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Retraction

Retracted: Novel Analysis of Fuzzy Fractional Klein-Gordon Model via Semianalytical Method

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Manipulated or compromised peer review

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] M. Alshammari, W. W. Mohammed, and M. Yar, "Novel Analysis of Fuzzy Fractional Klein-Gordon Model via Semianalytical Method," *Journal of Function Spaces*, vol. 2022, Article ID 4020269, 9 pages, 2022.

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Research Article

Novel Analysis of Fuzzy Fractional Klein-Gordon Model via Semianalytical Method

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The current article discusses the new fuzzy iterative transform method, a hybrid methodology based on fuzzy logic and an iterative transformation technique. We demonstrate the consistency of our technique by employing the Caputo derivative under generalized Hukuhara differentiability to construct fractional fuzzy Klein-Gordon equations with the initial fuzzy condition. The series produced result was calculated and compared to the exact result's recommended equations. Two problems were used to verify our method, with the results approximated in fuzzy form. The upper and lower half of the fuzzy results were approximated in each of the two examples using two distinct fractional orders between zero and one. Because it globalizes the dynamical behavior of the specified equation, it produces all forms of fuzzy results at any fractional order between 0 and 1. Since fuzzy numbers offer their results in a fuzzy form with lower and upper branches, the unknown amount also adds fuzziness. It is crucial to emphasize that the suggested fuzziness method is intended to demonstrate the efficiency and superiority of numerical solutions to nonlinear fractional fuzzy partial differential equations found in complex and physical structures.

1. Introduction

Fuzzy set theory is an excellent tool for modeling uncertain problems. As a result, fuzzy concepts have been applied to modeling various natural phenomena. The fractional fuzzy differential equation is a frequently used model in various scientific fields, including weapon system evaluation, population modeling, electro hydraulics, and civil engineering modeling. As a result, the concept of the fractional derivative is critical in fuzzy calculus [1–3]. As a result, fuzzy fractional differential equations have garnered considerable attention in mathematics and engineering. The first is a study on fuzzy fractional differential equations by Agarwal et al. [4]. They introduced the Riemann-Liouville idea under the Hukuhara concept to analyze fractional fuzzy differential equations. We still live in a world of confusion and uncertainty, which is reality. Many people are subject to doubting everything around them and wondering why it is for themselves or others? Because their reports are inadequate or erroneous, and they are not clear. Assume we are in a circumstance where there's a lot of erroneous information and there's a lot of ambiguity. We do not know how to answer many of our legitimate queries since they are based on incorrect facts. This mindset and this attitude of uncertainty are critical for scientists. Instead of trying to combat ambiguity, our goal is to figure out how to comprehend it and work around it. Because the development, resources, and life you desire are all in flux [5–8].

Recently, fractional calculus has been encouraged as a useful subject for obtaining exact solutions to engineering and mathematics problems such as signal processing, aerodynamics and control [9–12] systems, and biomathematical obstacles. Additionally, other authors have investigated fractional differential equations in fuzzy circumstances and solved them using a variety of methodologies [13–16]. In [17], Hoa examined fuzzy fractional differential equations with Caputo gH-differentiability. Simultaneously, Agarwal et al. researched the same subject in [18] to demonstrate its relevance to optimum control difficulties. Long et al. [19] established the solvability of fractional fuzzy differential equations, while Salahshour et al. [20] resolved the problem using fuzzy Laplace transforms.

Klein-Gordon equations (KGEs) play a critical role in physics, nonlinear optics, quantum field theory and solid-state physics, plasma kinematics, physics, mathematical biology, and initial state recurrence. Numerous phenomena, such as the behavior of fundamental particles and the dislocation of crystals, are important uses of (KGEs). To explore solitons [21], nonlinear wave equations [22], and condensed matter physics cite15, these equations drew academic attention. Mathematicians have made numerous significant efforts to find solutions to these problems over the last few years. Numerous methods for solving these equations have been introduced, including radial basis functions [23], B-spline collocation method [24], auxiliary approach [25], and exponential type potential. Additional techniques for solving these equations are discussed in [26–30]. To solve nonlinear KGEs, the task attracted considerable interest from scholars, and a variety of methods were created, as indicated in [30, 31]. Several further approaches include the stationary solution [32], the Homotopy perturbation technique [33], the Tanh technique [34], the variation iteration technique [35], and so on [36].

2. Basic Definition

Definition 1. Consider a continuous function fuzzy $\tilde{\upsilon}$ on $[0, \rho] \in R$, we express fractional fuzzy Riemann-Liouvilli integral in the presence of Φ as [37, 38]

$$\mathbf{I}^{\rho}\tilde{\upsilon} = \int_0^{\Phi} \frac{(\Phi-\eta)^{\rho-1}\tilde{\upsilon}(\eta)}{\Gamma(\rho)} d\eta, \rho, \eta \in (0,\infty). \tag{1}$$

Moreover, if $\tilde{\upsilon} \in C^F[0,\rho] \cap L^F[0,\rho]$, where $C^F[0,\rho]$ is the universe of fuzzy continuous function, and $L^F[0,\rho]$ is the space of continuous fuzzy functions. If the functions are Lebesgue integrable, then, the fuzzy fractional integral is given as

$$[\mathbf{I}^{\rho}\tilde{\upsilon}(\Phi)]_{\sigma} = [\mathbf{I}^{\rho}\underline{\upsilon}_{\sigma}, \mathbf{I}^{\rho}\bar{\upsilon}_{\sigma}], 0 \le \sigma \le 1, \tag{2}$$

such that

$$\mathbf{I}^{\rho}\underline{\upsilon}_{\sigma} = \int_0^{\Phi} \frac{(\Phi-n)^{\rho-1}\underline{\upsilon}_{\sigma}(\eta)}{\Gamma(\rho)}\eta, \rho, \eta \in (0,\infty), \tag{3}$$

$$\mathbf{I}^{\rho}\bar{\upsilon}_{\sigma} = \int_0^{\Phi} \frac{(\Phi-n)^{\rho-1}\bar{\upsilon}_{\sigma}(\eta)}{\Gamma(\rho)}\eta, \rho, \eta \in (0,\infty). \tag{4}$$

Definition 2. For a function $\tilde{\upsilon} \in C^F[0,\rho] \cap L_F[0,\rho]$, such that $\tilde{\upsilon} = [\underline{\upsilon}_{\sigma}(\Phi), \bar{\upsilon}_{\sigma}(\Phi)], \sigma \in [0,1]$ and $\Phi_0 \in (0,\rho)$, then, the fractional Caputo fuzzy derivative is given as [37, 38]

$$\left[D_{\rho}\tilde{\upsilon}(\Phi_0)\right]_{\sigma} = \left[D_{\rho}\underline{\upsilon}(\Phi_0), D_{\rho}\bar{\upsilon}(\Phi_0)\right], 0 < \rho \le 1, \tag{5}$$

where

$$D^{\rho}\underline{\upsilon}_{\sigma}(\Phi_0) = \left[\int_0^{\Phi} \frac{(\Phi-n)^{m-\rho-1}(d^m/d\eta^m)\underline{\upsilon}_{\sigma}(\eta)}{\Gamma(\rho)}\eta\right]_{\Phi=\Phi_0}, \tag{6}$$

$$D^{\rho}\bar{\upsilon}_{\sigma}(\Phi_0) = \left[\int_0^{\Phi} \frac{(\Phi-n)^{m-\rho-1}(d^m/d\eta^m)\bar{\upsilon}_{\sigma}(\eta)}{\Gamma(\rho)}\eta\right]_{\Phi=\Phi_0}, \tag{7}$$

in such a way that the integrating on the right sides convergence and $m = \lceil\rho\rceil$. Since $\rho \in (0,1] m = 1$.

Definition 3. The Laplace fuzzy transformation for $f(\xi)$, where $f(\xi)$ is value fuzzy term is defined as [37, 38]

$$G(\xi) = L[f(\xi)] = \int_0^{\infty} (\exp)^{-\xi\Phi} f(\Phi) d\Phi, \Phi > 0. \tag{8}$$

Definition 4. In terms of fuzzy convolution, a fuzzy Laplace transformation is described as [37, 38]

$$L[f_1 * f_2] = L[f_1] * L[f_2], \tag{9}$$

where $f_1 * f_2$, define the fuzzy convolution between f_1 and f_2, i.e.,

$$f_1 * f_2 = \int_0^{\xi} f_1(\Phi) * f_2(\xi - \Phi) d\Phi. \tag{10}$$

Definition 5. The "Mittag-Leffler function" $E_{\rho}(p)$ is defined as

$$E_{\rho}(\Phi) = \sum_{n=0}^{\infty} \frac{\Phi^n}{\Gamma(n\rho+1)}, \tag{11}$$

where $\rho > 0$.

Definition 6. Let $\kappa : \mathfrak{R} \longrightarrow [0,1]$ be a number of fuzzy which have the specified properties [37, 38]

(i) κ is an upper semicontinuous number

(ii) $\kappa\{\mu(\chi_1) + \mu(\chi_2)\} \ge \min\{\kappa(\chi_1), \kappa(\chi_2)\}$

(iii) $\exists \chi_0 \in \mathbf{R}$ such that $\kappa(\chi_0) = 1$, i.e., ν is normal

(iv) $cl\{\chi \in \mathfrak{R}, \kappa(\chi) > 0\}$ is compact

The set of all fuzzy numbers is represented by the notation E.

Definition 7. The aforementioned number can be written in parametric form as $[\underline{\kappa}(\gamma), \bar{\kappa}(\gamma)]$, so that $\gamma \in [0, 1]$ combined with the values [37, 38]

(i) $\underline{\kappa}(\sigma)$ from left is continuous, and bounded function increasing over $[0, 1]$

(ii) $\underline{\kappa}(\sigma)$ from right is continuous, and bounded function decreasing over $[0, 1]$

$$\underline{\kappa} \leq \bar{\kappa}. \tag{12}$$

3. Main Work with Applications

$$\begin{gathered} D^{\rho}_{\Phi}\tilde{v}(\xi, \Phi) = D^{2}_{\xi}\tilde{v}(\xi, \Phi) + \tilde{v}(\xi, \Phi) + \tilde{\kappa}(\sigma), 0 < \rho \leq 1, \\ \tilde{v}(\xi, 0) = \tilde{g}(\xi). \end{gathered} \tag{13}$$

In this case, we use the Laplace transform on (13) as

$$\mathscr{L}\left[D^{\rho}_{\xi}\tilde{v}(\xi, \Phi)\right] = \mathscr{L}\left[D^{2}_{\xi}\tilde{v}(\xi, \Phi) + \tilde{v}(\xi, \Phi) + \tilde{\kappa}\right], \tag{14}$$

with initial condition using, we get

$$\begin{aligned} s^{\rho}\mathscr{L}[\tilde{v}(\xi, \Phi)] &= s^{\rho-1}\tilde{g}(\xi) + \mathscr{L}\left[D^{2}_{\xi}\tilde{v}(\xi, \Phi) + \tilde{v}(\xi, \Phi) + \tilde{\kappa}\right], \\ \mathscr{L}[\tilde{v}(\xi, \Phi)] &= \frac{\tilde{g}(\xi)}{s} + \frac{1}{s^{\rho}}\mathscr{L}\left[D^{2}_{\xi}\tilde{v}(\xi, \Phi) + \tilde{v}(\xi, \Phi) + \tilde{\kappa}\right]. \end{aligned} \tag{15}$$

Suppose that the result as $\tilde{v}(\xi, \Phi) = \sum_{n=0}^{\infty} U_n(\xi, \Phi)$, then, (15) defines

$$\mathscr{L}\left[\sum_{n=0}^{\infty}\tilde{v}_n(\xi, \Phi)\right] = \frac{\tilde{g}(\xi)}{s} + \frac{1}{s^{\rho}}\mathscr{L}\left[D^{2}_{\xi}\sum_{n=0}^{\infty}\tilde{v}_n(\xi, \Phi) + \sum_{n=0}^{\infty}\tilde{v}_n(\xi, \Phi) + \tilde{\kappa}\right]. \tag{16}$$

On both sides' term comparisons, we get

$$\begin{aligned} \mathscr{L}[\tilde{v}_0(\xi, \Phi)] &= \frac{\tilde{g}(\xi)}{s} + \frac{1}{s^{\rho}}\mathscr{L}[\tilde{\kappa}], \\ \mathscr{L}[\tilde{v}_1(\xi, \Phi)] &= \frac{1}{s^{\rho}}\mathscr{L}\left[D^{2}_{\xi}\tilde{v}_0(\xi, \Phi) + \tilde{v}_0(\xi, \Phi)\right], \\ \mathscr{L}[\tilde{v}_2(\xi, \Phi)] &= \frac{1}{s^{\rho}}\mathscr{L}\left[D^{2}_{\xi}\tilde{v}_1(\xi, \Phi) + \tilde{v}_1(\xi, \Phi)\right], \\ &\vdots \\ \mathscr{L}[\tilde{v}_{n+1}(\xi, \Phi)] &= \frac{1}{s^{\rho}}\mathscr{L}\left[D^{2}_{\xi}\tilde{v}_n(\xi, \Phi) + \tilde{v}_n(\xi, \Phi)\right],\ n \geq 0. \end{aligned} \tag{17}$$

Using inverse Laplace transformation, we get

$$\begin{aligned} \tilde{v}_0(\xi, \Phi) &= \tilde{g}(\xi) + \mathscr{L}^{-1}\left[\frac{1}{s^{\rho}}\mathscr{L}[\tilde{\kappa}]\right], \\ \tilde{v}_1(\xi, \Phi) &= \mathscr{L}^{-1}\left[\frac{1}{s^{\rho}}\mathscr{L}\left[D^{2}_{\xi}\tilde{v}_0(\xi, \Phi) + \tilde{v}_0(\xi, \Phi)\right]\right], \\ &\vdots \\ \tilde{v}_{n+1}(\xi, \Phi) &= \mathscr{L}^{-1}\left[\frac{1}{s^{\rho}}\mathscr{L}\left[D^{2}_{\xi}\tilde{v}_n(\xi, \Phi) + \tilde{v}_n(\xi, \Phi)\right]\right], n \geq 0. \end{aligned} \tag{18}$$

As a consequence, the needed series result is obtained by

$$\tilde{v}(\xi, \Phi) = \tilde{v}_0(\xi, \Phi) + \tilde{v}_1(\xi, \Phi) + \tilde{v}_2(\xi, \Phi) + \cdots, \tag{19}$$

4. Numerical Result

Example 1. Consider fuzzy fractional Klein–Gordon equation with the fuzzy initial condition

$$\frac{\partial^{\rho}\tilde{v}(\xi, \Phi)}{\partial\Phi^{\rho}} - \frac{\partial^{2}\tilde{v}(\xi, \Phi)}{\partial\xi^{2}} + \tilde{v}(\xi, \Phi) = 0, 1 < \rho \leq 2, \tag{20}$$

with the initial conditions

$$\tilde{v}(\xi, 0) = \tilde{\kappa}(\sigma)(0), \tilde{v}_{\Phi}(\xi, 0) = \tilde{\kappa}(\sigma)\xi, \tag{21}$$

where $\tilde{\kappa}(\sigma) = [\underline{\kappa}(\sigma), \bar{\kappa}(\sigma)] = [\sigma - 1, 1 - \sigma], 0 \leq \sigma \leq 1$. Using the abovementioned procedure as described in (18), we obtain the following results.

$$\begin{aligned} &\underline{v}_0(\xi, \Phi) = \underline{\kappa}(\sigma)\Phi\xi, \bar{v}_0(\xi, \Phi) = \bar{\kappa}(\sigma)\Phi\xi, \\ &\underline{v}_1(\xi, \Phi) = -\underline{\kappa}(\sigma)\Phi\frac{\xi^{\rho+1}}{\Gamma(\rho+2)}, \bar{v}_1(\xi, \Phi) = -\bar{\kappa}(\sigma)\Phi\frac{\xi^{\rho+1}}{\Gamma(\rho+2)}, \\ &\underline{v}_2(\xi, \Phi) = \underline{\kappa}(\sigma)\Phi\frac{\xi^{2\rho+1}}{\Gamma(2\rho+2)}, \bar{v}_2(\xi, \Phi) = \bar{\kappa}(\sigma)\Phi\frac{\xi^{2\rho+1}}{\Gamma(2\rho+2)}, \\ &\underline{v}_3(\xi, \Phi) = -\underline{\kappa}(\sigma)\Phi\frac{\xi^{3\rho+1}}{\Gamma(3\rho+2)},\ \bar{v}_3(\xi, \Phi) = -\bar{\kappa}(\sigma)\Phi\frac{\xi^{3\rho+1}}{\Gamma(3\rho+2)}, \end{aligned} \tag{22}$$

and so forth, and more terms can be determined in this manner. As a result of (19), we can write the needed series result as an infinite series.

$$\tilde{v}(\xi, \Phi) = \tilde{v}_0(\xi, \Phi) + \tilde{v}_1(\xi, \Phi) + \tilde{v}_2(\xi, \Phi) + \tilde{v}_3(\xi, \Phi) + \cdots, \tag{23}$$

such that

$$\begin{aligned} \underline{v}(\xi, \Phi) &= \underline{v}_0(\xi, \Phi) + \underline{v}_1(\xi, \Phi) + \underline{v}_2(\xi, \Phi) + \underline{v}_3(\xi, \Phi) + \cdots, \\ \bar{v}(\xi, \Phi) &= \bar{v}_0(\xi, \Phi) + \bar{v}_1(\xi, \Phi) + \bar{v}_2(\xi, \Phi) + \bar{v}_3(\xi, \Phi) + \cdots. \end{aligned} \tag{24}$$

In general, we can write as follows:

$$\underline{\upsilon}(\xi,\Phi)=\underline{\kappa}(\sigma)\Phi\xi-\underline{\kappa}(\sigma)\Phi\frac{\xi+1^{\rho}}{\Gamma(\rho+2)}+\underline{\kappa}(\sigma)\Phi\frac{\xi^{2\rho+1}}{\Gamma(2\rho+2)}+\underline{\kappa}(\sigma)\Phi\frac{\xi^{3\rho+1}}{\Gamma(3\rho+2)}+\cdots,$$
$$\bar{\upsilon}(\xi,\Phi)=\bar{\kappa}(\sigma)\Phi\xi-\bar{\kappa}(\sigma)\Phi\frac{\xi+1^{\rho}}{\Gamma(\rho+2)}+\bar{\kappa}(\sigma)\Phi\frac{\xi^{2\rho+1}}{\Gamma(2\rho+2)}+\bar{\kappa}(\sigma)\Phi\frac{\xi^{3\rho+1}}{\Gamma(3\rho+2)}+\cdots. \quad (25)$$

The exact result is

$$\tilde{\upsilon}(\xi,\Phi)=\tilde{\kappa}(\sigma)\xi\sin(\Phi). \quad (26)$$

In Figure 1, first two-dimensional fuzzy upper branch graph and second bottom branch graph of an analytic series result of various fractional of ρ. In Figure 2, first two-dimensional fuzzy upper branch graph and second bottom branch graph of an analytic series result of various fractional of ρ with respect to time. In Figure 3, graph depicts a two-dimensional fuzzy upper and bottom branch graph of an analytic series result with respect to space and time in example 1.

Example 2. Consider fuzzy fractional Klein–Gordon equation

$$\frac{\partial^{\rho}\tilde{\upsilon}(\xi,\Phi)}{\partial\Phi^{\rho}}-\frac{\partial^{2}\tilde{\upsilon}(\xi,\Phi)}{\partial\xi^{2}}+\tilde{\upsilon}(\xi,\Phi)=2\sin(\xi),0<\xi,\Phi<0,1<\rho\leq 2, \quad (27)$$

with the initial conditions

$$\tilde{\upsilon}(\xi,0)=\tilde{\kappa}(\sigma)\sin(\Phi),\tilde{\upsilon}_{\Phi}(\xi,0)=\tilde{\kappa}(\sigma), \quad (28)$$

where $\tilde{\kappa}(\sigma)=[\underline{\kappa}(\sigma),\bar{\kappa}(\sigma)]=[\sigma-1,1-\sigma],0\leq\sigma\leq 1$. Using the abovementioned procedure as described in (18), we obtain the following results.

$$\underline{\upsilon}_0(\xi,\Phi)=\underline{\kappa}(\sigma)\left(\sin(\xi)+\Phi+\frac{2\xi^{\rho}\sin(\xi)}{\Gamma(\rho+1)}\right),\bar{\upsilon}_0(\xi,\Phi)=\bar{\kappa}(\sigma)\left(\sin(\xi)+\Phi+\frac{2\xi^{\rho}\sin(\xi)}{\Gamma(\rho+1)}\right),$$
$$\underline{\upsilon}_1(\xi,\Phi)=\underline{\kappa}(\sigma)\left(-\frac{2\Phi^{\rho}\sin(\xi)}{\Gamma(\rho+1)}-\frac{4\Phi^{2\rho}\sin(\xi)}{\Gamma(2\rho+1)}-\frac{\Phi^{\rho+1}}{\Gamma(\rho+2)}\right),$$
$$\bar{\upsilon}_1(\xi,\Phi)=\bar{\kappa}(\sigma)\left(-\frac{2\Phi^{\rho}\sin(\xi)}{\Gamma(\rho+1)}-\frac{4\Phi^{2\rho}\sin(\xi)}{\Gamma(2\rho+1)}-\frac{\Phi^{\rho+1}}{\Gamma(\rho+2)}\right),$$
$$\underline{\upsilon}_2(\xi,\Phi)=\underline{\kappa}(\sigma)\left(\frac{4\Phi^{2\rho}\sin(\xi)}{\Gamma(2\rho+1)}+\frac{8\Phi^{3\rho}\sin(\xi)}{\Gamma(3\rho+1)}+\frac{\Phi^{2\rho+1}}{\Gamma(2\rho+2)}\right),$$
$$\bar{\upsilon}_2(\xi,\Phi)=\bar{\kappa}(\sigma)\left(\frac{4\Phi^{2\rho}\sin(\xi)}{\Gamma(2\rho+1)}+\frac{8\Phi^{3\rho}\sin(\xi)}{\Gamma(3\rho+1)}+\frac{\Phi^{2\rho+1}}{\Gamma(2\rho+2)}\right),$$
$$\underline{\upsilon}_3(\xi,\Phi)=\underline{\kappa}(\sigma)\left(-\frac{8\Phi^{3\rho}\sin(\xi)}{\Gamma(3\rho+1)}-\frac{16\Phi^{4\rho}\sin(\xi)}{\Gamma(4\rho+1)}-\frac{\xi^{3\rho+1}}{\Gamma(3\rho+2)}\right),$$
$$\bar{\upsilon}_3(\xi,\Phi)=\bar{\kappa}(\sigma)\left(-\frac{8\Phi^{3\rho}\sin(\xi)}{\Gamma(3\rho+1)}-\frac{16\Phi^{4\rho}\sin(\xi)}{\Gamma(4\rho+1)}-\frac{\xi^{3\rho+1}}{\Gamma(3\rho+2)}\right), \quad (29)$$

and so forth, and more terms can be determined in this manner. As a result of (19), we can write the needed series result as an infinite series.

$$\tilde{\upsilon}(\xi,\Phi)=\tilde{\upsilon}_0(\xi,\Phi)+\tilde{\upsilon}_1(\xi,\Phi)+\tilde{\upsilon}_2(\xi,\Phi)+\tilde{\upsilon}_3(\xi,\Phi)+\cdots, \quad (30)$$

such that

$$\underline{\upsilon}(\xi,\Phi)=\underline{\upsilon}_0(\xi,\Phi)+\underline{\upsilon}_1(\xi,\Phi)+\underline{\upsilon}_2(\xi,\Phi)+\underline{\upsilon}_3(\xi,\Phi)+\cdots,$$
$$\bar{\upsilon}(\xi,\Phi)=\bar{\upsilon}_0(\xi,\Phi)+\bar{\upsilon}_1(\xi,\Phi)+\bar{\upsilon}_2(\xi,\Phi)+\bar{\upsilon}_3(\xi,\Phi)+\cdots. \quad (31)$$

In general, we can write as follows:

$$\underline{\upsilon}(\xi,\Phi)=\underline{\kappa}(\sigma)\left(\sin(\xi)+\Phi+\frac{2\xi^{\rho}\sin(\xi)}{\Gamma(\rho+1)}\right)+\underline{\kappa}(\sigma)\left(-\frac{2\Phi^{\rho}\sin(\xi)}{\Gamma(\rho+1)}-\frac{4\Phi^{2\rho}\sin(\xi)}{\Gamma(2\rho+1)}-\frac{\Phi^{\rho+1}}{\Gamma(\rho+2)}\right)+\underline{\kappa}(\sigma)\left(\frac{4\Phi^{2\rho}\sin(\xi)}{\Gamma(2\rho+1)}+\frac{8\Phi^{3\rho}\sin(\xi)}{\Gamma(3\rho+1)}+\frac{\Phi^{2\rho+1}}{\Gamma(2\rho+2)}\right)+\underline{\kappa}(\sigma)\left(-\frac{8\Phi^{3\rho}\sin(\xi)}{\Gamma(3\rho+1)}-\frac{16\Phi^{4\rho}\sin(\xi)}{\Gamma(4\rho+1)}-\frac{\xi^{3\rho+1}}{\Gamma(3\rho+2)}\right)+\cdots, \quad (32)$$

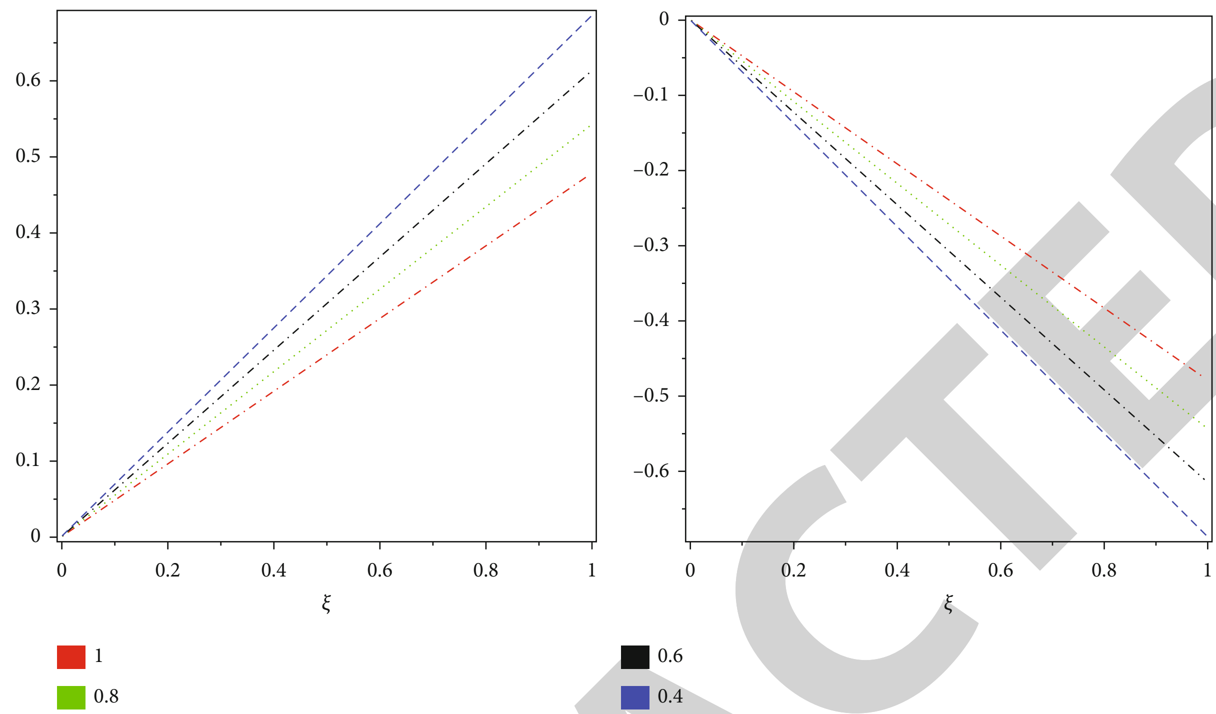

Figure 1: First two-dimensional fuzzy upper branch graph and second bottom branch graph of an analytic series result of various fractional of ρ.

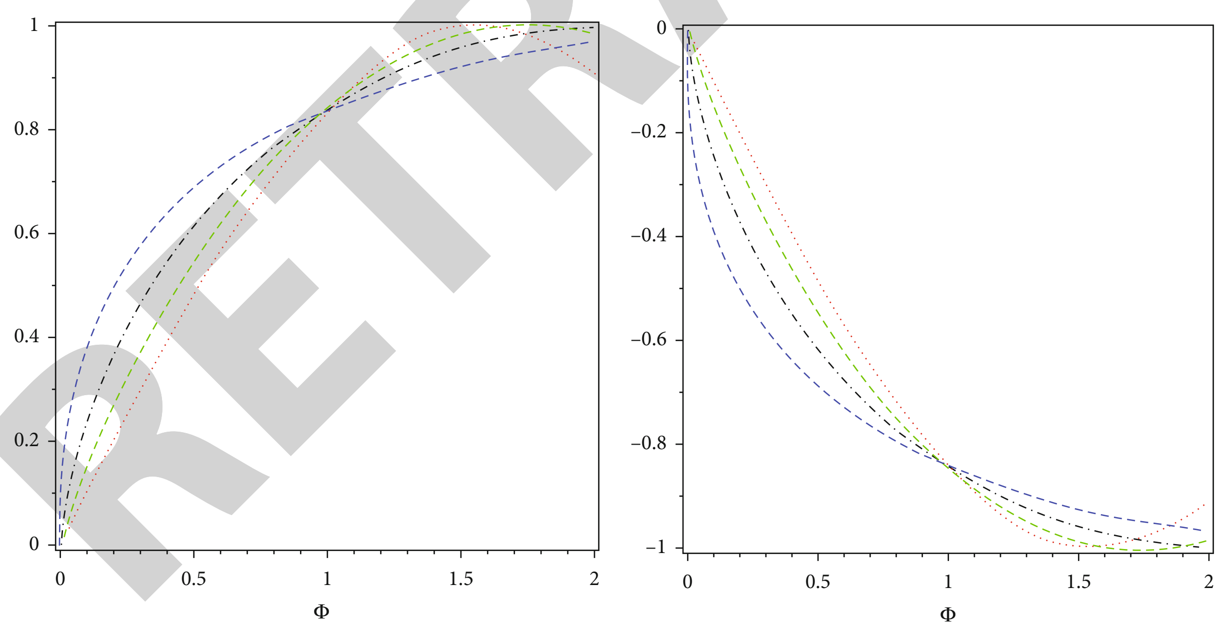

Figure 2: First two-dimensional fuzzy upper branch graph and second bottom branch graph of an analytic series result of various fractional of ρ with respect to time.

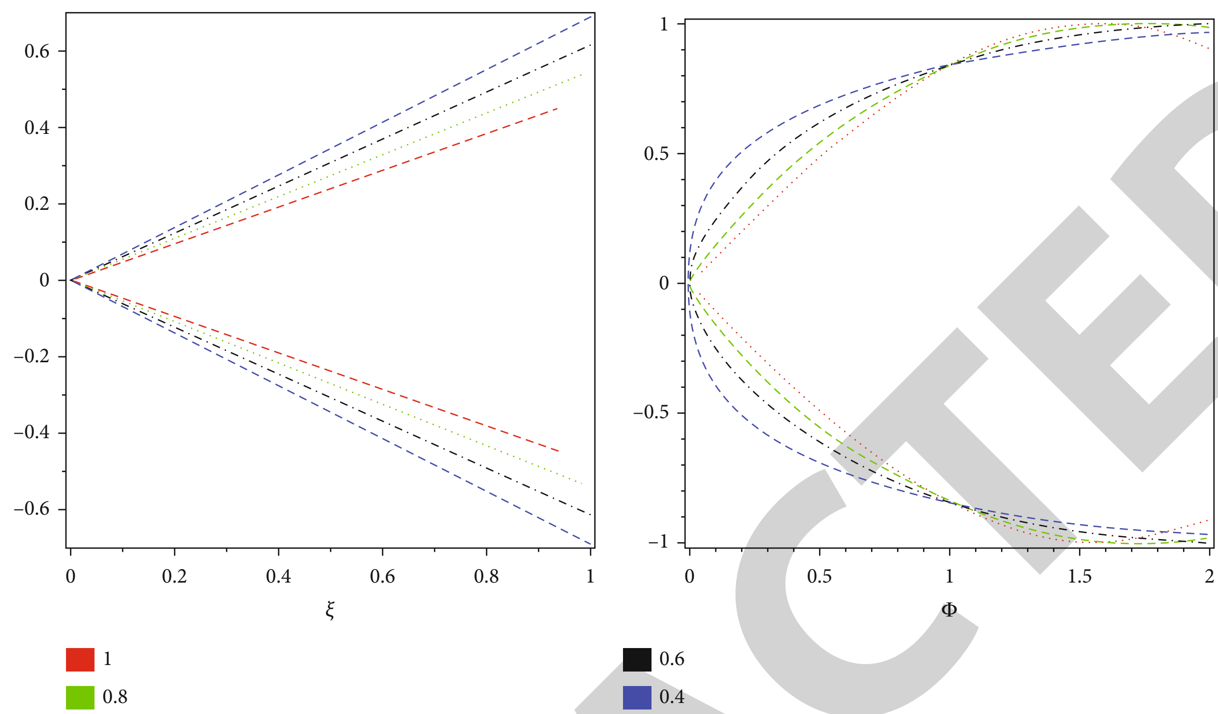

Figure 3: The first graph depicts a two-dimensional fuzzy upper and bottom branch graph of an analytic series result with respect to space and time.

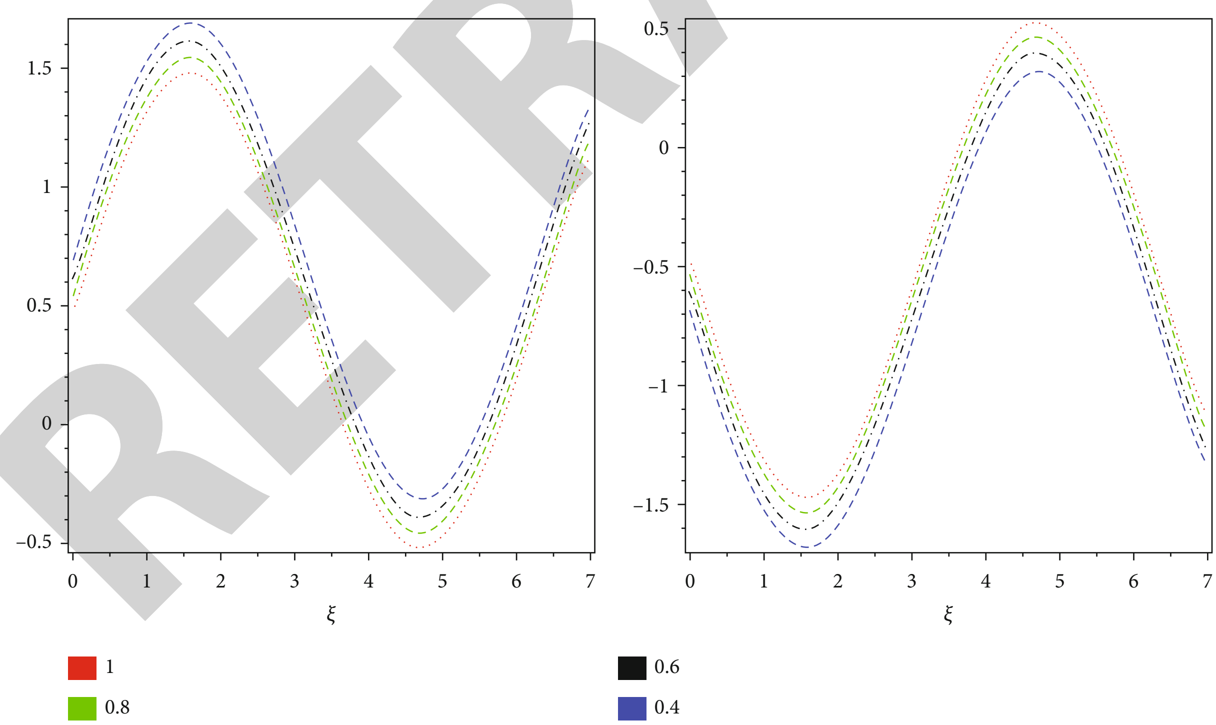

Figure 4: First two-dimensional fuzzy upper branch graph and second bottom branch graph of an analytic series result of various fractional of ρ.

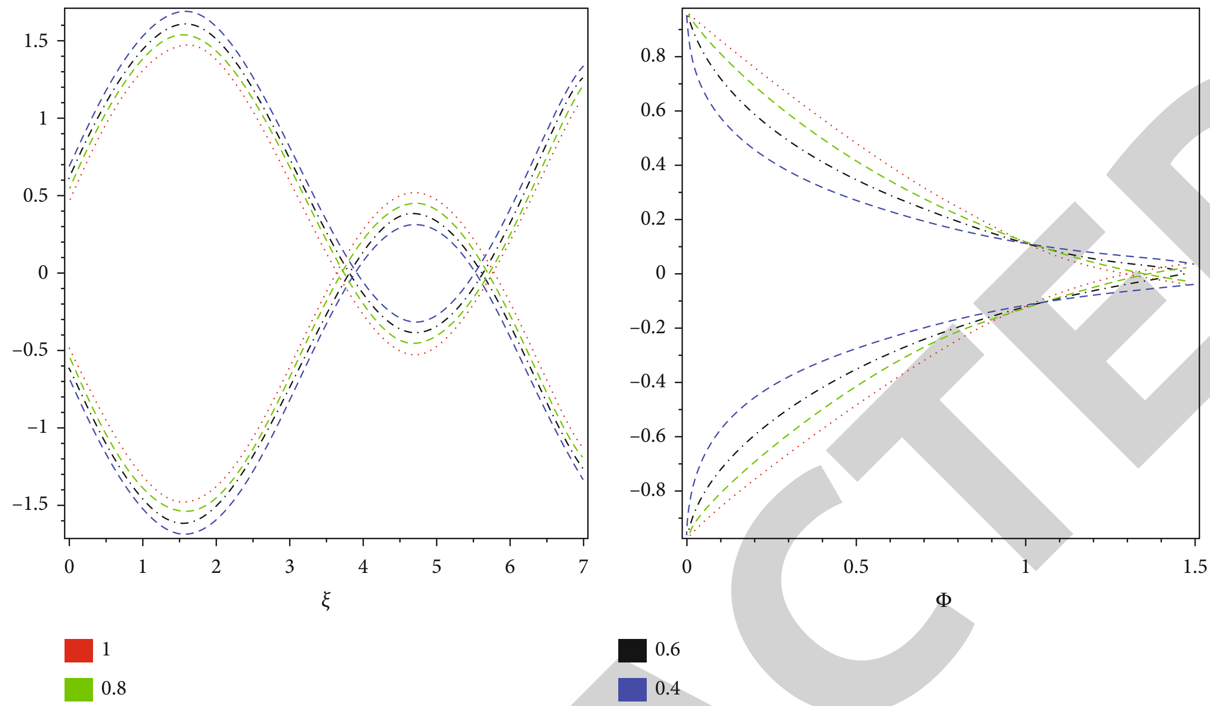

FIGURE 5: First two-dimensional fuzzy upper branch graph and second bottom branch graph of an analytic series result of various fractional of ρ with respect to time.

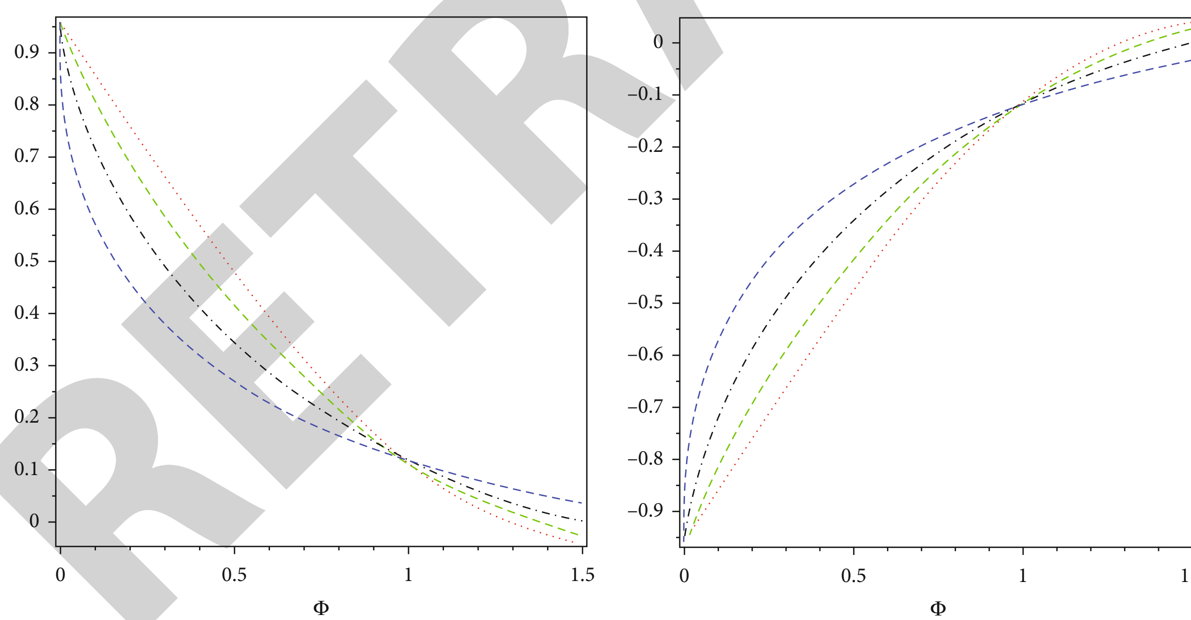

FIGURE 6: The first graph depicts a two-dimensional fuzzy upper and bottom branch graph of an analytic series result with respect to space and time.

$$\bar{\upsilon}(\xi,\Phi)=\bar{\kappa}(\sigma)\left(\sin(\xi)+\Phi+\frac{2\xi^{\rho}\sin(\xi)}{\Gamma(\rho+1)}\right)+\bar{\kappa}(\sigma)\left(-\frac{2\Phi^{\rho}\sin(\xi)}{\Gamma(\rho+1)}-\frac{4\Phi^{2\rho}\sin(\xi)}{\Gamma(2\rho+1)}-\frac{\Phi^{\rho+1}}{\Gamma(\rho+2)}\right)+\bar{\kappa}(\sigma)\left(\frac{4\Phi^{2\rho}\sin(\xi)}{\Gamma(2\rho+1)}+\frac{8\Phi^{3\rho}\sin(\xi)}{\Gamma(3\rho+1)}+\frac{\Phi^{2\rho+1}}{\Gamma(2\rho+2)}\right)+\bar{\kappa}(\sigma)\left(-\frac{8\Phi^{3\rho}\sin(\xi)}{\Gamma(3\rho+1)}-\frac{16\Phi^{4\rho}\sin(\xi)}{\Gamma(4\rho+1)}-\frac{\xi^{3\rho+1}}{\Gamma(3\rho+2)}\right)+\cdots. \tag{33}$$

The exact result is

$$\tilde{\upsilon}(\xi,\Phi)=\tilde{\kappa}(\sigma)(\sin(\xi)+\sin(\Phi)). \tag{34}$$

In Figure 4, first two-dimensional fuzzy upper branch graph and second bottom branch graph of an analytic series result of various fractional of ρ. In Figure 5, first two-dimensional fuzzy upper branch graph and second bottom branch graph of an analytic series result of various fractional of ρ with respect to time. In Figure 6, graph depicts a two-dimensional fuzzy upper and bottom branch graph of an analytic series result with respect to space and time in example 2.

5. Conclusion

This study is aimed to propose a semianalytic solution to the fuzzy fractional Klein-Gordon equations by employing Caputo fractional derivatives. An important example validated the conclusion reached. Additionally, we supplied graphs of the numerical solution in various fractional order. When illustrated in the pictures, the plots will close with the curve of classical order one as the fractional-order ρ approaches its integer value. As a result, we observed that fractional calculus accurately describes the dynamic global nature of the equations relating to the fuzzy idea. In future research, we intend to extend this approach to more dynamic fuzzy models. This technique may be utilized in the future to construct analytic and approximation solutions to perturbed fractional differential equations with nonclassical and integral initial conditions in the sense of the Caputo operator.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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