

Retraction

Retracted: Approximations of Intuitionistic Fuzzy Ideals over Dual Spaces by Soft Binary Relations

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

- (1) Discrepancies in scope
- (2) Discrepancies in the description of the research reported
- (3) Discrepancies between the availability of data and the research described
- (4) Inappropriate citations
- (5) Incoherent, meaningless and/or irrelevant content included in the article
- (6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

- [1] M. Z. Anwar, S. Bashir, M. Aslam, and M. Shabir, "Approximations of Intuitionistic Fuzzy Ideals over Dual Spaces by Soft Binary Relations," *Journal of Function Spaces*, vol. 2022, Article ID 3996256, 17 pages, 2022.

Research Article

Approximations of Intuitionistic Fuzzy Ideals over Dual Spaces by Soft Binary Relations

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The major advantage of this proposed work is to investigate roughness of intuitionistic fuzzy subsemigroups (RIFSs) by using soft relations. In this way, two sets of intuitionistic fuzzy (IF) soft subsemigroups, named lower approximation and upper approximation regarding aftersets and foresets, have been introduced. In RIFSs, incomplete and insufficient information is handled in decision-making problems like symptom diagnosis in medical science. In addition, this new technique is more effective as compared to the previous literature because we use intuitionistic fuzzy set (IFS) instead of fuzzy set (FS). Since the FS describes the membership degree only but often in real-world problems, we need the description of nonmembership degree. That is why an IFS is a more useful set due to its nonmembership degree and hesitation degree. The above technique is applied for left (right) ideals, interior ideals, and bi-ideals in the same manner as described for subsemigroups.

1. Introduction

We are facing critical problems involving uncertainty and impreciseness in everyday life. Inexact and incomplete information data has not complete and precise reasoning possibly. Nowadays, the gap between the world full of vagueness and traditional mathematics with precise concepts is going to reduce with highly appreciated mode. Researchers are keen to deal with different nature of uncertainty with different methods. In this regard, they studied many newly defined theories such as RSs, FSs, IFSs, and SSs. The FSs are very important to manage the uncertainties in real-world problems. They have numerous applications in several fields such as vacuum cleaner, control of subway systems, unmanned helicopters, transmission systems, models for new pricing, and weather forecasting systems. This logic has given new life to scientific fields and has been used in many fields such as electronics, image processing, and optimization [1]. The IFS theory introduced by Atanassov [2] is one of the generalizations of FS the-

ory. In IFS, membership degree, degree of nonmembership, and degree of hesitation of every element are expressed whose sum must be equal to 1. As we know, IFSs have degree of membership and degree of nonmembership which are more valuable in medical field. Intuitionistic fuzzy environment is more suitable to diagnose disease than FS due to its nonmembership degree. The FSs have only degree of membership but IF soft sets (IFSSs) keep a controlled degree of the vagueness, and they transform an imprecise pattern classification problem into a well-defined and precise optimization problem because an IFS gives out the uncertainty by a nonmembership degree. In real-life problems, IF logic has more effective use to control and overcome the uncertainty than fuzzy logic [3–6].

The rough sets (RSs) were introduced by Pawlak [7, 8]. They attract many researchers and specialists because they handle vagueness, uncertainty, and impreciseness in a non-customary manner and discover relationship of structures with FSs. They also help to make decisions in an eccentric way in daily life problems. The abdominal pain of children

is a very serious issue in medical field which is due to different reasons, and this is a challenging issue for researchers to diagnose the reasons correctly. Researchers can use the RS theory to diagnose it and give further consultations. This theory is also used for feature extraction, decision rule generation, data reduction, and feature selection and also applied successfully to pattern recognition, intelligent systems, machine learning, mereology, expert systems, signal analysis, decision analysis, and more other fields.

In RS theory, we define approximation spaces as set with multiple memberships, but in FS theory, it concerns only with partial membership. In RS theory, we use indiscernibility to remove useless data without using basic knowledge-based data. Usually, we approximate a crisp set into two formal approximations, lower and upper approximation. Upper approximation is a set which has elements having possible belonging with target set and lower approximated set has objectives having positive belonging with the target set. In simple words, we can say that a RS is a nonempty set regarding its boundary region, and in other case, this set is crisp set [7–20].

The soft set (SS) theory is presented by Molodtsov [21], and this is a very suitable approach to remove those associated difficulties which the other theories were unable to tackle, like FS theory, interval mathematics, and probability theory. In this theory, parameters can be chosen by researchers with their form according to their needs. In industrialized countries, more visible reason for cancer death in men is prostate cancer, which depends on many factors such as age, family cancer history, the level of prostate-specific antigen in blood and ethnic background, etc. Many researchers are working on finding the risks of prostate cancer with the help of FSs and SSs [22]. Nowadays, the SS theory has many study work by different authors rapidly. Aktas and Cagman described soft groups [23]. Jun connected SSs with ideal theory ([24, 25]). Maji et al. [26] worked on IFSSs. Razak and Mohamad [27] worked on decision-making regarding fuzzy soft sets (FSSs) in connection with SSs [28].

Al-shami et al. [29, 30] introduced extended form of FS called square-root FS and contrast square-root FS with IFS and Pythagorean FS. Hariwan et al. [31] proposed a useful concept of (3,2)-fuzzy sets with connection of other types of fuzzy sets and discussed its basic properties and operations. (3,2)-Fuzzy sets are more useful than IFSs and Pythagorean FSs due to their larger range of membership grades. Since we know RSs are useful to deal with incompleteness and IFSs are useful to deal vagueness, SSs have rich operations due to its parameters. So the combination of IFSs, RSs, and SSs is a valuable combination to deal with impreciseness [32–36].

1.1. Related Works. Binary relations are always very important in information sciences and mathematics both. Extended form of ordinary binary relations is soft binary relation which is a family of parameters of binary relation to a universe. Anwar et al. [37] presented a suitable model of IFRS in terms of soft relations and algorithm for real-world problems. Later on, Anwar et al. [38, 39] worked on [37] based on multisoft relations and its algebraic properties

with useful algorithms and introduced the optimistic multi-granulation intuitionistic fuzzy rough set (OMGIFRS) and pessimistic multi-granulation intuitionistic fuzzy rough set (PMGIFRS) in terms of multisoft relations.

1.2. Innovative Contribution. For several practical applications in real world, equivalence relation is much restrictive. Skowron and Stepaniuk [40] replaced equivalence relations by tolerance relations. The soft covering has also been discussed by Li et al. [41]. The rough approximation by soft binary relations handles multiple binary relations. Ali [42] presented the conceptual theory about soft binary relation, and he discussed the soft lower and soft upper approximation operations regarding soft equivalence relations. In rough approximations, only binary relations are addressed, but in any other case, there are different several binary relations in connection with rough approximations in terms of soft binary relations. Shabir et al. presented prime and semi-prime L-fuzzy hyperideals in terms of soft sets. They also discussed applications of semihypergroups in terms of L-fuzzy soft sets and fuzzy ideals in connection with rough fuzzy ternary subsemigroups and 3-dimensional congruence relation in their previous publications [41–49]. In 2019, Kanwal and Shabir discussed the fuzzy set of semigroup in terms of rough approximation with soft relations [50]. In this research, we have studied the rough approximation of IF set in semigroups based on soft relations.

1.3. Motivation. The main motivation of this research study is to extend the concept of fuzzy ideals into IF ideals based on soft relations in terms of semigroups. Its related properties are also discussed. The semigroup attracts many algebraists due to their applications to formal languages, automata theory and network analogy, etc. The connection of semigroup theory and theory of machines increases the importance of both theories during the past few decades. In association with the study of machines and automata, other areas of applications have been improved such as formal languages, and the software uses the language of modern algebra in terms of Boolean algebra, semigroups, and others. The semigroup theory contributes in biology, psychology, biochemistry, and sociology [51]. The FS describes only degree of membership of each element which is insufficient to tackle uncertainty in several real-world problems. Here, we need IFS to describe degree of membership as well as degree of nonmembership of each element to control vagueness and impreciseness in real-world problems. In [52], Bashir et al. discussed a useful model of regular ternary semirings based on bipolar fuzzy sets and its algebraic properties. In Kanwal and Shabir's paper [50], fuzzy ideals have been discussed, but in this paper, we discuss our proposed model in intuitionistic environment which is a more suitable environment to deal with incompleteness and uncertainty.

1.4. Organization of the Paper. The setting of the paper is as follows. In Section 2, a few basic concepts with SSs, soft binary relations, IFSs, and IF ideals are presented. In Section 3, we discussed an approximation of an IFS in terms of soft binary relations. We made approximations of IF set by the

foresets and aftersets, and we get two IFSSs, called the upper approximation and lower approximation regarding foresets and aftersets. After applying these concepts, approximations of IF subsemigroups, IF left (right) ideals, IF interior ideals, and IF bi-ideals of semigroups are discussed with examples. In Section 4, we present the comparison with the previous work. In Section 5, conclusion is described to present this research work and future work.

See Table 1 for the acronyms.

2. Preliminaries and Basic Concepts

In this section, basic notions about IFSSs, SSs, IFSSs, and some background materials are given. Throughout this paper, (U, \cdot) is semigroup and U_1 and U_2 represent two nonempty finite sets unless stated otherwise. Here, we recall some ideas and results which are useful for this paper.

We will denote the product of two elements $x, y \in U$ by xy instead of $x.y$. In what follows by subsets, we always means nonempty ones. Let us consider two subsets A and B of U , and then, the product AB is defined as $AB = \{ab : a \in A, b \in B\}$.

Definition 1 (see [53]). A binary relation T from U_1 to U_2 is a subset of $U_1 \times U_2$, and a subset of $U_1 \times U_1$ is said to be a binary relation on U_1 . If T is a binary relation on U_1 , then T is said to be reflexive if $(u, u) \in T$ for all $u \in U_1$, symmetric if $(u, v) \in T$ implies $(v, u) \in T$ for all $u, v \in U_1$, and transitive if $(u, v) \in T$ and $(v, w) \in T$ implies $(u, w) \in T$ for all $u, v, w \in U_1$. If a binary relation T is reflexive, symmetric, and transitive, then it is called an equivalence relation. A set is partitioned into disjoint classes by an equivalence relation.

Definition 2 (see [54]). For any subset A of U if $ab \in A$ for all $a, b \in A$, then A is called a subsemigroup of U . A left (right) ideal of U is a subset A of U such that $UA \subseteq A$ ($AU \subseteq A$). A two-sided ideal is a subset A of U which is a left as well as right ideal of U . A subsemigroup A of U is said to be an interior ideal of U if $UAU \subseteq A$. A subsemigroup A of U is said to be a bi-ideal of U if $AUA \subseteq A$.

Definition 3 (see [2]). An intuitionistic fuzzy set (IFS) B in U_1 is an object having the shape $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in U_1\}$, where $\mu_B : U_1 \rightarrow [0, 1]$ and $\gamma_B : U_1 \rightarrow [0, 1]$ satisfying $0 \leq \mu_B(x) + \gamma_B(x) \leq 1$ for all $x \in U_1$. The values $\mu_B(x)$ and $\gamma_B(x)$ are called membership degree and nonmembership degree of $x \in U_1$ to B , respectively. The number $\pi_B(x) = 1 - \mu_B(x) - \gamma_B(x)$ is called the hesitancy degree of $x \in U_1$ to B . The collection of all IFS in U_1 is denoted by $\text{IF}(U_1)$. In the remaining paper, we shall write an IFS by $B = \langle \mu_B, \gamma_B \rangle$ instead of $B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle : x \in U_1\}$. Let $B = \langle \mu_B, \gamma_B \rangle$ and $B_1 = \langle \mu_{B_1}, \gamma_{B_1} \rangle$ be two IFSs in U_1 . Then, $B \subseteq B_1$ if and only if $\mu_B(x) \leq \mu_{B_1}(x)$ and $\gamma_{B_1}(x) \leq \gamma_B(x)$ for all $x \in U_1$. Two IFSs B and B_1 are said to be equal if and only if $B \subseteq B_1$ and $B_1 \subseteq B$. The union and intersection of the two IFSs B and B_1 in U_1 are denoted and defined by $B \cup B_1 = \langle \mu_B \cup \mu_{B_1}, \gamma_B \cap \gamma_{B_1} \rangle$ and $B \cap B_1 = \langle \mu_B \cap \mu_{B_1}, \gamma_B \cup \gamma_{B_1} \rangle$, where $(\mu_B \cup \mu_{B_1})(x) = \sup \{\mu_B(x), \mu_{B_1}(x)\}$, $(\gamma_B \cap \gamma_{B_1})(x) = \inf \{\gamma_B(x), \gamma_{B_1}(x)\}$,

TABLE 1: List of acronyms.

Acronyms	Representations
FSs	Fuzzy sets
IFSs	Intuitionistic fuzzy sets
RSs	Rough sets
SSs	Soft sets
FSSs	Fuzzy soft sets
IFSSs	Intuitionistic fuzzy soft sets
IFRSs	Rough intuitionistic fuzzy sets
MGRS	Multigranulation rough set
OMGRS	Optimistic multigranulation rough set
PMGRS	Pessimistic multigranulation rough set
RIFs	Rough intuitionistic fuzzy subsemigroups

$$(\mu_B \cap \mu_{B_1})(x) = \inf \{\mu_B(x), \mu_{B_1}(x)\}, \text{ and } (\gamma_B \cup \gamma_{B_1})(x) = \sup \{\gamma_B(x), \gamma_{B_1}(x)\}.$$

Next, we define two special types of IFSs. The IF universe set $U = 1_U = \langle 1, 0 \rangle$ and IF empty set $\emptyset = 0_U = \langle 0, 1 \rangle$, where $1(x) = 1$ and $0(x) = 0$ for all $x \in U$. The complement of an IFS $A = \langle \mu, \gamma \rangle$ is denoted and defined as $A^c = \langle \gamma, \mu \rangle$.

Definition 4 (see [21]). A pair (σ, A) is said to be a soft set (SS) over U if $\sigma : A \rightarrow P(U)$, where $A \subseteq E$, E is the set of parameters and the set U has power set $P(U)$. Thus, $\sigma(e)$ is a subset of U for all $e \in A$. Hence, a SS over U is a parametrized collection of subsets of U .

Definition 5 (see [26, 55]). A pair (σ, A) is called an intuitionistic fuzzy soft set (IFSS) over U if $\sigma : A \rightarrow \text{IF}(U)$ and $A \subseteq E$ where E is the set of parameters. Thus, $\sigma(e)$ is an IFS in U for all $e \in A$. Hence, an IFSS over U is a parametrized collection of IFSs in U .

If (σ, A) and (ρ, B) are two IFSSs over U , we say that (σ, A) is an IF soft subset of (ρ, B) if (1) $A \subseteq B$ and (2) $\sigma(e)$ is an IF subset of $\rho(e)$ for all $e \in A$. If (σ, A) and (ρ, B) over U are IFSSs, then they called IF soft equal if (σ, A) is an IF soft subset of (ρ, B) and (ρ, B) is an IF soft subset of (σ, A) . The union of two IFSSs (σ, A) and (ρ, A) over the common universe U is the IFSS (λ, A) , where $\lambda(e) = \sigma(e) \cup \rho(e)$ for all $e \in A$. Over the common universe U , the intersection of two IFSSs (σ, A) and (ρ, A) is the IFSS (π, A) , where $\pi(e) = \sigma(e) \cap \rho(e)$ for all $e \in A$.

Definition 6 (see [21]). An IFS $B = \langle \mu_B, \gamma_B \rangle$ in U is called an IF subsemigroup of U if it satisfies the following:

- (1) $\mu_B(xy) \geq \mu_B(x) \wedge \mu_B(y)$
- (2) $\gamma_B(xy) \leq \gamma_B(x) \vee \gamma_B(y)$ for all $x, y \in U$

An IFS $B = \langle \mu_B, \gamma_B \rangle$ in a semigroup U is called an IF left (resp., right) ideal of U if it satisfies $\mu_B(xy) \geq \mu_B(y)$ (resp. $\mu_B(y)$)

$(xy) \geq \mu_B(x)$ and $\gamma_B(xy) \leq \gamma_B(y)$ (resp. $\gamma_B(xy) \leq \gamma_B(x)$) for all $x, y \in U$. An IF left ideal and IF right ideal are called fuzzy ideals. An IF subsemigroup $B = \langle \mu_B, \gamma_B \rangle$ in U is called an IF interior ideal of U if it satisfies $\mu_B(xay) \geq \mu_B(a)$ and $\gamma_B(xay) \leq \gamma_B(a)$ for all $x, y, a \in U$.

An IF subsemigroup $B = \langle \mu_B, \gamma_B \rangle$ in U is called an IF bi-ideal of U if it satisfies the following:

- (1) $\mu_B(xwy) \geq \mu_B(x) \wedge \mu_B(y)$
- (2) $\gamma_B(xwy) \leq \gamma_B(x) \vee \gamma_B(y)$ for all $x, y, w \in U$ [2, 26, 56–58]

Definition 7 (see [59]). An IFSS (σ, A) over U is called IF soft subsemigroup (left ideal, right ideal, interior ideal, and bi-ideal) over U if each $\sigma(e)$ is IF subsemigroup (left ideal, right ideal, interior ideal, and bi-ideal) of U for all $e \in A$.

Definition 8 (see [60]). A soft binary relation (σ, A) from U_1 to U_2 is a SS over $U_1 \times U_2$, that is, $\sigma : A \rightarrow P(U_1 \times U_2)$, where $A \subseteq E$.

Of course, (σ, A) is a parameterized collection of binary relations from U_1 to U_2 ; that is, for each $e \in A$, we have a binary relation $\sigma(e)$ from U_1 to U_2 .

Definition 9 (see [61]). A soft binary relation (σ, A) from a semigroup U_1 to a semigroup U_2 is said to be soft compatible if $(p, q), (r, s) \in \sigma(e)$ implies $(pr, qs) \in \sigma(e)$ for all $p, r \in U_1$ and $q, s \in U_2$.

In general, if (σ, A) is a soft compatible relation from U_1 to U_2 , then $p\sigma(e).q\sigma(e) \subseteq (pq)\sigma(e)$; indeed if $a \in p\sigma(e)$ and $b \in q\sigma(e)$, then $(p, a) \in \sigma(e)$ and $(q, b) \in \sigma(e)$. By compatibility of (σ, A) , $(pq, ab) \in \sigma(e)$; that is, $ab \in (pq)\sigma(e)$. Similarly, $\sigma(e)p.\sigma(e)q \subseteq \sigma(e)(pq)$. The following example shows that in general, $p\sigma(e).q\sigma(e) \neq (pq)\sigma(e)$ and $\sigma(e)p.\sigma(e)q \neq \sigma(e)(pq)$.

Example 1. Let $U_1 = \{p, q, r, s\}$ and $U_2 = \{1, 2, 3, 4\}$ be two semigroups, and their multiplication tables are as shown in Tables 2 and 3, respectively).

Let $A = \{e_1, e_2\}$. Define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\begin{aligned} \sigma(e_1) &= \{(p, 1), (q, 2), (r, 3), (s, 4), (p, 2), (p, 3), (q, 4), (p, 4), (q, 1), (s, 1), (s, 2), (s, 3)\}, \\ \sigma(e_2) &= \{(p, 2), (p, 3), (q, 2), (q, 3), (r, 2), (r, 3), (s, 2), (s, 3)\}. \end{aligned} \quad (1)$$

Then, (σ, A) is a soft compatible relation from U_1 to U_2 .

$$\begin{aligned} p\sigma(e_1) &= \{1, 2, 3, 4\}, \\ q\sigma(e_1) &= \{1, 2, 4\}, \\ r\sigma(e_1) &= \{3\}, \\ s\sigma(e_1) &= \{1, 2, 3, 4\} \\ p\sigma(e_2) &= \{2, 3\}, \\ q\sigma(e_2) &= \{2, 3\}, \\ r\sigma(e_2) &= \{2, 3\}, \\ s\sigma(e_2) &= \{2, 3\}. \end{aligned} \quad (2)$$

TABLE 2: Multiplication table for U_1 .

\cdot	p	q	r	s
p	p	p	p	s
q	p	q	p	s
r	p	p	r	s
s	s	s	s	s

TABLE 3: Multiplication table for U_2 .

\cdot	1	2	3	4
1	1	2	3	4
2	2	2	2	2
3	3	3	3	3
4	4	3	2	1

But

$$p\sigma(e_1).r\sigma(e_1) = \{2, 3\} \neq \{1, 2, 3, 4\} = (pr)\sigma(e_1). \quad (3)$$

Now,

$$\begin{aligned} \sigma(e_1)1 &= \{p, q, s\}, \\ \sigma(e_1)2 &= \{p, q, s\}, \\ \sigma(e_1)3 &= \{p, r, s\}, \\ \sigma(e_1)4 &= \{p, q, s\} \\ \sigma(e_2)1 &= \emptyset, \\ \sigma(e_2)2 &= \{p, q, r, s\}, \\ \sigma(e_2)3 &= \{p, q, r, s\}, \\ s\sigma(e_2) &= \emptyset. \end{aligned} \quad (4)$$

But

$$\sigma(e_1)3.\sigma(e_1)1 = \emptyset \neq \{p, q, r, s\} = \sigma(e_1)(31). \quad (5)$$

Definition 10 (see [61]). A soft compatible relation (σ, A) from a semigroup U_1 to a semigroup U_2 is said to be soft complete regarding aftersets if $p\sigma(e).q\sigma(e) = (pq)\sigma(e)$ for all $p, q \in U_1$ and $e \in A$ and is said to be soft complete relation regarding foresets if $\sigma(e)r.\sigma(e)s = \sigma(e)(rs)$ for all $r, s \in U_2$ and $e \in A$.

Definition 11 (see [37]). Let (σ, A) be a soft binary relation from U_1 to U_2 and $B = \langle \mu_B, \gamma_B \rangle$ be an IFS in U_2 . Then, the lower approximation $\underline{\sigma}^B = (\underline{\sigma}^{\mu_B}, \underline{\sigma}^{\gamma_B})$ and the upper approximation $\bar{\sigma}^B = (\bar{\sigma}^{\mu_B}, \bar{\sigma}^{\gamma_B})$ of $B = \langle \mu_B, \gamma_B \rangle$ are IFSSs over U_1 and defined as

$$\begin{aligned}
\underline{\sigma}^{\mu_B}(e)(u_1) &= \begin{cases} \bigwedge_{a \in u_1 \sigma(e)} \mu_B(a), & \text{if } u_1 \sigma(e) \neq \emptyset, \\ 1, & \text{if } u_1 \sigma(e) = \emptyset, \end{cases} \\
\underline{\sigma}^{\gamma_B}(e)(u_1) &= \begin{cases} \bigvee_{a \in u_1 \sigma(e)} \gamma_B(a), & \text{if } u_1 \sigma(e) \neq \emptyset, \\ 0, & \text{if } u_1 \sigma(e) = \emptyset, \end{cases} \\
\bar{\sigma}^{\mu_B}(e)(u_1) &= \begin{cases} \bigvee_{a \in u_1 \sigma(e)} \mu_B(a), & \text{if } u_1 \sigma(e) \neq \emptyset, \\ 0, & \text{if } u_1 \sigma(e) = \emptyset, \end{cases} \\
\bar{\sigma}^{\gamma_B}(e)(u_1) &= \begin{cases} \bigwedge_{a \in u_1 \sigma(e)} \gamma_B(a), & \text{if } u_1 \sigma(e) \neq \emptyset, \\ 1, & \text{if } u_1 \sigma(e) = \emptyset, \end{cases}
\end{aligned} \tag{6}$$

for all $u_1 \in U_1$, where $u_1 \sigma(e) = \{a \in U_2 : (u_1, a) \in \sigma(e)\}$ and is called the afterset of u_1 for $u_1 \in U_1$ and $e \in A$.

Definition 12 (see [37]). Let (σ, A) be a soft binary relation from U_1 to U_2 and $B = \langle \mu_B, \gamma_B \rangle$ be an IFS in U_1 . Then, the lower approximation ${}^B \underline{\sigma} = (\mu_B \underline{\sigma}, \gamma_B \underline{\sigma})$ and the upper approximation ${}^B \bar{\sigma} = (\mu_B \bar{\sigma}, \gamma_B \bar{\sigma})$ of $B = \langle \mu_B, \gamma_B \rangle$ are IFSSs over U_2 and defined as

$$\begin{aligned}
\mu_B \underline{\sigma}(e)(u_2) &= \begin{cases} \bigwedge_{a \in \sigma(e)u_2} \mu_B(a), & \text{if } \sigma(e)u_2 \neq \emptyset, \\ 1, & \text{if } \sigma(e)u_2 = \emptyset, \end{cases} \\
\gamma_B \underline{\sigma}(e)(u_2) &= \begin{cases} \bigvee_{a \in \sigma(e)u_2} \gamma_B(a), & \text{if } \sigma(e)u_2 \neq \emptyset, \\ 0, & \text{if } \sigma(e)u_2 = \emptyset, \end{cases} \\
\mu_B \bar{\sigma}(e)(u_2) &= \begin{cases} \bigvee_{a \in \sigma(e)u_2} \mu_B(a), & \text{if } \sigma(e)u_2 \neq \emptyset, \\ 0, & \text{if } \sigma(e)u_2 = \emptyset, \end{cases} \\
\gamma_B \bar{\sigma}(e)(u_2) &= \begin{cases} \bigwedge_{a \in \sigma(e)u_2} \gamma_B(a), & \text{if } \sigma(e)u_2 \neq \emptyset, \\ 1, & \text{if } \sigma(e)u_2 = \emptyset, \end{cases}
\end{aligned} \tag{7}$$

for all $u_2 \in U_2$ where $\sigma(e)u_2 = \{a \in U_1 : (a, u_2) \in \sigma(e)\}$ and is called the foreset of u_2 for $u_2 \in U_2$ and $e \in A$.

Of course, $\underline{\sigma}^B : A \rightarrow \text{IF}(U_1)$, $\bar{\sigma}^B : A \rightarrow \text{IF}(U_1)$ and ${}^B \underline{\sigma} : A \rightarrow \text{IF}(U_2)$, ${}^B \bar{\sigma} : A \rightarrow \text{IF}(U_2)$.

Theorem 13 (see [37]). Let (σ, A) be a soft binary relation from U_1 to U_2 that is $\sigma : A \rightarrow P(U_1 \times U_2)$. For any IFSs $D = \langle \mu_D, \gamma_D \rangle$, $D_1 = \langle \mu_{D_1}, \gamma_{D_1} \rangle$ and $D_2 = \langle \mu_{D_2}, \gamma_{D_2} \rangle$ of U_2 , the following hold.

- (1) $D_1 \leq D_2$ implies $\underline{\sigma}^{D_1} \leq \underline{\sigma}^{D_2}$
- (2) $D_1 \leq D_2$ implies $\bar{\sigma}^{D_1} \leq \bar{\sigma}^{D_2}$
- (3) $\underline{\sigma}^{D_1} \cap \underline{\sigma}^{D_2} = \underline{\sigma}^{D_1 \cap D_2}$
- (4) $\bar{\sigma}^{D_1} \cap \bar{\sigma}^{D_2} \supseteq \bar{\sigma}^{D_1 \cap D_2}$
- (5) $\underline{\sigma}^{D_1} \cup \underline{\sigma}^{D_2} \subseteq \underline{\sigma}^{D_1 \cup D_2}$
- (6) $\bar{\sigma}^{D_1} \cup \bar{\sigma}^{D_2} = \bar{\sigma}^{D_1 \cup D_2}$
- (7) $\underline{\sigma}^{1_{U_2}} = 1_{U_1}$ if $u\sigma(e) \neq \emptyset$
- (8) $\bar{\sigma}^{1_{U_2}} = 1_{U_1}$ if $u\sigma(e) \neq \emptyset$

$$(9) \underline{\sigma}^D = (\bar{\sigma}^{D^c})^C \text{ if } u\sigma(e) \neq \emptyset$$

$$(10) \bar{\sigma}^D = (\underline{\sigma}^{D^c})^C \text{ if } u\sigma(e) \neq \emptyset$$

$$(11) \underline{\sigma}^{0_{U_2}} = 0_{U_1} = \bar{\sigma}^{0_w} \text{ if } u\sigma(e) \neq \emptyset$$

Theorem 14 (see [37]). Let (σ, A) be a soft binary relation from U_1 to U_2 ; that is, $\sigma : A \rightarrow P(U_1 \times U_2)$. For any IFSs $D = \langle \mu_D, \gamma_D \rangle$, $D_1 = \langle \mu_{D_1}, \gamma_{D_1} \rangle$ and $D_2 = \langle \mu_{D_2}, \gamma_{D_2} \rangle$ of U_1 , the following are true.

- (1) If $D_1 \leq D_2^{D_1}$ implies $\underline{\sigma} \leq^{D_2} \underline{\sigma}$
- (2) If $D_1 \leq D_2$ implies ${}^{D_1} \bar{\sigma} \leq^{D_2} \bar{\sigma}$
- (3) ${}^{D_1} \underline{\sigma} \cap {}^{D_2} \underline{\sigma} = {}^{D_1 \cap D_2} \underline{\sigma}$
- (4) ${}^{D_1} \bar{\sigma} \cap {}^{D_2} \bar{\sigma} \supseteq {}^{D_1 \cap D_2} \bar{\sigma}$
- (5) ${}^{D_1} \underline{\sigma} \cup {}^{D_2} \underline{\sigma} \subseteq {}^{D_1 \cup D_2} \underline{\sigma}$
- (6) ${}^{D_1} \bar{\sigma} \cup {}^{D_2} \bar{\sigma} = {}^{D_1 \cup D_2} \bar{\sigma}$
- (7) $\underline{\sigma}^{1_{U_1}} = 1_{U_2}$ if $u\sigma(e) \neq \emptyset$
- (8) $\bar{\sigma}^{1_{U_1}} = 1_{U_2}$ if $u\sigma(e) \neq \emptyset$
- (9) ${}^D \underline{\sigma} = ({}^{D^c} \bar{\sigma})^C$ if $u\sigma(e) \neq \emptyset$
- (10) ${}^D \bar{\sigma} = ({}^{D^c} \underline{\sigma})^C$ if $u\sigma(e) \neq \emptyset$
- (11) ${}^{0_{U_1}} \underline{\sigma} = 0_{U_2} = {}^{0_{U_1}} \bar{\sigma}$

3. Approximations of IF Ideals in Semigroups by Soft Binary Relation

This is our major section of the paper. Our related work is narrated in this section. Here, we discuss the rough approximations of IF subsemigroup (IF left (right) ideal, IF interior ideal, and IF bi-ideal) in a semigroup regarding aftersets as well as regarding foresets by using soft compatible relation. We show that upper approximation of an IF subsemigroup (IF left (right), IF interior ideal, and IF bi-ideal) in a semigroup is an IF soft subsemigroup (IF soft left (right) ideal, IF soft interior ideal, and IF soft bi-ideal) and discuss examples which shows that its converse is not true. Similar results for lower approximation are also proved.

Theorem 15. Let (σ, A) be a soft compatible relation from a semigroup U_1 to a semigroup U_2 .

- (1) If B_2 is an IF subsemigroup of U_2 , then $(\bar{\sigma}^{B_2}, A)$ is an IF soft subsemigroup of U_1
- (2) If B_2 is an IF left (right, two-sided) ideal of U_2 , then $(\bar{\sigma}^{B_2}, A)$ is an IF soft left (right, two-sided) ideal of U_1

Proof.

- (1) We assume that B_2 is an IF subsemigroup of U_2 .
Now for $u, v \in U_1$,

$$\begin{aligned}\bar{\sigma}^{\mu_{B_2}}(e)(u) \wedge \bar{\sigma}^{\mu_{B_2}}(e)(v) &= \left(\bigvee_{x \in u\sigma(e)} \mu_{B_2}(x) \right) \wedge \left(\bigvee_{y \in v\sigma(e)} \mu_{B_2}(y) \right) \\ &= \bigvee_{x \in u\sigma(e)} \bigvee_{y \in v\sigma(e)} \left(\mu_{B_2}(x) \wedge \mu_{B_2}(y) \right) \leq \bigvee_{x \in u\sigma(e)} \bigvee_{y \in v\sigma(e)} \left(\mu_{B_2}(xy) \right) \\ &\leq \bigvee_{xy \in (uv)\sigma(e)} \left(\mu_{B_2}(xy) \right) = \bigvee_{a' \in (uv)\sigma(e)} \left(\mu_{B_2}(a') \right) = \bar{\sigma}^{\mu_{B_2}}(e)(uv).\end{aligned}\quad (8)$$

Similarly for $s, t \in U_1$,

$$\begin{aligned}\bar{\sigma}^{\gamma_{B_2}}(e)(s) \vee \bar{\sigma}^{\gamma_{B_2}}(e)(t) &= \left(\bigwedge_{a \in s\sigma(e)} \gamma_{B_2}(a) \right) \vee \left(\bigwedge_{b \in t\sigma(e)} \gamma_{B_2}(b) \right) \\ &= \bigwedge_{a \in s\sigma(e)} \bigwedge_{b \in t\sigma(e)} \left(\gamma_{B_2}(a) \vee \gamma_{B_2}(b) \right) \\ &\geq \bigwedge_{a \in s\sigma(e)} \bigwedge_{b \in t\sigma(e)} \left(\gamma_{B_2}(ab) \right) \geq \bigwedge_{ab \in (st)\sigma(e)} \left(\gamma_{B_2}(ab) \right) \\ &= \bigwedge_{b' \in (st)\sigma(e)} \left(\gamma_{B_2}(b') \right) = \bar{\sigma}^{\gamma_{B_2}}(e)(st).\end{aligned}\quad (9)$$

Hence, $\bar{\sigma}^{B_2}(e)$ is an IF subsemigroup of U_1 for all $e \in A$, so $(\bar{\sigma}^{B_2}, A)$ is an IF soft subsemigroup of U_1 .

- (2) Assume that B_2 is an IF left ideal of U_2 . Now for $x, y \in U_1$,

$$\begin{aligned}\bar{\sigma}^{\mu_{B_2}}(e)(y) &= \bigvee_{q \in y\sigma(e)} \mu_{B_2}(q) \leq \bigvee_{p \in x\sigma(e)} \bigvee_{q \in y\sigma(e)} \mu_{B_2}(pq) \\ &\leq \bigvee_{pq \in (xy)\sigma(e)} \mu_{B_2}(pq) = \bigvee_{a' \in (xy)\sigma(e)} \mu_{B_2}(a') = \bar{\sigma}^{\mu_{B_2}}(e)(xy).\end{aligned}\quad (10)$$

Similarly for $x, y \in U_1$,

$$\begin{aligned}\bar{\sigma}^{\gamma_{B_2}}(e)(y) &= \bigwedge_{q \in y\sigma(e)} \gamma_{B_2}(q) \geq \bigwedge_{p \in x\sigma(e)} \bigwedge_{q \in y\sigma(e)} \left(\gamma_{B_2}(pq) \right) \\ &\geq \bigwedge_{pq \in (xy)\sigma(e)} \left(\gamma_{B_2}(pq) \right) = \bigwedge_{b' \in (xy)\sigma(e)} \left(\gamma_{B_2}(b') \right) = \bar{\sigma}^{\gamma_{B_2}}(e)(xy).\end{aligned}\quad (11)$$

Hence, $\bar{\sigma}^{B_2}(e)$ is an IF left ideal of U_1 for each $e \in A$, so $(\bar{\sigma}^{B_2}, A)$ is an IF soft left ideal of U_1 . \square

In Theorem 15 from part 1, soft compatible relations from U_1 to U_2 are given, and B_2 is an IF subsemigroup in U_2 . After combining them, we get generalized IF soft subsemigroups in U_1 . Similarly, we take an IF left (right, two-sided) ideal B_2 of U_2 , and we get generalized IF soft left (right, two-sided) ideal of U_1 .

Theorem 16. Let (σ, A) be a soft compatible relation from a semigroup U_1 to a semigroup U_2 :

- (1) If B_1 is an IF subsemigroup of U_1 , then $({}^{B_1}\bar{\sigma}, A)$ is an IF soft subsemigroup of U_2
- (2) If B_1 is an IF left (right, two-sided) ideal of U_1 , then $({}^{B_1}\bar{\sigma}, A)$ is an IF soft left (right, two-sided) ideal of U_2

Proof. It follows from Theorem 15. \square

In Theorem 16 from part 1, soft compatible relations from U_1 to U_2 are given, and B_1 is an IF subsemigroup in U_1 . After combining them, we get generalized IF soft subsemigroups in U_2 . Similarly, we take an IF left (right, two-sided) ideal B_1 of U_1 , and we get generalized IF soft left (right, two-sided) ideal of U_2 .

Now, we show that the converses of parts of the above theorem do not hold in general.

Example 2. Let $U_1 = \{p, q, r, s, t\}$ and $U_2 = \{1, 2, 3, 4, 5\}$ be two semigroups, and their multiplication tables are as follows in Tables 4 and 5, respectively.

Let $A = \{e_1, e_2\}$. Define $\sigma : A \longrightarrow P(U_1 \times U_2)$ by

$$\begin{aligned}\sigma(e_1) &= \left\{ \begin{array}{l} (p, 1), (q, 2), (r, 3), (s, 4), (t, 5), \\ (q, 1), (r, 5), (q, 5), (s, 3), (s, 5), (s, 1), \end{array} \right\} \\ \sigma(e_2) &= \left\{ \begin{array}{l} (p, 1), (q, 2), (r, 3), (s, 4), (t, 5), \\ (q, 1), (r, 5), (q, 5), (s, 3), (s, 5), (s, 1), (q, 3). \end{array} \right\}\end{aligned}\quad (12)$$

Then, (σ, A) is a soft compatible relation from U_1 to U_2 .

$$\begin{aligned}p\sigma(e_1) &= \{1\}, \\ q\sigma(e_1) &= \{1, 2, 5\}, \\ r\sigma(e_1) &= \{3, 5\}, \\ s\sigma(e_1) &= \{1, 3, 4, 5\}, \\ t\sigma(e_1) &= \{5\} \\ p\sigma(e_2) &= \{1\}, \\ q\sigma(e_2) &= \{1, 2, 3, 5\}, \\ r\sigma(e_2) &= \{3, 5\}, \\ s\sigma(e_2) &= \{1, 3, 4, 5\}, \\ t\sigma(e_2) &= \{5\}, \\ \sigma(e_1)1 &= \{p, q, s\}, \\ \sigma(e_1)2 &= \{q\}, \\ \sigma(e_1)3 &= \{r, s\}, \\ \sigma(e_1)4 &= \{s\}, \\ \sigma(e_1)5 &= \{q, r, s, t\} \\ \sigma(e_2)1 &= \{p, q, s\}, \\ \sigma(e_2)2 &= \{q\}, \\ \sigma(e_2)3 &= \{q, r, s\}, \\ \sigma(e_2)4 &= \{s\}, \\ \sigma(e_2)5 &= \{q, r, s, t\}.\end{aligned}\quad (13)$$

- (1) Define $B_1 : U_2 \longrightarrow [0, 1]$ (given in Table 6)

TABLE 4: Multiplication table for U_1 .

.	p	q	r	s	t
p	q	q	s	s	s
q	q	q	s	s	s
r	s	s	r	s	r
s	s	s	s	s	s
t	s	s	r	s	r

TABLE 5: Multiplication table for U_2 .

.	1	2	3	4	5
1	1	5	3	4	5
2	1	2	3	4	5
3	1	5	3	4	5
4	1	5	3	4	5
5	1	5	3	4	5

TABLE 6: Intuitionistic fuzzy set B_1 .

B_1	1	2	3	4	5
μ_{B_1}	0.5	0.4	0.3	1	0.1
γ_{B_1}	0.4	0.5	0.7	0	0.8

Then, B_1 is not an IF subsemigroup of U_2 because if we take $x = 1, y = 2$, then $\mu_{B_1}(12) \not\geq \mu_{B_1}(1) \wedge \mu_{B_1}(2)$ and $\gamma_{B_1}(12) \not\leq \gamma_{B_1}(1) \vee \gamma_{B_1}(2)$. Upper approximation of B_1 is given in Table 7.

Clearly, $\bar{\sigma}^{B_1}(e_1)$ and $\bar{\sigma}^{B_1}(e_2)$ are IF subsemigroups of U_1 , so $(\bar{\sigma}^{B_1}, A)$ is an IF soft subsemigroup of U_1 .

(2) Define $B_2 : U_1 \rightarrow [0, 1]$ (given in Table 8)

Then, B_2 is not an IF subsemigroup of U_1 because if we take $x = t, y = t$, then $\mu_{B_2}(tt) \not\geq \mu_{B_2}(t) \wedge \mu_{B_2}(t)$ and $\gamma_{B_2}(tt) \not\leq \gamma_{B_2}(t) \vee \gamma_{B_2}(t)$. Upper approximation of B_2 is given in Table 9. Clearly, ${}^{B_2}\bar{\sigma}(e_1)$ and ${}^{B_2}\bar{\sigma}(e_2)$ are IF subsemigroups of U_2 , so $({}^{B_2}\bar{\sigma}, A)$ is an IF soft subsemigroup of U_2 .

(3) Define $B : U_2 \rightarrow [0, 1]$ (given in Table 10)

Then, B is not an IF left ideal of U_2 because if we take $x = 1, y = 2$, then $\mu_B(12) \not\geq \mu_B(2)$ and $\gamma_B(12) \not\leq \gamma_B(2)$. Upper approximation of B is given in Table 11. Clearly, $\bar{\sigma}^B(e_1)$ and $\bar{\sigma}^B(e_2)$ are IF left ideals of U_1 , so $(\bar{\sigma}^B, A)$ is an IF soft left ideal of U_1 .

(4) Define $H : U_1 \rightarrow [0, 1]$ (given in Table 12)

Then, H is not an IF left ideal of U_1 because if we take $x = t, y = t$, then $\mu_H(tt) \not\geq \mu_H(t)$ and $\gamma_H(tt) \not\leq \gamma_H(t)$. Upper approximation of H is given in Table 13.

TABLE 7: Upper approximation of B_1 .

.	p	q	r	s	t
$\bar{\sigma}^{\mu_{B_1}}(e_1)$	0.5	0.5	0.3	1	0.1
$\bar{\sigma}^{\gamma_{B_1}}(e_1)$	0.4	0.4	0.7	0	0.8
$\bar{\sigma}^{\mu_{B_1}}(e_2)$	0.5	0.5	0.3	1	0.1
$\bar{\sigma}^{\gamma_{B_1}}(e_2)$	0.4	0.4	0.7	0	0.8

TABLE 8: Intuitionistic fuzzy set B_2 .

B_2	p	q	r	s	t
μ_{B_2}	0.2	0.7	0.8	0	0.9
γ_{B_2}	0.7	0.3	0.2	1	0.1

Clearly, ${}^H\bar{\sigma}(e_1)$ and ${}^H\bar{\sigma}(e_2)$ are IF left ideals of U_2 , so $({}^H\bar{\sigma}, A)$ is an IF soft left ideal of U_2 .

Example 3. Consider the semigroups and soft binary relation of Example 2. Define $B : U_2 \rightarrow [0, 1]$ (given in Table 14).

Then, B is an IF left ideal of U_2 . Lower approximation of B is given in Table 15.

But $\underline{\sigma}^B(e_1)$ is not an IF left ideal of U_1 because if we take $x = r, y = t$, then $\mu_B(rt) \not\geq \mu_B(t)$ and $\gamma_B(rt) \not\leq \gamma_B(t)$.

This example shows that if the soft relation is compatible, then lower approximation of an IF left ideal is not IF soft left ideal. However, the following result is true.

Theorem 17. Let (σ, A) be a soft complete relation regarding aftersets from a semigroup U_1 to a semigroup U_2 .

- (1) If B is an IF subsemigroup of U_2 , then $(\underline{\sigma}^B, A)$ is an IF soft subsemigroup of U_1
- (2) If B is an IF left (right, two-sided) ideal of U_2 , then $(\underline{\sigma}^B, A)$ is an IF soft left (right, two-sided) ideal of U_1

Proof.

- (1) Assume that B is an IF subsemigroup of U_2 . Now for $x, y \in U_1$,

$$\begin{aligned} \underline{\sigma}^{\mu_B}(e)(xy) &= \bigwedge_{a' \in (xy)\sigma(e)} \mu_B(a') = \bigwedge_{a' \in x\sigma(e).y\sigma(e)} \mu_B(a') \\ &= \bigwedge_{p \in x\sigma(e).q \in y\sigma(e)} \mu_B(pq) \geq \bigwedge_{p \in x\sigma(e)} \bigwedge_{q \in y\sigma(e)} (\mu_B(p) \wedge \mu_B(q)) \\ &\geq \left(\bigwedge_{p \in x\sigma(e)} \mu_B(p) \right) \wedge \left(\bigwedge_{q \in y\sigma(e)} \mu_B(q) \right) = \underline{\sigma}^{\mu_B}(e)(x) \wedge \underline{\sigma}^{\mu_B}(e)(y). \end{aligned} \tag{14}$$

TABLE 9: Upper approximation of B_2 .

	1	2	3	4	5
$\mu_{B_2} \bar{\sigma}(e_1)$	0.7	0.7	0.8	0	0.9
$\gamma_{B_2} \bar{\sigma}(e_1)$	0.3	0.3	0.2	1	0.1
$\mu_{B_2} \bar{\sigma}(e_2)$	0.7	0.7	0.8	0	0.9
$\gamma_{B_2} \bar{\sigma}(e_2)$	0.3	0.3	0.2	1	0.1

TABLE 10: Intuitionistic fuzzy set B .

B	1	2	3	4	5
μ_B	0.5	0.4	0.3	1	0.1
γ_B	0.4	0.6	0.6	0	0.8

TABLE 11: Upper approximation of B .

	p	q	r	s	t
$\bar{\sigma}^{\mu_B}(e_1)$	0.5	0.5	0.3	1	0.1
$\bar{\sigma}^{\gamma_B}(e_1)$	0.4	0.4	0.6	0	0.8
$\bar{\sigma}^{\mu_B}(e_2)$	0.5	0.5	0.3	1	0.1
$\bar{\sigma}^{\gamma_B}(e_2)$	0.4	0.4	0.6	0	0.8

TABLE 12: Intuitionistic fuzzy set H .

H	p	q	r	s	t
μ_H	0.2	0.7	0.8	0	0.9
γ_H	0.7	0.2	0.1	1	0.1

TABLE 13: Upper approximation of H .

	1	2	3	4	5
$\mu_H \bar{\sigma}(e_1)$	0.7	0.7	0.8	0	0.9
$\gamma_H \bar{\sigma}(e_1)$	0.2	0.2	0.1	1	0.1
$\mu_H \bar{\sigma}(e_2)$	0.7	0.7	0.8	0	0.9
$\gamma_H \bar{\sigma}(e_2)$	0.2	0.2	0.1	1	0.1

Similarly, for $x, y \in U_1$,

$$\begin{aligned} \underline{\sigma}^{\gamma_B}(e)(xy) &= \bigvee_{a' \in (xy)\sigma(e)} \gamma_B(a') = \bigvee_{a' \in x\sigma(e).y\sigma(e)} \gamma_B(a') \\ &= \bigvee_{p \in x\sigma(e).q \in y\sigma(e)} \gamma_B(pq) \leq \bigvee_{p \in x\sigma(e)} \bigvee_{q \in y\sigma(e)} (\gamma_B(p) \vee \gamma_B(q)) \\ &\leq \left(\bigvee_{p \in x\sigma(e)} \gamma_B(p) \right) \vee \left(\bigvee_{q \in y\sigma(e)} \gamma_B(q) \right) = \underline{\sigma}^{\gamma_B}(e)(x) \vee \underline{\sigma}^{\gamma_B}(e)(y). \end{aligned} \quad (15)$$

Hence, $\underline{\sigma}^B(e)$ is an IF subsemigroup of U_1 for each $e \in A$, so $(\underline{\sigma}^B, A)$ is an IF soft subsemigroup of U_1 .

TABLE 14: Intuitionistic fuzzy set B .

B	1	2	3	4	5
μ_B	0.7	0.7	0.8	0	0.9
γ_B	0.2	0.3	0.1	1	0.1

TABLE 15: Lower approximation of B .

	p	q	r	s	t
$\underline{\sigma}^{\mu_B}(e_1)$	0.7	0.7	0.8	0	0.9
$\underline{\sigma}^{\gamma_B}(e_1)$	0.2	0.3	0.9	1	0.1

(2) Suppose B is an IF left ideal of U_2 . Now for $x, y \in U_1$,

$$\begin{aligned} \underline{\sigma}^{\mu_B}(e)(xy) &= \bigwedge_{a' \in (xy)\sigma(e)} \mu_B(a') = \bigwedge_{a' \in x\sigma(e).y\sigma(e)} \mu_B(a') \\ &= \bigwedge_{p \in x\sigma(e).q \in y\sigma(e)} \mu_B(pq) \geq \bigwedge_{q \in y\sigma(e)} \mu_B(q) = \underline{\sigma}^{\mu_B}(e)(y). \end{aligned} \quad (16)$$

Similarly, for $x, y \in U_1$,

$$\begin{aligned} \underline{\sigma}^{\gamma_B}(e)(xy) &= \bigvee_{a' \in (xy)\sigma(e)} \gamma_B(a') = \bigvee_{a' \in x\sigma(e).y\sigma(e)} \gamma_B(a') \\ &= \bigvee_{p \in x\sigma(e).q \in y\sigma(e)} \gamma_B(pq) \leq \bigvee_{q \in y\sigma(e)} \gamma_B(q) = \underline{\sigma}^{\gamma_B}(e)(y). \end{aligned} \quad (17)$$

Hence, $\underline{\sigma}^B(e)$ is an IF left ideal of U_1 for all $e \in A$, so $(\underline{\sigma}^B, A)$ is an IF soft left ideal of U_1 . \square

Theorem 18. Suppose (σ, A) is a soft complete relation regarding foresets from a semigroup U_1 to a semigroup U_2 . Then, the following are true:

- (1) If B is an IF subsemigroup of U_1 , then $({}^B\underline{\sigma}, A)$ is an IF soft subsemigroup of U_2
- (2) If B is an IF left (right, two-sided) ideal of U_1 , then $({}^B\underline{\sigma}, A)$ is an IF soft left (right, two-sided) ideal of U_2

Proof. It follows from Theorem 17. \square

Example 4. Consider the semigroups of Example 1.

Let $A = \{e_1, e_2\}$. Define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\begin{aligned} \sigma(e_1) &= \{(p, 2), (p, 3), (q, 2), (q, 3), (r, 2), (r, 3), (s, 2), (s, 3)\} \\ \sigma(e_2) &= \{(p, 2), (q, 2), (r, 2), (s, 2)\}. \end{aligned} \quad (18)$$

Then, (σ, A) is a soft complete relation from U_1 to U_2 with respect to the aftersets.

TABLE 16: Intuitionistic fuzzy set B .

B	1	2	3	4
μ_B	0.2	0.4	0.6	0.8
γ_B	0.7	0.5	0.2	0.1

$$\begin{aligned}
 p\sigma(e_1) &= \{2, 3\}, \\
 q\sigma(e_1) &= \{2, 3\}, \\
 r\sigma(e_1) &= \{2, 3\}, \\
 s\sigma(e_1) &= \{2, 3\} \\
 p\sigma(e_2) &= \{2\}, \\
 q\sigma(e_2) &= \{2\}, \\
 r\sigma(e_2) &= \{2\}, \\
 s\sigma(e_2) &= \{2\}.
 \end{aligned}
 \tag{19}$$

(1) Define $B : U_2 \rightarrow [0, 1]$ (given in Table 16)

Then, B is not an IF subsemigroup of U_2 because if we take $x = 4, y = 4$, then $\mu_B(44) \not\leq \mu_B(4) \wedge \mu_B(4)$ and $\gamma_B(44) \not\leq \gamma_B(4) \vee \gamma_B(4)$. Lower approximation of B is given in Table 17.

Clearly, $\underline{\sigma}^B(e_1)$ and $\underline{\sigma}^B(e_2)$ are IF subsemigroups of U_1 , so $(\underline{\sigma}^B, A)$ is an IF soft subsemigroup of U_1 .

(2) Define $B_1 : U_2 \rightarrow [0, 1]$ (given in Table 18)

Then, B_1 is not an IF left ideal of U_2 because if we take $x = 3, y = 4$, then $\mu_{B_1}(34) \not\leq \mu_{B_1}(3)$ and $\gamma_{B_1}(34) \not\leq \gamma_{B_1}(4)$. Lower approximations of B_1 are given in Table 19.

Clearly, $\underline{\sigma}^{B_1}(e_1)$ and $\underline{\sigma}^{B_1}(e_2)$ are IF left ideals of U_1 , so $(\underline{\sigma}^{B_1}, A)$ is an IF soft left ideal of U_1 .

Now define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\begin{aligned}
 \sigma(e_1) &= \{(s, 1), (s, 2), (s, 3), (s, 4)\} \\
 \sigma(e_2) &= \{(p, 1), (p, 2), (p, 3), (p, 4), (s, 1), (s, 2), (s, 3), (s, 4)\}.
 \end{aligned}
 \tag{20}$$

These are soft complete relations from U_1 to U_2 with respect to the foresets.

$$\begin{aligned}
 \sigma(e_1)1 &= \{s\}, \\
 \sigma(e_1)2 &= \{s\}, \\
 \sigma(e_1)3 &= \{s\}, \\
 \sigma(e_1)4 &= \{s\} \\
 \sigma(e_2)1 &= \{p, s\}, \\
 \sigma(e_2)2 &= \{p, s\}, \\
 \sigma(e_2)3 &= \{p, s\}, \\
 \sigma(e_2)4 &= \{p, s\}.
 \end{aligned}
 \tag{21}$$

TABLE 17: Lower approximation of B .

	p	q	r	s
$\underline{\sigma}^{\mu_B}(e_1)$	0.4	0.4	0.4	0.4
$\underline{\sigma}^{\gamma_B}(e_1)$	0.5	0.5	0.5	0.5
$\underline{\sigma}^{\mu_B}(e_2)$	0.4	0.4	0.4	0.4
$\underline{\sigma}^{\gamma_B}(e_2)$	0.5	0.5	0.5	0.5

TABLE 18: Intuitionistic fuzzy set B_1 .

B_1	1	2	3	4
μ_{B_1}	0.2	0.4	0.6	0.8
γ_{B_1}	0.7	0.5	0.2	0.1

(1) Define $B : U_1 \rightarrow [0, 1]$ (given in Table 20)

Then, B is not an IF subsemigroup of U_1 because if we take $x = r, y = q$, then $\mu_B(rq) \not\leq \mu_B(r) \wedge \mu_B(q)$ and $\gamma_B(rq) \not\leq \gamma_B(r) \vee \gamma_B(q)$. Lower approximation of B is given in Table 21.

Clearly, ${}^B\underline{\sigma}(e_1)$ and ${}^B\underline{\sigma}(e_2)$ are IF subsemigroups of U_2 , so $({}^B\underline{\sigma}, A)$ is an IF soft subsemigroup of U_2 .

(2) Define $B_1 : U_1 \rightarrow [0, 1]$ (given in Table 22)

Then, B_1 is not an IF left ideal of U_1 because if we take $x = r, y = q$, then $\mu_{B_1}(rq) \not\leq \mu_{B_1}(q)$ and $\gamma_{B_1}(rq) \not\leq \gamma_{B_1}(q)$. Lower approximation of B_1 is given in Table 23.

Clearly, ${}^{B_1}\underline{\sigma}(e_1)$ and ${}^{B_1}\underline{\sigma}(e_2)$ are IF left ideals of U_2 , so $({}^{B_1}\underline{\sigma}, A)$ is an IF soft left ideal of U_2 .

The next theorem shows that upper approximation of product of right and left IF ideals is contained in the intersection of their upper approximations.

Theorem 19. Suppose (σ, A) is a soft binary relation from a semigroup U_1 to a semigroup U_2 ; that is, $\sigma : A \rightarrow P(U_1 \times U_2)$. Then, for an IF right ideal $B_1 = \langle \mu_{B_1}, \gamma_{B_1} \rangle$ and for an IF left ideal $B_2 = \langle \mu_{B_2}, \gamma_{B_2} \rangle$ of U_2 , $\bar{\sigma}^{B_1 B_2} \subseteq \bar{\sigma}^{B_1} \cap \bar{\sigma}^{B_2}$.

Proof. Assume that B_1 is an IF right ideal and B_2 is IF left ideal of U_2 , so by definition, $B_1 B_2 \subseteq B_1 \cap B_2$.

It follows from Theorem 13,

$$\bar{\sigma}^{\mu_{B_1} \mu_{B_2}}(e) \subseteq \bar{\sigma}^{\mu_{B_1} \cap \mu_{B_2}}(e) \subseteq \bar{\sigma}^{\mu_{B_1}}(e) \cap \bar{\sigma}^{\mu_{B_2}}(e). \tag{22}$$

Hence,

$$\bar{\sigma}^{\mu_{B_1} \mu_{B_2}} \subseteq \bar{\sigma}^{\mu_{B_1}}(e) \cap \bar{\sigma}^{\mu_{B_2}}. \tag{23}$$

Also,

$$\bar{\sigma}^{\gamma_{B_1} \gamma_{B_2}}(e) \supseteq \bar{\sigma}^{\gamma_{B_1} \cap \gamma_{B_2}}(e) \supseteq \bar{\sigma}^{\gamma_{B_1}}(e) \cap \bar{\sigma}^{\gamma_{B_2}}(e). \tag{24}$$

TABLE 19: Lower approximation of B_1 .

	p	q	r	s
$\underline{\sigma}^{\mu_{B_1}}(e_1)$	0.4	0.4	0.4	0.4
$\underline{\sigma}^{\gamma_{B_1}}(e_1)$	0.5	0.5	0.5	0.5
$\underline{\sigma}^{\mu_{B_1}}(e_2)$	0.4	0.4	0.4	0.4
$\underline{\sigma}^{\gamma_{B_1}}(e_2)$	0.5	0.5	0.5	0.5

TABLE 20: Intuitionistic fuzzy set B .

	p	q	r	s
μ_B	0.1	0.3	0.5	0.7
γ_B	0.9	0.6	0.4	0.2

TABLE 21: Lower approximation of B .

	1	2	3	4
$\mu_B \underline{\sigma}(e_1)$	0.7	0.7	0.7	0.7
$\gamma_B \underline{\sigma}(e_1)$	0.2	0.2	0.2	0.2
$\mu_B \underline{\sigma}(e_2)$	0.1	0.1	0.1	0.1
$\gamma_B \underline{\sigma}(e_2)$	0.9	0.9	0.9	0.9

TABLE 22: Intuitionistic fuzzy set B_1 .

	p	q	r	s
μ_{B_1}	0.1	0.3	0.5	0.7
γ_{B_1}	0.8	0.6	0.5	0.2

Hence,

$$\bar{\sigma}^{\gamma_{B_1} \gamma_{B_2}} \supseteq \bar{\sigma}^{\gamma_{B_1}}(e) \cap \bar{\sigma}^{\gamma_{B_2}}. \quad (25)$$

□

Theorem 20. Suppose (σ, A) is a soft binary relation from a semigroup U_1 to a semigroup U_2 ; that is, $\sigma : A \rightarrow P(U_1 \times U_2)$. Then, for an IF right ideal $B_1 = \langle \mu_{B_1}, \gamma_{B_1} \rangle$ and for an IF left ideal $B_2 = \langle \mu_{B_2}, \gamma_{B_2} \rangle$ of U_1 , ${}^{B_1 B_2} \bar{\sigma} \subseteq {}^{B_1} \bar{\sigma} \cap {}^{B_2} \bar{\sigma}$.

Proof. It follows from Theorem 19. □

Theorem 21. Let (σ, A) be a soft binary relation from a semigroup U_1 to a semigroup U_2 ; that is, $\sigma : A \rightarrow P(U_1 \times U_2)$. Then, for an IF right ideal $B_1 = \langle \mu_{B_1}, \gamma_{B_1} \rangle$ and an IF left ideal $B_2 = \langle \mu_{B_2}, \gamma_{B_2} \rangle$ of U_2 , $\underline{\sigma}^{B_1 B_2} \subseteq \underline{\sigma}^{B_1} \cap \underline{\sigma}^{B_2}$.

Proof. Assume that B_1 is an IF right ideal and B_2 is IF left ideal of U_2 , so by definition $B_1 B_2 \subseteq B_1 \cap B_2$.

It follows from Theorem 13,

$$\underline{\sigma}^{\mu_{B_1} \mu_{B_2}}(e) \subseteq \underline{\sigma}^{\mu_{B_1} \cap \mu_{B_2}}(e) \subseteq \underline{\sigma}^{\mu_{B_1}}(e) \cap \underline{\sigma}^{\mu_{B_2}}(e). \quad (26)$$

TABLE 23: Lower approximation of B_1 .

	1	2	3	4
$\mu_{B_1} \underline{\sigma}(e_1)$	0.7	0.7	0.7	0.7
$\gamma_{B_1} \underline{\sigma}(e_1)$	0.2	0.2	0.2	0.2
$\mu_{B_1} \underline{\sigma}(e_2)$	0.1	0.1	0.1	0.1
$\gamma_{B_1} \underline{\sigma}(e_2)$	0.8	0.8	0.8	0.8

Hence,

$$\underline{\sigma}^{\mu_{B_1} \mu_{B_2}} \subseteq \underline{\sigma}^{\mu_{B_1}}(e) \cap \underline{\sigma}^{\mu_{B_2}}. \quad (27)$$

Also,

$$\underline{\sigma}^{\gamma_{B_1} \gamma_{B_2}}(e) \supseteq \underline{\sigma}^{\gamma_{B_1} \cap \gamma_{B_2}}(e) \supseteq \underline{\sigma}^{\gamma_{B_1}}(e) \cap \underline{\sigma}^{\gamma_{B_2}}(e). \quad (28)$$

Hence,

$$\underline{\sigma}^{\gamma_{B_1} \gamma_{B_2}} \supseteq \underline{\sigma}^{\gamma_{B_1}}(e) \cap \underline{\sigma}^{\gamma_{B_2}}. \quad (29)$$

□

Theorem 22. Suppose (σ, A) is a soft binary relation from a semigroup U_1 to a semigroup U_2 ; that is, $\sigma : A \rightarrow P(U_1 \times U_2)$. Then for IF right ideal $B_1 = \langle \mu_{B_1}, \gamma_{B_1} \rangle$ and IF left ideal $B_2 = \langle \mu_{B_2}, \gamma_{B_2} \rangle$ of U_1 , ${}^{B_1 B_2} \underline{\sigma} \subseteq {}^{B_1} \underline{\sigma} \cap {}^{B_2} \underline{\sigma}$.

Proof. It follows from Theorem 21. □

Now for IF interior ideals of a semigroup, we discuss a few properties.

Theorem 23. Let (σ, A) be a soft compatible relation from a semigroup U_1 to a semigroup U_2 . If B is an IF interior ideal of U_2 , then $(\bar{\sigma}^B, A)$ is an IF soft interior ideal of U_1 .

Proof. Suppose that B is an IF interior ideal of U_2 . Thus, B is an IF subsemigroup of U_2 , so by Theorem 15, $(\bar{\sigma}^B, A)$ is an IF soft subsemigroup of U_1 . Now for $x, a, y \in U_1$,

$$\begin{aligned} \bar{\sigma}^{\mu_B}(e)(a) &= \bigvee_{q \in a\sigma(e)} \mu_B(q) \leq \bigvee_{p \in x\sigma(e)} \bigvee_{q \in a\sigma(e)} \bigvee_{r \in y\sigma(e)} \mu_B(pqr) \\ &\leq \bigvee_{(pqr) \in (xay)\sigma(e)} \mu_B(pqr) = \bigvee_{a' \in xay\sigma(e)} \mu_B(a') = \bar{\sigma}^{\mu_B}(e)(xay). \end{aligned} \quad (30)$$

Similarly, for $s, b, t \in U_1$,

$$\begin{aligned} \bar{\sigma}^{\gamma_B}(e)(b) &= \bigwedge_{u \in s\sigma(e)} \gamma_B(u) \leq \bigwedge_{u \in s\sigma(e)} \bigwedge_{v \in b\sigma(e)} \bigwedge_{w \in t\sigma(e)} \gamma_B(uvw) \\ &\leq \bigwedge_{(uvw) \in (sbt)\sigma(e)} \gamma_B(uvw) = \bigwedge_{b' \in sbt\sigma(e)} \gamma_B(b') = \bar{\sigma}^{\gamma_B}(e)(sbt). \end{aligned} \quad (31)$$

□

Hence, $\bar{\sigma}^B(e)$ is an IF interior ideal of U_1 for all $e \in A$, so $(\bar{\sigma}^B, A)$ is an IF soft interior ideal of U_1 .

For converse of above result, we show the following example.

Example 5. Let $U_1 = \{1, 2, 3\}$ and $U_2 = \{p, q, r\}$ be two semi-groups, and their multiplication tables are as shown in Tables 24 and 25, respectively.

And $A = \{e_1, e_2\}$. Define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\begin{aligned} \sigma(e_1) &= \{(1, p), (2, q), (3, r), (1, q), (2, p), (1, r), (3, p)\} \\ \sigma(e_2) &= \{(1, p), (1, q), (1, r), (2, p), (2, q), (3, r)\}. \end{aligned} \tag{32}$$

Then, (σ, A) is a soft compatible relation from U_1 to U_2 .

$$\begin{aligned} 1\sigma(e_1) &= \{p, q, r\}, \\ 2\sigma(e_1) &= \{p, q\}, \\ 3\sigma(e_1) &= \{p, r\} \\ 1\sigma(e_2) &= \{p, q, r\}, \\ 2\sigma(e_2) &= \{p, q\}, \\ 3\sigma(e_2) &= \{r\}. \end{aligned} \tag{33}$$

Define $B : U_2 \rightarrow [0, 1]$ (given in Table 26).

Then, B is not an IF interior ideal of U_2 because if we take $x = q, a = r, y = p$, then $\mu_B(qrp) \not\leq \mu_B(r)$ and $\gamma_B(qrp) \not\leq \gamma_B(r)$. Upper approximation of B is given in Table 27.

Clearly, ${}^B\bar{\sigma}(e_1)$ and ${}^B\bar{\sigma}(e_2)$ are IF interior ideals of U_1 , so $({}^B\bar{\sigma}, A)$ is an IF soft interior ideal of U_1 .

Theorem 24. Let (σ, A) be a soft compatible relation from a semigroup U_1 to a semigroup U_2 . If B is an IF interior ideal of U_1 , then $({}^B\bar{\sigma}, A)$ is an IF soft interior ideal of U_2 .

Proof. It follows from Theorem 23. □

For converse of above result, we show the following example.

Example 6. Consider the semigroups of Example 5.

And $A = \{e_1, e_2\}$. Define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\begin{aligned} \sigma(e_1) &= \{(1, p), (2, q), (3, r), (1, q), (2, p), (1, r), (3, p)\} \\ \sigma(e_2) &= \{(1, p), (1, q), (1, r), (2, p), (2, q), (3, q)\}. \end{aligned} \tag{34}$$

Then, (σ, A) is a soft compatible relation from U_1 to U_2 .

$$\begin{aligned} \sigma(e_1)p &= \{1, 2, 3\}, \\ \sigma(e_1)q &= \{1, 2\}, \\ \sigma(e_1)r &= \{1, 3\} \\ \sigma(e_2)p &= \{1, 2\}, \\ \sigma(e_2)q &= \{2\}, \\ \sigma(e_2)r &= \{2, 3\}. \end{aligned} \tag{35}$$

Define $B : U_1 \rightarrow [0, 1]$ (given in Table 28).

TABLE 24: Multiplication table for U_1 .

.	1	2	3
1	1	2	3
2	1	2	3
3	1	2	3

TABLE 25: Multiplication table for U_2 .

.	p	q	r
p	p	p	r
q	p	q	r
r	p	r	r

TABLE 26: Intuitionistic fuzzy set B .

	p	q	r
μ_B	0	0.1	0.1
γ_B	0.9	0.8	0.9

Then, B is not an IF interior ideal of U_1 because if we take $x = 2, a = 3, y = 1$, then $\mu_B(231) \not\leq \mu_B(3)$ and $\gamma_B(231) \not\leq \gamma_B(3)$. Upper approximation of B is given in Table 29.

Clearly, ${}^B\bar{\sigma}(e_1)$ and ${}^B\bar{\sigma}(e_2)$ are IF interior ideals of U_2 , so $({}^B\bar{\sigma}, A)$ is an IF soft interior ideal of U_2 .

Now for lower approximations of an IF interior ideal of a semigroup, we discuss some results.

Theorem 25. Suppose (σ, A) is a soft complete relation regarding aftersets from a semigroup U_1 to a semigroup U_2 . If B is an IF interior ideal of U_2 , then $(\underline{\sigma}^B, A)$ is an IF soft interior ideal of U_1 .

Proof. Suppose that B is an IF interior ideal of U_2 . Thus, B is an IF subsemigroup of U_2 , so by Theorem 17, $(\underline{\sigma}^B, A)$ is an IF soft subsemigroup of U_1 . Now for $x, a, y \in U_1$,

$$\begin{aligned} \underline{\sigma}^{\mu_B}(e)(xay) &= \bigwedge_{a' \in (xay)\sigma(e)} \mu_B(a') = \bigwedge_{a' \in x\sigma(e).a\sigma(e).y\sigma(e)} \mu_B(a') \\ &= \bigwedge_{p \in x\sigma(e).q \in a\sigma(e).r \in y\sigma(e)} \mu_B(pqr) \geq \bigwedge_{q \in a\sigma(e)} \mu_B(q) = \underline{\sigma}^{\mu_B}(e)(a). \end{aligned} \tag{36}$$

Similarly, for $s, b, t \in U_1$,

$$\begin{aligned} \underline{\sigma}^{\gamma_B}(e)(sbt) &= \bigvee_{b' \in (sbt)\sigma(e)} \gamma_B(b') = \bigvee_{b' \in s\sigma(e).b\sigma(e).t\sigma(e)} \gamma_B(b') \\ &= \bigvee_{u \in s\sigma(e).v \in b\sigma(e).w \in t\sigma(e)} \gamma_B(uvw) \leq \bigvee_{u \in s\sigma(e)} \gamma_B(u) = \underline{\sigma}^{\gamma_B}(e)(s). \end{aligned} \tag{37}$$

□

Hence, $\underline{\sigma}^B(e)$ is an IF interior ideal of U_1 for all $e \in A$, so $(\underline{\sigma}^B, A)$ is an IF soft interior ideal of U_1 .

From following example, we show the converse of above theorem which is not true.

TABLE 27: Upper approximation of B .

	1	2	3
$\bar{\sigma}^{\mu_B}(e_1)$	0.1	0.1	0.1
$\bar{\sigma}^{\gamma_B}(e_1)$	0.8	0.8	0.9
$\bar{\sigma}^{\mu_B}(e_2)$	0.1	0.1	0.1
$\bar{\sigma}^{\gamma_B}(e_2)$	0.8	0.8	0.9

TABLE 28: Intuitionistic fuzzy set B .

	1	2	3
μ_B	0	0.1	0.1
γ_B	0.9	0.8	0.9

TABLE 29: Upper approximation of B .

	p	q	r
$^{\mu_B}\bar{\sigma}(e_1)$	0.1	0.1	0.1
$^{\gamma_B}\bar{\sigma}(e_1)$	0.8	0.8	0.8
$^{\mu_B}\bar{\sigma}(e_2)$	0.1	0.1	0.1
$^{\gamma_B}\bar{\sigma}(e_2)$	0.8	0.8	0.8

Example 7. Consider the semigroups of Example 1.

Let $A = \{e_1, e_2\}$. Define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\begin{aligned} \sigma(e_1) &= \{(p, 2), (p, 3), (q, 2), (q, 3), (r, 2), (r, 3), (s, 2), (s, 3)\} \\ \sigma(e_2) &= \{(p, 2), (q, 2), (r, 2), (s, 2)\}. \end{aligned} \tag{38}$$

Then, (σ, A) is a soft complete relation from U_1 to U_2 with respect to the aftersets.

$$\begin{aligned} p\sigma(e_1) &= \{2, 3\}, \\ q\sigma(e_1) &= \{2, 3\}, \\ r\sigma(e_1) &= \{2, 3\}, \\ s\sigma(e_1) &= \{2, 3\} \\ p\sigma(e_2) &= \{2\}, \\ q\sigma(e_2) &= \{2\}, \\ r\sigma(e_2) &= \{2\}, \\ s\sigma(e_2) &= \{2\}. \end{aligned} \tag{39}$$

(1) Define $B : U_2 \rightarrow [0, 1]$ such that $B = \langle \mu_B, \gamma_B \rangle$ (given in Table 30)

TABLE 30: Intuitionistic fuzzy set B .

	1	2	3	4
μ_B	0.4	0.6	0.8	1
γ_B	0.5	0.3	0.1	0

Then, B is not an IF interior ideal of U_2 , because if we take $x = 2, a = 3, y = 1$, then $\mu_B(231) \not\leq \mu_B(3)$ and $\gamma_B(231) \not\leq \gamma_B(3)$. Lower approximation of B is given in Table 31.

Clearly, $\underline{\sigma}^B(e_1)$ and $\underline{\sigma}^B(e_2)$ are IF interior ideals of U_1 , so $(\underline{\sigma}^B, A)$ is an IF soft interior ideal of U_1 .

Theorem 26. Suppose (σ, A) is a soft complete relation regarding foresets from a semigroup U_1 to a semigroup U_2 . If B is an IF interior ideal of U_1 , then $({}^B\sigma, A)$ is an IF soft interior ideal of U_2 .

Proof. It follows from Theorem 25. □

Example 8. Consider the semigroups of Example 1.

Define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\begin{aligned} \sigma(e_1) &= \{(s, 1), (s, 2), (s, 3), (s, 4)\}, \\ \sigma(e_2) &= \{(p, 1), (p, 2), (p, 3), (p, 4), (s, 1), (s, 2), (s, 3), (s, 4)\}. \end{aligned} \tag{40}$$

Then, (σ, A) is a soft complete relation from U_1 to U_2 with respect to the foresets.

$$\begin{aligned} \sigma(e_1)1 &= \{s\}, \\ \sigma(e_1)2 &= \{s\}, \\ \sigma(e_1)3 &= \{s\}, \\ \sigma(e_1)4 &= \{s\} \\ \sigma(e_2)1 &= \{p, s\}, \\ \sigma(e_2)2 &= \{p, s\}, \\ \sigma(e_2)3 &= \{p, s\}, \\ \sigma(e_2)4 &= \{p, s\}. \end{aligned} \tag{41}$$

Define $B : U_1 \rightarrow [0, 1]$ (given in Table 32).

Then, B is not an IF interior ideal of U_1 because if we take $x = q, a = r, y = p$, then $\mu_B(qrp) \not\leq \mu_B(r)$ and $\gamma_B(qrp) \not\leq \gamma_B(r)$. Lower approximation of B is given in Table 33.

Clearly, ${}^B\underline{\sigma}(e_1)$ and ${}^B\underline{\sigma}(e_2)$ are IF interior ideals of U_2 , so $({}^B\underline{\sigma}, A)$ is an IF soft interior ideal of U_2 .

Now, we discuss properties for IF bi-ideals of a semigroup.

Theorem 27. Suppose (σ, A) is a soft compatible relation from a semigroup U_1 to a semigroup U_2 . If B_1 is an IF bi-ideal of U_2 then $(\bar{\sigma}^{B_1}, A)$ is an IF soft bi-ideal of U_1 .

TABLE 31: Lower approximation of B .

	p	q	r	s
$\underline{\sigma}^{\mu_B}(e_1)$	0.6	0.6	0.6	0.6
$\underline{\sigma}^{\gamma_B}(e_1)$	0.3	0.3	0.3	0.3
$\underline{\sigma}^{\mu_B}(e_2)$	0.6	0.6	0.6	0.6
$\underline{\sigma}^{\gamma_B}(e_2)$	0.3	0.3	0.3	0.3

TABLE 32: Intuitionistic fuzzy set B .

	p	q	r	s
μ_B	0	0.3	0.5	0.7
γ_B	1	0.7	0.4	0.2

Proof. Suppose that B_1 is an IF bi-ideal of U_2 . Thus, B_1 is an IF subsemigroup of U_2 , so by Theorem 15, $(\bar{\sigma}^{B_1}, A)$ is an IF soft subsemigroup of U_1 . Now for $x, a, y \in U_1$,

$$\begin{aligned}
\bar{\sigma}^{\mu_{B_1}}(e)(x) \wedge \bar{\sigma}^{\mu_{B_1}}(e)(z) &= \left(\bigvee_{p \in x\sigma(e)} \mu_{B_1}(p) \right) \wedge \left(\bigvee_{q \in z\sigma(e)} \mu_{B_1}(q) \right) \\
&= \bigvee_{p \in x\sigma(e)} \bigvee_{q \in z\sigma(e)} \left(\mu_{B_1}(p) \wedge \mu_{B_1}(q) \right) \\
&\leq \bigvee_{p \in x\sigma(e)} \bigvee_{r \in y\sigma(e)} \bigvee_{q \in z\sigma(e)} \left(\mu_{B_1}(prq) \right) \\
&\leq \bigvee_{prq \in (xyz)\sigma(e)} \mu_{B_1}(prq) = \bigvee_{a' \in (xyz)\sigma(e)} \mu_{B_1}(a') \\
&= \bar{\sigma}^{\mu_{B_1}}(e)(xay).
\end{aligned} \tag{42}$$

Similarly, for $x, a, y \in U_1$,

$$\begin{aligned}
\bar{\sigma}^{\gamma_{B_1}}(e) \vee (x) \bar{\sigma}^{\gamma_{B_1}}(e)(z) &= \left(\bigwedge_{p \in x\sigma(e)} \gamma_{B_1}(p) \right) \vee \left(\bigwedge_{q \in z\sigma(e)} \gamma_{B_1}(q) \right) \\
&= \bigwedge_{p \in x\sigma(e)} \bigwedge_{q \in z\sigma(e)} \left(\gamma_{B_1}(p) \vee \gamma_{B_1}(q) \right) \\
&\geq \bigwedge_{p \in x\sigma(e)} \bigwedge_{r \in y\sigma(e)} \bigwedge_{q \in z\sigma(e)} \left(\gamma_{B_1}(prq) \right) \\
&\geq \bigwedge_{prq \in (xyz)\sigma(e)} \gamma_{B_1}(prq) = \bigwedge_{b' \in (xyz)\sigma(e)} \gamma_{B_1}(b') \\
&= \bar{\sigma}^{\gamma_{B_1}}(e)(xay).
\end{aligned} \tag{43}$$

□

Hence, $\bar{\sigma}^{B_1}(e)$ is an IF bi-ideal of U_1 for all $e \in A$, so $(\bar{\sigma}^{B_1}, A)$ is an IF soft bi-ideal of U_1 .

Theorem 28. Suppose (σ, A) is a soft compatible relation from a semigroup U_1 to a semigroup U_2 . If B_1 is an IF bi-ideal of U_1 , then $({}^{B_1}\bar{\sigma}, A)$ is an IF soft bi-ideal of U_2 .

Proof. It follows from Theorem 27. □

Example 9. Consider the semigroups and soft relations of Example 2.

Define $B_1 : U_1 \rightarrow [0, 1]$ (given in Table 34).

Then, B_1 is not an IF bi-ideal of U_1 because if we take $x = p, w = s, y = p$, then $\mu_B(psp) \not\leq \mu_B(p) \wedge \mu_B(p)$ and $\gamma_B(psp) \not\geq \gamma_B(p) \vee \mu_B(p)$. Upper approximation of B_1 is given in Table 35.

Clearly, ${}^{B_1}\bar{\sigma}(e_1)$ and ${}^{B_1}\bar{\sigma}(e_2)$ are IF bi-ideals of U_2 , so $({}^{B_1}\bar{\sigma}, A)$ is an IF soft bi-ideal of U_2 .

Theorem 29. Let (σ, A) be a soft complete relation regarding aftersets from a semigroup U_1 to a semigroup U_2 . If B_1 is an IF bi-ideal of U_2 , then $(\underline{\sigma}^{B_1}, A)$ is an IF soft bi-ideal of U_1 .

Proof. Suppose that B_1 is an IF bi-ideal of U_2 . Thus, B_1 is an IF subsemigroup of U_2 , so by Theorem 17, $(\underline{\sigma}^{B_1}, A)$ is an IF soft subsemigroup of U_1 . Now for $x, a, y \in U_1$,

$$\begin{aligned}
\underline{\sigma}^{\mu_{B_1}}(e)(xay) &= \bigwedge_{a' \in (xay)\sigma(e)} \mu_{B_1}(a') = \bigwedge_{a' \in x\sigma(e).a\sigma(e).y\sigma(e)} \mu_{B_1}(a') \\
&= \bigwedge_{p \in x\sigma(e).q \in a\sigma(e).r \in y\sigma(e)} \mu_{B_1}(pqr) \geq \left(\bigwedge_{p \in x\sigma(e)} \mu_{B_1}(p) \right) \wedge \left(\bigwedge_{r \in y\sigma(e)} \mu_{B_1}(r) \right) \\
&= \underline{\sigma}^{\mu_{B_1}}(e)(x) \wedge \underline{\sigma}^{\mu_{B_1}}(e)(y).
\end{aligned} \tag{44}$$

Similarly, for $x, a, y \in U_1$,

$$\begin{aligned}
\underline{\sigma}^{\gamma_{B_1}}(e)(xay) &= \bigvee_{b' \in (xay)\sigma(e)} \gamma_{B_1}(b') = \bigvee_{b' \in x\sigma(e).a\sigma(e).y\sigma(e)} \gamma_{B_1}(b') \\
&= \bigvee_{p \in x\sigma(e).q \in a\sigma(e).r \in y\sigma(e)} \gamma_{B_1}(pqr) \leq \left(\bigvee_{p \in x\sigma(e)} \gamma_{B_1}(p) \right) \vee \left(\bigvee_{r \in y\sigma(e)} \gamma_{B_1}(r) \right) \\
&= \underline{\sigma}^{\gamma_{B_1}}(e)(x) \vee \underline{\sigma}^{\gamma_{B_1}}(e)(y).
\end{aligned} \tag{45}$$

Hence, $\underline{\sigma}^{B_1}(e)$ is an IF bi-ideal of U_1 for each $e \in A$, so $(\underline{\sigma}^{B_1}, A)$ is an IF soft bi-ideal of U_1 .

Example 10. Taking the semigroups of Example 1.

Define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\begin{aligned}
\sigma(e_1) &= \{(p, 2), (p, 3), (q, 2), (q, 3), (r, 2), (r, 3), (s, 2), (s, 3)\} \\
\sigma(e_2) &= \{(p, 2), (q, 2), (r, 2), (s, 2)\}.
\end{aligned} \tag{46}$$

Then, (σ, A) is a soft complete relation from U_1 to U_2 with respect to the aftersets.

$$\begin{aligned}
p\sigma(e_1) &= \{2, 3\}, \\
q\sigma(e_1) &= \{2, 3\}, \\
r\sigma(e_1) &= \{2, 3\}, \\
s\sigma(e_1) &= \{2, 3\} \\
p\sigma(e_2) &= \{2\}, \\
q\sigma(e_2) &= \{2\}, \\
r\sigma(e_2) &= \{2\}, \\
s\sigma(e_2) &= \{2\}.
\end{aligned} \tag{47}$$

Define $B_1 : U_2 \rightarrow [0, 1]$ (given in Table 36).

TABLE 33: Lower approximation of B .

	1	2	3	4
$\mu_B \underline{\sigma}(e_1)$	0.7	0.7	0.7	0.7
$\gamma_B \underline{\sigma}(e_1)$	0.2	0.2	0.2	0.2
$\mu_B \underline{\sigma}(e_2)$	0	0	0	0
$\gamma_B \underline{\sigma}(e_2)$	1	1	1	1

TABLE 34: Intuitionistic fuzzy set B_1 .

	p	q	r	s	t
μ_{B_1}	1	0.5	0.7	0.9	0.2
γ_{B_1}	0	0.4	0.3	0.1	0.7

TABLE 35: Upper approximation of B_1 .

	1	2	3	4	5
$\mu_{B_1} \bar{\sigma}(e_1)$	1	0.5	0.9	0.9	0.9
$\gamma_{B_1} \bar{\sigma}(e_1)$	0	0.4	0.1	0.1	0.1
$\mu_{B_1} \bar{\sigma}(e_2)$	1	0.5	0.9	0.9	0.9
$\gamma_{B_1} \bar{\sigma}(e_2)$	0	0.4	0.1	0.1	0.1

TABLE 36: Intuitionistic fuzzy set B_1 .

	1	2	3	4
μ_{B_1}	0.7	0.2	0.1	0.4
γ_{B_1}	0.2	0.7	0.9	0.5

TABLE 37: Lower approximation of B_1 .

	p	q	r	s
$\underline{\sigma}^{\mu_{B_1}}(e_1)$	0.1	0.1	0.1	0.1
$\underline{\sigma}^{\gamma_{B_1}}(e_1)$	0.9	0.9	0.9	0.9
$\underline{\sigma}^{\mu_{B_1}}(e_2)$	0.2	0.2	0.2	0.2
$\underline{\sigma}^{\gamma_{B_1}}(e_2)$	0.7	0.7	0.7	0.7

TABLE 38: Intuitionistic fuzzy set B_2 .

	p	q	r	s
μ_{B_2}	0.1	0.8	0.6	0.7
γ_{B_2}	0.9	0.1	0.3	0.3

Then, B_1 is not an IF bi-ideal of U_2 because if we take $x = 1, w = 4, y = 1$, then $\mu_{B_1}(141) \not\leq \mu_{B_1}(1) \wedge \mu_{B_1}(1)$ and $\gamma_{B_1}(141) \not\geq \gamma_{B_1}(1) \vee \mu_{B_1}(1)$. Lower approximation of B_1 is given in Table 37.

Clearly, $\underline{\sigma}^{B_1}(e_1)$ and $\underline{\sigma}^{B_1}(e_2)$ are IF bi-ideals of U_1 , so $(\underline{\sigma}^{B_1}, A)$ is an IF soft bi-ideal of U_1 .

TABLE 39: Lower approximation of B_2 .

	1	2	3	4
$\mu_{B_2} \underline{\sigma}(e_1)$	0.7	0.7	0.7	0.7
$\gamma_{B_2} \underline{\sigma}(e_1)$	0.3	0.3	0.3	0.3
$\mu_{B_2} \underline{\sigma}(e_2)$	0.1	0.1	0.1	0.1
$\gamma_{B_2} \underline{\sigma}(e_2)$	0.9	0.9	0.9	0.9

Theorem 30. Let (σ, A) be a soft complete relation with respect to the foresets from a semigroup U_1 to a semigroup U_2 . If B_2 is an IF bi-ideal of U_1 , then $({}^{B_2}\sigma, A)$ is an IF soft bi-ideal of U_2 .

Proof. It follows from Theorem 29. □

Example 11. Taking the semigroups of Example 1.

Define $\sigma : A \rightarrow P(U_1 \times U_2)$ by

$$\sigma(e_1) = \{(s, 1), (s, 2), (s, 3), (s, 4)\}$$

$$\sigma(e_2) = \{(p, 1), (p, 2), (p, 3), (p, 4), (s, 1), (s, 2), (s, 3), (s, 4)\}. \tag{48}$$

Then, (σ, A) is a soft complete relation from U_1 to U_2 with respect to the foresets.

$$\begin{aligned} \sigma(e_1)1 &= \{s\}, \\ \sigma(e_1)2 &= \{s\}, \\ \sigma(e_1)3 &= \{s\}, \\ \sigma(e_1)4 &= \{s\}, \\ \sigma(e_2)1 &= \{p, s\}, \\ \sigma(e_2)2 &= \{p, s\}, \\ \sigma(e_2)3 &= \{p, s\}, \\ \sigma(e_2)4 &= \{p, s\}. \end{aligned} \tag{49}$$

Define $B_2 : U_1 \rightarrow [0, 1]$ (given in Table 38).

Then, B_2 is not an IF bi-ideal of U_1 because if we take $x = q, w = r, y = q$, then $\mu_{B_2}(qrq) \not\leq \mu_{B_2}(q) \wedge \mu_{B_2}(q)$ and $\gamma_{B_2}(qrq) \not\geq \gamma_{B_2}(q) \vee \mu_{B_2}(q)$. Lower approximation of B_2 is given in Table 39.

Clearly, ${}^{B_2}\underline{\sigma}(e_1)$ and ${}^{B_2}\underline{\sigma}(e_2)$ are IF bi-ideals of U_2 , so $({}^{B_2}\underline{\sigma}, A)$ is an IF soft bi-ideal of U_2 .

The flow chart of our proposed model is shown in Figure 1.

4. Comparison with Previous Work

Our research work is a generalized form of rough approximation of an FS in semigroups based on soft relation which is discussed by Kanwal and Shabir in [50]. In this paper, we use an IFS in semigroups instead of only an FS. As we know, an IFS is better than an FS to control the vagueness and

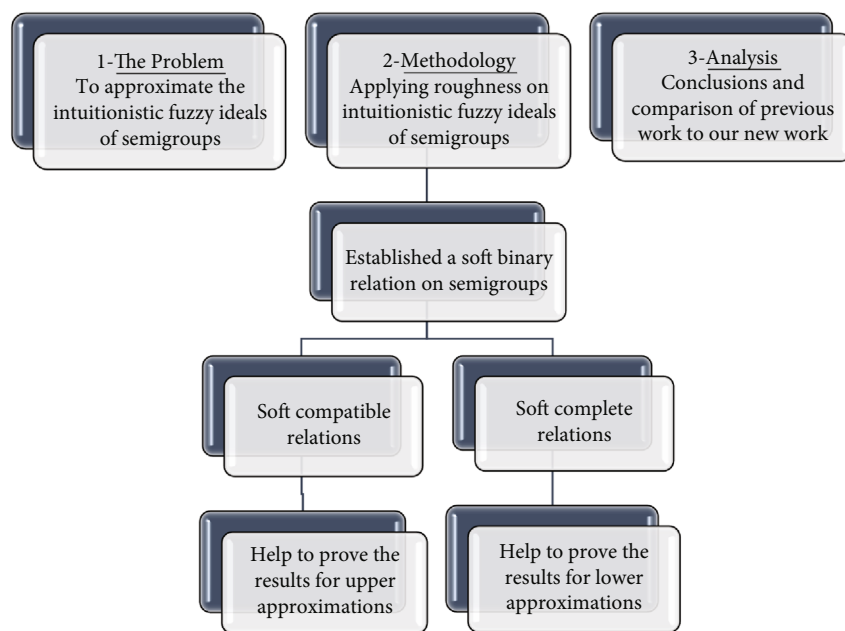


FIGURE 1: Flow chart of proposed model.

uncertainty in real-life problems. In FS theory, only membership function is discussed, but in IFS theory, functions of membership and nonmembership are also discussed. For example, if a person wants to buy a shirt with his choice, then he can buy his favourite shirt with specific design and colour by using this RIFS model. His preference will be calculated more effectively than Kanwal and Shabir's technique [50] which is about an FS. Cagman and Engino [62] redefined the operations of SSs and proposed the products of SSs and uni-int decision function and their corresponding decision-making algorithm. Maji et al. [63] proposed a decision-making algorithm using SSs with RSs. In comparison with these two above methods, our proposed model is more reliable due to IFSs. Similarly, IFSs play an important role in medical field to detect intestinal bacteria such as Shigella and Salmonella which cause dysentery and typhoid, respectively. This is a challenging situation for microbiologists to diagnose these such deceases. Khatibi and Montazer (2009) presented value approach towards the bacteria classification problem to examine capabilities of FSs and IFSs in facing vagueness and uncertainty in the medical pattern recognition by using their five similarity measures. After comparing and illustrating the measures by experimental results, they obtained better detection rates of IFS Hausdorff and Mitchell similarity measures with valuable results 95.27% and 94.48% detection rate, respectively. On the other hand, the FS Euclidean distance yielded 85% detection rate only [64]. We can say that we make better decision by using RIFS model.

5. Conclusion

This paper has developed RIFS model in terms of soft binary relations. By using this approach, we obtain the upper approximation and lower approximation in the form of two pairs of IFSSs regarding aftersets and foresets. After applying these con-

cepts, we discuss rough approximations of IF subsemigroups, IF left (right) ideals, IF interior ideals, and IF bi-ideals of semigroups. Earlier, rough approximation of an FS in semigroups based on soft binary relations has been discussed [50]. In an FS, only membership function is discussed, but in some critical situations of real world, we need nonmembership function to discuss uncertainty in good manner. Actually, the membership function alone is insufficient to deal vagueness. In such critical situations, an IFS is more useful set because its elements have membership function, nonmembership function, and the hesitation margin function. Since IFS has degree of membership, degree of nonmembership and hesitant degree with their sum of 1 but pythagorean FSs are more useful sets than IFSs because they cover more points that were not covered by IFSs. So Pythagorean FSs can describe the vagueness and impreciseness related to membership degree and nonmembership degree in a better way than IFSs.

5.1. Future Study. For future study, it is more considerable to make granulation roughness of an IFS and the roughness in interval valued set (IFS) under different several environments of soft binary relations. Study of this research work might be expanded to more outcomes in soft computing. For more research study, by taking on IF rough sets (IFRS), we can expand knowledge addition in IF information systems.

Data Availability

We did not use any data for this research work.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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