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Research Article

Analytical Approaches on the Attractivity of Solutions for Multiterm Fractional Functional Evolution Equations

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The most important objective of the current research is to establish some theoretical existence and attractivity results of solutions for a novel nonlinear fractional functional evolution equations (FFEE) of Caputo type. In this respect, we use a familiar Schauder's fixed-point theorem (SFPT) related to the method of measure of noncompactness (MNC). Furthermore, we consider the operator E and show that it is invariant and continuous. Moreover, we provide an application to show the capability of the achieved results.

1. Introduction

During the recent years, the study of fractional evolution equations (FEE) has attracted a lot of attention. Such class pulls out the interest of such countless creators toward itself, inspired by their broad use in numerical analysis. Fractional Calculus (FC), as much as classic analytics, has discovered significant examples in the study of problem in a thermal system and mechanical system. Also, in certain spaces of sciences like control hypothesis, a fractional differential operator appears to be more reasonable to model than the old style integer order operator. Because of this, FEE has been utilized in models about organic chemistry and medication.

In the last few years, the hypothesis of FEE has been scientifically explored by a major number of extremely fascinating and novel papers (see [1–3]). The existence of global attractivity solutions to the Ψ-Hilfer Cauchy fractional problem is investigated by several researchers (see [4]). Chang et al. [5] used fixed-point theorems to study the asymptotic decay of various operators, as well as the existence and uniqueness of a class of mild solutions of Sobolev fractional differential equations. In [6, 7], the theory of fractional differential equations was discussed. The Ψ-Hilfer fractional derivative was used to investigate the existence, uniqueness, and Ulam-Hyers stabilities of solutions of differential and integro-differential equations.

The existence and attractivity of solutions to the following coupled system of nonlinear fractional Riemann-Liouville-Volterra-Stieltjes quadratic multidelay partial integral equations are investigated by many authors. The properties of bounded variation functions are defined by them (see [8–10]). The attractivity of solutions to the Hilfer fractional stochastic evolution equations is discussed by Yang and others. In circumstances where the semigroup associated with the infinitesimal generator is compact, they establish sufficient criteria for the global attractivity of mild solutions (see [11]). Also, mild solutions for multiterm time-fractional differential equations with nonlocal initial conditions and fractional functional equations (FFE) have been researched (see [12, 13]).

A functional differential equation is a general name for a number for more specific types of DE that are used in different applications. There are delay differential equations (DDE), integro-differential equations, and so on. FC has been effectively applied in different applied zones like computational science and financial aspects. In specific circumstances, we need to solve FEE having more than one differential operator, and this kind of FEE is known as multiterm FEE. The researchers set up the existence of monotonic solution for multiterm PDE in Banach spaces, utilizing the RL-fractional derivative.

The greater part of the current work is concentrated on the existence and uniqueness of the solution for FEE (see [14–16]). The goal of this study is to investigate the existence of solutions to a class of multiterm FFEE on an unbounded interval in terms of bounded and consistent capacities. We also look at several key aspects of the arrangement that are relevant to the concept of attractivity of solution.

Consider IVP of the following FFEE:

$$\begin{cases} {}^{C}D^{\beta}v(t) = Bv(t) + \sum_{i=1}^{n} {}^{C}D^{\beta_i} f_i(t, v_t) + f_0(t, v_t), & t > t_0, 1 < \beta < 2, \\ v(t) = \phi(t) = \phi_0, v'(t) = \phi'(t) = \phi_1, & t_0 - \varrho \le t \le t_0, \end{cases} \tag{1}$$

where ${}^{C}D^{\beta}$ is the Caputo fractional derivative (CFD) of order $\beta > 0$, $\rho =$ constant, $\phi \in C([t_0 - \varrho, t_0), R)$, and $i = 1, 2, \cdots n$, ${}^{C}D^{\beta_i}$ is the CFD of order $0 < \beta_i < \beta$ and $f : H \times C([-\varrho, 0], R) \longrightarrow R$, in such a way that $H = (t_0,\infty)$ is a predefined function. We additionally consider for any $x \in H$ the function $v_t : [-\varrho, 0] \longrightarrow R$ given that $v_t(s) = v(t + s)$ for every $s \in [-\varrho, 0]$. We show that (1) has an attractive solution under the broad and favourable assumption using the SFPT and the concept of measure of noncompactness. We believe that by using classic SFPT and a control function, we can achieve a different result.

The following is the outline for this paper. We review some essential preliminaries in Section 2. In Section 3, we give a few supposition and lemmas or theorems to introduce the consequence of such section for (1) utilizing SFPT. In Section 4, we first review some assistant realities about the idea of MNC and related signs; at that point, we study the existence of solution for (1) applying a well-known Derbo-type fixed-point hypothesis along with the method of MNC. Finally, in Section 5, we discuss a useful application to represent our main result.

2. Preliminaries

In this section, we discuss some known definitions. Likewise, we define a few ideas identified with (1) along with SFPT.

Definition 1 (see [17]). For a function f, the fractional integral of order β with $t_0 \in R$ is defined as

$$I^{\beta}f(t) = \frac{1}{\Gamma(\beta)} \int_{t_0}^{t} \frac{f(s)}{(t-s)^{1-\beta}} ds, t > t_0, \beta > 0, \tag{2}$$

given that the R.H.S is pointwise characterized on $[t_0, \infty)$ where $\Gamma(\cdot)$ is the usual gamma function.

Definition 2 (see [17]). The RL-derivative of order $m - 1 < \beta < m$ with $t_0 \in R$ for a function $f \in C^m([x_0,\infty), R)$ can be composed as

$$D^{\beta}f(t) = \frac{1}{\Gamma(m-\beta)} \frac{d^m}{dt^m} \int_{t_0}^{t} \frac{f(s)}{(t-s)^{\beta+1-m}} ds, t > t_0, m \in \mathbb{N}. \tag{3}$$

Definition 3 (see [17]). Caputo derivative of order $m - 1 < \beta < m$ for a function $f \in C^{m+1}([t_0,\infty), R)$ can be composed as

$${}^{C}D^{\beta}f(t) = D^{\beta}\left(f(s) - \sum_{h=1}^{m-1} \frac{f^{(h)}(t_0)}{\Gamma(h-\beta+1)} (s-t_0)^{h-\beta}\right)(t), t > t_0, m \in \mathbb{N}. \tag{4}$$

Definition 4 (see [18]). The solution $v(x)$ of IVP (1) is supposed to be attractive if $\exists$ a constant term $c_0(t_0) > 0$ in such a way that

$$|\phi(s)| \le c_0(s \in [t_0 - \varrho, t_0]). \tag{5}$$

This means that $v(t) \longrightarrow 0$ as like $t \longrightarrow \infty$.

Definition 5 (see [19]). The solution $v(t)$ of IVP (1) is supposed to be attractive, if

$$\lim_{t \longrightarrow \infty} (v(t) - w(t)) = 0, \tag{6}$$

for some arrangement $w = w(t)$ of IVP (1).

Theorem 6 (SFP theorem [20]. *If V is nonempty, closed, bounded convex subset of Banach space Y and $K : V \longrightarrow V$ is totally continuous, at that point K has a fixed point in V.*

3. Attractivity of Solutions with Schauder's Fixed-Point Principle

The Schauder fixed-point theorem states that any compact convex nonempty subset of a normed space has the fixed-point property, which is one of the most well-known conclusions in fixed-point theory. It is also true in spaces that are locally convex. The Schauder fixed-point theorem has recently been extended to semilinear spaces. The Schauder fixed-point theorem is an extension of the Brouwer fixed-point theorem to topological vector spaces, which may be of infinite dimension.

This section contains the following information: we examine (1) utilizing the SFPT under the following suppositions:

(H1) The function $f_i(t, v_t)$ is Lebesgue measurable in terms of t for every $i = 1, 2, \cdots n$, on $[t_0, \infty)$, and $f_i(t, \phi)$ is continuous in terms of ϕ on $C([-\rho, 0], R)$.

(H2) There is a function that is strictly nonincreasing $\mathfrak{J} : R \longrightarrow R$ which disappears at infinity in such a way that

$$\left|\phi_m(t_0) + \sum_{i=1}^{n}\left(\frac{1}{\Gamma(\beta-\beta_i)}\int_{t_0}^{t}(t-s)^{\beta-\beta_i-1}f_i(s,v_s)ds\right)\right| \le \mathfrak{J}(t-t_0), \quad \forall t \in H = [t_0,\infty), m = 0, 1. \tag{7}$$

(H3) $\exists$ a constant α in such a way that for every $i = 1, 2, 3 \cdots n$, we have $f_i \in L^{1/\alpha}(H, C[-\varrho, 0], R)$ with

$$\alpha \in \left(0, \min_{0\le i\le n}\beta-\beta_i\right). \tag{8}$$

By condition (H1), IVP (1) is equal to the following condition:

$$v(x) = \begin{cases} \phi_m(t_0) + \sum_{i=1}^{n}\frac{1}{\Gamma(\beta-\beta_i)}\int_{t_0}^{t}(t-s)^{\beta-\beta_i-1}f_i(s,v_s)ds, & t > t_0, m = 0, 1, \\ \phi_m(t), & t \in [t_0-\varrho, t_0], \end{cases} \tag{9}$$

where $\beta_0 = 0$ and $0 < \beta_i < \beta$ for $i = 1, 2, 3 \cdots n$. We define the operator E as

$$[Ev](x) = \begin{cases} \phi_m(t_0) + \sum_{i=1}^{n}\frac{1}{\Gamma(\beta-\beta_i)}\int_{t_0}^{t}(t-s)^{\beta-\beta_i-1}f_i(s,v_s)ds, & t > t_0, m = 0, 1, \\ \phi_m(t), & t \in [t_0-\sigma, t_0], \end{cases} \tag{10}$$

for each $v \in C([t_0-\rho,\infty), R)$.

Consider the IVP of the following FFEE:

$$\begin{cases} {}^{C}D^{\beta}v(t) = Bv(t) + \sum_{i=1}^{n}{}^{C}D^{\beta_i}f_i(t,v_t) + f_0(t,v_t), & t > t_0, 1 < \beta < 2, \\ v(t) = \phi(t) = \phi_0, v'(t) = \phi'(t) = \phi_1, & t_0-\varrho \le t \le t_0. \end{cases} \tag{11}$$

The above system is equal to the following integral:

$$v(t) = \phi_0 + \phi_1 t + \sum_{i=1}^{n}\frac{1}{\Gamma(\beta-\beta_i)}\int_{t_0}^{t}(t-s)^{\beta-\beta_i-1}f_i(s,v_s)ds, t \in [0,\infty), \tag{12}$$

provided that the integral (12) exists.

Theorem 7. *If (12) holds, then*

$$v(t) = C_\beta(t)\phi_0 + K_\beta(t)\phi_1 + \int_{t_0}^{t}(t-s)^{\beta-1}P_\beta(t-s)f_i(s,v_s)ds, t \in [0,\infty), \tag{13}$$

where

$$C_\beta(t) = \int_0^\infty M_\beta(\theta)C\left(t^\beta\theta\right)d\theta, K_\beta(t) = \int_0^t C_\beta(s)ds, \quad P_\beta(t) = \int_0^\infty \beta\theta M_\beta(\theta)S\left(t^\beta\theta\right)d\theta. \tag{14}$$

Proof. Let $\lambda > 0$, then

$$v(\lambda) = \int_0^\infty e^{-\lambda s}v(s)ds, \mu(\lambda) = \int_0^\infty e^{-\lambda s}f_i(s)ds. \tag{15}$$

Applying the Laplace transform to (12), we get

$$v(\lambda) = \lambda^{\beta-1}\left(\lambda^\beta - B\right)^{-1}\phi_0 + \lambda^{\beta-2}\left(\lambda^\beta - B\right)^{-1}\phi_1 + \left(\lambda^\beta - B\right)^{-1}\mu(\lambda), \tag{16}$$

for $t \ge 0$.

$$v(\lambda) = \lambda^{(\beta/2)-1}\int_0^\infty e^{-\lambda^{\beta/2}t}C(t)\phi_0 dt + \lambda^{-1}\lambda^{(\beta/2)-1}\int_0^\infty e^{-\lambda^{\beta/2}t}C(t)\phi_1 dt + \int_0^\infty e^{-\lambda^{\beta/2}t}S(t)\mu(t)dt. \tag{17}$$

Let

$$\phi_\beta(\theta) = \frac{\beta}{\theta^{\beta+1}}M_\beta\left(\theta^{-\beta}\right), \theta \in (0,\infty), \tag{18}$$

and its Laplace transform is given by

$$\int_0^\infty e^{-\lambda\theta}\phi_\beta(\theta)d\theta = e^{-\lambda^\beta}, \beta \in \left(\frac{1}{2}, 1\right). \tag{19}$$

Using (19), we have

$$\begin{aligned} \lambda^{\beta-1}\int_0^\infty e^{-\lambda^\beta t}C(t)\phi_0 dt &= \int_0^\infty \beta(\lambda t)^{\beta-1}e^{-(\lambda t)^\beta}C\left(t^\beta\right)\phi_0 dt \\ &= \int_0^\infty -\frac{1}{\lambda}\frac{d}{dt}\left(\int_0^\infty e^{-\lambda t\theta}\phi_\beta(\theta)d\theta\right)C\left(t^\beta\right)\phi_0 dt \\ &= \int_0^\infty\int_0^\infty \frac{-\lambda\theta}{-\lambda}e^{-\lambda t\theta}\phi_\beta(\theta)C\left(t^\beta\right)\phi_0 dt \\ &= \int_0^\infty\int_0^\infty \theta\phi_\beta(\theta)e^{-\lambda t\theta}C\left(t^\beta\right)\phi_0 dt d\theta \\ &= \int_0^\infty\int_0^\infty \phi_\beta(\theta)e^{-\lambda t}C\left(\frac{t^\beta}{\theta^\beta}\right)\phi_0 d\theta dt \\ &= \int_0^\infty e^{-\lambda t}\left[\int_0^\infty \phi_\beta(\theta)C\left(\frac{t^\beta}{\theta^\beta}\right)\phi_0\right]d\theta dt \\ &= \mathscr{L}\left[\int_0^\infty M_\beta(\theta)C\left(t^\beta\theta\right)\phi_0 d\theta\right](\lambda) = \mathscr{L}\left[C_\beta(t)\phi_0\right](\lambda). \end{aligned} \tag{20}$$

Since $L[g_1(t)](\lambda) = \lambda^{-1}$, according to the Laplace convolution theorem, we have

$$\lambda^{-1}\lambda^{\beta-1}\int_0^\infty e^{-\lambda^\beta t}C(t)\phi_1 dt = \mathscr{L}[g_1(t)](\lambda) * \mathscr{L}\left[C_\beta(t)\phi_1\right](\lambda), \tag{21}$$

$$\lambda^{-1}\lambda^{\beta-1}\int_0^\infty e^{-\lambda^\beta t}C(t)\phi_1 dt = \mathscr{L}\left[\left(g_1 * C_\beta\right)(t)\phi_1\right](\lambda). \tag{22}$$

Similarly,

$$\begin{aligned}\int_0^\infty e^{-\lambda^\beta t}S(t)\mu(\lambda)dt &= \int_0^\infty \beta t^{\beta-1}e^{(-\lambda t)^\beta}S\left(t^\beta\right)\mu(\lambda)dt \\ &= \int_0^\infty\int_0^\infty \beta t^{\beta-1}\phi_\beta(\theta)e^{-\lambda t\theta}S\left(t^\beta\right)\mu(\lambda)dtd\theta \\ &= \int_0^\infty\int_0^\infty \beta\frac{t^{\beta-1}}{\theta^\beta}\phi_\beta(\theta)e^{-\lambda t\theta}S\left(\frac{t^\beta}{\theta^\beta}\right)\mu(\lambda)dtd\theta \\ &= \int_0^\infty e^{-\lambda t}\left[\int_0^\infty \beta\frac{t^{\beta-1}}{\theta^\beta}\phi_\beta(\theta)S\left(\frac{t^\beta}{\theta^\beta}\right)\mu(\lambda)d\theta\right]dt \\ &= \mathscr{L}\left[\int_0^\infty \beta t^{\beta-1}M_\beta(\theta)S\left(t^\beta\theta\right)d\theta\right](\lambda)\cdot L[f_i(t)](\lambda) \\ &= \mathscr{L}\left[\int_{t_0}^t (t-s)^{\beta-1}P_\beta(t-s)f_i(s)ds\right](\lambda).\end{aligned} \tag{23}$$

Combining equations (20), (22), and (23), we have

$$v(t) = C_\beta(t)\phi_0 + \int_{t_0}^t C_\beta(s)\phi_1 ds + \int_{t_0}^t (t-s)^{\beta-1}P_\beta(t-s)f_i(s, v_s)ds. \tag{24}$$

The above system can also be written as

$$\begin{aligned}v(t) &= C_\beta(t)\phi_0 + \int_{t_0}^t C_\beta(s)\phi_1 ds \\ &\quad + \sum_{i=1}^n \frac{1}{\Gamma(\beta-\beta_i)}\int_{t_0}^t (t-s)^{\beta-\beta_i-1}f_i(s, v_s)ds, t > t_0.\end{aligned} \tag{25}$$

Thus, the proof is complete. □

Lemma 8. *Assume that $f_i(t, v_t)$ fulfills conditions (H1)-(H3). At that point, (1) has minimum one solution in $C([t_0-\rho,\infty), R)$.*

Proof. Define a set $P \subset C([t_0-\varrho,\infty), R)$ by

$$P = \{v : v \in C([t_0-\varrho,\infty), R), |v(t)| \le \mathfrak{F}(t-t_0)\forall t \ge t_0\}. \tag{26}$$

□

P is clearly a nonempty, convex, closed, and bounded subset of $C([t_0-\varrho,\infty), R)$. To show that (1) has a solution, it just necessities to prove that in P, the operator E has a fixed point. To begin with, we prove that P is E-invariant. This is without any problem acquired by condition (H2). Now, we should explain that E is continuous. For this, suppose that $(v^m)_{m\in\mathbb{N}}$ is a sequence of a function to such an extent that $v^m \in P\forall m \in \mathbb{N}$ and $v^m \longrightarrow v$ as $m \longrightarrow \infty$. Clearly, by the continuity $f_i(t, v_t)$, we get

$$\lim_{m\longrightarrow\infty} f_i(t, v_t^m) = f_i(t, v_t)\forall t > t_0, i = 1, 2, \cdots n. \tag{27}$$

Assume that $\varepsilon > 0$ is given. After all, $\mathfrak{F}$ is strongly decreasing. At that point for some $T > t_0$, we have

$$\mathfrak{F}(t-t_0) < \frac{\varepsilon}{2}, \quad \forall t > T. \tag{28}$$

For $t_0 < t \le T$, we get

$$\begin{aligned}|[Ev^m](t) - [Ev](t)| &\le \sum_{i=1}^n \frac{1}{\Gamma(\beta-\beta_i)}\int_{t_0}^t (t-s)^{\beta-\beta_i-1}|f_i(s, v_s^m) - f(s, v_s)|ds \\ &\le \sum_{i=1}^n \frac{1}{\Gamma(\beta-\beta_i)}\left(\int_{t_0}^t (t-s)^{(\beta-\beta_i-1)/(1-\alpha)}\right)^{1-\alpha} \\ &\quad \cdot\left(\int_{t_0}^t |f_i(s, v_s^m) - f(s, v_s)|^{1/\alpha}ds\right)^\alpha \\ &\le \sum_{i=1}^n \frac{(1-\alpha)^{1-\alpha}}{\Gamma(\beta-\beta_i)(\beta-\beta_i-\alpha)^{1-\alpha}}(T-t_0)^{\beta-\beta_i-\alpha} \\ &\quad \cdot\left(\int_{t_0}^t |f_i(s, v_s^m) - f(s, v_s)|^{1/\alpha}ds\right)^\alpha \\ &\le \sum_{i=1}^n \frac{(1-\alpha)^{1-\alpha}}{\Gamma(\beta-\beta_i)(\beta-\beta_i-\alpha)^{1-\alpha}}(T-t_0)^{\beta-\beta_i},\end{aligned}$$

$$\sup_{t_0\le t\le T}|f_i(s, v_s^m) - f(s, v_s)|, \tag{29}$$

which disappear when $m \longrightarrow \infty$. Then again, since P in E -invariant, at that point, (28) yields

$$|[Ev^m](t) - [Ev](t)| \le 2\mathfrak{F}(t-t_0) < \varepsilon, \quad \forall t > T. \tag{30}$$

Thus, for $t > t_0$, this implies that

$$|[Ev^m](t) - [Ev](t)| \longrightarrow 0 \text{ as } m \longrightarrow \infty. \tag{31}$$

If $x \in [t_0-\rho, t_0]$, we clearly have $|[Ev^m](t) - [Ev](t)| = 0$. Therefore, the continuity E has been proven. Then, we prove that $E(P)$ is equicontinuous. Assume that $\varepsilon > 0$ is given, t_1, $t_2 \in (t_0, T]$ where $T > t_0$ is picked with the end of goal that (28) holds. Applying (H3), we get

$$\begin{aligned}
|(Ev)(t_2)-(Ev)(t_1)| &\le \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \left| \int_{t_0}^{t_2} (t_2-s)^{\beta-\beta_i-1} f_i(s,v_s)ds \right. \\
&\quad - \int_{t_0}^{t_1} (t_1-s)^{\beta-\beta_i-1} f_i(s,v_s)ds| \\
&\le \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \left[\int_{t_0}^{t_1} \left| (t_2-s)^{\beta-\beta_i-1} - (t_1-s)^{\beta-\beta_i-1} \right| |f_i(s,v_s)| ds \right. \\
&\quad \left. + \int_{t_1}^{t_2} (t_2-s)^{\beta-\beta_i-1} |f_i(s,v_s)| ds \right] \le \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \\
&\quad \cdot \left[\left(\int_{t_0}^{t_1} \left| (t_1-s)^{\beta-\beta_i-1} - (t_2-s)^{\beta-\beta_i-1} \right|^{1/(1-\alpha)} ds \right)^{1-\alpha} \right. \\
&\quad \cdot \left(\int_{t_0}^{t_1} |f_i(s,v_s)|^{1/\alpha} ds \right)^{\alpha} + \left(\int_{t_1}^{t_2} (t_2-s)^{(\beta-\beta_i-1)/(1-\alpha)} ds \right)^{1-\alpha} \\
&\quad \left. \cdot \left(\int_{t_1}^{t_2} |f_i(s,v_s)|^{1/\alpha} ds \right)^{\alpha} \right] \le \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \\
&\quad \cdot \left[\left(\int_{t_0}^{t_1} \left| (t_1-s)^{(\beta-\beta_i-1)/(1-\alpha)} - (t_2-s)^{(\beta-\beta_i-1)/(1-\alpha)} \right| du \right)^{1-\alpha} \right. \\
&\quad \cdot \left(\int_{t_0}^{t_1} |f_i(s,v_s)|^{1/\alpha} ds \right)^{\alpha} + \left(\int_{t_1}^{t_2} (t_2-s)^{(\beta-\beta_i-1)/(1-\alpha)} ds \right)^{1-\alpha} \\
&\quad \left. \cdot \left(\int_{t_1}^{t_2} |f_i(s,v_s)|^{1/\alpha} ds \right)^{\alpha} \right] \le \sum_{i=1}^{n} \frac{(1-\alpha)^{1-\alpha}}{\Gamma(\beta-\beta_i)(\beta-\beta_i-\alpha)^{1-\alpha}} \\
&\quad \cdot \left[\left(|t_2-t_1|^{(\beta-\beta_i-\alpha)/(1-\alpha)} + \left| (t_1-t_0)^{(\beta-\beta_i-\alpha)/(1-\alpha)} \right. \right. \right. \\
&\quad \left. \left. - (t_2-t_0)^{(\beta-\beta_i-\alpha)/(1-\alpha)} \right| \right)^{1-\alpha} \times \left(\int_{t_0}^{T} |f_i(s,v_s)|^{1/\alpha} ds \right)^{\alpha} \\
&\quad \left. + (t_2-t_1)^{\beta-\beta_i-\alpha} \left(\int_{t_0}^{T} |f_i(s,v_s)|^{1/\alpha} ds \right)^{\alpha} \right] \longrightarrow 0 as t_1 \longrightarrow t_2.
\end{aligned} \tag{32}$$

If $t_1, t_2 > T$, at that point, since P is E-invariant and using(28), we get

$$\begin{aligned}
|(Ev)(t_2)-(Ev)(t_1)| &\le \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \\
&\quad \times \left| \int_{t_0}^{t_2} (t_2-s)^{\beta-\beta_i-1} f_i(s,v_s)ds - \int_{t_0}^{t_1} (t_1-s)^{\beta-\beta_i-1} f_i(s,v_s)ds \right\} \Bigg| \\
&\le \mathfrak{J}(t_1-t_0) + \mathfrak{J}(t_2-t_0) < \varepsilon.
\end{aligned} \tag{33}$$

If $t_0 < t_1 < T < t_2$, it can be seen that $t_1 \longrightarrow t_2$ which implies that $(t_1 \longrightarrow T) \wedge (t_2 \longrightarrow T)$; then, according to the above discussion, we have got

$$\begin{aligned}
|(Ev)(t_2)-(Ev)(t_1)| &\le |(Ev)(t_2)-(Ev)(T)| \\
&\quad + |(Ev)(T)-(Ev)(t_1)| \longrightarrow 0 \text{ as } t_2 \longrightarrow t_1.
\end{aligned} \tag{34}$$

Thus, we resolve that $E(P)$ is equicontinuous on $[t_0, T]$ $\forall T > 0$. Since $E(P) \subset P$ and from the set P, it is clear that

$$\lim_{T\longrightarrow\infty} \sup_{v\in E(P)} (\sup \{|v(t)|: t > T\}) = 0. \tag{35}$$

Hence, $E(P)$ is a moderately smaller set in $C([t_0-\rho,\infty)$ $, R)$ and all requirements of SFPT are satisfied. In this set, the operator E maps on P and has a fixed point. This reality indicates that (1) has at least one solution in P.

Theorem 9. *Assume that conditions (H1)-(H2) are fulfilled; at that point, IVP (1) accepts at the minimum one attractive solution by Definition 4.*

Proof. The previous lemma states that there is at least one solution of (1) that belongs to P in (Lemma 8). Then, use the property of function $\mathfrak{J}$, to show attractivity. As a result, at ∞, all of the functions in P vanish, and therefore, the result of (1) is $\longrightarrow 0$ as $x \longrightarrow \infty$.

So, the proof is complete. □

Remark 10. The conclusion of Theorem 9 does not imply that solutions are globally attractive in the sense of Definition 5.

4. Uniform Local Attractivity of Solutions with Measure of Noncompactness

The purpose of this section is to look at the solution of (1) in the Banach space (BS), $BC(R_{t_0-\rho})$ consisting of every single real functions characterized, continuous as well as bounded on $R_{t_0-\rho} = [t_0-\rho,\infty)$ by means of the strategy of MNC. It is concentrated on an alternate method to develop some adequate conditions solvability of (1). We assemble a few definitions and assistant realities which will be required further on.

Let F be a BS and Conv$Y$ and $\bar{Y}$ represent the convex closure and closure of Y as a subset of F. Further, $\mathfrak{m}_F$ represents the group of all bounded subsets of E, and the $\mathfrak{n}_F$ represents its subfamily which contains all relatively compact sets. Also, assume that the closed ball is $B(y, r)$ where center $= y$, radius $= r$, and B_r represents the ball $B(\xi, r)$ with the end of goal that ξ is the zero component of the BS of F.

Definition 11. $\nu : \mathfrak{m}_F \longrightarrow R^+$ is supposed to be MNC in F if it fulfills the following criteria:

(i) The family $\ker\nu = \{Y \in \mathfrak{m}_F : \nu(Y) = 0\}$ is nonempty and $\ker\nu \in \mathfrak{n}_F$

(ii) $Y \subseteq Z \Rightarrow \nu(Y) \ge \nu(Z)$.

(iii) $\nu(Y) = \nu(\bar{Y})$

(iv) $\nu(\text{Conv}Y) = \nu(Y)$

(v) $\forall \lambda \in [0, 1]$

$$\nu(\lambda Y + (1-\lambda)Z) \le \lambda\nu(Y) + (1-\lambda)\nu(Z). \tag{36}$$

(vi) If $(Y_m)_{m\in\mathbb{N}}$ is a closed sequence set from $\mathfrak{m}_F$ in such a way that

$$Y_{m+1} \subset Y_m \forall m = 1, 2, \cdots, \lim_{m\longrightarrow\infty} \nu(Y_m) = 0, \tag{37}$$

then

$$Y_\infty = \bigcap_{m=1}^{\infty} Y_n \text{ is non-empty.} \quad (38)$$

As a result, the $\ker(\nu)$ family is referred to as the kernel of MNC of ν.

Definition 12. In F, let ν be an MNC. So the mapping $S : C \subseteq F \longrightarrow F$ is supposed to be a ν_F-contraction if $\exists$ a constant term $0 < b < 1$ in such way

$$\nu(S(D)) \le b\nu(D). \quad (39)$$

$D \subseteq C$ is a bounded closed subset.

Remark 13. As pointed out in [21], global attractivity of solutions implies local attractivity, while the converse is not true.

Theorem 14 (see [22]). *Suppose that C is a nonempty, bounded, convex, and closed subset of BS of F, and assume that $S : C \longrightarrow C$ is a continuous function which fulfills*

$$\nu(S(D)) \le \phi(\nu(D)), \quad (40)$$

for every $D \subseteq C$, where ν represents an arbitrary MNC and $\phi : R^+ \longrightarrow R^+$ represents a monotone nondecreasing function with $\lim_{m\longrightarrow\infty}\phi^m(t) = 0 \forall t \in R^+$. At that point, S has minimum one fixed point in C.

We will work in BS, $BC(R_{t_0-\rho})$ where t_0 and ρ are given in (1). The functional space is furnished with the standard norm which is $\|\nu\| = \sup\{\nu(t) : t \ge t_0 - \rho\}$. For this reason, we present a MNC in the space $BC(R_{t_0-\rho})$ which is built like the one in the space $BC(R^+)$. Suppose that B is a bounded subset in BS of $BC(R_{t_0-\rho})$ and $T > t_0 - \rho$ is given. For $\nu \in B$ and $\varepsilon > 0$, we denote by $\omega^T_{t_0-\rho}(\nu, \varepsilon)$ the modulus of continuity of the function ν on $[t_0 - \rho, T]$, i.e.,

$$\omega^T_{t_0-\rho}(\nu, \varepsilon) = \sup\{|\nu(t) - \nu(s)| : t, s \in [t_0 - \rho, T], |t - s| \le \varepsilon\}. \quad (41)$$

Now, suppose that we take

$$\begin{aligned} \omega^T_{t_0-\rho}(B, \varepsilon) &= \sup\left\{\omega^T_{t_0-\rho}(\nu, \varepsilon) : \nu \in B\right\}, \\ \omega^T_{t_0-\rho}(B) &= \lim_{\varepsilon\longrightarrow 0}\omega^T_{t_0-\rho}(B), \\ \omega_{t_0-\rho}(B) &= \lim_{T\longrightarrow\infty}\omega^T_{t_0-\rho}(B). \end{aligned} \quad (42)$$

If $t > t_0 - \rho$ is a fixed number, we use the term

$$B(t) = \{\nu(t) : \nu \in B\}, \quad (43)$$

as well as

$$\operatorname{diam} B(t) = \sup\{|\nu(t) - w(t)| : \nu, w \in B\}. \quad (44)$$

Consider ν defined on the family $\mathfrak{m}_{BC(R_{t_0-\rho})}$ by formula

$$\nu(B) = \omega_{t_0-\rho}(B) + \lim_{t\longrightarrow\infty} \sup \operatorname{diam} B(t). \quad (45)$$

(H4) For all $i = 1, 2, \cdots n, f_i : R_{t_0-\rho} \times C([t_0 - \rho, t_0], R) \longrightarrow R$ is continuous and there is a continuous function $e_i : R_{t_0-\rho} \longrightarrow R$ in such a way that

$$|f_i(t, \nu) - f_i(t, w)| \le e_i(t)\mathfrak{J}(\|\nu - w\|), \quad (46)$$

where $\mathfrak{J} : R^+ \longrightarrow R^+$ is a function that is superadditive, i.e, $\mathfrak{J}(b) + \mathfrak{J}(c) \le \mathfrak{J}(b + c), \forall b, c \ge 0$.

(H5) Assume that $\forall i = 1, 2 \cdots n$ such that the following constant exists:

$$\begin{aligned} B_i &= \sup_{t\in H}\int_{t_0}^{t}(t - s)^{\beta-\beta_i-1}e_i(s)ds < \infty, C_i \\ &= \sup_{t\in H}\int_{t_0}^{t}(t - s)^{\beta-\beta_i-1}|f_i(s, 0)|ds < \infty, \\ &\lim_{m\longrightarrow\infty}\lambda_B^m\mathfrak{J}^m(t) = 0 \forall t > 0, \end{aligned} \quad (47)$$

where

$$\lambda_B = \sum_{i=1}^{n}\frac{B_i}{\Gamma(\beta - \beta_i)} < 1. \quad (48)$$

(H6) $\exists$ a nonnegative result r_0 of the following inequality

$$\sup_{t\in[t_0-\rho, t_0]}|\phi(t)| + \lambda_B\mathfrak{J}(r) + \lambda_C \le r, \quad (49)$$

where

$$\lambda_B = \sum_{i=1}^{n}\frac{C_i}{\Gamma(\beta - \beta_i)}. \quad (50)$$

Theorem 15. *Under the supposition (H4)-(H6), equation (1) has minimum one solution in $BC(R_{t_0-\rho})$. In addition, solution of (1) is uniformly locally attractive.*

Proof. To begin, we will look at the operator E, which was defined by the formula in the previous section:

$$[E\nu](t) = \begin{cases} \phi_m(t_0) + \sum_{i=1}^{n}\frac{1}{\Gamma(\beta - \beta_i)}\int_{t_0}^{t}(t - s)^{\beta-\beta_i-1}f_i(s, \nu_s)ds, & t > t_0, m = 0, 1, \\ \phi_m(t), & t \in [t_0 - \sigma, t_0]. \end{cases} \quad (51)$$

□

$\forall v \in BC(R_{t_0-\rho})$. From the condition (H4)-(H6), the function Ev is continuous on $(R_{t_0-\rho})$. We note that $BC(R_{t_0-\rho})$ in E-invariant. For any $v \in BC(R_{t_0-\rho})$ and $t > t_0$, we have

$$\begin{aligned}
|[Ev](t)| &\leq |\phi_0(t_0)| + |\phi_1(t_0)| + \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \int_{t_0}^{t} (t-s)^{\beta-\beta_i-1} |f_i(s, v_s)| ds \\
&\leq |\phi_0(t_0)| + |\phi_1(t_0)| + \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \int_{t_0}^{t} (t-s)^{\beta-\beta_i-1} (|f_i(s, v_s) \\
&\quad - f_i(s,0)| + |f_i(s,0)|) ds \\
&\leq |\phi_0(t_0)| + |\phi_1(t_0)| + \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \int_{t_0}^{t} (t-s)^{\beta-\beta_i-1} e_i(s) \mathfrak{J}(\|v_s\|) ds \\
&\quad + \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \int_{t_0}^{t} (t-s)^{\beta-\beta_i-1} |f_i(s,0)| ds \\
&\leq |\phi_0(t_0)| + |\phi_1(t_0)| + \lambda_B \mathfrak{J}(\|v\|) + \lambda_C.
\end{aligned} \tag{52}$$

The above expression shows that Ev is bounded on the interval $[t_0, \infty)$, and connecting with the fact that $\phi \in C([t_0 - \rho, t_0], R)$, we infer that $Ev \in BC(R_{t_0-\rho})$; thus, E changes $BC(R_{t_0-\rho})$ into itself. Then again, utilize condition (H6)$\exists$ a number $r_0 > 0$ which appreciates in (49). For such digit, the operator E changes the ball B_{r_0} of $BC(R_{t_0-\rho})$ into itself. Consider a nonempty subset Y of the ball B_{r_0} as well as fix $x, y \in Y$ in whatever way you want. At that point, for fixed $t > t_0$, we get

$$\begin{aligned}
|[Ev](t) - [Ew](t)| &\leq \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \int_{t_0}^{t} (t-s)^{\beta-\beta_i-1} (|f_i(s, v_s) - f_i(s, w_s)| ds \\
&\leq \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \int_{t_0}^{t} (t-s)^{\beta-\beta_i-1} e_i(s) \mathfrak{J}(\|v_s - w_s\|) ds,
\end{aligned}$$

$$|[\mathrm{E}v](t) - [\mathrm{E}w](t)| \leq \lambda_B \mathfrak{J}(\|v - w\|), \tag{53}$$

which implies that

$$\lim_{t \longrightarrow \infty} \operatorname{diam}(E(Y))(t) \leq \lambda_B \mathfrak{J}\left(\lim_{t \longrightarrow \infty} \operatorname{diam} Y(t)\right). \tag{54}$$

Additionally, let us take $T > t_0$ as a fixed and $\varepsilon > 0$. Assume that $v \in Y$ is chosen and suppose that $t_1, t_2 \in (t_0, T]$ to such an extent that $|t_1 - t_2| \leq \varepsilon$. Without loss of consensus, we may suppose that $t_1 < t_2$. At that point, thinking about our hypothesis, we obtain

$$\begin{aligned}
|[Ev](t_2) - [Ew](t_1)| &\leq \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \left| \int_{t_0}^{t_2} (t_2 - s)^{\beta-\beta_i-1} f_i(s, v_s) ds \right. \\
&\quad \left. - \int_{t_0}^{t_1} (t_1 - s)^{\beta-\beta_i-1} f_i(s, v_s) ds \right| \leq \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \\
&\quad \times \left(\int_{t_0}^{t_1} \left| (t_1 - s)^{\beta-\beta_i-1} - (t_2 - s)^{\beta-\beta_i-1} \right| |f_i(s, v_s)| ds \right. \\
&\quad \left. + \int_{t_1}^{t_2} (t_2 - s)^{\beta-\beta_i-1} |f_i(s, v_s)| ds \right) \leq \sum_{i=1}^{n} \frac{1}{\Gamma(\beta-\beta_i)} \\
&\quad \times \left[\int_{t_0}^{t_1} \left| (t_1 - s)^{\beta-\beta_i-1} - (t_2 - s)^{\beta-\beta_i-1} \right| |f_i(s, v_s)| ds \right. \\
&\quad \left. + \int_{t_1}^{t_2} (t_2 - s)^{\beta-\beta_i-1} (e_i(s) \mathfrak{J}(\|v_s\|) + |f_i(s,0)|) ds \right] \\
&\leq \sum_{i=1}^{n} \frac{\omega_1^T(f_i, \beta_i, \varepsilon) + \omega_2^T(f_i, \beta_i, \varepsilon)}{\Gamma(\beta-\beta_i)} + \lambda_B \mathfrak{J}\left(\omega_{t_0-\rho}^T(v, \varepsilon)\right),
\end{aligned} \tag{55}$$

for $v \in Y \subseteq B_{r_0}$, where the symbols used in above term are given by

$$\omega_2^T(f_i, \beta_i, \varepsilon) = \sup \left\{ \int_{t_1}^{t_2} (t_2 - s)^{\beta-\beta_i-1} (e_i(s) \mathfrak{J}(r_0) + |f_i(s,0)|) ds : t_1, t_2 \in [t_0 - \rho, T], |t_1 - t_2| \leq \varepsilon \right\}. \tag{56}$$

Now, consider that the function $f_i(s, v_s)$ is uniformly continuous on $[t_0 - \rho, T] \times B_{r_0} \forall i = 1, 2, \cdots n$; we simply get the following expression:

$$\omega_{t_0-\rho}^T(\mathrm{E}(Y)) \leq \lambda_B \mathfrak{J}\left(\omega_{t_0-\rho}^T(Y)\right), \tag{57}$$

which along with (54) and superadditive of $\mathfrak{J}$ implies that

$$\begin{aligned}
\nu(E(Y)) &= \omega_{t_0-\rho}(E(Y)) + \lim_{t \longrightarrow \infty} \operatorname{supdiam}(E(Y))(t) \\
&\leq \lambda_B \mathfrak{J}(\omega_{t_0}(Y)) + \lambda_B \mathfrak{J}\left(\lim_{t \longrightarrow \infty} \operatorname{supdiam}(Y)(t)\right) \\
&\leq \lambda_B \mathfrak{J}(\nu(Y)).
\end{aligned} \tag{58}$$

Now, ν as given by (45) defines a MNC on $BC(R_{t_0-\rho})$; at that point, the inequality along with Theorem 14 shows that (1) has a solution in BS.

To show that all solutions of (1) are consistently locally attractive, let us put $B_{r_0}^1 = \mathrm{Conv}E(B_{r_0})$ and $B_{r_0}^2 = \mathrm{Conv}E(B_{r_0}^1)$ and so on, where $B_{r_0} = \text{ball}$ and $r_0 = \text{radius}$ and center $= 0$ in the space $BC(R_{t_0-\rho})$. We basically see that $B_{r_0}^1 \subset B_{r_0}$ and $B_{r_0}^{m+1} \subset B_{r_0}^m$ for $m = 1, 2, \cdots$, and furthermore, the set of this sequence is convex, closed, and nonempty. Moreover, in the light of the current inequality, we get

$$\nu\left(B_{r_0}^{m+1}\right) \leq \lambda_B^m \mathfrak{J}^m\left(\nu\left(B_{r_0}\right)\right) \text{for } n = 1, 2, \cdots. \tag{59}$$

Combining all the facts that $\nu(B_{r_0}) \geq 0$ as well as condition (H5) with the recent above inequality, we have

$$\lim_{m\longrightarrow\infty} \nu\left(B_{r_0}^m\right) = 0. \tag{60}$$

We can derive from the definition of MNC that $B = \bigcap_{m=1}^{\infty} B_{r_0}^m$ is convex, nonempty, closed, and bounded. B is an E-invariant set, and the operator E is continuous on it. In addition, remembering the reality that $B \in \ker\nu$ and the set belongs to $\ker\nu$, we infer that all solutions of (1) are consistently locally attractive.

Remark 16. Note that (1) has at least one attractive solution in the sense of Definition 4.

5. Example

Example 17. Consider the FFEE:

$$\begin{cases} {}^CD^{3/2}\nu(t) = {}^CD^{4/3}\left((s+1)^{-1/2}e^{-\sin\,\nu(s-1)}\right)(t) + \sin\dfrac{\nu(t-1)}{3}\left(|\nu(t-1)|+t+1\right)^{-4/5}, & t>0,\\ \nu(t) = te^{-t}, \nu'(t) = t^2e^{-t}, & t\in[-1,0]. \end{cases} \tag{61}$$

We can clearly see that condition (H1) holds. To show that condition (H2) is fulfilled, since $\nu(0)=0$ and $\nu'(0)=0$, we have the following expression $\forall t>0$:

$$\begin{aligned} &\left|\frac{1}{\Gamma(3/2)}\int_0^t (t-s)^{-1/2}\sin\frac{\nu(s-1)}{3}\left(|\nu(s-1)|+s+1\right)^{-4/5}ds \right.\\ &\quad\left. + \frac{1}{\Gamma(3/6)}\int_0^t (t-s)^{-5/6}(s+1)^{-1/2}e^{-\sin\,\nu(s-1)}ds\right|\\ &\le \frac{1}{\Gamma(3/2)}\int_0^t (t-s)^{-1/2}s^{-4/5}ds + \frac{1}{\Gamma(3/6)}\int_0^t (t-s)^{-5/6}s^{-1/2}ds\\ &= \frac{t^{-3/10}}{\Gamma(3/2)}\int_0^1 s^{-4/5}(1-s)^{-1/2}ds + \frac{t^{-1/3}}{\Gamma(3/6)}\int_0^1 s^{-1/2}(1-s)^{-5/6}ds. \end{aligned} \tag{62}$$

For any $\beta, \gamma \in R$, the following identity is obtained:

$$\int_0^1 s^{\beta-1}(1-s)^{\gamma-1}ds = \frac{\Gamma(\beta)\Gamma(\gamma)}{\Gamma(\beta+\gamma)}. \tag{63}$$

So (62) implies that

$$\begin{aligned} &\left|\frac{1}{\Gamma(3/2)}\int_0^t (t-s)^{-1/2}\sin\frac{\nu(s-1)}{3}\left(|\nu(s-1)|+s+1\right)^{-4/5}ds \right.\\ &\quad\left. + \frac{1}{\Gamma(3/6)}\int_0^t (t-s)^{-5/6}(s+1)^{-1/2}e^{-\sin\,\nu(s-1)}ds\right| \le \mathfrak{J}(t), \end{aligned} \tag{64}$$

where

$$\mathfrak{J}(t) = \frac{\Gamma(1/5)}{\Gamma(7/10)}t^{-3/10} + \frac{\sqrt{\pi}}{\Gamma(2/3)}t^{-1/3}, \quad t>0. \tag{65}$$

Clearly, this is a nonincreasing function on $\mathbb{R}$ and demonstrates that condition (H2) is true. Finally, it is necessary to show that condition (H3) is valid. Assume for a moment that $\gamma = 1/10 \in (0, \min\{1/2, 1/6\})$; at that point, we have

$$\int_0^\infty \left[\sin\frac{\nu(s-1)}{3}\left(|\nu(s-1)|+s+1\right)^{-4/5}\right]^{1/\gamma}ds \le \int_0^\infty (s+1)^{-8}ds = \frac{1}{7}, \tag{66}$$

which proves that $f_0(s,\nu_s) = \sin(\nu(s-1)/3)(|\nu(s-1)|+s+1)^{-4/5} \in L^{1/\gamma}(H, C([-1,0],R))$. Likewise, for $f_1(s,\nu_s) = (s+1)^{-1/2}e^{-\sin\,\nu(s-1)}$, we deduce that

$$\int_0^\infty \left[(s+1)^{-1/2}e^{-\sin\,\nu(s-1)}\right]^{1/\gamma}ds \le \int_0^\infty (s+1)^{-5}ds = \frac{1}{4}. \tag{67}$$

This shows that $f_1(s,\nu_s) \in L^{1/\gamma}(H, C([-1,0],R))$. Hence, all conditions are satisfied, so the solution of (1) is existent and also attractive.

6. Conclusion

The main conclusion of this study is that the multiterm fractional functional evolution equation belongs to a specific class of attractivity. The goal of this study is to investigate the existence of solutions to a class of multiterm FFEE on an unbounded interval in terms of bounded and consistent capacities. We look at some key aspects of the arrangement that are connected to the concept of solution attractivity. We use a familiar Schauder fixed-point theorem (SFPT) related to the method of measure of noncompactness (MNC). We go over some of the auxiliary realities surrounding the concept of MNC and related signs. Using a well-known Derbo-type fixed-point hypothesis and the MNC technique, we investigate the existence of a solution for (1).

Data Availability

No data were generated or analyzed during the current study.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

The authors declare that the study was realized in collaboration with equal responsibility. All authors read and approved the final manuscript.

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