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Research Article

New Fixed Point Theorems for Admissible Hybrid Maps

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The aim of this work is to investigate the concept of a new hybrid Suzuki contractive by using the Rus-Reich-Ćirić-type interpolative mappings in b-metric spaces. We seek the presence and uniqueness of a fixed point of such new contraction type mappings and prove some related results. We further give an application of Ulam-Hyers-type stability to show the well-posedness of our results.

1. Introduction and Preliminaries

Fixed point hypothesis has been a considerable area of research for mathematics and other sciences for the last century. It is the basis of functional analysis in mathematics, which is one of the critical topics of mathematics. The first concept of fixed point theory is knowing to appear in the work of Liouville in 1837 and Picard in 1890. But the main fixed point theorem was introduced by Banach [1]. The theorem is named after Banach. There are many generalizations of Banach theorem in the literature. In 1968, one of the most famous generalizations due to know, Kannan [2] introduced a new and useful contraction using Banach's theorem. Suzuki [3] introduced important extensions of Banach's main theorem, which we refer to [4–6]. In one of these studies [7], the researchers investigated a new extensive result by using simulation function. On the other hand, in [8], by using other auxiliary functions, called the Wardowski functions, they observed a contraction that combines both linear and nonlinear contractions. We also mention that in [9], the author obtained a fixed point theorem without the Picard operator. For more interesting results, see, e.g., [10–19]. In addition, Banach's fixed point theorem is a significant mean in the theory of metric spaces. The metric concept has been generalized from different angles. One of the significant generalizes is defined b-metric which was defined as follows.

Definition 1 (see [20, 21]). Let $\mathscr{L}$ be a (nonempty) set and $s \geq 1$ a real number. A function $b : \mathscr{L} \times \mathscr{L} \longrightarrow [0,\infty)$ is a b-metric space on $\mathscr{L}$ if following conditions are satisfied:

(i) $b(r, v) = 0$, if $r = v$

(ii) $b(r, v) = b(v, r)$

(iii) $b(r, v) \leq s[b(r, q) + b(q, v)]$, for every $r, v, q \in \mathscr{L}$

In this case, the pair $(\mathscr{L}, b, s)$ is called a b-metric space.

We recollect some basic notions that are used in our study.

A map $\varphi : [0,\infty) \longrightarrow [0,\infty)$ is defined as a comparison function if it is increasing and $\varphi^q(z) \longrightarrow 0, q \longrightarrow \infty$, for any $z \in [0, \infty)$. We state by Φ the class of all the comparison functions $\varphi : [0,\infty) \longrightarrow [0,\infty)$, see, e.g., [22–24]. Defined by $\Psi = \{\psi : [0,\infty) \longrightarrow [0,\infty) \,|\psi \text{ is the } b\text{-comparison function}\}$.

Lemma 2 (see [22, 23]). *For a comparison function, $\varphi : [0, \infty) \longrightarrow [0,\infty)$ satisfying the below statements take*

(1) every iterate φ^l of $\varphi, l \geq 1$ is a comparison function

(2) φ is continuous

(3) $\varphi(z) < z$, *for any* $z > 0$

Lemma 3 (see [25]). *If* $\varphi : [0,\infty) \longrightarrow [0,\infty)$ *is a b-comparison function, then,*

(1) *the series* $\sum_{l=0}^{\infty} s^l \varphi^l(z)$ *converges for any* $z \in [0,\infty)$

(2) *the function* $b_s : [0,\infty) \longrightarrow [0,\infty)$ *defined by* $b_s(z) = \sum_{l=0}^{\infty} s^l \varphi^l(z), z \in [0,\infty)$ *is increasing and continuous at 0.*

We state that any b-comparison function is a comparison function because of Lemma 2.3, and thus, in Lemma 2.2 any b-comparison function ψ *satisfies* $\psi(z) < z$.

Karapinar [26] introduced interpolation Kannan-type contraction generalized from the famous Kannan fixed point theorem by using interpolative operator. In the following, the common fixed point theorem for this contraction was obtained [27]. In [28], the authors extended the results in [26] by introducing the interpolative Reich-Rus-Ćirić contractive in a general framework, in the setting of partial metric space. In addition, the interpolative Hardy-Rogers-type contractive was defined and discussed in [28]. The contraction, defined in [29], was generalized in [30] by involving the admissibility into the contraction inequality. Furthermore, in [31], hybrid contractions were considered. Indeed, the notion of hybrid contraction here refers to combination of interpolative (nonlinear) contraction and linear contraction. For more interesting papers, see [32–34].

In 2019, inspired by interpolative contraction, researchers [35] obtained and published a hybrid contractive that integrates Reich-Rus-Ćirić-type contractive and interpolative-type mappings. In particular, this approach was applied for Pata-Suzuki-type contraction in [36]. On the other hand, by using hybrid contraction, a solution for a Volterra fractional integral equation was proposed in [37]. Furthermore, the hybrid contractions were discussed in a distinct abstract space, namely, Branciari-type distance space, in [38]. Another advance was recorded in [39] where the authors investigated the Ulam-type stability of this consideration. In addition, new hybrid contractions were developed in b-metric spaces [40]. As a result, as can be seen in the literature review, many papers were published on the subject of interpolative contraction and hybrid contraction inspired by it. The contractions are a current study topic for fixed point theory. Therefore, the results of the study contribute to the existing literature.

Now we give the idea of α-admissibility defined by Samet et al. [41] and Karapnar and Samet [42].

Definition 4. A mapping $\mathrm{M} : \mathscr{L} \longrightarrow \mathscr{L}$ is defined α-admissible if for each $r, v \in \mathscr{L}$ we have

$$\alpha(r, v) \geq 1 \Rightarrow \alpha(\mathrm{M}r, \mathrm{M}v) \geq 1, \tag{1}$$

where $\alpha : \mathscr{L} \times \mathscr{L} \longrightarrow [0,\infty)$ is a given function.

The mapping of w-orbital admissibility was presented by Popescu [43] as a modification of α-admissibility as follows:

Definition 5. Let $w : \mathscr{L} \times \mathscr{L} \longrightarrow [0,\infty)$ be a mapping and $\mathscr{L} \neq \varnothing$. A map $\mathrm{M} : \mathscr{L} \longrightarrow \mathscr{L}$ is defined w-orbital admissible if for every $r \in \mathscr{L}$, we get

$$w(r, \mathrm{M}r) \geq 1 \Rightarrow w\left(\mathrm{M}r, \mathrm{M}^2 r\right) \geq 1. \tag{2}$$

The following condition has often been considered on account of refraining from the continuity of the concerned contractive mappings.

Definition 6. A space $(\mathscr{L}, \mathrm{b}, s)$ is defined w-regular, if $\{r_{\mathrm{q}}\}$ is a sequence in $\mathscr{L}$ such that $\alpha(r_{\mathrm{q}}, r_{\mathrm{q}+1}) \geq 1$ for all $\mathrm{q} \in \mathbb{N}$ and $r_{\mathrm{q}} \longrightarrow r \in \mathscr{L}$ as $\mathrm{q} \longrightarrow \infty$; then, there exists a subsequence $\{r_{\mathrm{q}(p)}\}$ of $\{r_{\mathrm{q}}\}$ such that $w(r_{\mathrm{q}(p)}, r) \geq 1$ for all p.

The framework of this study is organized into four sections. After the first introduction section, in Section 2, we introduced the definitions, theorems, and some results on the Ćirić-Rus-Reich-Suzuki-type hybrid. In Section 3, we give an application Ulam-Hyers-type stability to show the well-posedness for our fixed point theorem. Finally, in the last section, the conclusions are drawn.

2. Main Results

We begin with the definition of the Ćirić-Rus-Reich-Suzuki-type hybrid contraction:

Definition 7. Let $(\mathscr{L}, \mathrm{b}, s)$ be a b-metric space and $w : \mathscr{L} \times \mathscr{L} \longrightarrow [0,\infty)$ be a function. A map $\mathrm{M} : \mathscr{L} \longrightarrow \mathscr{L}$ is a Ćirić-Rus-Reich-Suzuki-type hybrid contraction (CRRS-type hybrid contraction) if there exist $\psi \in \Psi$ such that

$$\frac{1}{2s} b(r, Mr) \leq b(r, v) \Rightarrow w(r, v) b(Mr, Mv) \leq \psi(\chi_M^a(r, v)), \tag{3}$$

for each $r, v \in \mathscr{L}$, where $a \geq 0$ and $\rho_i \geq 0, i = 1,2,3$, such that $\rho_1 + \rho_2 + \rho_3 = 1$,

$$\chi_{\mathrm{M}}^a(r, v) = \begin{cases} \left[\varrho_1 (b(r, v))^a + \varrho_2 (b(r, \mathrm{M}r))^a + \varrho_3 (b(v, \mathrm{M}v))^a\right]^{1/a}, \text{for } a > 0, r \neq v \\ (b(r, v))^{\rho_1} (b(r, \mathrm{M}r))^{\rho_2} (b(v, \mathrm{M}v))^{\rho_3}, \text{for } a = 0, r, v \in \mathscr{L} \setminus Fix(M). \end{cases} \tag{4}$$

Theorem 8. *Let* $(\mathscr{L}, b, s)$ *be a complete b-metric space and* w *-orbital admissible map also* $w(r_0, Mr_0) \geq 1$ *for some* $r_0 \in \mathscr{L}$. *Given that* $M : \mathscr{L} \longrightarrow \mathscr{L}$ *be a CRRS-type hybrid contraction satisfying one of the following conditions:*

(h_1) $(\mathscr{L}, b, s)$ *is* w*-regular*

(h_2) M *is continuous*

(h_3) M^2 *is continuous and* $w(r, Mr) \geq 1$, *where* $r \in Fix(M^2)$.

Thereupon, M *admits a fixed point in* $\mathscr{L}$.

Proof. We install an iterative sequence $\{r_q\}$ of points such that $M^q(r_0) = r_q$ for $q = 0,1,2,\cdots$ and $r_0 \in \mathscr{L}$ with $w(r_0, Mr_0) \geq 1$. If $r_{q_0} = r_{q_{0+1}}$ for some integers q_0, then $r_{q_0} = Mr_{q_0}$. Thus, suppose that $r_q \neq r_{q+1}$, as M is w-orbital admissible, then $w(r_0, Mr_0) = w(r_0, r_1) \geq 1$ implies that $w(r_1, Mr_1) = w(r_1, r_2) \geq 1$. Continuing this process, we get

$$w(r_q, r_{q+1}) \geq 1. \tag{5}$$

Condition 1: $a > 0$, by taking $\chi^a_M(r, v)$ choosing $r = r_{q-1}$ and $v = Mr_{q-1} = r_q$ in (3) we get

$$\frac{1}{2s} b(r_{q-1}, Mr_{q-1}) = \frac{1}{2s} b(r_{q-1}, r_q) \leq b(r_{q-1}, r_q) \Rightarrow, \tag{6}$$

$$w(r_{q-1}, r_q) b(Mr_{q-1}, Mr_q) \leq \psi(\chi^a_M(r_{q-1}, r_q)), \tag{7}$$

where

$$\begin{aligned}\chi^a_M(r_{q-1}, Mr_{q-1}) &= \left[\varrho_1(b(r_{q-1}, Mr_{q-1}))^a + \varrho_2(b(r_{q-1}, Mr_{q-1}))^a \right. \\ &\quad \left. + \varrho_3(b(Mr_{q-1}, M^2 r_{q-1}))^a\right]^{1/a} = \left[\varrho_1(b(r_{q-1}, r_q))^a \right. \\ &\quad \left. + \varrho_2(b(r_{q-1}, r_q))^a + \varrho_3(b(r_q, r_{q+1}))^a\right]^{1/a}.\end{aligned} \tag{8}$$

Whereupon, we deduce that

$$\begin{aligned}b(r_q, r_{q+1}) &\leq w(r_{q-1}, r_q) b(Mr_{q-1}, Mr_q) \leq \psi(\chi^a_M(r_{q-1}, r_q)) \\ &= \psi\left(\left[\varrho_1(b(r_{q-1}, r_q))^a + \varrho_2(b(r_{q-1}, r_q))^a + \varrho_3(b(r_q, r_{q+1}))^a\right]^{1/a}\right) \\ &= \psi\left(\left[(\varrho_1 + \varrho_2)(b(r_{q-1}, r_q))^a + \varrho_3(b(r_q, r_{q+1}))^a\right]^{1/a}\right).\end{aligned} \tag{9}$$

If we have given that $b(r_q, r_{q+1}) \geq b(r_{q-1}, r_q)$, then, accompanying that ψ is nondecreasing with (9), we get

$$\begin{aligned}b(r_q, r_{q+1}) &\leq \psi\left(\left[(\varrho_1 + \varrho_2)(b(r_{q-1}, r_q))^a + \varrho_3(b(r_q, r_{q+1}))^a\right]^{1/a}\right) \\ &\leq \psi\left(\left[(\varrho_1 + \varrho_2)(b(r_q, r_{q+1}))^a + \varrho_3(b(r_q, r_{q+1}))^a\right]^{1/a}\right) \\ &= \psi\left(\left[(\varrho_1 + \varrho_2 + \varrho_3)(b(r_q, r_{q+1}))^a\right]^{1/a}\right) \\ &= \psi\left(\left[(b(r_q, r_{q+1}))^a\right]^{1/a}\right) = \psi(b(r_q, r_{q+1})) < b(r_q, r_{q+1}),\end{aligned} \tag{10}$$

which is a contradiction. Thus, we obtain

$$b(r_q, r_{q+1}) < b(r_{q-1}, r_q). \tag{11}$$

As a result, from (9), we will turn into

$$b(r_q, r_{q+1}) \leq \psi(b(r_{q-1}, r_q)) < b(r_{q-1}, r_q), \tag{12}$$

and by similarly this process, we obtain that

$$b(r_q, r_{q+1}) \leq \psi^q(b(r_0, r_1)). \tag{13}$$

for any $q \in \mathbb{N}$.

We argue that $\{r_q\}$ is a fundamental sequence in $(\mathscr{L}, b, s)$. Then, let $q, l \in \mathbb{N}$ such that $l > q$ and using the triangle inequality with (13), we take

$$\begin{aligned}b(r_q, r_l) &\leq s b(r_q, r_{q+1}) + s^2 b(r_{q+1}, r_{q+2}) + \cdots + s^{l-q} b(r_{l-1}, r_l) \\ &\leq s\psi^q(b(r_0, r_1)) + s^2\psi^{q+1}(b(r_0, r_1)) + \cdots + s^{l-q}\psi^{l-1}(b(r_0, r_1)) \\ &= \frac{1}{s^{q-1}}\left(s^q\psi^q(b(r_0, r_1)) + s^{q+1}\psi^{q+1}(b(r_0, r_1)) + \cdots + s^{l-1}\psi^{l-1}(b(r_0, r_1))\right) \\ &= \frac{1}{s^{q-1}}\sum_{q=q}^{l-1} s^q\psi^q(b(r_1, r_0)).\end{aligned} \tag{14}$$

By using Lemma 3, the series $\sum_{q=0}^{\infty} s^q\psi^q(b(r_1, r_0))$ is convergent where $H_t = \sum_{q=0}^{t} s^q\psi^q(b(r_0, r_1))$, the above inequality finds

$$b(r_q, r_l) \leq \frac{1}{s^{q-1}}(H_{l-1} - H_{q-1}) \tag{15}$$

and $q, l \longrightarrow \infty$, we obtain

$$b(r_q, r_l) \longrightarrow 0. \tag{16}$$

Thus, $\{r_q\}$ is a fundamental sequence. Accompanying this together with the fact that the space $(\mathscr{L}, b, s)$ is complete, it will imply that there exists $p \in \mathscr{L}$ such that

$$\lim_{q \longrightarrow \infty} b(r_q, p) = 0. \tag{17}$$

We argue that p is a fixed point of M.

If the suppose (h_1) takes, we get $w(r_q, p) \geq 1$, and we assert that

$$\text{either} \frac{1}{2s} b(r_q, Mr_q) \leq b(r_q, p) \text{or} \frac{1}{2s} b(Mr_q, M(Mr_q)) \leq b(Mr_q, p), \tag{18}$$

for every $q \in \mathbb{N}$. Since, if we have given that

$$\frac{1}{2s} b(r_q, Mr_q) > b(r_q, p) \text{and} \frac{1}{2s} b(Mr_q, M(Mr_q)) > b(Mr_q, p), \tag{19}$$

then, by using conditions of b-metric spaces $(\mathscr{L}, b, s)$, since the sequence $\{b(r_q, r_{q+1})\}$ is decreasing, we write that

$$\begin{aligned}b(r_q, r_{q+1}) &= b(r_q, Mr_q) \leq s(b(r_q, p) + b(p, Mr_q)) < \frac{1}{2} b(r_q, Mr_q) + \frac{1}{2} b(Mr_q, M(Mr_q)) \\ &= \frac{1}{2} b(r_q, r_{q+1}) + \frac{1}{2} b(r_{q+1}, r_{q+2}) < \frac{1}{2} b(r_q, r_{q+1}) + \frac{1}{2} b(r_q, r_{q+1}) = b(r_q, r_{q+1})\end{aligned} \tag{20}$$

a contradiction. Therefore, for all $q \in \mathbb{N}$, either

$$\frac{1}{2s} b\left(r_q, Mr_q\right) \le b\left(r_q, p\right), \quad (21)$$

or

$$\frac{1}{2s} b\left(Mr_q, M\left(Mr_q\right)\right) \le b\left(Mr_q, p\right) \quad (22)$$

provides. In the condition that (21) takes, then by (3), we get

$$\begin{aligned} b\left(r_{q+1}, Mp\right) &\le w\left(r_q, p\right) b\left(Mr_q, Mp\right) \\ &\le \psi\left[\varrho_1\left(b\left(r_q, p\right)\right)^a + \varrho_2\left(b\left(r_q, Mr_q\right)\right)^a + \varrho_3(b(p, Mp))^a\right]^{1/a} \\ &< \left[\varrho_1\left(b\left(r_q, p\right)\right)^a + \varrho_2\left(b\left(r_q, Mr_q\right)\right)^a + \varrho_3(b(p, Mp))^a\right]^{1/a} \\ &= \left[\varrho_1\left(b\left(r_q, p\right)\right)^a + \varrho_2\left(b\left(r_q, r_{q+1}\right)\right)^a + \varrho_3(b(p, Mp))^a\right]^{1/a}. \end{aligned} \quad (23)$$

If the second condition, (22) holds, we obtain

$$\begin{aligned} b\left(r_{q+2}, Mp\right) &\le w\left(r_{q+1}, p\right) b\left(M^2 r_q, Mp\right) \\ &\le \psi\left[\varrho_1\left(b\left(Mr_q, p\right)\right)^a + \varrho_2\left(b\left(Mr_q, M^2 r_q\right)\right)^a + \varrho_3(b(p, Mp))^a\right]^{1/a} \\ &< \left[\varrho_1\left(b\left(Mr_q, p\right)\right)^a + \varrho_2\left(b\left(Mr_q, M^2 r_q\right)\right)^a + \varrho_3(b(p, Mp))^a\right]^{1/a} \\ &= \left[\varrho_1\left(b\left(r_{q+1}, p\right)\right)^a + \varrho_2\left(b\left(r_{q+1}, r_{q+2}\right)\right)^a + \varrho_3(b(p, Mp))^a\right]^{1/a}. \end{aligned} \quad (24)$$

Thereupon, taking $q \longrightarrow \infty$ in (23) and (24),

$$b(p, Mp) < \varrho_3^{1/a} b(p, Mp) \le b(p, Mp) \quad (25)$$

which is contraction. Therefore, we get that $b(p, Mp) = 0$ that is $p = Mp$.

If the presume (h_2) is correct, and the map M is continuous, we get

$$Mp = \lim_{q \longrightarrow \infty} Mr_q = \lim_{q \longrightarrow \infty} r_{q+1} = p. \quad (26)$$

In case that last supposition, (h_3) holds, from above, we write $M^2 p = \lim_{q \longrightarrow \infty} M^2 r_q = \lim_{q \longrightarrow \infty} r_{q+2} = p$, we want to show that $Mp = p$. Let us pretend otherwise, that is, $p \ne Mp$ from

$$\frac{1}{2s} b\left(Mp, M^2 p\right) = \frac{1}{2s} b(Mp, p) \le b(Mp, p) \quad (27)$$

using (3) we obtain that

$$\begin{aligned} b(p, Mp) = b\left(M^2 p, Mp\right) &\le w(Mp, p) b\left(M^2 p, Mp\right) \\ &\le \psi\left[\varrho_1(b(Mp, p))^a + \varrho_2\left(b\left(Mp, M^2 p\right)\right)^a + \varrho_3(b(p, Mp))^a\right]^{1/a} \\ &< \left[\varrho_1(b(Mp, p))^a + \varrho_2\left(b\left(Mp, M^2 p\right)\right)^a + \varrho_3(b(p, Mp))^a\right]^{1/a} \\ &= \left[(\varrho_1 + \varrho_2 + \varrho_3)(b(p, Mp))^a\right]^{1/a} = b(p, Mp) \end{aligned} \quad (28)$$

a contradiction. Eventually, $p = Mp$.

Condition 2: if $a = 0$, in the equation $\chi_M^a(r, v)$ taking $r = r_{q-1}$ and $v = Mr_{q-1} = r_q$ in (3) we write

$$\frac{1}{2s} b\left(r_{q-1}, Mr_{q-1}\right) = \frac{1}{2s} b\left(r_{q-1}, r_q\right) \le b\left(r_{q-1}, r_q\right) \Rightarrow, \quad (29)$$

$$\begin{aligned} b\left(r_q, r_{q+1}\right) &\le w\left(r_{q-1}, r_q\right) b\left(Mr_{q-1}, Mr_q\right) \le \psi\left(\chi_M^a\left(r_{q-1}, r_q\right)\right) \\ &= \psi\left(\left[b\left(r_{q-1}, r_q\right)\right]^{\rho_1} \cdot \left[b\left(r_{q-1}, Mr_{q-1}\right)\right]^{\rho_2} \cdot \left[b\left(r_q, Mr_q\right)\right]^{\rho_3}\right) \\ &< \left[b\left(r_{q-1}, r_q\right)\right]^{\rho_1} \cdot \left[b\left(r_{q-1}, r_q\right)\right]^{\rho_2} \cdot \left[b\left(r_q, r_{q+1}\right)\right]^{\rho_3}. \end{aligned} \quad (30)$$

From (30), we find

$$\left(b\left(r_q, r_{q+1}\right)\right)^{1-\varrho_3} < \left(b\left(r_{q-1}, r_q\right)\right)^{\varrho_1+\varrho_2} \quad (31)$$

and from $\varrho_1 + \varrho_2 + \varrho_3 = 1$, we attain that $b(r_q, r_{q+1}) < b(r_{q-1}, r_q)$ for every $q \in \mathbb{N}$. Using (30), we take

$$b\left(r_q, r_{q+1}\right) \le \psi\left(b\left(r_{q-1}, r_q\right)\right) \quad (32)$$

and as in condition 1, we can prove that

$$b\left(r_q, r_{q+1}\right) \le \psi^q\left(b\left(r_0, r_1\right)\right). \quad (33)$$

Since the equal methods as in the case of $a > 0$, we clearly prove that $\{r_q\}$ is a fundamental sequence in a complete b-metric space. Additionally, for $p \in \mathscr{L}$ so, $\lim_{q \longrightarrow \infty} b(r_q, p) = 0$ also we assert that $p = Mp$. In the meanwhile, $(\mathscr{L}, b, s)$ is w-regular; thus, as $\{r_q\}$ confirm (5), and $w(r_q, r_{q+1}) \ge 1$ for each $q \in \mathbb{N}$, we obtain $w(r_q, p) \ge 1$. Moreover, as in the proof of condition 1, we know that either

$$\frac{1}{2s} b\left(r_q, Mr_q\right) \le b\left(r_q, p\right), \quad (34)$$

or

$$\frac{1}{2s} b\left(Mr_q, M\left(Mr_q\right)\right) \le b\left(Mr_q, p\right) \quad (35)$$

holds, for each $q \in \mathbb{N}$. If (34) is taken, we conclude that

$$\begin{aligned} b\left(r_{q+1}, Mp\right) &\le w\left(r_q, p\right) b\left(Mr_q, Mp\right) \le \psi\left(\left[b\left(r_q, p\right)\right]^{\varrho_1} \cdot \left[b\left(r_q, Mr_q\right)\right]^{\varrho_2} \cdot [b(p, Mp)]^{\varrho_3}\right), \\ &= \psi\left(\left[b\left(r_q, p\right)\right]^{\varrho_1} \cdot \left[b\left(r_q, r_{q+1}\right)\right]^{\varrho_2} \cdot [b(p, Mp)]^{\varrho_3}\right), \\ &< \left[b\left(r_q, p\right)\right]^{\varrho_1} \cdot \left[b\left(r_q, r_{q+1}\right)\right]^{\varrho_2} \cdot [b(p, Mp)]^{\varrho_3}, \end{aligned} \quad (36)$$

Let us assume that inequality (35) is satisfied, then

$$\begin{aligned} b\left(r_{q+2}, Mp\right) &\le w\left(r_{q+1}, p\right) b\left(M^2 r_q, Mp\right) \\ &\le \psi\left(\left[b\left(Mr_q, p\right)\right]^{\varrho_1} \cdot \left[b\left(Mr_q, M^2 r_q\right)\right]^{\varrho_2} \cdot [b(p, Mp)]^{\varrho_3}\right), \\ &= \psi\left(\left[b\left(r_{q+1}, p\right)\right]^{\varrho_1} \cdot \left[b\left(r_{q+1}, r_{q+2}\right)\right]^{\varrho_2} \cdot [b(p, Mp)]^{\varrho_3}\right), \\ &< \left[b\left(r_{q+1}, p\right)\right]^{\varrho_1} \cdot \left[b\left(r_{q+1}, r_{q+2}\right)\right]^{\varrho_2} \cdot [b(p, Mp)]^{\varrho_3}, \end{aligned} \quad (37)$$

Then, getting to the limit, we conclude that $b(p, Mp) = 0$, and $p = Mp$. Now, the continuity of M implies $p = Mp$ (from condition 1). Therefore, supposition (h_3) lead to $M^2 p = \lim_{q \longrightarrow \infty} M^2 r_q = \lim_{q \longrightarrow \infty} r_{q+2} = p$. We will prove that $Mp = p$. Let's presume otherwise, that is, $p \neq Mp$

$$\frac{1}{2s} b\left(Mp, M^2 p\right) = \frac{1}{2s} b(Mp, p) \leq b(Mp, p) \tag{38}$$

using (3) we find that

$$\begin{aligned} b(p, Mp) = b\left(M^2 p, Mp\right) &\leq w(Mp, p) b\left(M^2 p, Mp\right) \\ &\leq \psi\left(\left[b(Mp, p)\right]^{\varrho_1} \cdot \left[b\left(Mp, M^2 p\right)\right]^{\varrho_2} \cdot \left[b(p, Mp)^{\varrho_3}\right]\right) \\ &< \left[b(Mp, p)\right]^{\varrho_1} \cdot \left[b(Mp, p)\right]^{\varrho_2} \cdot \left[b(p, Mp)^{\varrho_3}\right] = b(Mp, p), \end{aligned} \tag{39}$$

a contradiction. Consequently, $p = Mp$. Thus, the proof of the Theorem is completed. □

Theorem 9. *Adding $w(p, p^*) \geq 1$ for any $p, p^* \in Fix_M(\mathscr{L})$ and if supplying to all the hypothesis of Theorem 8, we prove the uniqueness of fixed point.*

Proof. Supposing that different p^* is fixed point of M, that is $Mp^* = p^*$ with $p \neq p^*$.In the case that $a > 0$, then, from (3) we have

$$\frac{1}{2s} b(p, Mp) = 0 \leq b(p, p^*) \text{implies} \tag{40}$$

$$\begin{aligned} b(p, p^*) &\leq w(p, p^*) b(Mp, Mp^*) \leq \psi(\chi_M^a(p, p^*)) < \chi_M^a(p, p^*) \\ &= \left[\varrho_1 (d(p, p^*))^a + \varrho_2 (b(p, Mp))^a + \varrho_3 (b(p^*, Mp^*))^a\right]^{1/a}. \end{aligned} \tag{41}$$

Thus,

$$b(p, p^*) < (\varrho_1)^{1/a} b(p, p^*) \leq b(p, p^*) \tag{42}$$

which is contradiction. In the case that $a = 0$, then, from (4) we get that

$$0 < b(p, p^*) < 0, \tag{43}$$

a contradiction. Eventually, $p = p^*$, so p is a unique fixed point of M. □

Example 1. Let $b : \mathscr{L} \times \mathscr{L} \longrightarrow [0,+\infty)$, $b(r, v) = |r - v|^2$ for every $r, v \in \mathscr{L}$ with $s = 2$ and

$$w(r, v) = \begin{cases} 4, & \text{if} r, v \in [0, 1] \\ 1, & \text{if} r = 0, v = 2 \\ 0, & \text{otherwise} \end{cases} \tag{44}$$

also, the function $\psi \in \Psi$ with $\psi(t) = t/4$. Define a mapping $M : \mathscr{L} \longrightarrow \mathscr{L}$ as

$$Mr = \begin{cases} \dfrac{1}{5}, & if r \in [0, 1] \\ \dfrac{r}{5}, & if r \in (1, 2] \end{cases} \tag{45}$$

also, $M^2 = r/10$, we get that M^2 is continuous but M is not continuous, where $\mathscr{L} = [0, 2]$.

We choose $a = 2$ and $\varrho_1 = \varrho_2 = \varrho_3 = 1/3$, then we obtain the following conditions:

(a) : For $r, v \in [0, 1]$ we get $b(Mr, Mv) = 0$, then, (3) holds

(b) : If $r = 0$ and $v = 2$

$$\frac{1}{2s} b(0, M0) = \frac{1}{100} < 4 = b(0, 2) \Rightarrow . \tag{46}$$

$$\begin{aligned} w(0, 2) b(M0, M2) = 0.04 \leq 0.3708099243547831 &= \frac{1}{4} \sqrt{\frac{1}{3}(2)^2 + \frac{1}{3}\left(\frac{1}{5}\right)^2 + \frac{1}{3}\left(2 - \frac{2}{5}\right)^2} \\ &= \psi \sqrt{\rho_1((b(r, v))^2 + \rho_2(b(r, Mr))^2 + \rho_3(b(v, Mv))^2}. \end{aligned} \tag{47}$$

Other conditions are confirmed, from $w(r, v) = 0$. Consequently, the assumptions of Theorem 8, being supplied, M has a fixed point ($r = 1/5$).

Corollary 10. *Let $(\mathscr{L}, b, s)$ be a complete b-metric space and let $M : \mathscr{L} \longrightarrow \mathscr{L}$ a continuous map satisfying the following inequality:*

$$\frac{1}{2s} b(r, Mr) \leq b(r, v) \text{ implies } b(Mr, Mv) \leq \psi(\chi_M^a(r, v)), \tag{48}$$

where $\chi_M^a(r, v)$ is defined by (4), $\psi \in \Psi$ and for all $r, v \in \mathscr{L}$, where $a \geq 0$ and $\varrho_i \geq 0, i = 1, 2, 3$ with $\varrho_1 + \varrho_2 + \varrho_3 = 1$. In the case of M or M^2 functions continuity, M admits a fixed point in $\mathscr{L}$.

Proof. It is sufficient to get $w(r, v) = 1$ for $r, v \in \mathscr{L}$ in Theorem 8. □

Corollary 11. *Let $(\mathscr{L}, b, s)$ be a complete b-metric space and let $M : \mathscr{L} \longrightarrow \mathscr{L}$ a continuous map satisfying the following inequality*

$$\frac{1}{2s} b(r, Mr) \leq b(r, v) \text{ implies } b(Mr, Mv) \leq \eta(\chi_M^a(r, v)), \tag{49}$$

where $\chi_M^a(r, v)$ is defined by (4), $\eta \in [0, 1)$ and for each $r, v \in \mathscr{L}$ where $a \geq 0$ and $\rho_i \geq 0, i = 1, 2, 3$ with $\rho_1 + \rho_2 + \rho_3 = 1$. In the event of M or M^2 functions continuity, M admits a fixed point in $\mathscr{L}$.

Proof. It is adequate get $\psi(v) = \eta v$ for any $v \geq 0$ in Corollary 10. □

Corollary 12. *Let* $(\mathscr{L}, b, s)$ *be a complete b-metric space and* $M : \mathscr{L} \longrightarrow \mathscr{L}$ *a continuous map. If there exist* $\eta \in [0, 1)$ *such that*

$$\frac{1}{2s} b(r, Mr) \le b(r, v) \text{implies} \tag{50}$$

$$b(Mr, Mv) \le \eta \sqrt[3]{b(r, v) b(r, Mr) b(v, Mv)} \tag{51}$$

for each $r, v \in \mathscr{L} \setminus Fix(M)$, *in the case of* M *or* M^2 *functions continuity,* M *admits a fixed point in* $\mathscr{L}$.

Proof. If $a = 0$, using Corollary 11, getting $\rho_1 = \rho_2 = \rho_3 = 1/3$. □

Corollary 13. *Let* $(\mathscr{L}, b, s)$ *be a complete b-metric space and* $M : \mathscr{L} \longrightarrow \mathscr{L}$ *a continuous map. If there exist* $\eta \in [0, 1)$ *such that*

$$\frac{1}{2s} b(r, Mr) \le b(r, v) \text{implies} \tag{52}$$

$$b(Mr, Mv) \le \frac{\eta}{3} (b(r, v) + b(r, Mr) + b(v, Mv)) \tag{53}$$

for each $r, v \in \mathscr{L} \setminus Fix(M)$, *in the case of* M *or* M^2 *functions continuity,* M *admits a fixed point in* $\mathscr{L}$.

Proof. By using Corollary 11, letting $\varrho_1 = \varrho_2 = \varrho_3 = 1/3$ and $a = 1$. □

Corollary 14. *Let* $(\mathscr{L}, b, s)$ *be a complete b-metric space and* $M : \mathscr{L} \longrightarrow \mathscr{L}$ *a continuous map. If there exist* $\eta \in [0, 1)$ *such that*

$$\frac{1}{2s} b(r, Mr) \le b(r, v) \text{implies} \tag{54}$$

$$b(Mr, Mv) \le \frac{\eta}{\sqrt{3}} \left(\sqrt{(b(r, v))^2 + (b(r, Mr))^2 + (b(v, Mv))^2} \right. \tag{55}$$

for each $r, v \in \mathscr{L} \setminus Fix(M)$, *in the case of* M *or* M^2 *functions continuity,* M *admits a fixed point in* $\mathscr{L}$.

Proof. By using Corollary 11, taking $\varrho_1 = \varrho_2 = \varrho_3 = 1/3$ and $a = 2$. □

3. An Application: Ulam-Hyers-Type Stability

The stability of the solution is a considerable important subject of nonlinear analysis. Recently, Ulam stability [44, 45] results in fixed point theory have been investigated heavily. In what follows. we investigate the Ulam stability of our main theorem.

Consider the following function:

$$\Upsilon : \{\gamma : [0,\infty) \longrightarrow [0,\infty) \textit{such that } \gamma \text{ is } \textit{continuous at zero with} \tag{56}$$

$$\gamma(0) = 0 \text{ and increasing}\} \tag{57}$$

Assume that $M : \mathscr{L} \longrightarrow \mathscr{L}$ is a map on a b-metric spaces $(\mathscr{L}, b, s)$. The fixed point problem of M is to notice an $r \in \mathscr{L}$ such that

$$r = Mr. \tag{58}$$

Equality (58) is also known as fixed point implication. The fixed point implication is called to be general Ulam-Hyers stable if and only if there exists a function $\gamma \in \Upsilon$ so that for all $\varepsilon > 0$ also for every $v_* \in \mathscr{L}$ which satisfies the following inequality,

$$b(v_*, Mv_*) \le \varepsilon \tag{59}$$

there exists $u \in \mathscr{L}$ providing the equation (58) such that

$$b(u, v_*) \le \gamma(\varepsilon). \tag{60}$$

Moreover, if there exists a $P > 0$ such that $\gamma(t) = Pt$ for all $t \in \mathbb{R}^+$, then the fixed point equation (58) is said to be Ulam-Hyers stable. On the b-metric spaces $(\mathscr{L}, b, s)$, fixed point problem (58) and $M : \mathscr{L} \longrightarrow \mathscr{L}$ are defined to be well-known if the following suppositions are satisfy:

(l_1) M has a unique fixed point $u \in \mathscr{L}$

(l_2) $\lim_{q \longrightarrow \infty} b(u, r_q) = 0$ for every sequence $r_q \in \mathscr{L}$ such that

$$\lim_{q \longrightarrow \infty} b(r_q, Mr_q) = 0 \tag{61}$$

Theorem 15. *Let* $(\mathscr{L}, b, s)$ *be a complete b-metric space. If we joint the condition* $a > 0$ *and* $e(a) s^a \rho_1 < 1$, *where* $e(a) = \max \{1, 2^{a-1}\}$ *and* $s^a \rho_1 + e(a) s^a (\rho_i + 1) < 1$, *where* $i = 1$ *or* $i = 2$ *or* $i = 3$, *also suppositions of Theorem 9, thus the following conditions hold:*

(a) *the fixed point problem (58) is Ulam-Hyers stable, if* $w(n, m) \ge 1$ *for any* n, m *satisfying the condition (59)*

(b) *the fixed point problem (58) is well-known, if* $w(r_q, u) \ge 1$ *for any* $\{r_q\}$ *in* $\mathscr{L}$ *such that* $\lim_{q \longrightarrow \infty} b(Mr_q, r_q) = 0$ *and* $Fix_M(\mathscr{L}) = u$.

Proof.

(a) Taking into account Theorem 9, we consider that there is a unique u in $\mathscr{L}$ such that $Mu = u$. Assume that v_* is a solution of (59), that is $b(v_*, Mv_*) \le \varepsilon$ for $\varepsilon > 0$. Clearly, u holds (59), then we get that $w(u, v_*) \ge 1$ and using triangular inequality satisfies

$$b(u, v_*) = b(Mu, v_*) \le s[b(Mu, Mv_*) + b(Mv_*, v_*)] \tag{62}$$

Since M is CRRS-type hybrid contraction, we obtain

$$\frac{1}{2s} b(u, Mu) = 0 \le b(u, v_*) implies \tag{63}$$

$$\begin{aligned} \mathrm{b}(u, v_*) &\le s[\mathrm{b}(\mathrm{M}u, \mathrm{M}v_*) + \mathrm{b}(\mathrm{M}v_*, v_*)] \\ &\le s[w(u, v_*)\mathrm{b}(\mathrm{M}u, \mathrm{M}v_*) + \mathrm{b}(\mathrm{M}v_*, v_*)] \\ &\le s[\psi(\chi_{\mathrm{M}}^a(u, v_*)) + \mathrm{b}(\mathrm{M}v_*, v_*)] \\ &< s[\chi_{\mathrm{M}}^a(u, v_*) + \mathrm{b}(\mathrm{M}v_*, v_*)] \\ &\le s[\rho_1(\mathrm{b}(u, v_*))^a + \rho_2(d(u, \mathrm{M}u))^a + \rho_3(\mathrm{b}(\mathrm{M}v_*, v_*))^a]^{1/a} \\ &\quad + s\mathrm{b}(\mathrm{M}v_*, v_*) \le s[\rho_1(\mathrm{b}(u, v_*))^a + \rho_3\varepsilon^a]^{1/a} + s\varepsilon \end{aligned} \tag{64}$$

Thus, we get

$$(\mathrm{b}(u, v_*))^a \le e(a)[s^a\varrho_1(\mathrm{b}(u, v_*))^a + s^a\varrho_3\varepsilon^a + s^a\varepsilon^a] \tag{65}$$

then,

$$(b(u, v_*))^a \le \frac{(1+\rho_3)e(a)s^a}{1-\rho_1 e(a)s^a}\varepsilon^a \tag{66}$$

$$\mathrm{b}(u, v_*) \le n\varepsilon \tag{67}$$

where $n = [(1+\varrho_3)e(a)s^a/1 - \varrho_1 e(a)s^a]^{1/a}$ for any $a > 0$ and $\varrho_1 \in [0, 1)$ such that $\varrho_1 < 1/e(a)s^a$..

(b) The Picard iterations is M-stable, that is, let $r_{\mathrm{q}} \in \mathscr{L}$ such that $\lim_{\mathrm{q}\longrightarrow\infty} \mathrm{b}(r_{\mathrm{q}+1}, \mathrm{M}r_{\mathrm{q}}) = 0$ and $Fix_{\mathrm{M}}(\mathscr{L}) = u$. From the triangular inequality, we can write

$$\mathrm{b}(r_{\mathrm{q}}, u) \le s\left[\mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}) + \mathrm{b}(\mathrm{M}r_{\mathrm{q}}, \mathrm{M}u)\right]. \tag{68}$$

Thus, M is a CRRS contraction, we have

$$\frac{1}{2s}\mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}) \le \mathrm{b}(r_{\mathrm{q}}, u)\text{implies} \tag{69}$$

$$\begin{aligned} \mathrm{b}(r_{\mathrm{q}}, u) &\le s\left[\mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}) + \mathrm{b}(\mathrm{M}r_{\mathrm{q}}, \mathrm{M}u)\right] \\ &\le s\left[\mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}) + w(r_{\mathrm{q}}, u)\mathrm{b}(\mathrm{M}r_{\mathrm{q}}, \mathrm{M}u)\right] \\ &\le s\left[\psi(\chi_{\mathrm{M}}^a(r_{\mathrm{q}}, u)) + \mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}})\right] < s\left[\chi_{\mathrm{M}}^a(r_{\mathrm{q}}, u) + \mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}})\right] \\ &\le s\left[\rho_1(\mathrm{b}(r_{\mathrm{q}}, u))^a + \rho_2(b(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}))^a + \rho_3(\mathrm{b}(\mathrm{M}u, u))^a\right]^{1/a} + s\mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}) \\ &\le s\left[\rho_1(\mathrm{b}(r_{\mathrm{q}}, u))^a + \rho_2(b(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}))^a\right]^{1/a} + s\mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}). \end{aligned} \tag{70}$$

Then, we calculate process

$$(\mathrm{b}(r_{\mathrm{q}}, u))^a \le e(a)\left[s^a\varrho_1(\mathrm{b}(r_{\mathrm{q}}, u))^a + s^a\varrho_2(b(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}))^a + s^a(b(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}))^a\right] \tag{71}$$

then,

$$(\mathrm{b}(r_{\mathrm{q}}, u))^a \le \frac{(1+\varrho_2)e(a)s^a}{1-\varrho_1 e(a)s^a}(b(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}))^a. \tag{72}$$

Taking $\mathrm{q} \longrightarrow \infty$ in the above inequality and using $\lim_{\mathrm{q}\longrightarrow\infty}\mathrm{b}(r_{\mathrm{q}}, \mathrm{M}r_{\mathrm{q}}) = 0$, we obtain $\lim_{\mathrm{q}\longrightarrow\infty}\mathrm{b}(r_{\mathrm{q}}, u) = 0$ the fixed point equation (58) is well posed. □

4. Conclusion

In this study, we present new hybrid fixed point theorems in b-metric spaces. We obtain the extended results of the interpolative Reich-Rus-Ćirić fixed point theorem by using w-orbital admissible and Suzuki-type mapping. We also offer an example to show the availability of introduced results. Further, we obtain Ulam-Hyers-type stability of the fixed point theorem which is the application of our study.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflict of interest.

Authors' Contributions

The authors contributed equally to this manuscript. All authors have read and agreed to the published version of the manuscript.

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