Research Article

A Fractional-Order Investigation of Vaccinated SARS-CoV-2 Epidemic Model with Caputo Fractional Derivative

Badr Saad T. Alkahtani

Department of Mathematics, College of Science, King Saud University, P.O. Box 1142, Riyadh 11989, Saudi Arabia

Correspondence should be addressed to Badr Saad T. Alkahtani; balqahtani1@ksu.edu.sa

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In this paper, we consider a fractional-order mathematical system comprising four different compartments for the recent pandemic of SARS-CoV-2 with regard to global and singular kernels of Caputo fractional operator. The SARS-CoV-2 fractional mathematical model is analyzed for series-type solution by Laplace–Adomian decomposition techniques (LADM) and homotopy perturbation method (HPM). The whole quantity of each compartment is divided into small parts, and then the sum of these all parts is written as a series solution for each agent of the system, while the nonlinear part is decomposed using the Adomian polynomial. The model is also checked for approximate solution by HPM through a comparison of the parameter power, $p$, for each equation. The numerical simulation for both methods is provided in different fractional orders along with comparison with each other as well as with natural order 1.

1. Introduction

The novel coronavirus (SARS-CoV-2), considered the most dangerous virus of this decade, belong to family of severe acute respiratory syndrome (SARS) [1, 2]. Therefore, this new virus is related to the viruses associated with the syndrome. It has become a new novel strain of the SARS family, which was recognised in humans before [3, 4]. SARS-CoV-2 affected not only humans but also several animals. This virus has been transmitted from human to human, and it occurs similarly in animals. But many times, it has become a mystery that how the animals have been affected by it in certain security. Infected humans and different species of various animals are also recognised as an active cause of the spreading of the virus [5]. In the past, some similar viruses like the Middle East respiratory syndrome coronavirus (MERS-CoV) were spread from camels to the human population, and for SARS-CoV-1, the civet cats were recognised as the source of spreading into humans [6].

Mathematical models concerning infections have vastly been used since the last century to study the dynamics and transmission of various pandemics and epidemics and to apply valuable techniques for their control or minimization. Scholars investigating pandemics in the various areas of sciences are working to control these epidemics or reduce these negative impacts to a stable situation [7–9]. They also give the concept of the globalization of the ODEs that have the natural-order differentiation providing more degrees of freedom at any order. The equations which contain the FO differential equation $\delta$, with $0 < \delta \leq 1$, may be studied in [10–12]. Mostly, in epidemiological problems, FDEs refer to models with memory effects [12]. Next, the fractional-order derivative has the term of integration providing the knowledge of the past spreading for an infection. We can expect the behavior of the transmission based on the previous results and studies. The hereditary and historical characteristics point to the past transmission of diseases, which is very beneficial for making predictions. Therefore, these characteristics can be tested by the application of noninteger order derivatives, and it impacts the transmission of an epidemic [10–12].

Modern calculus goods production arbitrary order may be used in different fields of clinical and physical sciences, such as goods production by engineering, control theory,
economics, financing, and infectious disease conditions. The extensive huge study of FODEs in modelling global phenomena is because of more attracting characteristics which are not explained in the natural order derivatives. The natural order differential equations are local in nature, while FODEs are concerned with nonlocality, which provides more globalization of their dynamics. Usage of integer operator is hot area and recently caught the researchers interest, whereas the noninteger order operators have been studied and used intensively. Infectious problems for the endemic are the most realistic area for the researchers as a research gap and are applied to test them in recent times. Moreover, the analysis for the mathematical models referring to the real-globe situation is made using the theory of stability, existence of solution, and with the optimal problem as in [13–15].

We proposed a new vaccinated SARS-CoV-2 epidemic model that has four quantities including the susceptible class \( W(t) \), the acutely infected class \( X(t) \), chronically infected class \( V(t) \), and the recovered class \( Z(t) \), which takes the following form:

\[
\begin{align*}
\frac{dW(t)}{dt} &= \Lambda - \beta W(t)X(t) - (\rho + \xi)W(t), \\
\frac{dX(t)}{dt} &= \beta W(t)X(t) - (\xi + \gamma + \lambda)X(t), \\
\frac{dV(t)}{dt} &= \lambda X(t) - (\xi + \delta + \kappa)V(t), \\
\frac{dZ(t)}{dt} &= \gamma X(t) + \kappa V(t) + \rho W(t) - \xi Z(t).
\end{align*}
\]

The parameters used in system (1) are described in Table 1:

The analysis of model (1) under fractional-order derivative with regard to the Caputo operator is given as

\[
\begin{align*}
C^\lambda D_1^\lambda (W(t)) &= \Lambda - \beta W(t)X(t) - (\rho + \xi)W(t), \\
C^\lambda D_1^\lambda (X(t)) &= \beta W(t)X(t) - (\xi + \gamma + \lambda)X(t), \\
C^\lambda D_1^\lambda (V(t)) &= \lambda X(t) - (\xi + \delta + \kappa)V(t), \\
C^\lambda D_1^\lambda (Z(t)) &= \gamma X(t) + \kappa V(t) + \rho W(t) - \xi Z(t),
\end{align*}
\]

with general initial approximation \( W(0) = N_1, X(0) = N_2, V(0) = N_3, \) and \( Z(0) = N_4.\)

In the seventeenth century, many researchers Euler, L'Hôpital, Fourier, Abels, Riemann–Liouville, etc., made fundamental contributions in this field of modern calculus and were known as the pioneers of fractional calculus. Besides them, many other scientists have made significant findings and discovered some fractional models as seen in [10–12].

Most of the basic properties needed for real phenomena such as memory, globality, and hereditary are involved in various fractional operators, while the integer-order differential operator have no such properties; therefore, modern calculus gives more realistic result. Modern calculus refers to the appliances of biological, physical, and engineering as in [11, 16–18]. Other properties of fractional operators like nonlocality, globality, singularity, and nonsingularity also attract the interest of many researchers. These properties are more applicable to most real-world problems.

It should be kept in mind that fractional operators do have not a unique definition and are formulated by different formulae. Most of the operators have the definite integral with singular and nonsingular kernels. These kernels can also be found in the various fractional integration formulae. These kernels are mostly taken from the Cauchy integral formula. These integral kernels present in the fractional operator may be analyzed by various techniques. Some researchers have used Laplace–Adomian decomposition techniques for both linear and nonlinear fractional differential models. For the terms of nonlinearity, Adomian polynomial is applied, which decomposes that term into a small one (see [19, 20]). Similarly, the homotopy perturbation techniques are also applied for a series solution, which can be obtained by perturbed by a small factor and then comparing the power of parameter \( \zeta \in [0, 1] \) on both sides of the equation. For the iterative numerical solution of integer-order models, mostly Runge–Kutta techniques (RK4 and RK2) were used. They can be used for fractional-order equations by involving the fractional terms. In this article, we will apply Laplace transformation along with Adomian decomposition techniques by converting the while quantity into small ones. We will also apply the homotopy perturbation techniques (HPM) for the series solution of the said problem [21, 22].

Different global real-life situations may be represented by noninteger differential equations (FODEs) such as physics problems, problems of control theory, chemistry problems (polymerization, rheology, and electronics), physical biology, heat, aerodynamics, infectious diseases, electro-statics, electrical circuits, and blood flow [11, 23–25]. In the last few years, the interesting description of the existence of solutions of FODEs by changing the boundary conditions counting integer order phenomena, nonlocal, nonsingular kernel, periodic/antiperiodic, and multipoints has been investigated and has provided good supposition, as can be seen in [26–28]. Different researchers have tested the solution of the coupled models by differential equations of noninteger orders which gives the key role in applied fields of mathematics. This is the establishment for the model developed from biochemistry, biology, ecology, and other areas of engineering and physical sciences such as in [29].

<table>
<thead>
<tr>
<th>Parameter symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Pi )</td>
<td>Rate of new birth or recruitment</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Rate of vaccination for SARS-CoV-2</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Rate of natural deaths</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Rate of death due to SARS-CoV-2</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Interaction rate of infectious and healthy population</td>
</tr>
<tr>
<td>( \gamma' )</td>
<td>Recovered rate for acute infection</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Recovered rate for chronic infection</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Transferring rate acute class to chronic infection</td>
</tr>
</tbody>
</table>

2. Basic Definition

In this section, some definitions are written from [19, 20, 30].

Definition 1. Consider \( \mathcal{Y} \in L^1([0, \infty) \mathbb{Z}) \) to be a mapping; then, the Riemann–Liouville noninteger order integral of order \( \zeta \) is as follows:

\[
I_0^\zeta \mathcal{Y}(t) = \frac{1}{\Gamma(\zeta)} \int_0^t \mathcal{Y}(\beta) (t - \beta)^{\zeta - 1} d\beta, \quad \zeta > 0,
\]

where the right integral must exist.

Definition 2. Let \( \mathcal{Y} \) be an operator, then the Caputo noninteger order derivative can be defined as

\[
C_D_t^\zeta \mathcal{Y}(t) = \frac{1}{\Gamma(n - \zeta)} \int_0^t (t - \beta)^{n-\zeta-1} \frac{d^n}{d\beta^n} \mathcal{Y}(\beta) d\beta,
\]

and the R.H.S of the integration must exist and \( n = [\zeta] + 1 \). If \( \zeta \in (0, 1) \), then someone has

\[
C_D_t^\zeta \mathcal{Y}(t) = \frac{1}{\Gamma(1 - \zeta)} \int_0^t \mathcal{Y}'(\beta) (t - \beta)^{\zeta - 1} d\beta.
\]

Lemma 1. From the noninteger order differential equation, the following is satisfied:

\[
I_0^\zeta C_D_t^\zeta p(t) = p(t) + \gamma_0 + \gamma_1 t + \gamma_2 t^2 + \cdots + \gamma_{n-1} t^{n-1}.
\]

Definition 3. With regard to the Caputo operator, the Laplace transformation can be written as

\[
\mathcal{L}\left[C_D_t^\zeta p(t)\right] = s^\zeta p(s) - \sum_{j=0}^{m-1} s^{\zeta - j - 1} p^{(j)}(0),
\]

where \( m - 1 < \zeta < m, m \in \mathbb{N} \).

Definition 4. On application of the homotopy perturbation techniques to an equation having linear \( L \) and nonlinear \( N \) classes, we may write a homotopy \( \nu(r, g) : \Omega \times [0 \times 1] \rightarrow \mathbb{Z} \).

\[
H(v, g) = (1 - g) [L(v) - L(u_0)] + g [L(v) + N(v) - f(r)] = 0,
\]

where \( r \in \Omega \) and \( q \in [0, 1] \) is the embedding parameter.

3. Series Solution of Problem (2) via LADM

This section is devoted to the analysis of general-typed series techniques for the considered model (2) along with some starting conditions. On the application of the Laplace transformation to the considered problem (2), we get the following:

\[
\begin{align*}
\mathcal{L}[\mathcal{W}(t)] &= \frac{N_1}{s} + \frac{1}{s^\beta} \mathcal{L}[L - \beta \mathcal{X} - (\rho + \xi)\mathcal{W}], \\
\mathcal{L}[\mathcal{X}(t)] &= \frac{N_2}{s} + \frac{1}{s^\rho} \mathcal{L}[\beta \mathcal{X} - (\xi + \gamma + \lambda)\mathcal{X}], \\
\mathcal{L}[\mathcal{Y}(t)] &= \frac{N_3}{s} + \frac{1}{s^\gamma} \mathcal{L}[\lambda \mathcal{X} - (\xi + \delta + \kappa)\mathcal{Y}], \\
\mathcal{L}[\mathcal{Z}(t)] &= \frac{N_4}{s} + \frac{1}{s^\kappa} \mathcal{L}[\gamma \mathcal{Y} + n \mathcal{Y} + \rho \mathcal{WW} - \xi \mathcal{Z}].
\end{align*}
\]

Applying the initial approximation, problem (9) becomes

\[
\begin{align*}
\mathcal{L}[\mathcal{W}(t)] &= \frac{N_1}{s} + \frac{1}{s^\beta} \mathcal{L}[\mathcal{W}], \\
\mathcal{L}[\mathcal{X}(t)] &= \frac{N_2}{s} + \frac{1}{s^\rho} \mathcal{L}[\mathcal{X}], \\
\mathcal{L}[\mathcal{Y}(t)] &= \frac{N_3}{s} + \frac{1}{s^\gamma} \mathcal{L}[\mathcal{Y}], \\
\mathcal{L}[\mathcal{Z}(t)] &= \frac{N_4}{s} + \frac{1}{s^\kappa} \mathcal{L}[\mathcal{W}].
\end{align*}
\]

Taking the infinite series solution for \( \mathcal{W}, \mathcal{X}, \mathcal{Y}, \) and \( \mathcal{Z} \) as

\[
\begin{align*}
\mathcal{W}(t) &= \sum_{n=0}^{\infty} \mathcal{W}_n(t), \mathcal{X}(t) = \sum_{n=0}^{\infty} \mathcal{X}_n(t), \\
\mathcal{Y}(t) &= \sum_{n=0}^{\infty} \mathcal{Y}_n(t), \mathcal{Z}(t) = \sum_{n=0}^{\infty} \mathcal{Z}_n(t),
\end{align*}
\]

the nonlinear term \( \mathcal{W}(t) \mathcal{X}(t) = \sum_{n=0}^{\infty} \mathcal{X}_n(t) \) can be decomposed as

\[
X_n(t) = \frac{1}{n!} \frac{d^n}{dt^n} \left[ \sum_{k=0}^{n} \lambda^k \mathcal{W}_k(t) \sum_{j=0}^{k} \lambda^j \mathcal{X}_j(t) \right]_{t=0}.
\]
Substitute equations (11) and (12) into equation (10), and on comparing the terms on each side, we obtain

\[
\mathcal{L}[W_0(t)] = \frac{N_1}{s}, \quad \mathcal{L}[X_0(t)] = \frac{N_2}{s}, \quad \mathcal{L}[Y_0(t)] = \frac{N_3}{s}, \quad \mathcal{L}[Z_0(t)] = \frac{N_4}{s},
\]

\[
\mathcal{L}[W_1(t)] = \frac{1}{s} \mathcal{L}[\Lambda - \beta W_0 A_0 - (\rho + \xi) W_0],
\]

\[
\mathcal{L}[X_1(t)] = \frac{1}{s} \mathcal{L}[\beta W_0 A_0 - (\xi + \gamma + \lambda) X_0],
\]

\[
\mathcal{L}[Y_1(t)] = \frac{1}{s} \mathcal{L}[\lambda X_0 - (\xi + \delta + \kappa) Y_0],
\]

\[
\mathcal{L}[Z_1(t)] = \frac{1}{s} \mathcal{L}[\gamma X_0 + \kappa Y_0 + \rho W_0 - \xi Z_0],
\]

\[
\vdots
\]

\[
\mathcal{L}[W_{n+1}(t)] = \frac{1}{s} \mathcal{L}[\Lambda - \beta W_n A_n - (\rho + \xi) W_n],
\]

\[
\mathcal{L}[X_{n+1}(t)] = \frac{1}{s} \mathcal{L}[\beta W_n A_n - (\xi + \gamma + \lambda) X_n],
\]

\[
\mathcal{L}[Y_{n+1}(t)] = \frac{1}{s} \mathcal{L}[\lambda X_n - (\xi + \delta + \kappa) Y_n],
\]

\[
\mathcal{L}[Z_{n+1}(t)] = \frac{1}{s} \mathcal{L}[\gamma X_n + \kappa Y_n + \rho W_n - \xi Z_n].
\]

Upon using the inverse Laplace transform in expression (14), we get
\[
W_0(t) = \mathcal{L}^{-1}\left[\frac{N_1}{s}\right] = N_1, \quad \Lambda_0(t) = \mathcal{L}^{-1}\left[\frac{N_2}{s}\right] = N_2, \quad W_0(t) = \mathcal{L}^{-1}\left[\frac{N_3}{s}\right] = N_3, \quad Z_0(t) = \mathcal{L}^{-1}\left[\frac{N_4}{s}\right] = N_4,
\]

\[
W_1(t) = [\Lambda - \beta N_1 N_2 - (\rho + \xi)N_4] t^{\xi} \Gamma(\xi + 1),
\]

\[
\Lambda_1(t) = [\beta N_1 N_2 - (\xi + \gamma + \lambda)N_2] t^{\xi} \Gamma(\xi + 1),
\]

\[
W_2(t) = \frac{\Lambda t^\xi}{\Gamma(\xi + 1)} \left[\beta(N_1 G_{11} + N_2 K_{11}) - (\rho + \xi)G_{11}\right] t^{2\xi} \Gamma(2\xi + 1),
\]

\[
\Lambda_2(t) = \left[\beta(N_1 G_{11} + N_2 K_{11}) - (\xi + \gamma + \lambda)K_{11}\right] \frac{t^{2\xi}}{\Gamma(2\xi + 1)},
\]

\[
W_3(t) = \left[\lambda K_{11} - (\xi + \delta + \kappa) L_{11}\right] t^{\xi} \Gamma(2\xi + 1),
\]

\[
\Lambda_3(t) = \left[\gamma K_{11} + \kappa L_{11} + \rho G_{11} - \xi M_{11}\right] \frac{t^{2\xi}}{\Gamma(2\xi + 1)}.
\]

In the same way, the remaining terms can be obtained, and the terms given in equation (14) are as follows:

\[
G_{11} = \Lambda - \beta N_1 N_2 - (\rho + \xi)N_1,
\]

\[
K_{11} = \beta N_1 N_2 - (\xi + \gamma + \lambda)N_2,
\]

\[
L_{11} = \lambda N_2 - (\xi + \delta + \kappa)N_3,
\]

\[
M_{11} = \gamma N_2 + \kappa N_3 + \rho N_1 - \xi N_4.
\]

4. **Approximate Solution for Problem (2) via HPM**

Furthermore, we apply the HPM to obtain the approximate solution for the proposed problem (2), according to [20, 21] as follows:

\[
\begin{align*}
(1 - q)\left[C D^\xi_t\left(W(t)\right) - C D^\xi_t\left(W_0(t)\right)\right] + q\left[C D^\xi_t\left(W(t)\right) - \Lambda + \beta SA + (\rho + \xi)W\right] &= 0, \\
(1 - q)\left[C D^\xi_t\left(\chi(t)\right) - C D^\xi_t\left(E_0(t)\right)\right] + q\left[C D^\xi_t\left(\chi(t)\right) - \beta SA + (\xi + \gamma + \lambda)\chi\right] &= 0, \\
(1 - q)\left[C D^\xi_t\left(\gamma(t)\right) - C D^\xi_t\left(I_0(t)\right)\right] + q\left[C D^\xi_t\left(\gamma(t)\right) - \lambda \chi + (\xi + \delta + \kappa)\gamma\right] &= 0, \\
(1 - q)\left[C D^\xi_t\left(Z(t)\right) - C D^\xi_t\left(Z_0(t)\right)\right] + q\left[C D^\xi_t\left(Z(t)\right) - \gamma \chi - \kappa \gamma - \rho W + \xi Z\right] &= 0.
\end{align*}
\]

By applying \( q = 0 \) in equation (16), a model of arbitrary differential equations can be obtained:
\[
{D}^\eta_t (W(t)) - \gamma D^\xi_t (W_0(t)) = 0, \\
{D}^\eta_t (X(t)) - \gamma D^\xi_t (X_0(t)) = 0, \\
{D}^\eta_t (Y(t)) - \gamma D^\xi_t (Y_0(t)) = 0, \\
{D}^\eta_t (Z(t)) - \gamma D^\xi_t (Z_0(t)) = 0.
\]

(17)

Solutions for the equation (17) are straightforward. If we use \( q = 1 \) in equation (16), we will obtain the same system as (2). The infinite series solution can be written in the following form as

\[
W(t) = \sum_{n=0}^{\infty} q^n S_n(t), X(t) = \sum_{n=0}^{\infty} q^n A_n(t), Y(t) = \sum_{n=0}^{\infty} q^n B_n(t), Z(t) = \sum_{n=0}^{\infty} q^n C_n(t).
\]

(18)

So, by comparing each term \( q = 1 \), in equation (18), we get the original model. Plugging equation (18) into equation (16) and on comparison of each terms with the same power of \( q \), we get

\[
q^0 : \{ W_0(t) = N_1, X_0(t) = N_2, Y_0(t) = N_3, Z_0(t) = N_4 \}
\]

Similarly,

\[
q^1 : \begin{cases}
W_1 = [\Lambda - \beta W_0 X_0 - (\rho + \xi) W_0] \frac{t^\xi}{\Gamma(\xi + 1)}, \\
X_1 = [\beta W_0 X_0 - (\xi + \gamma + \lambda) X_0] \frac{t^\xi}{\Gamma(\xi + 1)}, \\
Y_1 = [\lambda X_0 - (\xi + \delta + \kappa) Y_0] \frac{t^\xi}{\Gamma(\xi + 1)}, \\
Z_1 = [\gamma Y_0 + \kappa Y_0 + \rho W_0 + \rho Z_0] \frac{t^\xi}{\Gamma(\xi + 1)}.
\end{cases}
\]

(20)

\[
q^2 : \begin{cases}
W_2 = [\Lambda - \beta W_0 X_0 \{\beta W_0 X_0 - (\xi + \gamma + \lambda) X_0\} - (\beta X_0 + \rho + \xi) \{\Lambda - \beta W_0 X_0 - (\rho + \xi) W_0\}] \frac{t^{2\xi}}{\Gamma(2\xi + 1)}, \\
X_2 = [(W_0 - (\xi + \gamma + \lambda)) \{\beta W_0 X_0 - (\xi + \gamma + \lambda) X_0\} + X_0 \{\Lambda - \beta W_0 X_0 - (\rho + \xi) W_0\}] \frac{t^{2\xi}}{\Gamma(2\xi + 1)}, \\
Y_2 = [\lambda \beta W_0 X_0 - (\xi + \delta + \kappa) \{\lambda X_0 - (\xi + \delta + \kappa) Y_0\}] \frac{t^{2\xi}}{\Gamma(2\xi + 1)}, \\
Z_2 = [\gamma \beta W_0 X_0 - (\xi + \gamma + \lambda) X_0] + \kappa \{\lambda X_0 - (\xi + \delta + \kappa) Y_0\} + \rho \{\Lambda - \beta W_0 X_0 - (\rho + \xi) W_0\} \\
- \xi \{\gamma Y_0 + \kappa Y_0 + \rho W_0 - \xi Z_0\}] \frac{t^{2\xi}}{\Gamma(2\xi + 1)}.
\end{cases}
\]

(21)

Similarly, the high terms can be obtained, and the required terms are given in the part above. Hence, we obtain the same high terms as we obtained using LADM. Both the methods are applied as strong techniques for nonlinear fractional-order equations.
5. Discussion along with Numerical Simulation for the Proposed System (2)

Next, we compute the numerical solution for the considered system (2), by assigning different values for the parameter given in Table 1 used in problem (2).

The first four terms of the considered model (2), by using values from the table are given as follows:

\[
\begin{align*}
\mathcal{W}_0(t) &= 6, \mathcal{X}_0(t) = 3, \mathcal{Y}_0(t) = 2, \mathcal{Z}_0(t) = 0, \\
\mathcal{W}_1(t) &= \frac{-5.2}{\Gamma(\zeta + 1)} t^\zeta, \quad \mathcal{X}_1(t) = \frac{5.3}{\Gamma(\zeta + 1)} t^\zeta, \\
\mathcal{Y}_1(t) &= \frac{0.8}{\Gamma(\zeta + 1)} t^\zeta, \quad \mathcal{Z}_1(t) = 0, \\
\mathcal{W}_2(t) &= \frac{0.4}{\Gamma(\zeta + 1)} t^\zeta + \frac{4.7}{\Gamma(2\zeta + 1)} t^{2\zeta}, \\
\mathcal{X}_2(t) &= \frac{-5.7}{\Gamma(2\zeta + 1)} t^{2\zeta}, \quad \mathcal{Y}_2(t) = \frac{2.25}{\Gamma(2\zeta + 1)} t^{2\zeta}, \\
\mathcal{Z}_2(t) &= \frac{0.15}{\Gamma(2\zeta + 1)} t^{2\zeta}.
\end{align*}
\]

The solution for the first few terms are the following:

\[
\begin{align*}
\mathcal{W}(t) &= 6 - \frac{5.2}{\Gamma(\zeta + 1)} t^\zeta + \frac{4.876}{\Gamma(2\zeta + 1)} t^{2\zeta} + \frac{0.3687}{\Gamma(\zeta + 1)} t^{3\zeta} + \ldots, \\
\mathcal{X}(t) &= 3 + \frac{5.3}{\Gamma(\zeta + 1)} t^\zeta - \frac{5.765}{\Gamma(2\zeta + 1)} t^{2\zeta} - \frac{0.3465}{\Gamma(\zeta + 1)} t^{3\zeta} + \ldots, \\
\mathcal{Y}(t) &= 2 + \frac{0.8}{\Gamma(\zeta + 1)} t^\zeta + \frac{2.7654}{\Gamma(2\zeta + 1)} t^{2\zeta} - \frac{0.6753}{\Gamma(3\zeta + 1)} t^{3\zeta} + \ldots, \\
\mathcal{Z}(t) &= \frac{0.065}{\Gamma(2\zeta + 1)} t^{2\zeta} - \frac{0.0076}{\Gamma(3\zeta + 1)} t^{3\zeta} + \ldots.
\end{align*}
\]

Now, evaluating equation (23) with \(\zeta = 0.8\), we get

\[
\begin{align*}
\mathcal{W}(t) &= 6 - 5.765470670 t^{0.8} + 3.876379584 t^{1.8} - 0.507094532 t^{2.7} + \ldots, \\
\mathcal{X}(t) &= 2 - 4.543942654 t^{0.8} - 2.655544765 t^{1.8} - 0.5464427072 t^{2.7} + \ldots, \\
\mathcal{Y}(t) &= 2 + 0.8768770670 t^{0.8} + 0.876594565 t^{1.8} - 0.8766862398 t^{2.7} + \ldots, \\
\mathcal{Z}(t) &= 0.87656354145 t^{1.8} - 0.00872947616 t^{2.7} + \ldots.
\end{align*}
\]

Similarly, for \(\zeta = 0.6\), the approximate solution for (23) is
Table 2: Numerical values for the parameters given in model (2).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Figure 1: Graphical representation of the series solution of $(W(t), X(t), Y(t), Z(t))$ of the proposed system (2) for various arbitrary orders. (a) $W(t)$: susceptible population. (b) $X(t)$: acutely infected population. (c) $Y(t)$: chronically infected population. (d) $Z(t)$: recovered population.
\begin{align*}
W(t) &= 6 - 5.654737025t^{0.7} + 2.8759942t^{1.4} - 1.324229934t^{2.1} + \ldots, \\
X(t) &= 3 - 4.765353842t^{0.7} - 2.654389698t^{1.4} + 0.654351204t^{2.1} + \ldots, \\
Y(t) &= 2 + 0.05502737025t^{0.7} + 0.8473079812t^{1.4} - 0.4321037668t^{2.1} + \ldots, \\
Z(t) &= 0.098769077782t^{1.4} - 0.98767203392t^{2.1} + \ldots,
\end{align*}

and for \( \zeta = 0.4 \), the approximate solution for (23) is

Figure 2: Comparison of LADM and HPM for the series solution of (\( W(t) \), \( X(t) \)) for various arbitrary orders of the considered model (2). (a) Comparison of LADM and HPM for the series solution of \( W(t) \) for various arbitrary orders. (b) Comparison of LADM and HPM for the series solution of \( X(t) \) for various arbitrary orders.

Figure 3: Comparison of LADM and HPM for the series solution of (\( Y(t) \), \( Z(t) \)) for various arbitrary orders of the considered model (2). (a) Comparison of LADM and HPM for the series solution of \( Y(t) \) for various arbitrary orders. (b) Comparison of LADM and HPM for the series solution of \( Z(t) \) for various arbitrary orders.
For the verification of our semianalytical solution by both methods, we provide numerical simulations for problem (2). Our simulation pertains to the qualitative point analysis, and the parameters are considered through a biological feasibility approach. We take parameter numerical value and then different initial class sizes for each of the compartment, namely, susceptible \( W(t) \), acutely infectious \( X(t) \), chronic carries, \( Y(t) \), and recovered \( Z(t) \), from Table 2. Figure 1(a) shows a quick decline in the starting in the quantity of susceptible class at at different fractional orders. The occurred decline took much more time at low nonnatural order and very slow at high noninteger order. We observe that, as the arbitrary order values increased, the curve of simulation goes, converging to the order 1. Besides this, after few days, the said individuals show low growth and then show converging towards stability to the equilibrium point. Figure 1(b) represents the acute infectious class, \( X(t) \) increases for some beginning days at various fractional orders. Beside that, the curve declines and stay stable through all arbitrary orders and converges to an integer order. In Figure 1(c), one may see the chronical infectious cases. \( Y(t) \) decreases at the starting of few days and then moves up to the maximum value of 1.5 at different noninteger order as no treatment and precautionary measures are done. But after keeping the precautions, the infectious case declines to 0.9 and then their after show stability and convergency. From Figure 1(d) we see that the recovery class goes up to 2.7 at the starting at various fractional orders and by keeping precautions and treatment the recovered class also stabilized.

Next, we have provided the comparison of different agents of the considered model by both the methods of LADM and HPM, as given from Figures 2(a)–3(b). Figure 4 represents the comparison of various agents at \( \zeta = 1 \).

6. Conclusions

The current investigation is the development of the four compartmental fractional-order SARS-CoV-2 model with regard to the Caputo fractional-order operative of Caputo having a singular kernel. The analysis for the series type solution of the proposed problem has been successfully achieved by two methods, one is the Laplace transform along with the Adomian polynomial (LADM) for a nonlinear term and the other one is the homotopy perturbation method (HPM). The semianalytical type solution has been obtained by both the methods which are comparable with each other. The numerical simulation for a few terms has been plotted using the available data given in Table 1 for four different values of \( \zeta \). We also compare few fractional-order values with that of integer 1, and as increasing the fractional-order value, we achieved the behavior of order 1. A complete spectrum for all compartments has established, and we can use other fractional values between 0 and 1. As a result, we say that fractional-order analysis provides better results than those of the integer-order problems.

Data Availability

Data are available upon the request by email and according to the type of collaboration.
Conflicts of Interest
The author declares no conflicts of interest.

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