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Retraction

Retracted: Intuitionistic Fuzzy Prioritized Aggregation Operators Based on Priority Degrees with Application to Multicriteria Decision-Making

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This article has been retracted by Hindawi following an investigation undertaken by the publisher [1]. This investigation has uncovered evidence of one or more of the following indicators of systematic manipulation of the publication process:

(1) Discrepancies in scope

(2) Discrepancies in the description of the research reported

(3) Discrepancies between the availability of data and the research described

(4) Inappropriate citations

(5) Incoherent, meaningless and/or irrelevant content included in the article

(6) Peer-review manipulation

The presence of these indicators undermines our confidence in the integrity of the article's content and we cannot, therefore, vouch for its reliability. Please note that this notice is intended solely to alert readers that the content of this article is unreliable. We have not investigated whether authors were aware of or involved in the systematic manipulation of the publication process.

Wiley and Hindawi regrets that the usual quality checks did not identify these issues before publication and have since put additional measures in place to safeguard research integrity.

We wish to credit our own Research Integrity and Research Publishing teams and anonymous and named external researchers and research integrity experts for contributing to this investigation.

The corresponding author, as the representative of all authors, has been given the opportunity to register their agreement or disagreement to this retraction. We have kept a record of any response received.

References

[1] A. N. Al-Kenani, R. Anjum, and S. Islam, "Intuitionistic Fuzzy Prioritized Aggregation Operators Based on Priority Degrees with Application to Multicriteria Decision-Making," *Journal of Function Spaces*, vol. 2022, Article ID 4751835, 16 pages, 2022.

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Research Article

Intuitionistic Fuzzy Prioritized Aggregation Operators Based on Priority Degrees with Application to Multicriteria Decision-Making

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In practise, intuitionistic fuzzy numbers (IFNs) are particularly useful for describing ambiguous data. We look at multicriteria decision-making (MCDM) problems with a prioritising relationship between the parameters. The concept of a priority degree is presented. The aggregation operators (AOs) are formed by assigning nonnegative real numbers to stringent priority levels, known as priority degrees. As a result, we construct "intuitionistic fuzzy prioritized averaging operator with priority degrees" and "intuitionistic fuzzy prioritized geometric operator with priority degrees," which are both prioritized operators. The attributes of the existing method are frequently compared to those of other current approaches, stressing the superiority of the provided work over other methods now in use. In addition, the impact of priority degrees on the overall result is thoroughly investigated. Furthermore, in the intuitionistic fuzzy set (IFS) context, a decision-making strategy is proposed based on these operators. To highlight the efficacy of the proposed approach, an illustrative example relating to the selection of the best choice is considered.

1. Introduction

Aggregation operators (AOs) are used in a large number of practical multicriteria decision-making (MCDM) situations. Many systems rely on data aggregation and fusion, including machine learning, decision-making, image processing, and pattern recognition. The aggregation strategy, in a broader sense, combines various bits of data to arrive at a result or judgement. It has also been revealed that modelling working situations in human cognition mechanisms using simple data handling algorithms based on crisp integers is problematic. As a result of these techniques, decision-makers (DMs) are left with cloudy conclusions and confusing decisions. As a result, in order to cope with unclear and fuzzy circumstances that occur in the world, DMs demand a new ideology that allows them to comprehend ambiguous data values and sustain their decision making requirements in accordance with the situation. In this regard, Zadeh [1] has revolutionized the use of a fuzzy set theory to represent ambiguous data. Atanassov [2] revealed the notion of the intuitionistic fuzzy set (IFS).

Aggregation of data is important for decision-making corporate, administrative, social, medical, technological, psychological, and artificial intelligence fields. Awareness of the alternative has traditionally been seen as a crisp number or linguistic number. However, due to its uncertainty, the data cannot easily be aggregated. AOs, in fact, have a significant role in the context of MCDM issues the main goal of which is to aggregate a series of input to a single number. Ye [3] introduced the operational laws of single-valued neutrosophic sets (SVNSs) and suggested geometric and averaging AOs for SVNNs in this direction. Peng et al. [4] proposed upgraded SVNN operations and established their associated AOs. Nancy and Garg [5] established AOs by employing Frank operations. Liu et al. [6] created some AOs for SVNNs based on Hamacher operations. Zhang et al. [7] provided the AOs in the context of an interval-valued neutrosophic set. Li et al. [8] presented the novel idea of generalized simplified neutrosophic Einstein AOs. Wei and Wei [9] developed Dombi prioritized AOs for SVNSs. Liu [10] gave the idea of AOs based on archimedean t-norm and t-conorm for SVNSs. Garg and Nancy [11] gave the novel idea of prioritized muirhead mean AOs under NSs. AOs such as averaging and geometric operators for IFSs were proposed by Xu et al. [12–14]. Many studies extended aggregation operators to various fuzzy sets: Mahmood et al. [15], Wei et al. [16], Jose and Kuriaskose [17], Feng et al. [18], and Wang and Liu [19]. Liu and Liu [20] initiated the idea of q-rung orthopair (q-ROF) Bonferroni mean AOs. Liu et al. [21] proposed the idea of q-ROF Heronian mean AOs and application related to MCDM. Garg and Rani [22] constructed Bonferroni mean AOs for complex IFS and applied them to MCDM. Akram et al. [23] invented the linguistic q-ROF Einstein graph and applied it to real-world problems. Yager [24] introduced many prioritized AOs. Li and Xu [25] gave a novel idea of prioritized AOs based on the PDs. Wang et al. [26] gave the notion of power Heronian mean AOs related to q-ROFSs and their application towards MCDM. Rani and Garg [27] initiated the concept of complex intuitionistic fuzzy power AOs and their application to MCDM. Liang et al. [28] developed MULTIMOORA with interval-valued PFSs. Liu and Qin [29] introduced Maclaurin symmetric mean AOs related to IFSs. Gul [30] developed the notion of Fermatean fuzzy SAW, ARAS, and VIKOR with applications in COVID-19 testing laboratory selection problem. Ye et al. [31] introduced MCDM method based on fuzzy rough sets. Mu et al. [32] developed power Maclaurin symmetric mean AOs based on interval-valued Pythagorean fuzzy set. Batool et al. [33] gave the idea of Pythagorean probabilistic hesitant fuzzy AOs. Riaz et al. [34] introduced novel approach for third-party reverse logistic provider selection process under linear Diophantine fuzzy framework. Khan et al. [35] proposed new ranking technique for q-ROFSs based on the novel score function. Kamaci [36] proposed the idea of linguistic single-valued neutrosophic soft sets. Ashraf and Abdullah [37] presented some AOs related to the spherical fuzzy set. Karaaslan and Ozlu [38] introduced some correlation coefficients of dual type-2 hesitant fuzzy sets.

In our daily lives, we come across numerous situations where a mathematical function capable of reducing a collection of numbers to a single number is needed. As a result, the AO inquiry plays a significant role in MCDM problems. Because of their broad use in fields, many researchers have recently focused on how to aggregate data. However, we often come across cases where the points to be aggregated have a strict prioritization relationship. For example, if we want to buy a plot of land to build a house based on the parameters of utility access (P_1), location (P_2), and cost (P_3), we do not want to pay utility access for location and location for cost. That is, in this situation, there is strict prioritization among parameters, such as $P_1 > P_2 > P_3$, where > indicates preferred to. To deal this type of problem, Yu and Xu [39] introduced prioritized AOs with IFSs.

The concept of deciding priority degree (PD) among priority orders expands the versatility of the prioritized operators. The DM should choose the PD vector based on his or her preferences and the nature of the problem. Consider the preceding example of purchasing a plot to further illustrate the concept of PDs. Each priority level will be assigned a PD, which will be a true nonnegative number. Since$P_1 > P_2 > P_3$in the preceding case, the first priority order $P_1 > P_2$ is given a PD d_1 where $0 < d_1 {<} \infty$ and this prioritization relationship is written as $P_1 {>}_{d_1} P_2$. Correspondingly, the PD d_2 is allocated to the second priority order $P_2 {>}_{d_2} P_3$ and $0 < d_2 {<} \infty$. As a result, a two-dimensional vector $d = (d_1, d_2)$ is assigned to the prioritized criteria $P_1 > P_2 > P_3$, and the relationship is expressed as $P_1 {>}_{d_1} P_2 {>}_{d_2} P_3$. Now, we will look at three particular situations involving PDs:

(1) If the first parameter is to be given top priority, the first PD d_1 should be given a large value. Furthermore, we will illustrate in this paper that when $d_1 \longrightarrow \infty$, the consolidated value is calculated solely by the first criterion, with the other criterion values being ignored

(2) If we consider the PD vector to be zero, we can see that all of the parameters become equally as important, and no prioritization among the parameters remains

(3) There is natural prioritization among the parameters if each PD is equal to one. We will show Yu and Xu [39] proposed AOs and our proposed AOs based on PD is same

Taking into consideration the superiority of the IFNs set over the other sets (as discussed above) for dealing with MCDM issues, there is a need to build some new prioritized AOs based on PDs. To the best of our knowledge, no work has been performed in the context of establishing such operators that take PDs into account among strict priority levels in a IFS framework.

The rest of this article is arranged as follows. Section 2 contains several fundamental IFS notions. In Section 3, we looked at how the IF prioritized AOs based on the priority vector are working. In Section 4, we offer an approach for solving MCDM problems based on new AOs. In Section 5, you will find an application for selecting the agriculture land. Section 6 concludes with some final thoughts and recommendations for the future.

2. Certain Fundamental Concepts

In this section of the paper, we keep in mind a few basics and operational principles of IFNs.

Definition 1 (see [2]). Assume IFS $\tilde{\mathscr{T}}$ in $\mathcal{Q}$ is defined as

$$\mathscr{T} = \left\{ \left\langle \varsigma, \eta^{\gamma}{}_{\mathscr{T}}(\varsigma), \hbar^{\mathfrak{F}}{}_{\mathscr{T}}(\varsigma) \right\rangle : \varsigma \in \mathcal{Q} \right\}, \tag{1}$$

where $\eta^{\gamma}{}_{\mathcal{T}}, \hbar^{\mathfrak{I}}{}_{\mathcal{T}} : \mathcal{Q} \longrightarrow [0, 1]$ defines the MD and NMD of the alternative $\varsigma \in \mathcal{Q}$ and $\forall \varsigma$; we have

$$0 \leq \eta^{\gamma}{}_{\mathcal{T}}(\varsigma) + \hbar^{\mathfrak{I}}{}_{\mathcal{T}}(\varsigma) \leq 1. \tag{2}$$

Definition 2 (see [2]). Let $\xi^{\delta}{}_1 = \langle \eta^{\gamma}{}_1, \hbar^{\mathfrak{I}}{}_1 \rangle$ and $\xi^{\delta}{}_2 = \langle \eta^{\gamma}{}_2, \hbar^{\mathfrak{I}}{}_2 \rangle$ be IFNs. $\sigma > 0$, Then,

$$\begin{aligned}
\xi^{\delta c}{}_1 &= \left\langle \hbar^{\mathfrak{I}}{}_1, \eta^{\gamma}{}_1 \right\rangle, \\
\xi^{\delta}{}_1 \vee \xi^{\delta}{}_2 &= \left\langle \max\left\{\eta^{\gamma}{}_1, \hbar^{\mathfrak{I}}{}_1\right\}, \min\left\{\eta^{\gamma}{}_2, \hbar^{\mathfrak{I}}{}_2\right\} \right\rangle, \\
\xi^{\delta}{}_1 \wedge \xi^{\delta}{}_2 &= \left\langle \min\left\{\eta^{\gamma}{}_1, \hbar^{\mathfrak{I}}{}_1\right\}, \max\left\{\eta^{\gamma}{}_2, \hbar^{\mathfrak{I}}{}_2\right\} \right\rangle, \\
\xi^{\delta}{}_1 \oplus \xi^{\delta}{}_2 &= \left\langle \left(\eta^{\gamma 2}_1 + \eta^{\gamma 2}_2 - \eta^{\gamma 2}_1 \eta^{\gamma 2}_2\right), \hbar^{\mathfrak{I}}{}_1 \hbar^{\mathfrak{I}}{}_2 \right\rangle, \\
\xi^{\delta}{}_1 \otimes \xi^{\delta}{}_2 &= \left\langle \eta^{\gamma}{}_1 \eta^{\gamma}{}_2, \left(\hbar^{\mathfrak{I} 2}_1 + \hbar^{\mathfrak{I} 2}_2 - \hbar^{\mathfrak{I} 2}_1 \hbar^{\mathfrak{I} 2}_2\right) \right\rangle, \\
\sigma \xi^{\delta}{}_1 &= \left\langle \left(1 - \left(1 - \eta^{\gamma 2}_1\right)^{\sigma}\right), \hbar^{\mathfrak{I} \sigma}_1 \right\rangle, \\
\xi^{\delta \sigma}{}_1 &= \left\langle \eta^{\gamma \sigma}_1, \left(1 - \left(1 - \hbar^{\mathfrak{I} 2}_1\right)^{\sigma}\right) \right\rangle.
\end{aligned} \tag{3}$$

Definition 3 (see [2]). Let $\xi^{\delta} = \langle \eta^{\gamma}, \hbar^{\mathfrak{I}} \rangle$ be the IFN, score function Ξ of ξ^{δ} is defined as

$$\Xi\left(\xi^{\delta}\right) = \eta^{\gamma} - \hbar^{\mathfrak{I}}, \tag{4}$$

where $\Xi(\xi^{\delta}) \in [-1, 1]$. The IFN score shall decide its ranking, i.e., the maximum score shall determine the high IFN priority. In certain situations, the score function is not really beneficial for IFN. It is therefore not sufficient to use the score function to evaluate the IFNs. We are adding an additional function, i.e., an accuracy function.

Definition 4 (see [2]). Let $\xi^{\delta} = \langle \eta^{\gamma}, \hbar^{\mathfrak{I}} \rangle$ be the IFN, then an accuracy function H of ξ^{δ} is defines as

$$\begin{aligned}
H\left(\xi^{\delta}\right) &= \eta^{\gamma} + \hbar^{\mathfrak{I}}, \\
H\left(\xi^{\delta}\right) &\in [0, 1].
\end{aligned} \tag{5}$$

Definition 5. Consider $\xi^{\delta} = \langle \eta^{\gamma}{}_{\xi^{\delta}}, \hbar^{\mathfrak{I}}{}_{\xi^{\delta}} \rangle$ and $\beta = \langle \eta^{\gamma}{}_{\beta}, \hbar^{\mathfrak{I}}{}_{\beta} \rangle$ are two IFNs, and $\Xi(\xi^{\delta}), \Xi(\beta)$ are the score function of ξ^{δ} and β, and $H(\xi^{\delta}), H(\beta)$ are the accuracy function of ξ^{δ} and β, respectively, then

(a) If $\Xi(\xi^{\delta}) > \Xi(\beta)$, then $\xi^{\delta} > \beta$

(b) If $\Xi(\xi^{\delta}) = \Xi(\beta)$, then

If $H(\xi^{\delta}) > H(\beta)$ then $\xi^{\delta} > \beta$, and if $H(\xi^{\delta}) = H(\beta)$, then $\xi^{\delta} = \beta$.

It should always be noticed that the value of score function is between −1 and 1. We introduce another score function, to support this type of research, $\breve{\Xi}(R) = (1 + \eta^{\gamma}{}_R - \hbar^{\mathfrak{I}}{}_R)/2$. We can see that $0 \leq \breve{\Xi}(R) \leq 1$. This new score function satisfies all properties of score function defined in [2].

Definition 6 (see [12]). Assume that $\xi^{\delta}{}_g = \langle \eta^{\gamma}{}_g, \hbar^{\mathfrak{I}}{}_g \rangle$ is a family of IFNs, and IFWA : $\Lambda^n \longrightarrow \Lambda$, if

$$\text{IFWA}\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_u\right) = \sum_{g=1}^{u} \widehat{\mathfrak{Y}}_g \xi^{\delta}{}_g = \widehat{\mathfrak{Y}}_1 \xi^{\delta}{}_1 \oplus \widehat{\mathfrak{Y}}_2 \xi^{\delta}{}_2 \oplus \cdots, \widehat{\mathfrak{Y}}_u \xi^{\delta}{}_u, \tag{6}$$

where Λ^n is the set of all IFNs, and $\widehat{\mathfrak{Y}} = (\mathfrak{Y}\wedge_1, \mathfrak{Y}\wedge_2, \cdots, \mathfrak{Y}\wedge_u)^T$ is the weight vector of $(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots, \xi^{\delta}{}_u)$, such that $0 \leq \widehat{\mathfrak{Y}}_u \leq 1$ and $\sum_{g=1}^{u} \widehat{\mathfrak{Y}}_u = 1$. Then, the IFWA is called the intuitionistic weighted average operator.

Based on IFN operational rules, we can also consider IFWA by the theorem below.

Theorem 7 (see [12]). *Let $\xi^{\delta}{}_g = \langle \eta^{\gamma}{}_g, \hbar^{\mathfrak{I}}{}_g \rangle$ be the family of IFNs,we can find IFWG by*

$$IFWA\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_u\right) = \left\langle \left(1 - \prod_{g=1}^{u} \left(1 - \eta^{\gamma}{}_g\right)^{\mathfrak{Y}\wedge_g}\right), \prod_{g=1}^{u} \hbar^{\mathfrak{I}\widehat{\mathfrak{Y}}_g}_g \right\rangle. \tag{7}$$

Definition 8 (see [13]). Assume that $\xi^{\delta}{}_g = \langle \eta^{\gamma}{}_g, \hbar^{\mathfrak{I}}{}_g \rangle$ is the family of IFN, and IFWG : $\Lambda^n \longrightarrow \Lambda$, if

$$\text{IFWG}\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_u\right) = \sum_{g=1}^{u} \xi^{\delta \widehat{\mathfrak{Y}}_g}_g = \xi^{\delta \widehat{\mathfrak{Y}}_1}_1 \otimes \xi^{\delta \widehat{\mathfrak{Y}}_2}_2 \otimes \cdots, \xi^{\delta \widehat{\mathfrak{Y}}_u}_u, \tag{8}$$

where Λ^n is the set of all IFNs, and $\widehat{\mathfrak{Y}} = (\mathfrak{Y}\wedge_1, \mathfrak{Y}\wedge_2, \cdots, \mathfrak{Y}\wedge_u)^T$ is weight vector of $(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots, \xi^{\delta}{}_u)$, such that $0 \leq \widehat{\mathfrak{Y}}_u \leq 1$ and $\sum_{g=1}^{u} \widehat{\mathfrak{Y}}_u = 1$. Then, the IFWG is called the intuitionistic weighted geometric operator.

Based on IFNs operational rules, we can also consider IFWG by the theorem below.

Theorem 9 (see [13]). *Let $\xi^{\delta}{}_g = \langle \eta^{\gamma}{}_g, \hbar^{\mathfrak{I}}{}_g \rangle$ be the family of IFNs, we can find IFWG by*

$$IFWG\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_u\right) = \left\langle \prod_{g=1}^{u} \eta^{\gamma \widehat{\mathfrak{Y}}_g}_g, \left(1 - \prod_{g=1}^{u} \left(1 - \hbar^{\mathfrak{I}}{}_g\right)^{\mathfrak{Y}\wedge_g}\right) \right\rangle. \tag{9}$$

Definition 10 (see [39]). Let $\xi^{\delta}{}_{g} = \langle \eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g} \rangle$ be the family of IFNs, and IFPWA : $\Lambda^{n} \longrightarrow \Lambda$, be a n-dimension mapping. If

$$\begin{aligned} &\mathrm{IFPWA}\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \\ &= \left(\frac{\check{\mathrm{u}}_{1}}{\sum_{g=1}^{u} \check{\mathrm{u}}_{g}} \xi^{\delta}{}_{1} \oplus \frac{\check{\mathrm{u}}_{2}}{\sum_{g=1}^{u} \check{\mathrm{u}}_{g}} \xi^{\delta}{}_{2} \oplus \cdots, \oplus \frac{\check{\mathrm{u}}_{u}}{\sum_{g=1}^{u} \check{\mathrm{u}}_{g}} \xi^{\delta}{}_{u} \right), \end{aligned} \tag{10}$$

then the mapping IFPWA is called intuitionistic prioritized weighted averaging (IFPWA) operator, where $\check{\mathrm{u}}_{j} = \prod_{k=1}^{j-1} \breve{\Xi}(\xi^{\delta}{}_{k}) (j = 2 \cdots, n)$, $\check{\mathrm{u}}_{1} = 1$ and $\breve{\breve{\Xi}}(\xi^{\delta}{}_{k})$ is the score of k^{th} IFN.

Definition 11 (see [39]). Let $\xi^{\delta}{}_{p} = \langle \eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{p} \rangle$ be the family of IFNs, and IFPWG : $\Lambda^{n} \longrightarrow \Lambda$, be a n-dimension mapping. If

$$\begin{aligned} &\mathrm{IFPWG}\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \\ &= \left({\xi^{\delta}{}_{1}}^{\check{\mathrm{u}}_{1}/\sum_{g=1}^{u} \check{\mathrm{u}}_{g}} \otimes {\xi^{\delta}{}_{2}}^{\check{\mathrm{u}}_{2}/\sum_{g=1}^{u} \check{\mathrm{u}}_{g}} \otimes \cdots, \otimes {\xi^{\delta}{}_{u}}^{\check{\mathrm{u}}_{2}/\sum_{g=1}^{u} \check{\mathrm{u}}_{g}} \right), \end{aligned} \tag{11}$$

then the mapping IFPWG is called intuitionistic prioritized weighted geometric (IFPWG) operator, where $\check{\mathrm{u}}_{j} = \prod_{k=1}^{j-1} \breve{\Xi}(\xi^{\delta}{}_{k}) (j = 2 \cdots, n)$, $\check{\mathrm{u}}_{1} = 1$ and $\breve{\breve{\Xi}}(\xi^{\delta}{}_{k})$ is the score of k^{th} IFN.

3. Intuitionistic Fuzzy Prioritized Aggregation Operators with PDs

Within this section, we present the notion of intuitionistic prioritized averaging (IFPA$_{d}$) operator with PDs and intuitionistic prioritized geometric (IFPG$_{d}$) operator with PDs.

3.1. IFPA$_{d}$ Operator. Assume $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g}) (g = 1, 2 \cdots, u)$ is the assemblage of IFNs, there is a prioritization among these IFNs expressed by the strict priority orders $\xi^{\delta}{}_{1} \succ_{d_1} \xi^{\delta}{}_{2} \succ_{d_2} \cdots \succ_{d_{u-1}} \xi^{\delta}{}_{u-1}$, where $\xi^{\delta}{}_{u} \succ_{d_u} \xi^{\delta}{}_{u+1}$ indicates that the IFN $\xi^{\delta}{}_{u}$ has d_u higher priority than $\xi^{\delta}{}_{u+1}$. $d = (d_1, d_2, \cdots, d_{u-1})$ is the $(u-1)$-dimensional vector of PDs. The assemblage of such IFNs with strict priority orders and PDs is denoted by $\mathfrak{R}_{d}$.

Definition 12. A IFPA$_{d}$ operator is a mapping from $\mathfrak{R}_{d}^{u}$ to $\mathfrak{R}_{d}$ and defined as

$$\mathrm{IFPA_d}\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots, \xi^{\delta}{}_{u}\right) = \zeta_{1}^{(d)} \xi^{\delta}{}_{1} \oplus \zeta_{2}^{(d)} \xi^{\delta}{}_{2}, \cdots, \zeta_{u}^{(d)} \xi^{\delta}{}_{u}, \tag{12}$$

where $\zeta_{g}^{(d)} = T_{g}^{(d)} / \sum_{g=1}^{u} T_{g}^{(d)}$, $T_{g}^{(d)} = \prod_{q=1}^{g-1} \left(\breve{\Xi}(\xi^{\delta}{}_{q})\right)^{d_q}$, for each $g = (2, 3, \cdots, u)$, and $T_1 = 1$. Then, IFPA$_{d}$ is called intuitionistic prioritized averaging operators with PDs.

Theorem 13. *Assume $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g})$ is the assemblage of IFNs, we can also find IFPA$_{d}$ by*

$$\begin{aligned} &IFPA_{d}\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots, \xi^{\delta}{}_{u}\right) \\ &= \zeta_{1}^{(d)} \xi^{\delta}{}_{1} \oplus \zeta_{2}^{(d)} \xi^{\delta}{}_{2}, \cdots, \zeta_{u}^{(d)} \xi^{\delta}{}_{u} \\ &= 1 - \prod_{g=1}^{u} \left(1 - \eta^{\gamma}{}_{g}\right)^{\zeta_{g}^{(d)}}, \prod_{g=1}^{u} \left(\hbar^{\mathfrak{I}}{}_{g}\right)^{\zeta_{g}^{(d)}}, \end{aligned} \tag{13}$$

where $\zeta_{g}^{(d)} = T_{g}^{(d)} / \sum_{g=1}^{u} T_{g}^{(d)}$, $T_{g}^{(d)} = \prod_{q=1}^{g-1} \left(\breve{\Xi}(\xi^{\delta}{}_{q})\right)^{d_q}$, for each $g = (2, 3, \cdots, u)$, and $T_1 = 1$.

Proof. To prove this theorem, we use mathematical induction.

For $u = 2$,

$$\begin{aligned} \zeta_{1}^{(d)} \xi^{\delta}{}_{1} &= \left(1 - (1 - \eta^{\gamma}{}_{1})^{\zeta_{1}^{(d)}}, \hbar^{\mathfrak{I}\zeta_{1}^{(d)}}{}_{1}\right), \\ \zeta_{2}^{(d)} \xi^{\delta}{}_{2} &= \left(1 - (1 - \eta^{\gamma}{}_{2})^{\zeta_{2}^{(d)}}, \hbar^{\mathfrak{I}\zeta_{2}^{(d)}}{}_{2}\right). \end{aligned} \tag{14}$$

Then,

$$\begin{aligned} \zeta_{1}^{(d)} \xi^{\delta}{}_{1} \oplus \zeta_{2}^{(d)} \xi^{\delta}{}_{2} &= \left(1 - (1 - \eta^{\gamma}{}_{1})^{\zeta_{1}^{(d)}}, \hbar^{\mathfrak{I}\zeta_{1}^{(d)}}{}_{1}\right) \oplus \left(1 - (1 - \eta^{\gamma}{}_{2})^{\zeta_{2}^{(d)}}, \hbar^{\mathfrak{I}\zeta_{2}^{(d)}}{}_{2}\right) \\ &= \Big(1 - (1 - \eta^{\gamma}{}_{1})^{\zeta_{1}^{(d)}} + 1 - (1 - \eta^{\gamma}{}_{2})^{\zeta_{2}^{(d)}} \\ &\quad - \left(1 - (1 - \eta^{\gamma}{}_{1})^{\zeta_{1}^{(d)}}\right)\left(1 - \left((1 - \eta^{\gamma}{}_{2})^{\zeta_{2}^{(d)}}\right), \hbar^{\mathfrak{I}\zeta_{1}^{(d)}}{}_{1} . \hbar^{\mathfrak{I}\zeta_{2}^{(d)}}{}_{2}\right) \\ &= \Big(1 - (1 - \eta^{\gamma}{}_{1})^{\zeta_{1}^{(d)}} + 1 - (1 - \eta^{\gamma}{}_{2})^{\zeta_{2}^{(d)}} \\ &\quad - \Big(1 - (1 - \eta^{\gamma}{}_{2})^{\zeta_{2}^{(d)}} - (1 - \eta^{\gamma}{}_{1})^{\zeta_{1}^{(d)}} \\ &\quad + (1 - \eta^{\gamma}{}_{2})^{\zeta_{1}^{(d)}} (1 - \eta^{\gamma}{}_{1})^{\zeta_{1}^{(d)}}\Big), \hbar^{\mathfrak{I}\zeta_{1}^{(d)}}{}_{1} . \hbar^{\mathfrak{I}\zeta_{2}^{(d)}}{}_{2}\Big) \\ &= \left(1 - (1 - \eta^{\gamma}{}_{1})^{\zeta_{1}^{(d)}} (1 - \eta^{\gamma}{}_{2})^{\zeta_{2}^{(d)}}, \hbar^{\mathfrak{I}\zeta_{1}^{(d)}}{}_{1} . \hbar^{\mathfrak{I}\zeta_{2}^{(d)}}{}_{2}\right) \\ &= \left(1 - \prod_{g=1} \left(1 - \eta^{\gamma}{}_{g}\right)^{\zeta_{g}^{(d)}}, \prod_{g=1}^{u} \left(\hbar^{\mathfrak{I}}{}_{g}\right)^{\zeta_{g}^{(d)}}\right). \end{aligned} \tag{15}$$

This shows that Equation (13) is true for $u = 2$; now, let that Equation (13) holds for $u = b$, i.e.,

$$\mathrm{IFPA}_{d}\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{b}\right) = \left(1 - \prod_{g=1}^{b} \left(1 - \eta^{\gamma}{}_{g}\right)^{\zeta_{g}^{(d)}}, \prod_{g=1}^{b} \hbar^{\mathfrak{I}\zeta_{g}^{(d)}}{}_{g}\right). \tag{16}$$

Now $u = b + 1$, by operational laws of IFNs, we have

$$\begin{aligned}
&\mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_{b+1}\right) \\
&= \mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_b\right) \oplus \xi^{\delta}{}_{b+1} \\
&= \left(1 - \prod_{g=1}^{b}\left(1-\eta^{\gamma}{}_g\right)^{\zeta_g^{(d)}}, \prod_{g=1}^{b} \hbar^{\mathfrak{I}}{}_g^{\zeta_g^{(d)}}\right) \\
&\quad \oplus \left(1 - (1-\eta^{\gamma}{}_{b+1})^{\zeta_{b+1}^{(d+1)}}, \hbar^{\mathfrak{I}}{}_{b+1}^{\zeta_{b+1}^{(d+1)}}\right) \\
&= \left(1 - \prod_{g=1}^{b}\left(1-\eta^{\gamma}{}_g\right)^{\zeta_g^{(d)}} + 1 - (1-\eta^{\gamma}{}_{b+1})^{\zeta_{b+1}^{(d+1)}}\right. \\
&\quad - \left(1 - \prod_{g=1}^{b}\left(1-\eta^{\gamma}{}_g\right)^{\zeta_g^{(d)}}\right) \\
&\quad \left.\cdot\left(1 - (1-\eta^{\gamma}{}_{b+1})^{\zeta_{b+1}^{(d+1)}}\right), \prod_{g=1}^{b} \hbar^{\mathfrak{I}}{}_g^{\zeta_g^{(d)}} . \hbar^{\mathfrak{I}}{}_{b+1}^{\zeta_{b+1}^{(d+1)}}\right) \\
&= \left(1 - \prod_{g=1}^{b+1}\left(1-\eta^{\gamma}{}_g\right)^{\zeta_g^{(d)}}, \prod_{g=1}^{b+1} \hbar^{\mathfrak{I}}{}_g^{\zeta_g^{(d)}}\right).
\end{aligned} \tag{17}$$

This shows that for $u = b + 1$, Equation (13) holds. Then,

$$\mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_u\right) = \left(1 - \prod_{g=1}^{u}\left(1-\eta^{\gamma}{}_g\right)^{\zeta_g^{(d)}}, \prod_{g=1}^{u} \hbar^{\mathfrak{I}}{}_g^{\zeta_g^{(d)}}\right). \tag{18}$$

□□ □

Example 14. Let $\xi^{\delta}{}_1 = (0.73, 0.54)$, $\xi^{\delta}{}_2 = (0.53, 0.75)$, $\xi^{\delta}{}_3 = (0.82, 0.25)$, and $\xi^{\delta}{}_4 = (0.35, 0.64)$ be the four IFNs, there is strict prioritized relation in considered IFNs, such that $\xi^{\delta}{}_1 \succ_{d_1} \xi^{\delta}{}_2 \succ_{d_2} \xi^{\delta}{}_3 \succ_{d_3} \xi^{\delta}{}_4$. Priority vector $d = (d_1, d_2, d_3)$ is given as $(5, 1, 1)$, by Equation (13), and get

$$1 - \prod_{g=1}^{4}\left(1-\eta^{\gamma}{}_g\right)^{\zeta_g^{(d)}} = 0.719519,$$

$$\prod_{g=1}^{4}\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}} = 0.544041,$$

$$\begin{aligned}
&\mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \xi^{\delta}{}_3, \xi^{\delta}{}_4\right) \\
&= \left(1 - \prod_{g=1}^{4}\left(1-\eta^{\gamma}{}_g\right)^{\zeta_g^{(d)}}, \prod_{g=1}^{u}\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}}\right) \\
&= (0.719519, 0.544041).
\end{aligned} \tag{19}$$

Furthermore, the suggested IFPA_d operator is examined to ensure that it has idempotency and boundary properties. Their explanations are as follows.

Theorem 15. *Assume that $\xi^{\delta}{}_g = (\eta^{\gamma}{}_g, \hbar^{\mathfrak{I}}{}_g)$ is the assemblage of IFNs, and*

$$\begin{aligned}
\xi^{\delta^-} &= \left(min_g\left(\eta^{\gamma}{}_g\right), max_g\left(\hbar^{\mathfrak{I}}{}_g\right)\right), \\
\xi^{\delta^+} &= \left(max_g\left(\eta^{\gamma}{}_g\right), min_g\left(\hbar^{\mathfrak{I}}{}_g\right)\right).
\end{aligned} \tag{20}$$

Then,

$$\xi^{\delta^-} \le IFPA_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_n\right) \le \xi^{\delta^+}, \tag{21}$$

where $\zeta_g^{(d)} = T_g^{(d)} / \sum_{g=1}^{u} T_g^{(d)}$, $T_g^{(d)} = \prod_{q=1}^{g-1} (\breve{\Xi}(\xi^{\delta}{}_q))^{d_q}$, for each $g = (2, 3, \cdots, u)$ and $T_1 = 1$.

Proof. Since,

$$\min_g\left(\eta^{\gamma}{}_g\right) \le \eta^{\gamma}{}_g \le \max_g\left(\eta^{\gamma}{}_g\right), \tag{22}$$

$$\min_g\left(\hbar^{\mathfrak{I}}{}_g\right) \le \hbar^{\mathfrak{I}}{}_g \le \max_g\left(\hbar^{\mathfrak{I}}{}_g\right). \tag{23}$$

From Equation (22), we have

$$\begin{aligned}
&\min_g\left(\eta^{\gamma}{}_g\right) \le \eta^{\gamma}{}_g \le \max_g\left(\eta^{\gamma}{}_g\right) \\
&\Leftrightarrow \min_g\left(\eta^{\gamma}{}_g\right) \le \left(\eta^{\gamma}{}_g\right) \le \max_g\left(\eta^{\gamma}{}_g\right) \\
&\Leftrightarrow 1 - \max_g\left(\eta^{\gamma}{}_g\right) \le 1 - \left(\eta^{\gamma}{}_g\right) \le 1 - \min_g\left(\eta^{\gamma}{}_g\right) \\
&\Leftrightarrow \left(1 - \max_g\left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \le \left(1 - \left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \le \left(1 - \min_g\left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \\
&\Leftrightarrow \prod_{g=1}^{u}\left(1 - \max_g\left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \le \prod_{g=1}^{u}\left(1 - \left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \\
&\quad \le \prod_{g=1}^{u}\left(1 - \min_g\left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \\
&\Leftrightarrow 1 - \max_g\left(\eta^{\gamma}{}_g\right) \le \prod_{g=1}^{u}\left(1 - \left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \le 1 - \min_g\left(\eta^{\gamma}{}_g\right) \\
&\Leftrightarrow -1 + \min_j\left(\eta^{\gamma}{}_g\right) \le -\prod_{g=1}^{u}\left(1 - \left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \le -1 + \max_g\left(\eta^{\gamma}{}_g\right) \\
&\Leftrightarrow 1 - 1 + \min_j\left(\eta^{\gamma}{}_g\right) \le 1 - \prod_{g=1}^{u}\left(1 - \left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \le 1 - 1 + \max_g\left(\eta^{\gamma}{}_g\right) \\
&\Leftrightarrow \min_j\left(\eta^{\gamma}{}_g\right) \le 1 - \prod_{g=1}^{u}\left(1 - \left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \le \max_g\left(\eta^{\gamma}{}_g\right) \\
&\Leftrightarrow \min_j\left(\eta^{\gamma}{}_g\right) \le 1 - \prod_{g=1}^{u}\left(1 - \left(\eta^{\gamma}{}_g\right)\right)^{\zeta_g^{(d)}} \le \max_g\left(\eta^{\gamma}{}_g\right).
\end{aligned} \tag{24}$$

From Equation (23), we have

$$\begin{aligned}
&\min_g\left(\hbar^{\mathfrak{I}}{}_g\right) \le \hbar^{\mathfrak{I}}{}_g \le \max_g\left(\hbar^{\mathfrak{I}}{}_g\right)\\
&\Leftrightarrow \min_g\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}} \le \left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}} \le \max_g\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}}\\
&\Leftrightarrow \prod_{g=1}^{u}\min_g\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}} \le \prod_{g=1}^{u}\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}} \le \prod_{g=1}^{u}\max_g\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}}\\
&\Leftrightarrow \min_g\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}} \le \prod_{g=1}^{u}\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}} \le \max_g\left(\hbar^{\mathfrak{I}}{}_g\right)^{\zeta_g^{(d)}}.
\end{aligned} \tag{25}$$

Let

$$\mathrm{IFPA}_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_n\right) = \xi^\delta = \left(\eta^\gamma, \hbar^{\mathfrak{I}}\right). \tag{26}$$

Then, $\breve{\Xi}(\xi^\delta) = \eta^\gamma - \hbar^{\mathfrak{I}} \le \max_g(\eta^\gamma) - \min_j(\hbar^{\mathfrak{I}}) = \breve{\Xi}(\xi^\delta{}_{\max})$ So, $\breve{\Xi}(\xi^\delta) \le \breve{\Xi}(\xi^\delta{}_{\max})$.

Again, $\breve{\Xi}(\xi^\delta) = \eta^\gamma - \hbar^{\mathfrak{I}} \ge \min_g(\eta^\gamma) - \max_j(\hbar^{\mathfrak{I}}) = \breve{\Xi}(\xi^\delta{}_{\min})$. So, $\breve{\Xi}(\xi^\delta) \ge \breve{\Xi}(\xi^\delta{}_{\min})$.

If, $\breve{\Xi}(\xi^\delta) \le \breve{\Xi}(\xi^\delta{}_{\max})$ and $\breve{\Xi}(\xi^\delta) \ge \breve{\Xi}(\xi^\delta{}_{\min})$, then

$$\xi^\delta{}_{\min} \le \mathrm{IFPA}_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_n\right) \le \xi^\delta{}_{\max}. \tag{27}$$

If $\breve{\Xi}(\xi^\delta) = \breve{\Xi}(\xi^\delta{}_{\max})$, then $\eta^\gamma - \hbar^{\mathfrak{I}} = \max_g(\eta^\gamma) - \min_j(\hbar^{\mathfrak{I}})$

$$\begin{aligned}
&\Leftrightarrow \eta^\gamma - \hbar^{\mathfrak{I}} = \max_g(\eta^\gamma) - \min_g\left(\hbar^{\mathfrak{I}}\right),\\
&\Leftrightarrow \eta^\gamma = \max_g(\eta^\gamma), \hbar^{\mathfrak{I}} = \min_g\left(\hbar^{\mathfrak{I}}\right),\\
&\Leftrightarrow \eta^\gamma = \max_g \eta^\gamma, \hbar^{\mathfrak{I}} = \min_g \hbar^{\mathfrak{I}}.
\end{aligned} \tag{28}$$

Now, $H(\xi^\delta) = \eta^\gamma + \hbar^{\mathfrak{I}} = \max_g(\eta^\gamma) + \min_g(\hbar^{\mathfrak{I}}) = H(\xi^\delta{}_{\max})$

$$\mathrm{IFPA}_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_n\right) = \xi^\delta{}_{\max}. \tag{29}$$

If $\breve{\Xi}(\xi^\delta) = \breve{\Xi}(\xi^\delta{}_{\min})$, then $\eta^\gamma - \hbar^{\mathfrak{I}} = \min_g(\eta^\gamma) - \max_j(\hbar^{\mathfrak{I}})$

$$\begin{aligned}
&\Leftrightarrow \eta^\gamma - \hbar^{\mathfrak{I}} = \min_g(\eta^\gamma) - \max_g\left(\hbar^{\mathfrak{I}}\right),\\
&\Leftrightarrow \eta^\gamma = \min_g(\eta^\gamma), \hbar^{\mathfrak{I}} = \max_g\left(\hbar^{\mathfrak{I}}\right),\\
&\Leftrightarrow \eta^\gamma = \min_g \eta^\gamma, \hbar^{\mathfrak{I}} = \max_g \hbar^{\mathfrak{I}}.
\end{aligned} \tag{30}$$

Now, $H(\xi^\delta) = \eta^\gamma + \hbar^{\mathfrak{I}} = \min_g(\eta^\gamma) + \max_g(\hbar^{\mathfrak{I}}) = H(\xi^\delta{}_{\max})$

$$\mathrm{IFPA}_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_n\right) = \xi^\delta{}_{\min}. \tag{31}$$

Thus, from Equations (27), (29), and (31), we get

$$\xi^{\delta-} \le \mathrm{IFPA}_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_n\right) \le \xi^{\delta+}. \tag{32}$$

□ □

Theorem 16. *Assume that if $\xi^\delta{}_\diamond$ is a IFN satisfied the property, $\xi^\delta{}_g = \xi^\delta{}_\diamond, \forall g$ then*

$$IFPA_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_u\right) = \xi^\delta{}_\diamond. \tag{33}$$

Proof. Let $\xi^\delta{}_\diamond = (\eta^\gamma\diamond, \hbar^{\mathfrak{I}}{}_\diamond)$ be the IFN. Then, by assumption, we have $\xi^\delta{}_g = \xi^\delta{}_\diamond, \forall g$ gives $\eta^\gamma{}_g = \eta^\gamma\diamond$ and $\hbar^{\mathfrak{I}}{}_g = \hbar^{\mathfrak{I}}\diamond\forall g$. By Definition 12, we have $\sum_{g=1}^{u}\zeta_g^{(d)}$. Then, by using Theorem 13, we get

$$\begin{aligned}
&\mathrm{IFPA}_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_u\right)\\
&= \left(1 - \prod_{g=1}^{u}\left(1 - \eta^\gamma{}_\diamond\right)^{\zeta_g^{(d)}}, \prod_{g=1}^{u}\hbar^{\mathfrak{I}}{}_\diamond^{\zeta_g^{(d)}}\right)\\
&= \left(1 - \left(1 - \eta^\gamma{}_\diamond\right)^{\sum_{g=1}^{u}\zeta_g^{(d)}}, \hbar^{\mathfrak{I}}{}_\diamond^{\sum_{g=1}^{u}\zeta_g^{(d)}}\right)\\
&= \left(\eta^\gamma\diamond, \hbar^{\mathfrak{I}}{}_\diamond\right) = \xi^\delta{}_\diamond.
\end{aligned} \tag{34}$$

□ □

Corollary 17. *If $\xi^\delta{}_g = (\eta^\gamma{}_g, \hbar^{\mathfrak{I}}{}_g)$ is the assemblage of largest IFNs, i.e., $\xi^\delta{}_g = (1, 0)$ for all g, then*

$$IFPA_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_u\right) = (1, 0). \tag{35}$$

Proof. We can easily obtain a corollary similar to Theorem 16.□ □

Corollary 18. *If $\xi^\delta{}_1 = (\eta^\gamma{}_1, \hbar^{\mathfrak{I}}{}_1)$ is the smallest IFN, i.e., $\xi^\delta{}_1 = (0, 1)$, then*

$$IFPA_d\left(\xi^\delta{}_1, \xi^\delta{}_2, \cdots \xi^\delta{}_u\right) = (0, 1). \tag{36}$$

Proof. Here, $\xi^\delta{}_1 = (0, 1)$ then by definition of the score function, we have

$$\breve{\Xi}\left(\xi^\delta{}_1\right) = 0. \tag{37}$$

Since,

$$T_g^{(d)} = \prod_{q=1}^{g-1} \left(\breve{\Xi}\left({\xi^\delta}_q\right)\right)^{d_q}, \text{for each } g = (2, 3, \cdots, u) \text{ and } T_1 = 1. \quad (38)$$

We have

$$\begin{aligned} T_g^{(d)} &= \prod_{q=1}^{g-1} \left(\breve{\Xi}\left({\xi^\delta}_q\right)\right)^{d_q} \\ &= \left(\breve{\Xi}\left({\xi^\delta}_1\right)^{d_1}\right)\left(\breve{\Xi}\left({\xi^\delta}_2\right)^{d_2}\right) \cdots \left(\breve{\Xi}\left({\xi^\delta}_{g-1}\right)^{d_{g-1}}\right) = 0. \end{aligned} \quad (39)$$

From Definition 3, we have

$$\begin{aligned} \mathrm{IFPA}_d\left({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u\right) &= \zeta_1^{(d)}{\xi^\delta}_1 \oplus \zeta_2^{(d)}{\xi^\delta}_2, \cdots, \zeta_u^{(d)}{\xi^\delta}_u \\ &= 1\,{\xi^\delta}_1 \oplus 0\,{\xi^\delta}_2 \oplus \cdots 0\,{\xi^\delta}_n \\ &= {\xi^\delta}_1 = (0, 1). \end{aligned} \quad (40)$$

□ □

Theorem 19. *Assume that* ${\xi^\delta}_g = ({\eta^\gamma}_g, {\hbar^{\mathfrak{F}}}_g)$ *and* $\beta_g = (\phi_g, \varphi_g)$ *are two assemblages of IFNs, if* $r > 0$ *and* $\beta = ({\eta^\gamma}_\beta, {\hbar^{\mathfrak{F}}}_\beta)$ *is a IFN, then*

(1) $IFPA_d({\xi^\delta}_1 \oplus \beta, {\xi^\delta}_2 \oplus \beta, \cdots {\xi^\delta}_u \oplus \beta) = IFPA_d({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u) \oplus \beta$

(2) $IFPA_d(r{\xi^\delta}_1, r{\xi^\delta}_2, \cdots r{\xi^\delta}_u) = r\, IFPA_d({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u)$

(3) $IFPA_d({\xi^\delta}_1 \oplus \beta_1, {\xi^\delta}_2 \oplus \beta_2, \cdots {\xi^\delta}_u \oplus \beta_n) = IFPA_d({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_n) \oplus IFPA_d(\beta_1, \beta_2, \cdots \beta_u)$

(4) $IFPA_d(r{\xi^\delta}_1 \oplus \beta, r{\xi^\delta}_2 \oplus \beta, \cdots \oplus r{\xi^\delta}_u \oplus \beta) = r\, IFPA_d({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u) \oplus \beta$

Proof. This is trivial by definition.□ □

IFPA $_d$ operator satisfied following properties.

Property:1

Assume that ${\xi^\delta}_g = ({\eta^\gamma}_g, {\hbar^{\mathfrak{F}}}_g)$ is the assemblage of IFNs, then we have

$$\lim_{(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (1,1,\cdots,1)} \mathrm{IFPA}_d\left({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u\right) = \mathrm{IFPWA}\left({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u\right). \quad (41)$$

Proof. Given that $(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (1, 1, \cdots, 1)$, from this, we have

$$T_g^{(d)} = \prod_{q=1}^{g-1} \left(\breve{\Xi}\left({\xi^\delta}_q\right)\right)^{d_q} \longrightarrow \prod_{q=1}^{g-1} \left(\breve{\Xi}\left({\xi^\delta}_q\right)\right) = T_g, \quad (42)$$

by this, we obtain $\zeta_g^{(d)} \longrightarrow \zeta_g$

$$\begin{aligned} &\lim_{(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (1,1,\cdots,1)} \mathrm{IFPA}_d\left({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u\right) \\ &= \lim_{(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (1,1,\cdots,1)} \zeta_1^{(d)}{\xi^\delta}_1 \oplus \zeta_2^{(d)}{\xi^\delta}_2, \cdots, \zeta_u^{(d)}{\xi^\delta}_u \\ &= \zeta_1{\xi^\delta}_1 \oplus \zeta_2{\xi^\delta}_2, \cdots, \zeta_u{\xi^\delta}_u = \mathrm{IFPWA}\left({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u\right). \end{aligned} \quad (43)$$

□ □

Remark. When $d_1 = d_2 = \cdots, = d_{u-1} = 1$, Property:1 states that the existing IFPWA operator is a particular situation of the suggested IFPA_d operator. As a result, the IFPA_d operator is more generic than the IFPWA operator.

Property:2

Assume that ${\xi^\delta}_g = ({\eta^\gamma}_g, {\hbar^{\mathfrak{F}}}_g)$ is the assemblage of IFNs and $\breve{\Xi}({\xi^\delta}_g) \neq 0$ for all g, then we have

$$\begin{aligned} &\lim_{(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (0,0,\cdots,0)} \mathrm{IFPA}_d\left({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u\right) \\ &= \frac{1}{u}\left({\xi^\delta}_1 \oplus {\xi^\delta}_2 \oplus, \cdots, \oplus {\xi^\delta}_u\right). \end{aligned} \quad (44)$$

Proof. Given that $(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (0, 0, \cdots, 0)$, from this, we have

$$T_g^{(d)} = \prod_{q=1}^{g-1} \left(\breve{\Xi}\left({\xi^\delta}_q\right)\right)^{d_q} = 1, \quad (45)$$

and $\zeta_g^{(d)} = T_g^{(d)} / \sum_{g=1}^u T_g^{(d)} = 1/n$. Hence,

$$\begin{aligned} &\lim_{(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (0,0,\cdots,0)} \mathrm{IFPA}_d\left({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u\right) \\ &= \frac{1}{u}\left({\xi^\delta}_1 \oplus {\xi^\delta}_2 \oplus, \cdots, \oplus {\xi^\delta}_u\right). \end{aligned} \quad (46)$$

□ □

Property:3

Assume that ${\xi^\delta}_g = ({\eta^\gamma}_g, {\hbar^{\mathfrak{F}}}_g)$ is the assemblage of IFNs and $\breve{\Xi}({\xi^\delta}_1) \neq 0$ or 1, then we have

$$\lim_{d_1 \longrightarrow +\infty} \mathrm{IFPA}_d\left({\xi^\delta}_1, {\xi^\delta}_2, \cdots {\xi^\delta}_u\right) = {\xi^\delta}_1. \quad (47)$$

Proof. Here, $d_1 \longrightarrow +\infty$ for each $g = 2, 3, \cdots, u$, we have

$$T_g^{(d)} = \prod_{q=1}^{g-1} \left(\breve{\Xi}\left(\xi^{\delta}{}_q\right)\right)^{d_q} = \left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)^{+\infty}\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)^{d_2}\right)\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)^{d_{g-1}}\right) = 0, \tag{48}$$

because $0 < \breve{\Xi}(\xi^{\delta}{}_1) < 1$, $\sum_{g=1}^{u} T_g^{(d)} = T_1^{(d)} = 1 \Rightarrow \zeta_1^{(d)} = T_1^{(d)} / \sum_{g=1}^{u} T_1^{(d)} = 1$ and $\zeta_g^{(d)} = T_g^{(d)} / \sum_{g=1}^{u} T_g^{(d)}$ for each $g = 2, 3, \cdots, u$. Hence,

$$\lim_{d_1 \longrightarrow \infty} \mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_u\right) = \xi^{\delta}{}_1. \tag{49}$$

□

□

Remark. According to Property:3, when $d_1 \longrightarrow +\infty$, the PD d_1 of IFN $\xi^{\delta}{}_1$ is very high in comparison to the PDs of other IFNs. It indicates that IFN $\xi^{\delta}{}_1$ is extremely essential. As a result, $\xi^{\delta}{}_1$ determines the aggregation result obtained by using the proposed operator IFPA_d in this case.

Example 20. Let $\xi^{\delta}{}_1 = (0.73, 0.54)$, $\xi^{\delta}{}_2 = (0.53, 0.75)$, $\xi^{\delta}{}_3 = (0.82, 0.25)$, and $\xi^{\delta}{}_4 = (0.35, 0.64)$ be the four IFNs, it can easily compute that, $\breve{\Xi}_1 = 0.6158$, $\breve{\Xi}_2 = 0.3635$, $\breve{\Xi}_3 = 0.7679$, and $\breve{\Xi}_4 = 0.3904$. There is strict prioritized relation in considering IFNs, such that $\xi^{\delta}{}_1 \succ_{d_1} \xi^{\delta}{}_2 \succ_{d_2} \xi^{\delta}{}_3 \succ_{d_3} \xi^{\delta}{}_4$. In the corresponding portion, we will aggregate the IFNs for 4 distinct priority vectors $d = (d_1, d_2, d_3)$, keeping the values of PDs d_2, d_3 constant while varying the value of d_1 and discussing its effect on the aggregated results.

Case 1. when $d = (1, 1, 1)$,

$$\mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \xi^{\delta}{}_3, \xi^{\delta}{}_4\right) = (0.684742, 0.556094). \tag{50}$$

Case 2. when $d = (5, 1, 1)$,

$$\mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \xi^{\delta}{}_3, \xi^{\delta}{}_4\right) = (0.719519, 0.544041). \tag{51}$$

Case 3. when $d = (8, 1, 1)$,

$$\mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \xi^{\delta}{}_3, \xi^{\delta}{}_4\right) = (0.727349, 0.541027). \tag{52}$$

Case 4. when $d = (13, 1, 1)$,

$$\mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \xi^{\delta}{}_3, \xi^{\delta}{}_4\right) = (0.729769, 0.540074). \tag{53}$$

The consolidated findings from the preceding 4 cases show that as the PD d_1 referring to IFN $\xi^{\delta}{}_1$ rises, the aggregated value approaches the IFN $\xi^{\delta}{}_1$ ranking values.

Property:4

Assume that $\xi^{\delta}{}_g = (\eta^{\gamma}{}_g, \hbar^{\mathfrak{I}}{}_g)$ is the assemblage of IFNs and $\breve{\Xi}(\xi^{\delta}{}_g) \neq 0$ for all $g = 1, 2, \cdots, k+1$, and $\breve{\Xi}(\xi^{\delta}{}_{k+1}) \neq 1$, then we have

$$\lim_{(d_1, d_2, \cdots, d_k, d_{k+1}) \longrightarrow (0,0,\cdots,0,+\infty)} \mathrm{IFPA}_d\left(\xi^{\delta}{}_1, \xi^{\delta}{}_2, \cdots \xi^{\delta}{}_u\right) = \frac{1}{k+1}\left(\xi^{\delta}{}_1 \oplus \xi^{\delta}{}_2 \oplus, \cdots, \oplus \xi^{\delta}{}_{k+1}\right). \tag{54}$$

Proof. Given that $(d_1, d_2, \cdots, d_k, d_{k+1}) \longrightarrow (0, 0, \cdots, 0, +\infty)$. So,

$$T_g^{(d)} = \prod_{q=1}^{g-1} \left(\breve{\Xi}\left(\xi^{\delta}{}_q\right)\right)^{d_q} = \left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)^{d_1}\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)^{d_2}\right)\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)^{d_{g-1}}\right) \longrightarrow \left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)\right)^0\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)\right)^0\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)\right)^0 = 1, \tag{55}$$

for each $g = 2, 3, \cdots, k+1$.

$$T_g^{(d)} = \prod_{q=1}^{g-1} \left(\breve{\Xi}\left(\xi^{\delta}{}_q\right)\right)^{d_q} = \left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)^{d_1}\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)^{d_2}\right)\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)^{d_{g-1}}\right) \longrightarrow \left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)\right)^0\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)\right)^0\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_k\right)\right)^0\left(\breve{\Xi}\left(\xi^{\delta}{}_{k+1}\right)\right)^{+\infty}\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)^{d_{g-1}}\right) = 0,$$

$$\forall g = k+2, k+3, \cdots, u. \tag{56}$$

So,

$$\sum_{g=1}^{u} T_g^{(d)} = T_1^{(d)} = k+1 \text{ and } \zeta_g^{(d)} = \frac{T_g^{(d)}}{\sum_{g=1}^{u} T_g^{(d)}} \longrightarrow \frac{1}{k+1} \text{ for each } g = 1, 2, 3, \cdots, k+1,$$

$$\zeta_g^{(d)} = \frac{T_g^{(d)}}{\sum_{g=1}^{u} T_g^{(d)}} \longrightarrow \frac{0}{k+1} = 0 \text{ for each } g = k+2, k+3, \cdots, u. \tag{57}$$

Hence,

$$\lim_{(d_1,d_2,\cdots,d_k,d_{k+1})\longrightarrow(0,0,\cdots,0,+\infty)} \mathrm{IFPA}_d\left(\xi^{\delta}{}_1,\xi^{\delta}{}_2,\cdots\xi^{\delta}{}_u\right) = \frac{1}{k+1}\left(\xi^{\delta}{}_1\oplus\xi^{\delta}{}_2\oplus,\cdots,\oplus\xi^{\delta}{}_{k+1}\right). \tag{58}$$

□ □

Remark. When $(d_1,d_2,\cdots,d_k,d_{k+1})\longrightarrow(0,0,\cdots,0,+\infty)$, it means there is no prioritization association between the IFNs $\xi^{\delta}{}_1,\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_{k+1}$ and that all of these IFNs $\xi^{\delta}{}_1\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_{k+1}$ have a much higher priority than the IFNs $\xi^{\delta}{}_{k+2}\xi^{\delta}{}_{k+3},\cdots,\xi^{\delta}{}_u$. As a result, the aggregated value is solely dependent on IFNs $\xi^{\delta}{}_1\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_{k+1}$, and these IFNs $\xi^{\delta}{}_1\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_{k+1}$ have similar weightage in the aggregation method.

Property:5

Assume that $\xi^{\delta}{}_g=(\eta^{\gamma}{}_g,\hbar^{\mathfrak{J}}{}_g)$ is the assemblage of IFNs and $\breve{\Xi}(\xi^{\delta}{}_{k+1})\neq 1$ or 0 then, we have

$$\lim_{(d_1,d_2,\cdots,d_k,d_{k+1})\longrightarrow(1,1,\cdots,1,+\infty)} \mathrm{IFPA}_d\left(\xi^{\delta}{}_1,\xi^{\delta}{}_2,\cdots\xi^{\delta}{}_u\right) = \mathrm{IFPWA}\left(\xi^{\delta}{}_1\oplus\xi^{\delta}{}_2\oplus,\cdots,\oplus\xi^{\delta}{}_{k+1}\right). \tag{59}$$

Proof. Given that $(d_1,d_2,\cdots,d_k,d_{k+1})\longrightarrow(1,1,\cdots,1,+\infty)$. So,

$$\begin{aligned} T_g^{(d)} &= \prod_{q=1}^{g-1}\left(\breve{\Xi}\left(\xi^{\delta}{}_q\right)\right)^{d_q} \\ &= \left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)^{d_1}\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)^{d_2}\right)\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)^{d_{g-1}}\right) \\ &\longrightarrow\left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)\right)\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)\right)=T_g, \end{aligned} \tag{60}$$

for each $g=2,3,\cdots,k+1$.

$$\begin{aligned} T_g^{(d)} &= \prod_{q=1}^{g-1}\left(\breve{\Xi}\left(\xi^{\delta}{}_q\right)\right)^{d_q} \\ &= \left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)^{d_1}\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)^{d_2}\right)\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)^{d_{g-1}}\right) \\ &\longrightarrow\left(\breve{\Xi}\left(\xi^{\delta}{}_1\right)\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_2\right)\right)\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_k\right)\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_{k+1}\right)\right)^{+\infty} \\ &\quad\cdots\left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)\right)^{d_{g-1}}=0, \end{aligned}$$

$$\forall g=k+2,k+3,\cdots,u. \tag{61}$$

So,

$\sum_{g=1}^{u}T_g^{(d)}\longrightarrow\sum_{g=1}^{k+1}T_g$ and $\zeta_g^{(d)}=T_g^{(d)}/\sum_{g=1}^{u}T_g^{(d)}\longrightarrow T_g/\sum_{g=1}^{k+1}T_g$ for each $g=1,2,3,\cdots,k+1$.

$\zeta_g^{(d)}=T_g^{(d)}/\sum_{g=1}^{u}T_g^{(d)}\longrightarrow 0/\sum_{g=1}^{k+1}T_g=0$ for each $g=k+2,k+3,\cdots,u$.

Hence,

$$\lim_{(d_1,d_2,\cdots,d_k,d_{k+1})\longrightarrow(1,1,\cdots,1,+\infty)} \mathrm{IFPA}_d\left(\xi^{\delta}{}_1,\xi^{\delta}{}_2,\cdots\xi^{\delta}{}_u\right) = \mathrm{IFPWA}\left(\xi^{\delta}{}_1\oplus\xi^{\delta}{}_2\oplus,\cdots,\oplus\xi^{\delta}{}_{k+1}\right). \tag{62}$$

□ □

Remark. When $(d_1,d_2,\cdots,d_k,d_{k+1})\longrightarrow(1,1,\cdots,1,+\infty)$, it means there is normal prioritization association between the IFNs $\xi^{\delta}{}_1,\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_{k+1}$ and that all of these IFNs $\xi^{\delta}{}_1\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_{k+1}$ have a much higher priority than the IFNs $\xi^{\delta}{}_{k+2}\xi^{\delta}{}_{k+3},\cdots,\xi^{\delta}{}_u$. As a result, the aggregated value is solely dependent on IFNs $\xi^{\delta}{}_1\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_{k+1}$.

3.2. IFPG$_d$ Operator. Assume $\xi^{\delta}{}_g=(\eta^{\gamma}{}_g,\hbar^{\mathfrak{J}}{}_g)(g=1,2\cdots,u)$ is the assemblage of IFNs, there is a prioritization among these IFNs expressed by the strict priority orders $\xi^{\delta}{}_1\succ_{d_1}\xi^{\delta}{}_2\succ_{d_2}\cdots\succ_{d_{u-1}}\xi^{\delta}{}_{u-1}$, where $\xi^{\delta}{}_u\succ_{d_u}\xi^{\delta}{}_{u+1}$ indicates that the IFN $\xi^{\delta}{}_u$ has d_u higher priority than $\xi^{\delta}{}_{u+1}$. $d=(d_1,d_2,\cdots,d_{u-1})$ is the $(u-1)$-dimensional vector of PDs. The assemblage of such IFNs with strict priority orders and PDs is denoted by $\mathfrak{R}_d$.

Definition 21. A IFPG$_d$ operator is a mapping from $\mathfrak{R}_d^u$ to $\mathfrak{R}_d$ and defined as

$$\mathrm{IFPG}_d\left(\xi^{\delta}{}_1,\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_u\right)=\xi^{\delta\zeta_1^{(d)}}_1\oplus\xi^{\delta\zeta_2^{(d)}}_2,\cdots,\xi^{\delta\zeta_u^{(d)}}_u, \tag{63}$$

where $\zeta_g^{(d)}=T_g^{(d)}/\sum_{g=1}^{u}T_g^{(d)}$, $T_g^{(d)}=\prod_{q=1}^{g-1}(\breve{\Xi}(\xi^{\delta}{}_q))^{d_q}$, for each $g=(2,3,\cdots,u)$ and $T_1=1$. Then, IFPG$_d$ is called intuitionistic prioritized geometric operator with PDs.

Theorem 22. *Assume $\xi^{\delta}{}_g=(\eta^{\gamma}{}_g,\hbar^{\mathfrak{J}}{}_g)$ is the assemblage of IFNs, we can also find IFPG$_d$ by*

$$\begin{aligned} &IFPG_d\left(\xi^{\delta}{}_1,\xi^{\delta}{}_2,\cdots,\xi^{\delta}{}_u\right) \\ &=\xi^{\delta\zeta_1^{(d)}}_1\oplus\xi^{\delta\zeta_2^{(d)}}_2,\cdots,\xi^{\delta\zeta_u^{(d)}}_u \\ &=\left(\prod_{g=1}^{u}\left(\eta^{\gamma}{}_g\right)^{\zeta_g^{(d)}},1-\prod_{g=1}^{u}\left(1-\hbar^{\mathfrak{J}}{}_g\right)^{\zeta_g^{(d)}}\right), \end{aligned} \tag{64}$$

where $\zeta_g^{(d)}=T_g^{(d)}/\sum_{g=1}^{u}T_g^{(d)}$, $T_g^{(d)}=\prod_{q=1}^{g-1}(\breve{\Xi}(\xi^{\delta}{}_q))^{d_q}$, for each $g=(2,3,\cdots,u)$ and $T_1=1$.

Proof. To prove this theorem, we use mathematical induction.

For $u = 2$,

$$\begin{aligned}\xi^{\delta\zeta_1^{(d)}}_1 &= \left(\eta^{\gamma\zeta_1^{(d)}}_1, 1-\left(1-\hbar^{\mathfrak{J}}_1\right)^{\zeta_1^{(d)}}\right),\\ \xi^{\delta\zeta_2^{(d)}}_2 &= \left(\eta^{\gamma\zeta_2^{(d)}}_2, 1-\left(1-\hbar^{\mathfrak{J}}_2\right)^{\zeta_2^{(d)}}\right).\end{aligned} \tag{65}$$

Then,

$$\begin{aligned}\xi^{\delta\zeta_1^{(d)}}_1 \otimes \xi^{\delta\zeta_2^{(d)}}_2 &= \left(\eta^{\gamma\zeta_1^{(d)}}_1, 1-\left(1-\hbar^{\mathfrak{J}}_1\right)^{\zeta_1^{(d)}}\right) \otimes \left(\eta^{\gamma\zeta_2^{(d)}}_2, 1-\left(1-\hbar^{\mathfrak{J}}_2\right)^{\zeta_2^{(d)}}\right)\\ &= \left(\eta^{\gamma\zeta_1^{(d)}}_1 . \eta^{\gamma\zeta_2^{(d)}}_2, 1-\left(1-\hbar^{\mathfrak{J}}_1\right)^{\zeta_1^{(d)}} + 1-\left(1-\hbar^{\mathfrak{J}}_2\right)^{\zeta_2^{(d)}}\right.\\ &\quad \left. - \left(1-\left(1-\hbar^{\mathfrak{J}}_1\right)^{\zeta_1^{(d)}}\right)\left(1-\left(1-\hbar^{\mathfrak{J}}_2\right)^{\zeta_2^{(d)}}\right)\right)\\ &= \left(\eta^{\gamma\zeta_1^{(d)}}_1 . \eta^{\gamma\zeta_2^{(d)}}_2, 1-\left(1-\hbar^{\mathfrak{J}}_1\right)^{\zeta_1^{(d)}} + 1-\left(1-\hbar^{\mathfrak{J}}_2\right)^{\zeta_2^{(d)}}\right.\\ &\quad \left. - \left(1-\left(1-\hbar^{\mathfrak{J}}_2\right)^{\zeta_2^{(d)}} - \left(1-\hbar^{\mathfrak{J}}_1\right)^{\zeta_1^{(d)}} + \left(1-\hbar^{\mathfrak{J}}_2\right)^{\zeta_1^{(d)}}\left(1-\hbar^{\mathfrak{J}}_1\right)^{\zeta_1^{(d)}}\right)\right)\\ &= \left(\eta^{\gamma\zeta_1^{(d)}}_1 . \eta^{\gamma\zeta_2^{(d)}}_2, 1-\left(1-\hbar^{\mathfrak{J}}_1\right)^{\zeta_1^{(d)}}\left(1-\hbar^{\mathfrak{J}}_2\right)^{\zeta_2^{(d)}}\right)\\ &= \left(\prod_{g=1}^{u}\left(\eta^{\gamma}_g\right)^{\zeta_g^{(d)}}, 1-\prod_{g=1}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}}\right).\end{aligned} \tag{66}$$

This shows that Equation (64) is true for $u = 2$; now, let that Equation (64) holds for $u = b$, i.e.,

$$\mathrm{IFPG}_d\left(\xi^{\delta}_1, \xi^{\delta}_2, \cdots \xi^{\delta}_b\right) = \left(\prod_{g=1}^{b}\eta^{\gamma\zeta_g^{(d)}}_g, 1-\prod_{g=1}^{b}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}}\right). \tag{67}$$

Now $u = b + 1$, by operational laws of IFNs, we have

$$\begin{aligned}&\mathrm{IFPG}_d\left(\xi^{\delta}_1, \xi^{\delta}_2, \cdots \xi^{\delta}_{b+1}\right)\\ &= \mathrm{IFPG}_d\left(\xi^{\delta}_1, \xi^{\delta}_2, \cdots \xi^{\delta}_b\right) \otimes \xi^{\delta}_{b+1}\\ &= \left(\prod_{g=1}^{b}\eta^{\gamma\zeta_g^{(d)}}_g, 1-\prod_{g=1}^{b}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}}\right)\\ &\quad \otimes \left(\eta^{\gamma\zeta_{b+1}^{(d+1)}}_{b+1}, 1-\left(1-\hbar^{\mathfrak{J}}_{b+1}\right)^{\zeta_{b+1}^{(d+1)}}\right)\\ &= \left(\prod_{g=1}^{b}\eta^{\gamma\zeta_g^{(d)}}_g . \eta^{\gamma\zeta_{b+1}^{(d+1)}}_{b+1}, 1-\prod_{g=1}^{b}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}} + 1-\left(1-\hbar^{\mathfrak{J}}_{b+1}\right)^{\zeta_{b+1}^{(d+1)}}\right.\\ &\quad \left. -\left(1-\prod_{g=1}^{b}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}}\right)\left(1-\left(1-\hbar^{\mathfrak{J}}_{b+1}\right)^{\zeta_{b+1}^{(d+1)}}\right)\right)\\ &= \left(\prod_{g=1}^{b+1}\eta^{\gamma\zeta_g^{(d)}}_g, 1-\prod_{g=1}^{b+1}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}}\right).\end{aligned} \tag{68}$$

This shows that for $u = b + 1$, Equation (64) holds. Then,

$$\mathrm{IFPG}_d\left(\xi^{\delta}_1, \xi^{\delta}_2, \cdots \xi^{\delta}_u\right) = \left(\prod_{g=1}^{u}\eta^{\gamma\zeta_g^{(d)}}_g, 1-\prod_{g=1}^{u}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}}\right). \tag{69}$$

□ □

Example 23. Let $\xi^{\delta}_1 = (0.73, 0.54)$, $\xi^{\delta}_2 = (0.53, 0.75)$, $\xi^{\delta}_3 = (0.82, 0.25)$ and $\xi^{\delta}_4 = (0.35, 0.64)$ be the four IFNs, there is strict prioritized relation in considered IFNs, such that $\xi^{\delta}_1 \succ_{d_1} \xi^{\delta}_2 \succ_{d_2} \xi^{\delta}_3 \succ_{d_3} \xi^{\delta}_4$. Priority vector $d = (d_1, d_2, d_3)$ is given as $(5, 1, 1)$, by Equation (64)

$$\prod_{g=1}^{4}\left(\eta^{\gamma}_{\mathrm{g}}\right)^{\zeta_g^{(d)}} = 0.703208$$

$$1-\prod_{g=1}^{4}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}} = 0.565078,$$

$$\begin{aligned}&\mathrm{IFPG}_{\mathrm{d}}\left(\xi^{\delta}_1, \xi^{\delta}_2, \xi^{\delta}_3, \xi^{\delta}_4\right)\\ &= \left(\prod_{g=1}^{u}\left(\eta^{\gamma}_g\right)^{\zeta_g^{(d)}}, 1-\prod_{g=1}^{4}\left(1-\hbar^{\mathfrak{J}}_g\right)^{\zeta_g^{(d)}},\right)\\ &= (0.703208, 0.565078).\end{aligned} \tag{70}$$

Furthermore, the suggested IFPG $_d$ operator is examined to ensure that it has idempotency and boundary properties. Their explanations are as follows.

Theorem 24. *Assume that* $\xi^{\delta}_g = (\eta^{\gamma}_g, \hbar^{\mathfrak{J}}_g)$ *is the assemblage of IFNs, and*

$$\begin{aligned}\xi^{\delta^-} &= \left(min_g\left(\eta^{\gamma}_g\right), max_g\left(\hbar^{\mathfrak{J}}_g\right)\right),\\ \xi^{\delta^+} &= \left(max_g\left(\eta^{\gamma}_g\right), min_g\left(\hbar^{\mathfrak{J}}_g\right)\right).\end{aligned} \tag{71}$$

Then,

$$\xi^{\delta^-} \le IFPG_d\left(\xi^{\delta}_1, \xi^{\delta}_2, \cdots \xi^{\delta}_n\right) \le \xi^{\delta^+}, \tag{72}$$

where $\zeta_g^{(d)} = T_g^{(d)}/\sum_{g=1}^{u}T_g^{(d)}$, $T_g^{(d)} = \prod_{q=1}^{g-1}(\breve{\Xi}(\xi^{\delta}_q))^{d_q}$, *for each* $g = (2, 3, \cdots, u)$ *and* $T_1 = 1$.

Proof. Proof is same as Theorem 15.□ □

Theorem 25. *Assume that if* $\xi^{\delta}_{\diamond}$ *is a IFN satisfied the property,* $\xi^{\delta}_g = \xi^{\delta}_{\diamond}, \forall g$ *then*

$$IFPG_d\left(\xi^{\delta}_1, \xi^{\delta}_2, \cdots \xi^{\delta}_u\right) = \xi^{\delta}_{\diamond}. \tag{73}$$

Proof. Let $\xi^{\delta}{}_{\diamond} = (\eta^{\gamma}{\diamond}, \hbar^{\mathfrak{I}}{}_{\diamond})$ be the IFN. Then, by assumption, we have $\xi^{\delta}{}_{g} = \xi^{\delta}{}_{\diamond}, \forall g$ gives $\eta^{\gamma}{}_{g} = \eta^{\gamma}{\diamond}$ and $\hbar^{\mathfrak{I}}{}_{g} = \hbar^{\mathfrak{I}}{\diamond}\forall g$. By Definition 21, we have $\sum_{g=1}^{u} \zeta_g^{(d)}$. Then, by using Theorem 22, we get

$$\begin{aligned} &\mathrm{IFPG}_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \\ &= \left(\prod_{g=1}^{u} \eta^{\gamma}{}_{\diamond}^{\zeta_g^{(d)}}, 1 - \prod_{g=1}^{u}\left(1 - \hbar^{\mathfrak{I}}{}_{\diamond}\right)^{\zeta_g^{(d)}}\right) \\ &= \left(\eta^{\gamma}{}_{\diamond}^{\sum_{g=1}^{u} \zeta_g^{(d)}}, 1 - \left(1 - \hbar^{\mathfrak{I}}{}_{\diamond}\right)^{\sum_{g=1}^{u} \zeta_g^{(d)}}\right) \\ &= \left(\eta^{\gamma}{\diamond}, \hbar^{\mathfrak{I}}{}_{\diamond}\right) = \xi^{\delta}{}_{\diamond}. \end{aligned} \tag{74}$$

□ □

Corollary 26. *If $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g})$ is the assemblage of largest IFNs, i.e., $\xi^{\delta}{}_{g} = (1, 0)$ for all g, then*

$$IFPG_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) = (1, 0). \tag{75}$$

Proof. We can easily obtain a corollary similar to Theorem 25.□ □

Corollary 27. *If $\xi^{\delta}{}_{1} = (\eta^{\gamma}{}_{1}, \hbar^{\mathfrak{I}}{}_{1})$ is the smallest IFN, i.e., $\xi^{\delta}{}_{1} = (0, 1)$, then*

$$IFPG_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) = (0, 1). \tag{76}$$

Proof. Here, $\xi^{\delta}{}_{1} = (0, 1)$ then by definition of the score function, we have

$$\breve{\Xi}\left(\xi^{\delta}{}_{1}\right) = 0. \tag{77}$$

Since,

$$T_g^{(d)} = \prod_{q=1}^{g-1}\left(\breve{\Xi}\left(\xi^{\delta}{}_{q}\right)\right)^{d_q}, \text{for each } g = (2, 3, \cdots, u) \text{ and } T_1 = 1. \tag{78}$$

We have

$$\begin{aligned} T_g^{(d)} &= \prod_{q=1}^{g-1}\left(\breve{\Xi}\left(\xi^{\delta}{}_{q}\right)\right)^{d_q} \\ &= \left(\breve{\Xi}\left(\xi^{\delta}{}_{1}\right)^{d_1}\right)\left(\breve{\Xi}\left(\xi^{\delta}{}_{2}\right)^{d_2}\right) \cdots \left(\breve{\Xi}\left(\xi^{\delta}{}_{g-1}\right)^{d_{g-1}}\right) = 0. \end{aligned} \tag{79}$$

From Definition 3, we have

$$\begin{aligned} \mathrm{IFPG}_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) &= \xi^{\delta\zeta_1^{(d)}}_{1} \otimes \xi^{\delta\zeta_2^{(d)}}_{2}, \cdots, \xi^{\delta\zeta_u^{(d)}}_{u} \\ &= \xi^{\delta 1}_{1} \otimes \xi^{\delta 0}_{2} \otimes \cdots \xi^{\delta 0}_{u} = \xi^{\delta}{}_{1} = (0, 1). \end{aligned} \tag{80}$$

□ □

Theorem 28. *Assume that $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g})$ and $\beta_g = (\phi_g, \varphi_g)$ are two assemblages of IFNs, if $r > 0$ and $\beta = (\eta^{\gamma}{}_{\beta}, \hbar^{\mathfrak{I}}{}_{\beta})$ is a IFN, then*

$$\begin{aligned} &IFPG_d\left(\xi^{\delta}{}_{1} \oplus \beta, \xi^{\delta}{}_{2} \oplus \beta, \cdots \xi^{\delta}{}_{u} \oplus \beta\right) \\ &= IFPG_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \oplus \beta, \end{aligned}$$

$$IFPG_d\left(r\xi^{\delta}{}_{1}, r\xi^{\delta}{}_{2}, \cdots r\xi^{\delta}{}_{u}\right) = r\, IFPG_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right),$$

$$\begin{aligned} &IFPG_d\left(\xi^{\delta}{}_{1} \oplus \beta_1, \xi^{\delta}{}_{2} \oplus \beta_2, \cdots \xi^{\delta}{}_{u} \oplus \beta_n\right) \\ &= IFPG_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{n}\right) \oplus IFPG_d(\beta_1, \beta_2, \cdots \beta_u), \end{aligned}$$

$$\begin{aligned} &IFPG_d\left(r\xi^{\delta}{}_{1} \oplus \beta, r\xi^{\delta}{}_{2} \oplus \beta, \cdots \oplus r\xi^{\delta}{}_{u} \oplus \beta\right) \\ &= r\, IFPG_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \oplus \beta. \end{aligned} \tag{81}$$

Proof. This is trivial by definition.□ □

IFPG_d operator also satisfied following properties.

Property:1

Assume that $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g})$ is the assemblage of IFNs, then we have

$$\begin{aligned} &\lim_{(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (1,1,\cdots,1)} \mathrm{IFPG}_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \\ &= \mathrm{IFPWG}\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right). \end{aligned} \tag{82}$$

Property:2

Assume that $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g})$ is the assemblage of IFNs and $\breve{\Xi}(\xi^{\delta}{}_{g}) \neq 0$ for all g, then we have

$$\begin{aligned} &\lim_{(d_1, d_2, \cdots, d_{u-1}) \longrightarrow (0,0,\cdots,0)} \mathrm{IFPG}_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \\ &= \frac{1}{u}\left(\xi^{\delta}{}_{1} \otimes \xi^{\delta}{}_{2} \otimes, \cdots, \otimes \xi^{\delta}{}_{u}\right). \end{aligned} \tag{83}$$

Property:3

Assume that $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g})$ is the assemblage of IFNs and $\breve{\Xi}(\xi^{\delta}{}_{1}) \neq 0$ or 1, then we have

$$\lim_{d_1 \longrightarrow +\infty} \mathrm{IFPG}_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) = \xi^{\delta}{}_{1}. \tag{84}$$

Property:4

Assume that $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g})$ is the assemblage of IFNs and $\breve{\Xi}(\xi^{\delta}{}_{g}) \neq 0$ for all $g = 1, 2, \cdots, k+1$, and $\breve{\Xi}(\xi^{\delta}{}_{k+1}) \neq 1$ then we have

$$\begin{aligned} &\lim_{(d_1, d_2, \cdots, d_k, d_{k+1}) \longrightarrow (0,0,\cdots,0,+\infty)} \mathrm{IFPG}_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \\ &= \frac{1}{k+1}\left(\xi^{\delta}{}_{1} \otimes \xi^{\delta}{}_{2} \otimes, \cdots, \otimes \xi^{\delta}{}_{k+1}\right). \end{aligned} \tag{85}$$

Property:5

Assume that $\xi^{\delta}{}_{g} = (\eta^{\gamma}{}_{g}, \hbar^{\mathfrak{I}}{}_{g})$ is the assemblage of IFNs and $\breve{\Xi}(\xi^{\delta}{}_{k+1}) \neq 1$ or 0 then we have

$$\begin{aligned} &\lim_{(d_1, d_2, \cdots, d_k, d_{k+1}) \longrightarrow (1,1,\cdots,1,+\infty)} \mathrm{IFPG}_d\left(\xi^{\delta}{}_{1}, \xi^{\delta}{}_{2}, \cdots \xi^{\delta}{}_{u}\right) \\ &= \mathrm{IFPWG}\left(\xi^{\delta}{}_{1} \otimes \xi^{\delta}{}_{2} \otimes, \cdots, \otimes \xi^{\delta}{}_{k+1}\right). \end{aligned} \tag{86}$$

4. Methodology for MCDM Using Proposed AOs

Let $\overline{\coprod} = \{\overline{\coprod}_1, \overline{\coprod}_2 \cdots, \overline{\coprod}_m\}$ be the assemblage of alternatives and $\coprod\wedge' = \{\widehat{\coprod}'_1, \widehat{\coprod}'_2, \cdots, \widehat{\coprod}'_n\}$ is the assemblage of criterions, priorities are assigned between the criterions provided by strict priority orientation. $\widehat{\coprod}'_1 \succ_{d_1} \widehat{\coprod}'_2 \succ_{d_2} \widehat{\coprod}'_3 \cdots \succ_{d_{n-1}} \widehat{\coprod}'_n$ indicates criteria $\widehat{\coprod}'_J$ has a high priority than $\widehat{\coprod}'_{J+1}$ with degree d_q for $q \in \{1, 2, \cdots, (n-1)\}$. $K = \{K_1, K_2, \cdots, K_p\}$ is a assemblage of decision-makers (DMs). Priorities are assigned between the DMs provided by strict priority orientation, $K_1 \succ_{d_1}{}' K_2 \succ_{d_2}{}' K_3 \cdots \succ_{d_{p-1}}{}' K_p$. DMs give a matrix according to their own standpoints $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m\times n}$, where $\mathscr{B}_{ij}^{(p)}$ is given for the alternatives $\overline{\coprod}_i \in \overline{\coprod}$ with respect to the attribute $\widehat{\coprod}'_j \in \coprod\wedge'$ by K_p DM. If all performance criteria are the same kind, there is no need for normalization; however, since MCGDM has two different types of evaluation criteria (benefit kind attributes τ_b and cost kinds attributes τ_c), the matrix $D_{(p)}$ has been transformed into a normalize matrix using the normalization formula $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$,

$$\left(\mathscr{P}_{ij}^{(p)}\right)_{m\times n} = \begin{cases} \left(\mathscr{B}_{ij}^{(p)}\right)^c; & j \in \tau_c, \\ \mathscr{B}_{ij}^{(p)}; & j \in \tau_b. \end{cases} \tag{87}$$

where $(\mathscr{B}_{ij}^{(p)})^c$ show the compliment of $\mathscr{B}_{ij}^{(p)}$.

The suggested operators will be implemented to the MCGDM, which will require the preceding steps.

4.1. Algorithm

Step 1. Obtain the decision matrix $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m\times n}$ in the format of IFNs from DMs.

$$\begin{array}{cc} & \begin{array}{cccc} \widehat{\amalg}'_1 & \widehat{\amalg}'_2 & & \widehat{\amalg}'_n \end{array} \\ \begin{array}{cc} K_1 & \overline{\amalg}_1 \\ & \overline{\amalg}_2 \\ & \vdots \\ & \overline{\amalg}_m \\ K_2 & \overline{\amalg}_1 \\ \vdots & \overline{\amalg}_2 \\ & \vdots \\ & \overline{\amalg}_m \\ K_p & \overline{\amalg}_1 \\ & \overline{\amalg}_2 \\ & \vdots \\ & \overline{\amalg}_m \end{array} & \begin{bmatrix} (\eta^{\gamma 1}{}_{11}, \hbar^{\Im 1}{}_{11}) & (\eta^{\gamma 1}{}_{12}, \hbar^{\Im 1}{}_{12}) & \cdots\cdots & (\eta^{\gamma 1}{}_{1n}, \hbar^{\Im 1}{}_{1n}) \\ (\eta^{\gamma 1}{}_{21}, \hbar^{\Im 1}{}_{21}) & (\eta^{\gamma 1}{}_{22}, \hbar^{\Im 1}{}_{22}) & \cdots\cdots & (\eta^{\gamma 1}{}_{2n}, \hbar^{\Im 1}{}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\eta^{\gamma 1}{}_{m1}, \hbar^{\Im 1}{}_{m1}) & (\eta^{\gamma 1}{}_{m2}, \hbar^{\Im 1}{}_{m2}) & \cdots\cdots & (\eta^{\gamma 1}{}_{mn}, \hbar^{\Im 1}{}_{mn}) \\ (\eta^{\gamma}{}_{11}, \hbar^{\Im}{}_{11}) & (\eta^{\gamma}{}_{12}, \hbar^{\Im}{}_{12}) & \cdots\cdots & (\eta^{\gamma}{}_{1n}, \hbar^{\Im}{}_{1n}) \\ (\eta^{\gamma}{}_{21}, \hbar^{\Im}{}_{21}) & (\eta^{\gamma}{}_{22}, \hbar^{\Im}{}_{22}) & \cdots\cdots & (\eta^{\gamma}{}_{2n}, \hbar^{\Im}{}_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\eta^{\gamma}{}_{m1}, \hbar^{\Im}{}_{m1}) & (\eta^{\gamma}{}_{m2}, \hbar^{\Im}{}_{m2}) & \cdots\cdots & (\eta^{\gamma}{}_{mn}, \hbar^{\Im}{}_{mn}) \\ (\eta^{\gamma p}{}_{11}, \hbar^{\Im p}{}_{11}) & (\eta^{\gamma p}{}_{12}, \hbar^{\Im p}{}_{12}) & \cdots\cdots & (\eta^{\gamma p}{}_{1n}, \hbar^{\Im p}{}_{1n}) \\ (\eta^{\gamma p}{}_{21}, \hbar^{\Im p}{}_{21}) & (\eta^{\gamma p}{}_{22}, \hbar^{\Im p}{}_{22}) & \cdots\cdots & (\eta^{\gamma p}{}_{2n}, \hbar^{\Im}{}_{2n})^p \\ \vdots & \vdots & \ddots & \vdots \\ (\eta^{\gamma p}{}_{m1}, \hbar^{\Im p}{}_{m1}) & (\eta^{\gamma p}{}_{m2}, \hbar^{\Im p}{}_{m2}) & \cdots\cdots & (\eta^{\gamma p}{}_{mn}, \hbar^{\Im p}{}_{mn}) \end{bmatrix} \end{array} \tag{88}$$

Step 2. Two kinds of criterion are described in the decision matrix: (τ_c) cost type indicators and (τ_b) benefit type indicators. There is no need for normalization if all indicators are of the same kind, but in MCGDM, there may be two types of criteria. The matrix was updated to the transforming response matrix in this case $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ using the normalization formula Equation (87).

Step 3. Using one of provided AOs to combine all of the independent IF decision matrices $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ into one combined evaluation matrix of the alternatives $W^{(p)} = (\tilde{\chi}_{ij})_{m\times n}$.

$$\begin{aligned} \tilde{\chi}_{ij} &= \mathrm{IFPA}_d\left(\mathscr{P}_{ij}^{(1)}, \mathscr{P}_{ij}^{(2)}, \cdots \mathscr{P}_{ij}^{(p)}\right) \\ &= \left(1 - \prod_{z=1}^{p}\left(1 - \eta^{\gamma z}{}_{ij}\right)^{\zeta_{ij}^{(z)}}, \prod_{z=1}^{p}\left(\hbar^{\Im z}{}_{ij}\right)^{\zeta_{ij}^{(z)}}\right) \end{aligned} \tag{89}$$

$$\begin{aligned} \text{or } \tilde{\chi}_{ij} &= \mathrm{IFPG}_d\left(\mathscr{P}_{ij}^{(1)}, \mathscr{P}_{ij}^{(2)}, \cdots \mathscr{P}_{ij}^{(p)}\right) \\ &= \left(\prod_{z=1}^{p}\left(\eta^{\gamma z}{}_{ij}\right)^{\zeta_{ij}^{(z)}}, 1 - \prod_{z=1}^{p}\left(1 - \hbar^{\Im z}{}_{ij}\right)^{\zeta_{ij}^{(z)}}\right). \end{aligned} \tag{90}$$

Step 4. Aggregate the IF values $\tilde{\chi}_{ij}$ for each alternative $\overline{\coprod}_i$ by the IFPA_d (or IFPG_d) operator.

$$\begin{aligned} \tilde{\chi}_{ij} &= \mathrm{IFPA}_d(\mathscr{P}_{i1}, \mathscr{P}_{i2}, \cdots \mathscr{P}_{in}) \\ &= \left(1 - \prod_{j=1}^{n}\left(1 - \eta^{\gamma}{}_{ij}\right)^{\zeta_{ij}}, \prod_{j=1}^{n}\left(\hbar^{\Im}{}_{ij}\right)^{\zeta_{ij}}\right) \end{aligned} \tag{91}$$

Table 1: Rating given by DMs.

Experts	Alternatives	$\widehat{\coprod}'_1$	$\widehat{\coprod}'_2$	$\widehat{\coprod}'_3$	$\widehat{\coprod}'_4$	$\widehat{\coprod}'_5$
K_1	$\bar{\coprod}_1$	(0.80, 0.10)	(0.24, 0.71)	(0.77, 0.12)	(0.73, 0.23)	(0.80, 0.19)
	$\bar{\coprod}_2$	(0.78, 0.18)	(0.42, 0.71)	(0.43, 0.67)	(0.61, 0.30)	(0.50, 0.32)
	$\bar{\coprod}_3$	(0.74, 0.42)	(0.45, 0.46)	(0.62, 0.41)	(0.58, 0.46)	(0.55, 0.38)
	$\bar{\coprod}_4$	(0.43, 0.29)	(0.44, 0.69)	(0.47, 0.20)	(0.45, 0.37)	(0.57, 0.29)
K_2	$\bar{\coprod}_1$	(0.66, 0.33)	(0.80, 0.18)	(0.83, 0.15)	(0.83, 0.16)	(0.66, 0.33)
	$\bar{\coprod}_2$	(0.56, 0.19)	(0.34, 0.89)	(0.37, 0.78)	(0.11, 0.72)	(0.17, 0.29)
	$\bar{\coprod}_3$	(0.48, 0.27)	(0.56, 0.63)	(0.20, 0.10)	(0.77, 0.17)	(0.53, 0.27)
	$\bar{\coprod}_4$	(0.06, 0.93)	(0.40, 0.58)	(0.55, 0.44)	(0.55, 0.45)	(0.67, 0.23)
K_3	$\bar{\coprod}_1$	(0.88, 0.11)	(0.77, 0.22)	(0.73, 0.13)	(0.87, 0.11)	(0.84, 0.13)
	$\bar{\coprod}_2$	(0.76, 0.14)	(0.22, 0.75)	(0.35, 0.63)	(0.63, 0.36)	(0.57, 0.36)
	$\bar{\coprod}_3$	(0.74, 0.23)	(0.45, 0.49)	(0.69, 0.41)	(0.58, 0.39)	(0.57, 0.33)
	$\bar{\coprod}_4$	(0.46, 0.21)	(0.41, 0.61)	(0.43, 0.28)	(0.47, 0.33)	(0.54, 0.26)

Table 2: Normalized IF decision matrix.

Experts	Alternatives	$\widehat{\coprod}'_1$	$\widehat{\coprod}'_2$	$\widehat{\coprod}'_3$	$\widehat{\coprod}'_4$	$\widehat{\coprod}'_5$
K_1	$\bar{\coprod}_1$	(0.80, 0.10)	(0.71, 0.24)	(0.77, 0.12)	(0.73, 0.23)	(0.80, 0.19)
	$\bar{\coprod}_2$	(0.78, 0.18)	(0.71, 0.42)	(0.43, 0.67)	(0.61, 0.30)	(0.50, 0.32)
	$\bar{\coprod}_3$	(0.74, 0.42)	(0.46, 0.45)	(0.62, 0.41)	(0.58, 0.46)	(0.55, 0.38)
	$\bar{\coprod}_4$	(0.43, 0.29)	(0.69, 0.44)	(0.47, 0.20)	(0.45, 0.37)	(0.57, 0.29)
K_2	$\bar{\coprod}_1$	(0.66, 0.33)	(0.18, 0.80)	(0.83, 0.15)	(0.83, 0.16)	(0.66, 0.33)
	$\bar{\coprod}_2$	(0.56, 0.19)	(0.89, 0.34)	(0.37, 0.78)	(0.11, 0.72)	(0.17, 0.29)
	$\bar{\coprod}_3$	(0.48, 0.27)	(0.63, 0.56)	(0.20, 0.10)	(0.77, 0.17)	(0.53, 0.27)
	$\bar{\coprod}_4$	(0.06, 0.93)	(0.58, 0.40)	(0.55, 0.44)	(0.55, 0.45)	(0.67, 0.23)
K_3	$\bar{\coprod}_1$	(0.88, 0.11)	(0.22, 0.77)	(0.73, 0.13)	(0.87, 0.11)	(0.84, 0.13)
	$\bar{\coprod}_2$	(0.76, 0.14)	(0.75, 0.22)	(0.35, 0.63)	(0.63, 0.36)	(0.57, 0.36)
	$\bar{\coprod}_3$	(0.74, 0.23)	(0.49, 0.45)	(0.69, 0.41)	(0.58, 0.39)	(0.57, 0.33)
	$\bar{\coprod}_4$	(0.46, 0.21)	(0.61, 0.41)	(0.43, 0.28)	(0.47, 0.33)	(0.54, 0.26)

$$\text{or } \tilde{\chi}_{ij} = \mathrm{IFPG}_d(\mathscr{P}_{i1}, \mathscr{P}_{i2}, \cdots \mathscr{P}_{\mathrm{in}}) = \left(\prod_{j=1}^{n}\left({\eta^{\gamma}}_{ij}\right)^{\zeta_{ij}}, 1-\prod_{j=1}^{n}\left(1-{\hbar^{\mathfrak{I}}}_{ij}\right)^{\zeta_{ij}}\right). \tag{92}$$

Step 5. Analyze the score for all cumulative alternative assessments.

Step 6. The alternatives were classified by the score function, and the most suitable alternative was selected.

4.2. Numerical Illustration. Consider a decision-making problem of finding out the most appropriate agriculture land. Assume the assemblage of alternatives, $\bar{\coprod}_1$, $\bar{\coprod}_2$, $\bar{\coprod}_3$, and $\bar{\coprod}_4$. There are five criterions for evaluation of these alternatives $\widehat{\coprod}'_1$= soil (chemical and physical), $\widehat{\coprod}'_2$= cost, $\widehat{\coprod}'_3$= irrigation (water and canal), $\widehat{\coprod}'_4$= processing industry and market, and $\widehat{\coprod}'_5$= social-economic. Assume that the criterions have been prioritized in strict priority order $\widehat{\coprod}'_1 >_{d_1} \widehat{\coprod}'_2 >_{d_2} \widehat{\coprod}'_3 >_{d_3} \widehat{\coprod}'_4 >_{d_1} \widehat{\coprod}'_5$. The three-dimensional vector of PDs is $d = (2, 1, 3, 2)$. Here, three DMs K_1, K_2, and K_3 are involved; they have been prioritized in a strict priority order $K_1 >_{d_1{}'} K_2 >_{d_2{}'}$ and K_3, where $d' = (3, 4)$.

5. Algorithm

Step 1. Obtain the decision matrix $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m\times n}$ in the format of IFNs from DMs. The judgement values, given by three DMs, are described in Table 1.

Table 3: Combined evaluation matrix.

	$\widehat{\coprod}'_1$	$\widehat{\coprod}'_2$	$\widehat{\coprod}'_3$	$\widehat{\coprod}'_4$	$\widehat{\coprod}'_5$
$\bar{\coprod}_1$	(0.7861, 0.2172)	(0.6624, 0.3240)	(0.7844, 0.1455)	(0.6958, 0.2845)	(0.8019, 0.1904)
$\bar{\coprod}_2$	(0.7469, 0.1813)	(0.6071, 0.3835)	(0.4271, 0.6757)	(0.5760, 0.3505)	(0.4788, 0.3160)
$\bar{\coprod}_3$	(0.6543, 0.3412)	(0.5169, 0.3925)	(0.6249, 0.2907)	(0.5196, 0.4547)	(0.5472, 0.4289)
$\bar{\coprod}_4$	(0.4233, 0.3371)	(0.6909, 0.2893)	(0.5569, 0.2467)	(0.4654, 0.4035)	(0.5761, 0.3020)

Step 2. Normalize the decision matrixes acquired by DMs using Equation (87). In Table 1, there are two types of criterions. $\widehat{\coprod}'_2$ is cost type criteria and others are benefit type criterions. Normalized IF decision matrix is given in Table 2.

Step 3. Using IFPA $_d$ opeartor to combine all of the independent IF decision matrices $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ into one combined evaluation matrix of the alternatives $W^{(p)} = (\tilde{\chi}_{ij})_{m\times n}$ given in Table 3. First, we find $T_{ij}^{(1)} T_{ij}^{(2)}$ and $T_{ij}^{(3)}$, which are used in the calculation of IFPA $_d$ operator.

$$
\begin{aligned}
T_{ij}^{(1)} &= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}, \\
T_{ij}^{(2)} &= \begin{pmatrix} 0.4884 & 0.3035 & 0.3849 & 0.4727 & 0.4429 \\ 0.4112 & 0.2645 & 0.0590 & 0.2159 & 0.1628 \\ 0.2948 & 0.1273 & 0.1999 & 0.1654 & 0.1716 \\ 0.1568 & 0.2403 & 0.1645 & 0.1408 & 0.1955 \end{pmatrix}, \\
T_{ij}^{(3)} &= \begin{pmatrix} 0.1008 & 0.0003 & 0.1411 & 0.1776 & 0.1451 \\ 0.0479 & 0.1272 & 0.0004 & 0.0021 & 0.0094 \\ 0.0261 & 0.0106 & 0.0129 & 0.0351 & 0.0006 \\ 0.0001 & 0.1651 & 0.0045 & 0.0027 & 0.0010 \end{pmatrix}.
\end{aligned} \tag{93}
$$

Step 4. Aggregate the IF values $\tilde{\chi}_{ij}$ for each alternative $\bar{\coprod}_1$ by the IFPA$_d$ operator using Equation (91) given in Table 4.

$$
T_{ij} = \begin{pmatrix} 1 & 0.6109 & 0.3838 & 0.1554 & 0.0874 \\ 1 & 0.4975 & 0.3447 & 0.0196 & 0.0064 \\ 1 & 0.4186 & 0.2255 & 0.0511 & 0.0159 \\ 1 & 0.2691 & 0.1905 & 0.0369 & 0.0099 \end{pmatrix}. \tag{94}
$$

Step 5. Compute the score for all IF-aggregated values $\tilde{\chi}_i$.

$$
\begin{aligned}
\breve{\Xi}(\tilde{\chi}_1) &= 0.493599, \\
\breve{\Xi}(\tilde{\chi}_2) &= 0.440998, \\
\breve{\Xi}(\tilde{\chi}_3) &= 0.333649, \\
\breve{\Xi}(\tilde{\chi}_4) &= 0.211889.
\end{aligned} \tag{95}
$$

Table 4: IF-aggregated values $\tilde{\chi}_i$.

$\tilde{\chi}_1$	(0.741187, 0.247588)
$\tilde{\chi}_2$	(0.719510, 0.278512)
$\tilde{\chi}_3$	(0.647383, 0.313734)
$\tilde{\chi}_4$	(0.568093, 0.356204)

Step 6. Ranks according to score values.

$$
\begin{aligned}
\tilde{\chi}_1 \succ \tilde{\chi}_2 \succ \tilde{\chi}_3 \succ \tilde{\chi}_4, \\
\bar{\coprod}_1 \succ \bar{\coprod}_2 \succ \bar{\coprod}_3 \succ \bar{\coprod}_4.
\end{aligned} \tag{96}
$$

$\bar{\coprod}_1$ is the best alternative among all other alternatives.

6. Conclusion

In the current study, IFSs are used to handle ambiguity in data utilising MDs and NMDs. The IFS paradigm is extended by the IF framework. By considering stringent priority orders, we established the notions of intuitionistic prioritized averaging and intuitionistic prioritized geometric operators with PDs. Many theories about PD have been thoroughly researched, and they will be valuable in merging multiple IF data sets. A group MCDM strategy based on the proposed prioritized AOs has been established within the IF framework. An analogy is used to illustrate the proposed technique, and the methodology results are compared to several current AOs. Aside from that, the effect of PDs on aggregated outcomes is thoroughly explained. Furthermore, the impact of PDs on outcomes makes the proposed solution more robust since the DM can choose the PD vector based on his or her priorities and the complexity of the problem. We apply the proposed group MCDM approach on a case study of selection of agriculture land.

Data Availability

The data used to support the findings of the study are included with in the article.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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