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Research Article

On Coincidence Theorem in Intuitionistic Fuzzy *b*-Metric Spaces with Application

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The concept of intuitionistic fuzzy b-metric spaces (shortly, IFbMS) has been introduced and studied to generalize both the notion of intuitionistic fuzzy metric spaces and fuzzy *b*-metric spaces. The existence of coincident point and common fixed point for two self-mappings has been established. In order to show the strength of these results, some interesting examples are established as well. Our results generalize many previous results existing in literature. Some nontrivial examples are furnished as well as an application is created to give the strength of our main result.

1. Introduction

The advancement and the rich progress of fixed-point theorems in metric spaces (and in its various generalizations) have significant theoretic and useful applications. These developments in the last three decades were fabulous. Most of them used Banach's contraction theorem [1] in their reference result. Many problems in engineering and research can be solved by confining nonlinear equations to similar fixed-point cases. A fixed point $Fx = x$ can be proved for an operator sum $Gx = 0$, where F is a self-mapping in some relevant discipline. Fixed-point theory has various key modes for addressing difficulties arising from a variety of mathematical inspection offshoots, such as split feasibility concerns, supporting problems, equilibrium problems, and matching and selection issues. The theory of fixed points is a fascinating and energising field of study. This idea has already been identified as an over-the-top attempt to pack nonlinear analysis into a small amount of time.

In 1965, Zadeh [2] initiated the concept of FSs which is used to characterize/manipulate information and data having nonstatistical uncertainties. The objectives of the theory of FS are to offer rational and set hypothetical tools to deal with such problems where errors and degree of uncertainties are present. Later, the idea of intuitionistic-FSs was given by Atanassov [3] in 1986. This set theory not only defines the degree of membership but also degree of nonmembership which is a more generalized form of FS-theory. Many authors apply this concept in different fields of mathematics. Gulzar et al. (e.g., [4–6]) have applied this theory in groups and its characteristics. Akber [7] used this idea to define intuitionistic fuzzy mappings and obtained common fixed points for such types of mappings. Since the notion of distance function have a central part in approximation theory, therefore, FSs further have been applied to the classical notion of metric. In this direction, some authors [8–10] suggested the generalization of metric spaces to the fuzzy situations.

In 1975, the concept of FMS was offered by Kramosil and Michalek [11], which was further developed by George and Veeramani [12] in 1994, in order to build a Hausdorff topology using fuzzy metric. Rehman and Aydi [13] constructed results in fuzzy cone metric space. Bakhtin [14] first proposed the concept of *b*-metric. *b*-metric spaces are a broader category than metric spaces. Later, Javed et al. [15] established results on orthogonal partial *b*-metric spaces. Nădăban [16] proposed the notion of fuzzy *b*-metric space in 2016, which is a generalization of FMSs. Javed et al. [17] worked on finding fixed-point results in fuzzy b-metric spaces. Shazia et al. [18] established fixed points for various contractions in fuzzy strong b-metric spaces. Park [19] defined IFMSs as a refinement of FMSs by using theory of intuitionistic-FSs and continuous t-norm and continuous t-conorm in the year 2004. Jungck [20] enhanced Banach's theorem in 1976 by analyzing at coincidence and common fixed points of commuting mappings. Jungck [21] established the concept of compatible maps for a pair of self-mappings, as well as the existence of shared fixed points, in 1986. Turkoglu et al. [22] expanded Jungck's common fixed-point theorems in IFMSs in a paper published in 2006. Jungck and Rhoades [23] introduced weakly compatible mappings in 2006. Weakly compatible mappings are more generic since any pair of compatible mappings is weakly compatible, but not the other way around. Altun et al. [24,25] recently established excellent results on the best proximity points in 2021.

Grabiec [26] defined the completion of FMS in 1988. We proved the existence of the coincidence theorem and the common fixed-point theorem in IFbMS in this work. The structure of the paper is as follows.

After the preliminaries, in Section 3, the notion of IFbMSs has been defined, and this concept is clarified with the help of comprehensible examples. The conceptual definitions of convergent sequence, Cauchy sequence, and topology induced by an IFbMS are presented as well. In Section 4, we formulate and prove our main results regarding coincidence points and common fixed point of weakly compatible mappings in IFbMS and establish some nontrivial examples to justify the validity of our results. Section 5 consists of an application of our main result.

2. Preliminaries

For the reader's convenience, some definitions and results are recalled.

Definition 2.1 (see [27]). Let ζ be an arbitrary nonempty set and $s \geq 1$ be a given real number. A function $\varpi: \zeta \times \zeta \longrightarrow [0, \infty)$ is a b-metric on ζ if, for all $\omega, \upsilon, z \in \zeta$, the following conditions are satisfied:

(b_1) $\varpi(\omega, \upsilon) = 0 \Leftrightarrow \omega = \upsilon$

(b_2) $\varpi(\omega, \upsilon) = \varpi(\upsilon, \omega)$

(b_3) $\varpi(\omega, z) \leq s[\varpi(\omega, \upsilon) + \varpi(\upsilon, z)]$

The triple (ζ, ϖ, s) will be called b-metric space.

Example 1 (see [28]). The space $k_i\,(0 < i < 1)$,

$$k_i = \left\{ \left(\omega_j\right) \subset R : \sum_{j=1}^{\infty} \left|\omega_j\right|^i < \infty \right\}, \tag{1}$$

with a function $\varpi: k_i \times k_i \longrightarrow R$,

$$\varpi(\omega, \upsilon) = \left(\sum_{j=1}^{\infty} \left|\omega_j - \upsilon_j\right|^i \right)^{1/i}, \tag{2}$$

is a b-metric space, where $\omega = (\omega_j)$and$\upsilon = (\upsilon_j) \in k_i$. By some calculation, we obtain

$$\varpi(\omega, z) \leq 2^{1/i}[\varpi(\omega, \upsilon) + \varpi(\upsilon, z)]. \tag{3}$$

Here, $s = 2^{1/i} > 1$.

Example 2 (see [28]). The space $G_i\,(0 < i < 1)$, of all real functions $\omega(t), t \in [0, 1]$ such that $\int_0^1 |\omega(t)|^i dt < \infty$, is b-metric space if we take

$$\varpi(\omega, \upsilon) = \left[\int_0^1 |\omega(t) - \upsilon(t)|^i \right]^{1/i}, \tag{4}$$

for each $\omega, \upsilon \in G_i$.

Definition 2.2 (see [2]). Consider a nonempty set ζ. A mapping from ζ to $[0, 1]$ is called the fuzzy set. If K is a FS and $\eta \in \zeta$, then the function value $K(\eta)$ is called the grade of membership of η in K.

Definition 2.3 (see [3]). For a nonempty set ζ an intuitionistic-FS is defined as

$$A = \{\omega \in \zeta : \langle \mu_A(\omega), \nu_A(\omega) \rangle\}, \tag{5}$$

where $\mu_A: \zeta \longrightarrow [0, 1]$ is called the degree of membership and $\nu_A: \zeta \longrightarrow [0, 1]$ is called the degree of nonmembership of every ω to the set A such that $\mu_A(\omega) + \nu_A(\omega) \in [0, 1]$, for all $\omega \in \zeta$.

Definition 2.4 (see [29]). A binary operation $\odot: [0, 1] \times [0, 1] \longrightarrow [0, 1]$ is known as continuous-t-norm if the following axioms are satisfied:

(1) Associativity and commutativity properties are satisfied by $\odot$

(2) $\odot$ is a continuous function

(3) $\lambda \odot 1 = \lambda, \forall \lambda \in [0, 1]$

(4) If $\lambda \leq k$ and $\epsilon \leq l$ with $\lambda, \epsilon, k, l \in [0, 1]$, then $\lambda \odot \epsilon \leq k \odot l$

Example

(1) $\lambda \odot_1 \varepsilon = \min(\lambda, \varepsilon)$

(2) $\lambda \odot_2 \varepsilon = \lambda . \varepsilon$

(3) $\lambda \odot_3 \varepsilon = \max(\lambda + \varepsilon - 1, 0)$

Definition 2.5 (see [29]). A binary operation $\circ: [0, 1] \times [0, 1] \longrightarrow [0, 1]$ is known as continuous-t-conorm if the following axioms are satisfied:

(1) Associativity and commutativity properties are satisfied by $\circ$

(2) $\circ$ is continuous function

(3) $\lambda^{\circ} 0 = \lambda, \forall \lambda \in [0, 1]$

(4) $\lambda^{\circ} \epsilon \leq k^{\circ} l$, whenever $\lambda \leq k$ and $\epsilon \leq l$ for all $\lambda, \epsilon, k, l \in [0, 1]$

Example

(1) $\lambda \circ_1 \varepsilon = \min(\lambda + \varepsilon, 1)$

(2) $\lambda \circ_2 \varepsilon = \lambda + \varepsilon - \lambda\varepsilon$

(3) $\lambda \circ_3 \varepsilon = \max(\lambda, \varepsilon)$

Definition 2.6 (see [16]). Let ζ be a nonempty set. Let $s \in R(set\, of\, reals), s \geq 1$, and $\odot$ be a continuous-t-norm. A FS Φ on $\zeta \times \zeta \times [0, \infty)$ is called fuzzy b-metric if, for all $\omega, v, z \in \zeta$, the following conditions hold:

(bM1) $\Phi(\omega, v, 0) = 0$

(bM2) $\Phi(\omega, v, \iota) = 1, \forall \iota \geq 0 \Leftrightarrow S\omega = v$

(bM3) $\Phi(\omega, v, \iota) = \Phi(v, \omega, \iota), \forall \iota \geq 0$

(bM4) $\Phi(\omega, z, s(\iota + \theta)) \geq \Phi(\omega, v, \iota) \odot \Phi(v, z, \theta), \forall \iota, \theta \geq 0$

(bM5) $\lim_{\iota \longrightarrow \infty} \Phi(\omega, v, \iota) = 1$ and $\Phi(\omega, v, .)$: $[0, \infty) \longrightarrow [0, 1]$ is left continuous

The quadruple $(\zeta, \Phi, \odot, s)$ is known as fuzzy b-metric space.

3. Intuitionistic Fuzzy *b*-Metric Spaces and Coincidence Point Results

Definition 3.1. A 6-tuple $(\zeta, \Phi, \varphi, \odot, ^\circ, s)$ is said to be IFbMS if ζ is an arbitrary set, $s \geq 1$ is a given real number, $\odot$ is a continuous-t-norm, $^\circ$ is a continuous-t-conorm, Φ and φ are FSs on $\zeta^2 \times [0, \infty)$ satisfying the following conditions. For all $\omega, v, z \in \zeta$,

(a) $\Phi(\omega, v, \iota) + \varphi(\omega, v, \iota) \leq 1$

(b) $\Phi(\omega, v, 0) = 0$

(c) $\Phi(\omega, v, \iota) = 1, \forall \iota > 0$ iff $\omega = v$

(d) $\Phi(\omega, v, \iota) = \Phi(v, \omega, \iota), \forall \iota > 0$

(e) $\Phi(\omega, z, s(\iota + \theta)) \geq \Phi(\omega, v, \iota) \odot \Phi(v, z, \theta), \forall \iota, \theta > 0$

(f) $\Phi(\omega, v, .)$: $[0, \infty) \longrightarrow [0, 1]$ is left continuous and $\lim_{\iota \longrightarrow \infty} \Phi(\omega, v, \iota) = 1$

(g) $\varphi(\omega, v, 0) = 1$

(h) $\varphi(\omega, v, \iota) = 0, \forall \iota > 0$ iff $\omega = v$

(i) $\varphi(\omega, v, \iota) = \varphi(v, \omega, \iota), \forall \iota > 0$

(j) $\varphi(\omega, z, s(\iota + \theta)) \leq \varphi(\omega, v, \iota) ^\circ \varphi(v, z, \theta), \forall \iota, \theta > 0$

(k) $\lim_{\iota \longrightarrow \infty} \varphi(\omega, v, \iota) = 0$ and $\varphi(\omega, v, .)$: $[0, \infty) \longrightarrow [0, 1]$ is right continuous

Here, $\Phi(\omega, v, \iota)$ and $\varphi(\omega, v, \iota)$ denote the degree of nearness and the degree of nonnearness between ω and v with respect to t, respectively.

Example 3.2.1. Let (ζ, ϖ, s) be a b-metric space and $a = \min(a, b)$, $a ^\circ b = \max(a, b) \,\forall a, b \in [0, 1]$, and let Φ_ϖ, φ_ϖ be FSs on $\zeta^2 \times [0, \infty)$, defined as follows:

$$\Phi_\varpi(\omega, v, \iota) = \begin{cases} \dfrac{\iota}{\iota + \varpi(\omega, v)}, & \text{if } \iota > 0, \\ 0, & \text{if } \iota = 0, \end{cases}$$
$$\varphi_\varpi(\omega, v, \iota) = \begin{cases} \dfrac{\varpi(\omega, v)}{\iota + \varpi(\omega, v)}, & \text{if } \iota > 0, \\ 1, & \text{if } \iota = 0. \end{cases} \tag{6}$$

We check only axioms (e) and (j) of Definition 3.1 because verification of the other conditions is standard. Let $\omega, v, z \in \zeta$ and $\iota, u > 0$.

Without restraining the generality, we assume that

$$\Phi_\varpi(\omega, v, \iota) \leq \Phi_\varpi(v, z, u), \quad \varphi_\varpi(\omega, v, \iota) \geq \varphi_\varpi(v, z, u). \tag{7}$$

Thus,

$$\frac{\iota}{\iota + \varpi(\omega, v)} \leq \frac{u}{u + \varpi(v, z)}, \quad \frac{\varpi(\omega, v)}{\iota + \varpi(\omega, v)} \geq \frac{\varpi(v, z)}{u + \varpi(v, z)}, \tag{8}$$

i.e.,

$$\iota \varpi(v, z) \leq u \varpi(\omega, v). \tag{9}$$

On the contrary,

$$\begin{aligned} \Phi_\varpi(\omega, z, s(\iota + u)) &= \frac{s(\iota + u)}{s(\iota + u) + \varpi(\omega, z)} \\ &\geq \frac{s(\iota + u)}{s(\iota + u) + s[\varpi(\omega, v) + \varpi(v, z)]} \\ &= \frac{\iota + u}{\iota + u + \varpi(\omega, v) + \varpi(v, z)}. \end{aligned} \tag{10}$$

Also,

$$\begin{aligned} \varphi_\varpi(\omega, z, s(\iota + u)) &= \frac{\varpi(\omega, z)}{s(\iota + u) + \varpi(\omega, z)} \\ &\leq \frac{s[\varpi(\omega, v) + \varpi(v, z)]}{s(\iota + u) + s[\varpi(\omega, v) + \varpi(v, z)]} \\ &= \frac{\varpi(\omega, v) + \varpi(v, z)}{\iota + u + \varpi(\omega, v) + \varpi(v, z)}, \end{aligned}$$
$$\frac{\varpi(\omega, v) + \varpi(v, z)}{\iota + u + \varpi(\omega, v) + \varpi(v, z)} \leq \frac{\varpi(\omega, v)}{\iota + \varpi(\omega, v)}. \tag{11}$$

Hence, we will obtain that

$$\begin{aligned} \Phi_\varpi(\omega, z, s(\iota + u)) &\geq \Phi_\varpi(\omega, v, \iota) = \Phi_\varpi(\omega, v, \iota) \odot \Phi_\varpi(v, z, u), \\ \varphi_\varpi(\omega, z, s(\iota + u)) &\leq \varphi_\varpi(\omega, v, \iota) = \varphi_\varpi(\omega, v, \iota) ^\circ \varphi_\varpi(v, z, \mathrm{u}), \end{aligned} \tag{12}$$

which had to be verified. We remark that

$$\begin{aligned} &\frac{\iota + u}{\iota + u + \varpi(\omega, v) + \varpi(v, z)} \geq \frac{\iota}{\iota + \varpi(\omega, v)} \\ &\Leftrightarrow \iota^2 + u\iota + \iota\varpi(\omega, v) + u\varpi(\omega, v) \geq \iota^2 + u\iota + \iota\varpi(\omega, v) + \iota\varpi(v, z) \\ &\Leftrightarrow u\varpi(\omega, v) \geq \iota\varpi(v, z), \end{aligned} \tag{13}$$

which is true.

Also,

$$\frac{\varpi(\omega,v)+\varpi(v,z)}{\iota+u+\varpi(\omega,v)+\varpi(v,z)}\leq\frac{\varpi(\omega,v)}{\iota+\varpi(\omega,v)}$$

$$\begin{aligned}&\Leftrightarrow\iota\varpi(\omega,v)+\iota\varpi(v,z)+\varpi(\omega,v)\varpi(v,z)+(\varpi(\omega,v))^2\\&\leq\iota\varpi(\omega,v)+u\varpi(\omega,v)+\varpi(x,v)\varpi(v,z)+(\varpi(\omega,v))^2\\&\Leftrightarrow\iota\varpi(v,z)\leq u\varpi(\omega,v),\end{aligned}\tag{14}$$

which is true.

Hence, $(\zeta,\Phi_{\varpi},\varphi_{\varpi},\odot,^{\circ},s)$ is (standard) IFbMS.

Example 3.2.2. Let (ζ,ϖ,s) be a b-metric space and $a=\min(a,b)$ and $a^{\circ}b=\max(a,b)\,\forall a,b\in[0,1]$, and let Φ and φ be FSs on $\zeta^2\times[0,\infty)$, defined as follows:

$$\Phi(\omega,v,t)=\begin{cases}\left(\exp\left(\dfrac{\varpi(\omega,v)}{t}\right)\right)^{-1}, & \text{if } t>0,\\ 0, & \text{if } t=0,\end{cases}$$
$$\varphi\varpi(\omega,v,t)=\begin{cases}\dfrac{\exp(\varpi(\omega,v)/t)-1}{\exp(\varpi(\omega,v)/t)} & \text{if } t>0,\\ 1, & \text{if } t=0.\end{cases}\tag{15}$$

Then, $(\zeta,\Phi,\varphi,\odot,^{\circ},s)$ is an IFbMS.

Definition 3.3. Let $s\geq 1$ be a given real number. A function $f\colon R\longrightarrow R$ will be called s-nondecreasing if $t<u$ implies that $f(t)\leq f(su)$ and f is called s-nonincreasing if $t<u$ implies that $f(t)\geq f(su)$.

Proposition 3.4. *In an IFbMS $(\zeta,\Phi,\varphi,\odot,^{\circ},s)$, $\Phi(\omega,v,.)\colon[0,\infty)\longrightarrow[0,1]$ is s-nondecreasing and $\varphi(\omega,v,.)\colon[0,\infty)\longrightarrow[0,1]$ is s-nonincreasing, for all $\omega,v\in\zeta$.*

Proof. For $0<t<u$, we have

$$\begin{aligned}\Phi(\omega,v,su)&=\Phi(\omega,v,s(u-t+t)\\&\geq\Phi(\omega,\omega,u-t)\odot\Phi(\omega,v,t)\\&=1\odot\Phi(\omega,v,t)=\Phi(\omega,v,t).\end{aligned}\tag{16}$$

Also,

$$\begin{aligned}\varphi(\omega,v,su)&=\varphi(\omega,v,s(u-t+t)\\&\leq\varphi(\omega,\omega,u-t)^{\circ}\varphi(\omega,v,\mathrm{t})\\&=0^{\circ}\varphi(\omega,v,t)=\varphi(\omega,v,t).\end{aligned}\tag{17}$$

□

Definition 3.5. Let $(\zeta,\Phi,\varphi,\odot,^{\circ},s)$ be an IFbMS.

(a) Any sequence ω_n in ζ is said to be convergent if there exists $\omega\in\zeta$ such that $\lim_{n\longrightarrow\infty}\Phi(\omega_n,\omega,t)=1$ and $\lim_{n\longrightarrow\infty}\varphi(\omega_n,\omega,t)=0,\forall t>0$. ω is called the limit of the sequence ω_n, and it is written as $\lim_{n\longrightarrow\infty}\omega_n=\omega$, or $\omega_n\longrightarrow\omega$.

(b) Any sequence ω_n in $(\zeta,\Phi,\odot,^{\circ},s)$ is said to be a Cauchy sequence if, for every ϵ in $(0,1)$, there is $n_0\in\varphi$ such that $\Phi(\omega_n,\omega_m,t)>1-\epsilon$ *an d* $\varphi(\omega_n,\omega_m,t)<\epsilon$, for all $m,n\geq n_0$ and $t>0$.

(c) ζ is said to be complete if every Cauchy sequence in ζ is convergent in ζ.

In 2012, Tirado [30] proved that (standard) FMS is complete. It can be easily checked that (standard) IFbMS is also complete.

Definition 3.6. Let $(\zeta,\Phi,\varphi,\odot,^{\circ},s)$ be an IFbMS. An open ball $B(\omega,r,t)$ with center $\omega\in\zeta$ and radius r, $0<r<1$, and $t>0$ is defined as $B(\omega,r,t)=\{v\in\zeta\colon\Phi(\omega,v,t)>1-r,\varphi(\omega,v,t)<r\}$.

Definition 3.7. Let $(\zeta,\Phi,\varphi,\odot,^{\circ},s)$ be an IFbMS and A be a subset of ζ. A is said to be open if, for each $\omega\in A$, there is an open ball B (ω,r,t) contained in A.

Result: let $(\zeta,\Phi,\varphi,\odot,^{\circ},s)$ be an IFbMS. Define $\tau_{\Phi,\varphi}$ as $\tau_{\Phi,\varphi}=\{A\in P(\zeta)\colon\omega\in A\,iff\,\exists\iota>0\,an\,d\,r\in(0,1)\colon\ B(\omega,r,\iota)\subset A\}$, then $\tau_{\Phi,\varphi}$ is a **topology** on ζ, where $P(\zeta)$ is the power set of ζ.

4. Coincidence and Common Fixed-Point theorems

This section concerns with the constructing and proving of coincidence theorem and common fixed-point theorem in IFbMS. Many useful results existing in literature are presented here as corollaries of our results.

Definition 4.1. Let ζ be a nonempty set and $\Pi,\sigma\colon\zeta\longrightarrow\zeta$ be two mappings on ζ.

(i) A point $\omega\in\zeta$ is called a coincidence point of Π and σ if $\Pi(\omega)=\sigma(\omega)$

(ii) A point $v\in\zeta$ is called point of coincidence of Π and σ if there exists $\omega\in\zeta$ such that $v=\Pi(\omega)=\sigma(\omega)$

(iii) A point $z\in\zeta$ is known as common fixed point of Π and σ if $z=\Pi(z)=\sigma(z)$

Definition 4.2. Two self-maps $\Pi,\sigma\colon\zeta\longrightarrow\zeta$ are said to be weakly compatible if $\Pi\sigma(\omega)=\sigma\Pi(\omega)$ when $\Pi(\omega)=\sigma(\omega)$.

Theorem 4.1. *Let ζ be a nonempty set and $(Y,\Phi,\varphi,\odot,^{\circ},s)$ be an IFbMS and $\Pi,\sigma\colon\zeta\longrightarrow Y$ be mappings satisfying the following conditions:*

(1) $\sigma(\zeta)\subseteq\Pi(\zeta)$;

(2) There is k, $0\leq k<1$, such that, for all $\omega,v\in\zeta$,

$$\begin{aligned}\Phi(\sigma(\omega),\sigma(v),kt)&\geq\Phi(\Pi(\omega),\Pi(v),t),\\ \Pi\varphi(\sigma(\omega),\sigma(v),kt)&\leq\varphi(\Pi(\omega),\Pi(v),t).\end{aligned}\tag{18}$$

If $\Pi(\zeta)$ or $\sigma(\zeta)$ is complete, then there exists a point $z\in\zeta$ such that $\Pi(z)=\sigma(z)$. Moreover, Π and σ have a unique point of coincidence.

Proof. Let $\omega_0 \in \zeta$. By (1), we can find $\omega_1 \in \zeta$ such that $\Pi(\omega_1) = \sigma(\omega_0)$.

For $k = 0$,

$$\begin{aligned}
&\Phi(\sigma(\omega_0), \sigma(\omega_1), 0t) \geq \Phi((\Pi(\omega_0), \Pi(\omega_1), t),\\
&\varphi(\sigma(\omega_0), \sigma(\omega_1), 0t) \leq \varphi((\Pi(\omega_0), \Pi(\omega_1), t),\\
\Rightarrow &\Phi(\sigma(\omega_0), \sigma(\omega_1), 0t) = 1,\\
&\varphi(\sigma(\omega_0), \sigma(\omega_1), 0t) = 0.
\end{aligned} \tag{19}$$

Hence,

$$\begin{aligned}
&\sigma(\omega_0) = \sigma(\omega_1)\\
\Rightarrow &\Pi(\omega_1) = \sigma(\omega_1).
\end{aligned} \tag{20}$$

This implies that ω_1 is the coincidence point of Π and σ.

For $k \neq 0$, by induction, we can define a sequence $\{\omega_n\}$ in ζ such that $\Pi(\omega_n) = \sigma(\omega_{n-1})$:

$$\begin{aligned}
\Phi(\Pi(\omega_n), \Pi(\omega_{n+1}), t) &= \Phi(\sigma(\omega_{n-1}), \sigma(\omega_n), t)\\
&\geq \Phi(\Pi(\omega_{n-1}), \Pi(\omega_n), t/k)\\
&\geq \ldots\\
&\geq \Phi(\Pi(\omega_0), \Pi(\omega_1), t/k^n).
\end{aligned} \tag{21}$$

Clearly, $1 \geq \Phi(\Pi(\omega_n), \Pi(\omega_{n+1}), t) \geq \Phi(\Pi(\omega_0), \Pi(\omega_1), t/k^n) \longrightarrow 1$, when $n \longrightarrow \infty$.

Thus, $Lim_{n\longrightarrow\infty}\Phi(\Pi(\omega_n), \Pi(\omega_{n+1}), t) = 1$.

And

$$\begin{aligned}
\varphi(\Pi(\omega_n), \Pi(\omega_{n+1}), t) &= \varphi(\sigma(\omega_{n-1}), \sigma(\omega_n), t)\\
&\leq \varphi(\Pi(\omega_{n-1}), \Pi(\omega_n), t/k)\\
&\leq \ldots\\
&\leq \varphi(\Pi(\omega_0), \Pi(\omega_1), t/k^n).
\end{aligned} \tag{22}$$

Clearly, $0 \leq \varphi(\Pi(\omega_n), \Pi(\omega_{n+1}), t) \leq \varphi(\Pi(\omega_0), \Pi(\omega_1), t/k^n) \longrightarrow 0$, when $n \longrightarrow \infty$.

Thus, $Lim_{n\longrightarrow\infty}\varphi(\Pi(\omega_n), \Pi(\omega_{n+1}), t) = 0$.

Let $\tau_n(t) = \Phi(\Pi(\omega_n), \Pi(\omega_{n+1}), t)$ and $\mu_n(t) = \varphi(\Pi(\omega_n), \Pi(\omega_{n+1}), t)$, for all $n \in N \cup \{0\}$, $t > 0$.

Clearly, $\lim_{n\longrightarrow\infty}\tau_n(t) = 1$ and $\lim_{n\longrightarrow\infty}\mu_n(t) = 0$.

To show that $\Pi(\omega_n)$ is a Cauchy sequence, suppose it is not; then, there exists $0 < \epsilon < 1$ and two sequences $p(\eta)$ and $q(\eta)$ such that, for every $\eta \in N \cup \{0\}$, $t > 0$, $p(\eta) > q(\eta) \geq \eta$, $\Phi(\Pi(\omega_{p(\eta)}), \Pi(\omega_{q(\eta)}), t) \leq 1 - \varepsilon$, and $\varphi(\Pi(\omega_{p(\eta)}), \Pi(\omega_{q(\eta)}), t) \geq \varepsilon$.

Then, $\Phi(\Pi(\omega_{p(\eta)-1}), \Pi(\omega_{q(\eta)-1}), t) > 1 - \varepsilon$, $\Phi(\Pi(\omega_{p(\eta)-1}), \Pi(\omega_{q(\eta)}), t) > 1 - \varepsilon$, and $\varphi(\Pi(\omega_{p(\eta)-1}), \Pi(\omega_{q(\eta)-1}), t) < \varepsilon$, $\varphi(\Pi(\omega_{p(\eta)-1}), \Pi(\omega_{q(\eta)}), t) < \varepsilon$.

Now,

$$\begin{aligned}
1 - \varepsilon &\geq \Phi\left(\Pi\left(\omega_{p(\eta)}\right), \Pi\left(\omega_{q(\eta)}\right), t\right)\\
&\geq \Phi\left(\Pi\left(\omega_{p(\eta)-1}\right), \Pi\left(\omega_{p(\eta)}\right), t/2s\right) \odot \Phi\left(\Pi\left(\omega_{p(\eta)-1}\right), \Pi\left(\omega_{q(\eta)}\right), t/2s\right)\\
&> \tau_{p(\eta)-1}(t/2s) \odot (1 - \varepsilon),\\
\varepsilon &\leq \varphi\left(\Pi\left(\omega_{p(\eta)}\right), \Pi\left(\omega_{q(\eta)}\right), t\right)\\
&\leq \varphi\left(\Pi\left(\omega_{p(\eta)-1}\right), \Pi\left(\omega_{p(\eta)}\right), t/2s\right) \circ \varphi\left(\Pi\left(\omega_{p(\eta)-1}\right), \Pi\left(\omega_{q(\eta)}\right), t/2s\right)\\
&< \mu_{p(\eta)-1}(t/2s) \circ \varepsilon.
\end{aligned} \tag{23}$$

Since $\tau_{p(\eta)-1}(t/2s) \longrightarrow 1$ as $\eta \longrightarrow \infty$ and $\mu_{p(\eta)-1}(t/2s) \longrightarrow 0$ as $\eta \longrightarrow \infty$ for every t, supposing that $\eta \longrightarrow \infty$, we have $1 - \varepsilon \geq \Phi(\Pi(\omega_{p(\eta)}), \Pi(\omega_{q(\eta)}), t) > 1 - \varepsilon$, $\varepsilon \leq \varphi(\Pi(\omega_{p(\eta)}), \Pi(\omega_{q(\eta)}), t) < \varepsilon$.

Clearly, this leads to the contradiction.

Hence, $\Pi(\omega_n)$ is a Cauchy sequence in $\Pi(\zeta)$. □

Case I: suppose that $\Pi(\zeta)$ is complete; then, there exists a point $v \in \Pi(\zeta)$ such that $\lim_{n\longrightarrow\infty}\Pi(\omega_n) = v$.

This implies that there exists $z \in \zeta$ such that $v = \Pi(z)$.

Now,

$$\begin{aligned}
\Phi(\Pi(z), \sigma(z), t) &\geq \Phi(\Pi(z), \Pi(\omega_n), t/2s) \odot \Phi(\Pi(\omega_n), \sigma(z), t/2s)\\
&= \Phi(\Pi(z), \Pi(\omega_n), t/2s) \odot \Phi(\sigma(\omega_{n-1}), \sigma(z), t/2s) \geq \Phi(\Pi(z), \Pi(\omega_n), t/2s) \odot \Phi(\Pi(\omega_{n-1}), \Pi(z), t/2sk)\\
&\geq 1 \odot 1 = 1, \text{as } n \longrightarrow \infty,\\
\varphi(\Pi(z), \sigma(z), t) &\leq \varphi(\Pi(z), \Pi(\omega_n), t/2s) \circ \varphi(\Pi(\omega_n), \sigma(z), t/2s)\\
&= \varphi(\Pi(z), \Pi(\omega_n), t/2s) \circ \varphi(\sigma(\omega_{n-1}), \sigma(z), t/2s) \leq \varphi(\Pi(z), \Pi(\omega_n), t/2s) \circ \varphi(\Pi(\omega_{n-1}), \Pi(z), t/2sk)\\
&\leq 0 \circ 0 = 0, \text{as } n \longrightarrow \infty.
\end{aligned} \tag{24}$$

By (c) and (h) of Definition 3.1, it follows that $\Pi(z) = \sigma(z)$.

Hence, z is a coincidence point and v is the point of coincidence of Π and σ.

Case II : suppose that $\sigma(\zeta)$ is complete; then, there exists a point $v \in \sigma(\zeta)$ such that $\lim_{n\longrightarrow\infty}\Pi(\omega_n) = v$.

However, $\sigma(\zeta)\subseteq\Pi(\zeta)$; this implies that $v \in \Pi(\zeta)$, so there exists $z \in \zeta$ such that $v = \Pi(z)$.

Next onward, proof is the same as in case I.

Now, we show that the point of coincidence of Π and σ is unique.

Let v_1 be another point of coincidence of Π and σ. Then, $v_1 = \Pi(z_1) = \sigma(z_1)$ for some z_1 in ζ:

$$\begin{aligned} 1 &\geq \Phi(v, v_1, t) = \Phi(\sigma(z), \sigma(z_1), t) \\ &\geq \Phi(\Pi(z), \Pi(z_1), t/k) = \Phi(v, v_1, t/k) \\ &\geq \ldots \geq \Phi(v, v_1, t/k^n). \end{aligned} \tag{25}$$

Also,

$$\begin{aligned} 0 &\leq \varphi(v, v_1, t) = \varphi(\sigma(z), \sigma(z_1), t) \\ &\leq \varphi(\Pi(z), \Pi(z_1), t/k) = \varphi(v, v_1, t/k) \\ &\leq \ldots \leq \varphi(v, v_1, t/k^n). \end{aligned} \tag{26}$$

Thus, by (Π) and (k) of Definition 3.1, $\lim_{n\longrightarrow\infty}\Phi(v, v_1, t/k^n) = 1$ and $\lim_{n\longrightarrow\infty}\varphi(v, v_1, t/k^n) = 0$.

It follows that $1 \geq \Phi(v, v_1, t) \geq 1$ *and* $0 \leq \varphi(v, v_1, t) \leq 0$, which implies that $v = v_1$ by (c) and (h) of Definition 3.1. This completes the proof.

Note: the uniqueness of the coincidence point will be sure when Π or σ is one-one.

The following result gives common fixed point of Π and σ with the assumption of weakly compatibility.

Theorem 4.2. *Let $(\zeta, \Phi, \varphi, \odot, ^\circ, s)$ be a complete IFbMS and $\Pi, \sigma: \zeta \longrightarrow \zeta$ be mappings satisfying the following conditions:*

(1) $\sigma(\zeta) \subseteq \Pi(\zeta)$.

(2) There is k, $0 \leq k < 1$, such that, for all $\omega, v \in \zeta$,

$$\begin{aligned} &\Phi(\sigma(\omega), \sigma(v), kt) \geq \Phi(\Pi(\omega), \Pi(v), t) \\ &\varphi(\sigma(\omega), \sigma(v), kt) \leq \varphi(\Pi(\omega), \Pi(v), t). \end{aligned} \tag{27}$$

(3) Π and σ are weakly compatible.

Then, Π and σ have a unique-common fixed point in ζ.

Proof. By the above theorem, there is a unique point of coincidence of Π and σ in ζ. That is, we can get z, v in ζ such that $v = \Pi(z) = \sigma(z)$.

Since $v = \Pi(z)$ and Π and σ are weakly compatible, so $\sigma(v) = \sigma(\Pi(z)) = \Pi(\sigma(z)) = \Pi(v)$.

Let $u = \Pi(v) = \sigma(v)$; then, u is a point of coincidence of Π and σ. Since the point of coincidence is unique, this implies that $u = v \Rightarrow v = \Pi(v) = \sigma(v)$.

Hence, v is unique-common fixed point of Π and σ. This completes the proof. □

Corollary 1. *Let $(\zeta, \Phi, \varphi, \odot, ^\circ)$ be a complete IFMS and $\Pi, \sigma: \zeta \longrightarrow \zeta$ be mappings satisfying the following conditions:*

(1) $\sigma(\zeta)\subseteq\Pi(\zeta)$.

(2) There is k, $0 \leq k < 1$, such that, for all $\omega, v \in \zeta$,

$$\begin{aligned} &\Phi(\sigma(\omega), \sigma(v), kt) \geq \Phi(\Pi(\omega), \Pi(v), t), \\ &\varphi(\sigma(\omega), \sigma(v), kt) \leq \varphi(\Pi(\omega), \Pi(v), t). \end{aligned} \tag{28}$$

(3) Π and σ are weakly compatible.

Then, Π and σ have unique common fixed point in ζ.

Proof. By putting $s = 1$ in Theorem 4.2, we get the required result. □

Corollary 2. *Let $(\zeta, \Phi, \odot)$ be a complete fuzzy b-metric space and $\Pi, \sigma: \zeta \longrightarrow \zeta$ be mappings satisfying the following conditions:*

(1) $\sigma(\zeta) \subseteq \Pi(\zeta)$.

(2) There exist $k \in [0, 1)$ such that $\forall\, \omega, v \in \zeta$,

$$\Phi(\sigma(\omega), \sigma(v), kt) \geq \Phi(\Pi(\omega), \Pi(v), t). \tag{29}$$

(3) Π and σ are weakly compatible.

Then, Π and σ have unique-common fixed point in ζ.

Proof. By putting $\varphi = O$ (i.e., φ is a zero function) in Theorem 4.2, we get the required result. □

Corollary 3 (see [31]). *Let $(\zeta, \Phi, \odot)$ be a complete FMS and $\Pi, \sigma: \zeta \longrightarrow \zeta$ be mappings satisfying the following conditions:*

(1) $\sigma(\zeta)\subseteq\Pi(\zeta)$.

(2) There exist $k \in [0, 1)$ such that $\forall\, \omega, v \in \zeta$,

$$\Phi(\sigma(\omega), \sigma(v), kt) \geq \Phi(\Pi(\omega), \Pi(v), t). \tag{30}$$

(3) Π and σ are weakly compatible.

Then, Π and σ have unique-common fixed point in ζ.

Proof. By putting $\varphi = O$ (i.e., φ is a zero function) and $s = 1$ in Theorem 4.2, we get the required result. □

Example. Let $\zeta = R$ and $\Pi: \zeta \longrightarrow \zeta$ be a self-map on ζ defined as $\Pi(\omega) = 3\omega, \forall\omega \in \zeta$.

Define $\Phi, \varphi: \zeta^2 \times [0, \infty) \longrightarrow [0, 1]$ as

$$\begin{aligned} \Phi(\omega, v, t) &= \begin{cases} \dfrac{t}{t + |\omega - v|}, & \text{if } t > 0, \\ 0, & \text{if } t = 0, \end{cases} \\ \varphi(\omega, v, t) &= \begin{cases} \dfrac{|\omega - v|}{t + |\omega - v|}, & \text{if } t > 0, \\ 1, & \text{if } t = 0. \end{cases} \end{aligned} \tag{31}$$

By [30] and Example 3.2.1, it is clear that $(\zeta, \Phi, \varphi, \odot, ^\circ, s)$ is complete IFbMS, where $a = \min(a,b)$, $a^\circ b = \max(a,b)$, and $\forall a, b \in [0,1]$. Note that conventional Banach's contraction principle fails to find the fixed point of Π as Π is not a contraction.

Now, define $\sigma\colon \zeta \longrightarrow \zeta$ as $\sigma(\omega) = 2\omega, \forall \omega \in \zeta$.

It is evident that $\sigma(\zeta) \subseteq \Pi(\zeta)$ and Π and σ are weakly compatible. Then,

$$\begin{aligned}\Phi(\sigma(\omega), \sigma(v), 2t/3) &= \frac{2t/3}{2t/3 + |2\omega - 2v|} \\ &= \frac{t/3}{t/3 + |\omega - v|} \\ &\geq \frac{t}{t + |3\omega - 3v|}, \\ &= \Phi(\Pi(\omega), \Pi(v), t) \\ \varphi(\sigma(\omega), \sigma(v), 2t/3) &= \frac{|2\omega - 2v|}{2t/3 + |2\omega - 2v|} \\ &= \frac{|\omega - v|}{t/3 + |\omega - v|} \\ &\leq \frac{|3\omega - 3v|}{t + |3\omega - 3v|} \\ &= \varphi(\Pi(\omega), \Pi(v), t).\end{aligned} \tag{32}$$

Thus, all the conditions of Theorem 4.2 are satisfied for $k = 2/3$; hence, Π and σ have a unique-common fixed point: $0 = \Pi(0) = \sigma(0)$.

5. Application

Now, as an application of coincidence theorem, we give the following theorem.

Theorem. *Let $F, G\colon R \times I \longrightarrow R$ and $f\colon R \longrightarrow R$ be continuous mappings such that*

$$G(\omega, u) = F(\omega, u) + f(\omega), \tag{33}$$

where $I = \{u \in R\colon a \leq u \leq b, a, b \in R\}$.

Let $C(I)$ be the collection of all continuous functions defined from I into R. Suppose that, for each $\omega \in C(I)$, there exists $v \in C(I)$, such that $(fv)(u) = G(\omega(u), u)$ and $\{f\omega\colon \omega \in C(I)\}$ is complete. If there exists a number $k \in [0,1)$ such that, for all $\omega_1, \omega_2 \in C(I)$ and $u \in I$,

$$|G(\omega_1(u), u) - G(\omega_2(u), u)| \leq k|f(\omega_1(u)) - f(\omega_2(u))|, \tag{34}$$

then the equation,

$$F(\omega, u) = 0, \tag{35}$$

defines a continuous function ω in terms of u.

Proof. Let $\zeta = Y = C(I)$.

Define $\Phi, \varphi\colon \zeta^2 \times [0, \infty) \longrightarrow [0,1]$ as

$$\begin{aligned}\Phi_{\varpi}(\omega, v, t) &= \begin{cases} \dfrac{t}{t + \max_{u\in I}|\omega(u) - v(u)|}, & \text{if } t > 0, \\ 0, & \text{if } t = 0, \end{cases} \\ \varphi_{\varpi}(\omega, v, t) &= \begin{cases} \dfrac{\max_{u\in I}|\omega(u) - v(u)|}{t + \max_{u\in I}|\omega(u) - v(u)|}, & \text{if } t > 0, \\ 1, & \text{if } t = 0. \end{cases}\end{aligned} \tag{36}$$

Define mapping $\sigma\colon \zeta \longrightarrow \zeta$ as follows:

$$\sigma(\omega(u)) = G(\omega(u), u). \tag{37}$$

Then, by assumption, $f(\zeta) = \{f\omega\colon \omega \in \zeta\}$ is complete. Let $\omega^{\odot} \in \sigma(\zeta)$; then, $\omega^{\odot} = \sigma\omega$ for $\omega \in \zeta$ and $\omega^{\odot}(u) = \sigma\omega(u) = G(\omega(u), u)$.

By assumptions, there exists $v \in \zeta$ such that $\sigma\omega(u) = G(\omega(u), u) = fv(u)$.

Hence, $\sigma(\zeta) \subseteq f(\zeta)$.

Since

$$\begin{aligned}|(\sigma\omega)(u) - (\sigma v)(u)| &= |G(\omega(u), u) - G(v(u), u)| \\ &\leq k|(f\omega)(u) - (fv)(u)| \\ &\leq k(\max_{u\in I}|(f\omega)(u) - (fv)(u)|),\end{aligned} \tag{38}$$

it further implies that

$$\begin{aligned}&\max_{u\in I}|(\sigma\omega)(u) - (\sigma v)(u)| \leq k(\max_{u\in I}|(f\omega)(u) - (fv)(u)|) \\ &\Rightarrow \frac{\max_{u\in I}|(\sigma\omega)(u) - (\sigma v)(u)|}{kt} \leq \frac{(\max_{u\in I}|(f\omega)(u) - (fv)(u)|)}{t} \\ &\Rightarrow \frac{kt}{\max_{u\in I}|(\sigma\omega)(u) - (\sigma v)(u)|} \geq \frac{t}{(\max_{u\in I}|(f\omega)(u) - (fv)(u)|)} \\ &\Rightarrow \frac{kt}{kt + (\max_{u\in I}|(\sigma\omega)(u) - (\sigma v)(u)|)} \geq \frac{t}{t + (\max_{u\in I}|(f\omega)(u) - (fy)(u)|)} \Rightarrow \Phi(\sigma\omega, \sigma v, kt) \geq \Phi(f\omega, fv, t).\end{aligned} \tag{39}$$

Also, inequality (39) implies that

$$\frac{\max_{u\in I}|(\sigma\omega)(u)-(\sigma v)(u)|}{kt}\le\frac{(\max_{u\in I}|(f\omega)(u)-(fv)(u)|)}{t}$$
$$\Rightarrow\frac{\max_{u\in I}|(\sigma\omega)(u)-(\sigma v)(u)|}{kt+\max_{u\in I}|(\sigma\omega)(u)-(\sigma v)(u)|}\le\frac{(\max_{u\in I}|(f\omega)(u)-(fv)(u)|)}{t+(\max_{u\in I}|(f\omega)(u)-(fv)(u)|)} \tag{40}$$
$$\Rightarrow\ \varphi(\sigma\omega,\sigma v,kt)\le\varphi(f\omega,fv,t).$$

Hence, all the conditions of theorem (4.1) are satisfied to obtain a continuous function $z\colon I\longrightarrow R$ such that $\sigma z=fz$. Then,

$$G(z(u),u)-f(z(u))=0, \tag{41}$$

where z will be a solution of the equation $F(z,u)=0$. □

Remark. If we consider an implicit form $F(\omega,u)=10\omega^5(u-1)+u$, then, by the assumptions $G(\omega,u)=10\omega^5(u-1)+u+90\omega^5$ and $f(\omega(u))=90\omega^5$ in Theorem 4.3, we can easily obtain the explicit representation as $\omega=\sqrt{[5]}u/10(1-u)$.

For a nontrivial example, consider the implicit equation,

$$u+\sin\left(8\omega^5u\right)-\omega^5=0, \tag{42}$$

in the space $C([-1/9,t1/9])$. Let

$$F(\omega,u)=u+\sin\left(8\omega^5u\right)-\omega^5,$$
$$f(\omega)=5\omega^5-5, \tag{43}$$

where $F\colon R\times([-1/9,t1/9])\longrightarrow R$ and $f\colon R\longrightarrow R$. Then, let $G(\omega,u)=u+\sin(8\omega^5u)+4\omega^5-5$. Define $\sigma\colon C([-1/9,t1/9])\longrightarrow C([-1/9,t1/9])$ as

$$\sigma(\omega(u))=G(\omega(u),u)=u+\sin\left(8\omega^5(u)u\right)+4\omega^5(u)-5. \tag{44}$$

Here, $f(\omega)=5\omega^5-5$ implies that $f(R)=R$.

Now,

$$\left|\sigma\omega_1-\sigma\omega_2\right|=\left|G(\omega_1,u)-G(\omega_2,u)\right|=|u+\sin\left(8\omega_1^5u\right)+4\omega_1^5-5-u-\sin\left(8\omega_2^5u\right)-4\omega_2^5+5|$$
$$\le|\sin\left(8\omega_1^5u\right)-\sin\left(8\omega_2^5u\right)+4\omega_1^5-4\omega_2^5|$$
$$\le|\sin\left(8\omega_1^5u\right)-\sin\left(8\omega_2^5u\right)|+4|\omega_1^5-\omega_2^5| \tag{45}$$
$$\le 8|u||\omega_1^5-\omega_2^5|+4|\omega_1^5-\omega_2^5|$$
$$\le\frac{44}{45}|5\omega_1^5-5-5\omega_2^5+5|.$$

Hence, all the conditions of Theorem 4.3 are satisfied. To apply Theorem 4.1, choose an initial guess $\omega_0(u)=0$; then,

$$\sigma(\omega_0(u))=G(\omega_0(u),u)=u-5=f(\omega_1(u))=5\omega_1^5-5. \tag{46}$$

This implies that $\omega_1(u)=\sqrt{[5]}u/5$.

So,

$$\begin{aligned}
\sigma(\omega_1(u)) = G(\omega_1(u), u) &= u + \sin\left(8\omega_1^5 u\right) + 4\omega_1^5 - 5\\
&= u + \sin\left(8\frac{u^2}{5}\right) + 4\left(\frac{u}{5}\right) - 5,\\
f(\omega_2) &= u + \sin\left(8\frac{u^2}{5}\right) + 4\left(\frac{u}{5}\right) - 5,\\
5\omega_2^5(u) &= u + \sin\left(8\frac{u^2}{5}\right) + 4\left(\frac{u}{5}\right),\\
\Rightarrow \omega_2(u) &= \sqrt[5]{\frac{\sin\left(8u^2/5\right) + 9(u/5)}{5}}.
\end{aligned} \tag{47}$$

Now,

$$\begin{aligned}
\sigma(\omega_2(u)) &= G(\omega_2(u), u) = u + \sin\left(8\omega_2^5 u\right) + 4\omega_2^5 - 5,\\
f(\omega_3) &= u + \sin\, 8\left(\frac{u\, \sin\left(8\left(u^2/5\right)\right) + 9\left(u^2/5\right)}{5}\right) + 4\left(\frac{\sin\, 8\left(u^2/5\right) + 9(u/5)}{5}\right) - 5,\\
\Rightarrow \omega_3 &= \sqrt[5]{\frac{u + \sin\, 8\left(u\, \sin\left(8\left(u^2/5\right)\right) + 9\left(u^2/5\right)/5\right) + 4\left(\sin\, 8\left(u^2/5\right) + 9(u/5)/5\right)}{5}},
\end{aligned} \tag{48}$$

is an approximation of the explicit form of $F(\omega, u)$.

It is worthwhile to point out here that the application given in the above remark is not found in the literature even as an application of the following corollary of Theorem 4.1 regarding metric spaces.

Corollary. *Let ζ be a nonempty set and (Y, ϖ) be a b-metric space and $f, \sigma: \zeta \longrightarrow Y$ be mappings satisfying the following conditions:*

(1) $\sigma(\zeta) \subseteq f(\zeta)$.

(2) There exist $k \in [0, 1)$ such that $\forall\, \omega, v \in \zeta$:

$$\varpi(\sigma(\omega), \sigma(v)) \leq k\varpi(f(\omega), f(v)). \tag{49}$$

If $f(\zeta)$ or $\sigma(\zeta)$ is complete, then there exist a point $z \in \zeta$ such that $f(z) = \sigma(z)$. Moreover, f and σ have a unique point of coincidence.

6. Conclusion

Metric spaces play a vital role in functional analysis and its related concepts. Modern and latest developments are due to fuzzy theory, fuzzy logic, and its vast applications in almost all fields of research. This motivated us to define metric-type spaces in fuzzy version. We called this IFbMSs. Moreover, interesting nontrivial examples are created as well. These spaces generalize fuzzy b-metric spaces and IFMSs which are already generalized forms of classic metric spaces. Since fixed-point techniques have a lot of wonderful applications in science and technology, so in this research article, we intended to put our efforts in obtaining coincidence points and common fixed points in IFbMS. In such a way, many useful, present, and conventional results are presented as consequences of our results. Furthermore, as an application we have proved an implicit function theorem with the help of our main result. This activity will definitely motivate researchers to do further work in these spaces and fixed-point theory.

Abbreviations

FS: Fuzzy set
FMS: Fuzzy-metric-space
IFMS: Intuitionistic-fuzzy-metricspace
IFbMS: Intuitionistic-fuzzy-b-metric-space

Data Availability

The data used to support the finding of the study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this work.

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