Research Article

Finite Time Stability of 2D Fractional Hyperbolic System with Time Delay

Abdellatif Ben Makhlouf, Hassen Arfaoui, Salah Boulaaras, and Slim Dhahri

1Mathematics Department, College of Science, Jouf University, P.O. Box: 2014, Sakaka, Saudi Arabia
2Department of Mathematics, College of Sciences and Arts AR Ras, Qassim University, Saudi Arabia
3Department of Computer Engineering and Networks, College of Computer and Information Sciences, Jouf University, Sakaka, Saudi Arabia

Correspondence should be addressed to Abdellatif Ben Makhlouf; abmakhlouf@ju.edu.sa

Received 20 March 2022; Revised 1 May 2022; Accepted 13 May 2022; Published 27 May 2022

Academic Editor: Muhammad Gulzar

Copyright © 2022 Abdellatif Ben Makhlouf et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this work, a class of two-dimensional fractional hyperbolic differential linear system (2D-FHDLS) with time delay is investigated. By using generalized Gronwall’s inequality, sufficient conditions for the finite time stability (FTS) of two-dimensional fractional hyperbolic differential system with time delay are given. Numerical examples are also given to illustrate the stability result.

1. Introduction

Since twenty years, the area of fractional calculus has gained much attentions by the researchers, and numerous works have been published in this context. In fact, in [1], for a magnetic resonance imaging, a robust corner detection is developed. Authors have made a comparative experiment between the proposed methods and integer-order one. Furthermore, for the Hilfer stochastic delay fractional differential equations with the Poisson jumps, authors in [2] have analyzed the averaging principle. The author in [3] introduced a new approach for solving diffusive systems governed by the Caputo operator. Also, in [4], a hyperchaotic economic system was studied using fractional differential operator. Bayrak et al. in [5] established a novel approach for solving diffusive problems with conformable derivative. A new extension of the Hermite-Hadamard inequalities via generalized fractional integral has been given in [6]. Nagy and Ben Makhlouf in [7] studied the finite time stability of the linear Caputo-Katugampola fractional time delay systems.

Fractional differential equations have recently proved to be valuable tools in the modeling of many phenomena in different domain applications, whether in control theory, diffusion [5], viscoelasticity [8], or biology [9–11]. For example, in regard to the biology field, the pandemic transmission model of fractional-order COVID-19 type has been studied numerically by Higazy et al. in [9]. Regarding control theory field, a new adaptive surface control method based on fractional calculus is developed by Zouari et al. in [12]. It was found that all the variables, errors, and signals are practical finite time stability with an asymptotic convergence to zero of the tracking errors. Also, a regional observability for linear fractional systems has been studied in [13]. In [14], Xu et al. proved a global asymptotic stability for fractional neural networks with multiple time varying delay. A finite time stability for a class of fractional fuzzy neural networks with delay has been described and studied in [15]. In addition, authors in [16] have studied the FTS for fractional-order time delay systems.

For some basic results in the theory of fractional partial differential equations, the reader is referred to many various works. For example, for a perturbed partial fractional-order differential equations with finite delay, the Darboux problem is proposed by Abbas and Benchohra in [17]. A nonlinear fractional optimal control problem has been solved by generalized Bernoulli polynomials [18]. Wang and Zhang in
studied a Lyapunov inequality for PDE with mixed Caputo derivative. Also, Benchohra and Hessel in [20] proved a global uniqueness results for fractional partial hyperbolic differential equations with delay.

Motivated by the above interpretations, the main objective of this paper is to study the FTS for the linear Darboux fractional partial differential equations with delay or simply, as mentioned above, the 2D-FHDSLs with delay. In fact, we were able to establish a new result for the FTS of Caputo 2D-FHDSLs with delay. Indeed, thanks to the generalized Gronwall’s inequality, we have determined sufficient conditions for the FTS of the 2D-FHDSLs with delay. Recall that in [21], we have proved a similar result, but using a fixed point approach. By comparing the two methods, we have shown by numerical tests that the generalized Gronwall’s inequality method gives a wider stability interval than that given by fixed point method which proves that generalized Gronwall’s inequality method gives very satisfactory stability results.

The paper is organized as follows. In Section 2, some preliminaries are given. In Section 3, FTS results are presented. In Section 4, some numerical examples which show the efficiency of the results were presented.

2. Basic Results

**Definition 1** (see [22]). The Riemann-Liouville Fractional (RLF) integral of order \( \gamma = (y_1, y_2) \) of \( w \) is defined by

\[
I_{\xi}^{\gamma} w(\xi, \zeta) = [\Gamma(y_1) \Gamma(y_2)]^{-1} \int_{c_1}^{\xi} \int_{c_2}^{\zeta} (\xi - s)^{y_1-1} (\zeta - t)^{y_2-1} w(s, t) dt ds,
\]

where \( c = (c_1, c_2) \in \mathbb{R}^2 \), \( y_1, y_2 \) are strictly positive, and \( \Gamma(\cdot) \) is the Euler gamma function.

**Definition 2** (see [22]). The RLF derivative of order \( \gamma = (y_1, y_2) \) of \( w \) is defined by

\[
D_{\xi}^{\gamma} w(\xi, \zeta) = D_{\zeta}^{\gamma} w(\xi, \zeta) = [\Gamma(y_1) \Gamma(y_2)]^{-1} \int_{c_1}^{\xi} \int_{c_2}^{\zeta} (\xi - s)^{-y_1} (\zeta - t)^{-y_2} w(s, t) \frac{d^2}{d\xi d\zeta} dts,
\]

where \( c = (c_1, c_2) \in \mathbb{R}^2 \), \( y_1, y_2 \in (0, 1) \), and \( D_{\xi}^{\gamma} \) is defined as \( \partial^{y_1}\partial^{y_2} \).

**Definition 3** (see [22]). The Caputo fractional derivative (CFD) of order \( \gamma = (y_1, y_2) \) of \( w \) is defined by

\[
C_{\xi}^{y_1} D_{\xi}^{y_2} w(\xi, \zeta) = D_{\xi}^{y_2} w(\xi, \zeta) - w(\xi, c_2) - \int_{c_1}^{\xi} (\xi - s)^{-y_1} (\zeta - t)^{-y_2} w(s, t) \frac{d^2}{d\xi d\zeta} dts,
\]

where \( c = (c_1, c_2) \in \mathbb{R}^2 \), \( y_1, y_2 \in (0, 1) \), and \( D_{\xi}^{\gamma} \) is defined as \( \partial^{y_1}\partial^{y_2} \).

**Definition 4** (see [23]). Let \( m \in \mathbb{N} \) and \( \Theta \neq 0 \), \( \alpha_j, \beta_j, z, \rho \in \mathbb{C} \), such that \( \text{Re}(\alpha_j), \text{Re}(\beta_j) > 0 \) for \( j = 1, 2, \ldots, m \). The generalized Mittag-Leffler function (MLF) is defined by

\[
E_\rho^{(\alpha_j, \beta_j)}_{j=1}^{m} (z) = \sum_{k=0}^{\infty} \frac{(\rho)^k}{\Gamma(\alpha_j + \beta_j)_{\frac{k}{k!}}},
\]

where

\[
(\rho)^k = \rho(\rho + 1) \cdots (\rho + k - 1) = \frac{\Gamma(\rho + k)}{\Gamma(\rho)}.
\]

If \( m = 2 \) and \( \rho = 1 \), we get

\[
E_\rho^{(\alpha_j, \beta_j)}_{j=1}^{2} (z) = E^{(\alpha_j, \beta_j)}_{j=1}^{2} (z),
\]

\[
= \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha_1 + \beta_1) \Gamma(\alpha_2 + \beta_2)}.
\]

**Lemma 5** (see [24]). Let \( \hat{u}, \hat{v} \) be two integrable function and \( \hat{g} \) be a continuous function with domain \( j \). Assume that

1. \( \hat{u} \) and \( \hat{v} \) are nonnegative
2. \( \hat{v} \) is nondecreasing in each of its variables
3. \( \hat{g} \) is nonnegative and nondecreasing in each of its variables

If

\[
\hat{u}(r, \mu) \leq \hat{v}(r, \mu) + \hat{g}(r, \mu) \int_{a_j}^{b_j} (r - \xi)^{a_j-1} (\mu - \xi)^{b_j-1} \hat{u}(\xi, \mu) d\xi d\mu,
\]

Then

\[
\hat{u}(r, \mu) \leq \hat{v}(r, \mu) \mathcal{E}((\alpha_j, 1), (\alpha_j, 1)) \hat{g}(r, \mu) \Gamma(\alpha_j) \Gamma(\alpha_j)
\]

\[
\cdot (r - a_j)^{a_j}(\mu - a_j)^{b_j}.
\]

3. Main Result

In this paper, we are interested on the study of the initial value fractional-order linear system defined on the bounded domain \( \partial = [0, T] \times [0, T] \) as follows:

\[
C_{\mu}^{v_0} \theta(\xi, \zeta) = G \theta(\xi, \zeta) + K \theta(\xi - e_1(\xi), \zeta - e_2(\zeta)) + L \delta(\xi, \zeta),
\]

for all \( (\xi, \zeta) \in \partial \). The initial condition

\[
\theta(\xi, \zeta) = \psi(\xi, \zeta), \text{ for all } (\xi, \zeta) \in \mathcal{N},
\]

where \( C_{\mu}^{v_0} \) is the CFD of order \( v = (v_1, v_2) \), \( 0 < v_1, v_2 < 1 \). The functions \( e_1, e_2 \) are positive and continuous on \( [0, T] \) and \( [0, T] \), respectively. The matrices \( G, K \in \mathbb{R}^{n \times n} \) and \( L \in \mathbb{R}^{m \times m} \).
where the functions 
\[ \psi, \rho, \nu, \varphi, \zeta \in \mathcal{C}(\mathbb{R}^n) \],
and the constant \( M \) is given by
\[ M = \max\left( \frac{\mathcal{F}^{\nu_1}T^v}{\mathcal{F}^{\nu_1}T^v_2} \right) \]
of its variables. Then, for all \((r, \mu) \in [0, \xi] \times [0, \zeta]\):

\[
\| \vartheta(r, \mu) \| \leq \mathcal{M} + \frac{a + b}{\Gamma(v_1) \Gamma(v_2)} \int_0^\xi (\zeta - u)^{\nu - 1}(\zeta - v)^{\nu - 1} y(u, v) \, d\nu d\mu.
\]

Then, we get

\[
y(\xi, \zeta) = \sup_{(r, \mu) \in \mathcal{D}} \| \vartheta(r, \mu) \|,
\]

\[
\leq \max \left\{ \sup_{(r, \mu) \in \mathcal{D}} \| \vartheta(r, \mu) \|, \sup_{(r, \mu) \in \mathcal{D}} \| \vartheta(r, \mu) \| \right\}
\]

\[
\leq \max \left\{ \| \psi \|, \mathcal{M} + \frac{a + b}{\Gamma(v_1) \Gamma(v_2)} \int_0^\xi (\zeta - u)^{\nu - 1}(\zeta - v)^{\nu - 1} y(u, v) \, d\nu d\mu \right\}
\]

\[
\leq \mathcal{M} + \frac{a + b}{\Gamma(v_1) \Gamma(v_2)} \int_0^\xi (\zeta - u)^{\nu - 1}(\zeta - v)^{\nu - 1} y(u, v) \, d\nu d\mu.
\]

Now, using the generalized Gronwall inequality, we get

\[
y(\xi, \zeta) \leq \mathcal{M} E(v_1, v_2) \left( (a + b) \xi^v \xi^v \right)
\]

\[
\leq \frac{3}{2} \| \psi \| + c \rho \frac{1}{\Gamma(v_1 + 1) \Gamma(v_2 + 1)} \cdot E(v_1, v_2) \left( (a + b) \xi^v \xi^v \right) \leq y.
\]

for all \((\xi, \zeta) \in \mathcal{Q}\). The proof is completed. \(\square\)

**Remark 8.** Note that a similar result, to that given in Theorem 7, has been proved in ([21], Theorem 2) by a fixed point method.

### 4. Numerical Scheme

From relation (16), we have

\[
\vartheta(\xi, \zeta) = \rho(\xi, \zeta) + \frac{1}{\Gamma(v_1) \Gamma(v_2)} \int_0^\xi (\zeta - u)^{\nu - 1}(\zeta - v)^{\nu - 1} \psi(u, v) \, d\nu d\mu.
\]

for all \((\xi, \zeta) \in [0, \vartheta] \times [0, \varTheta]\), where the state \(\vartheta\) is the solution of system (9), and the functions \(\psi, \rho\) are given by relations (17) and (18). In this section, we study system (9) where \(\vartheta, \rho, \psi \in \mathbb{R}^2\). Then, let us assume that the solution \(\vartheta\) is of the following form:

\[
\vartheta(\xi, \zeta) = (\vartheta_1(\xi, \zeta), \vartheta_2(\xi, \zeta))^T \in \mathbb{R}^2.
\]

In this section, we use the same techniques of discretization and approximations that we have already used for the numerical resolution of the nonlinear problem in [21]. Thus, we build an uniform grid on the domain \([-m_1, \vartheta] \times [-m_2, \varTheta]\). Let \(r, h \in \mathbb{R}\) and \(\mathcal{F}, \mathcal{F}, n, m \in \mathbb{N}\) such that

\[
r = \frac{\vartheta}{n}, \quad h = \frac{\varTheta}{m}.
\]

Then, we introduce two sequences \((\xi_i)_i\) and \((\zeta_j)_j\) defined by

\[
\xi_i = ir, \forall i = -n, -n + 1, -n + 2, \cdots, -1, 0, \cdots, \vartheta,
\]

\[
\zeta_j = jh, \forall j = -m, -m + 1, -m + 2, \cdots, -1, 0, \cdots, \varTheta.
\]

So, the state \(\vartheta\) can be expressed at the point \((\xi_i, \zeta_j)\) as follows:

\[
\vartheta(\xi_i, \zeta_j) = \rho(\xi_i, \zeta_j) + \frac{1}{\Gamma(v_1) \Gamma(v_2)} \cdot \int_0^\xi (\zeta_i - u)^{\nu - 1}(\zeta_i - v)^{\nu - 1} \psi(u, v) \, d\nu d\mu,
\]

where \(\rho(\xi_i, \zeta_j) = \psi(0, \zeta_j) + \psi(\xi_i, 0) - \psi(0, 0)\). By considering the following approximations,

\[
\vartheta(\xi_i, \zeta_j) \approx \vartheta_{ij}, \quad \rho(\xi_i, \zeta_j) \approx \rho_{ij},
\]

\[
\psi(0, \zeta_j) \approx \psi_{0j}, \quad \psi(\xi_i, 0) \approx \psi_{i0}, \quad \psi(0, 0) \approx \psi_{00},
\]

we can rewrite equation (33) as follows:

\[
\vartheta_{ij} = \rho_{ij} + \frac{1}{\Gamma(v_1) \Gamma(v_2)} \int_0^\xi J_{ij} \int_0^\xi (\zeta_i - u)^{\nu - 1}(\zeta_i - v)^{\nu - 1} \psi(u, v) \, d\nu d\mu.
\]

Then, we deduce that

\[
\vartheta_{ij} = \rho_{ij} = \psi_{0j},
\]

\[
\vartheta_{ij} = \rho_{ij} = \psi_{i0}.
\]

By using the properties of integration, we can rewrite equation (35) as follows:

\[
\vartheta_{ij} = \rho_{ij} + \frac{1}{\Gamma(v_1) \Gamma(v_2)} \sum_{k=0}^{i-1} \sum_{l=0}^{j-1} \int_0^\xi (\zeta_i - u)^{\nu - 1}(\zeta_i - v)^{\nu - 1} \psi(u, v) \, d\nu d\mu.
\]

Now, using approximation proposed in [25], we obtain

\[
\vartheta_{ij} = \rho_{ij} + \frac{1}{\Gamma(v_1) \Gamma(v_2)} \sum_{k=0}^{i-1} \sum_{l=0}^{j-1} \psi_{ik} \int_0^\xi (\zeta_i - u)^{\nu - 1}(\zeta_i - v)^{\nu - 1} \psi_{kl} \, d\nu d\mu.
\]
where we have the approximation \( \Psi(\xi_k, \zeta_l) \approx \Psi_{kl} \) and

\[
\Psi(\xi_k, \zeta_l) = G\theta(\xi_k, \zeta_l) + K\theta(\xi_k - m_1, \zeta_l - m_2) + L \delta(\xi_k, \zeta_l),
\]

(39)

and the term

\[
\delta(\xi_k - m_1, \zeta_l - m_2) = \delta(kr - nr, lh - mh),
= \delta((k - n)r, (l - m)h),
= \delta(\xi_{k-n}, \zeta_{l-m}) \approx \delta_{k-n,l-m}.
\]

(40)

So, we deduce that

\[
\Psi^{kl} = G\delta_{kl} + K\delta_{k-n,l-m} + L \delta_{kl}.
\]

(41)

Calculating and simplifying the integral in equation (38) gives the following expression:

\[
\theta_{ij} = \rho_{ij} + \frac{r^\nu_1 h^\nu_2}{\Gamma(v_1 + 1)\Gamma(v_2 + 1)} \sum_{k=0}^{i-1} \sum_{l=0}^{j-1} \sigma_{ik} \tau_{lj} \Psi_{kl},
\]

(42)

where \( \sigma_{ik}, \tau_{lj} \) are given by

\[
\sigma_{ik} = (i - k - 1)^{\nu_1} - (i - k)^{\nu_1},
\tau_{lj} = (j - l - 1)^{\nu_2} - (j - l)^{\nu_2}.
\]

(43)

Remark 9. More details about the convergence, the consistency, and behavior of the error \( \|\theta(\xi_i, \zeta_j) - \theta_{ij}\| \) with respect to \( r, h, v_1, v_2 \) can be deduced from [25].

5. Numerical Simulation and Interpretation

In this section, we propose some numerical examples. With the help of these examples, we show that the solution of system (9) is consistent with Definition 6. Indeed, for any \( \varepsilon > 0 \),
\[ \forall \xi, \zeta (\xi, \zeta) \in [0, T] \times [0, T], \]

\[ \text{System (9) is defined by} \]

\[ ^CD_\nu \theta (\xi, \zeta) = G \theta (\xi, \zeta) + K \theta (\xi - m_1, \zeta - m_2) + L \delta (\xi, \zeta), \]

for all \( (\xi, \zeta) \in [0, T] \times [0, T], \nu = (\nu_1, \nu_2) \). The initial condition is given by

\[ \theta (\xi, \zeta) = \psi (\xi, \zeta), \forall (\xi, \zeta) \in [-m_1, 0] \times [-m_2, 0]. \]

Recall that we have denoted the solution \( \theta \) of system (45) as follows:

\[ \theta (\xi, \zeta) = (\theta_1 (\xi, \zeta), \theta_2 (\xi, \zeta))^T, \forall (\xi, \zeta) \in [0, T] \times [0, T]. \]

**Remark 10.** The following examples are established under the condition where the parameters \( \varepsilon, \gamma, \rho, T, \tau \) satisfy estimation (19).

We have chosen the following data:

\[
\begin{align*}
G &= \begin{pmatrix} -0.3 & -0.5 \\ 0.2 & 0.4 \end{pmatrix}, \\
K &= \begin{pmatrix} 0.1 & -0.5 \\ -0.3 & -0.1 \end{pmatrix}, \\
L &= \begin{pmatrix} 0.3 & 1 \\ 2 & -0.5 \end{pmatrix}, \\
\delta (t, s) &= (0.04, 0.01)^T.
\end{align*}
\]

The initial condition

\[ \psi (\xi, \zeta) = (0.002 \sin (11\pi \xi \zeta), 0)^T, \forall (\xi, \zeta) \in [-0.1, 0] \times [-0.2, 0]. \]
where \((m_1, m_2) = (0.1, 0.2)\) and we have \(\|\psi\| = 0.002 < \epsilon\). We consider the data: \(\mathcal{F} = 70, \mathcal{F} = 60, \epsilon = 0.1, \gamma = 10,\) and \(\rho = 0.55\). In the following, we have plotted the solution \(\theta(\xi, \zeta) = (\theta_1(\xi, \zeta), \theta_2(\xi, \zeta))^T, \forall (\xi, \zeta) \in \mathcal{F} \times [0, \mathcal{T}]\) for different values of \(v = (v_1, v_2)\) and \(\mathcal{F}, \mathcal{T}\), (see Figures 1, 2, 3, and 4). Also, in Table 1, we study the variation of \(\mathcal{F}, \mathcal{T}\), and \(\|\theta\|\) versus variation of \(v = (v_1, v_2)\). We conclude that the stability relation given in (44) is well satisfied \(\|\theta(\xi, \zeta)\| < 10\).

By using a fixed point method, we have proved in our work in [21] that system (9) is FTS w.r.t. \(\{c, \eta, \mathcal{F}, \mathcal{T}\}\) if there exists \(\eta_1, \eta_2 > 0\) such that \(a + b < \eta_1, \eta_2\) and the following inequality holds:

\[
3 \left[1 + (a + b)M_0(\eta_1, \eta_2)E_{\psi}((\eta_1, \mathcal{T}; \mathcal{F}) E_{\psi}((\eta_2, \mathcal{T}; \mathcal{F})) \right] \\
\cdot \psi + \left[cM_0(\eta_1, \eta_2)E_{\psi}((\eta_1, \mathcal{T}; \mathcal{F}) E_{\psi}((\eta_2, \mathcal{T}; \mathcal{F})) \right] \rho \leq \gamma,
\]

where \(E(\cdot)\) is the Mittag-Leffler function and \(M_0(\eta_1, \eta_2)\) is given by

\[
M_0(\eta_1, \eta_2) = \frac{\mathcal{T} \psi_1}{[1 + (a + b)/(\eta_1, \eta_2)]^\mathcal{T} \psi_2 + 1}.
\]

Now, we will compare the time interval \([0, \mathcal{T}_{FP}] \times [0, \mathcal{T}_{FP}]\) obtained by the fixed point method relation (50) and the time interval \([0, \mathcal{T}_{GG}] \times [0, \mathcal{T}_{GG}]\) obtained by the generalized Gronwall’s inequality relation (19). We consider the same data taken at the beginning of this section to which we add \((v_1, v_2) = (0.9, 0.5), \eta_1 = 1.5, \) and \(\eta_2 = 1\). Then, we fix \(\mathcal{T}_{GG} = \mathcal{T}_{FP} = 1.3\). From Table 1, we deduce that \(\mathcal{T}_{GG} = 0.815\). Now using relation (50), we can deduce by calculation that \(\mathcal{T}_{FP} = 0.00615\).

\[
[0, \mathcal{T}_{GG}] \times [0, \mathcal{T}_{GG}] = [0.13] \times [0.815], \\
[0, \mathcal{T}_{FP}] \times [0, \mathcal{T}_{FP}] = [0.13] \times [0.00615].
\]

We conclude from this experiment that the generalized Gronwall’s inequality gives a wider stability interval than that given by fixed point method which proves that generalized Gronwall’s inequality method gives very satisfactory stability results.

### Table 1: Variation of \(\mathcal{F}, \mathcal{T}\), and \(\|\theta\|\) versus variation of \(v = (v_1, v_2)\).

<table>
<thead>
<tr>
<th>(v = (v_1, v_2))</th>
<th>(\mathcal{F})</th>
<th>(\mathcal{T})</th>
<th>(|\theta|)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.9, 0.5)</td>
<td>1.3</td>
<td>0.815</td>
<td>3.3394</td>
</tr>
<tr>
<td>(0.7)</td>
<td>(\eta_{1, 2})</td>
<td>0.965</td>
<td>3.1741</td>
</tr>
<tr>
<td>(0.9)</td>
<td>(\eta_{1, 2})</td>
<td>1.0915</td>
<td>3.0458</td>
</tr>
<tr>
<td>(0.5, 0.5)</td>
<td>1.3</td>
<td>0.6335</td>
<td>3.5099</td>
</tr>
<tr>
<td>(0.7)</td>
<td>(\eta_{1, 2})</td>
<td>0.837</td>
<td>3.4191</td>
</tr>
<tr>
<td>(0.9)</td>
<td>(\eta_{1, 2})</td>
<td>1.0025</td>
<td>3.4392</td>
</tr>
<tr>
<td>(0.9, 0.5)</td>
<td>1.0026</td>
<td>1.3</td>
<td>3.4333</td>
</tr>
<tr>
<td>(0.7)</td>
<td>(\eta_{1, 2})</td>
<td>1.0309</td>
<td>3.2607</td>
</tr>
<tr>
<td>(0.9)</td>
<td>(\eta_{1, 2})</td>
<td>1.092</td>
<td>3.1271</td>
</tr>
</tbody>
</table>

6. Conclusion

In this work, we have proved the FTS for a class of two-dimensional fractional hyperbolic differential systems with time delay by using generalized Gronwall’s inequality. Sufficient conditions for the FTS of such systems are given. Moreover, numerical examples are given to illustrate the stability result.

In the coming works, we aspire to generalize the FTS for the 2D-FHDLs to several other types of well-known fractional derivatives.

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

This work was funded by the Deanship of Scientific Research at Jouf University under Grant Number (DSR2022-RG-0120).

### References


