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Research Article

Convolution and Coefficient Estimates for (p,q)-Convex Harmonic Functions Associated with Subordination

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We preface and examine classes of (p, q)-convex harmonic locally univalent functions associated with subordination. We acquired a coefficient characterization of (p, q)-convex harmonic univalent functions. We give necessary and sufficient convolution terms for the functions we will introduce.

1. Introduction

First, let us give the basic definitions and notations that we will use in our article. In order not to spoil the generality of this study, let us denote the continuous complex valued harmonic functions class with $\mathscr{H}$ which are harmonic in the open unit disk $\mathscr{U}=\{z : z\in\mathbb{C} \text{ and } |z|<1\}$ and let $\mathscr{A}$ be the subclass of $\mathscr{H}$ which consists of functions that are analytic in $\mathscr{U}$. A function harmonic in $\mathscr{U}$ can be written in the form $f=h+\bar{g}$, where h and g are analytic in $\mathscr{U}$. Here, the analytic part of the f function is h and the co-analytical part is g. The necessary and sufficient condition for f to be locally univalent and the sense-preserving in $\mathscr{U}$ is that $|h'(z)|>|g'(z)|$ ([1]). In the light of this information, we can write without losing generality as follows:

$$h(z)=z+\sum_{\jmath=2}^{\infty}a_\jmath z^\jmath \text{ and } g(z)=\sum_{\jmath=1}^{\infty}b_\jmath z^\jmath. \tag{1}$$

Let us denote the class of functions satisfying $f=h+\bar{g}$ which are harmonic, univalent, and sense-preserving in $\mathscr{U}$ for which $h(0)=h'(0)-1=0=g(0)$ conditions with $\mathscr{SH}$. From this point of view, we can easily say that $|b_1|<1$ if there is a sense-preserving feature.

In 1984 Clunie and Sheil-Small [1] defined and analyzed characteristic features of the class $\mathscr{SH}$. Over the years, many articles on the class of $\mathscr{SH}$ and its subclasses have been made by many researchers by referring to this article.

Many studies have been done on quantum calculus. As the importance of this subject can be understood from its multidisciplinary nature, it is known to be innovative and important in many fields. The quantum calculus is also known as q-calculus. We can roughly define this calculus as the traditional infinitesimal calculus. In fact, Euler and Jacobi first started to study the subject of q-calculus, they are also people who find many attractive implementations in several fields of mathematics and other sciences.

In the last study by Sahai and Yadav [2], the quantum calculus was based on two parameters (p,q). In fact, this two-parameter definition is the postquantum calculus denoted by (p,q)-calculus, which is the generalization of q-calculus. We will use the definition of (p,q)-calculus in this article as the basis of the article published by Chakrabarti and Jagannathan in 1991 [3]. Let $p>0, q>0$, for any non-negative integer $\jmath$, the (p,q)-integer number $\jmath$, denoted by $[\jmath]_{p,q}$ is

$$[\jmath]_{p,q}=\frac{p^\jmath-q^\jmath}{p-q}, (\jmath=1,2,3,\cdots), \quad [0]_{p,q}=0. \tag{2}$$

It can be seen that this twin-basic number defined above is a generalization of the q-number defined as follows:

$$[j]_q = \frac{1-q^j}{1-q}, (j = 1, 2, 3, \cdots), \quad q \neq 1. \tag{3}$$

In like manner, the (p, q)-differential operator of a function f, analytic in $|z| < 1$ is defined by

$$D_{p,q}f(z) = \frac{f(pz) - f(qz)}{(p-q)z}, p \neq q, z \in \mathcal{U}. \tag{4}$$

It may be easily shown that $D_{p,q}f(z) \longrightarrow f'(z)$ as $p \longrightarrow 1^-$ and $q \longrightarrow 1^-$. We can easily see that

$$\lim_{q\longrightarrow 1^-} \lim_{p\longrightarrow 1^-} [j]_{p,q} = j. \tag{5}$$

For more information and details on q-calculus and (p, q)-calculus, [2, 4] can be used as references. Apart from these, different studies have also been carried out [5–7].

Ismail et al. [8] and Ahuja et al. and Ahuja and Çetinkaya [6, 9]z) used q- calculus in the theory of analytic univalent functions. The q-difference operator's definition is

$$D_{p,q}h(z) = \begin{cases} \dfrac{h(pz) - h(qz)}{(p-q)z}, & z \neq 0, \\ h'(0), & z = 0, \end{cases}$$
$$D_{p,q}g(z) = \begin{cases} \dfrac{g(pz) - g(qz)}{(p-q)z}, & z \neq 0, \\ g'(0), & z = 0, \end{cases} \tag{6}$$

where h and g are of the form (1) which given in [4] and we get the following result for same h and g

$$D_{p,q}h(z) = 1 + \sum_{j=2}^{\infty} [j]_{p,q} a_j z^{j-1} \text{ and } D_{p,q}g(z) = \sum_{j=1}^{\infty} [j]_{p,q} b_j z^{j-1}. \tag{7}$$

For $f \in \mathcal{SH}$, let

$$\mathfrak{D}^1_{p,q}f(z) = zD_{p,q}h(z) - z\overline{D_{p,q}g(z)}, \tag{8}$$

$$\begin{aligned} \mathfrak{D}^2_{p,q}f(z) &= zD_{p,q}\left(zD_{p,q}h(z)\right) + zD_{p,q}\left(z\overline{D_{p,q}g(z)}\right) \\ &= z + \sum_{j=2}^{\infty} [j]^2_{p,q} a_j z^j + \sum_{j=1}^{\infty} [j]^2_{p,q} \overline{b_j z^j}. \end{aligned} \tag{9}$$

We say that an analytic function f is subordinate to an analytic function F and write $f \prec F$, if there exists a complex valued function ϖ which maps $\mathcal{U}$ into oneself with $\varpi(0) = 0$, such that $f(z) = F(\varpi(z)) (z \in \mathcal{U})$.

Additionally, if F is univalent in $\mathcal{U}$, then we can give the following result:

$$f(z) \prec F(z) \Leftrightarrow f(0) = F(0) \text{ and } f(\mathcal{U}) \subset F(\mathcal{U}). \tag{10}$$

Denote by $\mathcal{SHC}_{p,q}(A, B)$ the subclass of $\mathcal{H}$ consisting of functions f of the form (1) that satisfy the condition

$$\frac{\mathfrak{D}^2_{p,q}f(z)}{\mathfrak{D}^1_{p,q}f(z)} \prec \frac{1 + Az}{1 + Bz}, \tag{11}$$

$$(0 < p, q < 1, p \neq q, z \in \mathcal{U} \text{ and } -B \leq A < B \leq 1), \tag{12}$$

where is $\mathfrak{D}^1_{p,q}f(z)$ and $\mathfrak{D}^2_{p,q}f(z)$ are defined by (8) and (9).

By suitably specializing the parameters, the classes $\mathcal{SHC}_{p,q}(A, B)$ reduce to the various subclasses of harmonic univalent functions. That is, by assigning special values instead of p,q, A, and B, we are saying that they become classes that used to be studied. This is an indication that this article is a general subclass that includes other classes in harmonic functions. Such as

(i) $\mathcal{SHC}_{p,q}(A, B) = \mathcal{SHC}_q(A, B)$ for $p \longrightarrow 1^-$([10])

(ii) $\mathcal{SHC}_{p,q}(A, B) = \mathcal{SHC}(A, B)$ for $p \longrightarrow 1^-$ and $q \longrightarrow 1^-$([11]),

(iii) $\mathcal{SHC}_{p,q}((p+q)\vartheta - 1, p+q-1) = \mathcal{A}_{p,q}\mathcal{C}_H(\vartheta)$ for $0 \leq \vartheta < 1$ ([12]),

(iv) $\mathcal{SHC}_{p,q}((p+q)\alpha - 1, p+q-1) = \mathcal{SHC}_q(\alpha)$ for $p \longrightarrow 1^-$and $0 \leq \alpha < 1$ ([6, 13]),

(v) $\mathcal{SHC}_{p,q}((p+q)\alpha - 1, p+q-1) = \mathcal{SHC}(\alpha)$ for $p \longrightarrow 1^-$, $q \longrightarrow 1^-$ and $0 \leq \alpha < 1$([14, 15]),

(vi) $\mathcal{SHC}_{p,q}((p+q)\alpha - 1, p+q-1) = \mathcal{SHC}(0)$ for $p \longrightarrow 1^-$, $q \longrightarrow 1^-$ and $\alpha = 0$ ([16]).

Using the method that used by Dziok et al. [11, 17–19] we find necessary and sufficient conditions for the above defined class $\mathcal{SHC}_{p,q}(A, B)$.

2. Main Results

For functions $f_1$and $f_2 \in \mathcal{H}$ of the form

$$f_j(z) = z + \sum_{j=2}^{\infty} a_{j,j} z^j + \sum_{j=1}^{\infty} \overline{b_{j,j} z^j}, (j = 1, 2), \tag{13}$$

we define the Hadamard product of $f_1$and f_2 by

$$(f_1 * f_2)(z) = z + \sum_{j=2}^{\infty} a_{j,1} a_{j,2} z^j + \sum_{j=1}^{\infty} b_{j,1} \overline{b_{j,2} z^j}, z \in \mathcal{U}. \tag{14}$$

In the first theorem, we introduce a sufficient coefficient bound for harmonic functions in $\mathcal{SHC}_{p,q}(A, B)$.

Theorem 1. *Let us first assume that $f \in \mathcal{H}$. Then, $f \in \mathcal{SHC}_{p,q}(A, B)$ if and only if*

$$f(z) * \Xi_{p,q}(z; \zeta) \neq 0, (\zeta \in \mathbb{C}, |\zeta| = 1, z \in \mathcal{U} \setminus \{0\}), \tag{15}$$

where

$$\begin{aligned}
&\Xi_{p,q}(z;\zeta)\\
&=\frac{(B-A)\zeta z+\left[(p+q)(p+q-1)+\zeta\left(A(p+q)^2-B(p+q)+pq(B-A)\right)\right]z^2}{(1-pz)(1-qz)(1-p^2z)(1-q^2z)(1-pqz)}\\
&+\frac{1+pq-(p+q)(p+q+1)+(B+Apq-(p+q)[(p+q)A+B])\zeta}{(1-pz)(1-qz)(1-p^2z)(1-q^2z)(1-pqz)}pqz^3\\
&+\frac{1+pq+(B+Apq)\zeta}{(1-pz)(1-qz)(1-p^2z)(1-q^2)z(1-pqz)}p^2q^2z^4\\
&+\frac{[2+(A+B)\zeta]\bar{z}-\left[(p^2+q^2)(1+A\zeta)+(A-B)pq\zeta+(p+q)(1+B\zeta)\right]\bar{z}^2}{(1-p\bar{z})(1-q\bar{z})(1-p^2\bar{z})(1-q^2\bar{z})(1-pq\bar{z})}\\
&+\frac{\left[(p^2+q^2+pq)(1+A\zeta)+(1-p-q)(1+B\zeta)\right]}{(1-p\bar{z})(1-q\bar{z})(1-p^2\bar{z})(1-q^2\bar{z})(1-pq\bar{z})}pq\bar{z}^3\\
&+\frac{1+B\zeta-pq(1+A\zeta)}{(1-p\bar{z})(1-q\bar{z})(1-p^2\bar{z})(1-q^2\bar{z})(1-pq\bar{z})}p^2q^2\bar{z}^4.
\end{aligned} \tag{16}$$

Proof. Let $f\in\mathscr{H}$ be of the form (1). Then $f\in\mathcal{SHC}_{p,q}(A,B)$ if and only if it satisfies (11) or equivalently

$$\frac{\mathfrak{D}^2_{p,q}f(z)}{\mathfrak{D}^1_{p,q}f(z)}\neq\frac{1+A\zeta}{1+B\zeta}, \tag{17}$$

where $\zeta\in\mathbb{C}$, $|\zeta|=1$ and $z\in\mathscr{U}\setminus\{0\}$. Since

$$\begin{aligned}
zD_{p,q}h(z)&=h(z)*\frac{z}{(1-pz)(1-qz)},\\
zD_{p,q}g(z)&=g(z)*\frac{z}{(1-pz)(1-qz)},\\
zD_{p,q}\left(zD_{p,q}h(z)\right)&=h(z)*\frac{z(1+pqz)}{(1-p^2z)(1-q^2z)(1-pqz)},\\
zD_{p,q}\left(zD_{p,q}g(z)\right)&=g(z)*\frac{z(1+pqz)}{(1-p^2z)(1-q^2z)(1-pqz)},
\end{aligned} \tag{18}$$

the inequality (17) yields

$$\begin{aligned}
&(1+B\zeta)\mathfrak{D}^2_{p,q}f(z)-(1+A\zeta)\mathfrak{D}^1_{p,q}f(z)\\
&=(1+B\zeta)\left[zD_{p,q}\left(zD_{p,q}h(z)\right)+zD_{p,q}\left(z\bar{D}_{p,q}g(z)\right)\right]\\
&\quad-(1+A\zeta)\left[zD_{p,q}h(z)-zD_{p,q}\bar{g}(z)\right]\\
&=h(z)*\left\{\frac{(1+B\zeta)z(1+pqz)}{(1-p^2z)(1-q^2z)(1-pqz)}-\frac{(1+A\zeta)z}{(1-pz)(1-qz)}\right\}\\
&\quad+g(\bar{z})*\left\{\frac{(1+B\zeta)\bar{z}(1+pq\bar{z})}{(1-p^2\bar{z})(1-q^2\bar{z})(1-pq\bar{z})}+\frac{(1+A\zeta)\bar{z}}{(1-p\bar{z})(1-q\bar{z})}\right\}\\
&=f(z)*\Xi_{p,q}(z;\zeta)\neq0.
\end{aligned} \tag{19}$$

□

Theorem 2. *Let $f=h+\bar{g}$ be given by (1). If*

$$\begin{aligned}
&\sum_{j=1}^{\infty}[j]_{p,q}\left\{\left(\left|[j]_{p,q}-1\right|+\left|[j]_{p,q}B-A\right|\right)|a_j|\right.\\
&\quad\left.+\left([j]_{p,q}(1+B)+1+A\right)|b_j|\right\}\le2(B-A),
\end{aligned} \tag{20}$$

where $a_1=1$, $0<p,q<1$, $-B\le A<B\le1$, then, $f\in\mathcal{SHC}_{p,q}(A,B)$.

Proof. $f\in\mathcal{SHC}_{p,q}(A,B)$ if and only if there exists a complex valued function ϖ; $\varpi(0)=0$, $|\varpi(z)|<1$ $(z\in\mathscr{U})$ such that

$$\frac{zD_{p,q}\left(zD_{p,q}h(z)\right)+zD_{p,q}\left(z\bar{D}_{p,q}g(z)\right)}{zD_{p,q}h(z)-zD_{p,q}\bar{g}(z)}=\frac{1+A\varpi(z)}{1+B\varpi(z)}, \tag{21}$$

or equivalently

$$\left|\frac{\mathfrak{D}^2_{p,q}f(z)-\mathfrak{D}^1_{p,q}f(z)}{B\mathfrak{D}^2_{p,q}f(z)-A\mathfrak{D}^1_{p,q}f(z)}\right|<1. \tag{22}$$

The above inequality (20) holds, since for $|z|=r(0<r<1)$, we obtain

$$\begin{aligned}
&\left|zD_{p,q}\left(zD_{p,q}h(z)\right)-zD_{p,q}h(z)+zD_{p,q}\left(z\bar{D}_{p,q}g(z)\right)+zD_{p,q}\bar{g}(z)\right|\\
&\quad-\left|BD_{p,q}\left(zD_{p,q}h(z)\right)+BzD_{p,q}\left(z\bar{D}_{p,q}g(z)\right)\right.\\
&\quad\left.-AD_{p,q}h(z)+AD_{p,q}\bar{g}(z)\right|\\
&=\left|\sum_{j=2}^{\infty}[j]_{p,q}\left([j]_{p,q}-1\right)a_jz^j+\sum_{j=1}^{\infty}[j]_{p,q}\left([j]_{p,q}+1\right)b_j\bar{z}^j\right|\\
&\quad-\left|(B-A)z+\sum_{j=2}^{\infty}[j]_{p,q}\left([j]_{p,q}B-A\right)a_jz^j\right.\\
&\quad\left.+\sum_{j=1}^{\infty}[j]_{p,q}\left([j]_{p,q}B+A\right)b_j\bar{z}^j\right|\\
&\le\sum_{j=2}^{\infty}[j]_{p,q}\left(\left|[j]_{p,q}-1\right|\right)|a_j|r^j\\
&\quad+\sum_{j=1}^{\infty}[j]_{p,q}\left([j]_{p,q}+1\right)|b_j|r^j-(B-A)r\\
&\quad+\sum_{j=2}^{\infty}[j]_{p,q}\left|[j]_{p,q}B-A\right||a_j|r^j+\sum_{j=1}^{\infty}[j]_{p,q}\left([j]_{p,q}B+A\right)|b_j|r^j\\
&\le r\left\{\sum_{j=2}^{\infty}[j]_{p,q}\left(\left|[j]_{p,q}-1\right|+\left|[j]_{p,q}B-A\right|\right)|a_j|r^{j-1}\right.\\
&\quad\left.+\sum_{j=1}^{\infty}[j]_{p,q}\left([j]_{p,q}(1+B)+1+A\right)|b_j|r^{j-1}-(B-A)\right\}<0.
\end{aligned} \tag{23}$$

The harmonic function

$$f(z)=z+\sum_{j=2}^{\infty}\frac{(B-A)x_j}{[j]_{p,q}\left(\left|[j]_{p,q}-1\right|+\left|[j]_{p,q}B-A\right|\right)}z^j+\sum_{j=1}^{\infty}\frac{(B-A)y_j}{[j]_{p,q}\left([j]_{p,q}(1+B)+1+A\right)}\bar{z}^j, \tag{24}$$

where

$$\sum_{j=1}^{\infty}\left|x_j\right|+\sum_{j=1}^{\infty}\left|y_j\right|=1 \tag{25}$$

shows that the coefficient bound given by (20) is sharp. The functions of the form (8) are in $\mathcal{SHC}_{p,q}(A,B)$ because

$$\sum_{j=1}^{\infty}\frac{[j]_{p,q}\left(\left|[j]_{p,q}-1\right|+\left|[j]_{p,q}B-A\right|\right)}{2(B-A)}\left|a_j\right|+\frac{[j]_{p,q}\left([j]_{p,q}(1+B)+1+A\right)}{2(B-A)}\left|b_j\right|=\sum_{j=1}^{\infty}\left(\left|x_j\right|+\left|y_j\right|\right)=1. \tag{26}$$

□

Denote by $\mathcal{TSHC}_{p,q}(A,B)$ the subclass of $\mathcal{H}$ consisting of functions f of the form (1) that satisfy the inequality (20). It is clear that $\mathcal{TSHC}_{p,q}(A,B)\subset\mathcal{SHC}_{p,q}(A,B)$.

Theorem 3. *The class $\mathcal{TSHC}_{p,q}(A,B)$ is closed under convex combination.*

Proof. For $j=1,2,\cdots,$ suppose that $f_j\in\mathcal{TSHC}_{p,q}(A,B)$, where

$$f_j(z)=z+\sum_{j=2}^{\infty}\left|a_{j_j}\right|z^j+\sum_{j=1}^{\infty}\left|b_{j_j}\right|\bar{z}^j. \tag{27}$$

Then, by Theorem 2.,

$$\sum_{j=2}^{\infty}\frac{[j]_{p,q}\left(\left|[j]_{p,q}-1\right|+\left|[j]_{p,q}B-A\right|\right)}{B-A}\left|a_{j_j}\right|+\sum_{j=1}^{\infty}\frac{[j]_{p,q}\left([j]_{p,q}(1+B)+1+A\right)}{B-A}\left|b_{j_j}\right|\le 1. \tag{28}$$

For $\sum_{j=1}^{\infty}t_j=1, 0\le t_j\le 1$, we can write the convex combination of f_j as follows:

$$\sum_{j=1}^{\infty}t_jf_j(z)=z+\sum_{j=2}^{\infty}\left(\sum_{j=1}^{\infty}t_j\left|a_{j_j}\right|\right)z^j+\sum_{j=1}^{\infty}\left(\sum_{j=1}^{\infty}t_j\left|b_{j_j}\right|\right)\bar{z}^j. \tag{29}$$

Then, by (9),

$$\begin{aligned}&\sum_{j=1}^{\infty}\left\{\frac{[j]_{p,q}\left(\left|[j]_{p,q}-1\right|+\left|[j]_{p,q}B-A\right|\right)}{2(B-A)}\sum_{j=1}^{\infty}t_j\left|a_{j_j}\right|+\frac{[j]_{p,q}\left([j]_{p,q}(1+B)+1+A\right)}{2(B-A)}\sum_{j=1}^{\infty}t_j\left|b_{j_j}\right|\right\}\\&=\sum_{j=1}^{\infty}t_j\sum_{j=1}^{\infty}\left(\frac{[j]_{p,q}\left(\left|[j]_{p,q}-1\right|+\left|[j]_{p,q}B-A\right|\right)}{2(B-A)}\left|a_{j_j}\right|+\frac{[j]_{p,q}\left([j]_{p,q}(1+B)+1+A\right)}{2(B-A)}\left|b_{j_j}\right|\right)\\&\le\sum_{j=1}^{\infty}t_j=1,\end{aligned} \tag{30}$$

hence

$$\sum_{j=1}^{\infty}t_jf_j(z)\in\mathcal{TSHC}_{p,q}(A,B). \tag{31}$$

□

3. Conclusions

As a result, a general subclass has been defined in this article. Thus, with this study, which will be a good reference for the new results to be obtained, a subclass study has been made for harmonic functions using the (p,q) derivative, which is still popular today.

Data Availability

There is no data available.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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