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Retraction

Retracted: The Generalization Inverse Weibull Distribution Related to X-Gamma Generator Family: Simulation and Application for Breast Cancer

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] R. Alshenawy, "The Generalization Inverse Weibull Distribution Related to X-Gamma Generator Family: Simulation and Application for Breast Cancer," *Journal of Function Spaces*, vol. 2022, Article ID 4693490, 17 pages, 2022.

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Research Article

The Generalization Inverse Weibull Distribution Related to X-Gamma Generator Family: Simulation and Application for Breast Cancer

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The aim of this paper is to propose the new three-parameter X-Gamma inverse Weibull (XGAIW) distribution which generalizes the inverse Weibull model. The density function of the XGAIW can be expressed as a linear combination of the inverse Weibull densities. Some mathematical quantities (reliability and hazard rate properties) of the proposed XGAIW model are derived. Moreover, four estimation methods, namely, the maximum likelihood, maximum product spacing, least squares, and weighted least squares methods, are utilized to estimate the XGAIW parameters. The Monte Carlo simulation study has been performed to assess the performance of the proposed estimation methods using some criteria. The importance, flexibility, and potentiality of the XGAIW model are studied via a breast cancer data set application. The XGAIW model can produce better fits than some well-known distributions, so the proposed model can be used, as a good alternative to some existing distributions, in modeling several real data.

1. Introduction

In statistics literature, many new families of lifetime distributions are developed and commonly used to describe real-world phenomena. It is well known that adding an extra parameter to an existing family of distributions is very common in the statistical distribution theory. Often introducing an extra parameter brings additional flexibility to a class of probability distributions, and, in turn, it can be very useful for data analysis purposes. A common feature of these new classes of distributions is that they have more parameters and the model adequacy of the new generalized distribution performs better than the baseline distribution. Therefore, introducing new probability distributions and/or extending (or generalizing) existing probability distributions by adding extra parameters into its form has become a time-honored device for obtaining more flexible new families of distribution; see Alshenawy [1].

Inverse (or inverted) distributions are significant in many fields, including biological sciences, life test problems, and medical sciences, because of their applicability. Inverted conformation distributions have a different structure than noninverted conformation distributions in terms of density and hazard ratio. The inverse Weibull (IW) distribution is an important probability distribution which can be used to analyze the lifetime data with some monotone failure rates. It is a suitable model to describe degradation phenomena of mechanical components as mentioned by Keller et al. [2] and Alkarni et al. [3]. According to Nelson [4], the IW distribution provides a good fit to several data sets such as the times to breakdown of an insulating fluid subject to the action of a constant tension. The IW distribution has been used to model many real-life applications including medicine, reliability, and ecology. Some useful measures for the IW distribution have been discussed by Jiang et al. [5]. The IW distribution is appropriate model to a variety of failure characteristics such as wear-out period, infant mortality, and useful life; see Khan et al. [6], Hassan and Nassr [7], Subhradev [8], Khan and King [9], Biçer [10], and Ahmad and Almetwally [11].

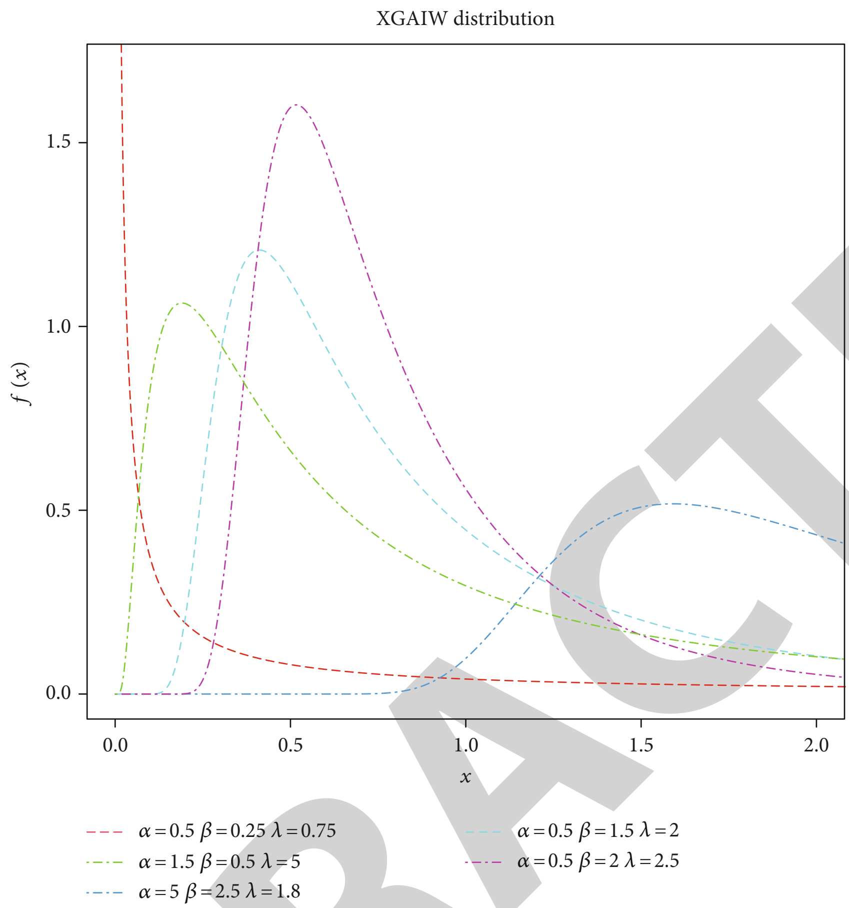

Figure 1: Plots of the PDF of the XGAIW distribution for different values of parameters.

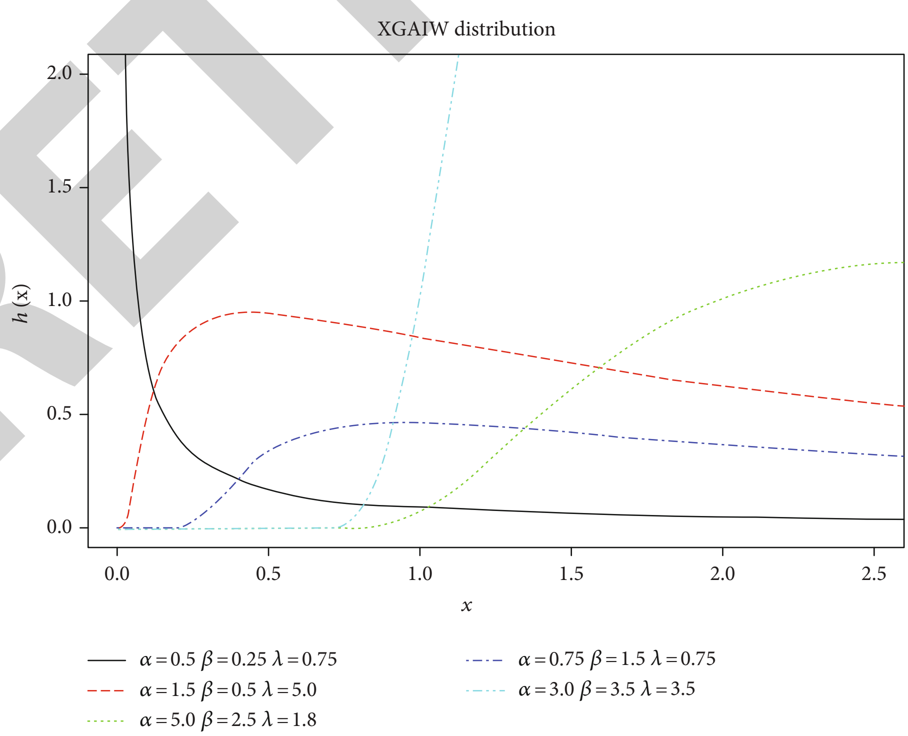

Figure 2: Plots of the HRF of the XGAIW distribution for different values of parameters.

Table 1: Bias and MSE values for α, β, and λ of the XGAIW distribution.

Parameters	MLEs		MPSEs		LSEs		WLSEs	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$n = 50$								
$\alpha = 0.5$	0.4377	0.4423	0.4433	0.2523	0.3643	0.2342	0.4321	0.2274
$\beta = 0.5$	0.6876	0.8352	0.4389	0.3142	0.3309	0.1564	0.3184	0.1322
$\lambda = 1$	-0.0798	0.1519	0.1692	0.1430	0.0698	0.1386	0.0371	0.1163
$n = 100$								
$\alpha = 0.5$	0.4366	0.3368	0.3937	0.1726	0.3620	0.1709	0.3984	0.1604
$\beta = 0.5$	0.4563	0.3728	0.3927	0.2847	0.3517	0.1382	0.3582	0.1488
$\lambda = 1$	-0.0226	0.0693	0.0463	0.0636	0.1726	0.0615	0.0298	0.0599
$n = 150$								
$\alpha = 0.5$	0.4169	0.2483	0.3983	0.1619	0.4835	0.1595	0.3618	0.1543
$\beta = 0.5$	0.4244	0.2692	0.3849	0.1694	0.3674	0.1274	0.3672	0.1170
$\lambda = 1$	-0.0292	0.0392	0.0246	0.0372	0.0582	0.0363	0.0130	0.0371

Table 2: Bias and MSE values for α, β, and λ of the XGAIW distribution.

Parameters	MLEs		MPSEs		LSEs		WLSEs	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$n = 50$								
$\alpha = 0.5$	0.9244	1.3274	0.8836	0.9353	0.7254	0.6418	0.8568	0.7255
$\beta = 1.5$	0.5243	1.1835	0.2194	0.2732	0.2464	0.1165	0.3639	0.1715
$\lambda = 1$	-0.0430	0.0986	0.0699	0.0974	0.0356	0.0918	0.1358	0.0934
$n = 100$								
$\alpha = 0.5$	0.8253	0.8399	0.7865	0.6967	0.7455	0.5965	0.7864	0.6732
$\beta = 1.5$	0.3844	0.4559	0.2863	0.1273	0.2817	0.1154	0.2720	0.1163
$\lambda = 1$	-0.0167	0.0498	0.0568	0.0473	0.0813	0.0428	0.0293	0.0459
$n = 150$								
$\alpha = 0.5$	0.7863	0.7368	0.7855	0.6467	0.7655	0.5265	0.8920	0.5808
$\beta = 1.5$	0.4355	0.5465	0.2753	0.1074	0.2863	0.0913	0.2864	0.0974
$\lambda = 1$	-0.0298	0.0269	0.0189	0.0316	0.0486	0.0277	0.0064	0.0253

Calabria and Pulcini [12] computed the maximum likelihood and least squares estimate of the parameters of the IW distribution. They also obtained the Bayes estimator of the model parameters as well as confidence limits for reliability and tolerance limits. See Calabria and Pulcini [13] for additional details. The random variable x has an IW distribution if its cumulative distribution function (CDF) takes the form

$$G_{\mathrm{IW}}(x\,;\alpha,\beta)=e^{-\alpha x^{-\beta}}\,;x\geq 0,\alpha,\beta>0, \tag{1}$$

where α and β are the scale and shape parameters, respectively. If $\alpha = 1$, we have the Fréchet distribution function. The corresponding probability density function (PDF) is given by

$$g_{\mathrm{IW}}(x\,;\alpha,\beta)=\alpha\beta x^{-\beta-1}e^{-\alpha x^{-\beta}}\,;x\geq 0,\alpha,\beta>0. \tag{2}$$

If $\beta = 1$, the IW (PDF) becomes inverse exponential (PDF), and when $\beta = 2$, the IW (PDF) is referred to as the inverse Raleigh (PDF).

The quantile function is $Q_F(x)=(-\ln(u)/\alpha)^{-1/\beta}\,;0<u<1$. In addition to that, the IW (PDF) satisfies

$$xf(x\,;\alpha,\beta)=\beta F(x\,;\alpha,\beta)(-\ln(F(x\,;\alpha,\beta))\,;x\geq 0,\alpha,\beta>0. \tag{3}$$

In recent years, many generalized classes of distributions have been proposed for modeling real-life data to provide great flexibility in modelling data in several applied fields such as reliability, engineering, biological studies, economics, medical sciences, environmental sciences, and finance. For example, Jiang et al. [14] presented the Weibull and Weibull inverse mixture models. Sultan et al. [15] discussed the mixture of two IW distributions. Khan et al. [6] studied

Table 3: Bias and MSE values for α, β, and λ of the XGAIW distribution.

Parameters	MLEs		MPSEs		LSEs		WLSEs	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$n = 50$								
$\alpha = 0.5$	1.2773	2.7154	0.8859	0.9610	0.8374	0.7843	0.7732	0.7416
$\beta = 1.5$	0.4794	1.9848	0.2864	0.4430	0.3249	0.3639	0.2642	0.1592
$\lambda = 1.5$	0.2283	0.2183	0.1365	0.1743	0.0622	0.1262	0.0174	0.1465
$n = 100$								
$\alpha = 0.5$	1.1846	2.1467	0.8473	0.8172	0.7528	0.6817	0.9415	0.6958
$\beta = 1.5$	0.3763	1.0990	0.2915	0.2127	0.2652	0.1073	0.2393	0.1474
$\lambda = 1.5$	0.1493	0.1062	0.0759	0.0922	0.0544	0.0863	0.0277	0.0767
$n = 150$								
$\alpha = 0.5$	0.9690	1.2168	0.7911	0.6276	0.8638	0.5836	0.2657	0.4290
$\beta = 1.5$	0.9315	1.0092	0.2015	0.1678	0.2830	0.1019	0.2054	0.0816
$\lambda = 1.5$	0.0426	0.0432	0.0817	0.0420	0.0183	0.0397	0.0373	0.0383

Table 4: Bias and MSE values for α, β, and λ of the XGAIW distribution.

Parameters	MLEs		MPSEs		LSEs		WLSEs	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$n = 50$								
$\alpha = 1$	0.5925	1.2282	0.2892	0.2593	0.2016	0.1056	0.1474	0.1576
$\beta = 1.5$	0.2336	1.4158	0.1668	0.3176	0.0278	0.1163	0.0369	0.2530
$\lambda = 1.5$	-0.0246	0.1968	0.0770	0.1839	0.2348	0.1816	0.0388	0.1561
$n = 100$								
$\alpha = 1$	0.5269	0.8388	0.2298	0.1695	0.2063	0.0967	0.1490	0.0929
$\beta = 1.5$	0.0614	0.7588	0.0676	0.1618	0.1388	0.1043	0.0478	0.0835
$\lambda = 1.5$	0.1318	0.1199	0.0567	0.1018	0.0063	0.0865	0.0389	0.0914
$n = 150$								
$\alpha = 1$	0.3289	0.4376	0.3479	0.1214	0.1414	0.0663	0.1732	0.0449
$\beta = 1.5$	0.1853	0.6479	-0.0179	0.1368	0.0768	0.0215	0.1287	0.0817
$\lambda = 1.5$	0.0383	0.0549	0.0163	0.0476	0.0739	0.0471	0.0016	0.0457

the flexibility of the IW distribution. De Gusmao et al. [16] proposed the generalized IW distribution and discussed several properties of this model with applications. Hemmati et al. [17] proposed the three-parameter Weibull-Poisson distribution as an aging class distribution. Khan et al. [18] proposed the new class of transmuted inverse Weibull distribution with application to reliability data. Baharith et al. [19] introduced the beta generalized inverse Weibull distribution. Finally, Kamel and Alqarni [20] proposed a new characterization of exponential distribution through minimum chi-squared divergence principle with some illustrative examples.

Additionally, the X-Gamma (XG) distribution is introduced by Sen et al. [21] and Sen et al. [22] as the probability distribution model with a single shape parameter. The XG distribution, which has many useful statistical features, is a probability distribution that could have the potential use for the modeling of lifetime data from a wide range of the field of science. Sen et al. [21] have studied many useful features of XG distribution. Although it has nice statistical properties, it is a disadvantage of XG that the distribution has only one parameter which plays a crucial role in determining the various behaviors of the distribution. Until today, various attempts have been made by several researchers to eliminate this disadvantage of the distribution. However, the XG distribution needs to be improved in an aspect of the ability to a model for a wide variety of data types especially the data with the hazard rates in different forms. Sen et al. [23] studied discrimination analysis between the Lindley and XG distribution. Finally, Yadav et al. [24] introduced inverse XG distribution using the transformation $Y = 1/X$

Table 5: Bias and MSE values for α, β, and λ of the XGAIW distribution.

Parameters	MLEs		MPSEs		LSEs		WLSEs	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$n = 50$								
$\alpha = 1.5$	0.0325	0.5954	-0.2983	0.2856	-0.2121	0.1190	-0.1949	0.1549
$\beta = 1.5$	0.0489	0.8856	-0.0941	0.2955	-0.2756	0.1665	-0.2513	0.2417
$\lambda = 1.5$	-0.0587	0.1890	0.0243	0.1648	0.1922	0.1346	-0.0187	0.1298
$n = 100$								
$\alpha = 1.5$	-0.0356	0.4279	-0.3986	0.1685	-0.2954	0.1299	-0.2289	0.0956
$\beta = 1.5$	-0.1759	0.5579	-0.2399	0.1899	-0.2579	0.1289	-0.1288	0.1785
$\lambda = 1.5$	-0.0379	0.0983	0.0085	0.0979	0.1199	0.0950	-0.0192	0.0913
$n = 150$								
$\alpha = 1.5$	-0.0665	0.3356	-0.3565	0.1359	-0.2529	0.0955	-0.2455	0.0908
$\beta = 1.5$	-0.0748	0.4486	-0.1364	0.0956	-0.2987	0.0875	-0.2190	0.0863
$\lambda = 1.5$	-0.0221	0.0557	-0.0285	0.0516	-0.0376	0.0465	0.0578	0.0499

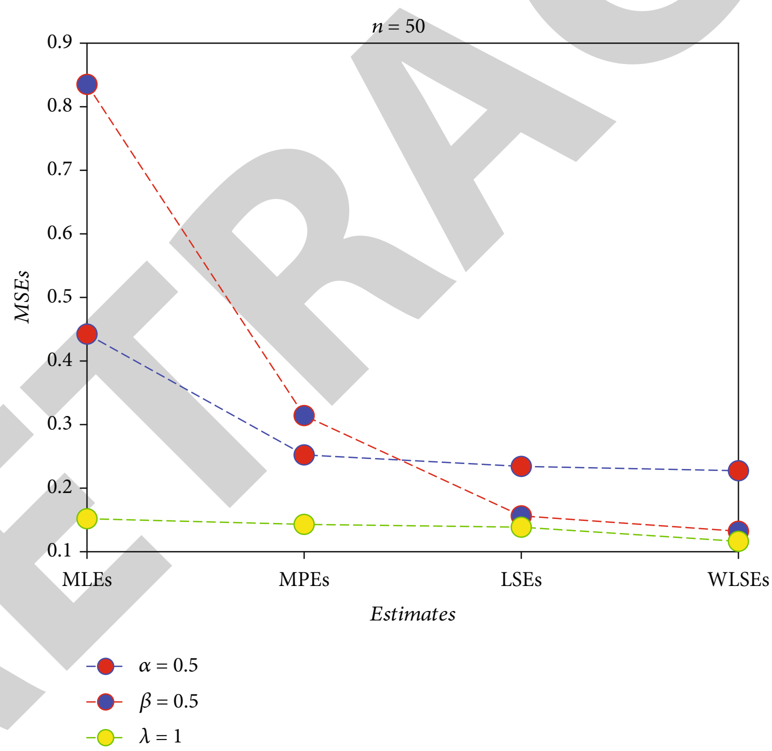

Figure 3: MSEs using $n = 50$: $\alpha = 0.5$, $\beta = 0.5$, and $\lambda = 1$.

Moreover, the XG-Generator (XG-G) family has been proposed by Cordeiro et al. [25] to incorporate any distribution into a larger family through an application of the XG (CDF). Based on the T–X transform defined by Alzaatreh et al. [26] and the XG (CDF), the XG-G family has flexible shapes to model various lifetime data sets. Additionally, its special models produce better fits than other well-known families. The XG-G family added a parameter which has one extra shape parameter $\lambda > 0$, and the CDF of XG-G family is given by

$$F(x;\lambda,\gamma) = 1 - \frac{[1-G(x;\gamma)]^{\lambda}}{\lambda+1}\{1+\lambda-\lambda \ln(1-G(x;\gamma)) + 0.5\lambda^2[\ln(1-G(x;\gamma))]^2\}, \quad (4)$$

where $G(x;\gamma)$ is a baseline CDF with a parameter vector γ.

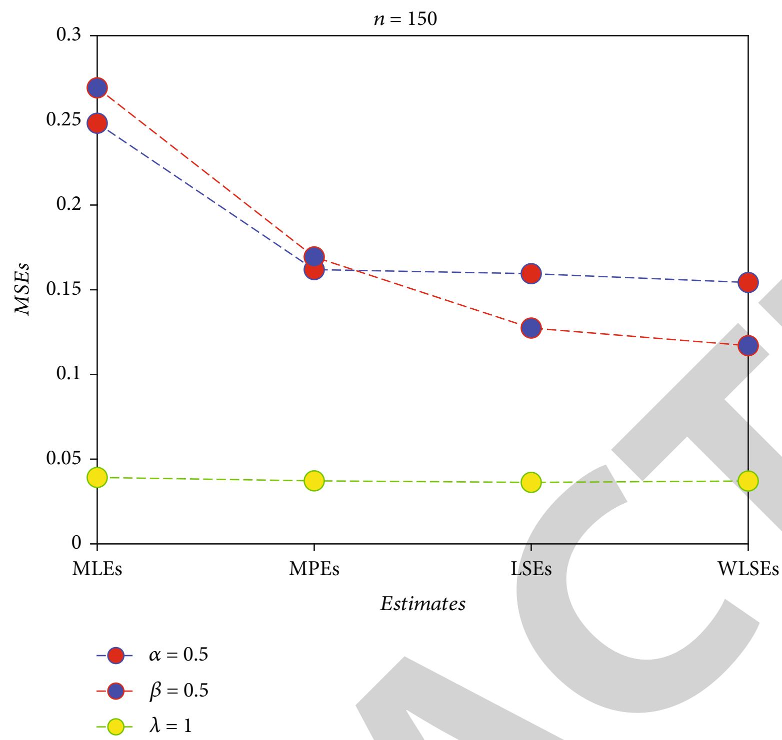

FIGURE 4: MSEs using $n = 150$: $\alpha = 0.5$, $\beta = 0.5$, and $\lambda = 1$.

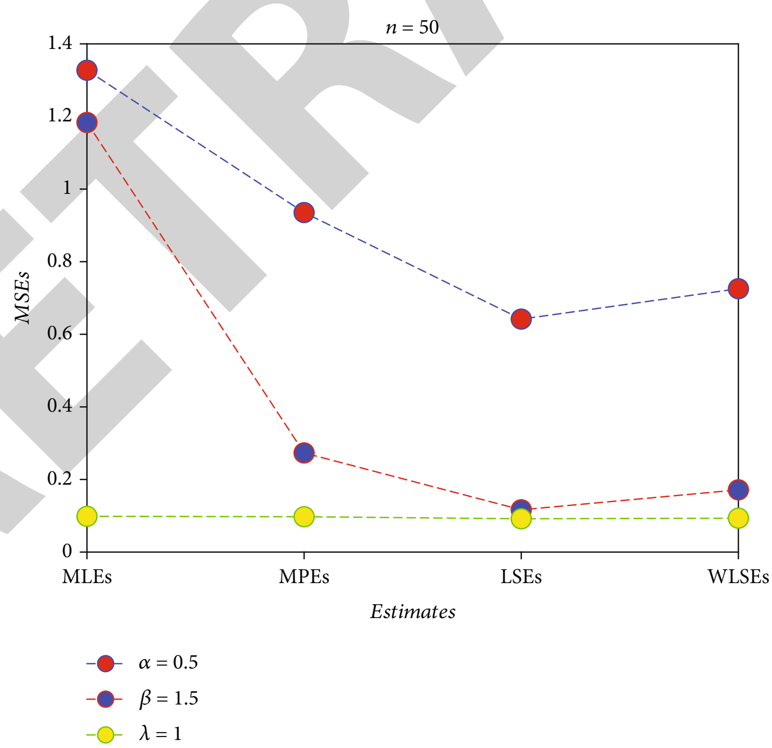

FIGURE 5: MSEs using $n = 50$: $\alpha = 0.5$, $\beta = 1.5$, and $\lambda = 1$.

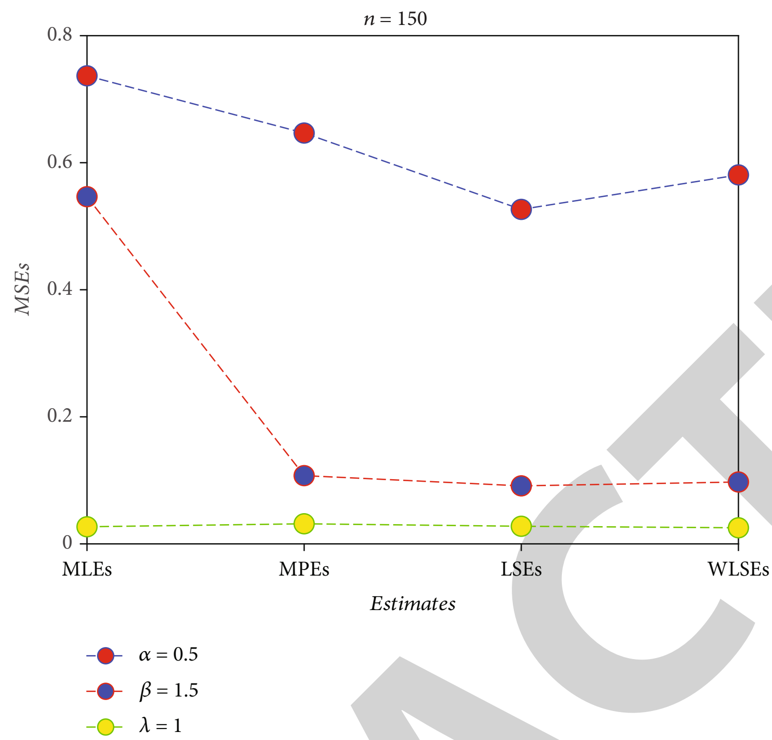

Figure 6: MSEs using $n = 150$: $\alpha = 0.5$, $\beta = 1.5$, and $\lambda = 1$.

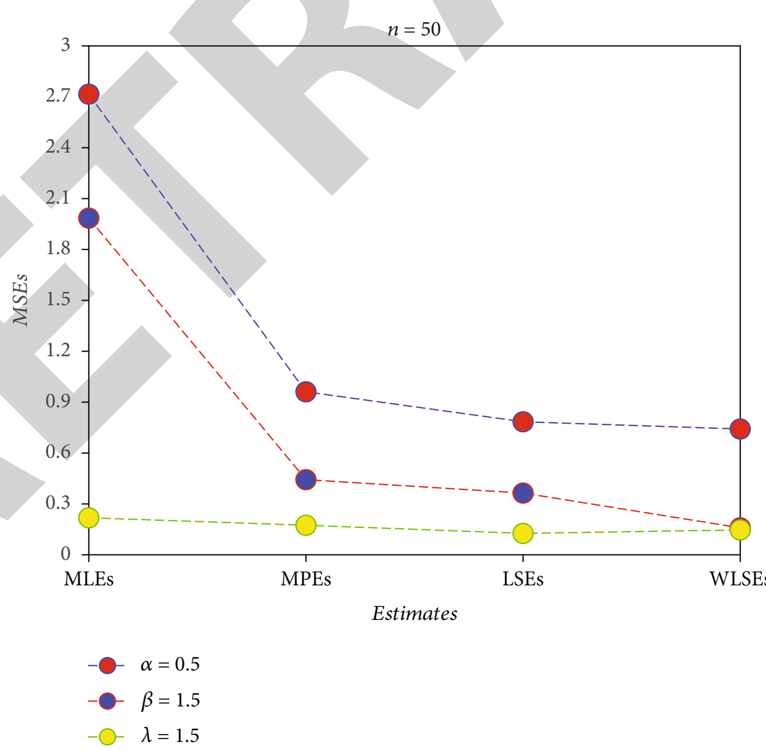

Figure 7: MSEs using $n = 50$: $\alpha = 0.5$, $\beta = 1.5$, and $\lambda = 1.5$.

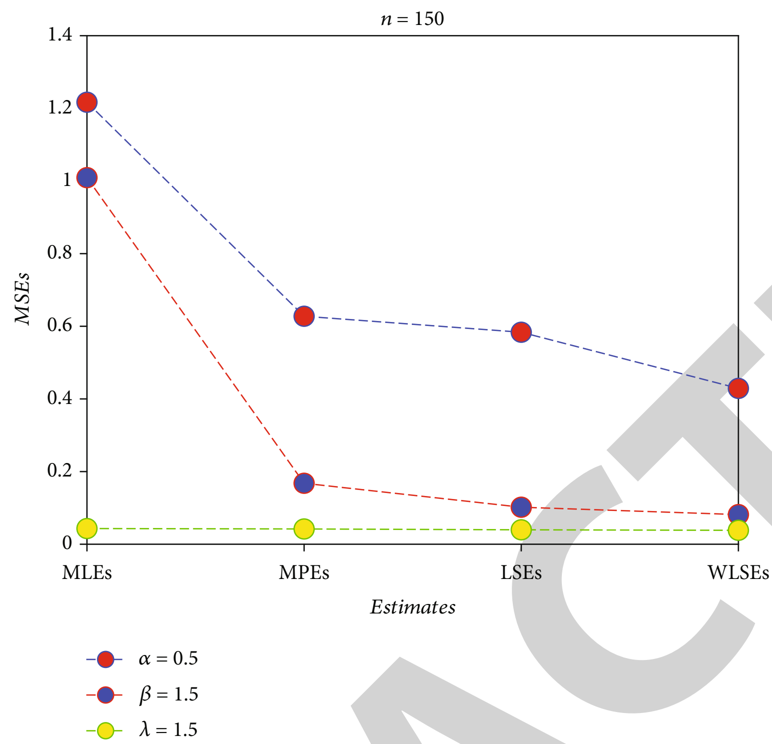

Figure 8: MSEs using $n = 150$: $\alpha = 0.5$, $\beta = 1.5$, and $\lambda = 1.5$.

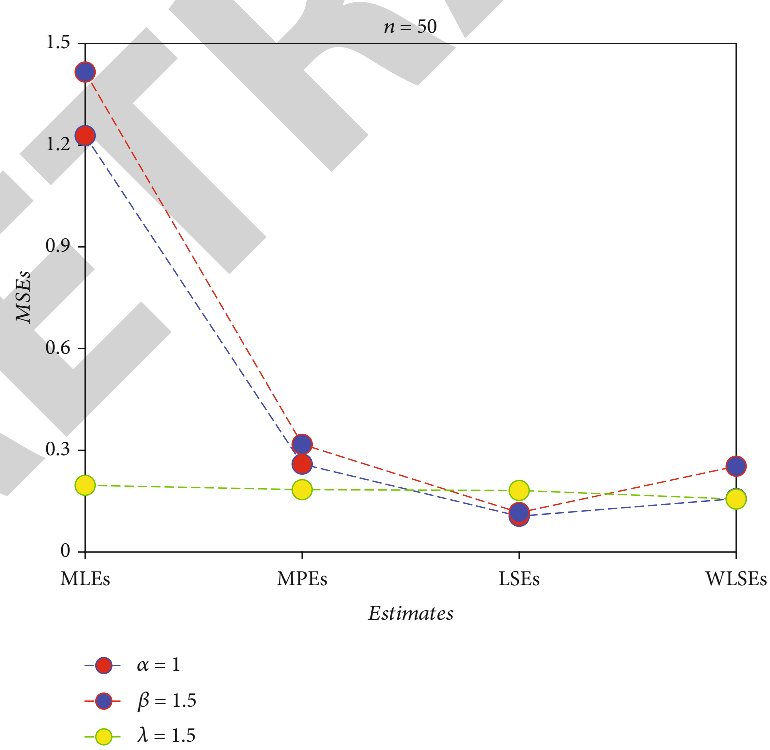

Figure 9: MSEs using $n = 50$: $\alpha = 1$, $\beta = 1.5$, and $\lambda = 1.5$.

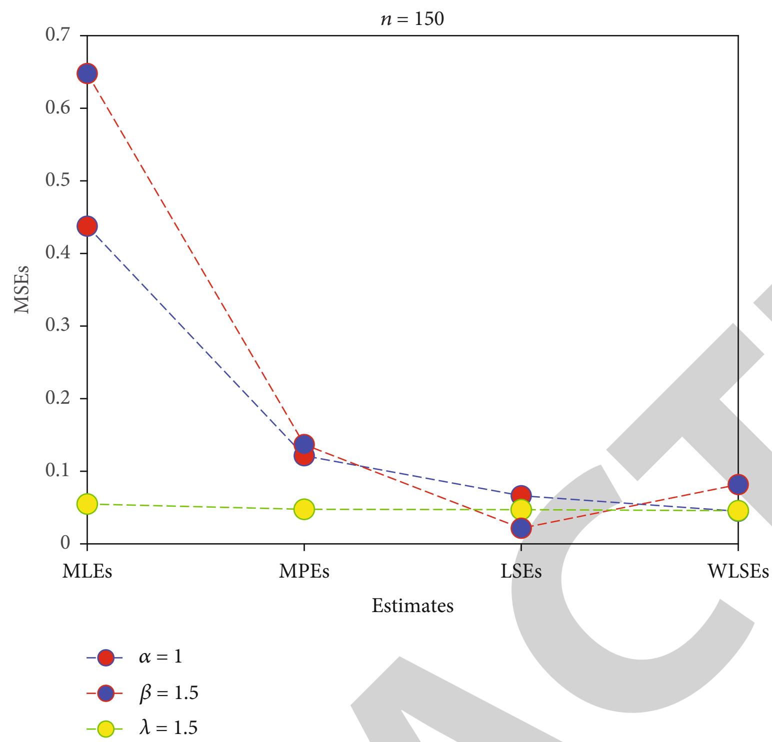

Figure 10: MSEs using $n = 150$: $\alpha = 1$, $\beta = 1.5$, and $\lambda = 1.5$.

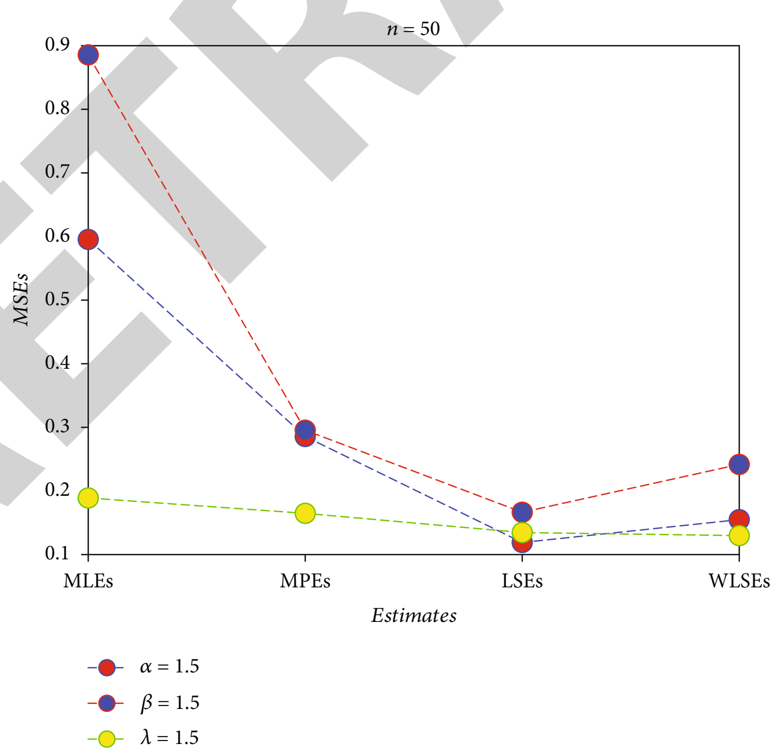

Figure 11: MSEs using $n = 50$: $\alpha = 1.5$, $\beta = 1.5$, and $\lambda = 1.5$.

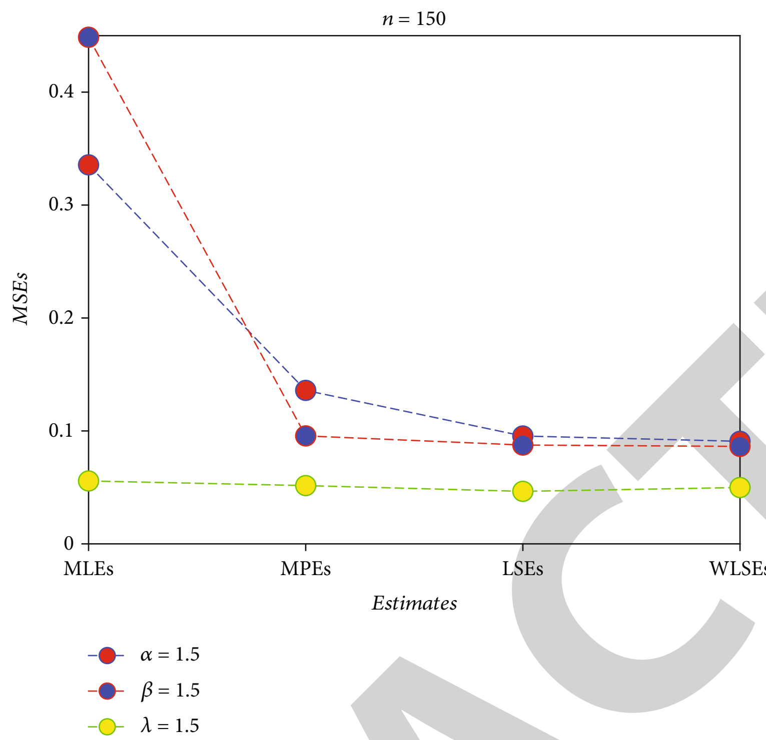

FIGURE 12: MSEs using $n = 150$: $\alpha = 1.5$, $\beta = 1.5$, and $\lambda = 1.5$.

TABLE 6: The lifetime number of days, in increasing order, for 65 patients suffering from breast cancer.

30, 31, 31, 33, 34, 35, 35, 36, 37, 38, 39, 40, 40, 41, 42, 42, 43, 44, 44, 45, 51, 52, 52, 53, 53, 54, 55, 57, 59, 60, 62, 64, 66, 67, 70, 71,73, 74, 75, 77, 78, 84, 85, 87, 90, 93, 100, 101, 103, 105, 108, 125, 130, 134, 150, 150,152, 154, 155, 171, 180, 262, 264, 290, 299.

The PDF of XG-G family can be expressed as

$$f(x;\lambda,\gamma) = \frac{\lambda}{\lambda+1} g(x;\gamma)[1-G(x;\gamma)]^{\lambda-1}\{\lambda 0.5\,\lambda^2 [\ln (1-G(x;\gamma))]^2\}, \tag{5}$$

where $g(x;\gamma) = dG(x;\gamma)/dx$.

We are motivated to introduce the new generalized inverse Weibull distribution because of the above generalizations in the exponentiated family of lifetime distributions. This paper introduces a new three parameter distribution called the X-Gamma inverse Weibull (XGAIW) distribution based on the XG-G family, which contains some lifetime distributions as special submodels. Also, this paper deliberates the comprehensive description of mathematical properties of the new model and presents a graphical analysis of some of its properties. We study different classical point estimation methods for the unknown parameters of XGAIW distribution. Some properties of the density function are discussed. Numerical methods are used to solve the obtained nonlinear equations. Simulation study is used to make comparison between those methods and also to determine which method is more efficient according to the Bias and mean square error (MSE) criteria. The proposed model can be used, as a good alternative to some existing distributions, in modeling several real data.

The rest of the paper is organized as follows. In Section 2, we define the XGAIW distribution and derive a useful representation for its PDF. The mathematical quantities (reliability and hazard rate properties) of the XGAIW distribution are derived in Section 3. In Section 4, the XGAIW parameters are estimated via four methods, namely, the maximum likelihood, maximum product spacing, least squares, and weighted least squares estimators. These estimators are compared via some simulations in Section 5. In Section 6, we illustrate the flexibility and potentiality of the XGAIW model using a real data set. Finally, some concluding remarks are offered in Section 7.

2. The XGAIW Distribution

In this section, we will introduce the XGAIW distribution and some of its submodels. The XG-G family and IW distribution have been used to generate XGAIW distribution. It is represented by the random variable $X \sim \text{XGAIW}(\lambda, \alpha, \beta)$. By using Equations (1)–(4), the CDF of the three-parameter XGAIW distribution takes this form:

$$F(x;\lambda,\alpha,\beta) = 1 - \frac{\left[1-e^{-\alpha x^{-\beta}}\right]^{\lambda}}{\lambda+1}\left\{1+\lambda-\lambda \ln\left(1-e^{-\alpha x^{-\beta}}\right) + 0.5\,\lambda^2\left[\ln\left(1-e^{-\alpha x^{-\beta}}\right)\right]^2\right\}, \tag{6}$$

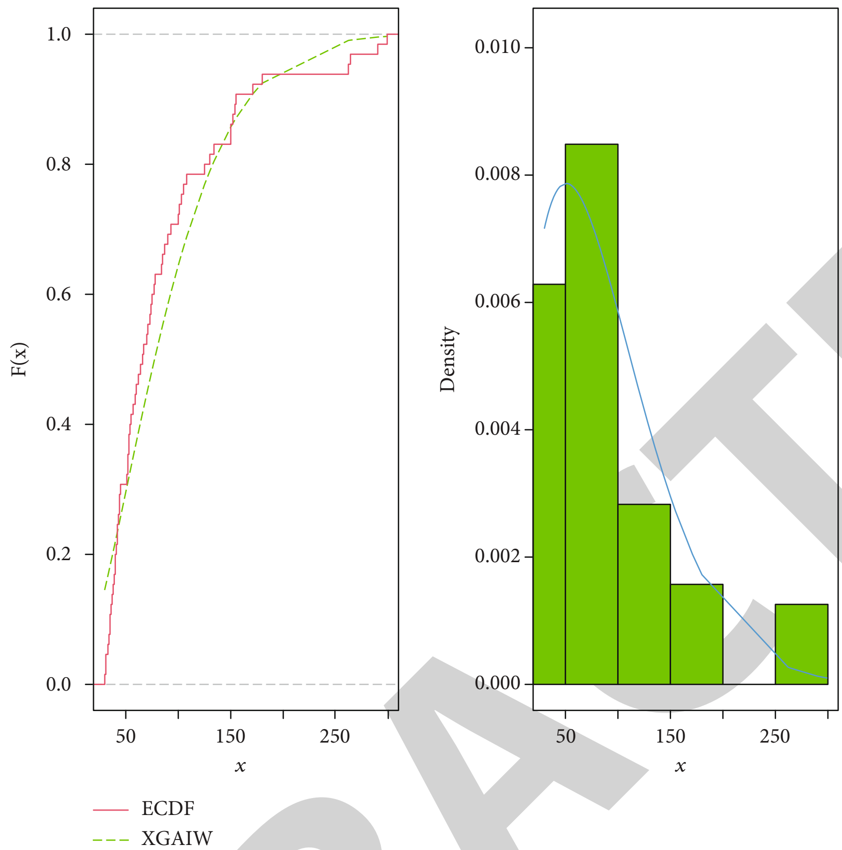

Figure 13: The fitted PDF and estimated CDF of the XGAIW model.

where $\lambda, \alpha, \beta > 0$ and $x > 0$. The corresponding PDF of XGAIW distribution is given as

$$f(x;\lambda,\alpha,\beta) = \frac{\lambda\alpha\beta}{\lambda+1} x^{-\beta-1} e^{-\alpha x^{-\beta}} \left[1 - e^{-\alpha x^{-\beta}}\right]^{\lambda-1} \left\{\lambda + 0.5\,\lambda^2 \left[\ln\left(1 - e^{-\alpha x^{-\beta}}\right)\right]^2\right\}. \quad (7)$$

The plots of density function are displayed in Figure 1. This figure provides some shapes of the PDF of the XGAIW distribution for some different values of the parameters.

The XGAIW distribution with three parameters (λ, α, β) with PDF in Equation (7) is a very flexible model that approaches to different distributions as special submodels:

(1) If $\alpha \longrightarrow 1$, then PDF in Equation (7) reduces to the two-parameter distribution, this is a new model, which can be denoted as X-Gamma inverse exponential (XGAIE) distribution

(2) If $\alpha \longrightarrow 2$, then PDF in Equation (7) reduces to the two-parameter distribution, this is a new model, which can be denoted as X-Gamma inverse Rayleigh (XGAIR) distribution

3. Reliability Analysis

The reliability function (survival function) of XGAIW distribution is given by

$$S(x;\lambda,\alpha,\beta) = \frac{\left[1 - e^{-\alpha x^{-\beta}}\right]^{\lambda}}{\lambda+1} \left\{1 + \lambda - \lambda \ln\left(1 - e^{-\alpha x^{-\beta}}\right) + 0.5\,\lambda^2 \left[\ln\left(1 - e^{-\alpha x^{-\beta}}\right)\right]^2\right\}. \quad (8)$$

The hazard rate function (failure rate) of a lifetime random variable X with XGAIW distribution is given by

$$h(x;\lambda,\alpha,\beta) = \frac{\lambda\alpha\beta x^{-\beta-1} e^{-\alpha x^{-\beta}} \left\{\lambda + 0.5\,\lambda^2 \left[\ln\left(1 - e^{-\alpha x^{-\beta}}\right)\right]^2\right\}}{\left(1 - e^{-\alpha x^{-\beta}}\right)\left\{1 + \lambda - \lambda \ln\left(1 - e^{-\alpha x^{-\beta}}\right) + 0.5\,\lambda^2 \left[\ln\left(1 - e^{-\alpha x^{-\beta}}\right)\right]^2\right\}}. \quad (9)$$

Figure 2 displays plots of the hazard rate function (HRF) of the XGAIW distribution for some values of the parameters as follows.

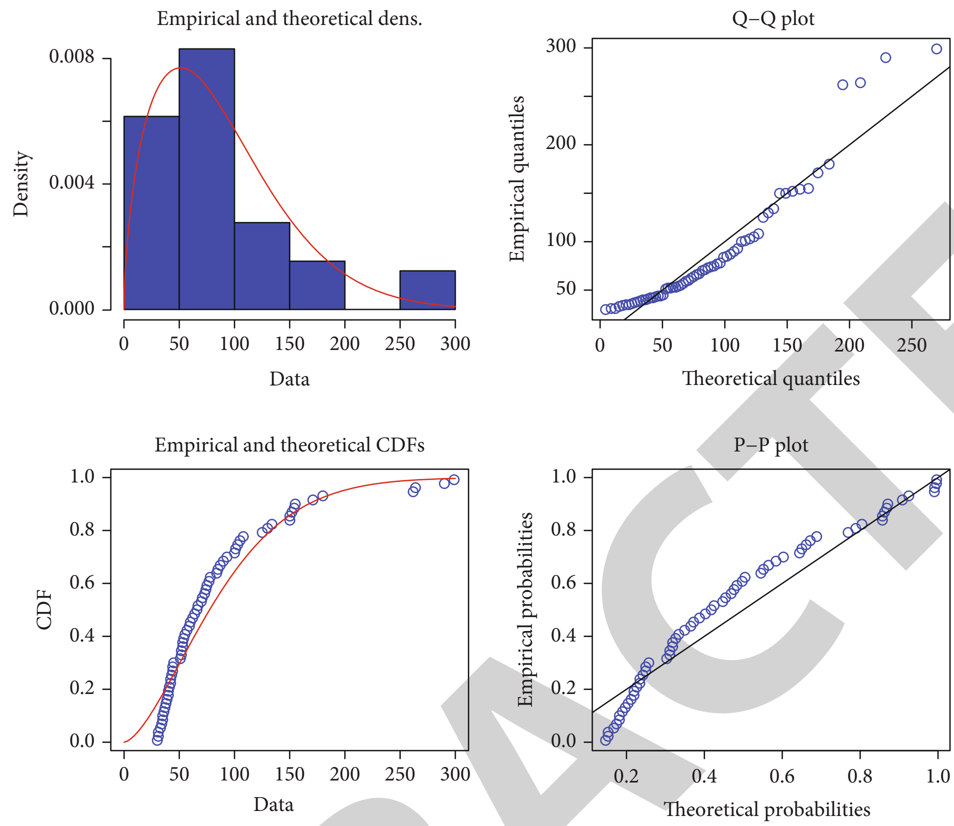

FIGURE 14: The estimated CDF, fitted PDF, P-P plot, and Q-Q plot of the XGAIW distribution for complete breast cancer data.

4. Estimation Methods for XGAIW Distribution

In this section, the parameter estimation of the XGAIW parameters is investigated using four methods of estimation, namely, the maximum likelihood estimators (MLEs), maximum product spacing estimators (MPSEs), least squares estimators (LSEs), and weighted least squares estimators (WLSEs), in the presence of complete sample which will be discussed in details.

4.1. Maximum Likelihood Estimators. We determine the MLEs of the XGAIW parameters. Let $(x_1, x_2, \cdots, x_n)$ be a random sample of size n from XGAIW (δ) where $\delta = (\lambda, \alpha, \beta)^T$. The log-likelihood function for δ of XGAIW distribution is given by

$$\begin{aligned}\ell(\lambda,\alpha,\beta) &= n\ln\left(\frac{\lambda}{\lambda+1}\right) + n[\ln(\beta)+\ln(\alpha)] \\ &\quad -(\beta+1)\sum_{i=1}^{n}\ln(x_i) - \alpha\sum_{i=1}^{n}x_i^{-\beta} \\ &\quad +(\lambda-1)\sum_{i=1}^{n}\ln\left(1-e^{-\alpha x_i^{-\beta}}\right) \\ &\quad +\sum_{i=1}^{n}\ln\left\{\lambda+0.5\lambda^2\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2\right\}.\end{aligned} \tag{10}$$

We can maximize the above log-likelihood equation by solving the nonlinear likelihood equations, which follow by differentiating it. Further, the resulting equations cannot be solved analytically, so some softwares can be used to solve them numerically via iterative techniques such as a Newton–Raphson algorithm. The associated components of the score vector

$$U_n(\delta) = \left(\frac{\partial l}{\partial\lambda}, \frac{\partial\ell}{\partial\alpha}, \frac{\partial\ell}{\partial\beta}\right)^T \tag{11}$$

are given by

$$\begin{aligned}\frac{\partial\ell}{\partial\lambda} &= \frac{n}{\lambda(\lambda+1)} + \sum_{i=1}^{n}\ln\left(1-e^{-\alpha x_i^{-\beta}}\right) + \sum_{i=1}^{n}\frac{1+\lambda\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2}{\lambda+0.5\lambda^2\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2}, \\ \frac{\partial\ell}{\partial\alpha} &= \frac{n}{\alpha} + \sum_{i=1}^{n}x_i^{-\beta} + (\lambda-1)\sum_{i=1}^{n}\frac{e^{-\alpha x_i^{-\beta}}x_i^{-\beta}}{1-e^{-\alpha x_i^{-\beta}}} \\ &\quad + \lambda^2\sum_{i=1}^{n}\frac{\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\left(e^{-\alpha x_i^{-\beta}}x_i^{-\beta}/1-e^{-\alpha x_i^{-\beta}}\right)}{\lambda+0.5\lambda^2\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2}, \\ \frac{\partial\ell}{\partial\beta} &= \frac{n}{\beta} - \sum_{i=1}^{n}\ln(x_i) + \alpha\sum_{i=1}^{n}x_i^{-\beta}\ln(x_i) - (\lambda-1)\alpha\sum_{i=1}^{n}\frac{e^{-\alpha x_i^{-\beta}}\ln(x_i)}{1-e^{-\alpha x_i^{-\beta}}} \\ &\quad - \lambda^2\sum_{i=1}^{n}\frac{\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\left(e^{-\alpha x_i^{-\beta}}\ln(x_i)/1-e^{-\alpha x_i^{-\beta}}\right)}{\lambda+0.5\lambda^2\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2}.\end{aligned} \tag{12}$$

4.2. Maximum Product Spacing Estimators. In statistics, the

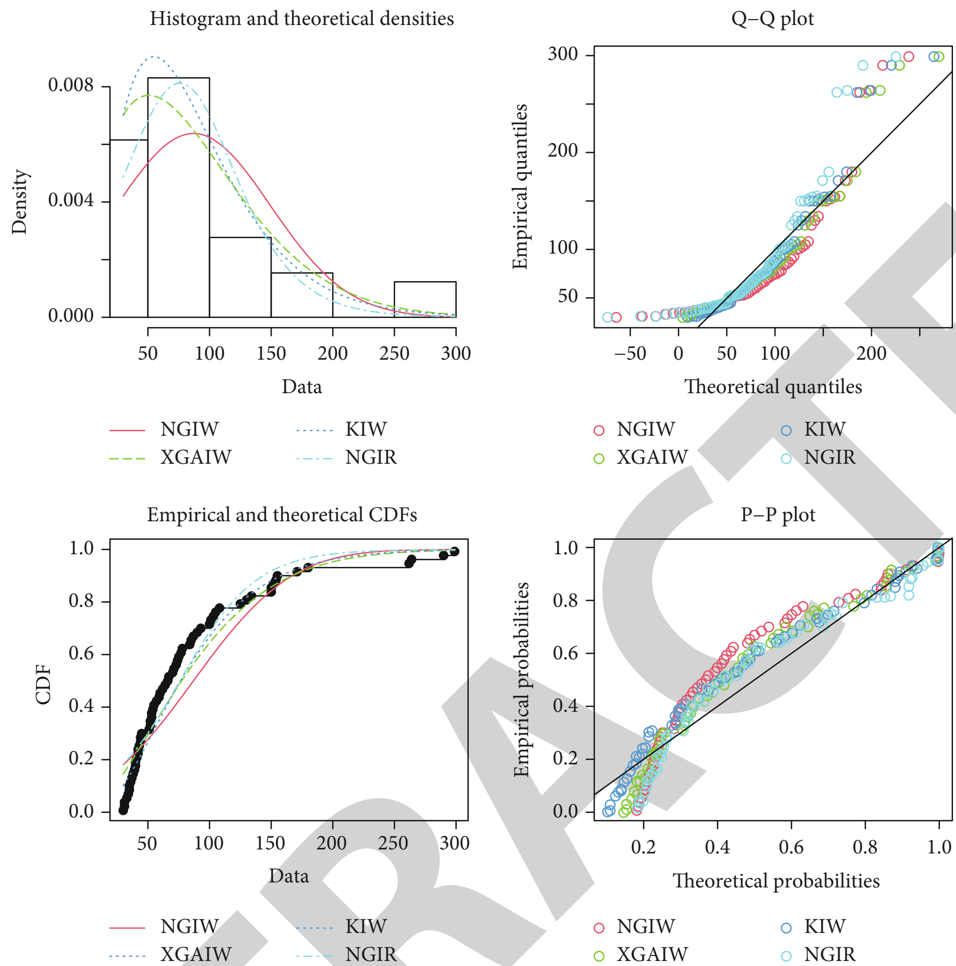

Figure 15: The estimated CDF, fitted PDF, P-P plot, and Q-Q plot of the NGIW, XGAIW, KIW, and NGIR distributions for complete breast cancer data.

Table 7: MLEs and SEs with different models for medical data.

Model	α Estimate	α SEs	β Estimate	β SEs	λ Estimate	λ SEs	η Estimate	η SEs
NGIW	187.248	30.495	1.136	0.091	219.64	18.397	18.7292	2.8363
EGIW	225.635	202.813	0.063	0.029	503.632	191.592	66.574	12.783
KIW	831.957	0.240	5.916	0.485	298.3765	38.780	115.865	5.0631
NGIR	100.249	191.960	0.638	0.336	151.423	12.817	—	—
XGAIW	204.139	163.945	0.495	0.068	62.4588	16.395	—	—
MIW	194.159	19.428	0.0232	0.025	750.835	1.494	—	—
IW	0.729	0.092	24.279	4.432	—	—	—	—

maximum product spacing estimator (MPSE) is a method for estimating the parameters of univariate statistical models. The method requires maximization of the geometric mean of spacings in the data, which are the differences between the values of the cumulative distribution function at neighboring data points. One of the most common methods for estimating the parameters of a distribution from data, the method of MLEs, can break down in various cases, such as involving certain mixtures of continuous distributions. In these cases, the MPSE method may be successful. The MPS method chooses the parameter values that make the observed data as uniform as possible, according to a specific quantitative measure of uniformity.

According to Cheng and Amin [27] and Alshenawy et al. [28] MPSE was introduced, and the uniform spacings of a

Table 8: Goodness-of-fit measures for medical data.

Model	-2ℓ	AIC	BIC	CAIC	HQIC	K-S	P value
NGIW	2209.752	2217.752	2217.004	2218.419	2211.819	0.995	0.006
EGIW	1531.662	1549.662	1538.914	1540.329	1533.792	0.105	0.217
NGIR	1433.051	1439.051	1438.490	1439.444	1434.601	0.056	0.905
XGAIW	1219.607	1225.607	1225.046	1226	1221.157	0.051	0.934
MIW	1539.854	1545.854	1545.293	1546.247	1541.404	0.066	0.778
KIW	1528.907	1536.907	1536.159	1537.574	1530.974	0.101	0.258
IW	2398.396	2402.396	2402.022	2402.49	2399.430	0.359	0.001

random sample $x_1<\cdots<x_n$ of size (n) from the XGAIW distribution can be defined by

$$D_i(\lambda,\alpha,\beta)=F(x_i,\lambda,\alpha,\beta)-F(x_{i-1},\lambda,\alpha,\beta)\,;\, i=1,2,\cdots,n+1, \tag{13}$$

where D_i refers to the uniform spacings and $\sum_{i=1}^{n+1}D_i=1$. The MPSEs can be obtained by maximizing

$$G(\lambda,\alpha,\beta)=\frac{1}{n+1}\sum_{i=1}^{n+1}\ln(D_i\,(\lambda,\alpha,\beta)). \tag{14}$$

The natural logarithm of the product spacing function for the MPSEs of XGAIW distribution is given by

$$\ln G(\lambda,\alpha,\beta)=\frac{1}{n+1}\left(\ln\left(\mathcal{O}[\lambda,\psi(x_n,\beta,\alpha)]\right)+\ln\left(1-\mathcal{O}[\lambda,\psi(x_1,\beta,\alpha)]\right)\right)+\sum_{i=2}^{n}\ln\left(\mathcal{O}[\lambda,\psi(x_{i-1},\beta,\alpha)]-\mathcal{O}[\lambda,\psi(x_i,\beta,\alpha)]\right)), \tag{15}$$

where $\psi(x_i\,;\beta,\alpha)=1-e^{-\alpha x_i^{-\beta}}$, $\mathcal{O}(\lambda,\psi)=((\psi)^{\lambda}/\lambda)\{1+\lambda-\lambda\ln(\psi)+0.5\lambda^2[\ln(\psi)]^2\}$.

To obtain the normal equations for the unknown parameters, we differentiate partially Equation (15) with respect to the vector parameter δ and equate them to zero. The estimators of δ can be obtained by solving the system of nonlinear equations, so the MPSEs of λ, α, and β can be found by using any iterative procedure technique such as conjugate-gradient algorithm solution.

4.3. Ordinary and Weighted Least Squares. Swain et al. [29] introduced the LSE and WLSE methods that are used to estimate the parameters of beta distribution. Let $x_{(1)}<x_{(2)}<\cdots<x_{(n)}$ be the order statistics of a sample from the XGAIW distribution, and then, the LSEs and WLSEs of λ, α, and β can be obtained by minimizing the following function with respect to λ, α, and β:

$$S(\lambda,\alpha,\beta)=\sum_{i=1}^{n}A_i\left[1-\frac{\left[1-e^{-\alpha x_i^{-\beta}}\right]^{\lambda}}{\lambda+1}\left\{1+\lambda-\lambda\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)+0.5\,\lambda^2\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2\right\}-\frac{i}{n+1}\right]^2, \tag{16}$$

where $(A_i=1)$ in the case of LSEs and $(A_i=((n+1)^2(n+2))/(i(n-i+1)))$ in the case of WLSEs. Further, the LSEs and WLSEs of the XGAIW parameters are also obtained by solving the following nonlinear equations simultaneously with respect to λ, α, and β:

$$\frac{\partial S(\lambda,\alpha,\beta)}{\partial\lambda}=\sum_{i=1}^{n}A_i\left[1-\frac{\left[1-e^{-\alpha x_i^{-\beta}}\right]^{\lambda}}{\lambda+1}\left\{1+\lambda-\lambda\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)+0.5\,\lambda^2\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2\right\}-\frac{i}{n+1}\right]\varphi_{1i=0},$$

$$\frac{\partial S(\lambda,\alpha,\beta)}{\partial\alpha}=\sum_{i=1}^{n}A_i\left[1-\frac{\left[1-e^{-\alpha x_i^{-\beta}}\right]^{\lambda}}{\lambda+1}\left\{1+\lambda-\lambda\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)+0.5\,\lambda^2\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2\right\}-\frac{i}{n+1}\right]\varphi_{2i=0},$$

$$\frac{\partial S(\lambda,\alpha,\beta)}{\partial\beta}=\sum_{i=1}^{n}A_i\left[1-\frac{\left[1-e^{-\alpha x_i^{-\beta}}\right]^{\lambda}}{\lambda+1}\left\{1+\lambda-\lambda\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)+0.5\,\lambda^2\left[\ln\left(1-e^{-\alpha x_i^{-\beta}}\right)\right]^2\right\}-\frac{i}{n+1}\right]\varphi_{3i=0}, \tag{17}$$

where

$$\varphi_{1i} = \left(\frac{-\phi_i^{\,2}}{\lambda+1}\right)\left[1-\ln\phi_i+\lambda(\ln\phi_i)^2+\frac{1}{\lambda+1}\right.$$
$$\left.\cdot\,(1-(\lambda+1)\ln\phi_i)\left(1+\lambda-\lambda\ln\phi_i+\lambda(\ln\phi_i)^2\right)\right],$$

$$\varphi_{2i} = \left(\lambda+0.5\,\lambda^2[\ln\phi_i]^2\right)\left(\frac{-\lambda x_i^{-\beta}\phi_i^{\lambda-1}(1-\phi_i)}{\lambda+1}\right),$$

$$\varphi_{3i} = \left(\lambda+0.5\,\lambda^2\,[\ln\phi_i]^2\right)\left(\frac{-\lambda\ln x_i\,\phi_i^{\lambda-1}(1-\phi_i)}{\lambda+1}\right),$$

$$\phi_i = 1-e^{-\alpha x_i^{-\beta}}. \tag{18}$$

5. Monte Carlo Simulation Study

In this section, we conduct a Monte Carlo simulation study to estimate the parameters based on complete sample by using MLE, MPSE, LSE, and WLSE methods. R software is used to perform our Monte Carlo simulation study; see Alshenawy et al. [30].

Monte Carlo experiments were carried out based on 10,000 random samples for following data generated form XGAIW distribution by using numerical analysis, where x_i is distributed as XGAIW distribution for different parameters (λ, α, β) with different actual values of parameter and for different sample sizes $n = 50, 100, \text{and}\, 150$. We compare the performances of the MLE, MPSE, LSE, and WLSE methods based on the Bias and mean squared errors (MSE) for different sample sizes. Therefore, we report all the results up to three decimal places. Remember that Bias estimator is $\text{Bias} = \widehat{\delta} - \delta$, where δ is the estimated value of δ, and the MSE of the estimator is $\text{MSE} = \text{Mean}\,(\widehat{\delta} - \delta)^2$.

We conclude remarks on the Monte Carlo simulation study as follows:

(1) The simulation outcomes are recorded in Tables 1–5. The following concluding remarks are noticed based on these tables as follows

(2) The Bias and MSE values of α, β, and λ for all estimation methods decrease as the sample size (n) increases

(3) The MPSE method has more relative efficiency than MLEs for most parameters of XGAIW distribution in all tables

(4) We can analyze that by increasing α, the MSE and Bias for the parameter β decrease while for λ decrease, in most cases

(5) The LSEs have the lowest MSE in most cases of α and β. Also, the WLSEs have the least MSE in most cases of λ for different sample sizes

Figures 3–12 present the values of MSEs corresponding with estimate methods MLE, MPE, LSE, and WLSE using Monte Carlo experiments which were carried out based on 10,000 random samples.

In Figures 3–12, it appears that the WLSE method is the best method.

6. An Application for Breast Cancer

In this section, we present an application to breast cancer data set to illustrate the performance and flexibility of the XGAIW distribution and show that XGAIW distribution can be a better model than some recently developed models where the particular data are utilized. This data set is obtained from the "Ministry of Health and Population Egypt" (see http://www.statista.com/statistics/1044734/egypt-number-of-cancer-prevalence-cases-general-population-by-type/ (2021)). The data set, given in Table 6, represents 65 patients suffering from breast cancer from one of the ministries of health hospitals in Egypt.

For these breast cancer data set, we compare the XGAIW model with some rival models, namely, the new generalized inverse Weibull distribution (NGIW) by Khan and King, exponentiated generalized inverse Weibull distribution (EGIW) by Elbatal and Muhammed [31], new generalized inverse Rayleigh (NGIR) by Malik and Ahmad [32], modified inverse Weibull (MIW) distribution by Khan and King [33], Kumaraswamy–inverse Weibull distribution (KIW) by Shahbaz M. Q. et al. [34], and inverse Weibull (IW) distribution; see Ibrahim and Almetwally [35].

Figures 13 and 14 offer the plots of estimated CDF, fitted PDF, Q-Q plot, and P-P plot for the XGAIW distribution for cancer data. Figure 15 indicates that the XGAIW distribution supply better fits to breast cancer data compared to some other distributions.

The selection of models for specific data is one of the basic tasks of the scientific study in choosing a predictive model from a group of candidate models. Several statistical methods are available to determine the fitness of competing distributions, where the most widely used are the Kolmogorov-Smirnov (K-S) statistic and corresponding P value, −2 log-likelihood function (-2ℓ), Akaike information criterion (AIC), the correct Akaike information criterion (CAIC), Bayesian information criterion (BIC), and Hannan-Quinn information criterion (HQIC). However, the better distribution corresponds to the smaller values of AIC, CAIC, BIC, HQIC, and K-S criteria and largest values of P value. These methods are determined according to the following formulas, respectively.

The AIC is evaluated as follows:

$$\text{AIC} = 2k - 2\ell. \tag{19}$$

The CAIC is

$$\text{CAIC} = \frac{2nk}{n-k-1} - 2\ell. \tag{20}$$

The BIC is given by

$$\text{BIC} = k\,\log(n) - 2\ell. \tag{21}$$

The HQIC is

$$\mathrm{HQIC} = 2k \log(\log(n)) - 2\ell, \quad (22)$$

where ℓ is the MLE log-likelihood function value, k is the number of parameters in any distribution, and n is considered as the size of the sample used in calculations.

The MLE estimates of the parameters with the corresponding standard errors (SEs) are reported in Table 7, while in Table 8, we list the values of AIC, CAIC, BIC, HQIC, and K-S and the P value statistics. We observe that the XGAIW model has the smallest AIC, CAIC, BIC, HQIC, and K-S values and has the largest P value as compared with those values of the other models. So, the XGAIW model seems to be a very competitive model to this data. More information is provided by a visual comparison of the histogram and estimated cumulative of the breast cancer data set as shown in Figures 13–15. It is clear from Figure 15 that the XGAIW distribution provides a better fit for breast cancer data set.

7. Concluding Remarks

In this paper, we proposed a three-parameter X-Gamma inverse Weibull (XGAIW) distribution, as a new extension of the IW model. The XGAIW density is a linear combination of the IW densities. Some explicit expressions for mathematical quantities of the XGAIW distribution are derived. The new distribution is much more flexible than the IW distribution and could have increasing, decreasing, and unimodal hazard rates. We consider four methods of estimation, namely, the MLEs, MPSEs, LSEs, and WLSEs, to estimate the XGAIW parameters. The performance of these proposed estimation methods is conducted via some simulations. A breast cancer data set application proves that the XGAIW model provides consistently better fits compared to some other rival models.

Data Availability

The data set, given in Table 6, represents 65 patients suffering from breast cancer from one of the ministry of health hospitals in Egypt.

Conflicts of Interest

The author declares no conflicts of interest.

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References

[1] R. Alshenawy, “A new one parameter distribution: properties and estimation with applications to complete and type II censored data,” *Journal of Taibah University for Science*, vol. 14, no. 1, pp. 11–18, 2020.

[2] A. Z. Keller, A. R. R. Kamath, and U. D. Perera, “Reliability analysis of CNC machine tools,” *Reliability Engineering*, vol. 3, no. 6, pp. 449–473, 1982.

[3] S. Alkarni, A. Z. Afify, I. Elbatal, and M. Elgarhy, “The extended inverse Weibull distribution: properties and applications,” *Complexity*, vol. 2020, Article ID 3297693, 11 pages, 2020.

[4] W. B. Nelson, *Applied Life Data Analysis*, vol. Vol. 521, John Wiley & Sons, 2003.

[5] R. Jiang, D. N. P. Murthy, and P. Ji, “Models involving two inverse Weibull distributions,” *Reliability Engineering & System Safety*, vol. 73, no. 1, pp. 73–81, 2001.

[6] M. S. Khan, G. R. Pasha, and A. H. Pasha, “Theoretical analysis of inverse Weibull distribution,” *WSEAS Transactions on Mathematics*, vol. 7, no. 2, pp. 30–38, 2008.

[7] A. S. Hassan and S. G. Nassr, “The inverse Weibull generator of distributions: properties and applications,” *Journal of Data Science*, vol. 16, no. 4, pp. 723–742, 2021.

[8] S. Subhradev, M. Ç. Korkmaz, M. C. Korkmaz, and H. M. Yousof, “The Quasi XGamma-Poisson Distribution: Properties and Application,” *Istatistik Journal of The Turkish Statistical Association*, vol. 11, no. 3, pp. 65–76, 2018.

[9] M. S. Khan and R. King, “New generalized inverse Weibull distribution for lifetime modeling,” *Communications for Statistical Applications and Methods*, vol. 23, no. 2, pp. 147–161, 2016.

[10] H. D. Biçer, “Properties and inference for a new class of XGamma distributions with an application,” *Mathematical Sciences*, vol. 13, no. 4, pp. 335–346, 2019.

[11] H. H. Ahmad and E. Almetwally, “Marshall-Olkin generalized Pareto distribution: Bayesian and non Bayesian estimation,” *Pakistan Journal of Statistics and Operation Research*, vol. 16, no. 1, pp. 21–33, 2020.

[12] R. Calabria and G. Pulcini, “On the maximum likelihood and least-squares estimation in the inverse Weibull distribution,” *Statistica Applicata*, vol. 2, no. 1, pp. 53–66, 1990.

[13] R. Calabria and G. Pulcini, “Bayes 2-sample prediction for the inverse Weibull distribution,” *Communications in Statistics-Theory and Methods*, vol. 23, no. 6, pp. 1811–1824, 1994.

[14] R. Jiang, M. J. Zuo, and H. X. Li, “Weibull and inverse Weibull mixture models allowing negative weights,” *Reliability Engineering & System Safety*, vol. 66, no. 3, pp. 227–234, 1999.

[15] K. S. Sultan, M. A. Ismail, and A. S. Al-Moisheer, “Mixture of two inverse Weibull distributions: properties and estimation,” *Computational Statistics & Data Analysis*, vol. 51, no. 11, pp. 5377–5387, 2007.

[16] F. R. De Gusmao, E. M. Ortega, and G. M. Cordeiro, “The generalized inverse Weibull distribution,” *Statistical Papers*, vol. 52, no. 3, pp. 591–619, 2011.

[17] F. Hemmati, E. Khorram, and S. Rezakhah, “A new three-parameter ageing distribution,” *Journal of Statistical Planning and Inference*, vol. 141, no. 7, pp. 2266–2275, 2011.

[18] M. S. Khan, R. King, and I. Hudson, “Characterizations of the transmuted inverse Weibull distribution,” *Anziam Journal*, vol. 55, pp. C197–C217, 2013.

[19] L. A. Baharith, S. A. Mousa, M. A. Atallah, and S. H. Elgayar, “The beta generalized inverse Weibull distribution,” *Journal of Advances in Mathematics and Computer Science*, vol. 4, pp. 252–270, 2014.

[20] A. R. Kamel and M. A. Alqarni, "A new characterization of exponential distribution through minimum chi-squared divergence principle," *Journal of Advanced Research in Applied Mathematics and Statistics*, vol. 5, no. 1-2, pp. 14–26, 2020.

[21] S. Sen, S. S. Maiti, and N. Chandra, "The xgamma distribution: statistical properties and application," *Journal of Modern Applied Statistical Methods*, vol. 15, no. 1, pp. 774–788, 2016.

[22] S. Sen and N. Chandra, "The quasi xgamma distribution with application in bladder cancer data," *Journal of data science*, vol. 15, no. 1, pp. 61–76, 2017.

[23] S. Sen, H. Al-Mofleh, and S. S. Maiti, "On discrimination between the Lindley and xgamma distributions," *Annals of Data Science*, vol. 8, no. 3, pp. 559–575, 2021.

[24] A. S. Yadav, S. S. Maiti, and M. Saha, "The inverse xgamma distribution: statistical properties and different methods of estimation," *Annals of Data Science*, vol. 8, no. 2, pp. 275–293, 2021.

[25] G. M. Cordeiro, E. Altun, M. C. Korkmaz, R. R. Pescim, A. Z. Afify, and H. M. Yousof, "The xgamma family: censored regression modelling and applications," *Revstat - Statistical Journal*, vol. 18, pp. 593–612, 2020.

[26] A. Alzaatreh, C. Lee, and F. Famoye, "A new method for generating families of continuous distributions," *Metron*, vol. 71, no. 1, pp. 63–79, 2013.

[27] R. C. H. Cheng and N. A. K. Amin, "Estimating parameters in continuous univariate distributions with a shifted origin," *Journal of the Royal Statistical Society: Series B (Methodological)*, vol. 45, no. 3, pp. 394–403, 1983.

[28] R. Alshenawy, M. A. Sabry, E. M. Almetwally, and H. M. Almongy, "Product spacing of stress–strength under progressive hybrid censored for exponentiated-gumbel distribution," *Computers, Materials & Continua*, vol. 66, no. 3, pp. 2973–2995, 2021.

[29] J. J. Swain, S. Venkatraman, and J. R. Wilson, "Least-squares estimation of distribution functions in Johnson's translation system," *Journal of Statistical Computation and Simulation*, vol. 29, no. 4, pp. 271–297, 1988.

[30] R. Alshenawy, N. Feroze, A. Al-Alwan, M. Saleem, and S. Islam, "A family of Bayesian estimators for the two-parametric Burr type II distribution," *Journal of Function Spaces*, vol. 2022, 12 pages, 2022.

[31] I. Elbatal and H. Z. Muhammed, "Exponentiated generalized inverse Weibull distribution," *Applied Mathematical Sciences*, vol. 8, no. 81, pp. 3997–4012, 2014.

[32] A. S. Malik and S. P. Ahmad, "A new generalized inverse Rayleigh distribution with applications to real life data," *International Journal of Statistics and Reliability Engineering*, vol. 5, no. 1, pp. 38–45, 2018.

[33] M. S. Khan and R. King, "Modified inverse Weibull distribution," *Journal of statistics applications & Probability*, vol. 1, no. 2, pp. 115–132, 2012.

[34] M. Q. Shahbaz, S. Shahbaz, and N. S. Butt, "The Kumaraswamy-inverse Weibull distribution," *Pakistan Journal of Statistics and Operation Research*, vol. 8, no. 3, pp. 479–489, 2012.

[35] G. M. Ibrahim and E. M. Almetwally, "The new extension of inverse Weibull distribution with applications of medicine data," *Scientific Journal for Financial and Commercial Studies and Researches (SJFCSR)*, vol. 2, 1, Part 1, pp. 577–598, 2021.