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Research Article

Fractional-View Analysis of Space-Time Fractional Fokker-Planck Equations within Caputo Operator

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In this article, we investigate the fractional-order Fokker-Planck equations with the help of the Yang transform decomposition method (YTDM). The YTDM combines Yang transform, Adomian decomposition method, and Adomian polynomials into one method. In the Caputo sense, fractional derivatives of space and time are studied. The convergent series form solution demonstrates the method's efficiency in resolving several types of fractional differential equations. Compared to other methods of finding approximate and exact solutions for nonlinear partial differential equations, this technique is more efficient and time-consuming.

1. Introduction

Fractional calculus, which can be thought of as a generalization of integer-order differentiation and integration, has received much attention in recent decades. Many definitions have been proposed for fractional derivatives, including Riesz, Grunwald-Letnikov, Caputo, Riemann-Liouville, and conformable fractional definitions [1–4]. Noninteger order integral and differential operators contain all historical conditions of the function in a weighted form known as the memory effect. In any case, fractional differential equations (FDEs), specifically fractional partial differential equations, are used to analyze a broad range of physical systems (FPDEs). FPDEs have gained attention due to their widespread application in electrical circuits, electrochemistry, quantum physics, and theoretical biology [5–8]. Furthermore, the nonlocal property of FPDEs is the most important feature for using them in such and other applications, whereas the differential operator having order integer is local. In this light, the next state of a fractional system is determined by both its current and historical states. This ensures that the mathematical model components in physical processes and dynamic systems are highly consistent. However, it is not easy to solve those FDEs, particularly for numerical calculations [9–11]. To handle partial differential equations (PDEs), having order fraction is of physical importance, and effective, trustworthy, and appropriate numerical methods are required [12–14]. Several major strategies have been utilized in this regard, including the fractional operational matrix method (FOMM) [15], Elzaki transform decomposition method (ETDM) [16, 17], homotopy analysis method (HAM) [18], homotopy perturbation method (HPM) [19, 20], iterative Laplace transform method [21], and variational iteration method (FVIM) [22].

The Fokker-Planck equation is a well-known statistical physics equation that Fokker and Planck first proposed to describe a particle's Brownian motion and the change in probability of a random function in time and space [23]. An uncontrolled, second-order truncation of the Kramers-Moyal expansion of the chemical master equation can also be used to obtain the chemical Fokker-Planck equation. This equation proves to be more accurate than the chemical master equation's linear-noise approximation. The Fokker-Planck equation appears in many natural science phenomena, such as probability flux, polymer dynamics, electron relaxation, solid-state systems, quantum optics, and other practical and theoretical models [24].

We have studied Fokker-Planck equations of fractional-order having general form as

$$\varphi^{\gamma}_{\mathfrak{I}}(\mu, \mathfrak{I}) = L\left(\varphi_{\mu}(\mu, \mathfrak{I}) + \varphi_{\mu\mu}(\mu, \mathfrak{I})\right) + N\varphi_{\mu\mu}(\mu, \mathfrak{I}), \mu, \mathfrak{I} > 0, \ \gamma \in (0, 1], \tag{1}$$

with the initial condition

$$\varphi(\mu, 0) = \zeta(\mu). \tag{2}$$

In biological molecules, chemical physics, energy consumption, and engineering, the fractional Fokker-Planck equation (F-FPE) has been successfully applied. Indeed, fractional diffusion, a special kind of F-FPE, has also been used in numerous scenarios such as frequency-dependent damping behaviour of materials, viscoelasticity, and diffusion processes [17]. Unfortunately, finding an accurate solution for FDEs, in general, is difficult. To approximate these solutions, various numerical and analytical techniques are used. Some of the advanced numerical and approximate methods used for F-FPEs include the Laplace transform method [18], the multistep reduced differential transform method [25], the predictor-corrector approach [26], the Adomian decomposition method (ADM) [27], and the variational iteration method (VIM) [28].

In this research, we used the Yang transform decomposition method (YTDM) to solve time-fractional F-FPEs. The Yang transform was proposed by Xiao-Jun Yang and can be utilized to solve a variety of differential equations with constant coefficients. The Adomian decomposition method [29] is a well-known methodology to solve linear and nonlinear differential and partial differential equations and integrodifferential and FDEs that yield accurate solutions in a concurrent series form. The results of the suggested strategy are convincing and offering specific solutions to the problems at work. The fractional problem results obtained through the given approach are also used to analyze the problems fractionally. It has been confirmed that the proposed technique can be implemented to solve various fractional PDEs and related systems.

2. Preliminaries

We covered several fundamental definitions of fractional calculus as well as Yang transform theory features in this part.

Definition 1. The fractional Caputo derivative is defined as

$$D^{\gamma}_{\varphi}\varphi(\mu, \mathfrak{I}) = \frac{1}{\Gamma(k-\gamma)}\int_0^{\mathfrak{I}} (\mathfrak{I} - \vartheta)^{k-\gamma-1}\varphi^{(k)}(\mu, \vartheta)d\vartheta, k-1 < \gamma \leq k, k \in N. \tag{3}$$

Definition 2. Xiao-Jun Yang introduced the Yang Laplace transform in 2018. $\varphi(\mathfrak{I})$ or $M(u)$ determines the Yang transform for a function $\varphi(\mathfrak{I})$ and is provided as

$$\mathbf{Y}\{\varphi(\mathfrak{I})\} = M(u) = \int_0^{\infty} e^{-\mathfrak{I}/u}\varphi(\mathfrak{I})d\mathfrak{I}, \mathfrak{I} > 0, u \in (-\mathfrak{I}_1, \mathfrak{I}_2). \tag{4}$$

The inverse Yang transform is given as

$$\mathbf{Y}^{-1}\{M(u)\} = \varphi(\mathfrak{I}). \tag{5}$$

Definition 3. For nth derivatives, the Yang transform is given as

$$\mathbf{Y}\{\varphi^{n}(\mathfrak{I})\} = \frac{M(u)}{u^n} - \sum_{k=0}^{n-1}\frac{\varphi^k(0)}{u^{n-k-1}}, \forall n = 1, 2, 3, \cdots. \tag{6}$$

Definition 4. For derivative having fractional order, the Yang transform is

$$\mathbf{Y}\{\varphi^{\gamma}(\mathfrak{I})\} = \frac{M(u)}{u^{\gamma}} - \sum_{k=0}^{n-1}\frac{\varphi^k(0)}{u^{\gamma-(k+1)}}, 0 < \gamma \leq n. \tag{7}$$

3. Idea of YTDM

The general methodology for solving fractional partial differential equations is given as

$$D^{\gamma}_{\mathfrak{I}}\varphi(\mu, \mathfrak{I}) = \mathscr{P}_1(\mu, \mathfrak{I}) + \mathscr{Q}_1(\mu, \mathfrak{I}), 0 < \gamma \leq 1, \tag{8}$$

with initial sources

$$\varphi(\mu, 0) = \varphi(\mu), \quad \frac{\partial}{\partial \mathfrak{I}}\varphi(\mu, 0) = \zeta(\mu), \tag{9}$$

where Caputo fractional derivative having order γ is represented by $D^{\gamma}_{\mathfrak{I}} = \partial^{\gamma}/\partial\mathfrak{I}^{\gamma}$; $\mathscr{P}_1$ and $\mathscr{Q}_1$ are linear and nonlinear functions, respectively.

On employing Yang transform, we get

$$\mathbf{Y}\left[D^{\gamma}_{\mathfrak{I}}\varphi(\mu, \mathfrak{I})\right] = \mathbf{Y}[\mathscr{P}_1(\mu, \mathfrak{I}) + \mathscr{Q}_1(\mu, \mathfrak{I})]. \tag{10}$$

By using Yang differentiation property, we have

$$\frac{1}{u^{\gamma}}\left\{M(u) - u\varphi(0) - u^2\varphi'(0)\right\} = \mathbf{Y}[\mathscr{P}_1(\mu, \mathfrak{I}) + \mathscr{Q}_1(\mu, \mathfrak{I})]. \tag{11}$$

From above equation

$$M(\varphi) = u\varphi(0) + u^2\varphi'(0) + u^{\gamma}\mathbf{Y}[\mathscr{P}_1(\mu, \mathfrak{I}) + \mathscr{Q}_1(\mu, \mathfrak{I})]. \tag{12}$$

On applying inverse Yang transform, we have

$$\varphi(\mu, \mathfrak{T}) = \varphi(0) + \varphi'(0) + \mathbf{Y}^{-1}[u^\gamma \mathbf{Y}[\mathscr{P}_1(\mu, \mathfrak{T}) + \mathscr{Q}_1(\mu, \mathfrak{T})]. \tag{13}$$

The solution in terms of infinite sequence $\varphi(\mu, \mathfrak{T})$ by means of YTDM is

$$\varphi(\mu, \mathfrak{T}) = \sum_{m=0}^{\infty} \varphi_m(\mu, \mathfrak{T}). \tag{14}$$

Now, the nonlinear terms by means of Adomian polynomials are decomposed as

$$\mathscr{Q}_1(\mu, \mathfrak{T}) = \sum_{m=0}^{\infty} \mathscr{A}_m. \tag{15}$$

The Adomian polynomials all forms of nonlinearity are given as

$$\mathscr{A}_m = \frac{1}{m!}\left[\frac{\partial^m}{\partial \ell^m}\left\{\mathscr{Q}_1\left(\sum_{k=0}^{\infty} \ell^k \mu_k, \sum_{k=0}^{\infty} \ell^k \mathfrak{T}_k\right)\right\}\right]_{\ell=0}. \tag{16}$$

By substituting Equation (35) and Equation (38) into (34), we have

$$\sum_{m=0}^{\infty} \varphi_m(\mu, \mathfrak{T}) = \varphi(0) + \varphi'(0) + \mathbf{Y}^{-1} u^\gamma \cdot \left[\mathbf{Y}\left\{\mathscr{P}_1\left(\sum_{m=0}^{\infty} \mu_m, \sum_{m=0}^{\infty} \mathfrak{T}_m\right) + \sum_{m=0}^{\infty} \mathscr{A}_m\right\}\right]. \tag{17}$$

The below terms are derived.

$$\begin{aligned} \varphi_0(\mu, \mathfrak{T}) &= \varphi(0) + \mathfrak{T}\varphi'(0), \\ \varphi_1(\mu, \mathfrak{T}) &= \mathbf{Y}^{-1}[u^\gamma \mathbf{Y}^+\{\mathscr{P}_1(\mu_0, \mathfrak{T}_0) + \mathscr{A}_0\}], \end{aligned} \tag{18}$$

thus for $m \geq 1$, the general term is given as

$$\varphi_{m+1}(\mu, \mathfrak{T}) = \mathbf{Y}^{-1}[u^\gamma \mathbf{Y}^+\{\mathscr{P}_1(\mu_m, \mathfrak{T}_m) + \mathscr{A}_m\}]. \tag{19}$$

Theorem 5. *Here, we will study the convergence analysis as same manner in [30] of the YTDM applied to the fractional order partial differential equation. Let us consider the Hilbert space H which may define by $H = L^2((\alpha, \beta)X[0, T])$ the set of applications:*

$$u : (\alpha, \beta)X[0, T] \longrightarrow with \int_{(\alpha,\beta)X[0,T]} u^2(x, s)\,ds\,d\theta < +\infty. \tag{20}$$

Now, we consider the fractional partial differential equation in the above assumptions and let us denote

$$\mathbf{Y}(u) = \frac{\partial^\gamma u}{\partial \mathfrak{T}^\gamma}, \tag{21}$$

then the fractional partial differential equation becomes in an operator form

$$\mathbf{Y}(u) = -\varphi \frac{\partial v(x, \mathfrak{T})}{\partial x} - w \frac{\partial^3 v(x, \mathfrak{T})}{\partial x^3}. \tag{22}$$

The YTDM is convergence if the following two hypotheses are satisfied:

H1: $(\mathbf{Y}(u) - \mathbf{Y}(v), u - v) \geq k\|u - v\|^2$; $k > 0, \forall u, v\varepsilon H$

H2: whatever may be $M > 0$, there exist a constant $C(M) > 0$ such that for $u, v\varepsilon H$ with $\|u\| \leq M$ and $\|v\| \leq M$ we have $(\mathbf{Y}(u) - \mathbf{Y}(v), u - v) \leq C(M)\|u - v\|\|w\|$ for every $w\varepsilon H$

4. Applications

Here, in this part, we implemented YTDM for solving various time-fractional Fokker-Planck equation.

Example 1. Consider F-FPEs of the form

$$\frac{\partial^\gamma}{\partial \mathfrak{T}^\gamma}(\varphi(\mu, \mathfrak{T})) + \frac{\partial}{\partial \mu}\left(\frac{\mu}{6}\varphi(\mu, \mathfrak{T})\right) - \frac{\partial^2}{\partial \mu^2}\left(\frac{\mu^2}{12}\varphi(\mu, \mathfrak{T})\right) = 0, \mu, \mathfrak{T} > 0, \ \gamma \in (0, 1], \tag{23}$$

with the initial condition

$$\varphi(\mu, 0) = \mu^2. \tag{24}$$

On employing Yang transform, we get

$$\mathbf{Y}\left\{\frac{\partial^\gamma \varphi}{\partial \mathfrak{T}^\gamma}\right\} = \mathbf{Y}\left[-\frac{\partial}{\partial \mu}\left(\frac{\mu}{6}\varphi(\mu, \mathfrak{T})\right) + \frac{\partial^2}{\partial \mu^2}\left(\frac{\mu^2}{12}\varphi(\mu, \mathfrak{T})\right)\right]. \tag{25}$$

By using Yang differentiation property, we have

$$\begin{aligned} \frac{1}{u^\gamma}\{M(u) - u\varphi(0)\} &= \mathbf{Y}\left[-\frac{\partial}{\partial \mu}\left(\frac{\mu}{6}\varphi(\mu, \mathfrak{T})\right) + \frac{\partial^2}{\partial \mu^2}\left(\frac{\mu^2}{12}\varphi(\mu, \mathfrak{T})\right)\right], \\ M(u) &= u\varphi(0) + u^\gamma \mathbf{Y}\left[-\frac{\partial}{\partial \mu}\left(\frac{\mu}{6}\varphi(\mu, \mathfrak{T})\right) + \frac{\partial^2}{\partial \mu^2}\left(\frac{\mu^2}{12}\varphi(\mu, \mathfrak{T})\right)\right]. \end{aligned} \tag{26}$$

On applying inverse Yang transform, we have

$$\varphi(\mu,\mathfrak{I}) = \varphi(0) + \mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left(-\frac{\partial}{\partial\mu}\left(\frac{\mu}{6}\varphi(\mu,\mathfrak{I})\right) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{12}\varphi(\mu,\mathfrak{I})\right)\right)\right\}\right],$$
$$\varphi(\mu,\mathfrak{I}) = \mu^2 + \mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left(-\frac{\partial}{\partial\mu}\left(\frac{\mu}{6}\varphi(\mu,\mathfrak{I})\right) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{12}\varphi(\mu,\mathfrak{I})\right)\right)\right\}\right]. \tag{27}$$

The solution in terms of infinite sequence $\varphi(\mu,\mathfrak{I})$ by means of YTDM is

$$\varphi(\mu,\mathfrak{I}) = \sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}),$$
$$\sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}) = \mu^2 + \mathbf{Y}^{-1}\left[u^{\gamma}\mathbf{Y}\left[-\frac{\partial}{\partial\mu}\left(\frac{\mu}{6}\varphi(\mu,\mathfrak{I})\right) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{12}\varphi(\mu,\mathfrak{I})\right)\right]\right]. \tag{28}$$

By comparing Equation (28) both sides, we get

$$\varphi_0(\mu,\mathfrak{I}) = \mu^2. \tag{29}$$

On $m = 0$,

$$\varphi_1(\mu,\mathfrak{I}) = \mu^2\frac{\mathfrak{I}^{\gamma}}{2\Gamma(\gamma+1)}. \tag{30}$$

On $m = 1$,

$$\varphi_2(\mu,\mathfrak{I}) = \mu^2\frac{\mathfrak{I}^{2\gamma}}{8\Gamma(2\gamma+1)}. \tag{31}$$

On $m = 2$,

$$\varphi_3(\mu,\mathfrak{I}) = \mu^2\frac{\mathfrak{I}^{3\gamma}}{24\Gamma(3\gamma+1)}. \tag{32}$$

The YTDM solution remaining components φ_m for $(m \geq 3)$ are calculated easily. Thus, we define the series form solution as

$$\varphi(\mu,\mathfrak{I}) = \sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}) = \varphi_0(\mu,\mathfrak{I}) + \varphi_1(\mu,\mathfrak{I}) + \varphi_2(\mu,\mathfrak{I}) + \varphi_3(\mu,\mathfrak{I}) + \cdots,$$
$$\varphi(\mu,\mathfrak{I}) = \mu^2 + \mu^2\frac{\mathfrak{I}^{\gamma}}{2\Gamma(\gamma+1)} + \mu^2\frac{\mathfrak{I}^{2\gamma}}{8\Gamma(2\gamma+1)} + \mu^2\frac{\mathfrak{I}^{3\gamma}}{24\Gamma(3\gamma+1)} + \cdots. \tag{33}$$

The YTDM solution at $\gamma = 1$ is

$$\varphi(\mu,\mathfrak{I}) = \mu^2\exp^{\mathfrak{I}/2}. \tag{34}$$

In Figure 1, the first graph shows the exact and second the analytical solution graph, which shows the close contact with each other. In Figure 1, the third and fourth graphs are the three- and two-dimensional graphs concerning different fractional order of problem 1. The figures show that the suggested technique agrees with the actual solution for the given problem. As fractional order approaches integer order, fractional-order solution surfaces converge to the integer-order surface, as depicted by graphs. It means that we may physically model any surface based on the physical events observed in nature.

Example 2. Consider F-FPEs of the form

$$\frac{\partial}{\partial\mathfrak{I}^{\gamma}}(\varphi(\mu,\mathfrak{I})) + \frac{\partial}{\partial\mu}(\mu\varphi(\mu,\mathfrak{I})) - \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{2}\varphi(\mu,\mathfrak{I})\right) = 0, \mu, \mathfrak{I} > 0, \ \gamma \in (0,1], \tag{35}$$

with the initial condition

$$\varphi(\mu,0) = \mu. \tag{36}$$

On employing Yang transform, we get

$$\mathbf{Y}\left\{\frac{\partial^{\gamma}\varphi}{\partial\mathfrak{I}^{\gamma}}\right\} = \mathbf{Y}\left[-\frac{\partial}{\partial\mu}(\mu\varphi(\mu,\mathfrak{I})) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{2}\varphi(\mu,\mathfrak{I})\right)\right]. \tag{37}$$

By using Yang differentiation property, we have

$$\frac{1}{u^{\gamma}}\{M(u) - u\varphi(0)\} = \mathbf{Y}\left[-\frac{\partial}{\partial\mu}(\mu\varphi(\mu,\mathfrak{I})) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{2}\varphi(\mu,\mathfrak{I})\right)\right], \tag{38}$$

$$M(u) = u\varphi(0) + u^{\gamma}\mathbf{Y}\left[-\frac{\partial}{\partial\mu}(\mu\varphi(\mu,\mathfrak{I})) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{2}\varphi(\mu,\mathfrak{I})\right)\right]. \tag{39}$$

On applying inverse Yang transform, we have

$$\varphi(\mu,\mathfrak{I}) = \varphi(0) + \mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left(-\frac{\partial}{\partial\mu}(\mu\varphi(\mu,\mathfrak{I})) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{2}\varphi(\mu,\mathfrak{I})\right)\right)\right\}\right],$$
$$\varphi(\mu,\mathfrak{I}) = \mu + \mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left(-\frac{\partial}{\partial\mu}(\mu\varphi(\mu,\mathfrak{I})) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{2}\varphi(\mu,\mathfrak{I})\right)\right)\right\}\right]. \tag{40}$$

The solution in terms of infinite sequence $\varphi(\mu,\mathfrak{I})$ by means of YTDM is

$$\varphi(\mu,\mathfrak{I}) = \sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}),$$
$$\sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}) = \mu + \mathbf{Y}^{-1}\left[u^{\gamma}\mathbf{Y}\left[-\frac{\partial}{\partial\mu}(\mu\varphi(\mu,\mathfrak{I})) + \frac{\partial^2}{\partial\mu^2}\left(\frac{\mu^2}{2}\varphi(\mu,\mathfrak{I})\right)\right]\right]. \tag{41}$$

By comparing Equation (41) both sides, we get

$$\varphi_0(\mu,\mathfrak{I}) = \mu. \tag{42}$$

On $m = 0$,

$$\varphi_1(\mu,\mathfrak{I}) = \mu\frac{\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}. \tag{43}$$

On $m = 1$,

$$\varphi_2(\mu,\mathfrak{I}) = \mu\frac{\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)}. \tag{44}$$

On $m = 2$,

$$\varphi_3(\mu,\mathfrak{I}) = \mu\frac{\mathfrak{I}^{3\gamma}}{\Gamma(3\gamma+1)}. \tag{45}$$

The YTDM solution remaining components φ_m with ($m \geq 3$) are calculated easily. Thus, we define the series form solution as

$$\varphi(\mu,\mathfrak{I}) = \sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}) = \varphi_0(\mu,\mathfrak{I}) + \varphi_1(\mu,\mathfrak{I}) + \varphi_2(\mu,\mathfrak{I}) + \varphi_3(\mu,\mathfrak{I}) + \cdots,$$
$$\varphi(\mu,\mathfrak{I}) = \mu + \mu\frac{\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)} + \mu\frac{\mathfrak{I}^{2\gamma}}{\Gamma(2\gamma+1)} + \mu\frac{\mathfrak{I}^{3\gamma}}{\Gamma(3\gamma+1)} + \cdots. \tag{46}$$

The YTDM solution at $\gamma = 1$ is

$$\varphi(\mu,\mathfrak{I}) = \mu\exp^{\mathfrak{I}}. \tag{47}$$

In Figure 2, the first graph shows the exact and second the analytical solution graph, which shows the close contact with each other. In Figure 2, the third and fourth graphs are the three- and two-dimensional graphs concerning different fractional order of problem 2. The figures show that the suggested technique agrees with the actual solution for the given problem. As fractional order approaches integer order, fractional-order solution surfaces converge to the integer-order surface, as depicted by graphs. It means that we may physically model any surface based on the physical events observed in nature.

Example 3. Consider F-FPEs of the form

$$\frac{\partial}{\partial\mathfrak{I}^{\gamma}}(\varphi(\mu,\mathfrak{I})) + \frac{\partial}{\partial\mu}\left(\frac{4}{\mu}\varphi^2(\mu,\mathfrak{I})\right) - \frac{\partial}{\partial\mu}\left(\frac{\mu}{3}\varphi(\mu,\mathfrak{I})\right) - \frac{\partial^2}{\partial\mu^2}\left(\varphi^2(\mu,\mathfrak{I})\right) = 0, \mu, \mathfrak{I} > 0, \gamma \in (0,1], \tag{48}$$

with the initial condition

$$\varphi(\mu,0) = \mu^2. \tag{49}$$

On employing Yang transform, we get

$$\mathbf{Y}\left\{\frac{\partial^{\gamma}\varphi}{\partial\mathfrak{I}^{\gamma}}\right\} = \mathbf{Y}\left[\frac{\partial}{\partial\mu}\left(\frac{\mu}{3}\varphi(\mu,\mathfrak{I})\right) + \frac{\partial^2}{\partial\mu^2}\left(\varphi^2(\mu,\mathfrak{I})\right) - \frac{\partial}{\partial\mu}\left(\frac{4}{\mu}\varphi^2(\mu,\mathfrak{I})\right)\right]. \tag{50}$$

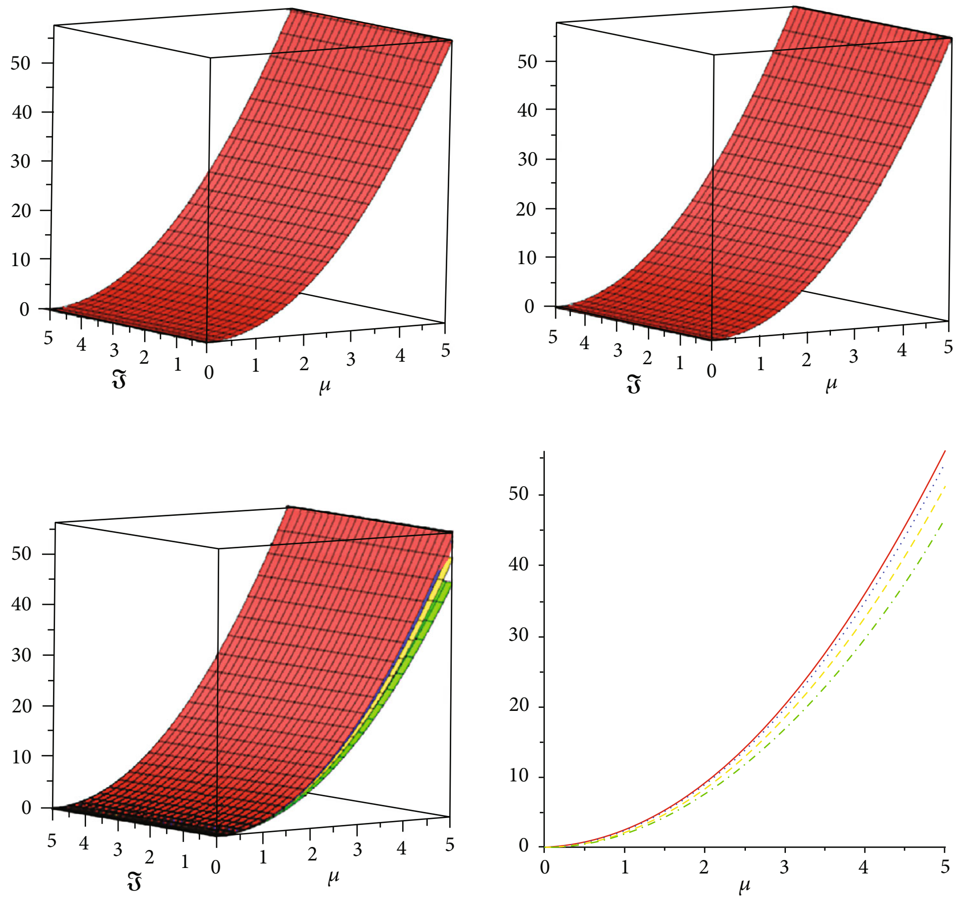

Figure 1: Nature of the exact solution, analytical solution, and solution at various fractional orders of problem 1.

By using Yang differentiation property, we have

$$\begin{aligned}\frac{1}{u^{\gamma}}\{M(u)-u\varphi(0)\} &= \mathbf{Y}\left[\frac{\partial}{\partial\mu}\left(\frac{\mu}{3}\varphi(\mu,\mathfrak{I})\right)+\frac{\partial^2}{\partial\mu^2}\left(\varphi^2(\mu,\mathfrak{I})\right)-\frac{\partial}{\partial\mu}\left(\frac{4}{\mu}\varphi^2(\mu,\mathfrak{I})\right)\right],\\ M(u) &= u\varphi(0)+u^{\gamma}\mathbf{Y}\left[\frac{\partial}{\partial\mu}\left(\frac{\mu}{3}\varphi(\mu,\mathfrak{I})\right)+\frac{\partial^2}{\partial\mu^2}\left(\varphi^2(\mu,\mathfrak{I})\right)-\frac{\partial}{\partial\mu}\left(\frac{4}{\mu}\varphi^2(\mu,\mathfrak{I})\right)\right].\end{aligned} \tag{51}$$

On applying inverse Yang transform, we have

$$\begin{aligned}\varphi(\mu,\mathfrak{I}) &= \varphi(0)+\mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left(\frac{\partial}{\partial\mu}\left(\frac{\mu}{3}\varphi(\mu,\mathfrak{I})\right)+\frac{\partial^2}{\partial\mu^2}\left(\varphi^2(\mu,\mathfrak{I})\right)-\frac{\partial}{\partial\mu}\left(\frac{4}{\mu}\varphi^2(\mu,\mathfrak{I})\right)\right)\right\}\right],\\ \varphi(\mu,\mathfrak{I}) &= \mu^2+\mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left(\frac{\partial}{\partial\mu}\left(\frac{\mu}{3}\varphi(\mu,\mathfrak{I})\right)+\frac{\partial^2}{\partial\mu^2}\left(\varphi^2(\mu,\mathfrak{I})\right)-\frac{\partial}{\partial\mu}\left(\frac{4}{\mu}\varphi^2(\mu,\mathfrak{I})\right)\right)\right\}\right].\end{aligned} \tag{52}$$

Now, by assuming that the infinite series form the function $\varphi(\mu,\mathfrak{I})$ which is unknown, it has the solution as

$$\varphi(\mu,\mathfrak{I})=\sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}). \tag{53}$$

Thus, the nonlinear terms are defined by the Adomian polynomial $\varphi^2=\sum_{m=0}^{\infty}\mathscr{A}_m$. Using specific concepts, Equation (52) can be rewritten in the form

$$\begin{aligned}\sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}) &= \varphi(\mu,0)+\mathbf{Y}^{-1}\left[u^{\gamma}\mathbf{Y}\left[\frac{\partial}{\partial\mu}\left(\frac{\mu}{3}\varphi(\mu,\mathfrak{I})\right)+\sum_{m=0}^{\infty}\mathscr{A}_m\right]\right],\\ \sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}) &= \mu^2+\mathbf{Y}^{-1}\left[u^{\gamma}\mathbf{Y}\left[\frac{\partial}{\partial\mu}\left(\frac{\mu}{3}\varphi(\mu,\mathfrak{I})\right)+\sum_{m=0}^{\infty}\mathscr{A}_m\right]\right].\end{aligned} \tag{54}$$

Now, by Adomian polynomial $\mathscr{Q}_1$, the nonlinear terms

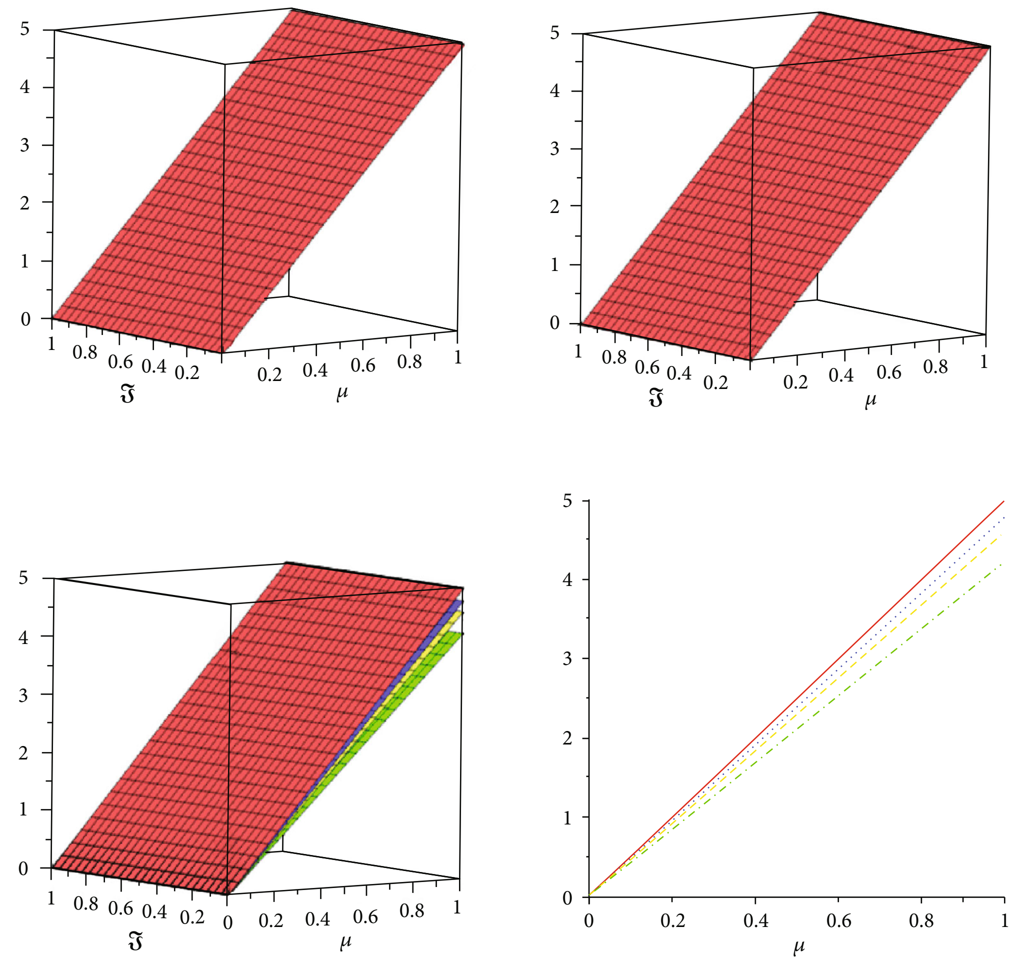

Figure 2: Nature of the exact solution, analytical solution, and solution at various fractional orders of problem 2.

are decomposed according to Equation (38),

$$\begin{aligned} \mathscr{A}_0 &= \varphi_0^2, \\ \mathscr{A}_1 &= 2\varphi_0\varphi_1, \\ \mathscr{A}_2 &= 2\varphi_0\varphi_2 + (\varphi_2)^2. \end{aligned} \tag{55}$$

By comparing Equation (54) both sides, we get

$$\varphi_0(\mu, \mathfrak{J}) = \mu^2. \tag{56}$$

On $m = 0$,

$$\varphi_1(\mu, \mathfrak{J}) = \mu^2 \frac{\mathfrak{J}^{\gamma}}{\Gamma(\gamma+1)}. \tag{57}$$

On $m = 1$,

$$\varphi_2(\mu, \mathfrak{J}) = \mu^2 \frac{\mathfrak{J}^{2\gamma}}{\Gamma(2\gamma+1)}. \tag{58}$$

On $m = 2$,

$$\varphi_3(\mu, \mathfrak{J}) = \mu^2 \frac{\mathfrak{J}^{3\gamma}}{\Gamma(3\gamma+1)}. \tag{59}$$

The YTDM solution remaining components φ_m with ($m \geq 3$) are calculated easily. Thus, we define the series form solution as

$$\begin{aligned} \varphi(\mu, \mathfrak{J}) &= \sum_{m=0}^{\infty} \varphi_m(\mu, \mathfrak{J}) = \varphi_0(\mu, \mathfrak{J}) + \varphi_1(\mu, \mathfrak{J}) \\ &\quad + \varphi_2(\mu, \mathfrak{J}) + \varphi_3(\mu, \mathfrak{J}) + \cdots, \\ \varphi(\mu, \mathfrak{J}) &= \mu^2 + \mu^2 \frac{\mathfrak{J}^{\gamma}}{\Gamma(\gamma+1)} + \mu^2 \frac{\mathfrak{J}^{2\gamma}}{\Gamma(2\gamma+1)} \\ &\quad + \mu^2 \frac{\mathfrak{J}^{3\gamma}}{\Gamma(3\gamma+1)} + \cdots. \end{aligned} \tag{60}$$

The YTDM solution at $\gamma = 1$ is

$$\varphi(\mu, \mathfrak{J}) = \mu^2 \exp^{\mathfrak{J}}. \tag{61}$$

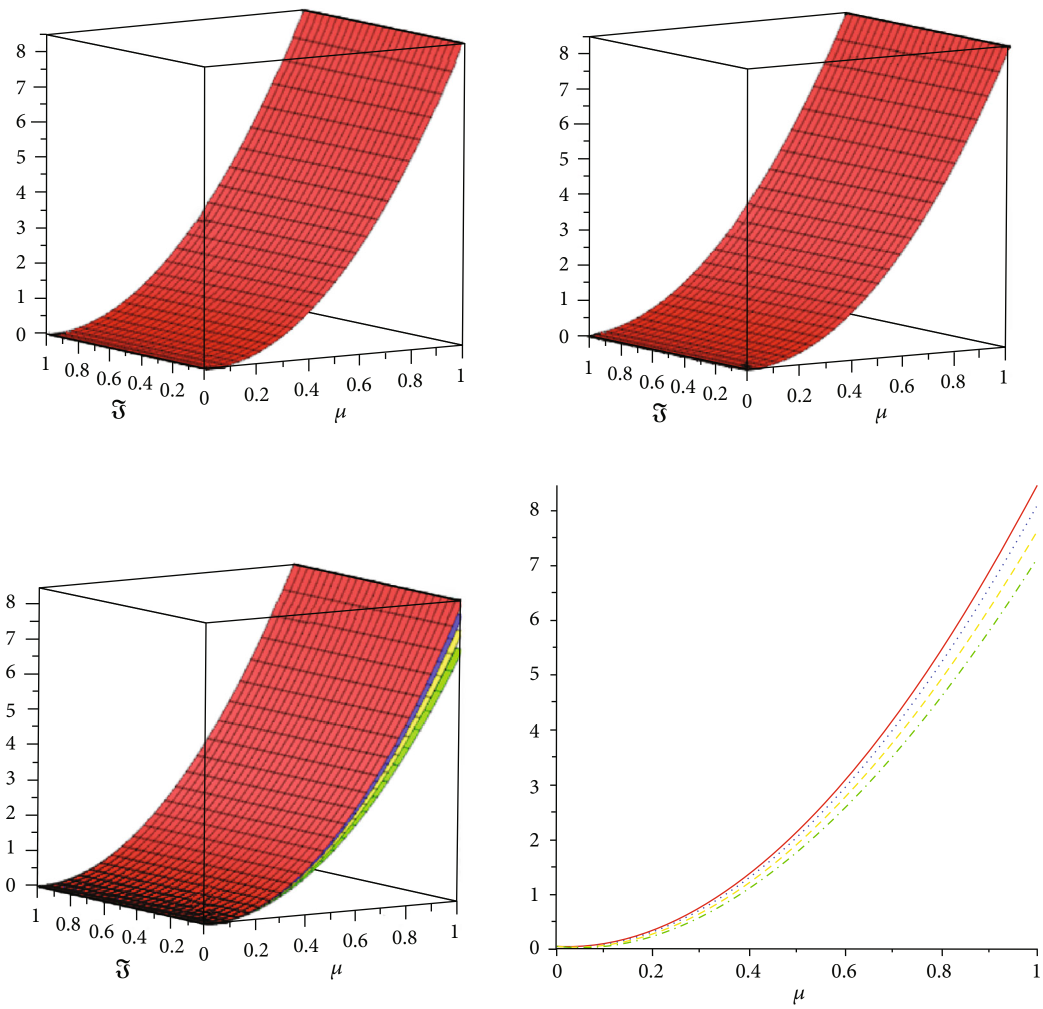

Figure 3: Nature of the exact solution, analytical solution, and solution at various fractional orders of problem 3.

In Figure 3, the first graph shows the exact and second the analytical solution graph, which shows the close contact with each other. In Figure 3, the third and fourth graphs are the three- and two-dimensional graphs concerning different fractional order of problem 3. The figures show that the suggested technique agrees with the actual solution for the given problem. As fractional order approaches integer order, fractional-order solution surfaces converge to the integer-order surface, as depicted by graphs. It means that we may physically model any surface based on the physical events observed in nature.

Example 4. Consider F-FPEs of the form

$$\frac{\partial}{\partial \mathfrak{I}^{\gamma}}(\varphi(\mu,\mathfrak{I})) - \frac{\partial}{\partial \mu}\varphi(\mu,\mathfrak{I}) - \frac{\partial^2}{\partial \mu^2}\varphi(\mu,\mathfrak{I}) = 0, \mathfrak{I} > 0, \ \gamma \in (0,1], \tag{62}$$

with the initial condition

$$\varphi(\mu, 0) = \mu. \tag{63}$$

On employing Yang transform, we get

$$\mathbf{Y}\left\{\frac{\partial^{\gamma}\varphi}{\partial \mathfrak{I}^{\gamma}}\right\} = \mathbf{Y}\left[\frac{\partial}{\partial \mu}\varphi(\mu,\mathfrak{I}) + \frac{\partial^2}{\partial \mu^2}\varphi(\mu,\mathfrak{I})\right]. \tag{64}$$

By using Yang differentiation property, we have

$$\begin{aligned} \frac{1}{u^{\gamma}}\{M(u) - u\varphi(0)\} &= \mathbf{Y}\left[\frac{\partial}{\partial \mu}\varphi(\mu,\mathfrak{I}) + \frac{\partial^2}{\partial \mu^2}\varphi(\mu,\mathfrak{I})\right], \\ M(u) &= u\varphi(0) + u^{\gamma}\mathbf{Y}\left[\frac{\partial}{\partial \mu}\varphi(\mu,\mathfrak{I}) + \frac{\partial^2}{\partial \mu^2}\varphi(\mu,\mathfrak{I})\right]. \end{aligned} \tag{65}$$

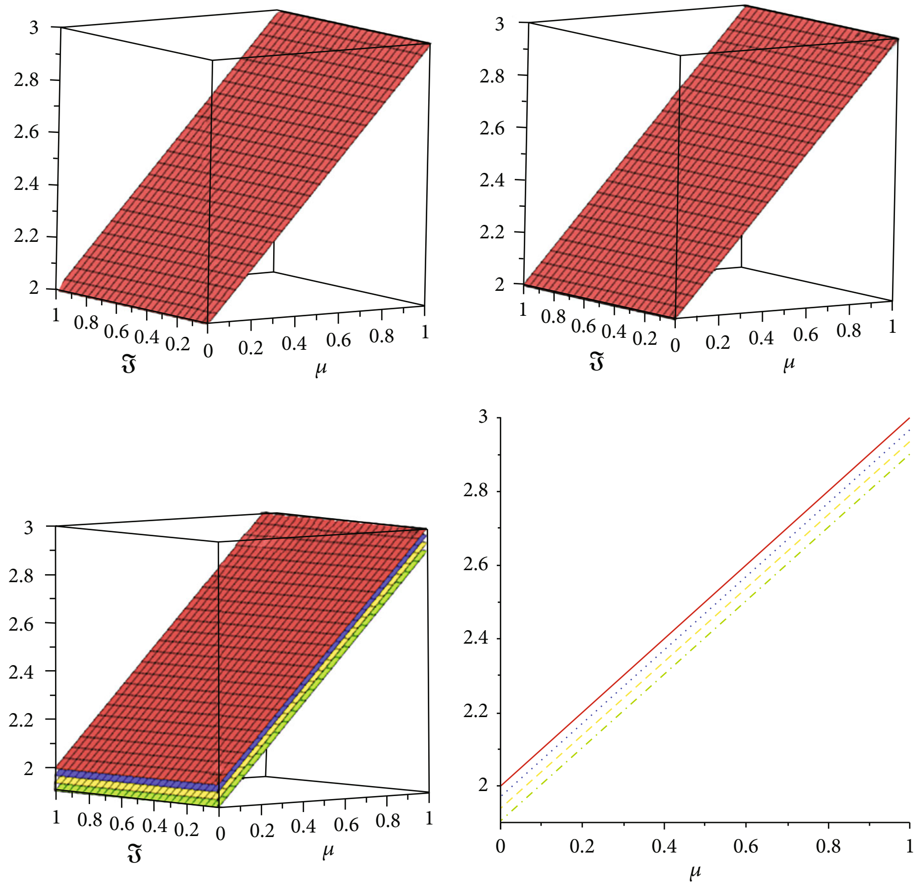

Figure 4: Nature of the exact solution, analytical solution, and solution at various fractional orders of problem 4.

On applying inverse Yang transform, we have

$$\varphi(\mu,\mathfrak{I}) = \varphi(0) + \mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left(\frac{\partial}{\partial\mu}\varphi(\mu,\mathfrak{I}) + \frac{\partial^2}{\partial\mu^2}\varphi(\mu,\mathfrak{I})\right)\right\}\right],$$
$$\varphi(\mu,\mathfrak{I}) = \mu + \mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left(\frac{\partial}{\partial\mu}\varphi(\mu,\mathfrak{I}) + \frac{\partial^2}{\partial\mu^2}\varphi(\mu,\mathfrak{I})\right)\right\}\right]. \tag{66}$$

The solution in terms of infinite sequence $\varphi(\mu,\mathfrak{I})$ by means of YTDM is

$$\varphi(\mu,\mathfrak{I}) = \sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}),$$
$$\sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}) = \mu + \mathbf{Y}^{-1}\left[u^{\gamma}\mathbf{Y}\left[\frac{\partial}{\partial\mu}\varphi(\mu,\mathfrak{I}) + \frac{\partial^2}{\partial\mu^2}\varphi(\mu,\mathfrak{I})\right]\right]. \tag{67}$$

By comparing Equation (67) both sides, we get

$$\varphi_0(\mu,\mathfrak{I}) = \mu. \tag{68}$$

On $m = 0$,

$$\varphi_1(\mu,\mathfrak{I}) = \frac{\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)}. \tag{69}$$

On $m = 1$,

$$\varphi_2(\mu,\mathfrak{I}) = 0. \tag{70}$$

On $m = 2$,

$$\varphi_3(\mu,\mathfrak{I}) = 0. \tag{71}$$

The YTDM solution remaining components φ_m with ($m \geq 3$) are calculated easily. Thus, we define the series form solution as

$$\varphi(\mu,\mathfrak{I}) = \sum_{m=0}^{\infty}\varphi_m(\mu,\mathfrak{I}) = \varphi_0(\mu,\mathfrak{I}) + \varphi_1(\mu,\mathfrak{I}) + \varphi_2(\mu,\mathfrak{I}) + \varphi_3(\mu,\mathfrak{I}) + \cdots, \tag{72}$$
$$\varphi(\mu,\mathfrak{I}) = \mu + \frac{\mathfrak{I}^{\gamma}}{\Gamma(\gamma+1)} + 0 + 0 + \cdots.$$

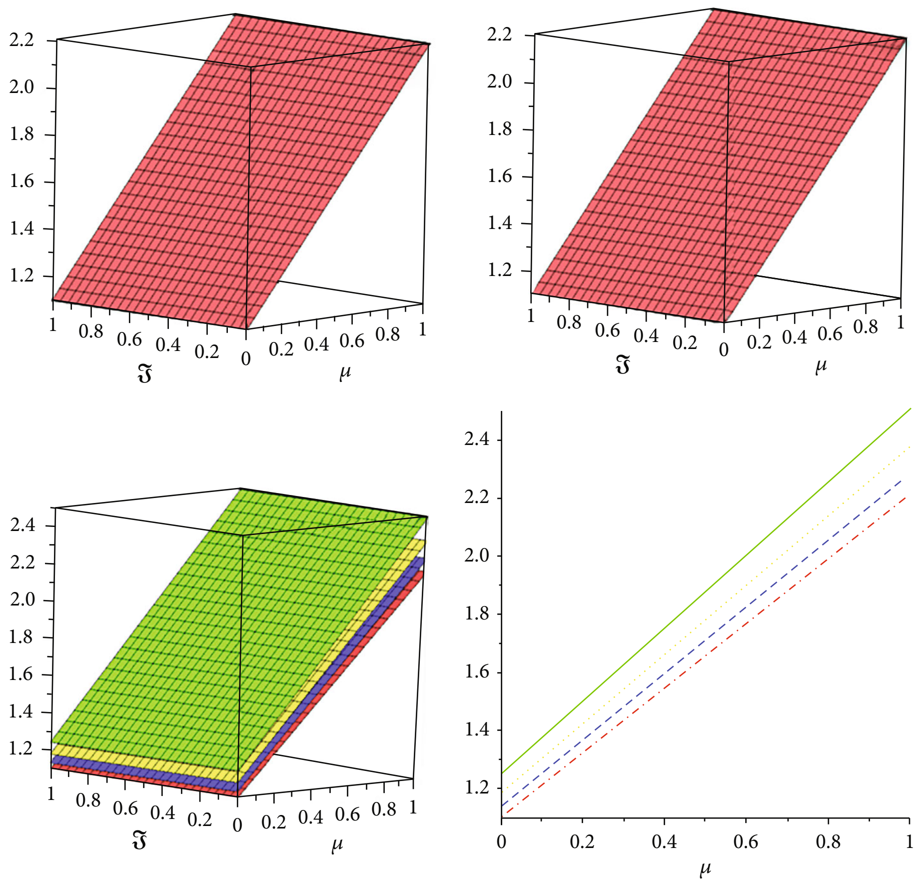

Figure 5: Nature of the exact solution, analytical solution, and solution at various fractional orders of problem 5.

The YTDM solution at $\gamma = 1$ is

$$\varphi(\mu, \mathfrak{J}) = \mu + \mathfrak{J}. \tag{73}$$

In Figure 4, the first graph shows the exact and second the analytical solution graph, which shows the close contact with each other. In Figure 4, the third and fourth graphs are the three- and two-dimensional graphs concerning different fractional order of problem 4. The figures show that the suggested technique agrees with the actual solution for the given problem. As fractional order approaches integer order, fractional-order solution surfaces converge to the integer-order surface, as depicted by graphs. It means that we may physically model any surface based on the physical events observed in nature.

Example 5. Consider F-FPEs of the form

$$\frac{\partial^\gamma}{\partial \mathfrak{J}^\gamma}(\varphi(\mu, \mathfrak{J})) - (1-\mu)\frac{\partial}{\partial \mu}\varphi(\mu, \mathfrak{J}) - \left(e^{\mathfrak{J}}\mu^2\right)\frac{\partial^2}{\partial \mu^2}\varphi(\mu, \mathfrak{J}) = 0, \mathfrak{J} > 0, \ \gamma \in (0, 1], \tag{74}$$

with the initial condition

$$\varphi(\mu, 0) = 1 + \mu. \tag{75}$$

On employing Yang transform, we get

$$\mathbf{Y}\left\{\frac{\partial^\gamma \varphi}{\partial \mathfrak{J}^\gamma}\right\} = \mathbf{Y}\left[(1-\mu)\frac{\partial}{\partial \mu}\varphi(\mu, \mathfrak{J}) + \left(e^{\mathfrak{J}}\mu^2\right)\frac{\partial^2}{\partial \mu^2}\varphi(\mu, \mathfrak{J})\right]. \tag{76}$$

By using Yang differentiation property, we have

$$\begin{aligned} \frac{1}{u^\gamma}\{M(u) - u\varphi(0)\} &= \mathbf{Y}\left[(1-\mu)\frac{\partial}{\partial \mu}\varphi(\mu, \mathfrak{J}) + \left(e^{\mathfrak{J}}\mu^2\right)\frac{\partial^2}{\partial \mu^2}\varphi(\mu, \mathfrak{J})\right], \\ M(u) &= u\varphi(0) + u^\gamma \mathbf{Y}\left[(1-\mu)\frac{\partial}{\partial \mu}\varphi(\mu, \mathfrak{J}) + \left(e^{\mathfrak{J}}\mu^2\right)\frac{\partial^2}{\partial \mu^2}\varphi(\mu, \mathfrak{J})\right]. \end{aligned} \tag{77}$$

On applying inverse Yang transform, we have

$$\varphi(\mu, \mathfrak{T}) = \varphi(0) + \mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left((1-\mu)\frac{\partial}{\partial \mu}\varphi(\mu, \mathfrak{T}) + \left(e^{\mathfrak{T}}\mu^2\right)\frac{\partial^2}{\partial \mu^2}\varphi(\mu, \mathfrak{T})\right)\right\}\right],$$
$$\varphi(\mu, \mathfrak{T}) = (1+\mu) + \mathbf{Y}^{-1}\left[u^{\gamma}\left\{\mathbf{Y}\left((1-\mu)\frac{\partial}{\partial \mu}\varphi(\mu, \mathfrak{T}) + \left(e^{\mathfrak{T}}\mu^2\right)\frac{\partial^2}{\partial \mu^2}\varphi(\mu, \mathfrak{T})\right)\right\}\right]. \tag{78}$$

The solution in terms of infinite sequence $\varphi(\mu, \mathfrak{T})$ by means of YTDM is

$$\varphi(\mu, \mathfrak{T}) = \sum_{m=0}^{\infty} \varphi_m(\mu, \mathfrak{T}), \tag{79}$$

$$\sum_{m=0}^{\infty} \varphi_m(\mu, \mathfrak{T}) = (1+\mu) + \mathbf{Y}^{-1}\left[u^{\gamma}\mathbf{Y}\left[(1-\mu)\frac{\partial}{\partial \mu}\varphi(\mu, \mathfrak{T}) + \left(e^{\mathfrak{T}}\mu^2\right)\frac{\partial^2}{\partial \mu^2}\varphi(\mu, \mathfrak{T})\right]\right]. \tag{80}$$

By comparing Equation (80) both sides, we get

$$\varphi_0(\mu, \mathfrak{T}) = 1 + \mu. \tag{81}$$

On $m = 0$,

$$\varphi_1(\mu, \mathfrak{T}) = (1+\mu)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)}. \tag{82}$$

On $m = 1$,

$$\varphi_2(\mu, \mathfrak{T}) = (1+\mu)\frac{\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)}. \tag{83}$$

On $m = 2$,

$$\varphi_3(\mu, \mathfrak{T}) = (1+\mu)\frac{\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)}. \tag{84}$$

The YTDM solution remaining components φ_m with ($m \geq 3$) are calculated easily. Thus, we define the series form solution as

$$\varphi(\mu, \mathfrak{T}) = \sum_{m=0}^{\infty} \varphi_m(\mu, \mathfrak{T}) = \varphi_0(\mu, \mathfrak{T}) + \varphi_1(\mu, \mathfrak{T}) + \varphi_2(\mu, \mathfrak{T}) + \varphi_3(\mu, \mathfrak{T}) + \cdots,$$
$$\varphi(\mu, \mathfrak{T}) = (1+\mu) + (1+\mu)\frac{\mathfrak{T}^{\gamma}}{\Gamma(\gamma+1)} + (1+\mu)\frac{\mathfrak{T}^{2\gamma}}{\Gamma(2\gamma+1)} + (1+\mu)\frac{\mathfrak{T}^{3\gamma}}{\Gamma(3\gamma+1)} + \cdots. \tag{85}$$

The YTDM solution at $\gamma = 1$ is

$$\varphi(\mu, \mathfrak{T}) = \exp^{\mathfrak{T}}(1+\mu). \tag{86}$$

In Figure 5, the first graph shows the exact and second the analytical solution graph, which shows the close contact with each other. In Figure 5, the third and fourth graphs are the three- and two-dimensional graphs concerning different fractional order of problem 5. The figures show that the suggested technique agrees with the actual solution for the given problem. As fractional order approaches integer order, fractional-order solution surfaces converge to the integer-order surface, as depicted by graphs. It means that we may physically model any surface based on the physical events observed in nature.

5. Conclusion

The Adomian decomposition approach was expanded in this paper to find explicit and numerical solutions to the F-FPEs. The proposed method is an effective and powerful strategy for solving the proposed equations. The plotted graphs confirm the strong relationship between the exact and analytical results. The approaches provide series form solutions with a higher convergence rate to exact results. While providing quantitatively accurate results, the Adomian decomposition method requires less computational work than existing approaches.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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