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Research Article

Decision Support System Based on Complex q-Rung Orthopair Fuzzy Rough Hamacher Aggregation Operator through Modified EDAS Method

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The best mathematical tools for combining numerous inputs into a single result are aggregation operators. The aggregation operators work to combine all of the individual evaluation values provided in a uniform form, and they are very useful for evaluating the options provided in the decision-making process. To provide a larger space for decision makers, complex q-rung orthopair fuzzy rough sets can express their uncertain information. As a generalization of the algebraic operations, the Einstein t-norm and t-conorm, Hamacher operations have become significant in aggregation theory. The Hamacher aggregation operator's major characteristic is that it can capture the interrelationship between several input arguments. In this article, some Hamacher aggregation operators for complex q-rung orthopair fuzzy rough sets are presented. We define a complex q-rung orthopair fuzzy rough Hamacher operation laws and a new score function. In addition, we propose a serious of averaging aggregation operators for complex q-rung orthopair fuzzy rough set. We present the essential properties of these operators. We use the defined operators and modified EDAS (evaluation based on distance from average solution) method to propose an approach for solving a multicriteria decision making problem. To demonstrate the practicality and effectiveness of our propose model, we consider a numerical example of area selection for an arboretum. Finally, a comparison between the suggested approach with existing operators has been presented for authenticity and reliability.

1. Introduction

In today's world, scientific and technological advancements have resulted in scientific and technological revolutions that have reduced the complications in our everyday lives. However, despite the advancement of science, which has made life easier, some issues, such as decision making (DM), remain complex. Over the last few years, DM, particularly multicriteria group decision making (MCGDM), has been widely used in a variety of fields where traditional methods have failed. As in real life, information is frequently indeterminate, and as the volume and complexity of information grows, more techniques are required. To deal with ambiguity in data, fuzzy sets [1] have been introduced. Fuzzy set theory is a variant of the traditional set theory that allows for more imprecise data. Fuzzy sets have been a very useful tool in DM for a few decades [2]. Over the last few decades, data aggregation has been used to improve the DM process' reliability and validity. To solve the decision-making problem where a single preference is required from multiple conflicting criteria, some aggregation operators have been introduced. Fundamentally, in multiattribute group decision making (MAGDM), the aggregation operator is the best way to handle the decision making process. Many aggregation operators have been developed by authors all over the world in this regard [3, 4]. Pythagorean fuzzy power

aggregation operators were developed in MCDM by Wei and Lu [5]. Xu [6] introduced the concept of an intuitionistic fuzzy aggregation operator. Xu and Yager [7] introduced ordered weighted aggregation (OWA) operator for MCDM. Yager [8] generalized OWA operator. Researchers have been using aggregation operators for fuzzy information for decades, with impressive results in DM. Seikh and Mandal [9] present a novel intuitionistic fuzzy Dombi weighted averaging and geometric operators approach that they used in DM. Cholewa [10], Dubios and Koning [11], and Van et al. [12] proposed some aggregation operators and decision making methods using fuzzy averaging operators. Salih et al. [13] developed a benchmarking AQM methods of network congestion control based on an extension of interval type-2 trapezoidal fuzzy decision by opinion score method.

Albahri et al. [14] developed a combination of fuzzy weighted zero inconsistency and fuzzy decision by opinion score methods in Pythagorean m-polar fuzzy environment and discussed the case study of sing language recognition systems. Al-Samarraay et al. [15] defined a new extension of FDOSM based on Pythagorean fuzzy environment for evaluating and benchmarking sign language recognition systems. Aggregated operators in fuzzy environment are not only useful in MCDM problem but also has astonishing results in generalized fuzzy sets like intuitionistic fuzzy set [16–20] and Pythagorean fuzzy sets [21] and q-rung orthopair fuzzy sets [22–27]. Alamoodi et al. [28] proposed a new development of FWZIC and FDOSM for benchmarking smart e-tourism applications based on neutrosophic fuzzy environment. Alsalem et al. [29] defined a combination of FWZIC and FDOSM for prioritising COVID-19 vaccine dose recipients based on T-spherical fuzzy environment.

Ramot et al. [30] defined the concept of complex fuzzy set (CFS). In a complex environment, an advanced form of the classical fuzzy set can be used to handle fuzziness in information. Because it contains both a phase and an amplitude term, a complex fuzzy value can deal with information in two ways. This feature of CFS allows for more data storage. Ramot et al. [31] define operations and relations on complex fuzzy sets. Hu et al. [32, 33] extend complex fuzzy sets with approximate paralleling and orthogonality relations. Zhang et al. [34] provide an example of δ-equalities between CFSs [35, 36]. According to Alkouri and Salleh [37], several distance measures for CFSs have been proposed. Bi et al. [38] initiate two classes of entropy measures in CFSs. Liu et al. [39] measure the distance between CFSs, their cross-entropy, and their applications in DM. The rotational invariance on complex fuzzy operations was generalized by Dai [40]. Some researchers have recently begun to work on complex fuzzy aggregation operators; for example, Merig et al. [41] present a complex fuzzy generalized aggregation operator and its applications in DM. Hu et al. [42] proposed a power aggregation operator for complex fuzzy information. Garg and Rani [43] are the first to propose a power aggregation operator and a ranking method for complex intuitionistic fuzzy sets (CIFSs), as well as their applications in DM. Garg and Rani [44] proposed generalized geometric aggregation operators based on t-norm operations for complex intuitionistic fuzzy sets and their applications in DM. Liu et al. [45] defined a group DM using complex q-rung orthopair (Cq-ROF) Bonferroni mean operator. Garg et al. [46] defined new logarithmic operation laws based complex q-rung orthopair fuzzy aggregation operators. Mahmood and Ali [47] proposed AOs and VIKOR method based on complex q-rung orthopair fuzzy uncertain linguistic information and their applications in MCDM. Naz et al. [48] defined a hybrid MADM model under complex q-rung orthopair fuzzy Hamy mean aggregation operators. Garg et al. [49] developed a generalized dice similarity measure for complex q-rung orthopair fuzzy sets (Cq-ROFSs) and its application.

One of the major milestones in DM is a rough set theory. Pawlak [50] came up with this brilliant idea, which is widely used to find hidden patterns in data sets. It can also be used for data mining, feature selections, data extraction, data deduction, decision rules generation, multiagent systems, and granular computing [51–54]. Rough set theory quickly drew researchers' attention to the concept of a hybrid form of fuzzy and rough sets [50, 55–57], because they are both useful in dealing with data imprecision. The idea of fuzzy rough sets was first proposed by Dubois and Prade [58]. The basic structure of rough set contains the upper and lower approximation. Alnoor et al. [59] defined a sustainable transportation industry and oil company benchmarking based on the extension of linear Diophantine fuzzy rough sets.

The operators for Hamacher weighted averaging, ordered weighted averaging, and hybrid averaging, as well as their key characteristics. The Hamacher aggregation operator based on interval valued intuitionistic fuzzy numbers was introduced by Liu [60], and its applications in DM problems were discussed. Darko and Liang [61] introduced the q-rung orthopair fuzzy Hamacher aggregation operators with the modified EDAS method and applied to MCGDM problem. In the CIF Hamacher aggregation operator framework, Akram et al. [62] introduced a new DM model. Mahmood and Ali [63] developed the Cq-ROF Hamacher aggregation operators and look into how it can be used to clean up gold mine production assessments. Huang [64] defined intuitionistic fuzzy logic, the Hamacher t-norm, and t-conorm concepts were used.

EDAS method was proposed by Ghorabaee et al. [65] and solved decision-making problems. In DM, the EDAS method produces remarkable results, especially when the criteria are in conflict. The top MCDM methods are TOPSIS (technique for order of preference by similarity to ideal solution) method [66]. And in VIKOR, the arrangements of alternatives take place and determine the solution which is the nearest to the ideal solution. EDAS method is based on calculating the best alternative from the list of possible choices based on positive distance from average solution (PDAS) and negative distance from average solution (NDAS) depending on the average solution (AS). The difference between each solution and the AS is indicated by the terms PDAS and NDAS. As a result, the good ones must have a higher PDAS value and a lower NDAS value. Ghorabaee et al. [67] pioneered the EDAS method for supplier selection based on intuitionistic fuzzy information. Using these operators to solve MCGDM problems, Zhang et al. [68] developed the picture fuzzy weighted averaging and weighted geometric operator, as well as the

EDAS method. According to Peng and Lui [69], the neutrosophic soft decision approach with similarity measure was introduced and applied to the EDAS method. Feng et al. [70] introduce the EDAS method by using it to analyse hesitant fuzzy data. According to Li et al. [71], the EDAS method was used to develop a hybrid operator and its applications in DM. Liang [72] presents an extended version of the EDAS method in an intuitionistic fuzzy environment, as well as its applications in energy conservation projects. EDAS method for site selection under intuitionistic fuzzy values was developed by Kahraman et al. [73]. Ilieva [74] pioneered the use of interval fuzzy data in the EDAS method for MCGDM. Kar Yan and Kahraman [75, 76] developed a new approach to the EDAS method based on interval-valued neutrosophic data. Stanujkic et al. [77] introduced the concept of grey numbers in the EDAS method. Ghorabaee et al. [78] proposed a novel approach of dynamic fuzzy method for MCGDM based on EDAS method. EDAS method for DM approach by using fuzzy data is represented by Stevic et al. [79]. Ghorabaee et al. [80] introduced the concept of rank reversal and examined a hybrid version of the EDAS and TOPSIS methods.

As a result of the above analysis, there is no use of the EDAS method in a complex q-rung orthopair fuzzy environment with hybrid Hamacher operators and rough set. Furthermore, existing operators only deal with the imprecision and ambiguity of real numbers, whereas the proposed approach in this article is for a Cq-ROF rough environment that is periodic. The complex q-rung orthopair fuzzy set has the ability to work in two dimensions. Thus, the main novelties of the current study are delineated as follows:

(1) To define some generalized operation rules for Cq-ROFRS by using Hamacher t-norm and Hamacher t-conorm, which can provide more choices for the DMs

(2) To originate complex q-rung orthopair fuzzy Hamacher arithmetic aggregation operators, including complex q-rung orthopair fuzzy rough Hamacher weighted average (Cq-ROFRHWA), complex q-rung orthopair fuzzy rough Hamacher ordered weighted average (Cq-ROFRHOWA), and complex q-rung orthopair fuzzy rough Hamacher hybrid average (Cq-ROFRHHA) operators. Further, some basic properties like idempotency, monotonicity, boundedness, and some limiting cases of these operators are also investigated

(3) To develop an MCGDM model based on the proposed Cq-ROFRS operator and modified EDAS method to handle the complex q-rung orthopair fuzzy rough decision problems

(4) The application of the advised strategy is illustrated through a real-world example involving the area selection for an arboretum problem

This manuscript's entire work is laid out as follows. We have added some basic definitions to Section 2, through which we will proceed with our work. We defined the Hamacher operator on complex q-rung orthopair fuzzy rough sets and the score function in Section 3. Furthermore, in Section 4, we proposed complex q-rung fuzzy rough Hamacher aggregation operator such as complex q-rung orthopair fuzzy rough Hamacher weighted average (Cq-ROFRHWA), complex q-rung orthopair fuzzy rough Hamacher ordered weighted average (Cq-ROFRHOWA), and complex q-rung orthopair fuzzy rough Hamacher hybrid average (Cq-ROFRHHA) operator. We have also talked about some of the operators' important properties. We presented a modified stepwise EDAS algorithm in Section 5. We used this method to solve an example of choosing the best location for an arboretum in Section 6. In Section 7, we discussed the conclusion of the paper.

2. Preliminaries

We have added some useful definitions to the current section that will help us connect to the new approach investigated in this paper. Complex fuzzy set, equivalence relation, rough set, complex fuzzy equivalence relation, and complex fuzzy rough set have been discussed.

Definition 1 (see [30]). Let a universe of discourse $\mathscr{X}$, a set $\mathscr{Q}$ is called a complex fuzzy set (CFS) and is defined as

$$\mathscr{Q} = \left(\left\langle x, \chi_{\mathscr{Q}}(x).e^{2\pi i \Phi_{\mathscr{Q}}(x)}\right\rangle | x \in \mathscr{X}\right), \tag{1}$$

where $i = \sqrt{-1}$, $\chi_{\mathscr{Q}}(x) \in [0, 1]$ represents the amplitude term, and $\Phi_{\mathscr{Q}}(x) \in [0, 2\pi]$ is known as phase term. Also, $\chi_{\mathscr{Q}}(x)$ denotes the membership value of an element of a CFS $\mathscr{Q}$.

Definition 2 (see [31]). The average of two complex fuzzy variables is defined as

$$\text{Avg} = \frac{\mathscr{A} + \mathscr{B}}{2} = \left(\frac{\chi_{\mathscr{A}}(x) + \chi_{\mathscr{B}}(x)}{2}\right) e^{2\pi i(\Phi_{\mathscr{A}}(x) + \Phi_{\mathscr{B}}(x)/2)}, \tag{2}$$

where $\mathscr{A}$ and B represent any two complex fuzzy values such that $\mathscr{A} = \chi_{\mathscr{A}}(x)e^{2\pi i \Phi_{\mathscr{A}}(x)}$ and $\mathscr{B} = \chi_{\mathscr{B}}(x)e^{2\pi i \Phi_{\mathscr{B}}(x)}$.

Definition 3 (see [81]). Consider a universal set $\mathscr{X}$ and $\Xi \in \mathscr{X} \times \mathscr{X}$ be any relation. Then,

(1) Ξ is said to be reflexive if $(u, u) \in \Xi$, $\forall u \in \mathscr{X}$

(2) Ξ is said to be symmetric if $\forall u, v \in \mathscr{X}$, $(u, v) \in \Xi$; then, $(v, u) \in \Xi$

(3) Ξ is said to be transitive if $\forall u, v, x \in \mathscr{X}$, $(u, v) \in \Xi$, and $(v, x) \in \Xi$; then, $(u, x) \in \Xi$

Definition 4 (see [81]). Let $\mathscr{X}$ be a nonempty and finite universe of discourse, and $\mathscr{P}$ be a fuzzy equivalence relation defined on $\mathscr{X} \times \mathscr{X}$. The pair $(\mathscr{X}, \mathscr{P})$ is called a fuzzy approximation space. For any $A \in F(\mathscr{X})$, the upper and lower approximation with $\mathscr{P}(A)$ respect to $(\mathscr{X}, \mathscr{P})$ is denoted by $\underline{\mathscr{P}}(A)$ and $\bar{\mathscr{P}}(A)$, respectively, and defined as

$$\underline{\mathscr{P}}(A) = \left\{ \left(x, \mu_{\underline{\mathscr{P}}(A)}(x) \right) | x \in \mathscr{X} \right\}, \tag{3}$$

$$\bar{\mathscr{P}}(A) = \left\{ \left(x, \mu_{\bar{\mathscr{P}}(A)}(x) \right) | x \in \mathscr{X} \right\}, \tag{4}$$

where

$$\mu_{\underline{\mathscr{P}}(A)}(x) = \wedge_{u \in \mathscr{X}}((1 - \mu_{\mathscr{P}}(x, u)) \vee \mu_A(u)), x \in \mathscr{X}, \tag{5}$$

$$\mu_{\underline{\mathscr{P}}(A)}(x) = \vee_{u \in \mathscr{X}}(\mu_{\mathscr{P}}(x, u) \wedge \mu_A(u)), x \in \mathscr{X}. \tag{6}$$

The pair $\mathscr{P}(A) = (\underline{\mathscr{P}}(A), \bar{\mathscr{P}}(A))$ is called the fuzzy rough set of A with respect to $(\mathscr{X}, \mathscr{P})$.

Definition 5 (see [81]). Let $\mathscr{G}$ be a universal set and $\mathscr{A} \in CFS(\mathscr{X} \times \mathscr{X})$ be complex fuzzy equivalence relation. Then,

(1) Ξ is reflexive if $\mu_{\Xi}(u, u) = 1, \forall u \in \mathscr{X}$

(2) Ξ is symmetric if for all $(u, v) \in (\mathscr{X} \times \mathscr{X})$, and $\mu_{\Xi}(u, v) = \mu_{\Xi}(v, u)$

(3) Ξ is transitive if for all $(u, v) \in \mathscr{X} \times \mathscr{X}$, and $\mu_{\Xi}(u, v) \geq \vee_{x \in \mathscr{X}}[\mu_{\Xi}(u, x) \wedge \mu_{\Xi}(x, v)]$

Definition 6 (see [82]). The notion of t-norm and t-conorm operators has the important role in fuzzy set theory. These two operators denote intersection and union of fuzzy sets, respectively. And t-norm and t-conorm represent Hamacher product $\otimes$ and Hamacher sum $\oplus$, respectively, and are defined as follows

$$T(l, m) = l \otimes m = \frac{lm}{\rho + (1 - \varrho)(l + m - lm)}, \varrho > 0, \tag{7}$$

$$S(l, m) = l \oplus m = \frac{l + m - lm - (1 - \varrho)lm}{1 - (1 - \varrho)lm}, \varrho > 0. \tag{8}$$

By taking different values of ϱ in the above equations, we will obtain different form of these equations. For example, if we put $\varrho = 1$ then the above equations become

$$T(l, m) = l \otimes m = lm, \tag{9}$$

$$S(l, m) = l \oplus m = l + m - lm. \tag{10}$$

This is called algebraic t-norm and t-conorm operators. Also by putting $\varrho = 2$, we will get

$$T(l, m) = l \otimes m = \frac{lm}{1 + (1 - l)(1 - m)}, \tag{11}$$

$$S(l, m) = l \oplus m = \frac{l + m}{1 + lm}. \tag{12}$$

This is called Einstein t-norm and t-conorm, respectively.

2.1. Complex q-Rung Orthopair Fuzzy Rough Set

Definition 7. Let $\mathscr{X}$ be the universal set and $\Xi \in \text{Cq} - \text{ROF}(\mathscr{X} \times \mathscr{X})$ be any complex q-rung orthopair fuzzy relation. Then, the pair $(\mathscr{X}, \Xi)$ is called complex q-rung orthopair fuzzy approximation space (Cq-ROFAS). Now, for any complex q-rung orthopair fuzzy set $\Lambda \in \text{Cq} - \text{ROF}(\mathscr{X})$, the lower and upper approximations of Λ with respect to $(\mathscr{X}, \Xi)$ are two complex q-rung orthopair fuzzy rough sets and defined as $\Xi(\Lambda) = \langle \underline{\Xi}(\Lambda), \bar{\Xi}(\Lambda) \rangle$;

$$\underline{\Xi}(\Lambda) = \left\{ \left\langle x, \chi_{\underline{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\chi_{\underline{\Xi}(\Lambda)}}(x)}, \xi_{\underline{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\xi_{\underline{\Xi}(\Lambda)}}(x)} \right\rangle | x \in \mathscr{X} \right\}, \tag{13}$$

$$\bar{\Xi}(\Lambda) = \left\{ \left\langle x, \chi_{\bar{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\chi_{\bar{\Xi}(\Lambda)}}(x)}, \xi_{\bar{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\xi_{\bar{\Xi}(\Lambda)}}(x)} \right\rangle | x \in \mathscr{X} \right\}, \tag{14}$$

where $\chi_{\underline{\Xi}(\Lambda)}(x), \xi_{\underline{\Xi}(\Lambda)}(x), \Phi_{\chi_{\underline{\Xi}(\Lambda)}}(x), \Phi_{\xi_{\underline{\Xi}(\Lambda)}}(x)$, and $\chi_{\bar{\Xi}(\Lambda)}(x)$, $\xi_{\bar{\Xi}(\Lambda)}(x), \Phi_{\chi_{\bar{\Xi}(\Lambda)}}(x), \Phi_{\xi_{\bar{\Xi}(\Lambda)}}(x) \in 0, 1]$.

Also,

$$\chi_{\underline{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\chi_{\underline{\Xi}(\Lambda)}}(x)} = \bigwedge_{y \in \mathscr{X}} \left\{ \begin{array}{c} \chi_{\underline{\Xi}(\Lambda)}(x, y) e^{2\pi i \Phi_{\chi_{\underline{\Xi}(\Lambda)}}(x)}(x, y) \\ \vee \chi_{\underline{\Xi}(\Lambda)}(y) e^{2\pi i \Phi_{\chi_{\underline{\Xi}(\Lambda)}}(x)}(y) \end{array} \right\}; \tag{15}$$

$$\xi_{\underline{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\xi_{\underline{\Xi}(\Lambda)}}(x)} = \bigvee_{y \in \mathscr{X}} \left\{ \begin{array}{c} \xi_{\underline{\Xi}(\Lambda)}(x, y) e^{2\pi i \Phi_{\xi_{\underline{\Xi}(\Lambda)}}(x)}(x, y) \\ \wedge \xi_{\underline{\Xi}(\Lambda)}(y) e^{2\pi i \Phi_{\xi_{\underline{\Xi}(\Lambda)}}(x)}(y) \end{array} \right\}; \tag{16}$$

$$\chi_{\bar{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\chi_{\bar{\Xi}(\Lambda)}}(x)} = \bigvee_{y \in \mathscr{X}} \left\{ \begin{array}{c} \chi_{\bar{\Xi}(\Lambda)}(x, y) e^{2\pi i \Phi_{\chi_{\bar{\Xi}(\Lambda)}}(x)}(x, y) \\ \wedge \chi_{\bar{\Xi}(\Lambda)}(y) e^{2\pi i \Phi_{\chi_{\bar{\Xi}(\Lambda)}}(x)}(y) \end{array} \right\}; \tag{17}$$

$$\xi_{\bar{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\xi_{\bar{\Xi}(\Lambda)}}(x)} = \bigwedge_{y \in \mathscr{X}} \left\{ \begin{array}{c} \xi_{\bar{\Xi}(\Lambda)}(x, y) e^{2\pi i \Phi_{\xi_{\bar{\Xi}(\Lambda)}}(x)}(x, y) \\ \vee \xi_{\bar{\Xi}(\Lambda)}(y) e^{2\pi i \Phi_{\xi_{\bar{\Xi}(\Lambda)}}(x)}(y) \end{array} \right\}; \tag{18}$$

such that $0 \leq \chi_{\underline{\Xi}(\Lambda)}(x) + \xi_{\underline{\Xi}(\Lambda)}(x) \leq 1, 0 \leq \Phi_{\chi_{\underline{\Xi}(\Lambda)}}(x) + \Phi_{\xi_{\underline{\Xi}(\Lambda)}}(x) \leq 1, 0 \leq \chi_{\bar{\Xi}(\Lambda)}(x) + \xi_{\bar{\Xi}(\Lambda)}(x) \leq 1$, and $0 \leq \Phi_{\chi_{\bar{\Xi}(\Lambda)}}(x) + \Phi_{\xi_{\bar{\Xi}(\Lambda)}}(x) \leq 1, \bar{\Xi}(\Lambda)$ and $\underline{\Xi}(\Lambda)$ are Cq-ROFRSs and $\underline{\Xi}(\Lambda), \bar{\Xi}(\Lambda)$: $\wp(x) \longrightarrow \wp(x)$ are lower and upper approximation operators.

Then, the pair

$$\Xi(\Lambda) = \left(\underline{\Xi}(\Lambda), \bar{\Xi}(\Lambda)\right) = \left\{ x, \left(\begin{array}{c} \left(\chi_{\underline{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\chi_{\underline{\Xi}(\Lambda)}}(x)}, \xi_{\underline{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\xi_{\underline{\Xi}(\Lambda)}}(x)}\right), \\ \left(\chi_{\bar{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\chi_{\bar{\Xi}(\Lambda)}}(x)}, \xi_{\bar{\Xi}(\Lambda)}(x) e^{2\pi i \Phi_{\xi_{\bar{\Xi}(\Lambda)}}(x)}\right) \end{array} \right) | x \in \mathcal{X} \right\}, \tag{19}$$

is called Cq-ROFRS. $\Xi(\Lambda)$ is also written as

$$\Xi(\Lambda) = \left(\left\langle \chi_{\underline{\Xi}(\Lambda)} e^{2\pi i \Phi_{\chi_{\underline{\Xi}(\Lambda)}}}, \xi_{\underline{\Xi}(\Lambda)} e^{2\pi i \Phi_{\xi_{\underline{\Xi}(\Lambda)}}} \right\rangle, \left\langle \chi_{\bar{\Xi}(\Lambda)} e^{2\pi i \Phi_{\chi_{\bar{\Xi}(\Lambda)}}}, \xi_{\bar{\Xi}(\Lambda)} e^{2\pi i \Phi_{\xi_{\bar{\Xi}(\Lambda)}}} \right\rangle \right), \tag{20}$$

and called as complex q-rung orthopair fuzzy rough variables (Cq-ROFRV).

3. Hamacher Operations on Complex q-Rung Orthopair Fuzzy Rough Set

We will present some new definitions for Hamacher operations on complex q-rung orthopair fuzzy rough set in this section, which will connect in with the rest of the research in this paper.

Definition 8. $\Xi(\Lambda_2) = (\underline{\Xi}(\Lambda_2), \bar{\Xi}(\Lambda_2))$ be any two complex q-rung orthopair fuzzy rough sets. Then,

$$\Xi(\Lambda_1) \oplus \Xi(\Lambda_2) = \left\{ \begin{array}{c} \left(\left(\sqrt[q]{\dfrac{\chi^q_{\underline{\Xi}(\Lambda_1)} + \chi^q_{\underline{\Xi}(\Lambda_2)} - \chi^q_{\underline{\Xi}(\Lambda_1)}\chi^q_{\underline{\Xi}(\Lambda_2)} - (1-\varrho)\chi^q_{\underline{\Xi}(\Lambda_1)}\chi^q_{\underline{\Xi}(\Lambda_2)}}{1-(1-\varrho)\chi^q_{\underline{\Xi}(\Lambda_1)}\chi^q_{\underline{\Xi}(\Lambda_2)}}}, e^{2\pi i\left(\frac{\chi_{\underline{\Xi}(\Lambda_1)}+\chi_{\underline{\Xi}(\Lambda_2)}-\chi_{\underline{\Xi}(\Lambda_1)}\chi_{\underline{\Xi}(\Lambda_2)}-(1-\varrho)\chi_{\underline{\Xi}(\Lambda_1)}\chi_{\underline{\Xi}(\Lambda_2)}}{1-(1-\varrho)\chi_{\underline{\Xi}(\Lambda_1)}\chi_{\underline{\Xi}(\Lambda_2)}}\right)} \right), \\ \left(\sqrt[q]{\dfrac{\chi^q_{\bar{\Xi}(\Lambda_1)} + \chi^q_{\bar{\Xi}(\Lambda_2)} - \chi^q_{\bar{\Xi}(\Lambda_1)}\chi^q_{\bar{\Xi}(\Lambda_2)} - (1-\varrho)\chi^q_{\bar{\Xi}(\Lambda_1)}\chi^q_{\bar{\Xi}(\Lambda_2)}}{1-(1-\varrho)\chi^q_{\bar{\Xi}(\Lambda_1)}\chi^q_{\bar{\Xi}(\Lambda_2)}}}, e^{2\pi i\left(\frac{\chi_{\bar{\Xi}(\Lambda_1)}+\chi_{\bar{\Xi}(\Lambda_2)}-\chi_{\bar{\Xi}(\Lambda_1)}\chi_{\bar{\Xi}(\Lambda_2)}-(1-\varrho)\chi_{\bar{\Xi}(\Lambda_1)}\chi_{\bar{\Xi}(\Lambda_2)}}{1-(1-\varrho)\chi_{\bar{\Xi}(\Lambda_1)}\chi_{\bar{\Xi}(\Lambda_2)}}\right)} \right) \right), \\ \left(\left(\dfrac{\xi_{\underline{\Xi}(\Lambda_1)}\xi_{\underline{\Xi}(\Lambda_2)}}{\sqrt[q]{\varrho + (1-\varrho)\left(\xi^q_{\underline{\Xi}(\Lambda_1)} + \xi^q_{\underline{\Xi}(\Lambda_2)} - \xi^q_{\underline{\Xi}(\Lambda_1)}\xi^q_{\underline{\Xi}(\Lambda_2)}\right)}}, e^{2\pi i\left(\frac{\xi_{\underline{\Xi}(\Lambda_1)}\xi_{\underline{\Xi}(\Lambda_2)}}{\sqrt[q]{\varrho+(1-\varrho)\left(\xi^q_{\underline{\Xi}(\Lambda_1)}+\xi^q_{\underline{\Xi}(\Lambda_2)}-\xi^q_{\underline{\Xi}(\Lambda_1)}\xi^q_{\underline{\Xi}(\Lambda_2)}\right)}}\right)} \right), \\ \left(\dfrac{\xi_{\bar{\Xi}(\Lambda_1)}\xi_{\bar{\Xi}(\Lambda_2)}}{\sqrt[q]{\varrho + (1-\varrho)\left(\xi^q_{\bar{\Xi}(\Lambda_1)} + \xi^q_{\bar{\Xi}(\Lambda_2)} - \xi^q_{\bar{\Xi}(\Lambda_1)}\xi^q_{\bar{\Xi}(\Lambda_2)}\right)}}, e^{2\pi i\left(\frac{\xi_{\bar{\Xi}(\Lambda_1)}\xi_{\bar{\Xi}(\Lambda_2)}}{\sqrt[q]{\varrho+(1-\varrho)\left(\xi^q_{\bar{\Xi}(\Lambda_1)}+\xi^q_{\bar{\Xi}(\Lambda_2)}-\xi^q_{\bar{\Xi}(\Lambda_1)}\xi^q_{\bar{\Xi}(\Lambda_2)}\right)}}\right)} \right) \end{array} \right\}, \tag{21}$$

$$\lambda\Xi(\Lambda_1) = \left\{ \begin{array}{c} \left(\left(\sqrt[q]{\dfrac{\left(1+(\varrho-1)\chi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda - \left(1-\chi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda}{\left(1+(\varrho-1)\chi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda + (\varrho-1)\left(1-\chi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda}}, e^{2\pi i\left(\frac{\left(1+(\varrho-1)\chi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda-\left(1-\chi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda}{\left(1+(\varrho-1)\chi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda+(\varrho-1)\left(1-\chi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda}\right)} \right), \\ \left(\dfrac{\sqrt[q]{\varrho}\left(\xi_{\underline{\Xi}(\Lambda_1)}\right)^\lambda}{\sqrt[q]{\left(1+(\varrho-1)\left(1-\xi^q_{\underline{\Xi}(\Lambda_1)}\right)\right)^\lambda + (\varrho-1)\left(\xi^q_{\underline{\Xi}(\Lambda_1)}\right)^\lambda}} \right) \right), \\ \left(\left(\sqrt[q]{\dfrac{\left(1+(\varrho-1)\chi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda - \left(1-\chi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda}{\left(1+(\varrho-1)\chi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda + (\varrho-1)\left(1-\chi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda}}, e^{2\pi i\left(\frac{\left(1+(\varrho-1)\chi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda-\left(1-\chi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda}{\left(1+(\varrho-1)\chi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda+(\varrho-1)\left(1-\chi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda}\right)} \right), \\ \left(\dfrac{\sqrt[q]{\varrho}\left(\xi_{\bar{\Xi}(\Lambda_1)}\right)^\lambda}{\sqrt[q]{\left(1+(\varrho-1)\left(1-\xi^q_{\bar{\Xi}(\Lambda_1)}\right)\right)^\lambda + (\varrho-1)\left(\xi^q_{\bar{\Xi}(\Lambda_1)}\right)^\lambda}} \right) \right) \end{array} \right\}. \tag{22}$$

Definition 9. The score function for Cq-ROFRV

$$\Xi(\Lambda) = \left(\left\langle \chi_{\underline{\Xi}(\Lambda)} e^{2\pi i \Phi_{\chi_{\underline{\Xi}(\Lambda)}}}, \xi_{\underline{\Xi}(\Lambda)} e^{2\pi i \Phi_{\xi_{\underline{\Xi}(\Lambda)}}} \right\rangle, \left\langle \chi_{\bar{\Xi}(\Lambda)} e^{2\pi i \Phi_{\chi_{\bar{\Xi}(\Lambda)}}}, \xi_{\bar{\Xi}(\Lambda)} e^{2\pi i \Phi_{\xi_{\bar{\Xi}(\Lambda)}}} \right\rangle \right), \tag{23}$$

is defined as

$$\mathcal{S}(\Xi(\Lambda)) = \frac{1}{4}\left(2 + \chi_{\underline{\Xi}(\Lambda)} + \xi_{\underline{\Xi}(\Lambda)} + \Phi_{\chi_{\underline{\Xi}(\Lambda)}} + \Phi_{\xi_{\underline{\Xi}(\Lambda)}} - \chi_{\bar{\Xi}(\Lambda)} - \xi_{\bar{\Xi}(\Lambda)} - \Phi_{\chi_{\bar{\Xi}(\Lambda)}} - \Phi_{\xi_{\bar{\Xi}(\Lambda)}}\right). \tag{24}$$

4. Complex q-Rung Orthopair Fuzzy Rough Hamacher Average Aggregation Operators

In this part, we will define some new type of Hamacher aggregation operator in Cq-ROF rough environment such as Cq-ROF rough Hamacher weighted average (Cq-ROFRHWA) operator, Cq-ROF rough Hamacher ordered weighted average (Cq-ROFRHOWA) operator, and Cq-ROF rough Hamacher hybrid average (Cq-ROFRHHA) operator. Furthermore, some characteristics related to these operators are also discussed.

4.1. Complex q-Rung Orthopair Fuzzy Rough Hamacher Weighted Average Operator

Definition 10. Let $\Xi(\Lambda_i) = (\underline{\Xi}(\Lambda_i), \bar{\Xi}(\Lambda_i))(i = 1, \cdots, n)$ be any collection of Cq-ROFRSs with weighted vector $\omega =$

$(\omega_1, \omega_2, \cdots, \omega_n)^T$ such that $\sum_{i=1}^{n}\omega_i = 1, 0 \le \omega_i \le 1$. Then, the Cq-ROF rough Hamacher weighted averaging (Cq-ROFRHWA) operator can be defined as follows;

$$\begin{aligned}&\mathrm{Cq-ROFRHWA}(\Xi(\Lambda_1), \Xi(\Lambda_2), \cdots, \Xi(\Lambda_n))\\&\quad= \left(\mathop{\oplus}_{i=1}^{n} \omega_i \underline{\Xi}(\Lambda_i),\ \mathop{\oplus}_{i=1}^{n} \omega_i \bar{\Xi}(\Lambda_i)\right).\end{aligned} \tag{25}$$

Based on the above definition, the aggregated result is illustrated through the following theorem.

Theorem 11. *Let $\Xi(\Lambda_i) = (\underline{\Xi}(\Lambda_i), \bar{\Xi}(\Lambda_i))(i = 1, \cdots, n)$ be any collection of Cq-ROFRSs with weighted vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ such that $\sum_{i=1}^{n}\omega_i = 1, 0 \le \omega_i \le 1$. Then, the Cq-ROF rough Hamacher weighted average (Cq-ROFRHWA) operator can be defined as follows;*

$$\begin{aligned}&Cq-ROFRHWA(\Xi(\Lambda_1), \Xi(\Lambda_2), \cdots, \Xi(\Lambda_n))\\&\quad= \left(\mathop{\oplus}_{i=1}^{n} \omega_i \underline{\Xi}(\Lambda_i),\ \mathop{\oplus}_{i=1}^{n} \omega_i \bar{\Xi}(\Lambda_i)\right)\\&\quad= \left\{\begin{array}{l}\left(\left(\sqrt[q]{\dfrac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} + (\varrho-1)\prod_{i=1}^{n}\left(1-\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}}\ e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} + (\varrho-1)\prod_{i=1}^{n}\left(1-\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}\right)}\right),\\ \left(\dfrac{\sqrt[q]{\varrho}\prod_{i=1}^{n}\left(\xi_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(1+(\varrho-1)\left(1-\xi_{\underline{\Xi}(\Lambda_i)}^{q}\right)\right)^{\omega_i} + (\varrho-1)\prod_{i=1}^{n}\left(\xi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}}\right)\end{array}\right\}\\&\quad= \left\{\begin{array}{l}\left(\left(\sqrt[q]{\dfrac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} + (\varrho-1)\prod_{i=1}^{n}\left(1-\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}}\ e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} + (\varrho-1)\prod_{i=1}^{n}\left(1-\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}\right)}\right),\\ \left(\dfrac{\sqrt[q]{\varrho}\prod_{i=1}^{n}\left(\xi_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(1+(\varrho-1)\left(1-\xi_{\bar{\Xi}(\Lambda_i)}^{q}\right)\right)^{\omega_i} + (\varrho-1)\prod_{i=1}^{n}\left(\xi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i}}}\right)\end{array}\right\}.\end{aligned} \tag{26}$$

The following are some characteristics of Cq-ROFRHWA operator which can be proved very easily.

Theorem 12 (Idempotency). *Let $\Xi(\Lambda_i) = (\underline{\Xi}(\Lambda_i), \bar{\Xi}(\Lambda_i))(i = 1, \cdots, n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega = (\omega_1, \cdots, \omega_n)^T$, such as $\sum_{i=1}^{n}\omega_i = 1$ and $0 \le \omega_i \le 1$. Then,*

$$Cq-ROFRHWA(\Xi(\Lambda_1), \cdots, \Xi(\Lambda_n)) = \Xi(\Lambda). \tag{27}$$

Theorem 13 (Monotonicity). *Let $\Xi(\Lambda_i) = (\underline{\Xi}(\Lambda_i), \bar{\Xi}(\Lambda_i))$ and $\Xi^{/}(\Lambda_i) = (\underline{\Xi}^{/}(\Lambda_i), \bar{\Xi}^{/}(\Lambda_i))(i = 1, \cdots, n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega = (\omega_1, \cdots, \omega_n)^T$, such as $\sum_{i=1}^{n}\omega_i = 1$ and $0 \le \omega_i \le 1$, if $\chi_{\underline{\Xi}(\Lambda)} \ge \chi^{/}_{\underline{\Xi}(\Lambda)}$, $\Phi_{\chi_{\underline{\Xi}(\Lambda)}} \ge \Phi^{/}_{\chi_{\underline{\Xi}(\Lambda)}}$, $\xi_{\underline{\Xi}(\Lambda)} \le \xi^{/}_{\underline{\Xi}(\Lambda)}$, $\Phi_{\xi_{\underline{\Xi}(\Lambda)}} \le \Phi^{/}_{\xi_{\underline{\Xi}(\Lambda)}}$, $\chi_{\bar{\Xi}(\Lambda)} \ge \chi^{/}_{\bar{\Xi}(\Lambda)}$, $\Phi_{\chi_{\bar{\Xi}(\Lambda)}} \ge \Phi^{/}_{\chi_{\bar{\Xi}(\Lambda)}}$, and $\xi_{\bar{\Xi}(\Lambda)} \le \xi^{/}_{\bar{\Xi}(\Lambda)}(u)$, $\Phi_{\xi_{\bar{\Xi}(\Lambda)}} \le \Phi^{/}_{\xi_{\bar{\Xi}(\Lambda)}}$. Then,*

$$\begin{aligned}&Cq-ROFRHWA(\Xi(\Lambda_1), \cdots, \Xi(\Lambda_n))\\&\quad\ge Cq-ROFRHWA\left(\frac{\Xi}{(\Lambda_1)}, \cdots, \frac{\Xi}{(\Lambda_n)}\right).\end{aligned} \tag{28}$$

Theorem 14 (Boundedness). *Let $\Xi(\Lambda_i) = (\underline{\Xi}(\Lambda_i), \bar{\Xi}(\Lambda_i))(i = 1, \cdots, n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega = (\omega_1, \cdots, \omega_n)^T$, such as $\sum_{i=1}^{n}\omega_i = 1$ and $0 \le \omega_i \le 1$, if $\Xi^{+}(\Lambda_i), \Xi^{-}(\Lambda_i)$ are the maximum and minimum Cq-ROFRVs. Then,*

$$\Xi^{+}(\Lambda_i) \le Cq-ROFRHWA(\Xi(\Lambda_1), \cdots, \Xi(\Lambda_n)) \le \Xi^{-}(\Lambda_i). \tag{29}$$

Now, we have studied some of the Cq-ROFRHWA operator's special cases in terms of the parameter.

(1) When $\rho = 1$, Cq-ROFRHWA operator reduces to the Cq-ROFRWA operator

$$\begin{aligned}&\mathrm{Cq-ROFRWA}(\Xi(\Lambda_1), \cdots, \Xi(\Lambda_n))\\&\quad= \left(\mathop{\oplus}_{i=1}^{n} \omega_i \underline{\Xi}(\Lambda_i),\ \mathop{\oplus}_{i=1}^{n} \omega_i \bar{\Xi}(\Lambda_i)\right)\\&\quad= \left\{\begin{array}{l}\left(\left(\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\chi_{\underline{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} e^{2\pi i\left(1-\prod_{i=1}^{n}\left(1-\Phi_{\chi_{\underline{\Xi}(\Lambda_i)}}^{q}\right)^{\omega_i}\right)}}\right),\\ \left(\prod_{i=1}^{n}\left(\xi_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i} e^{2\pi i\left(\prod_{i=1}^{n}\left(\Phi_{\xi_{\underline{\Xi}(\Lambda_i)}}\right)^{\omega_i}\right)}\right)\end{array}\right\},\\&\quad= \left\{\begin{array}{l}\left(\left(\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\chi_{\bar{\Xi}(\Lambda_i)}^{q}\right)^{\omega_i} e^{2\pi i\left(1-\prod_{i=1}^{n}\left(1-\Phi_{\chi_{\bar{\Xi}(\Lambda_i)}}^{q}\right)^{\omega_i}\right)}}\right),\\ \left(\prod_{i=1}^{n}\left(\xi_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i} e^{2\pi i\left(\prod_{i=1}^{n}\left(\Phi_{\xi_{\bar{\Xi}(\Lambda_i)}}\right)^{\omega_i}\right)}\right)\end{array}\right\}.\end{aligned} \tag{30}$$

(2) When $\rho = 2$, Cq-ROFRHWA operator reduces to the Cq-ROFR Einstein weighted average (Cq-ROFREWA) operator

$$\begin{aligned}
&\mathrm{Cq-ROFREWA}(\Xi(\Lambda_1),\Xi(\Lambda_2),\cdots,\Xi(\Lambda_n))\\
&=\left(\overset{n}{\underset{i=1}{\oplus}}\omega_i\underline{\Xi}(\Lambda_i),\ \overset{n}{\underset{i=1}{\oplus}}\omega_i\bar{\Xi}(\Lambda_i)\right)\\
&=\left\{\left(\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left(1+\chi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}}},\ e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+\chi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}}\right)}\right),\right.\right.\\
&\qquad\left.\left.\left(\frac{\sqrt[q]{2}\prod_{i=1}^{n}\left(\xi_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(2-\xi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}+\prod_{i=1}^{n}\left(\xi^q_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i}}}\right)\right)\right.\\
&=\left\{\left(\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left(1+\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}}},\ e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}}\right)}\right),\right.\right.\\
&\qquad\left.\left.\left(\frac{\sqrt[q]{2}\prod_{i=1}^{n}\left(\xi_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(2-\xi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}+\prod_{i=1}^{n}\left(\xi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}}}\right)\right)\right\}.
\end{aligned}\tag{31}$$

4.2. Complex q-Rung Orthopair Fuzzy Rough Hamacher Ordered Weighted Averaging Operator. In this subsection, we will define complex q-rung orthopair fuzzy rough Hamacher ordered weighted average aggregation operator.

Definition 15. Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$ be any collection of Cq-ROFRS with weighted vector $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ such that $\sum_{i=1}^{n}\omega_i=1, 0\le\omega_i\le 1$. Then, the Cq-ROF rough Hamacher ordered weighted average (Cq-ROFRHOWA) operator can be defined as follows;

$$\begin{aligned}
&\mathrm{Cq-ROFRHOWA}(\Xi(\Lambda_1),\Xi(\Lambda_2),\cdots,\Xi(\Lambda_n))\\
&=\left(\overset{n}{\underset{i=1}{\oplus}}\omega_i\underline{\Xi}\left(\Lambda_{\sigma(i)}\right),\ \overset{n}{\underset{i=1}{\oplus}}\omega_i\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)\right),
\end{aligned}\tag{32}$$

where $\sigma(1),\cdots,\sigma(n)$ be the permutation of the $(i=1,\cdots,n)$, for each $\Xi(\Lambda_{(\sigma)(i-1)})\ge\Xi(\Lambda_{\sigma(i)})$.

Based on the above definition, the aggregated result is illustrated through the following theorem.

Theorem 16. *Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$ be any collection of Cq-ROFRSs with weighted vector $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ such that $\sum_{i=1}^{n}\omega_i=1, 0\le\omega_i\le 1$. Then, the Cq-ROF rough Hamacher ordered weighted average (Cq-ROFRHOWA) operator can be defined as follows;*

$$\begin{aligned}
&Cq-ROFRHOWA(\Xi(\Lambda_1),\Xi(\Lambda_2),\cdots,\Xi(\Lambda_n))\\
&=\left(\overset{n}{\underset{i=1}{\oplus}}\omega_i\underline{\Xi}\left(\Lambda_{\sigma(i)}\right),\ \overset{n}{\underset{i=1}{\oplus}}\omega_i\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)\right)\\
&=\left\{\left(\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}},\ e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}\right)}\right),\right.\right.\\
&\qquad\left.\left.\left(\frac{\sqrt[q]{\varrho}\prod_{i=1}^{n}\left(\xi_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(1+(\varrho-1)\left(1-\xi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(\xi^q_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}\right)\right)\right.\\
&=\left\{\left(\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}},\ e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}\right)}\right),\right.\right.\\
&\qquad\left.\left.\left(\frac{\sqrt[q]{\varrho}\prod_{i=1}^{n}\left(\xi_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(1+(\varrho-1)\left(1-\xi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(\xi^q_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}\right)\right)\right\},
\end{aligned}\tag{33}$$

where $\sigma(1),\cdots,\sigma(n)$ be the permutation of the $(i=1,\cdots,n)$, for each $\Xi(\Lambda_{(\sigma)(i-1)})\ge\Xi(\Lambda_{\sigma(i)})$.

The following are some characteristics of Cq-ROFRHOWA operator which can be proved very easily.

Theorem 17 (Idempotency). *Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega=(\omega_1,\cdots,\omega_n)^T$, such as $\sum_{i=1}^{n}\omega_i=1$ and $0\le\omega_i\le 1$. Then,*

$$Cq-ROFRHOWA(\Xi(\Lambda_1),\cdots,\Xi(\Lambda_n))=\Xi(\Lambda).\tag{34}$$

Theorem 18 (Monotonicity). *Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))$ and $\Xi^{/}(\Lambda_i)=(\underline{\Xi}^{/}(\Lambda_i),\bar{\Xi}^{/}(\Lambda_i))(i=1,\cdots,n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega=(\omega_1,\cdots,\omega_n)^T$, such as $\sum_{i=1}^{n}\omega_i=1$ and $0\le\omega_i\le 1$, if $\chi_{\underline{\Xi}(\Lambda)}\ge\chi^{/}_{\underline{\Xi}(\Lambda)}$, $\Phi_{\chi_{\underline{\Xi}(\Lambda)}}\ge\Phi^{/}_{\chi_{\underline{\Xi}(\Lambda)}}$, $\xi_{\underline{\Xi}(\Lambda)}\le\xi^{/}_{\underline{\Xi}(\Lambda)}$, $\Phi_{\xi_{\underline{\Xi}(\Lambda)}}\le\Phi^{/}_{\xi_{\underline{\Xi}(\Lambda)}}$, $\chi_{\bar{\Xi}(\Lambda)}\ge\chi^{/}_{\bar{\Xi}(\Lambda)}$, $\Phi_{\chi_{\bar{\Xi}(\Lambda)}}\ge\Phi^{/}_{\chi_{\bar{\Xi}(\Lambda)}}$, $\xi_{\bar{\Xi}(\Lambda)}\le\xi^{/}_{\bar{\Xi}(\Lambda)}(u)$, and $\Phi_{\xi_{\bar{\Xi}(\Lambda)}}\le\Phi^{/}_{\xi_{\bar{\Xi}(\Lambda)}}$.*

Then,

$$\begin{aligned} &Cq-ROFRHOWA(\Xi(\Lambda_1),\cdots,\Xi(\Lambda_n)) \\ &\quad \geq Cq-ROFRHOWA\left(\Xi'(\Lambda_1),\cdots,\Xi'(\Lambda_n)\right). \end{aligned} \tag{35}$$

Theorem 19 (Boundedness). *Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega=(\omega_1,\cdots,\omega_n)^T$, such as $\sum_{i=1}^n\omega_i=1$ and $0\leq\omega_i\leq 1$, if $\Xi^+(\Lambda_i),\Xi^-(\Lambda_i)$ is the maximum and minimum Cq-ROFRVs. Then,*

$$\Xi^+(\Lambda_i)\leq Cq-ROFRHOWA(\Xi(\Lambda_1),\cdots,\Xi(\Lambda_n))\leq\Xi^-(\Lambda_i). \tag{36}$$

Now, we have studied some of the Cq-ROFRHOWA operator's special cases in terms of the parameter.

(1) When $\varrho=1$, Cq-ROFRHOWA operator reduces to the Cq-ROFROWA operator

$$\begin{aligned} &\mathrm{Cq-ROFROWA}(\Xi(\Lambda_1),\cdots,\Xi(\Lambda_n)) \\ &=\left(\mathop{\oplus}_{i=1}^{n}\omega_i\underline{\Xi}\left(\Lambda_{\sigma(i)}\right),\ \mathop{\oplus}_{i=1}^{n}\omega_i\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)\right) \\ &=\left\{\begin{array}{c}\left(\left(\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}\ e^{2\pi i\left(1-\prod_{i=1}^{n}\left(1-\Psi^q_{\chi_{\underline{\Xi}(\Lambda_{\sigma(i)})}}\right)^{\omega_i}\right)}\right),\right. \\ \left.\left(\prod_{i=1}^{n}\left(\xi_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}e^{2\pi i\left(\prod_{i=1}^{n}\left(\Psi_{\xi_{\underline{\Xi}(\Lambda_{\sigma(i)})}}\right)^{\omega_i}\right)}\right)\right) \\ \left(\left(\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}\ e^{2\pi i\left(1-\prod_{i=1}^{n}\left(1-\Psi^q_{\chi_{\bar{\Xi}(\Lambda_{\sigma(i)})}}\right)^{\omega_i}\right)}\right),\right. \\ \left.\left(\prod_{i=1}^{n}\left(\xi_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}e^{2\pi i\left(\prod_{i=1}^{n}\left(\Psi_{\xi_{\bar{\Xi}(\Lambda_{\sigma(i)})}}\right)^{\omega_i}\right)}\right)\right)\end{array}\right\}. \end{aligned} \tag{37}$$

(2) When $\varrho=2$, Cq-ROFRHOWA operator reduces to the Cq-ROFR Einstein ordered weighted average (Cq-ROFREOWA) operator

$$\begin{aligned} &\mathrm{Cq-ROFREOWA}(\Xi(\Lambda_1),\Xi(\Lambda_2),\cdots,\Xi(\Lambda_n)) \\ &=\left(\mathop{\oplus}_{i=1}^{n}\omega_i\underline{\Xi}\left(\Lambda_{\sigma(i)}\right),\ \mathop{\oplus}_{i=1}^{n}\omega_i\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)\right) \\ &=\left\{\begin{array}{c}\left(\left(\sqrt[q]{\dfrac{\prod_{i=1}^{n}\left(1+\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}}\ e^{2\pi i\left(\dfrac{\prod_{i=1}^{n}\left(1+\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\chi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}\right)}\right),\right. \\ \left.\left(\dfrac{\sqrt[q]{2}\prod_{i=1}^{n}\left(\xi_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(2-\xi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}+\prod_{i=1}^{n}\left(\xi^q_{\underline{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}}\right)\right) \\ \left(\left(\sqrt[q]{\dfrac{\prod_{i=1}^{n}\left(1+\chi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}}\ e^{2\pi i\left(\dfrac{\prod_{i=1}^{n}\left(1+\chi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^q_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}+\prod_{i=1}^{n}\left(1-\chi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}\right)}\right),\right. \\ \left.\left(\dfrac{\sqrt[q]{2}\prod_{i=1}^{n}\left(\xi_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(2-\xi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}+\prod_{i=1}^{n}\left(\xi^q_{\bar{\Xi}(\Lambda_{\sigma(i)})}\right)^{\omega_i}}}\right)\right)\end{array}\right\}. \end{aligned} \tag{38}$$

4.3. Complex q-Rung Orthopair Fuzzy Rough Hamacher Hybrid Weighted Average Operator. Next, we will defined complex q-rung orthopair fuzzy rough Hamacher ordered averaging aggregation operator.

Definition 20. Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$ be any collection of Cq-ROFRSs with weighted vector $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ such that $\sum_{i=1}^n\omega_i=1, 0\leq\omega_i\leq 1$. Then, the Cq-ROF rough Hamacher hybrid average (Cq-ROFRHHA) operator can be defined as follows;

$$\begin{aligned} &\mathrm{Cq-ROFRHHA}(\Xi(\Lambda_1),\Xi(\Lambda_2),\cdots,\Xi(\Lambda_n)) \\ &=\left(\mathop{\oplus}_{i=1}^{n}\omega_i\underline{\Xi}^*\left(\Lambda_{\sigma(i)}\right),\ \mathop{\oplus}_{i=1}^{n}\omega_i\bar{\Xi}^*\left(\Lambda_{\sigma(i)}\right)\right), \end{aligned} \tag{39}$$

where $\sigma(1),\cdots,\sigma(n)$ be the permutation of the $(i=1,\cdots,n)$, for each $\Xi(\Lambda_{(\sigma)(i-1)})\geq\Xi(\Lambda_{\sigma(i)})$, and $\Xi^*(\Lambda_{\sigma(i)})=(nw_i)\Xi(\Lambda_i)=(nw_i\underline{\Xi}(\Lambda_i),nw_i\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$, represents the largest value of permutation from the family of Cq-ROFRVs with $\sum_{i=1}^n w_i=1, 0\leq w_i\leq 1$, and n denotes the balancing coefficient.

Based on the above definition, the aggregated result is illustrated through the following theorem.

Theorem 21. *Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$ be any collection of Cq-ROFRSs with weighted vector $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ such that $\sum_{i=1}^n\omega_i=1, 0\leq\omega_i\leq 1$. Then, the*

Cq-ROF rough Hamacher hybrid average (Cq-ROFRHHA) operator can be defined as follows;

$$
\begin{aligned}
&Cq-ROFRHHA(\Xi(\Lambda_1),\Xi(\Lambda_2),\cdots,\Xi(\Lambda_n))\\
&\quad=\left(\underset{i=1}{\overset{n}{\oplus}}\omega_i\underline{\Xi}^*\left(\Lambda_{\sigma(i)}\right),\ \underset{i=1}{\overset{n}{\oplus}}\omega_i\bar{\Xi}^*\left(\Lambda_{\sigma(i)}\right)\right)\\
&\quad=\left\{\begin{array}{c}
\left(\left(\sqrt[q]{\dfrac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}\ ,\ e^{2\pi i\left(\dfrac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}\right)}\right),\\
\left(\dfrac{\sqrt[q]{\varrho}\prod_{i=1}^{n}\left(\xi^{*}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(1+(\varrho-1)\left(1-\xi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(\xi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}\right)
\end{array}\right\},\\
&\quad=\left\{\begin{array}{c}
\left(\left(\sqrt[q]{\dfrac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{*q}_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}\ ,\ e^{2\pi i\left(\dfrac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}-\prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{*q}_{\bar{\Xi}(\Lambda_i)}\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}\right)}\right),\\
\left(\dfrac{\sqrt[q]{\varrho}\prod_{i=1}^{n}\left(\xi^{*}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(1+(\varrho-1)\left(1-\xi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)\right)^{\omega_i}+(\varrho-1)\prod_{i=1}^{n}\left(\xi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}\right)
\end{array}\right\},
\end{aligned}
\tag{40}
$$

where $\sigma(1),\cdots,\sigma(n)$ be the permutation of the $(i=1,\cdots,n)$, for each $\Xi(\Lambda_{(\sigma)(i-1)})\geq\Xi(\Lambda_{\sigma(i)})$ for all $(i=1,\cdots,n)$, and $\Xi^*(\Lambda_{\sigma(i)})=(nw_i)\Xi(\Lambda_i)=(nw_i\underline{\Xi}(\Lambda_i),nw_i\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$, represents the largest value of permutation from the family of Cq-ROFRVs with $\sum_{i=1}^{n}w_i=1$, $0\leq w_i\leq 1$, and n denotes the balancing coefficient.

The following are some characteristics of Cq-ROFRHHA operator which can be proved very easily.

Theorem 22 (Idempotency). *Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega=(\omega_1,\cdots,\omega_n)^T$, such as $\sum_{i=1}^{n}\omega_i=1$ and $0\leq\omega_i\leq 1$. Then,*

$$Cq-ROFRHHWA(\Xi(\Lambda_1),\cdots,\Xi(\Lambda_n))=\Xi(\Lambda). \tag{41}$$

Theorem 23 (Monotonicity). *Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))$ and $\Xi^{/}(\Lambda_i)=(\underline{\Xi}^{/}(\Lambda_i),\bar{\Xi}^{/}(\Lambda_i))(i=1,\cdots,n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega=(\omega_1,\cdots,\omega_n)^T$, such as $\sum_{i=1}^{n}\omega_i=1$ and $0\leq\omega_i\leq 1$, if $\chi_{\underline{\Xi}(\Lambda)}\geq\chi^{/}_{\underline{\Xi}(\Lambda)}$, $\Phi_{\chi_{\underline{\Xi}(\Lambda)}}\geq\Phi^{/}_{\chi_{\underline{\Xi}(\Lambda)}}$, $\xi_{\underline{\Xi}(\Lambda)}\leq\xi^{/}_{\underline{\Xi}(\Lambda)}$, $\Phi_{\xi_{\underline{\Xi}(\Lambda)}}\leq\Phi^{/}_{\xi_{\underline{\Xi}(\Lambda)}}$, $\chi_{\bar{\Xi}(\Lambda)}\geq\chi^{/}_{\bar{\Xi}(\Lambda)}$, $\Phi_{\chi_{\bar{\Xi}(\Lambda)}}\geq\Phi^{/}_{\chi_{\bar{\Xi}(\Lambda)}}$, $\xi_{\bar{\Xi}(\Lambda)}\leq\xi^{/}_{\bar{\Xi}(\Lambda)}(u)$, and $\Phi_{\xi_{\bar{\Xi}(\Lambda)}}\leq\Phi^{/}_{\xi_{\bar{\Xi}(\Lambda)}}$. Then,*

$$
\begin{aligned}
&Cq-ROFRHHA(\Xi(\Lambda_1),\cdots,\Xi(\Lambda_n))\\
&\quad\geq Cq-ROFRHHA\left(\Xi^{/}(\Lambda_1),\cdots,\Xi^{/}(\Lambda_n)\right).
\end{aligned}
\tag{42}
$$

Theorem 24 (Boundedness). *Let $\Xi(\Lambda_i)=(\underline{\Xi}(\Lambda_i),\bar{\Xi}(\Lambda_i))(i=1,\cdots,n)$ be the set of Cq-ROFRVs in universal set $\mathcal{X}$ with the weight vector $\omega=(\omega_1,\cdots,\omega_n)^T$, such as $\sum_{i=1}^{n}\omega_i=1$ and $0\leq\omega_i\leq 1$, if $\Xi^+(\Lambda_i),\Xi^-(\Lambda_i)$ is the maximum and minimum Cq-ROFRVs. Then,*

$$\Xi^+(\Lambda_i)\leq Cq-ROFRHHA(\Xi(\Lambda_1),\cdots,\Xi(\Lambda_n))\leq\Xi^-(\Lambda_i). \tag{43}$$

Now, we have studied some of the Cq-ROFRHHA operator's special cases in terms of the parameter.

(1) When $\rho=1$, Cq-ROFRHHA operator reduces to the Cq-ROFRHA operator

$$
\begin{aligned}
&\mathrm{Cq-ROFRHA}(\Xi(\Lambda_1),\cdots,\Xi(\Lambda_n))\\
&\quad=\left(\underset{i=1}{\overset{n}{\oplus}}\omega_i\underline{\Xi}^*\left(\Lambda_{\sigma(i)}\right),\ \underset{i=1}{\overset{n}{\oplus}}\omega_i\bar{\Xi}^*\left(\Lambda_{\sigma(i)}\right)\right)\\
&\quad=\left\{\begin{array}{c}
\left(\left(\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}e^{2\pi i\left(1-\prod_{i=1}^{n}\left(1-\Phi^{*q}_{\chi_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}}\right)^{\omega_i}\right)}}\right),\\
\left(\prod_{i=1}^{n}\left(\xi^{*}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}e^{2\pi i\left(\prod_{i=1}^{n}\left(\Phi^{*}_{\xi_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}}\right)^{\omega_i}\right)}\right)
\end{array}\right\},\\
&\quad=\left\{\begin{array}{c}
\left(\left(\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}e^{2\pi i\left(1-\prod_{i=1}^{n}\left(1-\Phi^{*q}_{\chi_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}}\right)^{\omega_i}\right)}}\right),\\
\left(\prod_{i=1}^{n}\left(\xi^{*}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}e^{2\pi i\left(\prod_{i=1}^{n}\left(\Phi^{*}_{\xi_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}}\right)^{\omega_i}\right)}\right)
\end{array}\right\}.
\end{aligned}
\tag{44}
$$

(2) When $\rho=2$, Cq-ROFRHHA operator reduces to the Cq-ROFR Einstein hybrid average (Cq-ROFREHA) operator

$$
\begin{aligned}
&\mathrm{Cq-ROFREHA}(\Xi(\Lambda_1), \Xi(\Lambda_2), \cdots, \Xi(\Lambda_n)) \\
&= \left(\mathop{\oplus}_{i=1}^{n} \omega_i \underline{\Xi}^*\left(\Lambda_{\sigma(i)}\right), \mathop{\oplus}_{i=1}^{n} \omega_i \bar{\Xi}^*\left(\Lambda_{\sigma(i)}\right)\right) \\
&= \left\{\left(\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left(1+\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i} + \prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}, e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i} + \prod_{i=1}^{n}\left(1-\chi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}\right)}\right), \right. \\
&\qquad \left.\left(\frac{\sqrt[q]{2}\prod_{i=1}^{n}\left(\xi^{*}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(2-\xi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i} + \prod_{i=1}^{n}\left(\xi^{*q}_{\underline{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}\right)\right\} \\
&= \left\{\left(\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left(1+\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^{*q}_{\bar{\Xi}\left(\Lambda_{i}\right)}\right)^{\omega_i} + \prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}, e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\prod_{i=1}^{n}\left(1+\chi^{*q}_{\bar{\Xi}\left(\Lambda_{i}\right)}\right)^{\omega_i} + \prod_{i=1}^{n}\left(1-\chi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}\right)}\right), \right. \\
&\qquad \left.\left(\frac{\sqrt[q]{2}\prod_{i=1}^{n}\left(\xi^{*}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}{\sqrt[q]{\prod_{i=1}^{n}\left(2-\xi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i} + \prod_{i=1}^{n}\left(\xi^{*q}_{\bar{\Xi}\left(\Lambda_{\sigma(i)}\right)}\right)^{\omega_i}}}\right)\right\}.
\end{aligned}
\tag{45}
$$

5. Modified EDAS Method for MCGDM Based on Complex q-Rung Orthopair Fuzzy Rough Hamacher Aggregation Operators

In this section, we design an algorithm using the proposed complex q-rung orthopair fuzzy rough set and EDAS method to address the classical MCGDM problem.

This part shell developed a MCGDM method using the defined operator with the complex q-rung orthopair fuzzy rough environment. For conventional MCGDM problem, assume that $\wp = \{\wp_1, \cdots, \wp_m\}$ be a set of m alternatives and $\mathbb{Z} = \{\mathbb{Z}_1, \cdots, \mathbb{Z}_n\}$ be the set of n criteria. Let $E = \{E_1, \cdots, E_d\}$ be a set of d experts, who give their evaluation assessment for every alternative $\wp_i (i = 1, \cdots, m)$ against their criteria $\mathbb{Z}_j (1, \cdots, n)$. Let $\omega = (\omega_1, \cdots, \omega_n)^T$ be the weights for criteria $\mathbb{Z}_j$ and $\psi = (\psi_1, \cdots, \psi_d)^T$ be the weights for expert $E_l (l = 1, \cdots, t)$, such as $\sum_{j=1}^{n} \omega_j, \sum_{l=1}^{d} \psi_l$ and $0 \le \omega_j, \psi_l \le 1$. The main steps for algorithm are stated as follows.

Step 1. Collect the assessment data against their criteria $\mathbb{Z}_j$ for competent experts for each alternative $\wp_i$ and create a decision matrix that is given;

$$
M = \left[\Xi\left(\Lambda_{ij}^{l}\right)\right]_{m\times n}. \tag{46}
$$

Step 2. The collective expert data are aggregated against their weights by using the suggested method to obtain the aggregated decision matrix as

$$
M = \left[\Xi\left(\Lambda_{ij}\right)\right]_{m\times n}. \tag{47}
$$

Step 3. The decision matrix can be normalized by the following transformation and written as $M = [\Xi(\Lambda_{ij})]_{m\times n}$ to $M^n = [\Xi(\Lambda_{ij}^n)]_{m\times n}$.

$$
M^n = \begin{cases}
\Xi\left(\Lambda_{ij}\right) = \left(\left\langle \chi_{\underline{\Xi}(\Lambda)} e^{2\pi i \mp \chi_{\underline{\Xi}(\Lambda)}}, \xi_{\underline{\Xi}(\Lambda)} e^{2\pi i \mp \xi_{\underline{\Xi}(\Lambda)}} \right\rangle, \left\langle \chi_{\bar{\Xi}(\Lambda)} e^{2\pi i \mp \chi_{\bar{\Xi}(\Lambda)}}, \xi_{\bar{\Xi}(\Lambda)} e^{2\pi i \mp \xi_{\bar{\Xi}(\Lambda)}} \right\rangle\right) \text{ for benefit type,} \\
\Xi\left(\Lambda_{ij}\right) = \left(\left\langle \chi_{\bar{\Xi}(\Lambda)} e^{2\pi i \mp \chi_{\bar{\Xi}(\Lambda)}}, \xi_{\bar{\Xi}(\Lambda)} e^{2\pi i \mp \xi_{\bar{\Xi}(\Lambda)}} \right\rangle, \left\langle \chi_{\underline{\Xi}(\Lambda)} e^{2\pi i \mp \chi_{\underline{\Xi}(\Lambda)}}, \xi_{\underline{\Xi}(\Lambda)} e^{2\pi i \mp \xi_{\underline{\Xi}(\Lambda)}} \right\rangle\right) \text{ for cost type.}
\end{cases}
\tag{48}
$$

Step 4. Find the value of $Av\mathcal{S}$ for all alternatives by using the proposed approaches.

$$
\begin{aligned}
Av\mathcal{S} &= \left[Av\mathcal{S}_j\right]_{1\times n} = \left[\frac{1}{m}\sum_{i=1}^{n}\Xi\left(\Lambda_{ij}^{n}\right)\right]_{1\times n} \\
&= \left[\left(\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m} - \prod_{i=1}^{n}\left(1-\chi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m} + (\varrho-1)\prod_{i=1}^{n}\left(1-\chi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m}}}, e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{\omega_i} - \prod_{i=1}^{n}\left(1-\chi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m} + (\varrho-1)\prod_{i=1}^{n}\left(1-\chi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m}}\right)}\right), \right. \\
&\qquad \left.\left(\frac{\sqrt[qn]{\varrho}\prod_{i=1}^{n}\left(\xi^{n}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m}}{\sqrt[q]{\prod_{i=1}^{n}\left(1+(\varrho-1)\left(1-\xi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)\right)^{1/m} + (\varrho-1)\prod_{i=1}^{n}\left(\xi^{qn}_{\underline{\Xi}(\Lambda_i)}\right)^{1/m}}}\right)\right] \\
&\cdot \left[\left(\left(\sqrt[q]{\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m} - \prod_{i=1}^{n}\left(1-\chi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m} + (\varrho-1)\prod_{i=1}^{n}\left(1-\chi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m}}}, e^{2\pi i\left(\frac{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m} - \prod_{i=1}^{n}\left(1-\chi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m}}{\prod_{i=1}^{n}\left(1+(\varrho-1)\chi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m} + (\varrho-1)\prod_{i=1}^{n}\left(1-\chi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m}}\right)}\right), \right. \\
&\qquad \left.\left(\frac{\sqrt[qn]{\varrho}\prod_{i=1}^{n}\left(\xi^{n}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m}}{\sqrt[q]{\prod_{i=1}^{n}\left(1+(\varrho-1)\left(1-\xi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)\right)^{1/m} + (\varrho-1)\prod_{i=1}^{n}\left(\xi^{qn}_{\bar{\Xi}(\Lambda_i)}\right)^{1/m}}}\right)\right]_{1\times n}.
\end{aligned}
\tag{49}
$$

Step 5. Using the determined value of $Av\mathcal{S}$, we find the value of $PDA\mathcal{S}$ and $NDA\mathcal{S}$, using the following formula;

$$PDA\mathcal{S}_{ij} = \left[PDA\mathcal{S}_{ij}\right]_{m\times n} = \frac{\max\left(0, \left[\mathcal{S}\left(\Xi\left(\Lambda_{ij}^{n}\right)\right) - \mathcal{S}\left(Av\mathcal{S}_{j}\right)\right]\right)}{\mathcal{S}\left(Av\mathcal{S}_{j}\right)}, \tag{50}$$

$$NDA\mathcal{S}_{ij} = \left[NDA\mathcal{S}_{ij}\right]_{m\times n} = \frac{\max\left(0, \left[\mathcal{S}\left(Av\mathcal{S}_{j}\right) - \mathcal{S}\left(\Xi\left(\Lambda_{ij}^{n}\right)\right)\right]\right)}{\mathcal{S}\left(Av\mathcal{S}_{j}\right)}. \tag{51}$$

Step 6. In this step, we find the positive ($\mathcal{S}P_i$) and negative weight distance ($\mathcal{S}N_i$).

$$\mathcal{S}P_i = \sum_{j=1}^{n} \omega_j PDA\mathcal{S}_{ij}, \tag{52}$$

$$\mathcal{S}N_i = \sum_{j=1}^{n} \omega_j NDA\mathcal{S}_{ij}. \tag{53}$$

Step 7. Find the value $N\mathcal{S}P_i$ and $N\mathcal{S}N_i$ by using Equations (53) and (54);

$$N\mathcal{S}P_i = \frac{\mathcal{S}P_i}{\max_i(\mathcal{S}P_i)}, \tag{54}$$

and

$$N\mathcal{S}N_i = 1 - \frac{\mathcal{S}N_i}{\max_i(\mathcal{S}N_i)}. \tag{55}$$

Step 8. Using $N\mathcal{S}P_i$ and $N\mathcal{S}N_i$ values, find the value of $A\mathcal{S}$ (appraisal score) by the following equation:

$$A\mathcal{S}_i = \frac{1}{2}\left(N\mathcal{S}P_i + N\mathcal{S}N_i\right). \tag{56}$$

Step 9. Give ranking to alternatives based on the value of $A\mathcal{S}_i$.

6. Example

An arboretum is a large area designated for the cultivation and effective display of a wide variety of valuable ornamental trees, shrubs, wild, medicinal, herbaceous, and other plants that can be cultivated in the given area, as well as plant maintenance, proper labelling, and research. These are important centers for systematic research and study. They make it easier for researchers to work on the region's flora, as well as plant hybridization and propagation.

Plant diversity and many species have been harmed and forced to face extinction as a result of forest destruction and overuse. As a result, the need for these types of gardens will grow as plants and trees play an increasingly important role in the ecosystem. Furthermore, these are the locations where plants will be preserved for future generations. To demonstrate the importance of the proposed model in MCGDM, we will use the selection of the best location for an arboretum as an example (botanical garden).

Assume the government wishes to create an arboretum in the country. Four sites have been chosen for this purpose, and they will be further evaluated to determine the best location for the garden. The proposed model will be used to carry out this evaluation. To appraise these four areas, they invite a team of three professional experts $E_k(k = 1, 2, 3)$ with an associated weight vector $\psi = (0.2, 0.3, 0.5)^T$, and they will evaluate these sites according to the three criteria: $\mathbb{Z}_1$ = soil acidity, $\mathbb{Z}_2$ = size of the area, $\mathbb{Z}_3$ = environmental benefit, and $\mathbb{Z}_4$ = economic benefits.

The weight vector for the above criteria is given as $\omega = (0.32, 0.23, 0.21, 0.24)^T$. Each expert $E_k(k = 1, 2, 3)$ gives the assessment report for each $\wp_i(i = 1, 2, 3, 4)$ according to their corresponding criteria in the form of Cq-ROFRVs. Further, we will use the proposed aggregation operators (Cq-ROFRHWA, Cq-ROFRHOWA, and Cq-ROFRHHA) and the above stepwise algorithm of the EDAS method to find the best optimal site for the arboretum amongst the alternatives.

Step 1. The collective information given by every expert for the alternative $\wp_i$ with criteria $\mathbb{Z}_j$ are given in Tables 1–3.

Step 2. Table 4 shows the results of aggregating the information provided by experts against their weights using the Cq-ROFRHWA operators.

Step 3. Normalization is not required because all of the criteria are of the same type (benefit).

Step 4. Calculate the Av$\mathcal{S}$ value for each alternative using the proposed approach and the criteria listed in Table 5.

Step 5. Using the determined value of Av$\mathcal{S}$ from the Table 5, to find the scores of Av$\mathcal{S}_i(i = 1, \cdots, 4)$ as

$$Av\mathcal{S}_1 = 0.3441, Av\mathcal{S}_2 = 0.5320, Av\mathcal{S}_3 = 0.2425, Av\mathcal{S}_4 = 0.3541. \tag{57}$$

Now, find the PDA$\mathcal{S}$ value as given in Table 6.

Now, find the NDA$\mathcal{S}$ value as given in Table 7.

Step 6. In this step, find the $\mathcal{S}P_i$ and $\mathcal{S}N_i$ by utilizing criteria weights $\omega = (0.16, 0.28, 0.36, 0.20)^T$ is given in Table 8.

Step 7. Now to normalize the $\mathcal{S}P_i$ and $\mathcal{S}N_i$, we have

$$N\mathcal{S}P_1 = 1.00, N\mathcal{S}P_2 = 0.2610, N\mathcal{S}P_3 = 0.3523, N\mathcal{S}P_4 = 0.3734, \tag{58}$$

$$N\mathcal{S}N_1 = 1.00, N\mathcal{S}N_2 = 0.3420, N\mathcal{S}N_3 = 0.2410, N\mathcal{S}N_4 = 0.1903. \tag{59}$$

Table 1: Cq-ROFR information given by expert E^1.

	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$
$\wp_1$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.7)}, 0.5e^{2\pi i(0.6)}\rangle, \\ \langle 0.5e^{2\pi i(0.8)}, 0.3e^{2\pi i(0.3)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.7)}, 0.6e^{2\pi i(0.5)}\rangle, \\ \langle 0.3e^{2\pi i(0.4)}, 0.7e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3e^{2\pi i(0.7)}, 0.7e^{2\pi i(0.5)}\rangle, \\ \langle 0.9e^{2\pi i(0.5)}, 0.5e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.4)}, 0.2e^{2\pi i(0.7)}\rangle, \\ \langle 0.5e^{2\pi i(0.8)}, 0.8e^{2\pi i(0.3)}\rangle\end{pmatrix}$
$\wp_2$	$\begin{pmatrix}\langle 0.3e^{2\pi i(0.8)}, 0.7e^{2\pi i(0.5)}\rangle, \\ \langle 0.6e^{2\pi i(0.7)}, 0.5e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.3)}, 0.7e^{2\pi i(0.4)}\rangle, \\ \langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.6)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.3)}, 0.7e^{2\pi i(0.4)}\rangle, \\ \langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.6)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4e^{2\pi i(0.5)}, 0.6e^{2\pi i(0.8)}\rangle, \\ \langle 0.7e^{2\pi i(0.6)}, 0.5e^{2\pi i(0.2)}\rangle\end{pmatrix}$
$\wp_3$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.7)}, 0.6e^{2\pi i(0.5)}\rangle, \\ \langle 0.3e^{2\pi i(0.8)}, 0.7e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.5)}, 0.8e^{2\pi i(0.2)}\rangle, \\ \langle 0.3e^{2\pi i(0.8)}, 0.7e^{2\pi i(0.5)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4e^{2\pi i(0.5)}, 0.6e^{2\pi i(0.6)}\rangle, \\ \langle 0.5e^{2\pi i(0.6)}, 0.7e^{2\pi i(0.3)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.6)}, 0.1e^{2\pi i(0.4)}\rangle, \\ \langle 0.6e^{2\pi i(0.5)}, 0.5e^{2\pi i(0.6)}\rangle\end{pmatrix}$
$\wp_4$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.8)}\rangle, \\ \langle 0.5e^{2\pi i(0.9)}, 0.6e^{2\pi i(0.4)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3e^{2\pi i(0.4)}, 0.6e^{2\pi i(0.7)}\rangle, \\ \langle 0.5e^{2\pi i(0.2)}, 0.4e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.5)}, 0.8e^{2\pi i(0.7)}\rangle, \\ \langle 0.9e^{2\pi i(0.2)}, 0.4e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.5)}\rangle, \\ \langle 0.5e^{2\pi i(0.9)}, 0.6e^{2\pi i(0.4)}\rangle\end{pmatrix}$

Table 2: Cq-ROFR evaluation information given by expert E^2.

	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$
$\wp_1$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.5)}, 0.8e^{2\pi i(0.7)}\rangle, \\ \langle 0.3e^{2\pi i(0.8)}, 0.4e^{2\pi i(0.5)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.5e^{2\pi i(0.4)}, 0.7e^{2\pi i(0.6)}\rangle, \\ \langle 0.5e^{2\pi i(0.8)}, 0.4e^{2\pi i(0.3)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3e^{2\pi i(0.7)}, 0.7e^{2\pi i(0.5)}\rangle, \\ \langle 0.9e^{2\pi i(0.5)}, 0.5e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.5)}, 0.8e^{2\pi i(0.7)}\rangle, \\ \langle 0.2e^{2\pi i(0.2)}, 0.4e^{2\pi i(0.8)}\rangle\end{pmatrix}$
$\wp_2$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.7)}, 0.6e^{2\pi i(0.5)}\rangle, \\ \langle 0.3e^{2\pi i(0.4)}, 0.7e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.7)}, 0.5e^{2\pi i(0.6)}\rangle, \\ \langle 0.5e^{2\pi i(0.8)}, 0.8e^{2\pi i(0.3)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.4)}, 0.7e^{2\pi i(0.5)}\rangle, \\ \langle 0.8e^{2\pi i(0.3)}, 0.3e^{2\pi i(0.9)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.7)}, 0.5e^{2\pi i(0.6)}\rangle, \\ \langle 0.3e^{2\pi i(0.8)}, 0.6e^{2\pi i(0.5)}\rangle\end{pmatrix}$
$\wp_3$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.8)}\rangle, \\ \langle 0.5e^{2\pi i(0.9)}, 0.6e^{2\pi i(0.4)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4e^{2\pi i(0.6)}, 0.8e^{2\pi i(0.4)}\rangle, \\ \langle 0.6e^{2\pi i(0.5)}, 0.5e^{2\pi i(0.6)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.6)}, 0.8e^{2\pi i(0.4)}\rangle, \\ \langle 0.6e^{2\pi i(0.5)}, 0.5e^{2\pi i(0.6)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.5)}, 0.4e^{2\pi i(0.7)}\rangle, \\ \langle 0.5e^{2\pi i(0.4)}, 0.3e^{2\pi i(0.8)}\rangle\end{pmatrix}$
$\wp_4$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.3)}, 0.7e^{2\pi i(0.4)}\rangle, \\ \langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.6)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.7)}, 0.6e^{2\pi i(0.5)}\rangle, \\ \langle 0.3e^{2\pi i(0.8)}, 0.7e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.4)}, 0.6e^{2\pi i(0.7)}\rangle, \\ \langle 0.5e^{2\pi i(0.2)}, 0.4e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.5)}\rangle, \\ \langle 0.5e^{2\pi i(0.9)}, 0.6e^{2\pi i(0.4)}\rangle\end{pmatrix}$

Table 3: Cq-ROFR evaluation information given by expert E^3.

	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$
$\wp_1$	$\begin{pmatrix}\langle 0.3e^{2\pi i(0.7)}, 0.7e^{2\pi i(0.5)}\rangle, \\ \langle 0.4e^{2\pi i(0.5)}, 0.5e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.3e^{2\pi i(0.8)}, 0.7e^{2\pi i(0.5)}\rangle, \\ \langle 0.2e^{2\pi i(0.7)}, 0.5e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.4)}, 0.7e^{2\pi i(0.5)}\rangle, \\ \langle 0.8e^{2\pi i(0.3)}, 0.3e^{2\pi i(0.9)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.8)}\rangle, \\ \langle 0.5e^{2\pi i(0.9)}, 0.6e^{2\pi i(0.4)}\rangle\end{pmatrix}$
$\wp_2$	$\begin{pmatrix}\langle 0.4e^{2\pi i(0.6)}, 0.8e^{2\pi i(0.4)}\rangle, \\ \langle 0.6e^{2\pi i(0.5)}, 0.5e^{2\pi i(0.6)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4e^{2\pi i(0.5)}, 0.6e^{2\pi i(0.8)}\rangle, \\ \langle 0.7e^{2\pi i(0.6)}, 0.5e^{2\pi i(0.2)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.7)}, 0.5e^{2\pi i(0.6)}\rangle, \\ \langle 0.3e^{2\pi i(0.8)}, 0.3e^{2\pi i(0.5)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.3)}, 0.6e^{2\pi i(0.4)}\rangle, \\ \langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.6)}\rangle\end{pmatrix}$
$\wp_3$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.3)}, 0.7e^{2\pi i(0.4)}\rangle, \\ \langle 0.8e^{2\pi i(0.6)}, 0.4e^{2\pi i(0.6)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.5)}, 0.8e^{2\pi i(0.7)}\rangle, \\ \langle 0.3e^{2\pi i(0.8)}, 0.5e^{2\pi i(0.5)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4e^{2\pi i(0.5)}, 0.6e^{2\pi i(0.6)}\rangle, \\ \langle 0.5e^{2\pi i(0.6)}, 0.7e^{2\pi i(0.3)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.8e^{2\pi i(0.7)}, 0.6e^{2\pi i(0.5)}\rangle, \\ \langle 0.3e^{2\pi i(0.4)}, 0.7e^{2\pi i(0.8)}\rangle\end{pmatrix}$
$\wp_4$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.4)}, 0.6e^{2\pi i(0.7)}\rangle, \\ \langle 0.5e^{2\pi i(0.2)}, 0.4e^{2\pi i(0.8)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.6e^{2\pi i(0.7)}, 0.5e^{2\pi i(0.6)}\rangle, \\ \langle 0.5e^{2\pi i(0.8)}, 0.8e^{2\pi i(0.3)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.4e^{2\pi i(0.6)}, 0.5e^{2\pi i(0.7)}\rangle, \\ \langle 0.3e^{2\pi i(0.8)}, 0.3e^{2\pi i(0.3)}\rangle\end{pmatrix}$	$\begin{pmatrix}\langle 0.7e^{2\pi i(0.6)}, 0.1e^{2\pi i(0.4)}\rangle, \\ \langle 0.6e^{2\pi i(0.5)}, 0.5e^{2\pi i(0.6)}\rangle\end{pmatrix}$

Step 8. Based on $N\mathcal{S}P_i$ and $N\mathcal{S}_iN$, determin the appraisal score value ($A\mathcal{S}$) as

$$A\mathcal{S}_1 = 1.00, A\mathcal{S}_2 = 0.6510, A\mathcal{S}_3 = 0.5671, A\mathcal{S}_4 = 0.6808. \tag{60}$$

Step 9. Based on EDAS method, the score values of the defined model are given in Table 9. It is clear from Table 9 that the ordered of the ranking is same, and the best choice remains the same using different aggregation operators. Hence, the best alternative is $\wp_3$.

6.1. Comparison Analysis. This section discusses the proposed model's comparison to existing models in the context of Cq-ROF. Analytical factors were extracted from relevant literature and intuition and considered from both theoretical

Table 4: Cq-ROFR aggregated decision matrix using Cq-ROFRHWA operator.

	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$
$\wp_1$	$\begin{pmatrix} \langle 0.281e^{2\pi i(0.189)}, 0.176e^{2\pi i(0.324)} \rangle, \\ \langle 0.310e^{2\pi i(0.123)}, 0.302e^{2\pi i(0.217)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.212e^{2\pi i(0.215)}, 0.328e^{2\pi i(0.156)} \rangle, \\ \langle 0.375e^{2\pi i(0.123)}, 0.209e^{2\pi i(0.327)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.235e^{2\pi i(0.173)}, 0.328e^{2\pi i(0.212)} \rangle, \\ \langle 0.419e^{2\pi i(0.310)}, 0.386e^{2\pi i(0.219)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.517e^{2\pi i(0.152)}, 0.236e^{2\pi i(0.321)} \rangle, \\ \langle 0.398e^{2\pi i(0.287)}, 0.152e^{2\pi i(0.237)} \rangle \end{pmatrix}$
$\wp_2$	$\begin{pmatrix} \langle 0.410e^{2\pi i(0.154)}, 0.318e^{2\pi i(0.328)} \rangle, \\ \langle 0.216e^{2\pi i(0.100)}, 0.190e^{2\pi i(0.215)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.198e^{2\pi i(0.217)}, 0.276e^{2\pi i(0.218)} \rangle, \\ \langle 0.435e^{2\pi i(0.265)}, 0.418e^{2\pi i(0.209)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.197e^{2\pi i(0.376)}, 0.274e^{2\pi i(0.236)} \rangle, \\ \langle 0.316e^{2\pi i(0.281)}, 0.198e^{2\pi i(0.390)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.124e^{2\pi i(0.416)}, 0.276e^{2\pi i(0.176)} \rangle, \\ \langle 0.264e^{2\pi i(0.231)}, 0.412e^{2\pi i(0.327)} \rangle \end{pmatrix}$
$\wp_3$	$\begin{pmatrix} \langle 0.218e^{2\pi i(0.217)}, 0.424e^{2\pi i(0.132)} \rangle, \\ \langle 0.420e^{2\pi i(0.215)}, 0.310e^{2\pi i(0.221)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.188e^{2\pi i(0.328)}, 0.219e^{2\pi i(0.298)} \rangle, \\ \langle 0.224e^{2\pi i(0.221)}, 0.461e^{2\pi i(0.287)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.512e^{2\pi i(0.208)}, 0.487e^{2\pi i(0.298)} \rangle, \\ \langle 0.277e^{2\pi i(0.383)}, 0.387e^{2\pi i(0.196)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.216e^{2\pi i(0.324)}, 0.376e^{2\pi i(0.218)} \rangle, \\ \langle 0.421e^{2\pi i(0.132)}, 0.287e^{2\pi i(0.323)} \rangle \end{pmatrix}$
$\wp_4$	$\begin{pmatrix} \langle 0.318e^{2\pi i(0.317)}, 0.287e^{2\pi i(0.301)} \rangle, \\ \langle 0.219e^{2\pi i(0.209)}, 0.511e^{2\pi i(0.188)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.317e^{2\pi i(0.213)}, 0.531e^{2\pi i(0.321)} \rangle, \\ \langle 0.327e^{2\pi i(0.147)}, 0.276e^{2\pi i(0.165)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.421e^{2\pi i(0.219)}, 0.216e^{2\pi i(0.265)} \rangle, \\ \langle 0.198e^{2\pi i(0.410)}, 0.327e^{2\pi i(0.185)} \rangle \end{pmatrix}$	$\begin{pmatrix} \langle 0.343e^{2\pi i(0.216)}, 0.432e^{2\pi i(0.167)} \rangle, \\ \langle 0.254e^{2\pi i(0.265)}, 0.289e^{2\pi i(0.319)} \rangle \end{pmatrix}$

Table 5: The aggregated value of $A\nu\mathcal{S}$.

$\mathbb{Z}_1$	$\left(\left(\left\langle 0.342e^{2\pi i(0.276)}, 0.328e^{2\pi i(0.201)}\right\rangle\right), \left(\left\langle 0.198e^{2\pi i(0.376)}, 0.421e^{2\pi i(0.152)}\right\rangle\right)\right)$
$\mathbb{Z}_2$	$\left(\left(\left\langle 0.217e^{2\pi i(0.290)}, 0.278e^{2\pi i(0.381)}\right\rangle\right), \left(\left\langle 0.183e^{2\pi i(0.346)}, 0.387e^{2\pi i(0.254)}\right\rangle\right)\right)$
$\mathbb{Z}_3$	$\left(\left(\left\langle 0.326e^{2\pi i(0.217)}, 0.521e^{2\pi i(0.310)}\right\rangle\right), \left(\left\langle 0.541e^{2\pi i(0.167)}, 0.287e^{2\pi i(0.288)}\right\rangle\right)\right)$
$\mathbb{Z}_4$	$\left(\left(\left\langle 0.487e^{2\pi i(0.212)}, 0.491e^{2\pi i(0.220)}\right\rangle\right), \left(\left\langle 0.341e^{2\pi i(0.301)}, 0.345e^{2\pi i(0.265)}\right\rangle\right)\right)$

Table 6: The values of PDA$\mathcal{S}_{ij}$ matrix.

	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$
$\wp_1$	0.000	0.1253	0.2134	0.2100
$\wp_2$	0.2180	0.000	0.1809	0.000
$\wp_3$	0.1922	0.0142	0.000	0.000
$\wp_4$	0.000	0.1805	0.0982	0.1864

Table 7: The values of NDA$\mathcal{S}_{ij}$ matrix.

	$\mathbb{Z}_1$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_4$
$\wp_1$	0.1326	0.000	0.000	0.000
$\wp_2$	0.000	0.0216	0.000	0.0874
$\wp_3$	0.000	0.000	0.1653	0.1165
$\wp_4$	0.2161	0.000	0.000	0.000

Table 8: The values of $\mathcal{S}P_i$ and $\mathcal{S}N_i$.

$\mathcal{S}P_1 = 0.1954$	$\mathcal{S}N_1 = 0.000$
$\mathcal{S}P_2 = 0.1180$	$\mathcal{S}N_2 = 0.1525$
$\mathcal{S}P_3 = 0.1230$	$\mathcal{S}N_3 = 0.0926$
$\mathcal{S}P_4 = 0.0968$	$\mathcal{S}N_4 = 0.1178$

Table 9: Score value and their ranking.

Proposed operators	Appraisal score values of alternatives				Ranking
	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	
Cq – ROFRHWA	1.00	0.6510	0.5671	0.6808	$\wp_1 > \wp_4 > \wp_2 > \wp_3$
Cq – ROFRHOWA	1.00	0.7182	0.5922	0.7514	$\wp_1 > \wp_4 > \wp_2 > \wp_3$
Cq – ROFRHHA	1.00	0.7417	0.6235	0.7891	$\wp_1 > \wp_4 > \wp_2 > \wp_3$

and numerical perspectives. State-of-the-art models that use Cq-ROFS are considered in order to consider homogeneity in comparison. The following are some of the existing models that are being investigated. Table 10 compares these models to the proposed model in order to determine which is superior.

A comparative study was conducted in the context of some existing methods ([45–49, 63]) in order to demonstrate the superiority of our investigated Cq-ROFR modified EDAS method.

Table 10 shows the aggregated results of the comparative analysis of existing models utilizing our approach, using

Table 10: Different methods and their ranking.

Methods	Appraisal scores of alternatives	Ranking
Garg et al. [46]	Inaccessible	×
Garg et al. [49]	Inaccessible	×
Liu et al. [45]	Inaccessible	×
Mahmood and Ali [47]	Inaccessible	×
Naz et al. [48]	Inaccessible	×
Mahmood and Ali [63]	Inaccessible	×

Table 5 and the weight vector $\omega = (0.32, 0.23, 0.21, 0.24)^T$. Table 10 shows that other existing approaches based on Cq-ROFS aggregation operators are unable to solve the defined illustrated example of Section 6 using Cq-ROF rough values. Table 10 reveals that there is a scarcity of rough data on current methods, and that these methods are incapable of solving and ranking the established example. As a result, the established methodology is more capable and reliable than current methods.

From the above comparisons, it is clear that the proposed methods in this paper such as Cq-ROFWA, Cq-ROFROWA, and Cq-ROFRHA operators are more general than Cq-ROFS. Therefore, the proposed methods in this manuscript are more suitable to solve the MAGDM problems.

7. Conclusion

The novelty and contribution of this study can be elaborated as the development of a new MCGDM framework based on Cq-ROFRS to benchmark the applications of smart grade system and tackle the uncertainty problem thoroughly.

The aims of this article and the concept of complex q-rung orthopair fuzzy sets (Cq-ROFSs) and rough set are combined to propose the novel approach of complex q-rung orthopair fuzzy rough sets (Cq-ROFRSs) and their fundamental laws. In the procedure of decision making, owing to the increasing complexity and uncertainty of real life scenarios, the decision information is more suitably expressed in terms of Cq-ROFRSs. Though several information fusion techniques have been explored to aggregate complex q-rung orthopair fuzzy information, all these techniques are limited to algebraic or Einstein t-norm and t-conorm, and some of them have certain limitations. Motivated by these defects and beneficial characteristics of Hamacher t-norm and t-conorm, we explored basic operational rules of Cq-ROFRSs to build complex q-rung orthopair fuzzy rough aggregation operators that comply with the principles of Hamacher t-norm and t-conorm. Some desirable properties and special cases of these operators are also studied comprehensibly. We demonstrated the reliability and superiority of the proposed work using weighted averaging operators, and we also discussed its benefits by contrasting it with other existing work. In this manuscript, it is also discussed how the proposed method compares to current methodologies.

This method could be extended to other aggregation operators in the future, such as Einstein operations, Dombi operations, Hamacher operations, power aggregation operators, Maclaurin's symmetric mean operator, and complex fuzzy ordered weighted quadratic averaging operator.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors participated in every stage of the research, and all authors read and approved the final manuscript.

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