilde{A}_2\,;\mu_2\rangle)$ be three P-CFNs and $\delta_1, \delta_2 > 0$ where $A_1 = [a_1, b_1]$, $\tilde{A}_1 = [\tilde{a}_1, \tilde{b}_1]$, $A_2 = [a_2, b_2]$, $\tilde{A}_2 = [\tilde{a}_2, \tilde{b}_2]$, $A = [a, b]$, and $\tilde{A} = [\tilde{a}, \tilde{b}]$; then, the following hold in P-CFNs:*

$$\begin{aligned}
P_{c_1} \oplus P_{c_2} &= P_{c_2} \oplus P_{c_1},\\
P_{c_1} \otimes P_{c_2} &= P_{c_2} \otimes P_{c_1},\\
\delta\left(P_{c_1} \oplus P_{c_2}\right) &= \left(\delta P_{c_1} \oplus \delta P_{c_2}\right),\\
(\delta_1 + \delta_2)p_c &= \delta_1 p_c \oplus \delta_2 p_c,\\
\left(P_{c_1} \otimes P_{c_2}\right)^\delta &= \left(P_{c_1}\right)^\delta \otimes \left(P_{c_2}\right)^\delta.
\end{aligned} \tag{17}$$

Proof. Straight forward. □

Definition 16 (see [19]). Let $P_c = (\langle A\,;\lambda\rangle, \langle \tilde{A}\,;\mu\rangle)$ be a Pythagorean Fuzzy number; the score value of a PFN can be obtained through the following score function:

$$S(P_c) = \left(\frac{a + b - \lambda}{3}\right)^2 - \left(\frac{\tilde{a} + \tilde{b} - \mu}{3}\right)^2, \tag{18}$$

where $S(P_c) \in [-1, 1]$, such that a, b and $\tilde{a}, \tilde{b}$ are membership and nonmembership values of P_c.

3. Fermatean Cubic Fuzzy Numbers (F-CFNs) and Related Properties

In this section, we state the analogy of new and more approachable concept in fuzzy system called "Fermatean cubic fuzzy numbers (F-CFNs)" and will also discuss their important properties by using Pythagorean cubic fuzzy numbers and their properties. It must also be noted that in the rest of the paper we will denote a Fermatean cubic fuzzy number by F_c.

Definition 17. Let X be a fixed set; then, a F-CFS denoted by F_c can be defined as

$$F_c = \{x, \Gamma F_c(x), \Psi F_c(x) | x \in X\}, \tag{19}$$

where $\Gamma F_c(x) = \langle A_{F_c}(x)\,;\lambda_{F_c}(x)\rangle = \langle [a_{F_c}, b_{F_c}]\,;\lambda_{F_c}(x)\rangle$ and $\Psi F_c(x) = \langle \tilde{A}_{F_c}(x)\,;\mu_{F_c}(x)\rangle = \langle [\tilde{a}_{F_c}, \tilde{b}_{F_c}]\,;\mu_{F_c}(x)\rangle$, i.e., $\Gamma F_c(x)$ and $\Psi F_c(x)$ both are cubic fuzzy numbers and also denote the membership and nonmembership degrees of F-CFNs with the following conditions:

$$\begin{aligned}
0 &\le (\Gamma F_c(x))^3 + (\Psi F_c(x))^3 \le 1,\\
0 &\le \left(\lambda_{F_c}(x)\right)^3 + \left(\mu_{F_c}(x)\right)^3 \le 1.
\end{aligned} \tag{20}$$

The overhead restriction can also be scripted as

$$0 \le (\sup \Gamma F_c(x))^3 + (\sup \Psi F_c(x))^3 \le 1. \tag{21}$$

The degree of indeterminacy for F-CFNs can be defined as

$$\pi_{F_c} = \left\langle \sqrt[3]{1 - \left(\sup\left(A_{F_c}(x)\right)\right)^3 - \left(\sup\left(\tilde{A}_{F_c}(x)\right)\right)^3}\,; \cdot \sqrt{31 - \lambda^3(x) - \mu^3(x)}\right\rangle. \tag{22}$$

It is clear that if the power on the membership and nonmembership degrees is raised to 1, the above definitions reduce to I-CFS and indeterminacy function for I-CFS, while if it is raised to "2", then they works as P-CFS and indeterminacy for P-CFS.

Example 18. Consider a fixed set $Z = \{Z_1, Z_2, Z_3\}$ and suppose we have a formulation in Z by the following texture:

$$F_c = \left\{\begin{array}{l} (Z_1, \langle [.5, .6]\,; .7), [.6, .7]\,; .5)\rangle, \\ (Z_2, \langle [.4, .7]\,; .6), [.5, .8]\,; .6)\rangle, \\ (Z_3, \langle [.4, .6]\,; .7), [.5, .9]\,; .6)\rangle \end{array}\right\}. \tag{23}$$

Equation (23) shows that each term in Z is graded on the basis of Definition 17 F-CFS criteria. Each membership and

nonmembership grade of Z_1, Z_2, and Z_3 is restricted to the condition of F-CFNs. If Equation (23) is compared with Equation (19), then for Z_1 we have

$$\begin{aligned}&[.5, .6]\,;.7 = \Gamma F_c(x) \text{ membership},\\ &[.6, .7]\,;.5 = \Psi F_c(x) \text{ nonmembership}.\end{aligned} \tag{24}$$

The same is also for Z_2 and $Z_3 \in F_c$.

3.1. Basic Operations on F-CFNs

Definition 19. Let $F_c = (\langle A\,;\lambda\rangle, \langle \tilde{A}\,;\mu\rangle)$, $F_{c_1} = (\langle A_1\,;\lambda_1\rangle, \langle \tilde{A}_1\,;\mu_1\rangle)$, and $F_{c_2} = (\langle A_2\,;\lambda_2\rangle, \langle \tilde{A}_2\,;\mu_2\rangle)$ be three F-CFNs and $\delta > 0$ also $A_1 = [a_1, b_1]$, $\tilde{A}_1 = [\tilde{a}_1, \tilde{b}_1]$, $A_2 = [a_2, b_2]$, and $\tilde{A}_2 = [\tilde{a}_2, \tilde{b}_2]$; then, the following are the operational laws regarding Fermatean cubic fuzzy numbers:

$$\begin{aligned}
F_{c_1} \oplus F_{c_2} &= \left\{ \begin{array}{c} \left\langle \sqrt[3]{(a_1)^3 + (a_2)^3 - (a_1)^3(a_2)^3}, \sqrt[3]{b_1^3 + b_2^3 - b_1^3 b_2^3}\,;\sqrt[3]{\lambda_1^3 + \lambda_2^3 - \lambda_1^3\lambda_2^3}\right\rangle, \\ \left\langle \left[\tilde{a}_1\tilde{a}_2, \tilde{b}_1\tilde{b}_2\right]\,;\mu_1\mu_2\right\rangle \end{array} \right\},\\
F_{c_1} \otimes F_{c_2} &= \left\{ \begin{array}{c} \langle [a_1a_2, b_1b_2]\,;\lambda_1\lambda_2\rangle, \\ \left\langle \sqrt[3]{(a_1)^3 + (a_2)^3 - (a_1)^3(a_2)^3}, \sqrt[3]{b_1^3 + b_2^3 - b_1^3 b_2^3}\,;\sqrt[3]{\lambda_1^3 + \lambda_2^3 - \lambda_1^3\lambda_2^3}\right\rangle \end{array} \right\},\\
\delta F_{c_1} &= \left\{ \begin{array}{c} \left\langle \sqrt[3]{1 - \left(1 - a_1^3\right)^\delta}, \sqrt[3]{1 - \left(1 - b_1^3\right)^\delta}\,;\sqrt[3]{1 - (1 - \lambda_1)^\delta}\right\rangle, \\ \left\langle (\tilde{a}_1)^\delta, \left(\tilde{b}_1\right)^\delta\right]\,;(\mu_1)^\delta\right\rangle \end{array} \right\},\\
F_{c_1}^\delta &= \left\{ \begin{array}{c} \left\langle (a_1)^\delta, (b_1)^\delta\right]\,;(\mu_1)^\delta\right\rangle, \\ \left\langle \sqrt[3]{1 - \left(1 - \tilde{a}_1^3\right)^\delta}, \sqrt[3]{1 - \left(1 - \tilde{b}_1^3\right)^\delta}\,;\sqrt[3]{1 - (1 - \tilde{\mu}_1)^\delta}\right\rangle \end{array} \right\},\\
F_c^c &= \left\langle \tilde{A}\,;\mu\right\rangle, \langle A\,;\lambda\rangle.
\end{aligned} \tag{25}$$

Theorem 20. *Let $F_c = (\langle A\,;\lambda\rangle), \langle \tilde{A}\,;\mu\rangle)$, $F_{c_1} = (\langle A_1\,;\lambda_1\rangle, \langle \tilde{A}_1\,;\mu_1\rangle)$, and $F_{c_2} = (\langle A_2\,;\lambda_2\rangle, \langle \tilde{A}_2\,;\mu_2\rangle)$ be three F-CFNs and for $\delta > 0$, $\delta_1 > 0$, and $\delta_2 > 0$ with $A_1 = [a_1, b_1]$, $\tilde{A}_1 = [\tilde{a}_1, \tilde{b}_1]$, $A_2 = [a_2, b_2]$, and $\tilde{A}_2 = [\tilde{a}_2, \tilde{b}_2]$; then, the following properties hold in Fermatean cubic fuzzy numbers:*

(1) $F_{c_1} \oplus F_{c_2} = F_{c_2} \oplus F_{c_1}$,

(2) $F_{c_1} \otimes F_{c_2} = F_{c_2} \otimes F_{c_1}$,

(3) $\delta(F_{c_1} \oplus F_{c_2}) = \delta F_{c_1} \oplus \delta F_{c_2}$,

(4) $(\delta_1 + \delta_2)F_c = \delta_1 F_c \oplus \delta_2 F_c$,

(5) $(F_{c_1} \otimes F_{c_2})^\delta = F_{c_1}^\delta \otimes F_{c_2}^\delta$,

(6) $F_c^{(\delta_1+\delta_2)} = F_c^{\delta_1,} \otimes F_c^{\delta_2}$.

Following the proofs in [19] these properties can easily be proved for F-CFNs. The score function is an essential tool to demonstrate the space analogy and to compare two or more fuzzy numbers. We define a score function as well as accuracy function for F-CFNs.

Definition 21. Let $F_{c_i} = (\langle C_i\,;\lambda_i\rangle), \langle \tilde{C}_i\,;\mu_i\rangle)$ where $C = [a, b]$ and $\tilde{C} = [\tilde{a}, \tilde{b}]$ for $(i = 1, 2, 3 \cdots n)$ be the family of F-CFNs; then, the score function for F-CFNs Fermatean cubic fuzzy numbers is scripted below:

$$S(F_c) = \left(\frac{a + b - \lambda}{3}\right)^3 - \left(\frac{\tilde{a} + \tilde{b} - \mu}{3}\right)^3, \tag{26}$$

where $S(F_c) \in -[1, 1]$; the following situations should be kept

in mind while finding score function for two or more than two F-CFNs:

(1) If $S(F_{c_1}) < S(F_{c_2})$, then $F_{c_1} < F_{c_2}$,

(2) If $S(F_{c_1}) > S(F_{c_2})$, then $F_{c_1} > F_{c_2}$,

(3) If $S(F_{c_1}) = S(F_{c_2})$, then $F_{c_1} \sim F_{c_2}$.

Definition 22. Suppose $F_c = (\langle C\,;\lambda\rangle, \langle \tilde{C}\,;\mu\rangle)$ where $C = [a, b]$ and $\tilde{C} = [\tilde{a}, \tilde{b}]$, the accuracy function for F_c is $\alpha(F_c)$ and is defined below:

$$\alpha(F_c) = \left(\frac{a+b-\lambda}{3}\right)^3 + \left(\frac{\tilde{a}+\tilde{b}-\mu}{3}\right)^3, \tag{27}$$

where $\alpha(F_c) \in -[1, 1]$ and clearly for $q = 1$ it is the accuracy degree for I-CFNs, and for $q = 2$, it is an accuracy degree for P-CFNs. Furthermore, an important factor in fuzzy system is the distance measuring function which is our focal point too.

Definition 23. We define the distance measuring function between two F-CFNs by the following relation:

$$D(F_{c_1}, F_{c_2}) = \frac{1}{6}\left[\left|a_1^3 - a_2^3\right| + \left|b_1^3 - b_2^3\right| + \left|\tilde{a}_1^3 - \tilde{a}_2^3\right| + \left|\tilde{b}_1^3 - \tilde{b}_2^3\right| + \left|\lambda_1^3 - \lambda_2^3\right| + \left|\mu_1^3 - \mu_2^3\right|\right]. \tag{28}$$

Example 24. Let us consider we have two F-CFNs such that $F_{c_1} = (\langle[0.6,0.7]\,;0.3\rangle, \langle[0.5,0.7]\,;0.8\rangle)$ and $F_{c_2} = (\langle[0.5,0.6]\,;0.4\rangle, \langle[0.4,0.7]\,;0.5\rangle)$ and then using Equation (28) to find the distance measure as given below.

$$D(F_{c_1}, F_{c_2}) = \frac{1}{6}\left[\left|0.6^3 - 0.5^3\right| + \left|0.7^3 - 0.6^3\right| + \left|0.5^3 - 0.4^3\right| + \left|0.7^3 - 0.7^3\right| + \left|0.3^3 - 0.4^3\right| + \left|0.8^3 - 0.5^3\right|\right] = \frac{1}{6}[0.629] = 0.10483. \tag{29}$$

The above value is the required distance between F_{c_1} and F_{c_2} obtained by using Fermatean cubic fuzzy distance measuring operator.

4. Aggregation Operators under Fermatean Cubic Fuzzy Environment

In this section, we present some aggregation operators under Fermatean cubic fuzzy environment such as F-CFWA, F-CFWG, and F-CFOWG in F-CFN environment.

Definition 25. Let F_{c_i} for $i = 1, 2, 3, \cdots, n$ be the collection of F-CFNs. Suppose $V = (v_1, v_2, v_3, \cdots, v_n)^T$ be the weight vector of F_{c_i} for $i = 1, 2, 3, \cdots, n$ with $v_i \geq 0$ and $v_i \in [0, 1]$. Also, $\sum_{i=1}^{n} v_i = 1$; then, Fermatean cubic fuzzy-weighted averaging F-CFWA operator is defined as

$$\begin{aligned}&\text{F-CFWA}(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n}),\\ &\quad = v_1 F_{c_1} \oplus v_2 F_{c_2} \oplus v_3 F_{c_3} \oplus \cdots. \oplus v_n F_{c_n}.\end{aligned} \tag{30}$$

The above results can be easily proved by studying the basics of fuzzy algebra.

Theorem 26. *Let $F_c = (\langle C_i\,;\lambda\rangle, \langle \tilde{C}_i\,;\mu\rangle)$ be a group of F-CFNs for $i = 1, 2, 3, \cdots, n$ and $V = (v_1, v_2, v_3, \cdots, v_n)^T$ be the weight vector associated with F_{c_i} for $v_i \geq 0$, $v_i \in [0, 1]$, and $\sum v_i = 1$. Then, the aggregation result according to our definition is also a F-CFN that is given below.*

$$\begin{aligned}&\text{F-CFWA}(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, Fc_n)\\ &= \left(\begin{array}{c}\left\langle\left[\sqrt[3]{1 - \prod_{i=1}^{n}\left(1 - a_i^3\right)^{v_i}}, \sqrt[3]{1 - \prod_{i=1}^{n}\left(1 - b_i^3\right)^{v_i}}\,; \sqrt[3]{1 - \prod_{i=1}^{n}\left(1 - \lambda_i^3\right)^{v_i}}\right]\right\rangle,\\ \left\langle\left[\prod_{i=1}^{n}\tilde{a}_i^{v_i}, \prod_{i=1}^{n}\tilde{b}_i^{v_i}\,; \prod_{i=1}^{n}\mu_i^{v_i}\right]\right\rangle\end{array}\right).\end{aligned} \tag{31}$$

Proof. By mathematical induction, for $n = 2$ we have

$$v_1 Fc_1 = \left\langle \left[\sqrt[3]{1-\left(1-a_1^3\right)^{v_1}}, \sqrt[3]{1-\left(1-b_1^3\right)^{v_1}}; \sqrt[3]{1-\left(1-\lambda_1^3\right)^{v_1}} \right] \right\rangle, \left\langle \left[\tilde{a}_1^{v_1}, \tilde{b}_1^{v_1}; \mu_1^{v_1} \right] \right\rangle,$$
$$v_2 Fc_2 = \left\langle \left[\sqrt[3]{1-\left(1-a_2^3\right)^{v_2}}, \sqrt[3]{1-\left(1-b_2^3\right)^{v_2}}; \sqrt[3]{1-\left(1-\lambda_2^3\right)^{v_2}} \right] \right\rangle, \left\langle \left[\tilde{a}_2^{v2}, \tilde{b}_1^{v2}; \mu_2^{v_2} \right] \right\rangle. \tag{32}$$

Thus,

$$v_1 Fc_1 \oplus v_2 Fc_2 = \left\langle \left[\sqrt[3]{1-\left(1-a_1^3\right)^{v_1}}, \sqrt[3]{1-\left(1-b_1^3\right)^{v_1}}; \sqrt[3]{1-\left(1-\lambda_1^3\right)^{v_1}} \right] \right\rangle, \left\langle \left[\tilde{a}_1^{v_1}, \tilde{b}_1^{v_1}; \mu_1^{v_1} \right] \right\rangle$$
$$\oplus \left\langle \left[\sqrt[3]{1-\left(1-a_2^3\right)^{v_2}}, \sqrt[3]{1-\left(1-b_2^3\right)^{v_2}}; \sqrt[3]{1-\left(1-\lambda_2^3\right)^{v_2}} \right] \right\rangle, \left\langle \left[\tilde{a}_2^{v2}, \tilde{b}_1^{v2}; \mu_2^{v_2} \right] \right\rangle$$
$$= \left(\left\langle \left[\sqrt[3]{1-\prod_{i=1}^{2}\left(1-a_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{2}\left(1-b_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{2}\left(1-\lambda_i^3\right)^{v_i}} \right] \right\rangle, \left\langle \left[\prod_{i=1}^{2}\tilde{a}_i^{v_i}, \prod_{i=1}^{2}\tilde{b}_i^{v_i}; \prod_{i=1}^{2}\mu_i^{v_i} \right] \right\rangle \right). \tag{33}$$

From the above equation it is clear that the result is true. We suppose it is true for $n = k$ where k is any positive integer:

$$\text{F-CFWA}(Fc_1, Fc_2, Fc_3, \cdots, Fc_k) = \left(\left\langle \left[\sqrt[3]{1-\prod_{i=1}^{k}\left(1-a_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{k}\left(1-b_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{k}\left(1-\lambda_i^3\right)^{v_i}} \right] \right\rangle, \left\langle \left[\prod_{i=1}^{k}\tilde{a}_i^{v_i}, \prod_{i=1}^{k}\tilde{b}_i^{v_i}; \prod_{i=1}^{k}\mu_i^{v_i} \right] \right\rangle \right). \tag{34}$$

Let the result is true for $n = k + 1$; then,

$$\text{F-CFWA}(Fc_1, Fc_2, Fc_3, \cdots, Fc_{k+1}) = \left(\left\langle \left[\sqrt[3]{1-\prod_{i=1}^{k}\left(1-a_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{k}\left(1-b_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{k}\left(1-\lambda_i^3\right)^{v_i}} \right] \right\rangle, \left\langle \left[\prod_{i=1}^{k}\tilde{a}_i^{v_i}, \prod_{i=1}^{k}\tilde{b}_i^{v_i}; \prod_{i=1}^{k}\mu_i^{v_i} \right] \right\rangle \right)$$
$$\oplus \left(\left\langle \left[\sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-a_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-b_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-\lambda_i^3\right)^{v_i}} \right] \right\rangle, \left\langle \left[\prod_{i=1}^{k+1}\tilde{a}_i^{v_i}, \prod_{i=1}^{k+1}\tilde{b}_i^{v_i}; \prod_{i=1}^{k+1}\mu_i^{v_i} \right] \right\rangle \right)$$
$$\cdot \left(\left\langle \left[\sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-a_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-b_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-\lambda_i^3\right)^{v_i}} \right] \right\rangle, \left\langle \left[\prod_{i=1}^{k+1}\tilde{a}_i^{v_i}, \prod_{i=1}^{k+1}\tilde{b}_i^{v_i}; \prod_{i=1}^{k+1}\mu_i^{v_i} \right] \right\rangle \right). \tag{35}$$

This clearly exhibits that the result holds true for F-CFWA operator for k, where k is any positive integer and i is index set. □

Definition 27. Let F_{c_i} for $i = 1, 2, 3, \cdots, n$ be the collection of F-CFNs and $v = (v_1, v_2, v_3, \cdots, v_n)^T$ be the weight vector of F_{c_i} for $i = 1, 2, 3, \cdots, n$ with $v_i \geq 0$ $v_i \in [0, 1]$ and $\sum_{i=1}^{n} v_i = 1$; then, Fermatean cubic fuzzy-weighted geometric (F-CFWG) operator is defined as

$$\text{F-CFWG}\left(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n}\right) = F_{c_1}^{v_1} \oplus F_{c_2}^{v_2} \oplus F_{c_3}^{v_3} \oplus, \cdots, \oplus F_{c_n}^{v_n}. \tag{36}$$

Theorem 28. *Let $F_{c_i} = (\langle C_i; \lambda\rangle, \langle \tilde{C}_i; \mu\rangle)$ be a group of F-CFNs for $i = 1, 2, 3, \cdots, n$ and $V = (v_1, v_2, v_3, \cdots, v_n)^T$ be the weight vector of F_{c_i} with $v_i \geq 0$, $v_i \in [0, 1]$, and $\sum v_i = 1$; then, the aggregated result according to our definition is also a F-CFN given by*

$$\text{F-CFWG}\left(F_{c_1}, F_{c_2}, F_{c_3}, \cdots \cdots F_{c_n}\right) = \left(\left\langle \left[\prod_{i=1}^{n} a_i^{v_i}, \prod_{i=1}^{n} b_i^{v_i}; \prod_{i=1i}^{n} \lambda_i^{v_i} \right] \right\rangle, \left\langle \left[\sqrt[3]{1-\prod_{i=1}^{n}\left(1-\tilde{a}_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{n}\left(1-\tilde{b}_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{n}\left(1-\mu_i^3\right)^{v_i}} \right] \right\rangle \right). \tag{37}$$

Proof. By Mathematical induction, first we check it for $n = 2$,

$$F_{c_1}^{v_1} = \left\langle \sqrt[3]{1-\left(1-\tilde{a}_1^3\right)^{v_1}}, \sqrt[3]{1-\left(1-\tilde{b}_1^3\right)^{v_1}}; \sqrt[3]{1-\left(1-\tilde{\mu}_1\right)v_1} \right\rangle \begin{matrix} \langle (a_1)^{v_1}, (b_1)^{v_1}]; (\mu_1)^{v_1} \rangle, \\ \\ \end{matrix},$$

$$F_{c_2}^{v_2} = \left\langle \sqrt[3]{1-\left(1-\tilde{a}_2^3\right)^{v_2}}, \sqrt[3]{1-\left(1-\tilde{b}_2^3\right)^{v_2}}; \sqrt[3]{1-\left(1-\tilde{\mu}_2^3\right)v_2} \right\rangle \begin{matrix} \langle (a_2)^{v_2}, (b_2)^{v_2}]; (\mu_2)^{v_2} \rangle, \\ \\ \end{matrix},$$

$$F_{c_1}^{v_1} \otimes F_{c_2}^{v_2} = \left\langle \sqrt[3]{1-\left(1-\tilde{a}_1^3\right)^{v_1}}, \sqrt[3]{1-\left(1-\tilde{b}_1^3\right)^{v_1}}; \sqrt[3]{1-\left(1-\tilde{\mu}_1\right)v_1} \right\rangle \begin{matrix} \langle (a_1)^{v_1}, (b_1)^{v_1}]; (\mu_1)^{v_1} \rangle, \\ \\ \end{matrix}$$
$$\otimes \left\langle \sqrt[3]{1-\left(1-\tilde{a}_2^3\right)^{v_2}}, \sqrt[3]{1-\left(1-\tilde{b}_2^3\right)^{v_2}}; \sqrt[3]{1-\left(1-\tilde{\mu}_2^3\right)v_2} \right\rangle \begin{matrix} \langle (a_2)^{v_2}, (b_2)^{v_2}]; (\mu_2)^{v_2} \rangle, \\ \\ \end{matrix},$$

$$F_{c_1}^{v_1} \otimes F_{c_2}^{v_2} = \left(\begin{matrix} \left\langle \left[\prod_{i=1}^{2} a_i^{v_i}, \prod_{i=1}^{2} b_i^{v_i}; \prod_{i=1i}^{2} \lambda_i^{v_i} \right] \right\rangle, \\ \left\langle \left[\sqrt[3]{1-\prod_{i=1}^{2}\left(1-\tilde{a}_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{2}\left(1-\tilde{b}_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{2}\left(1-\mu_i^3\right)^{v_i}} \right] \right\rangle \end{matrix} \right). \tag{38}$$

We suppose the result is true for $n = k$, where k is any positve integer.

$$\text{F-CFWG}\left(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_k}\right) = \left(\begin{matrix} \left\langle \left[\prod_{i=1}^{k} a_i^{v_i}, \prod_{i=1}^{k} b_i^{v_i}; \prod_{i=1i}^{k} \lambda_i^{v_i} \right] \right\rangle, \\ \left\langle \left[\sqrt[3]{1-\prod_{i=1}^{k}\left(1-\tilde{a}_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{k}\left(1-\tilde{b}_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{k}\left(1-\mu_i^3\right)^{v_i}} \right] \right\rangle \end{matrix} \right). \tag{39}$$

Next, we prove the result for $n = k + 1$,

$$\begin{aligned} &\text{F-CFWG}\left(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_{k+1}}\right) \\ &= \left(\begin{matrix} \left\langle \left[\prod_{i=1}^{k} a_i^{v_i}, \prod_{i=1}^{k} b_i^{v_i}; \prod_{i=1i}^{k} \lambda_i^{v_i} \right] \right\rangle, \\ \left\langle \left[\sqrt[3]{1-\prod_{i=1}^{k}\left(1-\tilde{a}_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{k}\left(1-\tilde{b}_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{k}\left(1-\mu_i^3\right)^{v_i}} \right] \right\rangle \end{matrix} \right) \\ &\otimes \left(\begin{matrix} \left\langle \left[\prod_{i=1}^{k+1} a_i^{v_i}, \prod_{i=1}^{k+1} b_i^{v_i}; \prod_{i=1i}^{k+!} \lambda_i^{v_i} \right] \right\rangle, \\ \left\langle \left[\sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-\tilde{a}_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-\tilde{b}_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-\mu_i^3\right)^{v_i}} \right] \right\rangle \end{matrix} \right) \\ &= \left(\begin{matrix} \left\langle \left[\prod_{i=1}^{k+1} a_i^{v_i}, \prod_{i=1}^{k+1} b_i^{v_i}; \prod_{i=1i}^{k+!} \lambda_i^{v_i} \right] \right\rangle, \\ \left\langle \left[\sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-\tilde{a}_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-\tilde{b}_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{k+1}\left(1-\mu_i^3\right)^{v_i}} \right] \right\rangle \end{matrix} \right). \end{aligned} \tag{40}$$

Hence, the result is true for all integers. □

Theorem 29. *Let F_{c_i} for $i = 1, 2, 3, \cdots, n$ be the collection of F-CFNs with $v = (v_1, v_2, v_3, \cdots, v_n)^T$ be the weight vector of F_{c_i} for $i = 1, 2, 3, \cdots, n$ with $v_i \geq 0$ and $v_i \in [0, 1]$. Also, $\sum_{i=1}^{n} v_i = 1$; then, the following properties are satisfied for the operators defined under Fermatean cubic fuzzy environment.*

(1) Idempotency: for F_{c_i} when $F_{c_i} = F_c$ for all $i = 1, 2, 3, , \cdots, n$; then,

$$\text{F-CFWA}\left(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n}\right) = \text{F-CFWG}\left(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n}\right) = F_c. \tag{41}$$

Proof. Given that $F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n} = F_c$

$$\begin{aligned} &\text{F-CFWA}\left(F_{c_1}, F_{c_2} F_{c_3}, \cdots, F_{c_n}\right) \\ &= v_1 F_{c_1} \oplus v_2 F_{c_2} \oplus v_3 F_{c_3}, \cdots, v_n F_{c_n} \\ &= v_1 F_c \oplus v_2 F_c \oplus v_3 F_c, \cdots, v_n F_c \\ &= F_c(v_1 \oplus v_2 \oplus v_3, \cdots, v_n) = F_c(1) = F_c. \end{aligned} \tag{42}$$

Or

$$\text{F-CFWA}\left(F_{c_1}, F_{c_2} F_{c_3}, \cdots, F_{c_n}\right) = \left(\begin{matrix} \left\langle \left[\sqrt[3]{1-\prod_{i=1}^{n}\left(1-a_i^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{n}\left(1-b_i^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{n}\left(1-\lambda_i^3\right)^{v_i}} \right] \right\rangle, \\ \left\langle \left[\prod_{i=1}^{n} \tilde{a}_i^{v_i}, \prod_{i=1}^{n} \tilde{b}_i^{v_i}; \prod_{i=1}^{n} \mu_i^{v_i} \right] \right\rangle \end{matrix} \right). \tag{43}$$

If $Fc_i = F_c$ for $i = 1, 2, 3, \cdots, n$, i.e., $a_1 = a_2 = a_3 = a$, $b_1 = b_2 = b_3 = b$, and $\lambda_i = \lambda$ as well as $\tilde{a}_i = \tilde{a}$, $\tilde{b}_i = \tilde{b}$, and $\mu_i = \mu$. Then, the above equation becomes

$$\begin{aligned} &= \left(\begin{matrix} \left\langle \left[\sqrt[3]{1-\prod_{i=1}^{n}\left(1-a^3\right)^{v_i}}, \sqrt[3]{1-\prod_{i=1}^{n}\left(1-b^3\right)^{v_i}}; \sqrt[3]{1-\prod_{i=1}^{n}\left(1-\lambda^3\right)^{v_i}} \right] \right\rangle, \\ \left\langle \left[\prod_{i=1}^{n} \tilde{a}^{v_i}, \prod_{i=1}^{n} \tilde{b}^{v_i}; \prod_{i=1}^{n} \mu^{v_i} \right] \right\rangle \end{matrix} \right) \\ &= \left(\begin{matrix} \left\langle \left[\sqrt[3]{1-\left(1-a^3\right)^{\sum v_i}}, \sqrt[3]{1-\left(1-b^3\right)^{\sum v_i}}; \sqrt[3]{1-\left(1-\lambda^3\right)^{\sum v_i}} \right] \right\rangle, \\ \left\langle \left[\tilde{a}^{\sum v_i}, \tilde{b}^{\sum v_i}; \mu^{\sum v_i} \right] \right\rangle \end{matrix} \right). \end{aligned} \tag{44}$$

$\sum v_i = 1$ for $i = 1, 2, 3, \cdots, n$

$$\begin{aligned} &= \left(\begin{matrix} \left\langle \left[\sqrt[3]{1-\left(1-a^3\right)}, \sqrt[3]{1-\left(1-b^3\right)}; \sqrt[3]{1-\left(1-\lambda^3\right)} \right] \right\rangle, \\ \left\langle \left[\tilde{a}, \tilde{b}; \mu \right] \right\rangle \end{matrix} \right) \\ &= \left(\begin{matrix} \left\langle \left[\sqrt[3]{1-1+a^3)}, \sqrt[3]{1-1+b^3)}; \sqrt[3]{1-1+\lambda^3)} \right] \right\rangle, \\ \left\langle \left[\tilde{a}, \tilde{b}; \mu \right] \right\rangle \end{matrix} \right) \\ &= \left(\begin{matrix} \left\langle \left[\sqrt[3]{a^3)}, \sqrt[3]{b^3)}; \sqrt[3]{\lambda^3)} \right] \right\rangle, \\ \left\langle \left[\tilde{a}^{v_i}, \tilde{b}^{v_i}; \mu^{v_i} \right] \right\rangle \end{matrix} \right) = \left(\langle [a, b; \lambda] \rangle, \left\langle \left[\tilde{a}, \tilde{b}; \mu \right] \right\rangle \right), \end{aligned} \tag{45}$$

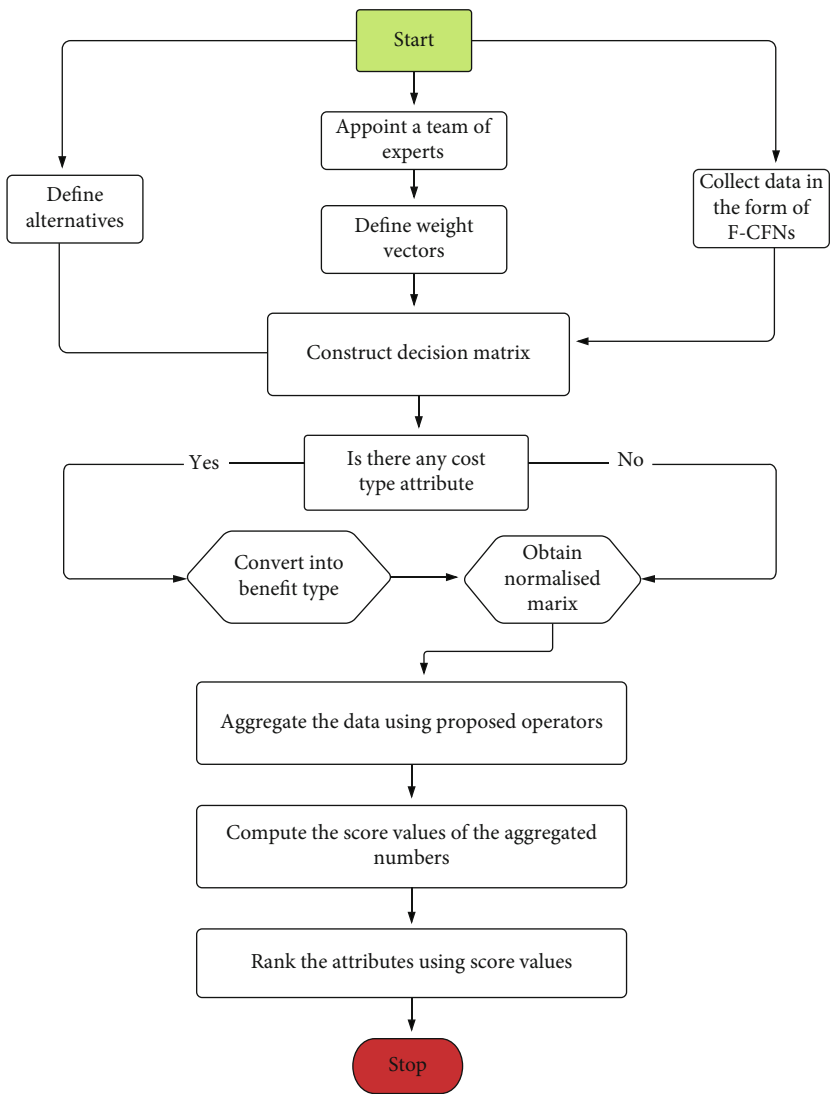

Figure 1: Pictorial representation of decision-making approach.

which is a F-CFN and this completes the proof. Same procedure can be repeated for F-CFWG operator.

(2) Boundary: $F_{c\,\min} \leq \text{F-CFWA}(F_{c_1}, F_{c_2}, \cdots, F_{c_n})$, $\text{F-CFWG}(F_{c_1}, F_{c_2}, \cdots, F_{c_n}) \leq F_{c(\max)}$ for all n, where $F_{c(\min)} = (\min(\Gamma_i), \max(\Psi_i))$ and $F_{c(\max)} = (\max(\Gamma_i), \min(\Psi_i))$.

(3) Monotonicity: let $F_c = (\langle C_i; \lambda_i\rangle, \langle \tilde{C}_i; \mu_i\rangle)$ and $F_c = (\langle C_i; \lambda_i\rangle, \langle \tilde{C}_i; \mu_i\rangle)$ for $i = 1, 2, 3, \cdots, n$ be the group of F-CFNs if we have $C_i \leq C_i\lambda_i \leq \lambda_i \tilde{C}_i \leq C_i$, $\mu_i \leq \mu_i$; then,

$$\begin{aligned}&\text{F-CFWA}(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n}), \text{F-CFWG}(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n})\\ &\quad \leq \text{F-CFWA}(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n}), \text{F-CFWG}(F_{c_1}, F_{c_2}, F_{c_3}, \cdots, F_{c_n}).\end{aligned} \tag{46}$$

□

The proofs of boundary and monotonicity are straightforward based on [19].

Lemma 30. *Consider $F_{c_i} > 0$ for $w_i > 0 (i = 1, 2, 3, \cdots, n)$ and $\sum w_i = 1$; then,*

$\prod (F_{c_i})^{w_i} \leq \sum w_i F_{c_i}$ for $i = 1, 2, 3, \cdots, n$; here, the equality holds only if $F_{c_1} = F_{c_2} = F_{c_3}, \cdots, = F_{c_n}$.

Theorem 31. *Let F_{c_i} be the group of all F-CFNs; then, F-CFOWG$(F_{c_1}, F_{c_1}, F_{c_1}, \cdots . F_{cn}) \leq$ F-CFOWA$(F_{c_1}, F_{c_1}, F_{c_1}, \cdots . F_{cn})$, where $W = (w_1, w_2, w_3, \cdots, w_n)^T$ is the associated weight vector of F_{c_i} such that $w_i \in [0, 1]$ and $\sum w_i = 1$ for $i = 1, 2, 3, \cdots, n$.*

Proof. The proof of this theorem is based on Lemma 30. □

Table 1: Rating values by decision-maker d_1 about four alternatives.

Fc_{ij}	U_1	U_2	U_3	U_4
C_1	$\begin{pmatrix}\langle[.6,.8];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.6\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.4];.6\rangle,\\ \langle[.8,.9];.7\rangle\end{pmatrix}$
C_2	$\begin{pmatrix}\langle[.5,.7];.5\rangle,\\ \langle[.4,.5];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.4];.8\rangle,\\ \langle[.8,.9];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.7];.8\rangle,\\ \langle[.4,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.8\rangle,\\ \langle[.5,.6];.5\rangle\end{pmatrix}$
C_3	$\begin{pmatrix}\langle[.5,.6];.8\rangle,\\ \langle[.4,.7];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.6\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.2\rangle,\\ \langle[.5,.6];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.7\rangle,\\ \langle[.5,.6];.6\rangle\end{pmatrix}$
C_4	$\begin{pmatrix}\langle[.6,.8];.6\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.9\rangle,\\ \langle[.5,.6];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.7];.7\rangle,\\ \langle[.4,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.5];.8\rangle,\\ \langle[.7,.8];.6\rangle\end{pmatrix}$
C_5	$\begin{pmatrix}\langle[.8,.9];.3\rangle,\\ \langle[.3,.4];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.9\rangle,\\ \langle[.5,.6];.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.5\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$

Table 2: Rating values by decision-maker d_2 about four alternatives.

Fc_{ij}	U_1	U_2	U_3	U_4
C_1	$\begin{pmatrix}\langle[.4,.6];.5\rangle,\\ \langle[.7,.8];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.6];.6\rangle,\\ \langle[.7,.8];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.8,.9];.5\rangle,\\ \langle[.3,.4];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.6\rangle,\\ \langle[.4,.5];.5\rangle\end{pmatrix}$
C_2	$\begin{pmatrix}\langle[.5,.6];.8\rangle,\\ \langle[.7,.8];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.5];.8\rangle,\\ \langle[.7,.9];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.4\rangle,\\ \langle[.4,.5];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.6\rangle,\\ \langle[.3,.6];.5\rangle\end{pmatrix}$
C_3	$\begin{pmatrix}\langle[.7,.8];.4\rangle,\\ \langle[.4,.5];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.5];.7\rangle,\\ \langle[.7,.8];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.8];.7\rangle,\\ \langle[.6,.7];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.6];.4\rangle,\\ \langle[.6,.7];.6\rangle\end{pmatrix}$
C_4	$\begin{pmatrix}\langle[.7,.8];.7\rangle,\\ \langle[.4,.5];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.5\rangle,\\ \langle[.4,.5];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.7\rangle,\\ \langle[.5,.7];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.5];.7\rangle,\\ \langle[.7,.8];.5\rangle\end{pmatrix}$
C_5	$\begin{pmatrix}\langle[.7,.8];.6\rangle,\\ \langle[.4,.5];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.7\rangle,\\ \langle[.5,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.5];.7\rangle,\\ \langle[.5,.7];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.5\rangle,\\ \langle[.4,.5];.6\rangle\end{pmatrix}$

5. Application in MADM Problem

Here, we present a MADM problem and apply our modified aggregation operators to prove the applicability and smoothness of our newly introduced operators. To complete the process and find out a solution to a decision-making problem, the following is a flowchart in Figure 1.

5.1. Illustrative Example. To show the application of the proposed method to MADM, we consider a practical example of university ranking. Currently, universities pay considerable attention to their ranking because it significantly affects government investment, social donations, recruitment, etc. There are many systems to rank the universities throughout the world. Here, we give four more influential and authoritative evaluation systems. Assume that $U = \{U_1, U_2, U_3, U_4\}$ be the set of four universities or alternatives that are to be ranked upon the following set of five criteria C_j $(j = 1, 2, 3, 4, 5)$. Namely, they are

(1) Stability of the indicator system (C_1)

(2) Stability of weight setting (C_2)

(3) Correlation of historical ranking data (C_3)

(4) Consistency of data trends (C_4)

(5) Consistency of teacher students ratio (C_4)

To evaluate the stability of the evaluation system, first we obtained expert's opinions in the form of F-CFNs. There are a number of parameters to be considered, but the above set of criterion C_j was established by the four experts/decision-makers $D = \{d_1, d_2, d_3, d_4\}$. This means that the decision makers will give their opinions in the form of F-CFNs according to the set C_j of attributes. It is obvious that, in a set of criteria, the importance of criterion may not be the same, and hence, we take the weight vector $w = (0.21, 0.26, 0.23, 0.30)^T$ that gives the weight or importance of each criterion. To evaluate the best university from the

Table 3: Rating values by decision-maker d_3 about four alternatives.

Fc_{ij}	U_1	U_2	U_3	U_4
C_1	$\begin{pmatrix}\langle[.6,.7];.7\rangle,\\ \langle[.5,.6];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.6\rangle,\\ \langle[.4,.5];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.5];.5\rangle,\\ \langle[.7,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.6];.8\rangle,\\ \langle[.6,.7];.5\rangle\end{pmatrix}$
C_2	$\begin{pmatrix}\langle[.6,.8];.2\rangle,\\ \langle[.4,.5];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.5];.4\rangle,\\ \langle[.7,.8];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.8\rangle,\\ \langle[.5,.6];.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.8\rangle,\\ \langle[.5,.6];.4\rangle\end{pmatrix}$
C_3	$\begin{pmatrix}\langle[.7,.8];.4\rangle,\\ \langle[.4,.5];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.4\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.8\rangle,\\ \langle[.5,.6];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.7\rangle,\\ \langle[.4,.6];.4\rangle\end{pmatrix}$
C_4	$\begin{pmatrix}\langle[.6,.7];.7\rangle,\\ \langle[.5,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.5\rangle,\\ \langle[.4,.5];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.7\rangle,\\ \langle[.4,.5];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.9\rangle,\\ \langle[.4,.5];.3\rangle\end{pmatrix}$
C_5	$\begin{pmatrix}\langle[.5,.6];.6\rangle,\\ \langle[.6,.7];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.5\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.4\rangle,\\ \langle[.4,.5];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.5\rangle,\\ \langle[.4,.6];.7\rangle\end{pmatrix}$

Table 4: Rating values by decision-maker d_4 about four alternatives.

Fc_{ij}	U_1	U_2	U_3	U_4
C_1	$\begin{pmatrix}\langle[.7,.8];.6\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.4];.6\rangle,\\ \langle[.8,.9];.7\rangle\end{pmatrix}$
C_2	$\begin{pmatrix}\langle[.5,.7];.5\rangle,\\ \langle[.4,.5];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.6];.8\rangle,\\ \langle[.4,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.8\rangle,\\ \langle[.5,.6];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.4];.8\rangle,\\ \langle[.8,.9];.6\rangle\end{pmatrix}$
C_3	$\begin{pmatrix}\langle[.6,.8];.2\rangle,\\ \langle[.5,.6];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.6\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.7\rangle,\\ \langle[.5,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.6];.8\rangle,\\ \langle[.4,.7];.6\rangle\end{pmatrix}$
C_4	$\begin{pmatrix}\langle[.6,.8];.6\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.7];.7\rangle,\\ \langle[.4,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.9\rangle,\\ \langle[.5,.6];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.5];.8\rangle,\\ \langle[.7,.8];.6\rangle\end{pmatrix}$
C_5	$\begin{pmatrix}\langle[.8,.9];.3\rangle,\\ \langle[.3,.4];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.5\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.9\rangle,\\ \langle[.5,.6];.3\rangle\end{pmatrix}$

Table 5: Aggregated values by F-CFWA operator.

Fc_{ij}	U_1	U_2	U_3	U_4
C_1	$\begin{pmatrix}\langle[.6,.7];.8\rangle,\\ \langle[.7,.4];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.4];.6\rangle,\\ \langle[.8,.9];.7\rangle\end{pmatrix}$
C_2	$\begin{pmatrix}\langle[.5,.9];.5\rangle,\\ \langle[.4,.5];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.6];.8\rangle,\\ \langle[.4,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.8\rangle,\\ \langle[.5,.6];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.4];.8\rangle,\\ \langle[.8,.9];.6\rangle\end{pmatrix}$
C_3	$\begin{pmatrix}\langle[.6,.2];.9\rangle,\\ \langle[.8,.7];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.6\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.7\rangle,\\ \langle[.5,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.6];.8\rangle,\\ \langle[.4,.7];.6\rangle\end{pmatrix}$
C_4	$\begin{pmatrix}\langle[.6,.5];.7\rangle,\\ \langle[.8,.4];.2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.7];.7\rangle,\\ \langle[.4,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.9\rangle,\\ \langle[.5,.6];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.5];.8\rangle,\\ \langle[.7,.8];.6\rangle\end{pmatrix}$
C_5	$\begin{pmatrix}\langle[.7,.8];.8\rangle,\\ \langle[.5,.4];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.8];.5\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.9\rangle,\\ \langle[.5,.6];.3\rangle\end{pmatrix}$

set $U = \{U_1, U_2, U_3, U_4\}$, expert's opinions were obtained in the form of F-CFNs and following decision matrices were constructed on the basis of their opinions.

Step 1. In the following, Tables 1–4 are the rating values arranged in matrix form that were assigned by the experts about the four alternatives, considering the set of parameters C_j.

Table 6: Aggregated values by F-CFWG operator.

Fc_{ij}	U_1	U_2	U_3	U_4
C_1	$\begin{pmatrix}\langle[.4,.5];.9\rangle,\\ \langle[.7,.4];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.4];.5\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.4];.8\rangle,\\ \langle[.7,.3];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.3,.4];.6\rangle,\\ \langle[.8,.9];.7\rangle\end{pmatrix}$
C_2	$\begin{pmatrix}\langle[.5,.3];.4\rangle,\\ \langle[.7,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.6];.4\rangle,\\ \langle[.4,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.9];.8\rangle,\\ \langle[.3,.7];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.7];.6\rangle,\\ \langle[.5,.9];.7\rangle\end{pmatrix}$
C_3	$\begin{pmatrix}\langle[.9,.5];.6\rangle,\\ \langle[.4,.2];.9\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.6\rangle,\\ \langle[.5,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.8];.7\rangle,\\ \langle[.5,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.5,.6];.8\rangle,\\ \langle[.4,.7];.6\rangle\end{pmatrix}$
C_4	$\begin{pmatrix}\langle[.3,.9];.7\rangle,\\ \langle[.4,.9];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.7];.7\rangle,\\ \langle[.4,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.9\rangle,\\ \langle[.5,.6];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.7];.8\rangle,\\ \langle[.7,.8];.6\rangle\end{pmatrix}$
C_5	$\begin{pmatrix}\langle[.7,.8];.8\rangle,\\ \langle[.7,.4];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.4,.9];.3\rangle,\\ \langle[.6,.8];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.6\rangle,\\ \langle[.5,.6];.8\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.7,.6];.7\rangle,\\ \langle[.5,.4];.3\rangle\end{pmatrix}$

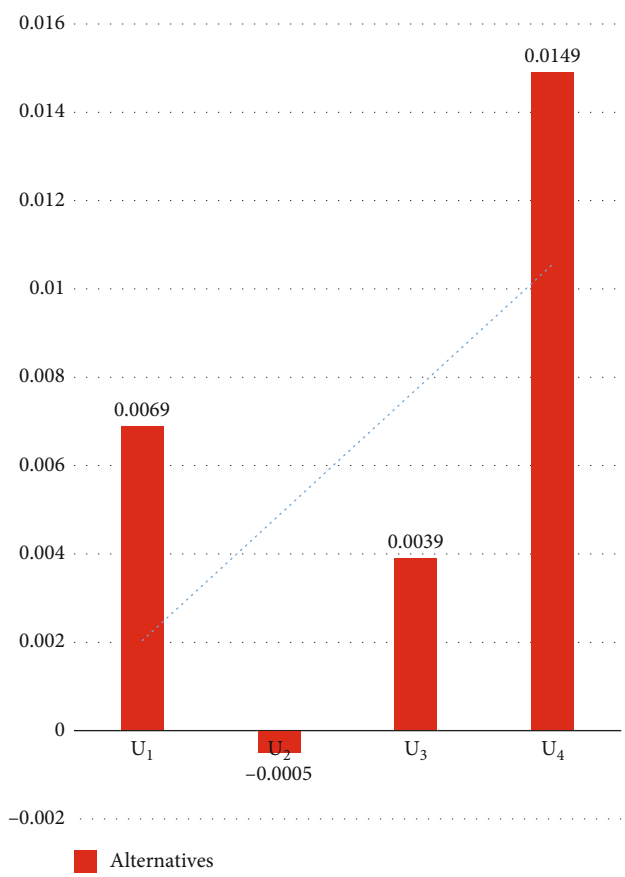

Figure 2: Graphical view of results by F-CFWA operator.

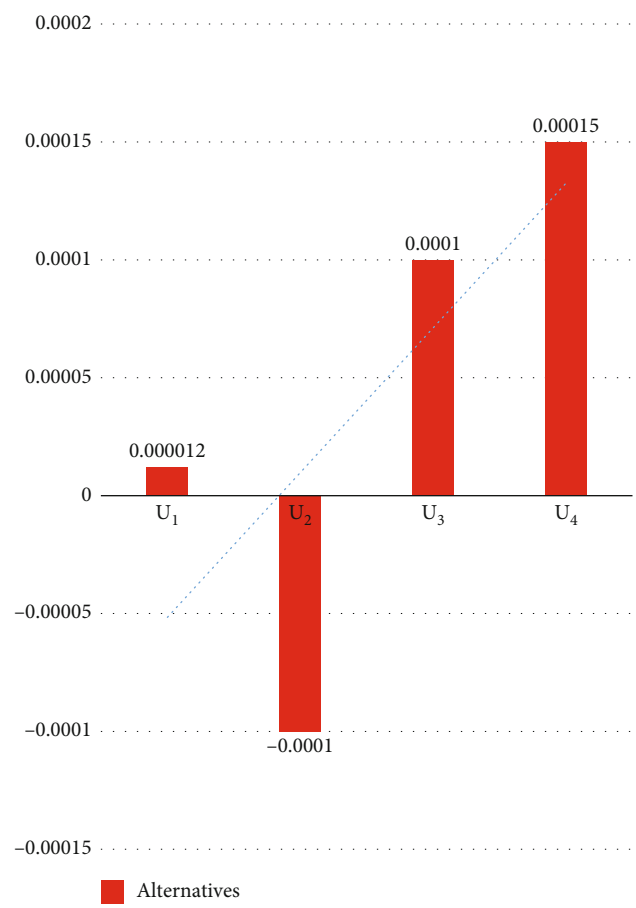

Figure 3: Graphical view of results by F-CFWG operator.

Step 2. In this step, the data is examined for the purpose of normalization. The normalization is done by converting the cost type parameters into benefit type if there is any. Since, in our case, there is no cost type parameter, so this step is skipped.

Step 3. This is the aggregation phase and so we first use the novel F-CFWA operator (and later F-CFWG operator) in

Table 7: Aggregated values by operators defined in [38].

Alternatives	Performance index value	Ranking
A_1	−0.02774	4
A_2	0.079762	2
A_3	0.76802	3
A_4	0.8566	1

Table 8: Aggregated results by F-CFWA operator.

$F-CFWA$	C_1	C_2	C_3	C_4
A_1	$\begin{pmatrix}\langle[.11,.17];.6\rangle,\\ \langle[.4,.5];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.08,.15];.05\rangle,\\ \langle[.4,.5];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.04,.15];.12\rangle,\\ \langle[.5,.6];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.6,.7];.18\rangle,\\ \langle[.6,.7];.6\rangle\end{pmatrix}$
A_2	$\begin{pmatrix}\langle[.07,.13];.07\rangle,\\ \langle[.4,.5];.5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.06,.11];.11\rangle,\\ \langle[.67,.56];.69\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.07,.11];.08\rangle,\\ \langle[.46,.65];.71\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.04,.74];.1\rangle,\\ \langle[.52,.63];.4\rangle\end{pmatrix}$
A_3	$\begin{pmatrix}\langle[.07,.13];.15\rangle,\\ \langle[.5,.6];.6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.09,.15];.06\rangle,\\ \langle[.43,.53];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.12,.09];.8\rangle,\\ \langle[.50,.42];.49\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.06,.11];.2\rangle,\\ \langle[.4,.6];.4\rangle\end{pmatrix}$
A_4	$\begin{pmatrix}\langle[.06,.10];.05\rangle,\\ \langle[.5,.6];.3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.04,.09];.07\rangle,\\ \langle[.4,.6];.7\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.08,.03];.01\rangle,\\ \langle[.5,.7];.4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[.06,.1];.09\rangle,\\ \langle[.6,.7];.6\rangle\end{pmatrix}$

order to aggregate the data about the alternatives. Table 5 is the merged matrix or aggregated matrix by utilizing the F-CFWA operator on Tables 1–4 and is given below.

Further, the F-CFWA operator is once again used to aggregate Table 5 and the following preference values of the alternatives are obtained:

$$\begin{aligned} U_1 &= \langle[.6,.8];.6\rangle, \langle[.5,.6];.6\rangle,\\ U_2 &= \langle[.5,.6];.7\rangle, \langle[.5,.6];.4\rangle,\\ U_3 &= \langle[.6,.7];.6\rangle, \langle[.5,.6];.6\rangle,\\ U_4 &= \langle[.6,.8];.6\rangle, \langle[.4,.6];.6\rangle. \end{aligned} \tag{47}$$

Step 3. In this step, we utilize the score function to find the score values of U_i for $(i = 1, 2, 3, 4,)$,

$$\begin{aligned} S(U_1) &= 0.0069,\\ S(U_2) &= -0.0005,\\ S(U_3) &= 0.0039,\\ S(U_4) &= 0.0149. \end{aligned} \tag{48}$$

Step 4. All the alternatives are ranked in descending order on the basis of their score values. Therefore,

$$U_4 > U_1 > U_3 > U_2. \tag{49}$$

Using the score function, we find that U_1 is the best choice. Thus, U_4 is the best university upon the set of five criteria. Furthermore, U_1 can be selected as the best option in case U_4 is not available.

5.2. Using Fermatean Cubic Fuzzy-Weighted Geometric Operator (F-CFWG). Similarly, utilizing F-CFWG on Tables 1–4, we obtained the following aggregated decision matrix (as given in Table 6).

Note that same steps are followed for the whole calculations.

Table 9: Preference values of alternatives.

Alternatives	Preference values
A_1	$\begin{pmatrix}\langle[.0009,.00015];.00007\rangle,\\ \langle[.47,.57];.59\rangle\end{pmatrix}$
A_2	$\begin{pmatrix}\langle[.00042,.0002];.000089\rangle,\\ \langle[.58,.58];.57\rangle\end{pmatrix}$
A_3	$\begin{pmatrix}\langle[.00012,.00024];.00002\rangle,\\ \langle[.44,.53];.52\rangle\end{pmatrix}$
A_4	$\begin{pmatrix}\langle[.000048,.000064];.000019\rangle,\\ \langle[.49,.65];.49\rangle\end{pmatrix}$

Table 10: Score values and final ranking.

Alternatives	Score values	Ranking
A_1	−0.0033757	3
A_2	−0.00551	4
A_3	−0.0033740	2
A_4	−0.0001	1

The preferences values obtained through the proposed F-CFWG operator are given below:

$$\begin{aligned} U_1 &= \langle[.4,.3];.9\rangle, \langle[.3,.5];.6\rangle,\\ U_2 &= \langle[.4,.5];.7\rangle, \langle[.2,.3];.4\rangle.\\ U_3 &= \langle[.5,.6];.6\rangle, \langle[.5,.6];.7\rangle,\\ U_4 &= \langle[.4,.8];.5\rangle, \langle[.4,.6];.9\rangle. \end{aligned} \tag{50}$$

The score function is used to find the score values of U_i for $(i = 1, 2, 3, 4,)$,

$$\begin{aligned} S(U_1) &= 0.000012,\\ S(U_2) &= -0.00010,\\ S(U_3) &= 0.00011,\\ S(U_4) &= 0.00015. \end{aligned} \tag{51}$$

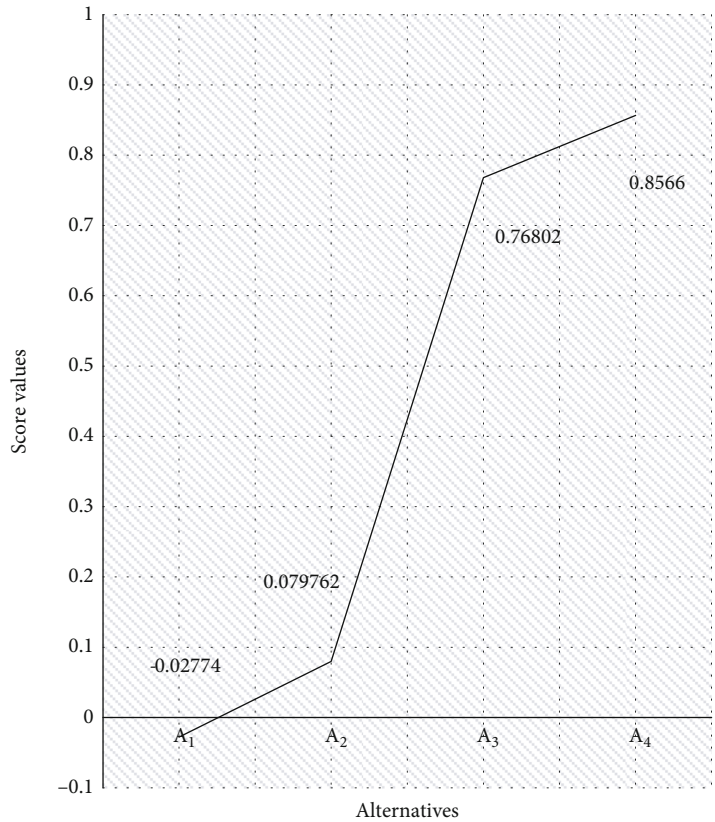

Figure 4: Graphical representation of score values by Hamacher aggregation operator defined in [38].

Therefore, the final ranking is:

$$U_4 > U_1 > U_3 > U_2. \tag{52}$$

Again, we see that the ranking order is the same as the one obtained through F-CFWA operator and that U_4 is the best alternative.

The graphical interpretation of the decision-making process to best examine the nature and behavior of the modified operators and the results obtained through these operators on the basis of associated score values are presented in Figures 2 and 3. They illustrate the graphical behavior of the results obtained through the F-CFWA and F-CFWG, respectively.

We can easily exhibit the conclusion that in Figures 2 and 3, the alternative U_4 has a peak output; thus, U_4 is the best choice with respect to averaging and geometric operators. For further rectification and demolishing the novel operator, we now compare our results with some existing operators that are developed in the environment of cubic fuzzy set.

6. Comparison with Aggregation Operators under P-CFNs

In this section, we compare our results with aggregation operators of Pythagorean cubic fuzzy numbers such as Pythagorean cubic fuzzy Hamacher-weighted averaging (PCFHWA), Pythagorean cubic fuzzy Hamacher order-weighted averaging (PCFHOWA), Pythagorean Cubic fuzzy Hamacher hybrid averaging (PCFHHA), Pythagorean Cubic fuzzy Hamacher-weighted Geometric (PCFHWG), Pythagorean Cubic fuzzy Hamacher order-weighted Geometric (PCFHOWG), and Pythagorean Cubic fuzzy Hamacher hybrid geometric (PCFHHA) operators, Abdullah et al. in [38]. The performance index values of the alternatives using aggregation operators as abovementioned, each alternative's output index value, and their respective ranking are calculated regarding all criteria. It indicates that in Table 7, with the highest performance index of 0.8566, alternative A_4 has the highest performance value.

Next, we have utilized the newly developed operator (F-CFWA) for the data given in [38] (from Tables 1–3). The aggregated results using the novel operator are given in Table 8 and their corresponding preference values are tabulated in Table 9.

Finally, the ranking order on the basis of the score values are given in Table 10. The ranking orders of the F-CFWA operator are identical with the ranking orders of the alternatives in Table 7. Overall comparison revealed to us that the best choice is C_4 under criteria A_j $(j = 1, 2, 3, 4)$ with weighted vector $w = (0.21, 0.26, 0.23, 0.3)^T$.

This shows the stability of our proposed operators in Fermatean cubic fuzzy environment, and hence, the presented work can be utilized instead of other methods. The developed operators are based on novel concept of F-CFS having greater capability of supporting ambiguous situations

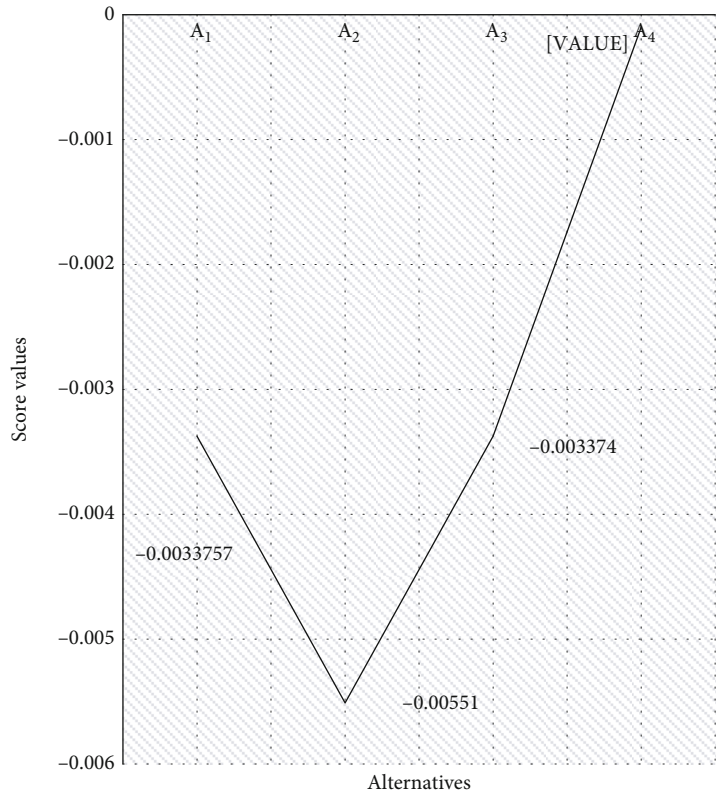

Figure 5: Graphical representation of score values by F-CFWA operator.

as the spatial scope of the F-CFS is lager compare to other cubic fuzzy sets. Hence, the advantage of the presented work is that the F-CFWA and F-CFWG operators yield better approximation than other existing operators.

Figures 4 and 5, respectively, elaborates that the ranking orders of the Hamacher aggregation operators defined for Pythagorean cubic fuzzy numbers [38] are identical and hence support the stability of the presented work.

7. Conclusion and Future Work

Fermatean cubic fuzzy set has prominent applications in MADM and is a highly efficient tool to deal with data involving fuzziness as compared to I-CFS and P-CFS. The aggregation operators are very effective for evaluating the given alternatives in decision-making process as they combine the different values about individuals into a unified form. In this article, we have designed Fermatean cubic fuzzy aggregation operators such as F-CFWA and F-CFWG. Some functional characteristics such as idempotency, boundary, and monotonicity of these novel operators have been discussed. We have defined Fermatean cubic fuzzy distance operator between two Fermatean cubic fuzzy numbers and illustrated it through an example showing better result as compare to I-CFS and P-CFS. Since the power of decimals in F-CFS is higher than I-CFS and P-CFS, so by expansion, their results lead to a more precised value. On the basis of F-CFWA and F-CFWG operators, we have proposed a MADM approach to show the effectiveness of the developed operators. A numerical example is also provided in order to show that the proposed operators give us a more precise way to resolve the MADM problem. Also, the stability of the proposed work is provided by comparing it with some existing work. We concluded that the I-CFS and P-CFS are subsequent to F-CFS. Furthermore, we developed the operational laws associated to it such as associativity, commutativity with respect to algebraic sum, and algebraic product. In the future, we plan to develop more generalized aggregation operators like Fermatean cubic fuzzy hybrid aggregation operators, Fermatean cubic fuzzy Einstein aggregation operators, and Fermatean cubic fuzzy Hamacher aggregation operators. We also plan to generalize our presented work to introduce q-rung orthopair cubic fuzzy set and its application in MADM problem of medical diagnosis.

Data Availability

The data sets generated during and/or analyzed during the current study are available from the corresponding authors on reasonable request.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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